Class – X
Subject – MATHEMATICS

[Time – 3 Hrs] **[Full Marks – 100]**

Answer question 1 (compulsory)and five other questions from section A

and answer any two questions from either section B or section C.

Section – A

Question – 1.

(i) Construct a 2 X 3 matrix whose elements aij are given by :

$a_{ij} = 2i - j.$ [3]

(ii) Evaluate : $\sin[2 \cos^{-1}(-3/5)]$ [3]

(iii) The foci of a hyperbola coincide with the foci of the ellipse $9x2 + 25y2 = 225$. Find the equation of the hyperbola if its eccentricity is 2. [3]

(iv) Evaluate : $\text{Lt}_{x \to 0} (e^{ax} - e^{bx})/x.$ [3]

(v) Evaluate : $\int[(1 - \tan x)/(1 + \tan x)]dx$ [3]

(vi) Differentiate w. r. t. x : $x \tan x \log_5 x.$ [3]

(vii) A card is drawn at random from a pack of 52 playing cards. Find the probability of getting a king or a heart or a red card. [3]

(viii) Find the coefficient of correlation from the regression lines given by :

$x - 2y + 3 = 0,\ 4x - 5y + 1 = 0.$ [3]

(ix) Solve the equation : $2z = |z| + 2i.$ [3]

(x) Solve the differential equation :

$(x^2 - y x^2)\, dy + (y^2 + x y^2)\, dx = 0$ [3]

Question – 2.

(a) Find the integers k for which the system of equations :

$x + 2y - 3z = 1$

$2x - ky - 3z = 2$

$x + 2y + kz = 3$

has a unique solution. Find the solution for k = 0. [5]

(b) Solve the system of equation using matrix method :

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$. [5]

Question – 3.

(a) It is given that for the function $f(x) = x^3 + b x^2 + a x + 5$ on [1, 3]. Rolle's theorem holds with $c = 2 + 1/\sqrt{3}$. Find the value of a and b. [5]

(b) The equation of the directrix of the parabola is $3x + 2y + 1 = 0$. The focus is (2, 1). Find the equation of the parabola. [5]

Question – 4.

(a) Prove that : $\sin^{-1} 4/5 + 2 \tan^{-1} 1/3 = \pi/2$. [5]

(b) Prove that the current will flow through the network represented by the function [AB(A'B + AB')]' irrespective of whether A and / or B are closed or open. [5]

Question – 5.

(a) If $y = \tan^{-1} 5ax/(a^2 - 6x^2)$, prove that $dy/dx = 3a/(a^2 + 9x^2) + 2a/(a^2 + 4x^2)$. [5]

(b) Find the maximum surface area of a circular cylinder that can be inscribed in a sphere of radius R. [5]

Question – 6.

(a) Evaluate : $\int_0^{\pi/2} \sin 2x \log \tan x \, dx$ [5]

(b) Draw a rough sketch of the graph of the function $y = 2\sqrt{(1 - x^2)}$, $x \in [0, 1]$ and evaluate the area enclosed between the curve and the x-axis. [5]

Question – 7.

(a) Marks obtained by nine students in Physics and Mathematics are given below :

Physics	35	23	47	17	10	43	9	6	28
Mathematics	30	33	45	23	8	49	12	4	31

Calculate Spearman's coefficient of rank correlation and interpret the result. [5]

(b) Treating x as independent variable, find the line of best fit for the following data :

x	15	12	11	14	13
Y	25	28	24	22	30

Hence, predict the value of y when x = 10. [5]

Question – 8.

(a) Assume that each child born is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that that both are girls, given that

(i) the youngest is a girl (ii) at least one is a girl. [5]

(b) Two bags P and Q contains 4 white, 3 black balls and 2 white, 2 black balls respectively. From bag P two balls are transferred to bag Q. Find the probability of drawing 1 white and 1 black ball from bag Q. [5]

Question – 9.

(a) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$,

Prove that $x^2 + y^2 + z^2 + 2xyz = 1$. [5]

(b) Find the locus of a complex number $z = x + iy$ satisfying the relation $|2z + 3i| = |2z + 5|$. Illustrate the locus in Argand plane. [5]

Section – B

Question – 10.

(a) Find the shortest distance between the lines l_1 and l_2 whose vector equations are :

$\vec{r} = i + j + \lambda(2i - j + k)$ and $\vec{r} = 2i + j - k + \mu(3i - 5j + 2k)$. [5]

(b) Find the equation of the plane passing through the point $(1, -1, -1)$ and perpendicular to each of the planes

$2(x - 2y) + 2(y - z) - (6z + x) = 0$ and $2(y - x) + 3(y + z) + 4(x - z) = 0$. [5]

Question – 11.

(a) Prove the following :

(i) $[\vec{a} + \vec{b}\ \ \vec{b} + \vec{c}\ \ \vec{c} + \vec{a}] = 2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

(ii) $[\vec{a} - \vec{b}\ \ \vec{b} - \vec{c}\ \ \vec{c} - \vec{a}] = 0$. [5]

(b) Show that the diagonals of a rhombus bisect each other at right angles. [5]

Question – 12.

(a) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both spades. Find the probability of the lost card being a spade. [5]

(b) Four bad eggs are mixed with 10 good ones. If 3 eggs are drawn one by one without replacement, find the probability distribution of the number of bad eggs drawn. [5]

Section – C

Question – 13.

(a) A man has Rs 15000 for purchase of rice and wheat. A bag of rice and a bag of wheat cost Rs 1800 and Rs 1200 respectively. He has storage capacity of 10 bags only. He earns a profit of Rs 110 and Rs 90 per bag of rice and wheat respectively. Formulate an L. P. P. to maximize the profit and solve it. [5]

(b) Mr Gupta has been accumulating a fund at 8% effective, which will provide him with an annual income of Rs 30000 for 3 years, the first payment being paid on his 60^{th} birthday. If he wishes to reduce the number of payments to 10, find how much annual income will he receive. [5]

(c)

Question – 14.

(a) A firm has the following total cost and demand functions :

$C(x) = x^3/3 - 7x^2 + 111x + 50$

$x = 100 - p$

Find the profit maximizing output. [5]

(b) A bill of Rs 1000 drawn on May 7, 2012 for 6 months was discounted on August 29, 2012 for cash payment of Rs 988. Find the rate of interest charged by the bank. [5]

Question – 15.

(a) Taking 2008 as the base year, with an index number 100, calculate the index number for 2012, based on weighted average of price relatives derived from the table given below :

Commodity	A	B	C	D
Weights	30	15	25	30
Price per unit in 2008	20	10	5	40
Price per unit in 2012	25	20	30	40

The weights are now changed so that the weight for A is 40 and C is 10 and the total weight is 100. If the value of the index number in 2012 with the changed weight is 182, calculate the weights applied to B and D. [5]

(b) Daily absence from a school during 3 weeks is recorded as follows :

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1.	23	28	21	33	40
Week 2.	38	52	43	58	63
Week 3.	52	54	61	51	51

Draw a graph, illustrating these figures. Calculate 5 day moving average and plot them on the same graph. [5]