ICSE Board
Class X Mathematics
Sample Paper 5

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1

(a) Find the value of a and b if (x – 1) and (x – 2) are factors of $x^3 - ax + b$. [3]

(b) A point P(a, b) is reflected on the y-axis as P'(−3, 1)
Write down the values of a and b.
P'' is the image of P when reflected on the x –axis.
Write down the coordinates of P''
P''' is the image of P when reflected on the line x = 5
Write down the coordinates of P'''. [3]

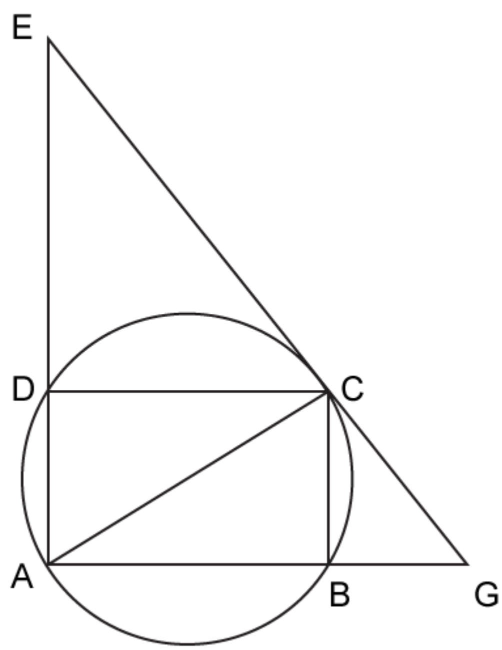
(c) The area of the base of a right circular cone is 28.26 sq. cm. If its height is 4 cm, find its volume and the curved surface area. (use π = 3.14) [4]

Q. 2.

(a) A bag contains 20 balls, of which x balls are blue. If one ball is drawn at random, find the probability of drawing a blue ball. If 10 more blue balls are put in the bag, and a ball is drawn at random then the probability of getting a blue ball is double the probability obtained before. Find x. [3]

(b) Solve the equation: $2y^{-4} + 5y^{-2} - 3 = 0$ [3]

(c) In the diagram given below m$\angle$EDC = 90°. The tangent drawn to the circle at C makes an angle of 50° with AB produced. Find the measure of $\angle$ACB. [4]



Q. 3.

(a) Mr. Arora has two years recurring deposit account in State Bank of India and deposits Rs.1500 per month. If he receives Rs. 37,875 at the time of maturity, find the rate of interest. [3]

(b) If m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then show that the sum of the mn terms is $\frac{1}{2}(mn + 1)$.

(c) Find the equation of a line passing through the point (2, 3) and the point of intersection of the lines $4x - 3y = 7$ and $3x + 4y + 1 = 0$.

Q. 4.

(a) Without using trigonometric table calculate [3]

$$4\frac{\sin 32^{o}}{\cos 58^{o}} + 5\frac{\tan 48^{o}}{\cot 42^{o}} - 8\frac{\sec 72^{o}}{\operatorname{cosec} 18^{o}}$$

(b) Solve the given equation and graph the solution on the number line. [3]

$2y - 3 \leq y + 1 \leq 4y + 7; y \in R$

(c) Calculate the arithmetic mean, correct to one decimal place, for the following frequency distribution of marks obtained in a geometry test. [4]

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	7	13	15	12	3

SECTION – B (40 Marks)

*(Answer **any four questions** from this Section)*

Q. 5.

(a) The diameter of a metallic sphere is 6 cm. It is melted and drawn into a wire having a cross sectional diameter of 0.2 cm. Find the length of the wire. [3]

(b) A shopkeeper buys a camera at a discount of 20% from the wholesaler, the printed price of the camera being Rs. 1,600 and the rate of sales tax is 6%. The shopkeeper sells it to the buyer at the printed price and charges tax at the same rate. Find

i. The price at which the camera can be bought

ii. The VAT paid by the shopkeeper [3]

(c) Find the equation of the altitudes AD of the triangle whose vertices are (7, −1), (−2, 8) and (1, 2). [4]

Q. 6.

(a) Use a graph paper to solve this question.

The points A(2, 3), B(4, 5) and C(7, 2) are the vertices of ΔABC.

i. Write down the co-ordinates of A', B', C' if Δ A'B'C' is the image of ΔABC, when reflected on the origin.

ii. Write down the co-ordinates of A", B", C" if ΔA"B"C" is the image of ΔABC, when reflected on the x-axis.

iii. Mention the special name of the quadrilateral BCC"B" and find its area. [3]

(b) Factorize : $(a^2 - a)(4a^2 - 4a - 5) - 6$ [3]

(c) The sum of three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms. [4]

Q. 7.

(a) If 3x + 4y : 4x + 7y = 5 : 7, find x : y. [3]

(b) Find the matrix M, such that $-A + 3B + M = 0$, where $A = \begin{bmatrix} -2 & 6 \\ 5 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$. [3]

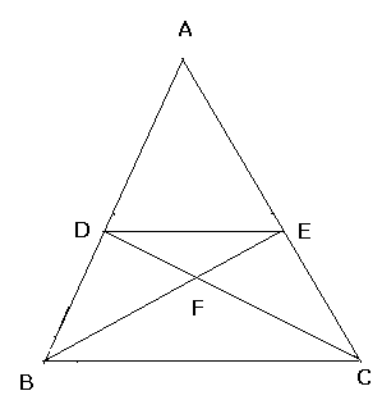
(c) In the given figure, ABC is a triangle, DE || BC and $\frac{AD}{BD} = \frac{3}{2}$ [4]

i. Find out $\frac{AD}{AB}$

ii. Prove that the ΔADE is similar to ΔABC and write down the ratio $\frac{DE}{BC}$.

iii. Prove that ΔDEF is similar to ΔCFB

iv. Find out the ratio of $\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta DEC}$



Q. 8.

(a) The following table shows the distribution of the heights of a group of students:

Height (cm)	140-145	145-150	150-155	155-160	160-165	165-170	170-175
No. of students	8	12	18	22	26	10	4

Use a graph sheet to draw an Ogive curve for the distribution.
Use the Ogive curve to find

i. the inter-quartile range
ii. the number of students whose height is more than 168cm [6]

(b) If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, show that $x^2 + y^2 = 1$ [4]

Q. 9.

(a) Use a rule and a pair of compasses to construct m∠ABC = 75°. Mark a point D on BC, such that BD = 5 cm. Construct a circle to touch AB at B and also to pass through D. Measure and record its radius. [3]

(b) Prove that: $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{1}{\sin\theta} = \dfrac{1}{\sin\theta} - \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$ [3]

(c) The daily wages of 30 workers in company are distributed as follows:

Daily wages	0-10	10-20	20-30	30-40	40-50	50-60
No. of workers	1	8	10	5	4	2

Draw the histogram and calculate the modal daily wage from the histogram. [4]

Q. 10.

(a) Construct a quadrilateral ABCD in which m ∠BAD = 45°, AD = AB = 6 cm, BC = 3.6 cm and CD = 5 cm.

i. Measure ∠BCD.
ii. Locate the point P on BD which equidistant from BC and CD. [3]

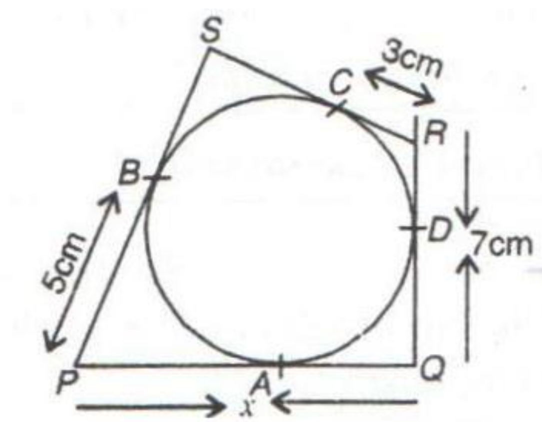
(b) If y is the mean proportional between x and z, prove that xy + yz is the mean proportional between $x^2 + y^2$ and $y^2 + z^2$. [3]

(c) The angle of elevation of an airplane from a point P on the ground is 60°. After 12 seconds from the same point P, the angle of elevation of the same plane changes to 30°. If the plane is flying horizontally at a speed of $600\sqrt{3}$ km/h, find the height the plane is flying at. [4]

Q. 11.

(a) Ram wishes to start a 200 m^2 rectangular vegetable garden. Since he has only 50 m of barbed wire, he fences three sides of the rectangular garden letting his house compound wall act as the fourth side of the fence. Find the dimensions of the garden. [3]

(b) In the given figure, quadrilateral PQRS is circumscribed around a circle, find x. [3]



(c) A straight line passes through the points P(–1, 4) and Q(5, –2). It intersects the co-ordinate axes at point A and B. M is the midpoint of the segment AB. Find

i. The equation of the line
ii. The co-ordinates of A and B
iii. The co-ordinates of M [4]

Solution

SECTION – A (40 Marks)

Q. 1.

(a) Since (x – 1) is a factor of $x^3 - ax + b$,

Thus, for x = 1, we have

$(1)^3 - a(1) + b = 0$

$\Rightarrow 1 - a + b = 0$

$\Rightarrow a - b = 1$ ----(1)

Also, (x – 2) is a factor of $x^3 - ax + b$,

$\therefore$ For x = 2, $(2)^3 - a(2) + b = 0$

$\Rightarrow 8 - 2a + b = 0$

$\Rightarrow 2a - b = 8$ ----(2)

Now subtracting equation (1) from equation (2), we get a = 7

Substituting a = 7 in equation (1), we have

7 – b = 1

b = 6

$\therefore$ a = 7 and b = 6

(b) $P(a, b) \xrightarrow{\text{y-axis}} P'(-3, 1)$

$\Rightarrow a = 3, b = 1$

$P(3, 1) \xrightarrow{\text{x-axis}} P''(3, -1)$

$\therefore$ Co-ordinates of P'' are (3, –1)

$P(3, 1) \xrightarrow{\text{Line } x = 5} P'''(7, 1)$

And co-ordinates of P''' are (7, 1)

(c)

Area of base of cone $= 28.26 \text{ cm}^2$

$\Rightarrow \pi r^2 = 28.26 \Rightarrow r^2 = \dfrac{28.26}{3.14} = 9 \Rightarrow r = 3 \text{ cm}$

Volume of the cone $= \dfrac{1}{3}\pi r^2 h = 37.68 \text{ cm}^3$

Curved surface area of the cone $= \pi r \ell = \pi r\sqrt{r^2 + h^2} = 3.14 \times 3 \times \sqrt{3^2 + 4^2} = 3.14 \times 3 \times 5 = 47.10 \text{ cm}^2$

Q. 2.

(a) Since there are 20 balls out of which x are blue.

The probability of drawing a blue ball = P(a blue ball) = $\frac{x}{20}$

When 10 more blue balls are added, total number of balls = 20 + 10 = 30

No. of blue balls = x + 10

New probability of drawing a blue ball $= P(\text{Blue ball}) = \frac{x+10}{30}$

According to question,

$\frac{x+10}{30} = 2 \times \frac{x}{20} \Rightarrow \frac{x+10}{30} = \frac{x}{10}$

Cross multiplying, we get

10x + 100 = 30x

$\Rightarrow$ 20x = 100 $\Rightarrow$ x = 5

(b) $2y^{-4} + 5y^{-2} - 3 = 0$

Let $y^{-2} = x \therefore y^{-4} = x^2$

So, $2x^2 + 3x + 5 = 0 \Rightarrow (2x - 1)(x + 3) = 0$

$\Rightarrow x = \frac{1}{2}, x = -3 \Rightarrow y^{-2} = \frac{1}{2}\ \ y^{-2} = -3$

$\Rightarrow \frac{1}{y^2} = \frac{1}{2}, \frac{1}{y^2} = -3$

$\Rightarrow y^2 = 2, y^2 = \frac{1}{-3}$

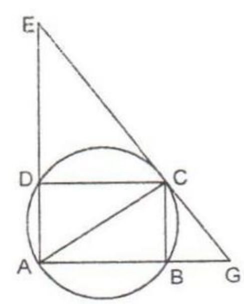
$\Rightarrow y = \pm\sqrt{2}$ ($y^2 = \frac{1}{-3}$ does not exist)

(c) $\angle EDC = 90°$

Now, $\angle ABC = \angle EDC$ [exterior angle of a cyclic quadrilateral = interior opposite angle]

$\Rightarrow \angle ABC = 90°$

$\angle CGB = 50°$ [given]



And, $\angle ABC + \angle CBG = 180°$ [linear pair angles]

Thus, $90° + \angle CBG = 180°$

$\Rightarrow \angle CBG = 180° - 90° = 90°$

Now in ΔBCG, $\angle CBG + \angle CGB + \angle BCG = 180°$ [Angle sum property]

$\Rightarrow 90° + 50° + \angle BCG = 180° \Rightarrow 140° + \angle BCG = 180° \Rightarrow \angle BCG = 40°$

Also, $\angle CAB = \angle BCG$ [Angle in alternate segment]

$\Rightarrow \angle CAB = 40°$

And, $\angle CBG = \angle CAB + \angle ACB$ [exterior angle of a Δ]

Thus, $90° = 40° + \angle ACB$

$\Rightarrow \angle ACB = 50°$

Q. 3.

(a) Monthly deposit = Rs. 1500

Amount deposited in 2 years (24 months) = Rs. 1500 × 24 = Rs. 36,000

Amount received on maturity = Rs. 37,875

$\therefore$ Interest (I) = Rs. (37,875 – 36,000)

Now n = 24, P = Rs. 1500 and r = ?

$$\text{Equivalent principal for one month} = P \times \frac{n(n+1)}{2}$$

$$= \frac{1500 \times 24(24+1)}{2}$$

$$= 1500 \times 12 \times 25$$

$$\therefore \text{ Interest (I)} = 1500 \times 12 \times 25 \times \frac{r}{100} \times \frac{1}{12}$$

$$\text{Or } 1{,}875 = 1500 \times 12 \times 25 \times \frac{r}{100 \times 12} \Rightarrow r = 5\%$$

$\therefore$ Rate of interest = 5% p.a.

(b) Let a and d respectively be the first term and the common difference of the A.P.

$$a + (m-1)d = \frac{1}{n} \quad \text{.........(i)}$$

$$a + (n-1)d = \frac{1}{m} \quad \text{........(ii)}$$

On solving (i) and (ii) we get,

$$(m-n)d = \frac{1}{n} - \frac{1}{m}$$

$$\Rightarrow (m-n)d = \frac{m-n}{mn}$$

$$\Rightarrow d = \frac{1}{mn}$$

$$\text{Therefore, } a = \frac{1}{n} - \frac{m-1}{mn} = \frac{m-m+1}{mn} = \frac{1}{mn}$$

$$S_{mn} = \frac{mn}{2}\left[2 \times \frac{1}{mn} + (mn-1)\left(\frac{1}{mn}\right)\right]$$

$$= \frac{1}{2}[2 + (mn-1)]$$

$$= \frac{1}{2}(mn+1)$$

(c) $4x - 3y = 7$ ... (i)

$3x + 4y = -1$ ... (ii)

After multiplying eq. (i) by 4 and eq. (ii) by 3 we get

$16x - 12y = 28$

$\Rightarrow 9x + 12y = -3$

On adding $25x = 25$

$\Rightarrow x = 1$

Put $x = 1$, in equation $4x - 3y = 7$, we get

$\Rightarrow 4x(1 - 3y) = 7$

$\Rightarrow -3y = 7 - 4$

$\Rightarrow -3y = 3$

$\Rightarrow y = -1$

Point of intersection = $(1, -1)$

Equation of line passing through (2, 3) and $(1, -1)$

$$\Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow y - 3 = \frac{-1 - 3}{1 - 2}(x - 2)$$

$$\Rightarrow y - 3 = 4(x - 2)$$

$$\Rightarrow y - 3 = 4x - 8$$

$$\Rightarrow 4x - y - 5 = 0$$

Q. 4.

(a) $4\frac{\sin 32^\circ}{\cos 58^\circ} + 5\frac{\tan 48^\circ}{\cot 42^\circ} - 8\frac{\sec 72^\circ}{\mathrm{cosec}\,18^\circ}$

$$= 4\frac{\sin 32^\circ}{\sin(90^\circ - 58^\circ)} + 5\frac{\cot(90^\circ - 48^\circ)}{\cot 42^\circ} - 8\frac{\mathrm{cosec}(90^\circ - 72^\circ)}{\mathrm{cosec}\,18^\circ}$$

$$= 4\frac{\sin 32^\circ}{\sin 32^\circ} + 5\frac{\cot 42^\circ}{\cot 42^\circ} - 8\frac{\mathrm{cosec}\,18^\circ}{\mathrm{cosec}\,18^\circ}$$

$= 4 + 5 - 8$

$= 9 - 8$

$= 1$

(b) $2y - 3 \le y + 1 \le 4y + 7; y \in R$

$\Rightarrow 2y - 3 - 1 \le y \le 4y + 7 - 1$

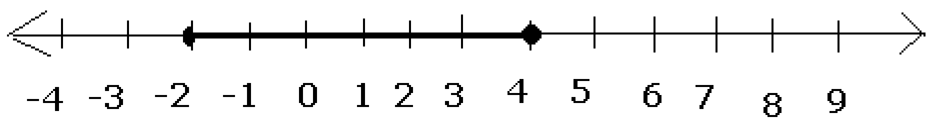
$\Rightarrow 2y - 4 \le y \le 4y + 6$

$\Rightarrow 2y - 4 \le y$ and $y \le 4y + 6$

$\Rightarrow y \le 4$ and $-6 \le 3y$

$\therefore -2 \le y \le 4$

Graphical solution is given by



(c) Drawing table and Calculating values,

Marks	x	f	$u = \frac{x-25}{10}$	fu
0-10	5	7	−2	−14
10-20	15	13	−1	−13
20-30	25	15	0	0
30-40	35	12	1	12
40-50	45	3	2	6
Total		$\sum f = 50$		$\sum fu = -9$

Here, a = 25, h = 10, $\sum f = 50$ and $\sum fu = -9$

$\therefore \text{Mean} = (\overline{x})$

$= a + h\frac{\sum fu}{\sum f}$

$= 25 + \frac{10(-9)}{50}$

$= 25 - 1.8$

$= 23.2$

SECTION – B (40 Marks)

Q. 5.

(a) For the sphere, D = 6 cm, R = $\frac{6}{2} = 3$ cm

For cylindrical wire, d = 0.2 cm, r = $\frac{0.2}{2} = 0.1$ cm

Let the length of wire be x cm.

So, according to problem

Volume of cylindrical wire = Volume of sphere

$$\Rightarrow \pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow \pi \times 0.1 \times 0.1 \times x = \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$\Rightarrow 0.1 \times 0.1 \times x = \frac{4}{3} \times 3 \times 3 \times 3$$

$$\Rightarrow x = \frac{4 \times 3 \times 3 \times 3}{3 \times 0.1 \times 0.1}$$

$$\Rightarrow x = 3600 \text{ cm} \quad \text{or} \quad 36 \text{ m}$$

(b)

i. Price at which the camera can be bought

= 80% of 1,600 + 6% of 80% of 1,600

= Rs. 1,280 + Rs. 76.80

= Rs. 1,356.80

ii. For wholesaler VAT = 6% of 1,280

= Rs. 76.80

For buyer VAT = 6% of 1,600

$= \frac{6}{100} \times 1,600$

= Rs. 96

VAT paid by the shopkeeper = Rs. 96 – 76.80 = Rs. 19.20

(c)

Slope of (BC) $=\frac{2-8}{1+2}=\frac{-6}{3}=-2$ So,

Slope of (AD) = $\frac{1}{2}$

Equation of AD, $y=\frac{1}{2}x+c$

Since the altitude AD passes through the point A(7, −1), we have

$-1=\frac{1}{2}\times 7+c$

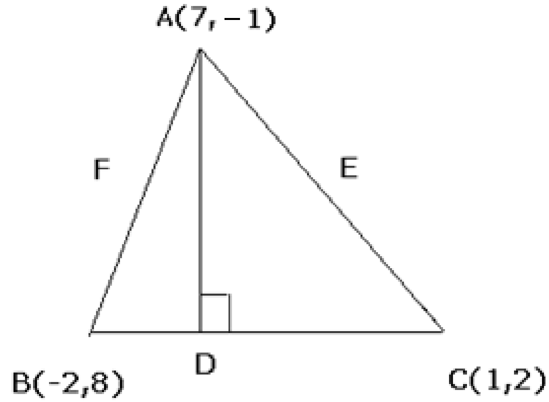
$\Rightarrow \ c=-1-\frac{7}{2}=\frac{-9}{2}$

Equation of AD

$\Rightarrow y=\frac{1}{2}x-\frac{9}{2}$

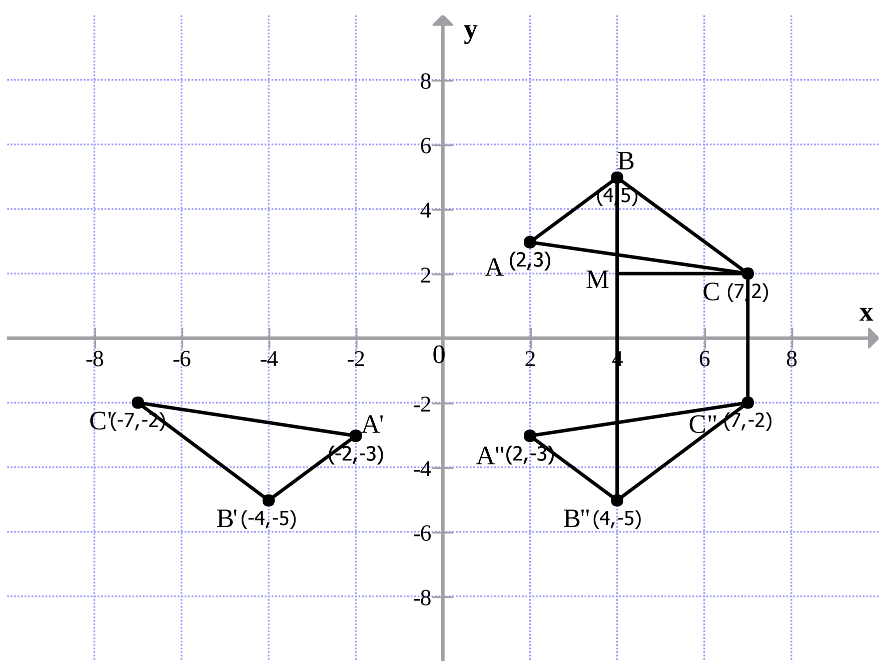
$\Rightarrow 2y=x-9$

$\Rightarrow x-2y-9=0$



Q. 6.

(a) Graph:



i. The coordinates of A’, B’ and C’ are (−2, −3), (−4, −5) and (−7, −2).

ii. The coordinates of A”, B” and C” are (2, −3), (4, −5) and (7, −2)

iii. BCC”B” is a trapezium

Area of trapezium $=\frac{1}{2}(BB''+CC'')\times CM=\frac{1}{2}(10+4)\times 3=\frac{1}{2}\times 14\times 3=$ 21 sq. units

(b) Let us assume $a^2 - a = x$

Then the given expression is

$(a^2 - a)(4a^2 - 4a - 5) - 6$

$= x(4x - 5)$

$= 4x^2 - 5x - 6$

$= 4x^2 - 8x + 3x - 6$

$= 4x(x - 2) + 3(x - 2)$

$= (4x + 3)(x - 2)$

$= (4(a^2 - a) + 3)(a^2 - a - 2)$ [Substituting value of x]

$= (4a^2 - 4a + 3)(a^2 - a - 2)$

$= (4a^2 - 4a + 3)(a^2 - 2a + a - 2)$

$= (4a^2 - 4a + 3)(a - 2)(a + 1)$

(c)

Let the 3 terms of the G.P. be $\frac{a}{r}$, a, ar,where r is the common ratio.

It is given that the product of the terms is 1.

Thus,

$$\frac{a}{r} \times a \times ar = 1$$

$$\Rightarrow a^3 = 1$$

$$\Rightarrow a = 1$$

Also, it is given that the sum of the terms is 39/10. Thus,

$$\frac{a}{r} + a + ar = \frac{39}{10}$$

$$\Rightarrow a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$$

$$\Rightarrow 1 \times \left(\frac{1 + r + r^2}{r}\right) = \frac{39}{10}$$

$$\Rightarrow 10 + 10r + 10r^2 = 39r$$

$$\Rightarrow 10r^2 - 29r + 10 = 0$$

$$\Rightarrow r = \frac{5}{2} \text{ or } \frac{2}{5}$$

Thus, the common ratio is $\frac{5}{2}$ or $\frac{2}{5}$.

The terms are $\frac{2}{5}$, 1, $\frac{5}{2}$ or $\frac{5}{2}$, 1, $\frac{2}{5}$.

Q. 7.

(a) Given: $\frac{3x + 4y}{4x + 7y} = \frac{5}{7}$

$\Rightarrow 7(3x + 4y) = 5(4x + 7y)$

$\Rightarrow 21x + 28y = 20x + 35y$

$\Rightarrow 21x - 20x = 35y - 28y$

$\Rightarrow x = 7y$

$\Rightarrow \frac{x}{y} = \frac{7}{1}$

$\Rightarrow x : y = 7 : 1$

(b) $-A + 3B + M = 0$

$\Rightarrow M = A - 3B$

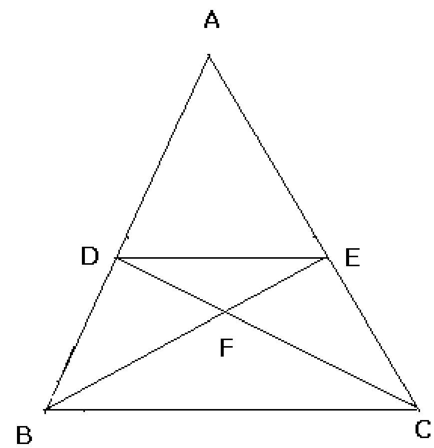
$$= \begin{bmatrix} -2 & 6 \\ 5 & 8 \end{bmatrix} - 3\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 6 \\ 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ -6 & 9 \end{bmatrix}$$

$$= \left[\begin{bmatrix} -2-3 & 6-6 \\ 5+6 & 8-9 \end{bmatrix}\right.$$

$$= \begin{bmatrix} -5 & 0 \\ 11 & -1 \end{bmatrix}$$

(c)



(i) $\frac{AD}{BD} = \frac{3}{2}$

Let AD = 3x

Then DB = 2x

$$\therefore \frac{AD}{AB} = \frac{3x}{AD + DB}$$

$$= \frac{3x}{3x + 2x}$$

$$= \frac{3x}{5x}$$

$$= \frac{3}{5}$$

(ii) In ΔADE and ΔABC

∠ADE = ∠ABC [Corresponding angles]

∠AED = ∠ACB [Corresponding angles]

$\Rightarrow \Delta ADE \sim \Delta ABC$

$$\Rightarrow \frac{DE}{BC} = \frac{AD}{DB} = \frac{3}{5}$$

(iii) In ΔDEF and ΔCFB

∠DEF = ∠CBF [Alternate interior angles]

∠DFE = ∠CFB [vertically opposite angles]

$\therefore \Delta DEF \sim \Delta CFB$ [AA Similarity]

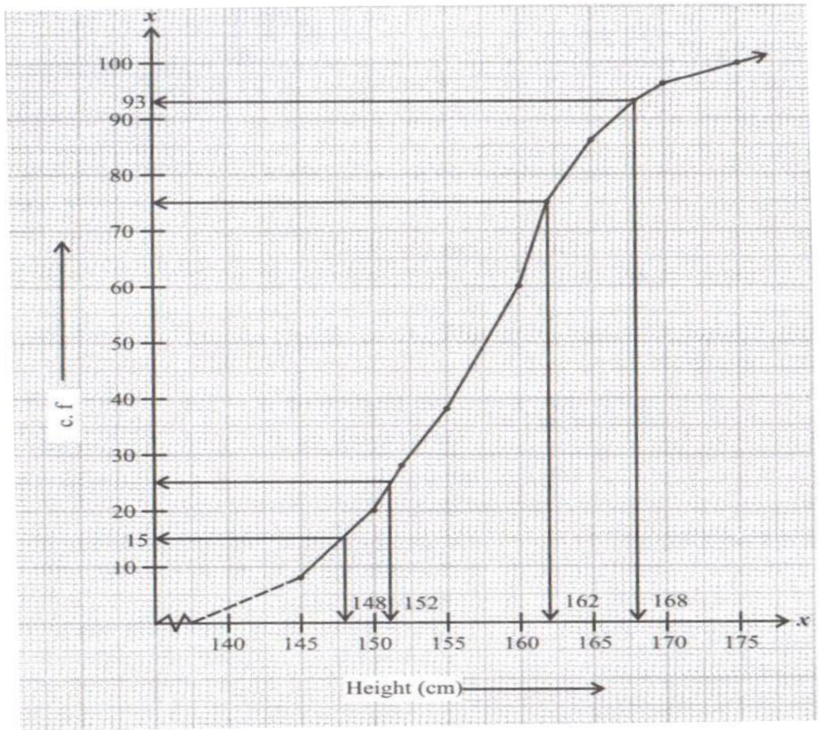
(iv) $\frac{DE}{BC} = \frac{DF}{FC} = \frac{3}{5}$

$$\Rightarrow \frac{DF}{DC} = \frac{3}{8}$$

$$\frac{\text{Area of } \Delta DFE}{\text{Area of } \Delta DEC} = \frac{\frac{1}{2} \times DF \times h}{\frac{1}{2} \times DC \times h} = \frac{DF}{DC} = \frac{3}{8}$$

Q. 8.

(a) Ogive for the distribution,



Height (in cm)	Number of students (f)	Cumulative Frequency(c.f.)
140-145	8	8
145-150	12	20
150-155	18	38
155-160	22	60
160-165	26	86
165-170	10	96
170-175	4	100
	N = 100	

i. Lower Quartile,

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{ observation} = \left(\frac{100}{4}\right)^{th} \text{ observation} = 25^{th} \text{ observation} = 152 \text{ cm}$$

Upper quartile,

$$Q_3 = \left(\frac{3N}{4}\right)^{th} \text{ observation} = \left(\frac{3 \times 100}{4}\right)^{th} \text{ observation} = 75^{th} \text{ observation} = 162 \text{ cm}$$

So, Inter Quartile Range, $Q_3 - Q_1 = 162 - 152 = 10$ cm

ii. The number of students whose height is more than 168 cm = 100 – 93 = 7

b) Given, $x \sin \theta = y \cos \theta$ -----(1)

$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

or $(x\sin\theta)\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

$y\cos\theta\sin^2\theta + y\cos^3\theta = \sin\theta\cos\theta$

$y\cos\theta(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow y\cos\theta = \sin\theta\cos\theta$

$\Rightarrow y = \sin\theta$

Put the value of y in equation (1)

$\Rightarrow x\sin\theta = \sin\theta.\cos\theta$

$\Rightarrow x = \cos\theta$

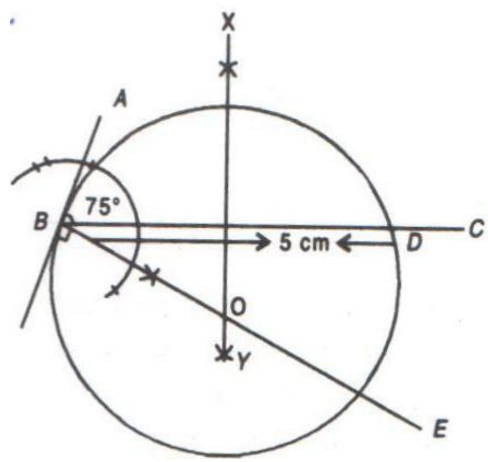
$\Rightarrow$ Now $x^2 + y^2 = \cos^2\theta + \sin^2\theta$

$\Rightarrow x^2 + y^2 = 1$

Hence proved

Q. 9.

(a)



Radius OB = 2.4 cm

(b)

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

if $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} + \dfrac{1}{\operatorname{cosec}\theta + \cot\theta} = \dfrac{1}{\sin\theta} + \dfrac{1}{\sin\theta}$

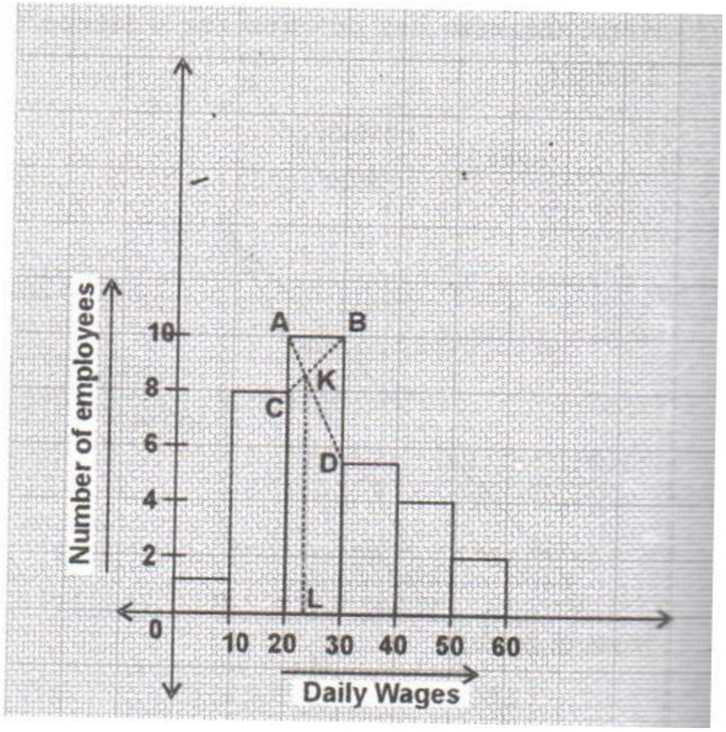
if $\dfrac{\operatorname{cosec} + \cot\theta + \operatorname{cosec} - \cot\theta}{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec} + \cot\theta)} = \dfrac{2}{\sin\theta}$

if $\dfrac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2 - \cot^2\theta} = \dfrac{2}{\sin\theta}$

if $\dfrac{2\operatorname{cosec}\theta}{1} = 2\operatorname{cosec}\theta$

Or $2 \operatorname{cosec} \theta = 2 \operatorname{cosec} \theta$

(c) The Histogram is as follows:



Steps:

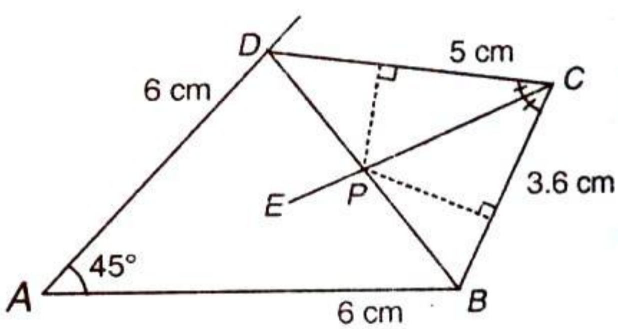
i. Mark the upper corner of the rectangle with maximum frequency as A and B.
ii. Mark the inner corner of adjacent rectangles to the above rectangle as C and D.
iii. Join AC and BD to intersect at K. From K, draw KL perpendicular to x-axis meet at L.
iv. The value of L on the x-axis represents the mode.

$\therefore$ Modal daily wage = Rs 23.

Q. 10.

(a) Draw the quadrilateral ABCD.

i. Measure m $\angle BCD = 65^o$
ii. Draw bisector of $\angle BCD$, which intersects BD at P.



(b) y is the mean proportional between x and z $\Rightarrow y^2 = xz$

To prove: $(x^2+y^2)(y^2+z^2) = (xy+yz)^2$

L.H.S. = $(x^2+y^2)(y^2+z^2)$

$= (x^2+xz)(xz+z^2)$

$= x(x+z)\,z(x+z)$

$= xz(x+z)^2$

$= y^2(x+z)^2$

$= (xy+yz)^2$

= R.H.S.

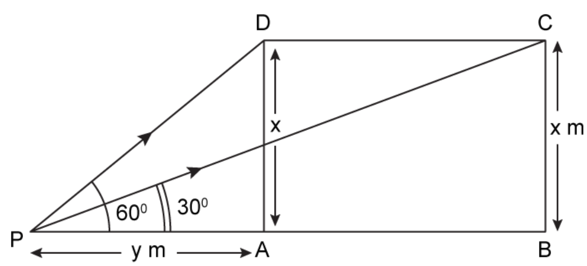
Hence Proved

(c) Let the height of the plane be = x m

Speed of plane = $600\sqrt{3}$ km/h

$= \frac{600\sqrt{3}\times1000}{3600}$ m/sec

$= \frac{500}{3}\sqrt{3}$ m/sec



Distance (CD) = Speed × time

$= \frac{500}{3}\sqrt{3}\times12\,\text{m} = 2000\sqrt{3}\,\text{m}$

Let PA = y m

In ΔPAD, $\tan 60° = \frac{x}{y} = \frac{AD}{PA} \Rightarrow \sqrt{3} = \frac{x}{y} \Rightarrow x = y\sqrt{3}$ or $y = \frac{x}{\sqrt{3}}$

In ΔPBC,

$\tan 30° = \frac{BC}{PB} = \frac{x}{PA+AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{PA+AB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{y+2000\sqrt{3}}$

$\Rightarrow y + 2000\sqrt{3} = x\sqrt{3}$

$\Rightarrow \frac{x}{\sqrt{3}} + 2000\sqrt{3} = x\sqrt{3}$

$\Rightarrow \frac{x+6000}{\sqrt{3}} = x\sqrt{3}$

$\Rightarrow 3x = x + 6000$

$\Rightarrow 2x = 6000$

$\Rightarrow x = 3000$

So, the height at which the plane is flying = 3000 m.

Q. 11.

(a) Let ABCD be the rectangular garden and CD act the house's compound wall

Let length AB = x m

Breadth BC = y m

According to question

y = 200 .... (1)

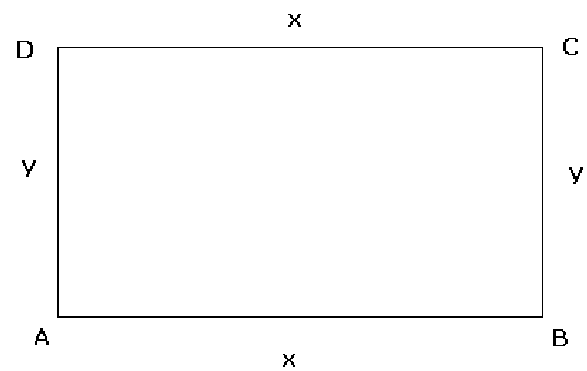
x + 2y = 50 .... (2)

From equation (2), we get

x = 50 − 2y

By putting value of x in equation (1), we get

xy = 200



$\Rightarrow (50 - 2y)y = 200$

$\Rightarrow 50y - 2y^2 = 200$

$\Rightarrow 2y^2 - 50y + 200 = 0$

$\Rightarrow y^2 - 25y + 100 = 0$

$\Rightarrow (y - 20)(y - 5) = 0$

So, y = 5 or 20

And, when y = 5, then x = 50 – 2y = 50 – 2 × 5 = 40

When y = 20, then x = 50 – 2y = 50 – 2 × 20 = 10

So, dimension of the garden = 10m and 20 m or 40m and 5m

(b) We know that, tangents from a point outside the circle are equal.

So, PA = PB

PA = 5 cm $(\because PB = 5cm)$

Now, AQ = PQ – PA = x – 5

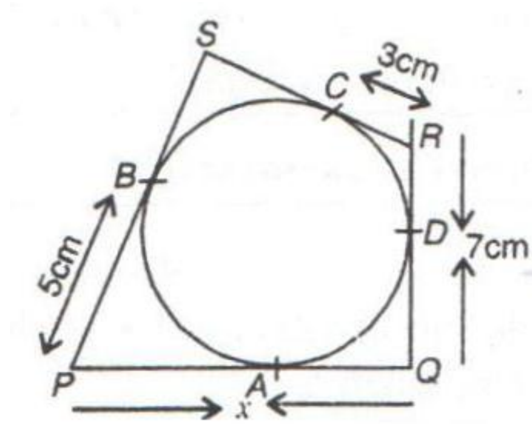
DQ = AQ

DQ = x – 5

Similarly, RD = CR

RD = 3cm $(\because CR = 3cm)$

Now, RQ = RD + DQ



$\Rightarrow 7 = 3 + x - 5$

$\Rightarrow 7 = x - 2$

$\Rightarrow 7 + 2 = x$

$\Rightarrow 9 = x$

$\Rightarrow x = 9$ cm

(c) Using two points form a line:

i. Equation of line passing through (x_1, y_1) and (x_2, y_2) is

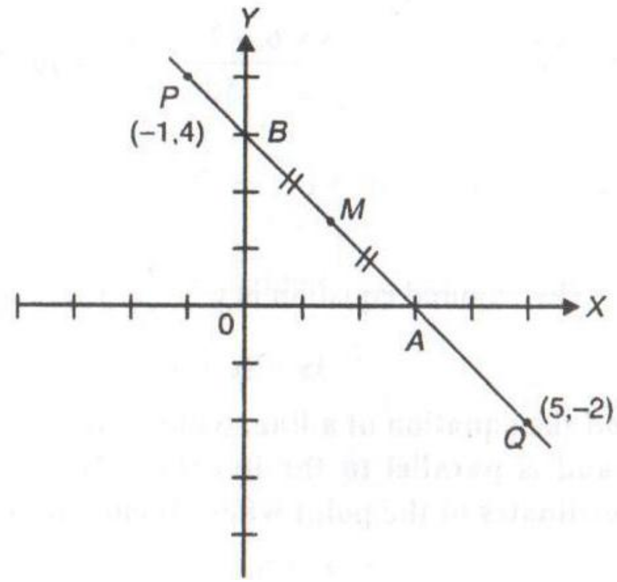
$$y - y_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)}(x - x_1)$$

$$y - 4 = \frac{-6}{5+1}(x+1)$$

$$\Rightarrow y - 4 = \frac{-6}{6}(x+1)$$

$$\Rightarrow y - 4 = -x - 1$$

$$\Rightarrow x + y - 3 = 0$$



ii. The equation of line is x + y – 3 = 0

For y = 0, x + 0 − 3 = 0 ⇒ x = 3

∴ Co-ordinates of A are (3, 0)

And for x = 0,

0 + y – 3 = 0

⇒ y = 3

∴ Co-ordinates of B are (0, 3).

iii. Co-ordinates of M = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$= \left(\frac{3+0}{2}, \frac{0+3}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{3}{2}\right)$$