ICSE Board
Class X Mathematics
Sample Paper 4

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1.

(a) If (x - 2) is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by (x - 3), it leaves a remainder 52, find the values of a and b

[3]

(b)

If $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$. [3]

Find: $AB - 5C$.

(c) The sum of three consecutive terms of an A.P. is 21 and the sum of their squares is 165. Find these terms

[4]

Q. 2.

(a) A die has 6 faces marked by the given numbers as shown below:

1	2	3	-1	-2	-3

The die is thrown once. What is the probability of getting

i. a positive integer?
ii. an integer greater than -3?
iii. the smallest integer?

[3]

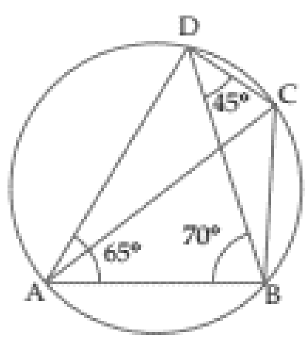
(b) Find the value of 'p', if the following quadratic equations have equal roots :

$x^2 + (p - 3)x + p = 0$ [3]

(c) In the given figure, ∠BAD = 65°, ∠ABD = 70°, ∠BDC = 45°

(i) Prove that AC is a diameter of the circle.

(ii) Find ∠ACB.



[4]

Q. 3.

(a) Mr. Britto deposits a certain sum of money each month in a Recurring Deposit Account of a bank. If the rate of interest is of 8% per annum and Mr. Britto gets 8,088 from the bank after 3 years, find the value of his monthly instalment. [3]

(b) A metal pipe has a bore (inner diameter) of 5 cm. The pipe is 5 mm thick all round.Find the weight, in kg, of 2 metres of the pipe if 1 cm^3 of the metal weights 7.7g. [3]

(c) Find the length of the median through the vertex A of triangle ABC whose vertices are A (7, -3), B(5, 3) and C(3, -1). [4]

Q. 4.

(a) Solve the following inequation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$$ [3]

(b) Evaluate

$\sin^2 34° + \sin^2 56° + 2\tan 18° \tan 72° - \cot^2 30°$ [3]

(c) For the following frequency distribution, draw a histogram. Hence, calculate the mode.

Class :	5-10	10-15	15-20	20-25	25-30
Frequency	7	18	10	8	5

[4]

SECTION – B (40 Marks)

*(Answer **any four questions** from this Section)*

Q. 5.

(a) The marked price of two articles A and B together is ` 6,000. The sales tax on article A is 8% and that on article B is 10%. If on selling both the articles, the total sales tax collected is ` 552, find the marked price of collected of the articles A and B. [3]

(b) The radius of a solid right circular cylinder increases by 20% and its height decreases by 20%. Find the percentage change in its volume. [3]

(c) Point A(1,-5) is mapped as A' on reflection in the line y=1. The point B(-5,1) is mapped as B' on reflection in the line y=4. Write the co-ordinates of A' and B'. Calculate AB'. [4]

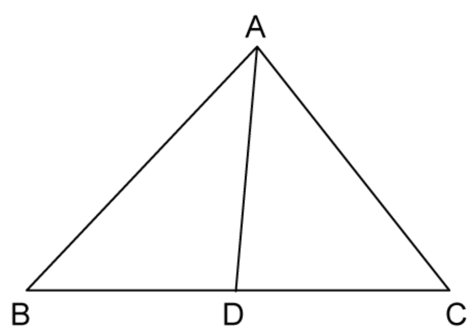
Q. 6.

(a) ABCD is a square. If the coordinates of A and C are (5, 4) and (-1, 6); find the coordinates of B and D. [3]

(b) Solve : $\frac{x}{3}+\frac{3}{6-x}=\frac{2(6+x)}{15};(x\neq 6)$ [3]

(c) In ΔABC, ∠ABC = ∠DAC, AB = 8 cm, AC = 4 cm and AD = 5 cm.

Find area of ΔACD : area of ΔABC.



[4]

Q. 7.

(a)

If $(p-x)\ :\ (q-x)$ be the duplicate ratio of $p:q$

then show that: $\frac{1}{p}+\frac{1}{q}=\frac{1}{x}$ [3]

(b) If $A=\begin{bmatrix}3 & x\\0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}9 & 16\\0 & -y\end{bmatrix}$, find x and y when $A^2=B$. [3]

(c) Find the arithmetic mean (correct to the nearest whole-number) by using step-deviation method.

x	5	10	15	20	25	30	35	40	45	50
f	20	43	75	67	72	45	39	9	8	6

[4]

Q. 8.

(a) The income of the parents of 100 students in a class in a certain university are tabulated below.

Income (in thousand Rs)	0-8	8-16	16-24	24-32	32-40
No. of students	8	35	35	14	8

(i) Draw a cumulative frequency curve to estimate the median income.

(ii) If 15% of the students are given freeships on the basis of the basis of the income of their parents, find the annual income of parents, below which the freeships will be awarded.

(iii) Calculate the Arithmetic mean. [6]

(b) In Δ ABC, angle ABC is equal to twice the angle ACB, and bisector of angle ABC meets the opposite side at point P. Show that :CB : BA = CP : PA [4]

Q. 9.

(a) Mrs. Kulkarni invests `1, 31,040 in buying `100 shares at a discount of 9%. She sells shares worth `72,000 at a premium of 10% and the rest at a discount of 5%. Find her total gain or loss on the whole.. [3]

(b) Construct a triangle ABC with AB = 5.5 cm, AC = 6 cm and ∠BAC = 105°. Hence :

i. Construct the locus of points equidistant from BA and BC.
ii. Construct the locus of points equidistant from B and C.
iii. Mark the point which satisfies the above two loci as P. Measure and write the length of PC. [3]

(c) Prove the identity $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \mathrm{cosec}\,\theta$. [4]

Q. 10.

(a)

If the m^{th} term of an A.P is $\frac{1}{n}$ and the n^{th} term of it is $\frac{1}{m}$,

show that: $(mn)^{th}$ term of this A.P. is 1.

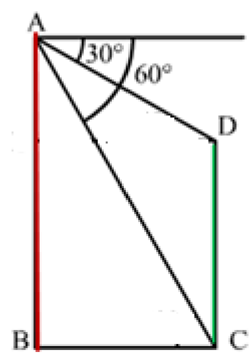
[3]

(b) If a, b and c are in G.P., prove that: $\log a^n$, $\log b^n$ and $\log c^n$ are in A.P. [3]

(c) In the given figure, from the top of a building AB = 60 m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively. Find :

(i) the horizontal distance between AB and CD.

(ii) the height of the lamp post. [4]

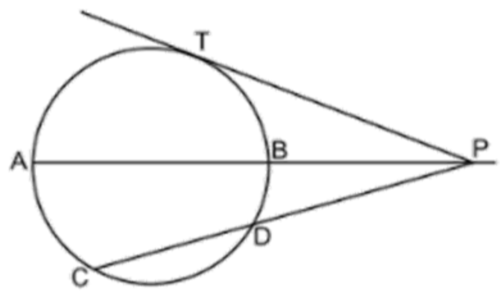


Q. 11.

(a) In the given figure, diameter AB and chord CD of a circle meet at P. PT is a tangent to the circle at T. CD = 7.8 cm, PD = 5 cm, PB = 4 cm. Find

(i) AB.

(ii) the length of tangent PT.



[3]

(b) Solve : $\frac{x}{3}+\frac{3}{6-x}=\frac{2(6+x)}{15};(x \neq 6)$.

[3]

(c) Find the equation of line through the intersection of lines 2x - y = 1 and 3x + 2y = -9 and making an angle of 30° with positive direction of x-axis.

[4]

Solution

SECTION – A (40 Marks)

Q. 1.

(a)

Since $(x-2)$ is a factor of polynomial $2x^3+ax^2+bx-14,$ we have

$2(2)^3+a(2)^2+b(2)-14=0$

$\Rightarrow 16+4a+2b-14=0$

$\Rightarrow 4a+2b+2=0$

$\Rightarrow 2a+b+1=0$

$\Rightarrow 2a+b=-1 \qquad ....(i)$

On dividing by $(x-3),$ the polynomial $2x^3+ax^2+bx-14$ leaves remainder 52,

$\Rightarrow 2(3)^3+a(3)^2+b(3)-14=52$

$\Rightarrow 54+9a+3b-14=52$

$\Rightarrow 9a+3b+40=52$

$\Rightarrow 9a+3b=12$

$\Rightarrow 3a+b=4 \qquad ....(ii)$

Subtracting (i) from (ii), we get

$a=5$

Substituting $a=5$ in (i), we get

$2\times 5+b=-1$

$\Rightarrow 10+b=-1$

$\Rightarrow b=-11$

Hence, $a=5$ and $b=-11.$

(b)

Given: $A = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix}$

Now,

$$AB = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 5 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 3\times 0 + 7\times 5 & 3\times 2 + 7\times 3 \\ 2\times 0 + 4\times 5 & 2\times 2 + 4\times 3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 + 35 & 6 + 21 \\ 0 + 20 & 4 + 12 \end{bmatrix}$$
$$= \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix}$$

$$5C = 5\begin{bmatrix} 1 & -5 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix}$$

$$\therefore\ AB - 5C = \begin{bmatrix} 35 & 27 \\ 20 & 16 \end{bmatrix} - \begin{bmatrix} 5 & -25 \\ -20 & 30 \end{bmatrix} = \begin{bmatrix} 30 & 52 \\ 40 & -14 \end{bmatrix}$$

(c)

Let the three consecutive terms in A.P. be $a - d$, a and $a + d$.

$\therefore (a - d) + a + (a + d) = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$ ....(i)

Also, $(a - d)^2 + a^2 + (a + d)^2 = 165$

$\Rightarrow a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 165$

$\Rightarrow 3a^2 + 2d^2 = 165$

$\Rightarrow 3 \times (7)^2 + 2d^2 = 165$ ....[From (i)]

$\Rightarrow 3 \times 49 + 2d^2 = 165$

$\Rightarrow 147 + 2d^2 = 165$

$\Rightarrow 2d^2 = 18$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

When $a = 7$ and $d = 3$

Required terms $= a - d$, a and $a + d$

$= 7 - 3, 7, 7 + 3$

$= 4, 7, 10$

When $a = 7$ and $d = -3$

Required terms $= a - d$, a and $a + d$

$= 7 - (-3), 7, 7 + (-3)$

$= 10, 7, 4$

Q. 2.

(a)

Given that the die has 6 faces marked by the given numbers as below:

3	2	1	–1	–2	–3

When a die is rolled, total number of possible outcomes = 6

(i) For getting a positive integer, the favourable outcomes are: 1, 2, 3

$\Rightarrow$ Number of favourable outcomes = 3

$\Rightarrow$ Required probability $= \frac{3}{6} = \frac{1}{2}$

(ii) For getting an integer greater than -3, the favourable outcomes are: -2, -1, 1, 2, 3

$\Rightarrow$ Number of favourable outcomes = 5

$\Rightarrow$ Required probability $= \frac{5}{6}$

(iii) For getting a smallest integer, the favourable outcomes are: -3

$\Rightarrow$ Number of favourable outcomes = 1

$\Rightarrow$ Required probability $= \frac{1}{6}$

(b) $x^2 + (p - 3)x + p = 0$

Here, a = 1, b = (p – 3), c = p

Since, the roots are equal,

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (p - 3)^2 - 4(1)(p) = 0$

$\Rightarrow p^2 + 9 - 6p - 4p = 0$

$\Rightarrow p^2 - 10p + 9 = 0$

$\Rightarrow p^2 - 9p - p + 9 = 0$

$\Rightarrow p(p - 9) - 1(p - 9) = 0$

$\Rightarrow (p - 9)(p - 1) = 0$

$\Rightarrow p - 9 = 0$ or $p - 1 = 0$

$\Rightarrow p = 9$ or $p = 1$

(c)

(i) In ΔABD,

$\angle DAB + \angle ABD + \angle ADB = 180^\circ$

$\Rightarrow 65^\circ + 70^\circ + \angle ADB = 180^\circ$

$\Rightarrow 135^\circ + \angle ADB = 180^\circ$

$\Rightarrow \angle ADB = 180^\circ - 135^\circ = 45^\circ$

Now, $\angle ADC = \angle ADB + \angle BDC = 45^\circ + 45^\circ = 90^\circ$

Since $\angle ADC$ is the angle of semicircle, so AC is a diameter of the circle.

(ii) $\angle ACB = \angle ADB$ ....(angles in the same segment of a circle)

$\Rightarrow \angle ACB = 45^\circ$

i. Now, $m\angle CQP + m\angle CPQ + m\angle PCQ = 180^\circ$

$54^\circ + 54^\circ + m\angle PCQ = 180^\circ$

$108^\circ + m\angle PCQ = 180^\circ$

$\Rightarrow m\angle PCQ = 180^\circ - 108^\circ = 72^\circ$

Q. 3.

(a)

Let the value of the monthly instalment be Rs. P.

Since rate of interest (r) = 8%,

Number of months, $n = 3 \times 12 = 36$

Maturity value (M.V.) = Rs. 8088

$\therefore M.V. = P \times n + P \times \frac{n(n+1)}{2} \times \frac{r}{12 \times 100}$

$\Rightarrow 8088 = P \times 36 + P \times \frac{36 \times 37}{2} \times \frac{8}{12 \times 100}$

$\Rightarrow 8088 = 36P + 4.44P$

$\Rightarrow 8088 = 40.44P$

$\Rightarrow P = \frac{8088}{40.44} = 20$

Thus, the value of his onthly instalment is Rs. 200.

(b)

Inner radius of the pipe $= r = \frac{5}{2}$ cm $= 2.5$ cm

External radius of the pipe $= R =$ Inner radius of the pipe $+$ Thickness of the pipe

$= 2.5$ cm $+ 0.5$ cm

$= 3$ cm

Length of the pipe $= h = 2$ m $= 200$ cm

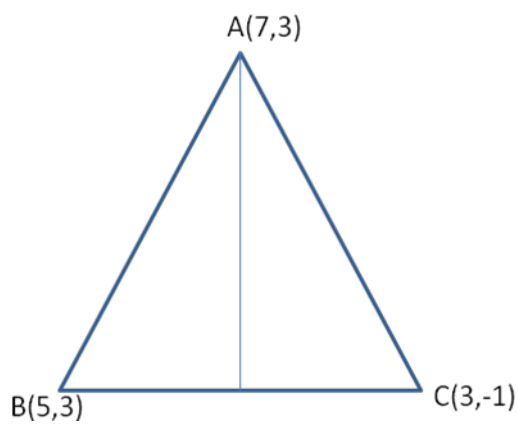
Volume of the pipe $=$ External Volume $-$ Internal Volume

$$= \pi R^2 h - \pi r^2 h$$
$$= \pi\left(R^2 - r^2\right)h$$
$$= \pi(R - r)(R + r)h$$
$$= \frac{22}{7}(3 - 2.5)(3 + 2.5) \times 200$$
$$= \frac{22}{7} \times 0.5 \times 5.5 \times 200$$
$$= 1728.6 \text{ cm}^3$$

Since 1cm^3 of the metal weights 7.7 g,

$\therefore$ Weight of the pipe $= (1728.6 \times 7.7)$ g $= \left(\frac{1728.6 \times 7.7}{1000}\right)$ kg $= 13.31$ kg

(c)



we know that the median of triangle bisects the opposite side

$\therefore$ BD : DC = 1 : 1

Coordinates of D are,

$$D(x, y) = D\left(\frac{5+3}{2}, \frac{3-1}{2}\right) = D(4,1)$$

Length of median AD $= \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = \sqrt{25} = 5$ units

Q. 4.

(a)

The given inequation is

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$$

$$\Rightarrow -\frac{x}{3} \le \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

Now,

$$-\frac{x}{3} \le \frac{x}{2} - \frac{4}{3}$$

$$\Rightarrow -\frac{x}{3} - \frac{x}{2} \le -\frac{4}{3}$$

$$\Rightarrow \frac{2x+3x}{6} \ge \frac{4}{3}$$

$$\Rightarrow \frac{5x}{6} \ge \frac{4}{3}$$

$$\Rightarrow 5x \ge 8$$

$$\Rightarrow x \ge \frac{8}{5}$$

$$\Rightarrow x \ge 1.6$$

$$\frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$

$$\Rightarrow \frac{x}{2} < \frac{1+4\times 2}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{1+8}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{9}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{3}{2}$$

$$\Rightarrow x < 3$$

$\therefore$ Solution set $= \{x : 1.6 \le x < 3\}$

It can be represented on a number line as follows:

(b)

$\sin^2 34^\circ + \sin^2 56^\circ + 2\tan 18^\circ \tan 72^\circ - \cot^2 30^\circ$

$= \sin^2 34^\circ + \sin^2(90^\circ - 34^\circ) + 2\tan 18^\circ \tan(90^\circ - 72^\circ) - \cot^2 30^\circ$

$= \sin^2 34^\circ + \cos^2 34^\circ + 2\tan 18^\circ \cot 18^\circ - \cot^2 30^\circ$

$= (\sin^2 34^\circ + \cos^2 34^\circ) + 2\tan 18^\circ \times \frac{1}{\tan 18^\circ} - \cot^2 30^\circ$

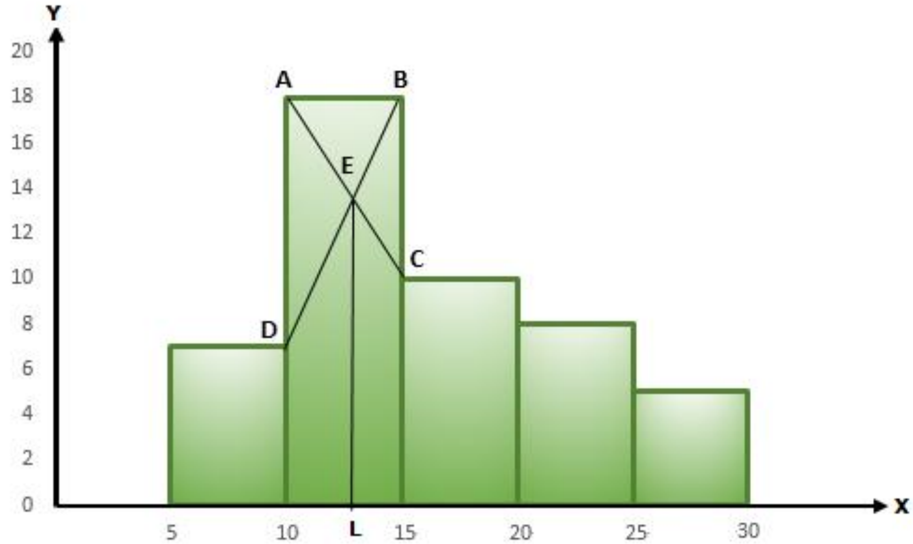
$= 1 + 2\times 1 - \left(\sqrt{3}\right)^2$

$= 1 + 2 - 3$

$= 3 - 3$

$= 0$

(c)



In the highest rectangle which represents modal class draw two lines AC and BD intersecting at E.

From E, draw a perpendicular to x-axis meeting at L.

Value of L is the mode. Hence, mode = 13

SECTION – B (40 Marks)

Q. 5.

(a)

Let the marked price for article A be Rs. x and for article B be Rs. y.

The marked price of two articles A and B together is Rs. 6,000.

$\Rightarrow x + y = 6000 \quad ....(i)$

The sales tax on article A is 8% and that on article B is 10%.

Also the total sales tax collected by selling both the articles is Rs. 552.

$\Rightarrow 8\% \text{ of } x + 10\% \text{ of } y = 552$

$\Rightarrow \frac{8}{100} \times x + \frac{10}{100} \times y = 552$

$\Rightarrow 8x + 10y = 55200 \quad ....(ii)$

Multiplying equation (i) by 8, we get

$8x + 8y = 48000 \quad ....(iii)$

Subtracting (iii) from (ii), we get

$2y = 7200$

$\Rightarrow y = 3600$

Substituting $y = 3600$ in (i), we get

$x + 3600 = 6000$

$\Rightarrow x = 2400$

Thus, the marked price for article A is Rs. 2,400

and for article B is Rs. 3,600.

(b)

Let the radius of a solid right circular cylinder be $r = 100$ cm

And, let the height of a solid right circular cylinder be $h = 100$ cm

$\therefore$ Volume (original) of a solid right circular cylinder $= \pi r^2 h$

$$= \pi \times (100)^2 \times 100$$
$$= 1000000\pi \text{ cm}^3$$

New radius $= r' = 120$ cm

New height $= h' = 80$ cm

$\therefore$ Volume (New) of a solid right circular cylinder $= \pi r'^2 h'$

$$= \pi \times (120)^2 \times 80$$
$$= 1152000\pi \text{ cm}^3$$

$\therefore$ Increase in Volume = New Volume $-$ Original Volume

$$= 1152000\ \pi \text{ cm}^3 - 1000000\pi \text{ cm}^3$$
$$= 152000\ \pi \text{ cm}^3$$

Thus, Percentage change in volume $= \dfrac{\text{Increase in Volume}}{\text{Original Volume}} \times 100\%$

$$= \frac{152000\ \pi \text{ cm}^3}{1000000\ \pi \text{ cm}^3} \times 100\%$$
$$= 15.2\%$$

(c) A(1,-5), the co-ordinates of A' = (1, 2x1-(-5)) = (1,7)

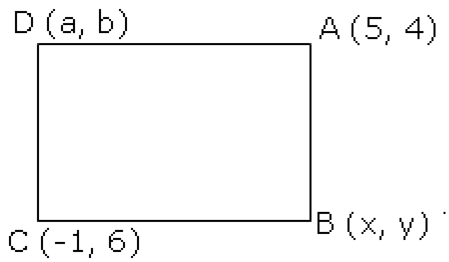
B(-5,1), the co-ordinates of B' = (-5, 2x4-(1)) = (-5,7)

The distance AB' =

$$= \sqrt{(-5-1)^2 + (7-(-5))^2}$$
$$= \sqrt{(-6)^2 + 12^2}$$
$$= \sqrt{36+144}$$
$$= \sqrt{180}$$
$$= 13.41 \text{units}$$

Q. 6.

(a)



Given ABCD is a square.

$\therefore AB = BC$ (all sides of a square are equal)

$\sqrt{(x-5)^2 + (y-4)^2} = \sqrt{(x+1)^2 + (y-6)^2}$

Squaring both sides,

$x^2 + 25 - 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow -12x + 4y + 4 = 0$

$\Rightarrow -3x + y + 1 = 0$

$y = 3x - 1 \qquad .... (1)$

Also, each angle in a square measures 90^0

By pythogoras theorem,

$AB^2 + BC^2 = AC^2$

$\Rightarrow (5-x)^2 + (4-y)^2 + (x+1)^2 + (y-6)^2 = 36 + 4$

$\Rightarrow 25 + x^2 - 10x + 16 + y^2 - 8y + x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow 2x^2 + 2y^2 - 8x - 20y + 38 = 0$

$\Rightarrow x^2 + y^2 - 4x - 10y + 19 = 0$

$\Rightarrow x^2 + (3x-1)^2 - 4x - 10(3x-1) + 19 = 0$

$\Rightarrow x^2 + 9x^2 + 1 - 6x - 4x - 30x + 10 + 19 = 0$

$\Rightarrow 10x^2 - 40x + 30 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x-3) - 1(x-3) = 0$

$\Rightarrow (x-1)(x-3) = 0$

$x = 1, 3$

When, $x = 1$, $y = 3(1) - 1 = 2 \qquad (1, 2)$

$x = 3$, $y = 3(3) - 1 = 8 \qquad (3, 8)$

Thus, coordinates of B and D are $(1, 2)$ and $(3, 8)$.

(b)

$$\frac{x}{3}+\frac{3}{6-x}=\frac{2(6+x)}{15}$$

$$\Rightarrow \frac{x(6-x)+3\times 3}{3(6-x)}=\frac{12+2x}{15}$$

$$\Rightarrow \frac{x(6-x)+3\times 3}{6-x}=\frac{12+2x}{5}$$

$$\Rightarrow \frac{6x-x^2+9}{6-x}=\frac{12+2x}{5}$$

$$\Rightarrow 30x-5x^2+45=72+12x-12x-2x^2$$

$$\Rightarrow 30x-5x^2+45=72-2x^2$$

$$\Rightarrow 3x^2-30x+27=0$$

$$\Rightarrow x^2-10x+9=0$$

$$\Rightarrow x^2-9x-x+9=0$$

$$\Rightarrow x(x-9)-1(x-9)=0$$

$$\Rightarrow (x-9)(x-1)=0$$

$$\Rightarrow x-9=0 \text{ or } x-1=0$$

$$\Rightarrow x=9 \text{ or } x=1$$

(c) Scale factor

In ΔACD and ΔBCA,

$\angle DAC = \angle ABC$ ....(given)

$\angle ACD = \angle BCA$ ....(common angles)

$\Delta ACD \sim \Delta BCA$ ....(AA criterion for Similarity)

$$\Rightarrow \frac{ar(\Delta ACD)}{ar(\Delta ABC)}=\frac{AD^2}{AB^2}$$

$$\Rightarrow \frac{ar(\Delta ACD)}{ar(\Delta ABC)}=\frac{5^2}{8^2}=\frac{25}{64}$$

Q.7.

(a)

We have,

$$\frac{(p-x)}{(q-x)} = \frac{p^2}{q^2}$$

$\Rightarrow q^2(p-x) = p^2(q-x)$

$\Rightarrow pq^2 - q^2x = p^2q - p^2x$

$\Rightarrow p^2x - q^2x = p^2q - pq^2$

$\Rightarrow x(p^2 - q^2) = pq(p-q)$

$\Rightarrow x(p-q)(p+q) = pq(p-q)$

$\Rightarrow x = \frac{pq}{p+q}$

$\Rightarrow \frac{p+q}{pq} = \frac{1}{x}$

$\Rightarrow \frac{p}{pq} + \frac{q}{pq} = \frac{1}{x}$

$\Rightarrow \frac{1}{q} + \frac{1}{p} = \frac{1}{x}$

$\Rightarrow \frac{1}{p} + \frac{1}{q} = \frac{1}{x}$

(b)

Given : $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$ and $A^2 = B$

Now, $A^2 = A \times A$

$$= \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 3x + x \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

We have $A^2 = B$

Two matrices are equal if each and every corresponding element is equal.

$\Rightarrow \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$

$\Rightarrow 4x = 16$ and $1 = -y$

$\Rightarrow x = 4$ and $y = -1$

(c)

Let the assumed mean A = 30

x	f	d = x – A	$t = \frac{x - A}{i} = \frac{x - 30}{5}$	ft
5	20	–25	–5	–100
10	43	–20	–4	–172
15	75	–15	–3	–225
20	67	–10	–2	–134
25	72	–5	–1	–72
A = 30	45	0	0	0
35	39	5	1	39
40	9	10	2	18
45	8	15	3	24
50	6	20	4	24
	$\sum f = 384$			$\sum ft = -598$

$$\therefore \text{Mean} = A + \frac{\sum ft}{\sum f} \times i$$

$$= 30 + \frac{(-598)}{384} \times 5$$

$$= 30 - \frac{299}{192} \times 5$$

$$= 30 - \frac{1495}{192}$$

$$= \frac{5760 - 1495}{192}$$

$$= \frac{4265}{192}$$

$$= 22.21$$

$$= 22$$

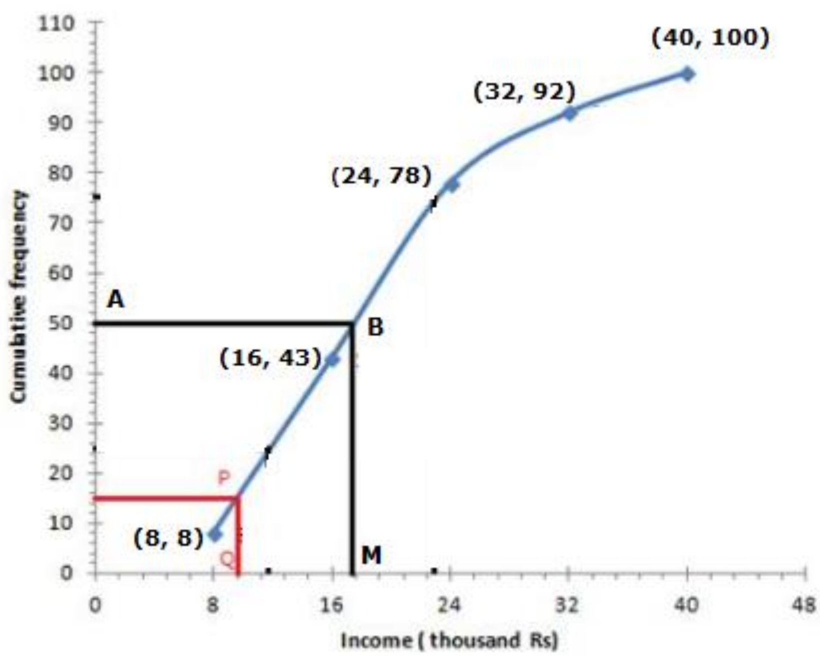
Q.8.

(a)

(i) Cumulative Frequency Curve

Income (in thousand Rs.)	No. of students f	Cumulative Frequency	Class mark x	fx
0 – 8	8	8	4	32
8 – 16	35	43	12	420
16 – 24	35	78	20	700
24 – 32	14	92	28	392
32 – 40	8	100	36	288
	$\sum f = 100$			$\sum fx = 1832$

We plot the points (8, 8), (16, 43), (24, 78), (32, 92) and (40, 100) to get the curve as follows:



$$\text{Here, } N = 100 \Rightarrow \frac{N}{2} = 50$$

At y = 50, affix A.

Through A, draw a horizontal line meeting the curve at B.

Through B, a vertical line is drawn which meets OX at M.

OM = 17.6 units

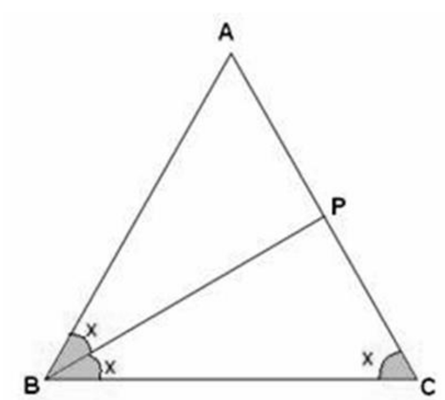
Hence, median income = 17.6 thousands

(ii) 15% of 100 students $= \frac{15 \times 100}{100} = 15$

From c.f. 15, draw a horizontal line which intersects the curve at P.
From P, draw a perpendicular to x-axis meeting it at Q
which is equal to 9.6
Therefore, freeship will be awarded to students provided annual
income of their parents is upto 9.6 thousands.

(iii) Mean $= \frac{\sum fx}{\sum f} = \frac{1832}{100} = 18.32$

(b)



In ΔABC,
$\angle ABC = 2\angle ACB$
Let $\angle ACB = x$
$\Rightarrow \angle ABC = 2\angle ACB = 2x$
Given BP is bisector of $\angle ABC$.
Hence $\angle ABP = \angle PBC = x$.
Using the angle bisector theorem,
that is, the bisector of an angle divides the side opposite to it
in the ratio of other two sides.
Hence, $CB : BA = CP : PA$.

Q. 9.

(a)

Investment = Rs. 131040

N.V. of 1 share = Rs. 100

Discount = 9% of Rs. 100 = Rs. 9

$\therefore$ M.V. of 1 share = Rs. 100 − Rs. 9 = Rs. 91

$\therefore$ Number of shares purchased = $\frac{\text{Investment}}{\text{M.V. of 1 share}} = \frac{131040}{91} = 1440$

Number of shares worth Rs. 72000 = $\frac{72000}{100} = 720$

$\therefore$ Mrs. Kulkarni sells 720 shares at a premium of 10%

M.V. of 1 share = Rs.100 + Rs. 10 = Rs. 110

$\therefore$ Selling price of 720 shares = 720 × Rs. 110 = Rs. 79200

Number of remaining shares = 1440 − 720 = 720

She sells 720 shares at a discount of 5%

M.V. of 1 share = Rs. 100 − Rs.5 = Rs. 95

$\therefore$ Selling price of 720 shares = 720 × Rs. 95 = Rs. 68400

$\therefore$ Total selling price = Rs. (79200 + 68400) = Rs. 147600

$\therefore$ Total gain = Total selling price − Total investment

= Rs. (147600 − 131040)

= Rs. 16560

(b)

Steps of construction:

1) Draw AB = 5.5 cm

2) Construct $\angle$BAR = 105°

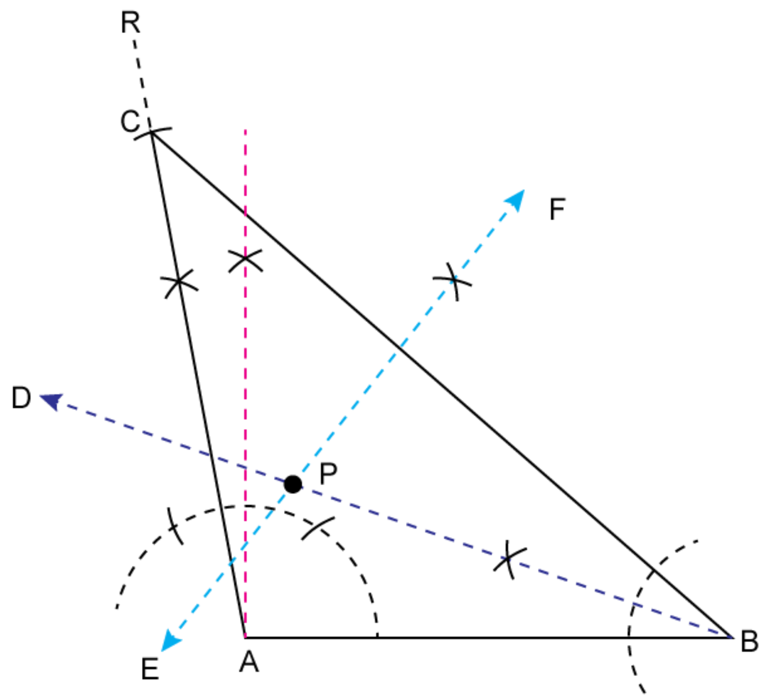
3) With centre A and radius 6 cm, cut off arc on AR at C.

4) Join BC. ABC is the required triangle.

(i) Draw angle bisector BD of $\angle$ABC, which is the locus of points equidistant from BA and BC.

(ii) Draw perpendicular bisector EF of BC, which is the locus of points equidistant from B and C.

(iii) BD and EF intersect each other at point P.
Thus, P satisfies the above two loci.
By measurement, PC = 4.8 cm



(c)

$$\text{L.H.S.} = (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$
$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$$
$$= \frac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}$$
$$= \frac{\sin\theta}{\cos\theta\sin\theta} + \frac{\cos\theta}{\cos\theta\sin\theta}$$
$$= \frac{1}{\cos\theta} + \frac{1}{\sin\theta}$$
$$= \sec\theta + \operatorname{cosec}\theta$$
$$= \text{R.H.S.}$$

Q.10.

(a)

Let 'a' be the first term and 'd' be the common difference of given A.P.

m^{th} term $= \frac{1}{n}$

$\Rightarrow a + (m-1)d = \frac{1}{n}$ ....(i)

n^{th} term $= \frac{1}{m}$

$\Rightarrow a + (n-1)d = \frac{1}{m}$ ....(ii)

Subtracting (ii) from (i), we get

$(m-1)d - (n-1)d = \frac{1}{n} - \frac{1}{m}$

$\Rightarrow md - d - nd + d = \frac{m-n}{mn}$

$\Rightarrow (m-n)d = \frac{m-n}{mn}$

$\Rightarrow d = \frac{1}{mn}$

Substituting value of d in (i), we get

$a + (m-1) \times \frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a = \frac{1}{n} - \frac{m-1}{mn} = \frac{m-m+1}{mn} = \frac{1}{mn}$

Now,

$(mn)^{th}$ term $= a + (mn-1)d$

$$= \frac{1}{mn} + (mn-1) \times \frac{1}{mn}$$

$$= \frac{1 + mn - 1}{mn}$$

$$= \frac{mn}{mn}$$

$$= 1$$

(b)

Here, a, b, c are in G.P.

$\Rightarrow b^2 = ac$

$\Rightarrow (b^2)^n = (ac)^n$

$\Rightarrow b^{2n} = a^n c^n$

Taking log on both sides, we get

$\log (b^{2n}) = \log (a^n c^n)$

$\Rightarrow \log (b^n)^2 = \log a^n + \log c^n$

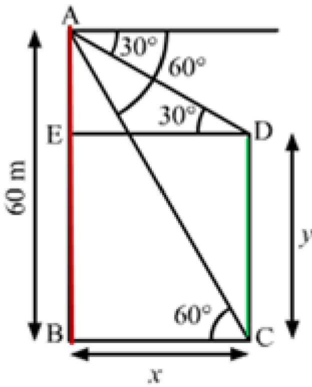
$\Rightarrow 2 \log b^n = \log a^n + \log c^n$

$\Rightarrow \log b^n + \log b^n = \log a^n + \log c^n$

$\Rightarrow \log b^n - \log a^n = \log c^n - \log b^n$

$\Rightarrow \log a^n$, $\log b^n$ and $\log c^n$ are in A.P.

(c)



Given that AB is a building that is 60 m, high.

Let BC = DE = x and CD = BE = y

$\Rightarrow$ AE = AB − BE = 60 − y

(i) In right ΔAED,

$$\tan 30^\circ = \frac{AE}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60 - y}{x}$$

$$\Rightarrow x = 60\sqrt{3} - y\sqrt{3} \quad ...(1)$$

In right ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60\sqrt{3}}{3}$$

$$\Rightarrow x = 20\sqrt{3}$$

$$\Rightarrow x = 20 \times 1.732 = 34.64 \text{ m}$$

Thus, the horizontal distance between AB and CD is 34.64 m.

(ii) From (i), we get the height of the lamp post = CD = y

$$x = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow 20\sqrt{3} = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow 20 = 60 - y$$

$$\Rightarrow y = 40 \text{ m}$$

Thus, the height of the lamp post is 40 m.

Q.11.

(a)

(i) $PA = AB + BP = (AB + 4)$ cm

$PC = PD + CD = 5 + 7.8 = 12.8$ cm

Since $PA \times PB = PC \times PD$

$\Rightarrow (AB + 4) \times 4 = 12.8 \times 5$

$\Rightarrow AB + 4 = \dfrac{12.8 \times 5}{4}$

$\Rightarrow AB + 4 = 16$

$\Rightarrow AB = 12$ cm

(ii) Since $PT^2 = PC \times PD$

$\Rightarrow PT^2 = 12.8 \times 5$

$\Rightarrow PT^2 = 64$

$\Rightarrow PT = 8$ cm

(b)

$\dfrac{x}{3} + \dfrac{3}{6 - x} = \dfrac{2(6 + x)}{15}$

$\Rightarrow \dfrac{x(6 - x) + 3 \times 3}{3(6 - x)} = \dfrac{12 + 2x}{15}$

$\Rightarrow \dfrac{x(6 - x) + 3 \times 3}{6 - x} = \dfrac{12 + 2x}{5}$

$\Rightarrow \dfrac{6x - x^2 + 9}{6 - x} = \dfrac{12 + 2x}{5}$

$\Rightarrow 30x - 5x^2 + 45 = 72 + 12x - 12x - 2x^2$

$\Rightarrow 30x - 5x^2 + 45 = 72 - 2x^2$

$\Rightarrow 3x^2 - 30x + 27 = 0$

$\Rightarrow x^2 - 10x + 9 = 0$

$\Rightarrow x^2 - 9x - x + 9 = 0$

$\Rightarrow x(x - 9) - 1(x - 9) = 0$

$\Rightarrow (x - 9)(x - 1) = 0$

$\Rightarrow x - 9 = 0 \;\text{ or }\; x - 1 = 0$

$\Rightarrow x = 9 \;\text{ or }\; x = 1$

(c)

Slope of line AB $= m = \frac{2-3}{0-(-1)} = \frac{-1}{1} = -1$

Using the slope-point form, the equation of line AB is given by

$y - y_1 = m(x - x_1)$

i.e. $y - 3 = -1[x - (-1)]$

i.e. $y - 3 = -1(x + 1)$

i.e. $y - 3 = -x - 1$

i.e. $x + y = 2$

Now, slope of line BC $= \frac{1-2}{1-0} = \frac{-1}{1} = -1$

Since, Slope of line AB = Slope of line BC, points A, B and C are collinear.