ICSE Board
Class X Mathematics
Sample Paper 3

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1.

(a) The remainder obtained by dividing $kx^2 - 3x + 6$ by $(x - 2)$ is twice the remainder obtained by dividing $3x^2 + 5x - k$ by $(x + 3)$. Find the value of k. [3]

(b) If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, show that $AB \neq BA$. [3]

(c) In a school, students thought of planting trees in an around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class in which they are studying, e.g., a section of class-I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students? [4]

Q. 2.

(a) From a pack of 52 playing cards, the Jack, Queen and King of clubs are removed, and the pack is well shuffled. From the remaining cards, a card is drawn. Find the probability of getting

i. a club

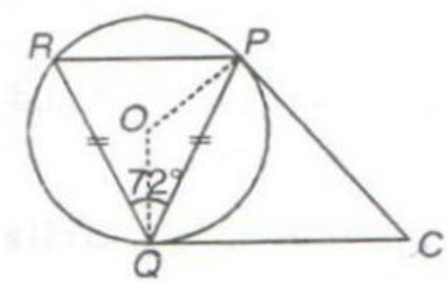
ii. ii. a red face card [3]

(b) Solve the equation: $2x - \frac{3}{x} = 5$ [3]

(c) In the given figure, QR = PQ, $\angle PQR = 72^\circ$. PC and QC are tangents to the circle with centre O. Calculate

i. the angle subtended by the chord PQ at the centre

ii. $\angle PCQ$ [4]



Q. 3.

(a) Manu has a 5 years recurring deposit account and deposits Rs. 240 per month. If he receives Rs. 17,694 at the time of maturity, find the rate of interest. [3]

(b) A sphere of diameter 6 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 6 cm. If the sphere is completely submerged in water, find by how much the surface level of water will be raised. [3]

(c) A(1, 4), B(3, 2) and C(7, 5) are the vertices of ΔABC.
Find

i. The coordinates of the centroid G of ΔABC.

ii. The equation of a line, through G and parallel to AB. [4]

Q. 4.

(a) Solve the following inequation and graph the solution on the number line:
$-2\frac{2}{3} \le x + \frac{1}{3} < 3\frac{1}{3}; x \in R$ [3]

(b) Without using trigonometric table, find the value of
$\frac{\cos 70°}{\sin 20°} + \frac{\cos 59°}{\sin 31°} - 8\sin^2 30°$ [3]

(c) Draw a histogram of the following frequency distribution and use it to calculate the mode. [4]

C.I.	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	15	10	5	12	8

SECTION – B (40 Marks)

*(Answer **any four questions** from this Section)*

Q. 5.

(a) Satwika purchases a dress for Rs. 2,298.24 which includes two successive discounts of 20% and 5% respectively on the marked price and then 8% sales tax on the remaining price. Find the marked price of the dress. [3]

(b) A steel wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the mass of the wire, assuming the density of steel to be 8.88 g per cm^3. [3]

(c) Find the image of the point (−8, 12) with respect to the line mirror 4x + 7y + 13 = 0. [4]

Q. 6.

(a) The points (4, 1), (4, −1), (−4, 1) and (−4, −1) are the vertices of a rectangle. If the rectangle is reflected in the line x = 5, find the coordinates of the reflected rectangle. Also, find the area and perimeter of the reflected rectangle. [3]

(b) Using the properties of proportion, solve for x, given

$$\frac{x^4 + 1}{2x^2} = \frac{17}{8}$$ [3]

(c) The scale of a map is 1 : 200000. A plot of land of area 20 km^2 is to be represented on the map. Find

i. the number of kilometers on the ground which is represented by 1 cm on the map.
ii. the area in sq. km that can be represented by 1 cm^2.
iii. the area of the map that represents the plot of land. [4]

Q. 7.

(a) Using the properties of proportion, solve for x:

$$\frac{\sqrt{a + x} + \sqrt{a - x}}{\sqrt{a + x} - \sqrt{a - x}} = b$$ [3]

(b) Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, satisfies the equation $A^3 - 4A^2 + A = 0$ [3]

(c) Calculate the mean of the following frequency distribution by the step-deviation method: [4]

X	15	20	25	30	35	40	45	50	55
F	5	8	11	20	23	18	13	3	1

Q. 8.

(a) The table below shows the distribution of the scores obtained by 120 shooters in a competition. Using a graph sheet, draw an Ogive curve for the distribution.
Use the Ogive curve to estimate the:

i. Median
ii. Inter-quartile range
iii. Number of shooters who obtained more than 75% scores [6]

Scores	0 – 10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90-100
No. of Shooters	5	9	16	22	26	18	11	6	4	3

(b) Through the midpoint M, of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD produced in E. Prove that EL = 2BL [4]

Q. 9.

(a) A man invests Rs. 20,020 in buying shares having a nominal value of Rs. 26 at 10% premium. The dividend on the shares is 15% per annum. Calculate

i. The number of shares he buys.
ii. The dividend he receives annually
iii. The rate of interest he gets on his money. [3]

(b) Draw a pair of tangents to a circle of any convenient radius, which are inclined to the line joining the centre of the circle and intersect at a point forming an angle of 45° with the line. [3]

(c) Prove the identity $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A} = \sec A . \operatorname{cosec} A + 1$ [4]

Q. 10.

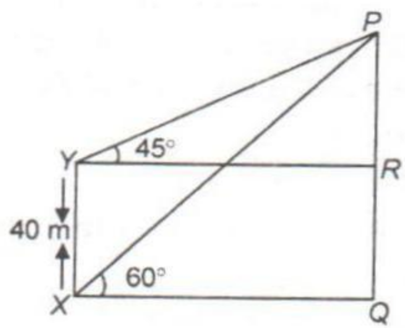
(a) Find the numbers such that their mean proportion is 14 and third proportion is 112. [3]

(b) If the product of the first four consecutive terms of a G.P. is 256 and if the common ratio is 4 and the first term is positive, then find its 3rd term. [3]

(c) In the figure, the angle of elevation of the top P of a vertical tower from a point X is 60°. At a point y, 40 m vertically above X the angle of elevation is 45°. Find the [4]

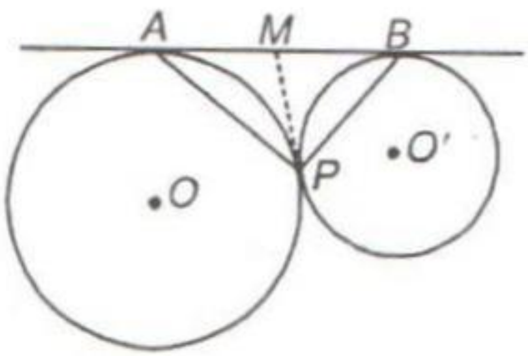
i. height PQ.

ii. distance XQ.



Q. 11.

(a) In the given figure,



Two circles touch each other externally at point P. AB is the direct common tangent of these circles. Prove that:

i. m∠APB = 90°

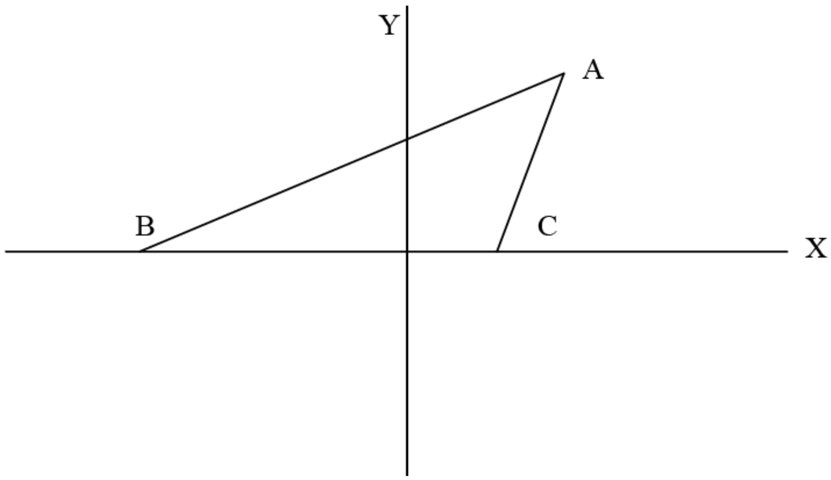
ii. Tangent at point P bisects AB [3]

(b) A person on tour has Rs. 360 for his expenses. If he extends his tour for 4 days, he has to cut down his daily expenses by Rs. 3. Taking the original duration of tour as x, form an equation in x and solve it. [3]

(c) In the diagram given below, equation of AB is $x - \sqrt{3}y + 1 = 0$ and equation of AC is x – y – 2 = 0.

i. Write down the angles that the lines AC and AB make with the positive direction of the x-axis.

ii. Find m∠BAC [4]



Solution

SECTION – A (40 Marks)

Q. 1.

(a) Put $x - 2 = 0 \Rightarrow x = 2$ in $kx^2 - 3x + 6$

By Factor Theorem,

the remainder obtained on dividing $kx^2 - 3x + 6$ by $x - 2 = k(2)^2 - 3(2) + 6$

Put $x + 3 = 0 \Rightarrow x = -3$ in $3x^2 + 5x - k$

Similarly, the remainder obtained by dividing $3x^2 + 5x - k$ by $x + 3 = 3(-3)^2 + 5(-3) - k$

Now, $k(2)^2 - 3(2) + 6 = 2[3(-3)^2 + 5(-3) - k]$

$\Rightarrow 4k - 6 + 6 = 2[27 - 15 - k]$

$\Rightarrow 4k = 2(12 - k)$

$\Rightarrow 4k = 24 - 2k$

$\Rightarrow 4k + 2k = 24$

$\Rightarrow 6k = 24$

$\Rightarrow k = 4$

(b)

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+0 & 1+0 \\ 0-1 & 0+0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0-1 \\ 1+0 & 0-0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Thus, $AB \neq BA$

(c) There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections × 1 = 3 × 1 = 3

The number of trees planted by class II = number of sections × 2 = 3 × 2 = 6

The number of trees planted by class III = number of sections × 3 = 3 × 3 = 9

Therefore, we have the sequence: 3, 6, 9, ..., (12 terms)

To find total number of trees planted by all the students, we need to find sum of the 12 terms of the sequence.

First term = a = 3 and Common difference = d = 6 – 3 = 3

n = 12

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow S_{12} = \frac{12}{2}[2\times3+(12-1)3] = 6[6+33] = 6\times39 = 234$$

Thus, in total 234 trees will be planted by the students.

Q. 2.

(a) Number of cards removed = 3 (face cards of clubs)

Remaining number of cards = 52 – 3 = 49

i. Remaining number of club cards = 13 – 3 = 10

$\therefore$ P(a club) = $\frac{10}{49}$

ii. Number of red face cards = 6 (i.e. 3 of diamonds and 3 of hearts)

P(a red face card) = $\frac{6}{49}$

(b) $2x - \frac{3}{x} = 5$

$\Rightarrow 2x^2 - 5x - 3 = 0$

$\Rightarrow 2x^2 - 6x + x - 3 = 0$

$\Rightarrow 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow 2x + 1 = 0$ or $x - 3 = 0$

$\Rightarrow x = \frac{-1}{2}$ or $x = 3$

(c)

i. Given: QR = QP

$\Rightarrow \angle QPR = \angle QRP$

$m\angle QPR + m\angle RPQ + m\angle PQR = 180°$

$2m\angle QPR + 72° = 180°$

$2m\angle QPR = 108°$

$m\angle QPR = 54°$

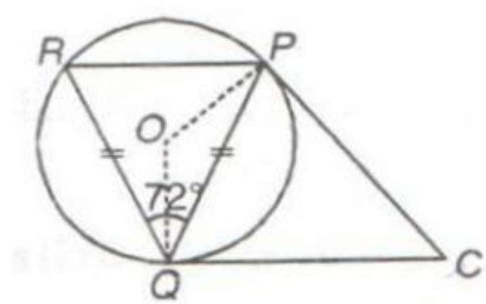
$\therefore m\angle QRP = m\angle RPQ = 54°$

Now, $m\angle POQ = 2m\angle QRP = 108°$

For angles in alternate segment

$\therefore m\angle CQP = m\angle QRP = 54°$

And $m\angle CPQ = m\angle QPC = 54°$



ii. Now, $m\angle CQP + m\angle CPQ + m\angle PCQ = 180°$

$54° + 54° + m\angle PCQ = 180°$

$108° + m\angle PCQ = 180°$

$\Rightarrow m\angle PCQ = 180° - 108° = 72°$

Q. 3.

(a) Total amount deposited by Manu in 5 years = Rs. 240 × 60 = Rs. 14,400

Equivalent principal for 1 month = $\text{Rs. } 240 \times \dfrac{60(60+1)}{2} = \text{Rs. } 4{,}39{,}200$

Let the rate of interest be r%

Interest on Rs. 4,39,200 for 1 month $= \text{Rs. } 439200 \times \dfrac{1}{12} \times \dfrac{r}{100} = \text{Rs. } 366r$

Maturity amount = 17694

Or 14400 + 366r = 17694

$\Rightarrow r = 9\%$

(b)Diameter of sphere = 6 cm

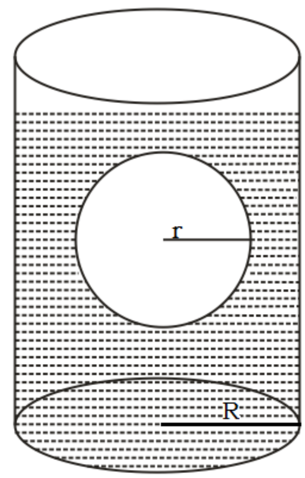
Radius of sphere = r = 3 cm

Radius of cylinder = R = 6 cm

Let the height of water raised be h cm.

Then, volume of water thus raised = $\pi R^2 h$

$\therefore$ Volume of water raised = volume of sphere



$\Rightarrow \pi R^2 h = \dfrac{4}{3}\pi r^3$

$\Rightarrow R^2 h = \dfrac{4}{3} r^3$

$\Rightarrow 36h = \dfrac{4}{3} \times 27$

$\Rightarrow h = 1 \text{ cm}$

Therefore, water will be raised by 1 cm.

(c)

i. Co-ordinates of centroid, $x = \dfrac{x_1 + x_2 + x_3}{3} = \dfrac{1+3+7}{3} = \dfrac{11}{3}$

And, $y = \dfrac{y_1 + y_2 + y_3}{3} = \dfrac{4+2+5}{3} = \dfrac{11}{3}$

So, $G = \left(\dfrac{11}{3}, \dfrac{11}{3}\right)$

ii. The equation of a line, through G and parallel to AB.

Slope of line $AB = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-4}{3-1} = -\dfrac{2}{2} = -1$

Now, equation of line through G and parallel to AB,

$\Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow y - \dfrac{11}{3} = -1\left(x - \dfrac{11}{3}\right)$

$\Rightarrow 3x + 3y - 22 = 0$

Q. 4.

(a) $-2\frac{2}{3} \le x + \frac{1}{3} < 3\frac{1}{3}; x \in R.$

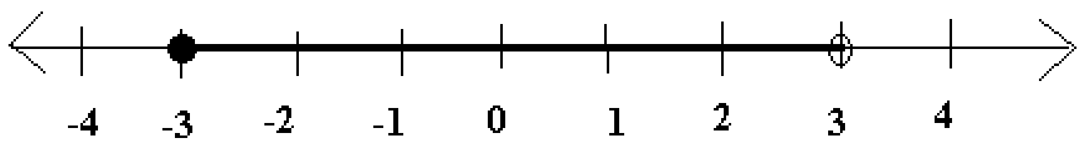
$\Rightarrow \frac{-8}{3} \le x + \frac{1}{3} < \frac{10}{3}$ [by subtracting $\frac{1}{3}$]

$\Rightarrow \frac{-8}{3} - \frac{1}{3} \le x + \frac{1}{3} - \frac{1}{3} < \frac{10}{3} - \frac{1}{3}$

$\Rightarrow \frac{-9}{3} \le x < \frac{9}{3}$

$\Rightarrow -3 \le x < 3$

Solution is $\{x : x \in R, -3 \le x < 3\}$



(b) $\frac{\cos 70°}{\sin 20°} + \frac{\cos 59°}{\sin 31°} - 8\sin^2 30°$

$= \frac{\sin(90° - 70°)}{\sin 20°} + \frac{\sin(90° - 59°)}{\sin 31°} - 8\left(\frac{1}{2}\right)^2$

$= \frac{\sin 20°}{\sin 20°} + \frac{\sin 31°}{\sin 31°} - 8 \times \frac{1}{4}$

$= 1 + 1 - 2$

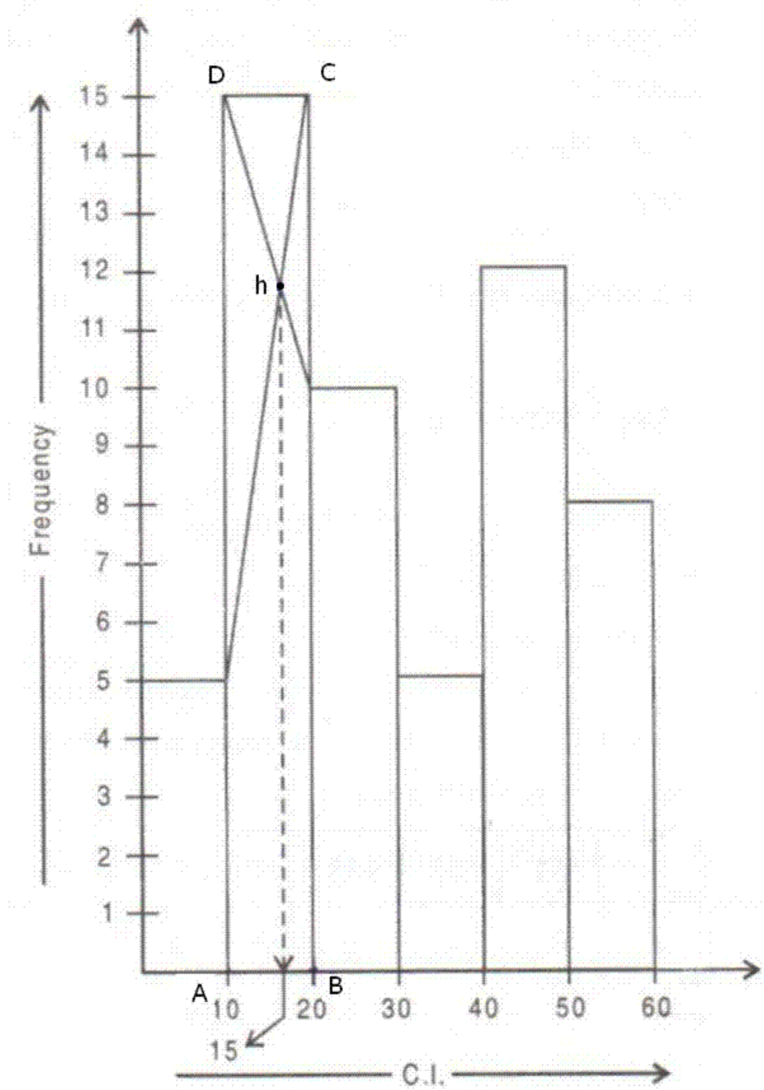
$= 0$

(c) To locate the mode from the histogram, we proceed as follows:

i Find the modal class. Rectangle ABCD is the largest rectangle. It represents the modal class, that is, the mode lies in this rectangle. The modal class is 10–20.

ii Draw two lines diagonally from the vertices C and D to the upper corners of the two adjacent rectangles. Let these rectangles intersect at point H.

iii The x-value of the point H is the mode. Thus, mode of the given data is approximately 15.



SECTION – B (40 Marks)

Q. 5.

(a) M.P. = Rs. x, Discount = 20% of x = $\text{Rs.}\frac{x}{5}$

New M.P. after discount = $\text{Rs.}\,x - \frac{x}{5} = \text{Rs.}\frac{4x}{5}$

Again discount = 5% of $\frac{4x}{5} = \frac{1}{20}\left(\frac{4x}{5}\right) = \text{Rs.}\frac{x}{25}$

Changed M.P. after 2nd discount = $\frac{4x}{5} - \frac{x}{25} = \text{Rs.}\frac{19x}{25}$

Now, 2298.24 = $\frac{19x}{25} + 8\%\text{ of }\frac{19x}{25}$

$$2298.24 = \frac{19x}{25} + \frac{8}{100} \times \frac{19x}{25}$$

$$\Rightarrow 2298.24 = \frac{19x}{25} + \frac{38x}{625}$$

$$\Rightarrow 2298.24 = \frac{475x + 38x}{625}$$

$$\Rightarrow 2298.24 = \frac{513x}{625}$$

$$\Rightarrow \frac{2298.24 \times 625}{513} = x$$

$$\Rightarrow x = \text{Rs.}\,2800$$

$$\therefore \text{M.P.} = \text{Rs.}\,2800$$

(b)

Length of the wire used in one round = 3 mm = 0.3 cm

Number of rounds required to cover 12 cm of length = $\frac{12}{0.3} = 40$

Diameter of the cylinder = 10 cm $\Rightarrow$ Radius of the cylinder = 5 cm

Length of wire required for one round = 2πr = 2π × 5 cm = 10π cm

$\therefore$ Length of wire required for 40 rounds = 10π × 40 = 400π cm = 400 × 3.14 = 1256 cm

Now,radius of the wire = $\frac{3}{2}$ mm= $\frac{3}{20}$ cm

Volume of the wire = $\pi r^2 h = \pi\left(\frac{3}{20}\right)^2 \times 1256\ \text{cm}^3$

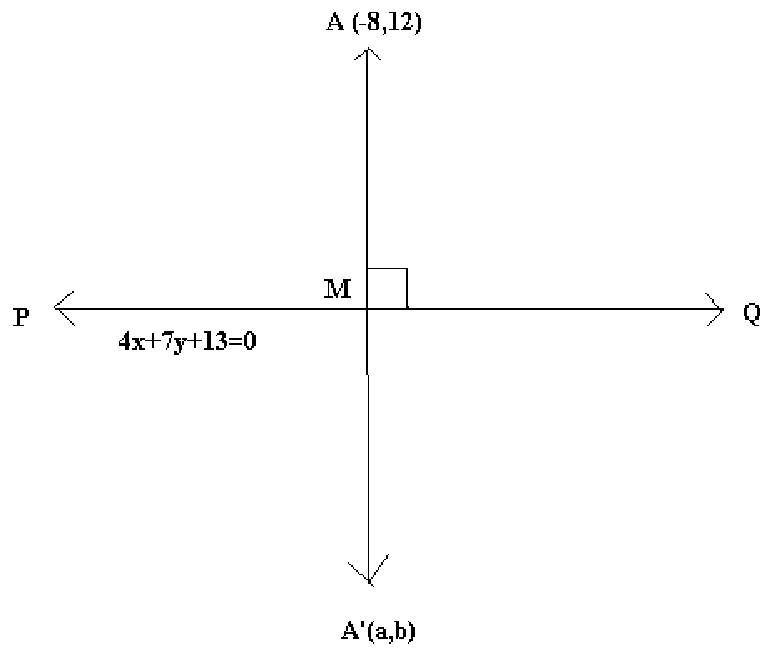
Weight of the wire = Volume of the wire × 8.88 gm $= 3.14 \times \left(\frac{3}{20}\right)^2 \times 1256 \times 8.88 = 787.98$ gm

(c) $M = \left(\frac{a-8}{2}, \frac{b+12}{2}\right)$

It lies on $4x + 7y + 13 = 0$

$\therefore 4\left(\frac{a-8}{2}\right) + 7\left(\frac{b+12}{2}\right) + 13 = 0$

$\Rightarrow 4a + 7b + 78 = 0$



Since AA' $\perp$ PQ, so

Slope (PQ) $\times$ Slope (AA') = -1

$\frac{-4}{7} \times \frac{b-12}{a+8} = -1$

$\Rightarrow 7a - 4b + 104 = 0$

$4a + 7b = -78$ ....(1)

$7a - 4b = -104$ ....(2)

Multiplying equation (1) by 4 and equation (2) by 7 we get

$16a + 28b = -312$

$49a - 28b = -728$

$65a = -1040 \Rightarrow a = \frac{-1040}{65} = -16,$

Put $a = -16$ in equation, we get

$4a + 7b + 78 = 0$

$\Rightarrow 4 \times -16 + 7b + 78 = 0$

$\Rightarrow -64 + 7b + 78 = 0$

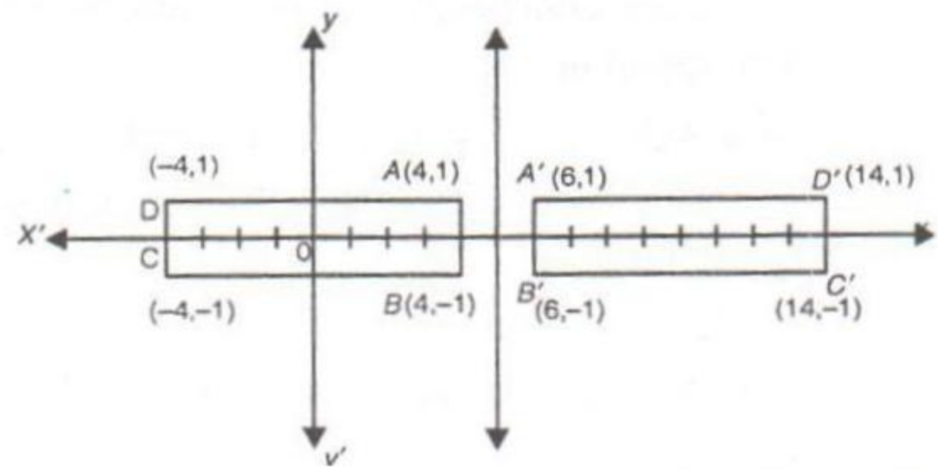
$\Rightarrow 7b + 14 = 0 \Rightarrow 7b = -14 \Rightarrow b = -2$

Image = $(-16, -2)$

Q. 6.

(a) First the coordinates A(4, 1), B(4, −1), D(−4, 1) and C(−4, −1) are plotted to form a rectangle.

The rectangle so formed and the reflected rectangle are shown in the figure:



It is reflected through x = 5, then the coordinates of the new rectangle become A'(6, −1), B'(6, 1), C'(14, −1) and D'(14, 1)

Perimeter of A'B'C'D' = A'B' + B'C' + C'D' + D'A'

= 2 + 8 + 2 + 8

= 20 units

Area = L × B = B'C' × C'D' = 8 × 2 = 16 sq. units

(b) $\frac{x^4 + 1}{2x^2} = \frac{17}{8}$

Using componendo and dividendo,

$\Rightarrow \frac{x^4 + 2x^2 + 1}{x^4 - 2x^2 + 1} = \frac{17 + 8}{17 - 8}$

$\Rightarrow \frac{(x^2 + 1)^2}{(x^2 - 1)^2} = \frac{25}{9}$

$\Rightarrow \frac{(x^2 + 1)}{(x^2 - 1)} = \frac{5}{3}$

Using componendo and dividendo again,

$\Rightarrow \frac{x^2 + 1 + x^2 - 1}{x^2 + 1 - x^2 + 1} = \frac{5 + 3}{5 - 3}$

$\Rightarrow \frac{2x^2}{2} = \frac{8}{2}$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

(c) Scale factor

$k = \frac{1}{200000}$ and $1\,cm = \frac{1}{200000} \times$ Length of the ground

i. Length of the ground $= \frac{200000}{100 \times 1000}$ km

ii. Area of map $= k^2$ times the corresponding area of the ground

$$1\ cm^2 = \left(\frac{1}{200000}\right)^2 \times \text{Area of ground}$$

$$\Rightarrow \text{Area of ground} = 200000 \times 200000 = 4\,km^2$$

iii. Total area of the map $= k^2$ times the total area of the plot

$$= \left(\frac{1}{200000}\right)^2 \times 20\ km^2$$

$$= \frac{1}{40000000000} \times 20 \times (100 \times 1000)^2\ cm^2$$

$$= 5\ cm^2$$

Q.7.

(a) Given, $\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}} = \frac{b}{1}$

By using the componendo and dividendo, we get

$$\frac{2\sqrt{a+x}}{2\sqrt{a-x}} = \frac{b+1}{b-1}$$

Now squaring both sides, we get

$$\frac{a+x}{a-x} = \frac{b^2+1+2b}{b^2+1-2b},$$

Again use componendo and dividendo, to get the value of x.

$$\frac{a+x+a-x}{a+x-a+x} = \frac{b^2+1+2b+b^2+1-2b}{b^2+1+2b-b^2-1+2b}$$

$$\Rightarrow \frac{2a}{2x} = \frac{2b^2+2}{4b}$$

$$\Rightarrow \frac{a}{x} = \frac{b^2+1}{2b}$$

$$\Rightarrow x = \frac{2ab}{b^2+1}$$

(b)

$$A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Now, L.H.S. = $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - \begin{bmatrix} 28 & 48 \\ 16 & 28 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$= \text{R.H.S.}$$

(c) Here we need to find the mean by the mean deviation method.

x_i	f_i	$u_i = \frac{x_i - A}{h}, h = 5$	$f_i u_i$
15	5	−4	−20
20	8	−3	−24
25	11	−2	−22
30	20	−1	−20
A = 35	23	0	0
40	18	1	18
45	13	2	26
50	3	3	9
55	1	4	4
	$\sum f_i = 102$		$\sum f_i u_i = -29$

From the table, $A = 35, \sum f_i = 102, h = 5, \sum f_i u_i = -29$

So, Mean $(\bar{x})$ = $A + h\frac{\sum f_i u_i}{\sum f_i}$

$= 35 + 5 \times \frac{-29}{102}$

$= 35 - \frac{145}{102}$

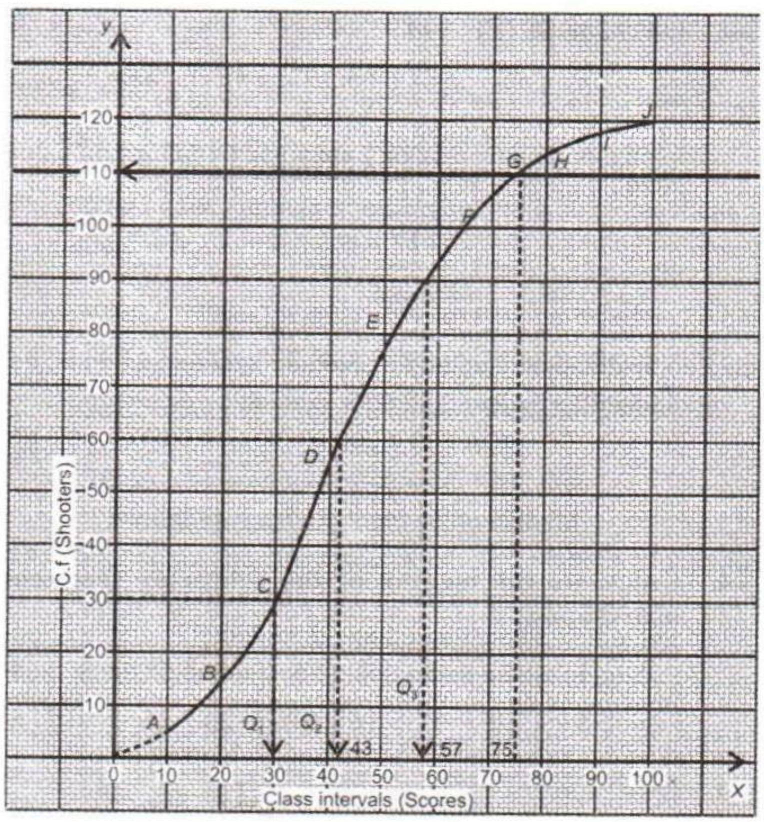
$= 35 - 1.42$

$= 33.58$

Q.8.

(a)

Scores Obtained	Number of Shooters	C.f.
0-10	5	5
10-20	9	14
20-30	16	30
30-40	22	52
40-50	26	78
50-60	18	96
60-70	11	107
70-80	6	113
80-90	4	117
90-100	3	120
	n = 120	



i. n = 120 (even)

The position of the median is given by $\frac{n}{2} = \frac{120}{2} = 60$

So, from the graph, median = 43 scores

ii. The positions of Q_1 is given by $\frac{n}{4} = \frac{120}{4} = 30$

So, from the graph, Q_1 = 30 scores

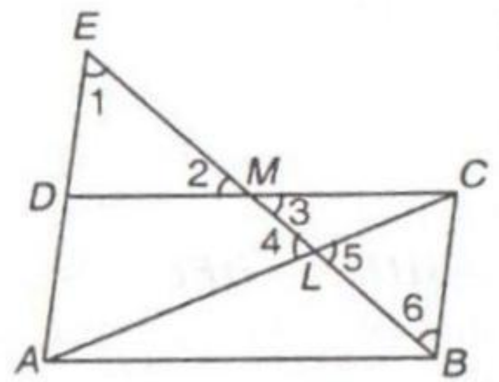
The position of Q_3 is given by $\frac{3n}{4} = \frac{3 \times 120}{4} = 90$

So, from the graph, Q_3 = 57 scores

Now, Inter-quartile range = $Q_3 - Q_1 = 57 - 30 = 27$ scores

iii. Number of shooters who obtained more than 75% scores = 120 – 110 = 10

(b) From the figure,



$\angle 1 = \angle 6$ [Alternate interior ∠'s]

$\angle 2 = \angle 3$ [Vertically opposite ∠'s]

DM = MC

∴ ΔDEM ≅ ΔBMC [AAS congruency]

⇒ DE = BC [by c.p.c.t]

Also AD = BC [opposite sides of a || gm]

⇒ AE = AD + DE = 2BC

Now, $\angle 1 = \angle 6$ and $\angle 4 = \angle 5$ ∴ ΔELA ~ ΔBLC

$$\Rightarrow \frac{EL}{BL} = \frac{AE}{BC} \Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} \Rightarrow EL = 2BL$$

Q. 9.

(a) Market value of 1 share = $Rs.\left(\frac{26 \times 110}{100}\right) = Rs.28.80$

i. Number of shares bought = $\frac{20,020}{28.60} = \frac{20,02,000}{2860} = 700$

ii. Dividend on one share = $Rs.\frac{26 \times 15}{100} = Rs.\frac{39}{10}$

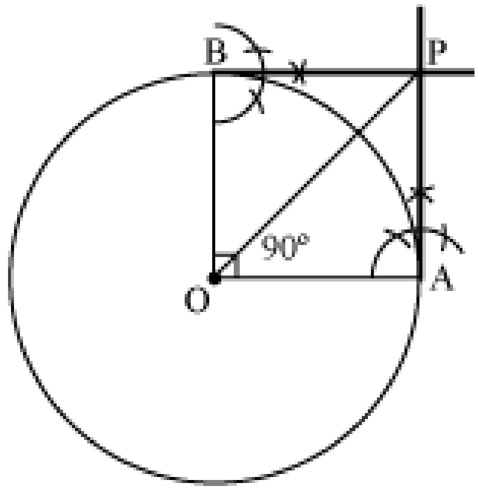
∴ Dividend on 700 shares = $700 \times \frac{39}{10}$ = Rs.2730

iii. Rate of interest = $\frac{2730}{20020} \times 100 = 13.63\%$

(b) The steps of construction are as follows:

i. Draw a circle of any convenient radius with centre O.
ii. (Take a point A on the circumference of the circle and join OA. Draw a perpendicular to OA at point A.
iii. Draw radius OB, making an angle of 90° with OA.
iv. Draw a perpendicular to OB at point B. Let both the perpendiculars intersect at point P.
v. Join OP.

PA and PB are the required tangents, which make an angle of 45° with OP.



(c)

L.H.S.

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^2 A}{(\tan A - 1)} - \frac{1}{\tan A(\tan A - 1)}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)}$$

$$= \tan A + 1 + \cot A$$

$$= 1 + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= 1 + \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} = 1 + \frac{1}{\sin A \cos A}$$

$$= 1 + \operatorname{cosec} A \sec A$$

$$= \text{R.H.S}$$

Hence Proved

Q.10.

(a)

Let the numbers be x and y.

$xy = 14^2$

$\Rightarrow xy = 196$

$\Rightarrow x = \frac{196}{y}$

And $112x = y^2$

Put $x = \frac{196}{y}$ in $112x = y^2$, we get

$\Rightarrow \frac{112 \times 196}{y} = y^2$

$\Rightarrow y = 28$

And $x = \frac{196}{28} = 7$

So, numbers are = 7, 14 and 28.

(b) Let 'a' be the first term and r be the common ratio of the G.P.

Here, r = 4.

Also,

$a \cdot ar \cdot ar^2 \cdot ar^3 = 256$

$\Rightarrow a^4r^6 = 256$

$\Rightarrow (a^2r^3)^2 = 256$

$\Rightarrow a^2r^3 = 16$

$\Rightarrow a^2(4)^3 = 16$

$\Rightarrow a^2 = \frac{16}{64}$

$\Rightarrow a^2 = \frac{1}{4}$

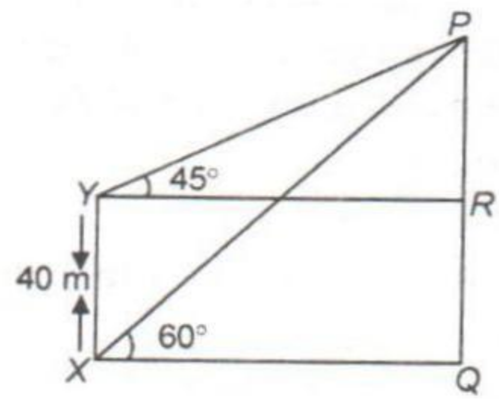
$\Rightarrow a = \frac{1}{2}$

Thus, the third term is: $ar^2 = \frac{1}{2} \times 4^2 = 8$

(c)

Let XQ = x m and PR = h m then YR = XQ = x m

And YX = RQ = 40 m



In ΔPRY, $\frac{h}{x} = \tan 45° \Rightarrow h = x$

In ΔPQX, $\frac{h+40}{x} = \tan 60° \Rightarrow h + 40 = \sqrt{3}x$

$\Rightarrow h + 40 = \sqrt{3}h$ [since h = x]

$40 = \sqrt{3}h - h$

$40 = h(\sqrt{3} - 1)$

Or $h = \frac{40}{\sqrt{3} - 1}$

$= \frac{40}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$

$= \frac{40(1.73 + 1)}{2}$

= 20 × 2.730

= 2 × 27.3

= 54.6

i. Height PQ = h + 40 = 54.6 + 40 = 94.6 m
ii. The distance XQ = x = h = 54.6 m

Q.11.

(a)

i. AM = MP [tangents from an external point]

$\Rightarrow \angle MAP = \angle MPA$

Similarly, MP = MB

$\therefore \angle MBP = \angle MPB$

Now in ΔABP,

$m\angle PAB + m\angle ABP + m\angle APB = 180°$

$m\angle MPA + m\angle MPB + m\angle MPA + m\angle MPB = 180°$

$(m\angle MPA + m\angle MPB) = 180°$

$\Rightarrow m\angle APB = 90°$

ii. AM = MP and MP = MB

$\Rightarrow$ AM = MB

$\therefore$ Tangent at P bisects AB.

(b) Total money for expenses = Rs. 360

Original duration of tour = x days

Final duration of tour = x + 4 days

Originally, 1 day expenses = Rs. $\frac{360}{x}$

Finally, 1 day expenses = Rs. $\frac{360}{x+4}$

According to the question,

$$\frac{360}{x} = \frac{360}{x+4} + 3$$

$\Rightarrow 360x + 1440 - 360x = 3x^2 + 12x$

$\Rightarrow 3x^2 + 12x - 1440 = 0$

$\Rightarrow x^2 + 4x - 480 = 0$

$\Rightarrow x^2 + 24x - 20x - 480 = 0$

$\Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow (x - 20)(x + 24) = 0$

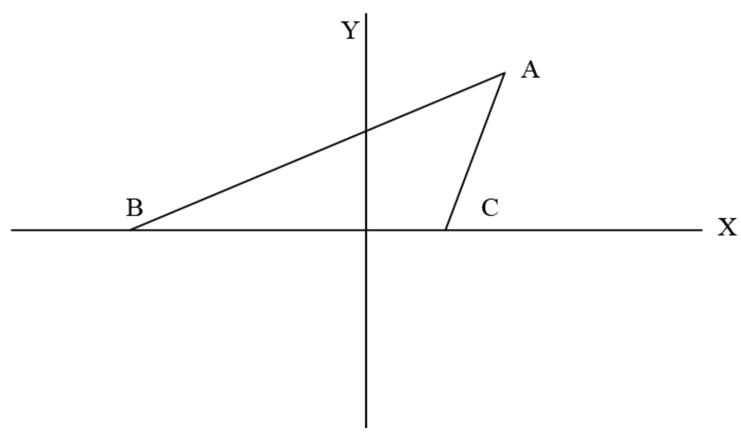
Thus, x = 20, −24

Days cannot be negative.

Hence, x = 20.

(c) Let, $\angle ACX = \theta$ and $\angle ABX = \phi$

i. Given, equation of AB, $x - \sqrt{3}y + 1 = 0$



$\therefore$ Slope of AB $= \frac{-\text{coefficient of x}}{\text{coefficient of y}}$

$= \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

So, $(m_1) = \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

$\therefore$ Angle made by the line AB = 30°

Now, equation of AC, x − y − 2 = 0

$\therefore$ Slope of AC = $= \frac{-\text{coefficient of x}}{\text{coefficient of y}} = \frac{-1}{-1} = 1$

So, $(m_2) = \tan\phi = 1$

$\Rightarrow \phi = 45°$

Angle made by the line AC = 45°

ii. m $\angle BAC$ = m $\angle C - \angle B = 45° - 30° = 15°$