# ICSE Board <br> Class IX Mathematics <br> Sample Paper 4 

Time: $\mathbf{2 1}^{1 ⁄ 2}$ hrs
Total Marks: $\mathbf{8 0}$

## General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will NOT be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt all questions from Section A and any four questions from Section B.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks
7. Mathematical tables are provided.

## SECTION - A (40 Marks) <br> (Answer all questions from this Section)

Q. 1.
(a) If $\frac{3 \sqrt{2}+2 \sqrt{3}}{5 \sqrt{2}-4 \sqrt{3}}=x-y \sqrt{6}$, find $x$.
(b) How many bricks having dimensions $20 \mathrm{~cm} \times 5 \mathrm{~cm} \times 5 \mathrm{~cm}$ are required to make a wall 2.5 m long, 0.5 m broad and 5 m in height?
(c) In two successive years interest on a certain sum at C.I. payable annually is Rs. 350 and Rs.420. Find the rate of interest.
Q. 2.
(a) If $a^{2}-3 a+1=0$, find
(i) $a^{2}+\frac{1}{a^{2}}$ (ii) $a^{3}+\frac{1}{a^{3}}$
(b) Factories: $20-45(\mathrm{~m}+\mathrm{n})^{2}$
(c) Without using tables evaluate:

$$
\frac{5 \sin 62^{\circ}}{\cos 28^{\circ}}-\frac{2 \sec 34^{\circ}}{\operatorname{cosec} 56^{\circ}}
$$

Q. 3.
(a) The perimeter of a square is $4(p+3 q)$. Find its area.
(b) In figure below, ABCD is a rhombus in which the diagonal DB is produced to E . If $\mathrm{m} \angle \mathrm{ABE}=160^{\circ}$ then find $\mathrm{x}, \mathrm{y}$ and z .

(c) If $x=2^{\frac{1}{3}}+2^{\frac{-1}{3}}$, prove that $2 x^{3}=6 x+5$

## Q. 4.

(a) Solve for $a$ and $b$ :

$$
\frac{\log (\mathrm{a}-\mathrm{b})}{\log 5}=\frac{\log 4}{\log \frac{1}{2}}=\frac{\log (\mathrm{a}+\mathrm{b})}{\log 2}
$$

(b) $A D$ is perpendicular to the side $B C$ of an equilateral $\triangle A B C$.
(c) Sum of the external angles of a regular polygon is $\frac{1}{6}$ of the sum of interior angles.

Find the number of sides.

## SECTION - B (40 Marks) <br> (Answer any four questions from this Section)

Q. 5.
(a) Solve: $3 x-7=\frac{1}{y}, x+\frac{1}{y}=1$
(b) In the fig., $\angle \mathrm{R}=\angle \mathrm{S}$ and $\angle \mathrm{RPQ}=\angle \mathrm{PQS}$. Prove that $\mathrm{PS}=\mathrm{QR}$.

(c) In an equilateral triangle with side a, prove that
(i) Altitude $=\frac{a \sqrt{3}}{2}$
(ii) Area $=\frac{\sqrt{3}}{4} a^{2}$
Q. 6.
(a) The points $A(4,-1), B(6,0), C(7,2)$ and $D(5,1)$ are the vertices of a rhombus. Is $A B C D$ also a square?
(b) Factories: $(e-y)^{3}+(y-g)^{3}+(g-e)^{3}$
(c) If $\sin \theta=\frac{5}{13}$ where $\theta<90^{\circ}$, find the value of $\tan \theta+\frac{1}{\cos \theta}$

## Q. 7.

(a) Show that the median of a triangle divides it into two triangles of equal area.

(b) In the figure of $\triangle \mathrm{PQR}, \angle \mathrm{P}=\theta^{\circ}$ and $\angle \mathrm{R}=\phi^{\circ}$ Find (i) $(\sqrt{x+1}) \cot \phi$
(ii) $\left(\sqrt{\mathrm{x}^{3}+\mathrm{x}^{2}}\right) \tan \theta$
(iii) $\cos \theta$

[3]
(c) A road, 14 m wide surrounds a circular ground whose circumference is 704 m Find the surface area of the road. Also, find the cost of paving the road at Rs. 100 per $\mathrm{m}^{2}$.

Q. 8.
(a) In the $\triangle A B C, D, E, F$ are the mid-points of $B C, C A$ and $A B$ respectively. Given $A B=5.8$ $\mathrm{cm} E F=6 \mathrm{~cm}$ and $\mathrm{DF}=5 \mathrm{~cm}$. Calculate BC and CA .

(b) Factorise: $x^{3}+y^{3}+z^{3}-3 x y z$
(c) Padma invested Rs. 30,000 in a finance company and received Rs. 39,930 after $1 \frac{1}{2}$ years. Find the rate of interest per annum compound semi-annually.

## Q. 9.

(a) In the given fig., $\mathrm{AD}=\mathrm{AB}$ and AE bisects $\angle \mathrm{A}$. Prove that: $\mathrm{BE}=\mathrm{ED}$.

(b) Find $x, \frac{x-b-c}{a}+\frac{x-c-a}{b}+\frac{x-a-b}{c}=3$, if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \neq 0$.
(c) If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal.
Q. 10.
(a) The distribution of weight (in kg ) of 40 students in a class is as given below:

| Weight (kg) | $36-40$ | $41-45$ | $46-50$ | $51-55$ | $56-60$ | $61-65$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| No.of students | 3 | 6 | 5 | 10 | 9 | 7 |

i. Draw a histogram for the distribution
ii. Draw a frequency polygon for the distribution
(b) If the numerator of a fraction is increased by 2 and the denominator by 1 , it becomes $\frac{5}{8}$ and if the numerator and denominator of the same fraction are each increased by 1 , the fraction becomes $\frac{1}{2}$. Find the fraction.

## Q. 11.

(a) Construct a rectangle ABCD in which $\mathrm{AB}=\mathrm{CD}=5.2 \mathrm{~cm}$ and $\mathrm{AC}=\mathrm{BD}=5.7 \mathrm{~cm}$ and angle B measures 90 degrees.
(b) Using a scale of $1 \mathrm{~cm}=1$ unit on both axes, draw the graphs of the following equation: $4 x-y=13,5 x+y=14$
From the graph find,
i. The co-ordinates of the point where two lines intersect
ii. The area of the triangle between the lines and the x -axis.
(c) A rope is wound round the outside of a circular drum whose diameter is 70 cm and a bucket is tied to the other end of the rope. Find the number of revolutions made by the drum, if the bucket is raised by 11 m .

## Solution

## SECTION - A (40 Marks)

Q. 1.
(a) Consider

$$
\begin{aligned}
& \frac{3 \sqrt{2}+2 \sqrt{3}}{5 \sqrt{2}-4 \sqrt{3}}=x-y \sqrt{6} \\
& \Rightarrow \frac{3 \sqrt{2}+2 \sqrt{3}}{5 \sqrt{2}-4 \sqrt{3}} \times \frac{5 \sqrt{2}+4 \sqrt{3}}{5 \sqrt{2}+4 \sqrt{3}}=x-y \sqrt{6} \\
& \Rightarrow \frac{30+12 \sqrt{6}+10 \sqrt{6}+24}{50-48}=x-y \sqrt{6} \\
& \Rightarrow \frac{54+22 \sqrt{6}}{2}=\frac{2(27+11 \sqrt{6})}{2}=x-y \sqrt{6} \\
& \Rightarrow 27+11 \sqrt{6}=x-y \sqrt{6} \\
& \Rightarrow x=27, y=-11
\end{aligned}
$$

(b) Dimensions of the brick:

Length (l) $=20 \mathrm{~cm}$, breadth (b) $=5 \mathrm{~cm}$ and height (h) $=5 \mathrm{~cm}$
Volume of brick (V) $=\mathrm{lbh}=500 \mathrm{~cm}^{3}$
Dimensions of the wall:
Length ( l ) $=2.5 \mathrm{~m}=250 \mathrm{~cm}$, breadth (b) $=0.5 \mathrm{~m}=50 \mathrm{~cm}$ and
height ( h ) $=5 \mathrm{~m}=500 \mathrm{~cm}$
Volume of the wall $\left(\mathrm{V}_{1}\right)=\mathrm{LBH}=6250000 \mathrm{~cm}^{3}$
Let N be the number of bricks required to make the wall.
Then, $\mathrm{N} \times \mathrm{V}=\mathrm{V}_{1}$
$\Rightarrow \mathrm{N}=\frac{6250000}{500}=12500$
Thus, 12500 bricks are required to make the wall.
(c) Let $\mathrm{P}=$ Rs. x , rate $=\mathrm{r} \%$

Then, $\frac{\mathrm{x} \times \mathrm{r} \times 1}{100}=350 \Rightarrow \mathrm{rx}=35000$
Now, principal for second year $=$ Rs. $(\mathrm{x}+350)$
$\Rightarrow r x+350 r=42000$
$\Rightarrow 35000+350 \mathrm{r}=42000$
[From (1)]
$\Rightarrow 350 \mathrm{r}=42000-35000$
$\Rightarrow 350 \mathrm{r}=7000$
$\Rightarrow \mathrm{r}=\frac{7000}{350}=20 \%$

## Q. 2.

(a) $a^{2}-3 a+1=0$

On dividing equation (1) by a, we get, $a-3+\frac{1}{a}=0$
(i) $a+\frac{1}{a}=3$

On squaring equation (2), we have

$$
\begin{aligned}
& \left(a+\frac{1}{a}\right)^{2}=3^{2} \\
& a^{2}+\frac{1}{a^{2}}+2 \times a \times \frac{1}{a}=9 \\
& a^{2}+\frac{1}{a^{2}}+2=9 \\
& a^{2}+\frac{1}{a^{2}}=9-2=7
\end{aligned}
$$

(ii) Cubing equation (2), we have

$$
\begin{aligned}
& \left(a+\frac{1}{a}\right)^{3}=3^{3} \\
& a^{3}+\frac{1}{a^{3}}+3 \times a \times \frac{1}{a}\left(a+\frac{1}{a}\right)=27 \\
& a^{3}+\frac{1}{a^{3}}+3(3)=27 \\
& a^{3}+\frac{1}{a^{3}}=27-9=18
\end{aligned}
$$

(b)

$$
\begin{aligned}
& 20-45(\mathrm{~m}+\mathrm{n})^{2}=5\left[4-9(\mathrm{~m}+\mathrm{n})^{2}\right] \\
& =5\left[(2)^{2}-\{3(\mathrm{~m}+\mathrm{n})\}^{2}\right] \\
& =5[2+3(\mathrm{~m}+\mathrm{n})][2-3(\mathrm{~m}+\mathrm{n})] \\
& =5(2+3 \mathrm{~m}+3 \mathrm{n})(2-3 \mathrm{~m}-3 \mathrm{n})
\end{aligned}
$$

(c) Here
$\frac{5 \sin 62^{\circ}}{\cos 28^{\circ}}=\frac{5 \sin \left(90^{\circ}-28^{\circ}\right)}{\cos 28^{\circ}}=\frac{5 \cos 28^{\circ}}{\cos 28^{\circ}}=5$
And $\frac{2 \sec 34^{\circ}}{\operatorname{cosec} 56^{\circ}}=\frac{2 \sec \left(90^{\circ}-56^{\circ}\right)}{\operatorname{cosec} 56^{\circ}}=\frac{2 \operatorname{cosec} 56^{\circ}}{\operatorname{cosec} 56^{\circ}}=2$
$\therefore \frac{5 \sin 62^{\circ}}{\cos 28^{\circ}}-\frac{2 \sec 34^{\circ}}{\operatorname{cosec} 56^{\circ}}=5-2=3$
Q. 3
(a) Let the side of square be xcm
$\therefore$ Its perimeter is 4 x cm
Given, $4 x=4(p+3 q)$
$\Rightarrow x=\frac{4(p+3 q)}{4}=(p+3 q) \mathrm{cm}$
$\therefore$ Area $=(\text { side })^{2}=(p+3 q)^{2}$
$=\left(p^{2}+9 q^{2}+6 p q\right) \mathrm{cm}^{2}$
(b) $\mathrm{m} \angle \mathrm{AOB}=\mathrm{z}=90^{\circ}$ [Diagonals of a rhombus bisect each other at right angle]
$\mathrm{m} \angle \mathrm{ABO}=180-160=20^{\circ}$

In $\triangle \mathrm{AOB}, \mathrm{m} \angle \mathrm{BAO}+\mathrm{m} \angle \mathrm{AOB}+\mathrm{m} \angle \mathrm{ABO}=180^{\circ}$
$\mathrm{m} \angle \mathrm{BAO}+90^{\circ}+20^{\circ}=180^{\circ}$
$\mathrm{m} \angle \mathrm{BAO}=70^{\circ}$


Also, $\mathrm{x}=\mathrm{m} \angle \mathrm{BAO}=70^{\circ}$ [Alternate interior angles are equal as $\mathrm{AB} \| \mathrm{DC}$ ]
In $\triangle \mathrm{ADB}, \mathrm{AD}=\mathrm{AB}$
$\angle \mathrm{ABD}=\angle \mathrm{ADB}$ (Angles opposites to equal sides are equal)
Therefore, $\mathrm{y}=20^{\circ}$
(c) $\mathrm{x}=2^{\frac{1}{3}}+2^{\frac{-1}{3}}$

On cubing both sides

$$
x^{3}=\left(2^{\frac{1}{3}}\right)^{3}+\left(2^{\frac{-1}{3}}\right)^{3}+3 \times 2^{\frac{1}{3}} \times 2^{\frac{-1}{3}}\left(2^{\frac{1}{3}}+2^{\frac{-1}{3}}\right)
$$

[By using $(a+b)^{3}=a^{3}+b^{3}+3 a b(a+b)$ ]
$\Rightarrow \mathrm{x}^{3}=2+2^{-1}+3 \times 2^{\circ} \times \mathrm{x}$
$\mathrm{x}^{3}=2+\frac{1}{2}+3 \mathrm{x}$
$\Rightarrow 2 x^{3}=5+6 x$ or $2 x^{3}=6 x+5$
Q. 4.
(a) We have $\frac{\log 4}{\log \frac{1}{2}}=\frac{\log 2^{2}}{\log 2^{-1}}=\frac{2 \log 2}{-1 \log 2}=-2$

$$
\text { Now } \frac{\log (a-b)}{\log 5}=-2
$$

$$
\Rightarrow \log (\mathrm{a}-\mathrm{b})=-2 \log 5
$$

$$
\Rightarrow \log (\mathrm{a}-\mathrm{b})=\log 5^{-2}
$$

$$
\Rightarrow \mathrm{a}-\mathrm{b}=\frac{1}{5^{2}}
$$

$$
\begin{equation*}
\Rightarrow a-b=\frac{1}{25} \tag{1}
\end{equation*}
$$

Again, $\frac{\log (a+b)}{\log 2}=-2$
$\Rightarrow \log (\mathrm{a}+\mathrm{b})=-2 \log 2$
$\Rightarrow \log (\mathrm{a}+\mathrm{b})=\log 2^{-2}$
$\Rightarrow \mathrm{a}+\mathrm{b}=\frac{1}{4}$
From (1) and (2), we get
$\mathrm{a}=\frac{29}{200}, \mathrm{~b}=\frac{21}{200}$
(b) Given: In the equilateral $\triangle A B C, A D$ is perpendicular to $B C$.

To prove: $4 A D^{2}=3 A B^{2}$
Proof: $\mathrm{AD} \perp \mathrm{BC} \quad$ [Given]
$B D=D C$
[In an equilateral triangle $\perp$ from the vertex bisects the base] In right $\triangle A D B, A D^{2}+B D^{2}=A B^{2} \quad$ [Pythagoras theorem]
$\Rightarrow \mathrm{AD}^{2}+\left[\frac{1}{2} \mathrm{BC}\right]^{2}=\mathrm{AB}^{2} \quad\left[\because \mathrm{BD}=\frac{1}{2} \mathrm{BC}\right]$

$\Rightarrow \mathrm{AD}^{2}+\frac{\mathrm{BC}^{2}}{4}=\mathrm{AB}^{2}$
$\Rightarrow \mathrm{AD}^{2}+\frac{\mathrm{AB}^{2}}{4}=\mathrm{AB}^{2}[\because \mathrm{AB}=\mathrm{BC}]$
$\Rightarrow 4 \mathrm{AD}^{2}+\mathrm{AB}^{2}=4 \mathrm{AB}^{2}$
$\Rightarrow 4 \mathrm{AD}^{2}=3 \mathrm{AB}^{2}$
(c)

Sum of exterior angles $=\frac{1}{6}^{\text {th }}$ of the sum of interior angles
$360=\frac{1}{6} \times(2 n-4) \times 90$
$\Rightarrow 4 \times 6=2 n-4$
$\Rightarrow 24=2 \mathrm{n}-4$
$\Rightarrow 2 \mathrm{n}=28 \Rightarrow \mathrm{n}=14$

## SECTION - B

Q. 5.
(a)

$$
\begin{align*}
& 3 x-7=\frac{1}{y}  \tag{1}\\
& x+\frac{1}{y}=1 \tag{2}
\end{align*}
$$

Substituting $\frac{1}{y}=a$
$3 x-a=7 \ldots .(3)$
$x+a=1$
Applying 1(3)-3(4), we get

$$
-4 a=4
$$

$$
\Rightarrow \mathrm{a}=\frac{-4}{4}=-1
$$

$$
\therefore \frac{1}{y}=-1
$$

$$
\Rightarrow \mathrm{y}=-1
$$

Substituting value of $y$ in equation (2) we get
$\mathrm{x}=2$
Hence, $\mathrm{x}=2$ and $\mathrm{y}=-1$.
(b) Given: $\angle \mathrm{R}=\angle \mathrm{S}$ and $\angle \mathrm{RPQ}=\angle \mathrm{PQS}$

To prove: $\mathrm{PS}=\mathrm{QR}$
Proof: In $\triangle P Q S$ and $\triangle P Q R$, we have

$$
\begin{aligned}
& \mathrm{PQ}=\mathrm{PQ} \quad \text { [Common] } \\
& \angle \mathrm{PSQ}=\angle \mathrm{PRQ} \text { [Given] } \\
& \angle \mathrm{RPQ}=\angle \mathrm{PQS} \text { [Given] }
\end{aligned}
$$

Hence, $\triangle \mathrm{PQS} \cong \triangle \mathrm{PQR} \quad$ [AAS]
$\therefore \mathrm{PS}=\mathrm{PR}$ [CPCT]

(c) Let ABC be an equilateral triangle whose sides measure ' a ' units each. Draw $\mathrm{AD} \perp \mathrm{BC}$. Then, D is the mid-point of BC .

$$
\Rightarrow \quad \mathrm{AB}=\mathrm{a}, \mathrm{BD}=\frac{1}{2} \mathrm{BC}=\frac{\mathrm{a}}{2}
$$

Since $\triangle \mathrm{ABD}$ is a right triangle right - angled at D .

$$
\begin{aligned}
& \therefore \mathrm{AB}^{2}=\mathrm{AD}^{2}+\mathrm{BD}^{2} \\
& \Rightarrow \mathrm{a}^{2}=\mathrm{AD}^{2}+\left(\frac{\mathrm{a}}{2}\right)^{2} \\
& \Rightarrow \mathrm{AD}^{2}=\mathrm{a}^{2}-\frac{\mathrm{a}^{2}}{4}=\frac{3 \mathrm{a}^{2}}{4} \\
& \Rightarrow \mathrm{AD}=\frac{\sqrt{3} \mathrm{a}}{2} \\
& \therefore \quad \text { Altitude }=\frac{\sqrt{3}}{2} \mathrm{a}
\end{aligned}
$$



Now,
Area of $\Delta \mathrm{ABC}=\frac{1}{2}($ Base $\times$ Height $)$
Area of $\triangle A B C=\frac{1}{2}(B C \times A D)$
$\Rightarrow$ Area of $\triangle \mathrm{ABC}=\frac{1}{2} \times \mathrm{a} \times \frac{\sqrt{3}}{2} \mathrm{a}=\frac{\sqrt{3}}{4} \mathrm{a}^{2}$

## Q. 6.

(a) The given points are $\mathrm{A}(4,-1), \mathrm{B}(6,0), \mathrm{C}(7,2)$ and $\mathrm{D}(5,1)$.

Using distance formula,
Diagonal AC $=\sqrt{(4-7)^{2}+(-1-2)^{2}}=\sqrt{9+9}=\sqrt{18}$
Diagonal $\mathrm{BD}=\sqrt{(6-5)^{2}+(0-1)^{2}}=\sqrt{1+1}=\sqrt{2}$
Since, $A C \neq B D, A B C D$ is not a square.
(b)
$(e-y)^{3}+(y-g)^{3}+(g-e)^{3}$
Here, $e-y+y-g+g-e=0$
Here, by the result, if $a+b+c=0$, then, $a^{3}+b^{3}+c^{3}=3 a b c$

$$
(e-y)^{3}+(y-g)^{3}+(g-e)^{3}=3 \times(e-y)(y-g)(g-e)
$$

(c)

$$
\begin{aligned}
& \left.(\text { Per. })^{2}+(\text { Base })^{2}=(\text { Hyp. })^{2} \quad \text { [By Pythagoras theorem }\right] \\
& \Rightarrow 5^{2}+(\text { Base })^{2}=13^{2} \\
& \Rightarrow(\text { Base })^{2}=13^{2}-5^{2} \\
& \Rightarrow \text { Base }=\sqrt{169-25}=\sqrt{144}=12 \\
& \therefore \tan \theta=\frac{5}{12}, \frac{1}{\cos \theta}=\sec \theta=\frac{13}{12} \\
& \Rightarrow \tan \theta+\frac{1}{\cos \theta}=\frac{5}{12}+\frac{13}{12}=\frac{5+13}{12}=\frac{18}{12}=\frac{3}{2}
\end{aligned}
$$

## Q. 7.

(a) Given: $\triangle \mathrm{ABC}$ in which AD is the median.

To prove: $\operatorname{ar}(\triangle A B C)=\operatorname{ar}(\triangle A D C)$
Construction: Draw AL $\perp \mathrm{BC}$.
Proof: Since D is the mid-point of $B C$, we have $B D=D C$
$\Rightarrow \frac{1}{2} \mathrm{BD} \times \mathrm{AL}=\frac{1}{2} \mathrm{DC} \times \mathrm{AL}$
$\Rightarrow \operatorname{ar}(\triangle \mathrm{ABD})=\operatorname{ar}(\triangle \mathrm{ADC})$
Hence, a median of a triangle divides in into two triangles of equal area.
(b) Let the width of concrete wall be $\mathrm{x} m$

In $\triangle \mathrm{PQR}, \angle \mathrm{Q}=90^{\circ}, \angle \mathrm{P}=\theta^{\circ}$ and $\angle \mathrm{R}=\phi^{\circ}$
By Pythagoras theorem, we have

$$
\begin{aligned}
& \mathrm{PQ}^{2}=\mathrm{PR}^{2}-\mathrm{QR}^{2} \\
& \Rightarrow \mathrm{PQ}^{2}=(\mathrm{x}+2)^{2}-\mathrm{x}^{2} \\
& \Rightarrow \mathrm{PQ}^{2}=\mathrm{x}^{2}+4 \mathrm{x}+4-\mathrm{x}^{2} \\
& \Rightarrow \mathrm{PQ}^{2}=4(\mathrm{x}+1) \\
& \Rightarrow \mathrm{PQ}=2 \sqrt{\mathrm{x}+1}
\end{aligned}
$$

Now, $\cot \phi=\frac{\mathrm{QR}}{\mathrm{PQ}}=\frac{\mathrm{x}}{2 \sqrt{\mathrm{x}+1}}$ and $\tan \theta=\frac{\mathrm{QR}}{\mathrm{PQ}}=\frac{\mathrm{x}}{2 \sqrt{\mathrm{x}+1}}$
(i) $(\sqrt{x+1}) \cot \phi=(\sqrt{x+1}) \times \frac{x}{2 \sqrt{x+1}}=\frac{x}{2}$
(ii) $\left(\sqrt{\mathrm{x}^{3}+\mathrm{x}^{2}}\right) \tan \theta=\left(\sqrt{\mathrm{x}^{2}(\mathrm{x}+1)}\right) \tan \theta=\mathrm{x}(\sqrt{\mathrm{x}+1}) \times \frac{\mathrm{x}}{2 \sqrt{\mathrm{x}+1}}=\frac{\mathrm{x}^{2}}{2}$
(iii) $\cos \theta=\frac{P Q}{P R}=\frac{2 \sqrt{x+1}}{x+2}$
(c) Given, circumference $=704 \mathrm{~m}$
$\therefore 2 \pi r=704$
$\Rightarrow 2 \times \frac{22}{7} \times r=704$
$\Rightarrow \mathrm{r}=\frac{7 \times 704}{2 \times 22}=112 \mathrm{~m}$
andR $=112+14=126 \mathrm{~m}$
$\therefore$ Surface area of road $=\pi R^{2}-\pi r^{2}$

$$
=\pi(\mathrm{R}+\mathrm{r})(\mathrm{R}-\mathrm{r})
$$

$$
=\frac{22}{7}(126+112)(126-112)
$$

$$
=\frac{22}{7} \times 238 \times 14=10472 \mathrm{~m}^{2}
$$

Given Rate of paving $=$ Rs. 100 per $\mathrm{m}^{2}$
$\therefore$ Total cost $=10472 \times 100=$ Rs. 1047200
Q. 8.
(a) Given: $\mathrm{A} \triangle \mathrm{ABC}$ and $\mathrm{D}, \mathrm{E}, \mathrm{F}$ are the mid-points of $\mathrm{BC}, \mathrm{CA}$ and DF $\mathrm{AB}=5.8 \mathrm{~cm}, \mathrm{EF}=6 \mathrm{~cm}$ and $\mathrm{DF}=5 \mathrm{~cm}$
To find: BC and CA
$\mathrm{EF} \| \mathrm{BC}$ [As E and F are mid-points of AC and AB ]
Also, $\mathrm{EF}=\frac{1}{2} \mathrm{BC}$
$\mathrm{BC}=2 \times \mathrm{EF}=2 \times 6=12 \mathrm{~cm}$
Thus, $\mathrm{BC}=12 \mathrm{~cm}$
DF || AC [As D and F are mid-points of AB and BC respectively]
And $\mathrm{DF}=\frac{1}{2} \mathrm{AC} \Rightarrow 5=\frac{1}{2} \mathrm{AC} \Rightarrow \mathrm{AC}=10 \mathrm{~cm}$
(b)

$$
\begin{aligned}
& x^{3}+y^{3}+z^{3}-3 x y z \\
& =(x+y)^{3}-3 x y(x+y)+z^{3}-3 x y z \\
& =(x+y)^{3}+z^{3}-3 x y(x+y)-3 x y z \\
& =(x+y+z)\left[(x+y)^{2}-(x+y) z+z^{2}\right]-3 x y(x+y+z) \\
& =(x+y+z)\left(x^{2}+2 x y+y^{2}-x z-y z+z^{2}-3 x y\right) \\
& =(x+y+z)\left(x^{2}+y^{2}+z^{2}-y z-x z-x y\right)
\end{aligned}
$$

(c) $\mathrm{P}=$ Rs. $30,000, \mathrm{~A}=$ Rs. $39,930, \mathrm{~T}=3$ half years $\Rightarrow \mathrm{n}=3$

$$
\begin{aligned}
& A=P\left(1+\frac{r}{100}\right)^{n} \\
& 39,930=30,000\left(1+\frac{r}{100}\right)^{3} \\
& \Rightarrow \frac{39930}{30000}=\left(1+\frac{r}{100}\right)^{3} \\
& \Rightarrow \frac{1331}{1000}=\left(1+\frac{r}{100}\right)^{3} \\
& \Rightarrow\left(\frac{11}{10}\right)^{3}=\left(1+\frac{r}{100}\right)^{3} \\
& \Rightarrow \frac{r}{100}=\frac{11}{10}-1=\frac{1}{10} \\
& \Rightarrow \quad r=10 \%
\end{aligned}
$$

So, rate of interest per annum $=20 \%$

## Q. 9.

(a) Given: $\mathrm{AD}=\mathrm{AB}, \mathrm{AE}$ bisects $\angle \mathrm{A}$

Construction: Join DE
To prove: $\mathrm{BE}=\mathrm{ED}$
Proof: In $\triangle A B E$ and $\triangle A D E$
$\mathrm{AE}=\mathrm{AE}$ [Common]
$\mathrm{AD}=\mathrm{AB}$ [Given]
And $\angle \mathrm{BAE}=\angle \mathrm{DAE} \quad$ [AE bisects $\angle \mathrm{A}$ ]
$\Rightarrow \triangle \mathrm{ABE} \cong \triangle \mathrm{ADE} \quad$ [S.A.S. Congruency]
So, $\mathrm{BE}=\mathrm{ED}$ [CPCT]

(b)

Given: $\frac{x-b-c}{a}+\frac{x-c-a}{b}+\frac{x-a-b}{c}=3$
$\Rightarrow \frac{\mathrm{x}-\mathrm{b}-\mathrm{c}}{\mathrm{a}}-1+\frac{\mathrm{x}-\mathrm{c}-\mathrm{a}}{\mathrm{b}}-1+\frac{\mathrm{x}-\mathrm{a}-\mathrm{b}}{\mathrm{c}}-1=3-3$
$\Rightarrow \frac{\mathrm{x}-\mathrm{a}-\mathrm{b}-\mathrm{c}}{\mathrm{a}}+\frac{\mathrm{x}-\mathrm{c}-\mathrm{a}-\mathrm{b}}{\mathrm{b}}+\frac{\mathrm{x}-\mathrm{a}-\mathrm{b}-\mathrm{c}}{\mathrm{c}}=0$
$\Rightarrow(\mathrm{x}-\mathrm{a}-\mathrm{b}-\mathrm{c})\left(\frac{1}{\mathrm{a}}+\frac{1}{\mathrm{~b}}+\frac{1}{\mathrm{c}}\right)=0$
$\Rightarrow \mathrm{x}-\mathrm{a}-\mathrm{b}-\mathrm{c}=0 \quad\left(\right.$ as $\left.\frac{1}{\mathrm{a}}+\frac{1}{\mathrm{~b}}+\frac{1}{\mathrm{c}} \neq 0\right)$
$\Rightarrow \mathrm{x}=\mathrm{a}+\mathrm{b}+\mathrm{c}$
(c) Given that AB and CD are two chords of a circle with centre 0 , intersecting at a point $E . P Q$ is the diameter through E , such that $\angle \mathrm{AEQ}=\angle \mathrm{DEQ}$.
To prove that $\mathrm{AB}=\mathrm{CD}$.
Draw perpendiculars OL and OM on chords AB and CD respectively.
Now, $\mathrm{m} \angle \mathrm{LOE}=180^{\circ}-90^{\circ}-\mathrm{m} \angle \mathrm{LEO}$... [Angle sum property of a triangle]
$=90^{\circ}-\mathrm{m} \angle \mathrm{LEO}$
$\Rightarrow \mathrm{m} \angle \mathrm{LOE}=90^{\circ}-\mathrm{m} \angle \mathrm{AEQ}$
$\Rightarrow \mathrm{m} \angle \mathrm{LOE}=90^{\circ}-\mathrm{m} \angle \mathrm{DEQ}$
$\Rightarrow \mathrm{m} \angle \mathrm{LOE}=90^{\circ}-\mathrm{m} \angle \mathrm{MEQ}$
$\Rightarrow \angle \mathrm{LOE}=\angle \mathrm{MOE}$
In $\triangle$ OLE and $\triangle O M E$,
$\angle \mathrm{LEO}=\angle \mathrm{MEO}$
$\angle \mathrm{LOE}=\angle \mathrm{MOE}$
$\mathrm{EO}=\mathrm{EO}$
$\Delta \mathrm{OLE} \cong \triangle \mathrm{OME}$

$\mathrm{OL}=\mathrm{OM}$
Therefore, cords AB and CD are equidistant from the centre.
Hence AB = CD

## Q. 10.

(a) Adjustment factor $=\frac{41-40}{2}=0.5$

| C.I | C.I after <br> Adjustment | Frequency |
| :---: | :---: | :---: |
| $36-40$ | $35.5-40.5$ | 3 |
| $41-45$ | $40.5-45.5$ | 6 |
| $46-50$ | $45.5-50.5$ | 5 |
| $51-55$ | $50.5-55.5$ | 10 |
| $56-60$ | $55.5-60.5$ | 9 |
| $61-65$ | $60.5-65.5$ | 7 |


(b) Let the numerator be $x$ and denominator be $y$

Then, the required fraction is $\frac{x}{y}$
According to the given conditions

$$
\begin{align*}
& \frac{x+2}{y+1}=\frac{5}{8} \\
& \Rightarrow 8 x+16=5 y+5 \\
& \Rightarrow 8 x-5 y=-11  \tag{1}\\
& \text { And } \frac{x+1}{y+1}=\frac{1}{2} \\
& \Rightarrow 2 x+2=y+1 \\
& \quad \Rightarrow 2 x-y=-1 \tag{2}
\end{align*}
$$

On Solving (1) and (2), we get
$y=7$ and $x=3$
Hence, the required fraction is $\frac{3}{7}$

## Q. 11.

(a)
(i) Draw $\mathrm{AB}=5.2 \mathrm{~cm}$
(ii) At B construct $\mathrm{m} \angle \mathrm{ABP}=90^{\circ}$
(iii) With A as the centre and radius 5.7 cm , draw an arc to cut BP at C.
(iv) With C as centre and radius equal to 5.2 cm draw an arc.
(v) With B as centre and radius equal to 5.7 cm , cut the previous arc at D
(vi) Join AD and DC

(b) $4 x-y=13 \Rightarrow 4 x=13+y \Rightarrow x=\frac{13+y}{4}$

Taking convenient values of $y$, we get

| $x$ | 3 | 4 | 2 |
| :--- | :--- | :--- | :--- |
| $y$ | -1 | 3 | -5 |

And $5 x+y=14 \Rightarrow 5 x=14-y \Rightarrow x=\frac{14-y}{5}$
Taking convenient values of $y$, we get

| $x$ | 3 | 2 | 4 |
| :--- | :--- | :--- | :--- |
| $y$ | -1 | 4 | -6 |

Now plot these points on the graph paper,

i. From graph, the coordinates of the point of intersection of two lines are $(3,-1)$.
ii. In $\triangle A B C, B C=0.6 \mathrm{~cm}, A D=1 \mathrm{~cm}$
$\therefore$ Area $(\triangle \mathrm{ABC})=\frac{1}{2} \times \mathrm{BC} \times \mathrm{AD}=\frac{1}{2} \times 0.6 \times 1=0.3 \mathrm{~cm}^{2}$
(c) Radius of the drum $=\frac{70}{2}=35 \mathrm{~cm}$
$\therefore$ No. of revolution $=\frac{\text { Distance by which the bucket is raised }}{\text { Circumference of the drum }}$

$$
=\frac{11 \times 100}{2 \pi \times 35}=\frac{11 \times 100 \times 7}{2 \times 35 \times 22}=5
$$

No. of revolutions $=5$

