ICSE Board
Class IX Mathematics
Sample Paper 3

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1.

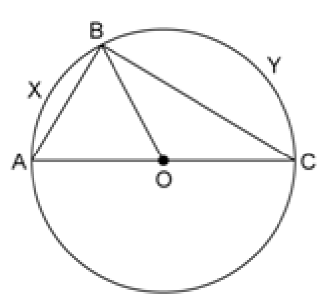
(a) Without using tables, find the value of $\dfrac{\sin 30° - \sin 90° + 2\cos 0°}{\tan 30° \times \tan 60°}$ [3]

(b) Evaluate: $\dfrac{3 \times 27^{n+1} + 9 \times 3^{n-1}}{8 \times 3^{3n} - 5 \times 27^{n}}$ [3]

(c) If x and y are rational numbers and $\dfrac{2+\sqrt{3}}{2-\sqrt{3}} = x + y\sqrt{3}$, find the value of x and y. [4]

Q. 2.

(a) In the given figure, AOC is a diameter of a circle with centre O and arc AXB = $\dfrac{1}{2}$ arc BYC. Find ∠BOC. [3]

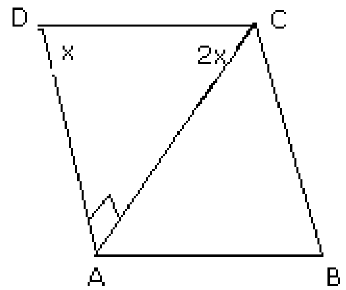


(b) Find xy, if x + y = 6 and x – y = 4. [3]

(c) If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$, prove that $a^a.b^b.c^c = 1$ [4]

Q. 3.

(a) From the given figure, find the angles of the parallelogram ABCD. [3]



(b) Express $5.3\overline{47}$ in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0. [3]

(c) The table below classifies the days of the months of June, July and August according to the rainfall received in a locality. [3]

Rain (mm)	**Days**
10 – 20	8
20 – 30	10
30 – 40	14
40 – 50	20
50 – 60	15
60 – 70	8
70 – 80	7
80 – 90	6
90 – 100	4

Q. 4.

(a) There are two regular polygons with number of sides equal to (n − 1) and (n + 2). Their external angles differ by 6°. Find the value of n. [3]

(b) ABCD is a parallelogram, E is the midpoint of AB and F is the mid-point of CD. PQ is any line that intersects AD, EF and BC at P, G and Q. Prove that PG = GQ. [3]

(c) A man borrows Rs. 5000 at 12% p.a. compound interest. He repays Rs. 2000 at the end of each year. Calculate the amount he has to pay at the end of the third year. [4]

SECTION – B (40 Marks)

(Answer ***any four questions*** *from this Section)*

Q. 5.

(a) A wire is bent to form a square enclosing an area of 484 m2. Using the same wire, a circle is formed. Find the area of the circle. [3]

(b) Given, $\sin\theta = \frac{p}{q}$, find $\cos\theta + \sin\theta$ in terms of p and q. [3]

(c) If the points (a, 0), (0, b) and (1, 1) are collinear, then prove that [4]

$$\frac{1}{a}+\frac{1}{b}=1$$

Q. 6.

(a) Factorise: $\left(x^2+y^2-z^2\right)^2-4x^2y^2$ [3]

(b) Prove that if the diagonals of a parallelogram cut at right angles, it is a rhombus. [4]

(c) If $3a = p\left(\frac{x}{2}-y\right)$, make 'y' the subject. Find y, when x = 4, p = 5. [3]

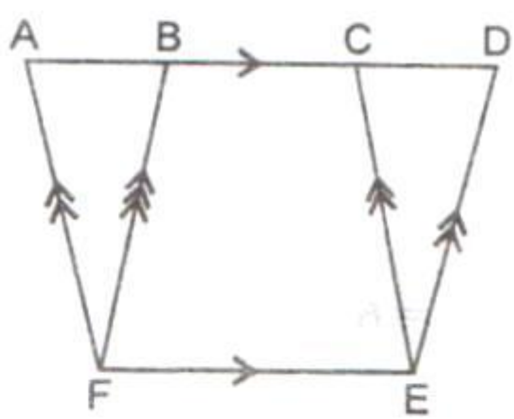
Q. 7.

(a) Draw the graph of the equations 2x – 3y = 7 and x + 6y =11, taking 1 cm = 1 unit on both axes and find their solutions. [6]

(b) In the given figure, area of parallelogram AFEC is $140\ cm^2$. Find the area of

i. Parallelogram BFED

ii. ΔBFD [4]



Q. 8.

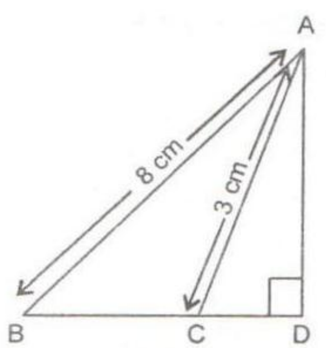
(a) Show that in any quadrilateral the sum of all the four sides exceeds the sum of the diagonals. [4]

(b) A and B start at the same time from two places 30 km apart. If they walk in the same directions, A overtakes B in 10 hours and if they walk in opposite directions they meet in 2hours. Find the rates of walking of A and B. [6]

Q. 9.

(a) The mean height of the 10 girls in a class is 1.38 m and the mean height of the 40 boys is 1.44 m. Find the mean height of the 50 students of the class. [3]

(b) In the given fig., m∠D = 90°, AB = 8 cm, BC = 6 cm and CA = 3 cm. Find CD. [4]



(c) A rectangular water-tank measuring 80 cm × 60 cm × 60 cm is filled form a pipe of cross-sectional area 1.5 cm^2, the water emerging at 3.2 m/s. How long does it take to fill the tank? [3]

Q. 10.

(a) Find the value of

$$\frac{\sec(90^\circ-\theta).\operatorname{cosec}\theta-\tan(90^\circ-\theta)\cot\theta+\cos^2 25^\circ+\cos^2 65^\circ}{3\tan 27^\circ \tan 63^\circ}$$ [3]

(b) Construct a rhombus ABCD in which AB = 4.5 cm and m∠A = 60°. [3]

(c) Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and on opposite sides of the centre. If the distance between AB and CD is 6 cm, find the radius of the circle. [4]

Q. 11.

(a) If $a + \frac{1}{a} = p$, show that $a^3 + \frac{1}{a^3} = p(p^2 - 3)$. [3]

(b) Solve: $\frac{5}{x+y} + \frac{3}{x-y} = 4, \frac{2}{x+y} + \frac{5}{x-y} = 5\frac{2}{5}$ [4]

(c) Find the mean of the following data: [3]

x	25	35	45	55	65	75
f	10	6	8	12	5	9

Solution

SECTION – A (40 Marks)

Q. 1.

(a) $$\frac{\sin 30^\circ - \sin 90^\circ \ + 2\cos 0^\circ}{\tan 30^\circ \times \tan 60^\circ} = \frac{\frac{1}{2} - 1 + 2}{\frac{1}{\sqrt{3}} \times \sqrt{3}} = \frac{1}{2} + 1 = \frac{3}{2} = 1\frac{1}{2}$$

(b) $$\frac{3\times 27^{n+1} + 9\times 3^{n-1}}{8\times 3^{3n} - 5\times 27^{n}} = \frac{3\times 3^{3n+3} + 9\times 3^{3n-1}}{8\times 3^{3n} - 5\times 3^{3n}}$$

$$= \frac{3^{3n}\left(3\times 3^{3} + 9\times 3^{-1}\right)}{3^{3n}\left(8-5\right)}$$

$$= \frac{3\times 27 + 9\times \frac{1}{3}}{3}$$

$$= \frac{81+3}{3}$$

$$= \frac{84}{3}$$

$$= 28$$

(c) Given, $\frac{2+\sqrt{3}}{2-\sqrt{3}} = x + y\sqrt{3}$

Rationalize the denominator

$$\frac{\left(2+\sqrt{3}\right)\left(2+\sqrt{3}\right)}{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)} = x + y\sqrt{3}$$

$$\Rightarrow \frac{4+3+4\sqrt{3}}{4-3} = x + y\sqrt{3}$$

$$\Rightarrow 7 + 4\sqrt{3} = x + y\sqrt{3}$$

Comparing the real and irrational parts on both sides, we get

x = 7, y = 4

Q. 2.

(a)

Given that,

arc AXB = $\frac{1}{2}$ arc BYC

$\Rightarrow \angle AOB = \frac{1}{2}\angle BOC$

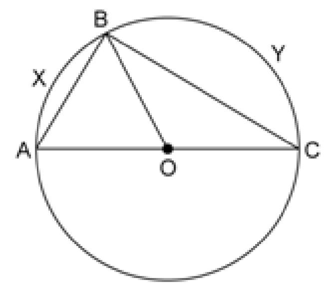
Since AOC is a straight line,

$\angle AOB + \angle BOC = 180^\circ$

$\therefore \frac{1}{2}\angle BOC + \angle BOC = 180^\circ$

$\therefore \frac{3}{2}\angle BOC = 180^\circ$

$\therefore \angle BOC = 180^\circ \times \frac{2}{3} = 120^\circ$



(b) Given, x + y = 6; x – y = 4

We know that

$(x+y)^2 = (x-y)^2 + 4xy$

$(6)^2 = (4)^2 + 4xy$

$\Rightarrow 36 - 16 = 4xy$

$\Rightarrow 20 = 4xy$

$\Rightarrow xy = 5$

(c) Let, $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$

$\Rightarrow \log a = k(b-c)$, $\log b = k(c-a)$ and $\log c = k(a-b)$

Now, $A = a^a.b^b.c^c$

Taking log both sides

$\log A = a \log a + b \log b + c \log c$

$= a.k(b-c) + b.k(c-a) + c.k(a-b)$

$= k[ab - ac + bc - ab + ac - bc]$

$= k \times 0 = 0$

$\Rightarrow \log A = 0$

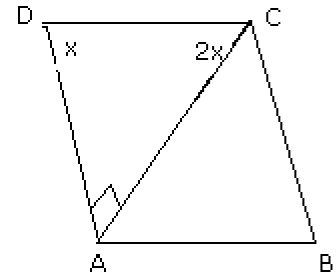
$\Rightarrow \log A = \log 1 \Rightarrow A = 1$

$\therefore a^a.b^b.c^c = 1$

Hence Proved.

Q. 3.

(a) In ΔADC,



$x + 2x + 90° = 180°$ (sum of all angles in a Δ is $180°$)

$\Rightarrow 3x = 90°$

$\Rightarrow x = 30°$

$\Rightarrow m\angle D = 30°$

$\therefore$ $m\angle B = m\angle D = 30°$ [Opposite angles of ||gm are equal]

And $m\angle A = m\angle C = 180° - 30°$

[sum of co- interior angles = $180°$ in a ||gm]

$\Rightarrow m\angle A = m\angle C = 150°$

Thus, the angles of a parallelogram are 150°, 30°, 150° and 30°.

(b) Let $x = 5.3\overline{47} = 5.34747.....$ ....(i)

Multiplying (i) by 10, we get

$10x = 53.4747....$ ....(ii)

Multiplying (ii) by 100, we get

$1000x = 5347.47.....$ ....(iii)

Subtracting (ii) from (iii), we get

$1000x - 10x = 5347.47.... - 53.47...$

$\Rightarrow 990x = 5294$

$$\Rightarrow x = \frac{5294}{990}$$

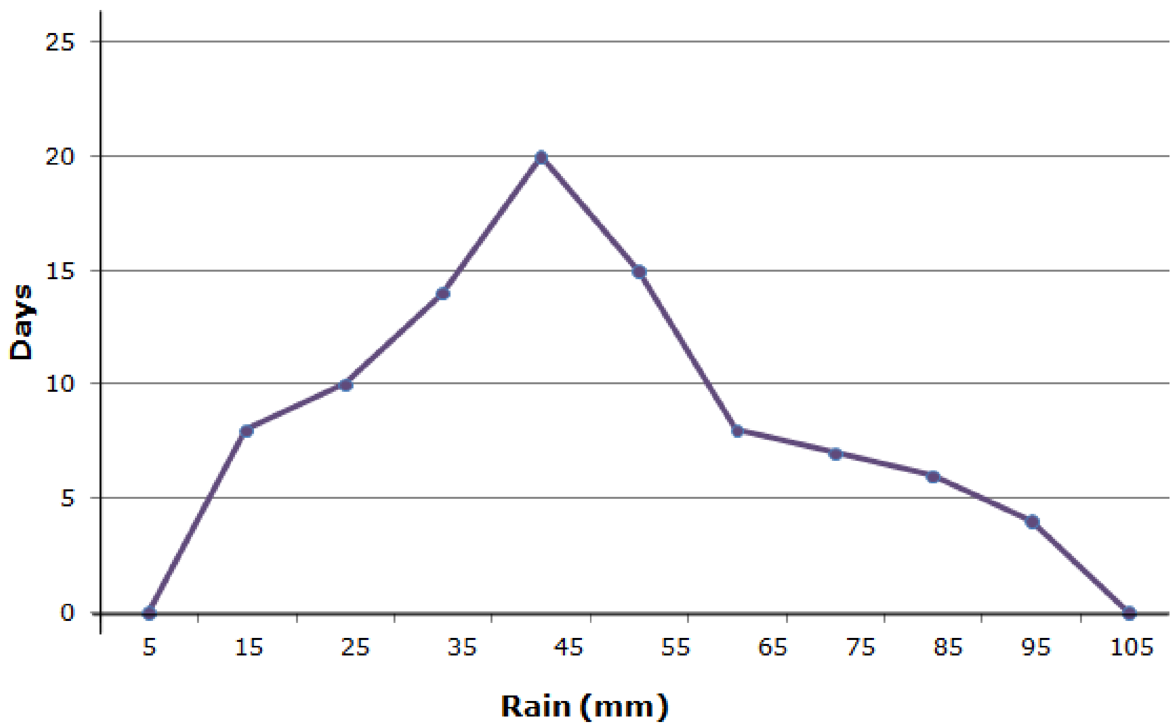
$$\Rightarrow x = \frac{2647}{495}$$

$$\therefore 5.3\overline{47} = \frac{2647}{495}$$

(c) While drawing frequency polygon, we represent each class by its mid-value.

Classes Rain (mm)	Class Marks	Days
0 – 10	5	0
10 – 20	15	8
20 – 30	25	10
30 – 40	35	14
40 – 50	45	20
50 – 60	55	15
60 – 70	65	8
70 – 80	75	7
80 – 90	85	6
90 – 100	95	4
100 – 110	105	0

The frequency polygon is as follows:



(d)

Q. 4.

(a) Each exterior angle of first polygon = $\frac{360^\circ}{n-1}$

Each exterior angle of second polygon = $\frac{360^\circ}{n+2}$

$$\frac{360^\circ}{n-1}-\frac{360^\circ}{n+2}=6$$

$$\Rightarrow \frac{1}{n-1}-\frac{1}{n+2}=\frac{1}{60}$$

$$\Rightarrow \frac{n+2-n+1}{(n-1)(n+2)}=\frac{1}{60}$$

$$\Rightarrow \frac{3}{n^2+2n-n-2}=\frac{1}{60}$$

$$\Rightarrow n^2+n-2=180$$

$$\Rightarrow n^2+n-182=0$$

$$\Rightarrow n^2+14n-13n-182=0$$

$$\Rightarrow n(n+14)-13(n+14)=0$$

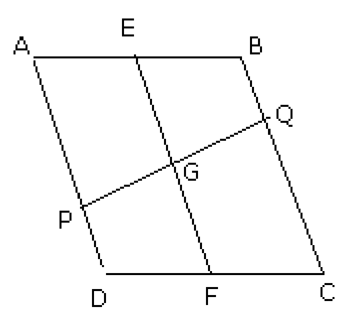
$$\Rightarrow (n+14)(n-13)=0$$

$n=-14$ is not applicable

$\therefore n-13=0$

$\therefore n=13$

(b)



Given: A ||gm ABCD, E and F are the midpoints of AB and DC. Line PQ meets AD, EF and BC at P, G and Q respectively.

To prove: PG = GQ

Proof: $AE=\frac{1}{2}AB$ and $DF=\frac{1}{2}DC$

[E and F are the mid-points]

$\Rightarrow$ AE = DF

Also, AE || DF [As AB || DC]

AE and DF are the parts of AB and DC.

$\therefore$ AEFD is a parallelogram

[$\because$ Opposite sides of parallelogram are equal and parallel]

$\therefore$ AD || EF || BC

$\therefore$ PG = GQ

[By intercept theorem AD, EF and BC are parallel, PQ cuts them]

Hence proved.

(c) P = Rs. 5000, r = 12%, T = 1 year

Amount at the end of first year

$$= 5000 + \left(\frac{5000 \times 12 \times 1}{100}\right)$$

= Rs. 5000 + 600 = Rs. 5600

Amount at the end of first year after payment of Rs. 2000

= Rs. 5600 – Rs. 2000 = Rs. 3600

Amount at the end of second year

$$= 3600 + \left(\frac{3600 \times 12 \times 1}{100}\right)$$

$$= Rs.3600 + 432 = Rs.4032$$

Amount at the end of second year after payment of Rs. 2000 = Rs. 4032 – Rs. 2000 = Rs. 2032

Amount at the end of third year

$$= 2032 + \left(\frac{2032 \times 12 \times 1}{100}\right)$$

$$= Rs.\ 2032 + Rs.\ 243.84$$

$$= Rs.\ 2275.84$$

SECTION – B (40 Marks)

Q. 5.

(a) Area of a square formed = 484 m^2

$\Rightarrow$ $(Side)^2 = 484$

$\Rightarrow$ Side = 22 m

Thus, perimeter of a square = 4 × Side = 4 × 22 = 88 m

Let r be the radius of the circle formed.

Now,

Circumference of a circle = Perimeter of a square

$$\Rightarrow 2 \times \frac{22}{7} \times r = 88$$

$$\Rightarrow r = \frac{88 \times 7}{2 \times 22}$$

$$\Rightarrow r = 14\ m$$

$$\therefore \text{ Area of a circle} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616\ m^2$$

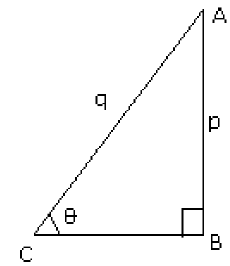
(b) $\sin\theta = \frac{p}{q} = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC}$

By Pythagoras theorem,

$BC^2 = AC^2 - AB^2 = q^2 - p^2$

$BC = \sqrt{q^2 - p^2}$

$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{q^2 - p^2}}{q}$



Now,

$\sin\theta + \cos\theta = \frac{p}{q} + \frac{\sqrt{q^2 - p^2}}{q} = \frac{\sqrt{q^2 - p^2} + p}{q}$

(c) Points (a, 0), (0, b) and (1, 1) are collinear. So, area of triangle formed by these points will be 0.

$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow [a(b - 1) + 0(1 - 0) + 1(0 - b)] = 0$

$\Rightarrow ab - a - b = 0$

$\Rightarrow ab = a + b$

On dividing throughout by ab, we get

$1 = \frac{1}{a} + \frac{1}{b}$

Q. 6.

(a)

$(x^2 + y^2 - z^2)^2 - (2xy)^2$

$= (x^2 + y^2 - z^2 + 2xy)(x^2 + y^2 - z^2 - 2xy)$

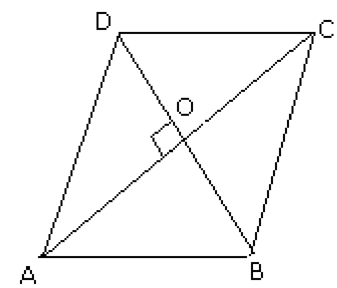
$= (x^2 + y^2 + 2xy - z^2)(x^2 + y^2 - 2xy - z^2)$

$= \{(x + y)^2 - (z)^2\}\{(x - y)^2 - (z)^2\}$

$= (x + y + z)(x + y - z)(x - y - z)(x - y + z)$

(b) Given: A parallelogram ABCD, such that $BD \perp AC$.

To prove: ABCD is a rhombus



Proof:

In ΔOAB and ΔOBC

OA = OC [Diagonals of a ||gm bisect each]

$\angle AOB = \angle BOC$ [Each 90°]

OB = OB

$\therefore \Delta OAB \cong \Delta OBC$ [SAS axioms of congruency]

AB = BC [C.P.C.T]

BC = AD and AB = DC [Given]

$\Rightarrow$ AB = DC = BC = AD

Hence ABCD is a rhombus.

(c) $3a = p\left(\frac{x}{2} - y\right)$

$\Rightarrow \frac{x}{2} - y = \frac{3a}{p}$

$\Rightarrow y = \left(\frac{x}{2} - \frac{3a}{p}\right)$

$\Rightarrow y = \left(\frac{px - 6a}{2p}\right)$

Given a = 32, x = 4, p = 5

$\Rightarrow y = \frac{5 \times 4 - 6 \times 32}{2 \times 5}$

$\Rightarrow y = \frac{20 - 192}{10}$

$\Rightarrow y = -17.2$

Q. 7.

(a) $2x - 3y = 7 \Rightarrow 2x = 7 + 3y$

$$\Rightarrow x = \frac{7+3y}{2}$$

Taking convenient values of y, we get

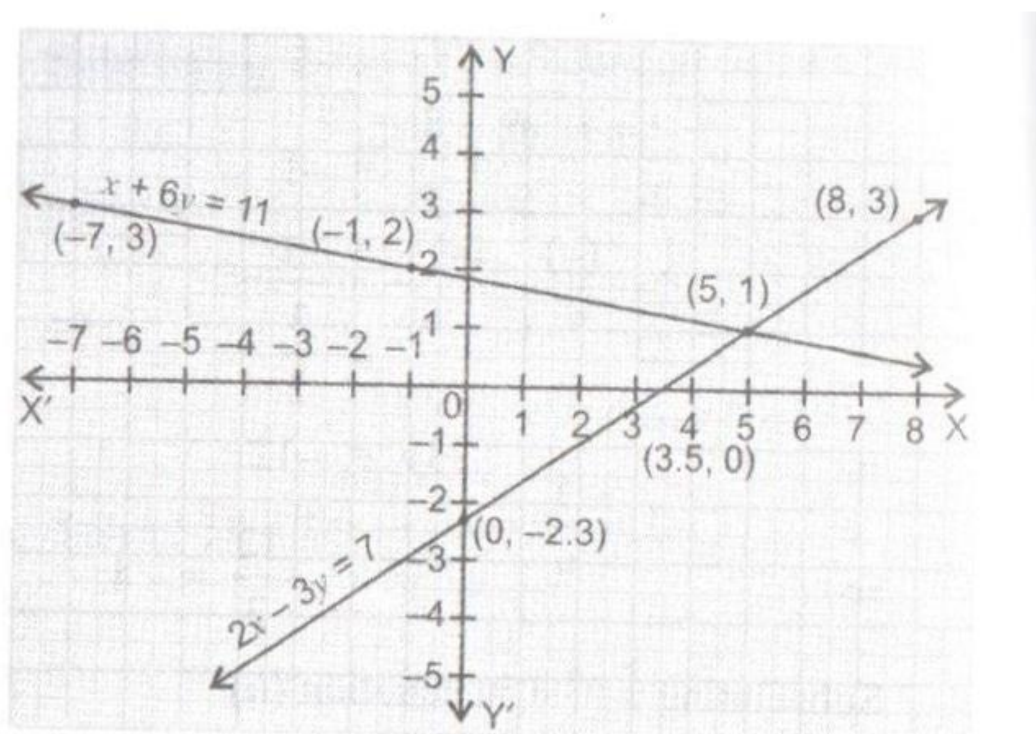
x	3.5	5	8
y	0	1	3

And $x + 6y = 11 \Rightarrow x = 11 - 6y$

Taking convenient values of y, we get

x	5	-1	-7
y	1	2	3

Now we plot these points on graph paper as follows:



The point of intersection of the two lines is (5, 1).

Hence the solution set is $x = 5$, $y = 1$.

(b) (i) Parallelograms BFED and AFEC are on the same base FE and between the same parallels AD || EF. Thus, they are equal in area.

$\therefore$ ar (||gm BFED) = ar (||gm AFEC) = 140 cm^2

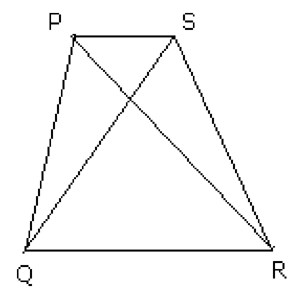
(ii) ΔBFD and ||gm BFED are on the same base BD and between the same parallels BD and FE.

$\therefore$ A(ΔBFD) = $\frac{1}{2} \times$ A(||gm BFED) = $\frac{1}{2} \times 140 = 70$ cm^2

Q. 8.

(a) Given: PQRS is a quadrilateral, and PR and QS are its diagonals.

To Prove: (PQ + QR + RS + SP) > (PR + QS)



Proof:

1. In ΔPQS, SP + PQ > QS
2. In ΔPQR, PQ + QR > PR
3. In ΔQRS, (QR + RS) > QS
4. In ΔRSP, (RS + SP) > PR

[Sum of the two sides of a Δ is greater than the third side]

Adding 1, 2, 3 and 4

2(PQ + QR + RS + SP) > 2(PR + QS)

$\Rightarrow$ PQ + QR + RS + SP > PR + QS [Hence Proved]

(b) Let the two places $P_1$and P_2 be 30 km apart and let A start from P_1 and B fromP_2.

Let A's speed of walking be x km/ hr and B's speed be y km/ hr.

According to the question, when they walk in the same direction

$10(x - y) = 30$ ....(i)

When they walk in opposite directions

$2(x + y) = 30$ ....(ii)

On dividing (i) by (ii), we get

$\frac{5(x-y)}{x+y} = 1 \Rightarrow 5x - 5y = x + y$

$\Rightarrow 4x - 6y = 0$ ....(iii)

On multiplying (ii) by 2, we get

$4x + 4y = 60$ ....(iv)

Subtracting (iii) from (iv), we get

$10y = 60 \Rightarrow y = 6$

Substituting value of y in (iv),

$4x + 4(6) = 60$

$\Rightarrow 4x + 24 = 60 \Rightarrow 4x = 36$

$\Rightarrow x = \frac{36}{4} = 9$

$\therefore$ A's speed = 9 km/ hr and B's speed = 6 km/ hr

Q. 9.

(a) Mean height of the 10 girls = 1.38 m

Sum of heights of 10 girls = $1.38 \times 10 = 13.8$ m

Mean height of 40 boys = 1.44 m

Sum of heights of 40 boys = $1.44 \times 40 = 57.6$ m

Sum of heights of 50 students (10 girls + 40 boys) = 13.8 + 57.6 = 71.4 m

$\therefore$ Mean height of 50 students = $\frac{71.4}{50} = 1.428$ m

(b) Given: $m\angle D = 90°$, AB = 8 cm, BC = 6 cm and CA = 3 cm

To find: length of CD

Let CD = x cm

In ΔADC,

$AD^2 + CD^2 = AC^2$ [By Pythagoras theorem]

$AD^2 = AC^2 - DC^2 = 3^2 - x^2$

$AD^2 = 9 - x^2$

In ΔADB,

$AD^2 + BD^2 = AB^2$ [By Pythagoras theorem]

$9 - x^2 + (6+x)^2 = 8^2$ [$\because AD^2 = 9 - x^2$ and $BD = 6 + x$]

$\Rightarrow 9 - x^2 + 36 + x^2 + 12x = 64$

$\Rightarrow 12x = 64 - 45 = 19$

$\therefore \; x = \frac{19}{12}$

$CD = \frac{19}{12} \text{cm} = 1\frac{7}{12}$

(c) Volume of rectangular tank = $80 \times 60 \times 60$ cm^3 = 288000 cm^3

One liter = 1000 cm^3

Volume of water flowing in per sec

$= 1.5 \text{ cm}^2 \times 3.2 \frac{\text{m}}{\text{s}}$

$= 15. \text{ cm}^2 \times \frac{(3.2 \times 100) \text{ cm}}{\text{s}}$

$= 480 \frac{\text{cm}^3}{\text{s}}$

Volume of water flowing in 1 min = 480×60 = 28800 cm^3

Hence,

28800 cm^3 of water can be filled in 1 min

$\Rightarrow$ Time required to fill 288000 cm^3 of water $= \left(\frac{1}{28800} \times 288000 \right) \text{min} = 10 \text{ min}$

Q. 10.

(a) Using sec (90° – θ) = cosec θ, tan (90° – θ) = cot θ

And cos (90° – θ) = sin θ

$$\frac{\sec(90^0-\theta).\operatorname{cosec}\theta-\tan(90^0-\theta)\cot\theta+\cos^2 25^0+\cos^2 65^0}{3\tan 27^0\tan 63^0}$$

$$=\frac{\operatorname{cosec}\theta.\operatorname{cosec}\theta-\cot\theta.\cot\theta+\cos^2(90^0-65^0)+\cos^2 65^0}{3\tan(90^0-63^0)\tan 63^0}$$

$$=\frac{\operatorname{cosec}^2\theta-\cot^2\theta+\sin^2 65^0+\cos^2 65^0}{3\cot 63^0\tan 63^0}$$

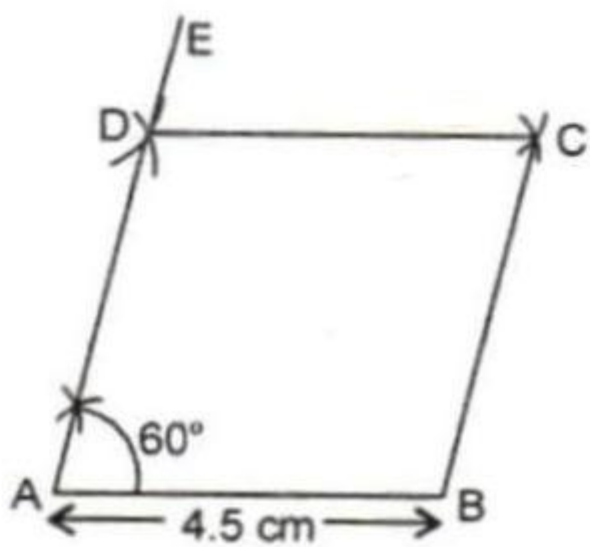
$$=\frac{1+1}{3}$$

$$=\frac{2}{3}$$

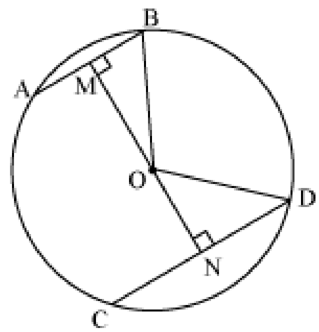
(b) Steps of construction:

1. We draw AB = 4.5 cm
2. Now we construct m∠BAE = 60°
3. Then we cut off AD = 4.5 cm from AE
4. We draw two arcs of radius 4.5 cm, one with centre at D and other at B.
5. Let them cut at C.
6. Join DC and BC.

ABCD is the required rhombus.



(c) Construction: Draw $OM \perp AB$ and $ON \perp CD$. Join OB and OD.



$BM = \frac{AB}{2} = \frac{5}{2}$ and $ND = \frac{CD}{2} = \frac{11}{2}$ (Perpendicular from centre bisects the chord)

Let ON be x.

Then, OM will be $6 - x$.

In ΔMOB,

$OM^2 + MB^2 = OB^2$

$\therefore (6-x)^2 + \left(\frac{5}{2}\right)^2 = OB^2$

$\therefore 36 + x^2 - 12x + \frac{25}{4} = OB^2$ ....(1)

In ΔNOD, $ON^2 + ND^2 = OD^2$

$\therefore OD^2 = x^2 + \left(\frac{11}{2}\right)^2 = x^2 + \frac{121}{4}$ ....(2)

We have $OB = OD$ ....(radii of same circle)

$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$ ....[From (1) and (2)]

$\therefore 12x = 36 + \frac{25}{4} - \frac{121}{4} = \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$

$\therefore 12x = 12$

$\Rightarrow x = 1$

From equation (2),

$OD^2 = (1)^2 + \left(\frac{121}{4}\right) = 1 + \frac{121}{4} = \frac{125}{4}$

$\Rightarrow OD = \frac{5}{2}\sqrt{5}$

Hence, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

Q. 11.

(a) Given : $a + \frac{1}{a} = p$,

Cubing on both the sides, we get

$$\left(a + \frac{1}{a}\right)^3 = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\left(a + \frac{1}{a}\right) = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3p = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} = p^3 - 3p = p\left(p^2 - 3\right)$$

Hence Proved.

(b) Let $\frac{1}{x+y} = a$, $\frac{1}{x-y} = b$

Then, we have

$5a + 3b = 4$ ....(i)

$2a + 5b = \frac{27}{5}$ ....(ii)

Multiplying equation (i) by 2 and equation (ii) by 5, we get

$10a + 6b = 8$ ....(iii)

$10a + 25b = 27$ ....(iv)

Subtracting (iv) from (iii), we get $-19b = -19$

$b = \frac{-19}{-19} = 1$

Substituting value of b in equation (i), we get

$5a + 3(1) = 4$

$\Rightarrow 5a + 3 = 4 \Rightarrow 5a = 1 \Rightarrow a = \frac{1}{5}$

Now $\frac{1}{x+y} = a$, $\frac{1}{x-y} = b$

$\therefore \frac{1}{x+y} = \frac{1}{5}$, $\frac{1}{x-y} = 1$

$\Rightarrow x + y = 5$ ....(v)

$x - y = 1$ ....(vi)

Adding (v) and (vi), we get $2y = 4 \Rightarrow y = 2$

Substituting the value of y in equation (vi), we get $x – 2 = 1 \Rightarrow x = 3$

Hence, $x = 3$, $y = 2$.

(c)

x	25	35	45	55	65	75	Total
f	10	6	8	12	5	9	50
fx	250	210	360	660	325	675	2,480

Here, $\sum f = 50, \sum fx = 2480$

$\therefore$ Mean = $\dfrac{\sum fx}{\sum f} = \dfrac{2480}{50} = 49.6$