ICSE Board
Class IX Mathematics
Sample Paper 1

Time: 2½ hrs **Total Marks: 80**

General Instructions:

1. Answers to this paper must be written on the paper provided separately.
2. You will **NOT** be allowed to write during the first 15 minutes. This time is to be spent in reading the question paper.
3. The time given at the head of this paper is the time allowed for writing the answers.
4. This question paper is divided into two Sections. Attempt **all** questions from **Section A** and any **four** questions from **Section B**.
5. Intended marks for questions or parts of questions are given in brackets along the questions.
6. All working, including rough work, must be clearly shown and should be done on the same sheet as the rest of the answer. Omission of essential working will result in loss of marks.
7. Mathematical tables are provided.

SECTION – A (40 Marks)

*(Answer **all questions** from this Section)*

Q. 1.

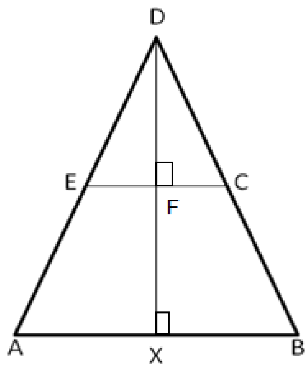
(a) Calculate the amount and the compound interest on Rs. 6000 at 10% p.a. for $1\frac{1}{2}$ years, when the interest is compounded half yearly. [3]

(b) If $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}} = x - y\sqrt{77}$, find the values x and y. [3]

(c) Sonu and Monu had adjacent triangular fields with a common boundary of 25 m. The other two sides of Sonu's field were 52 m and 63 m, while Monu's were 114 m and 101 m. If the cost of fertilization is Rs 20 per sq m, then find the total cost of fertilization for both of Sonu and Monu together. [4]

Q. 2.

(a) Calculate the area of fig., ABCDE. Given DX = 9 cm, DC = 5 cm FC = 4 cm and XB = 6 cm. Also F and X are the mid-points of EC and AB respectively. [3]



(b) If two intersecting chords of a circle make equal angles with the diameter passing through their point of intersection, prove that the chords are equal. [4]

(c) Find x, if $\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \left(\sqrt[3]{8}\right)^{5}$. [3]

Q. 3.

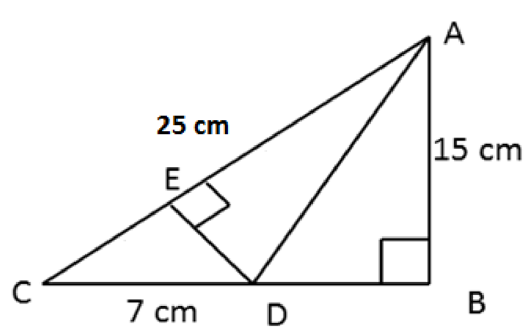
(a) Given log x = a + b and log y = a – b, find the value of $\log\frac{10x}{y^2}$ in terms of 'a' and 'b'. [3]

(b) The bisector of $\angle A$ of a ΔABC meets BC at D and BC is produced to E. prove that $\angle ABC + \angle ACE = 2\angle ADC$. [3]

(c) Using ruler and compass only, construct a trapezium ABCD, in which the parallel sides AB and DC are 3.3 cm apart; AB = 4.5 cm, $\angle A = 120°$, BC = 3.6 cm and $\angle B$ is obtuse. [4]

Q. 4.

(a) The following figure shows a right-angled triangle ABC with $\angle B = 90°$, AB = 15 cm and AC = 25 cm. D is a point in side BC and CD = 7cm. If DE ⊥ AC, find the length of DE. [4]



(b) Prove that the interior angle of a regular pentagon is three times the exterior angle of a regular decagon. [3]

(c) If $\tan\theta + \cot\theta = 3$, find the value of $\tan^2\theta + \cot^2\theta$. [3]

SECTION – B (40 Marks)

*(Answer **any four questions** from this Section)*

Q. 5.

(a) Graphically solve the simultaneous equations:

$x - 2y = 1; x + y = 4$ [4]

(b) A and B together can do a piece of work in 15 days. If A's one day's work is $\frac{3}{2}$ times B's one day's work; in how many days can A and B do the work alone? [3]

(c) How many sides does a regular polygon have, each angle of which is of measure 108^o? [3]

Q. 6.

(a) A person invests Rs. 5600 at 14% p.a. compound interest for 2 years. Calculate

i. The interest for the first year.

ii. The amount at the end of the first year.

iii. The interest for the second year corrected to the nearest rupee. [3]

(b) A point P lies on the x-axis and another point Q lies on the y-axis. [3]

i. Write the ordinate of point P.

ii. Write the abscissa of point Q.

iii. If the abscissa of point P is -12 and the ordinate of point Q is –16; calculate the length of line segment PQ.

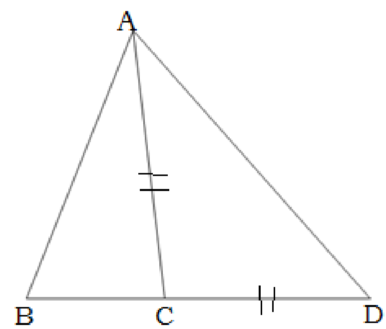
(c) ΔABC is right angled at B. If $m\angle A = 30°$ and BC = 8 cm. Find the remaining angles and sides. [4]

Q. 7.

(a) Simplify: $\dfrac{3^{n+1}}{3^{n(n-1)}} \div \dfrac{9^{n+1}}{\left(3^{n+1}\right)^{n-1}}$ [4]

(b) The area of an isosceles triangle is 12 cm^2 and the base is 8 cm in length. Find the perimeter of the triangle. [3]

(c) In the figure, AC = CD. Prove that BC < CD. [3]



Q. 8.

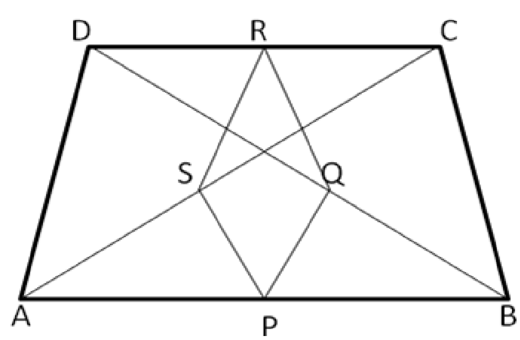
(a) If $\cos \theta = \dfrac{2\sqrt{mn}}{m+n}$, find the value of $\sin \theta$ (> (given m > n). [4]

(b) The mean of 5 numbers is 20. If one number is excluded the mean of the remaining numbers becomes 23. Find the excluded number. [3]

(c) 3 equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of three cubes. [3]

Q. 9.

(a) In the given quadrilateral AD = BC, and P, Q, R and S are the midpoints of the sides AB, BD, CD and AC, respectively. Prove that PQRS is a rhombus. [4]



(b) The distance (in km) of 40 engineers from their residence to place of work were found as follows: [3]

5	3	10	20	25	11	13	7	12	31
2	19	10	12	17	18	11	32	17	16
3	7	9	7	8	3	5	12	15	18
12	12	14	2	9	6	15	15	7	6

Construct a grouped frequency distribution table with class size 5 for the data given above taking the first interval as 0 - 5 (5 not included). What main feature do you observe from this tabular representation?

(c) Solve: $\log_x(8x-3)-\log_x 4=2$ [3]

Q. 10.

(a) Prove that $\sqrt{5}$ is an irrational number. [4]

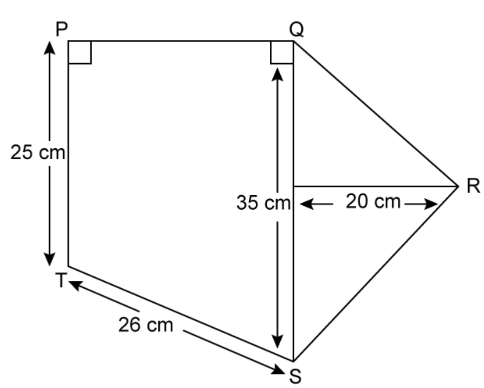
(b) If $\tan(\theta_1+\theta_2)=\dfrac{\tan\theta_1+\tan\theta_2}{1-\tan\theta_1\tan\theta_2}$, find the value of $(\theta_1 + \theta_2)$ given that $\tan\theta_1=\dfrac{1}{2}$ and $\tan\theta_2=\dfrac{1}{3}$. [3]

(c) If $\dfrac{x^2+1}{x}=4$, find the value of $2x^3+\dfrac{2}{x^3}$. [3]

Q. 11.

(a) Factorise: $4a^3b - 44a^2b + 112b$ [3]

(b) Find the area of following figure: [3]



(c) In ΔABC, AB = AC = x, BC = 20 cm and the area of the triangle is 250 cm^2. Find x. [4]

Solution

SECTION – A (40 Marks)

Q. 1.

(a) P = Rs. 6000, R = 10% p.a., n = $1\frac{1}{2}$ years $=\frac{3}{2}$ years

$$A = P\left(1+\frac{R}{2\times 100}\right)^{2n} \quad (\because \text{Interest is compounded half yearly})$$

$$= 6000\left(1+\frac{10}{2\times 100}\right)^{3}$$

$$= 6000\left(1+\frac{5}{100}\right)^{3}$$

$$= 6000\times(1.05)^{3}$$

$$= \text{Rs. } 6945.75$$

Amount = Rs. 6945.75

C.I. = 6945.75 – 6000 = Rs. 945.75

(b) We have

$$\frac{\left(\sqrt{11}-\sqrt{7}\right)\left(\sqrt{11}-\sqrt{7}\right)}{\left(\sqrt{11}+\sqrt{7}\right)\left(\sqrt{11}-\sqrt{7}\right)} = x - y\sqrt{77}$$

$$\Rightarrow \frac{11-\sqrt{77}-\sqrt{77}+7}{11-7} = x - y\sqrt{77}$$

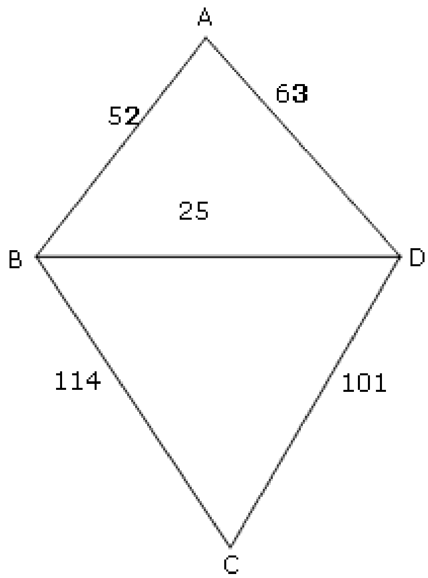
$$\Rightarrow \frac{18-2\sqrt{77}}{4} = x - y\sqrt{77}$$

$$\Rightarrow \frac{9}{2}-\frac{1}{2}\sqrt{77} = x - y\sqrt{77}$$

$$\Rightarrow x = \frac{9}{2},\ y = \frac{1}{2}$$

(c)

Sonu and Monu's field together form a quadrilateral ABCD.



Sonu's field is ΔABD,

$$s = \frac{a+b+c}{2} = \frac{52+25+63}{2} = 70$$

$s - a = 70 - 52 = 18$, $s - b = 70 - 25 = 45$ and $s - c = 70 - 63 = 7$

Area of ΔABD =

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{70.18.45.7} = 630 \text{ sq m}$$

Monu's field is ΔBCD,

$$s = \frac{a+b+c}{2} = \frac{114+25+101}{2} = 120$$

$s - a = 120 - 114 = 6$, $s - b = 120 - 25 = 95$ and $s - c = 120 - 101 = 19$

Area of ΔBCD =

$$\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{120.6.95.19} = 1140 \text{ sq m}$$

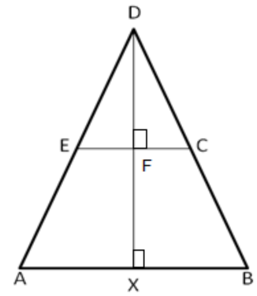
Total area is = 630 + 1140 = 1770 sq m

The cost of fertilization is Rs 20 per sq m.

Therefore the total cost is = 1770 x 20 = Rs 35,400.

Q. 2.

(a) In ΔDFC, $DC^2 = DF^2 + FC^2$ [Pythagoras Theorem]



$\Rightarrow 5^2 = DF^2 + 4^2$

$\Rightarrow 5^2 - 4^2 = DF^2 \Rightarrow DF^2 = 25 - 16 \Rightarrow DF = 3$

$\therefore$ Area of $\Delta DEC = \frac{1}{2} \times (4+4) \times 3 = \frac{1}{2} \times 8 \times 3 = 12\ cm^2$

FX = DX – DF = 9 – 3 = 6 cm

Area of trapezium CEBA = $\frac{1}{2} \times (4+4+6+6) \times 6 = \frac{1}{2} \times 20 \times 6 = 60\ cm^2$

$\therefore$ Area of figure ABCDE = area of ΔDEC + area of trapezium ECBA = 12 + 60 = 72 cm^2

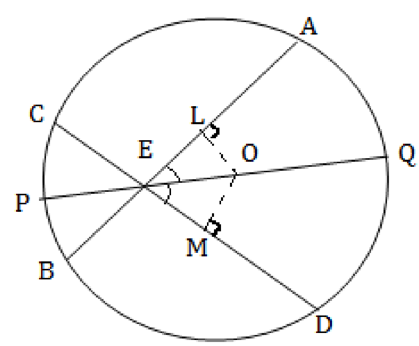
(b) Given that AB and CD are two chords of a circle with centre O, intersecting at a point E. PQ is the diameter through E, such that ∠AEQ = ∠DEQ.

To prove that AB = CD.

Draw perpendiculars OL and OM on chords AB and CD respectively.

Now, m∠LOE = 180° – 90° – m∠LEO ... [Angle sum property of a triangle]

= 90° – m∠LEO



⇒ m∠LOE = 90° – m∠AEQ

⇒ m∠LOE = 90° – m∠DEQ

⇒ m∠LOE = 90° – m∠MEQ

⇒ ∠LOE = ∠MOE

In ΔOLE and ΔOME,

∠LEO = ∠MEO

∠LOE = ∠MOE

EO = EO

ΔOLE ≅ ΔOME

OL = OM

Therefore, cords AB and CD are equidistant from the centre.

(c)

$$\left(\sqrt[3]{4}\right)^{2x+\frac{1}{2}} = \left(\sqrt[3]{8}\right)^{5}$$

$$\Rightarrow \left[(4)^{1/3}\right]^{2x+\frac{1}{2}} = \left[8^{1/3}\right]^{5}$$

$$\Rightarrow \left(2^{2/3}\right)^{\left(2x+\frac{1}{2}\right)} = \left[8^{1/3}\right]^{5}$$

$$\Rightarrow (2)^{\frac{2}{3}\left(2x+\frac{1}{2}\right)} = (2)^{5}$$

$$\Rightarrow \frac{2}{3}\left(2x+\frac{1}{2}\right) = 5$$

$$\Rightarrow 4x+1 = 15$$

$$\Rightarrow x = \frac{7}{2}$$

Q. 3.

(a) Given, log x = a + b and log y = a – b

$\log\frac{10x}{y^2}$ = log 10x – log y^2 [Using quotient law]

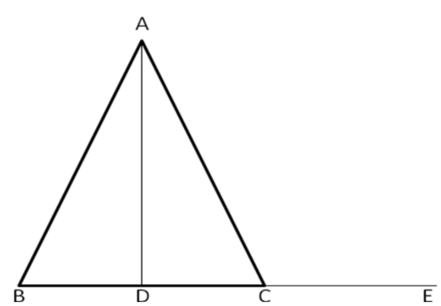
= log 10 + log x – 2 log y

= 1 + (a + b) – 2(a – b)

= 1 + a + b – 2a + 2b

= 1 – a + 3b

(b) Given : In ΔABC, AD is the bisector of ∠BAC and BC is produced to E



To Prove: ∠ABC + ∠ACE = 2∠ADC

Proof:

Let ∠BAD = ∠DAC = x and ∠ABC = y

Now, ∠ACE = ∠ABC + ∠BAC ....[Exterior angle = Sum of interior opposite ∠s]

⇒ ∠ACE = y + 2x

In ΔABD, ∠ADC = x + y ....[Exterior angle = Sum of interior opposite ∠s]

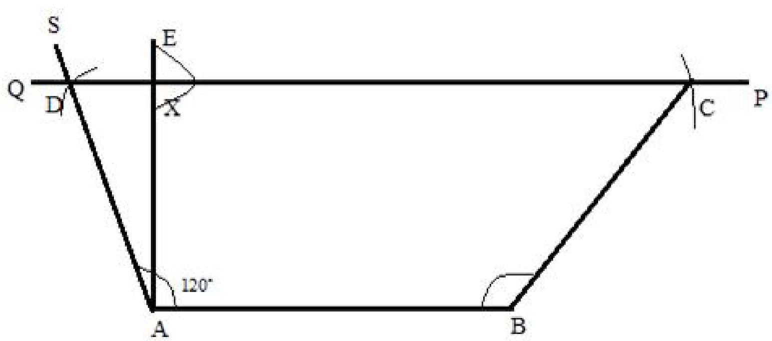
∴ ∠ABC + ∠ACE = y + y + 2x = 2(x + y)

⇒ ∠ABC + ∠ACE = 2∠ADC

(c) Steps of Construction:

1. Draw AB = 4.5 cm.
2. Draw $\angle BAS = 120°$ and draw $EA \perp AB$.
3. From A, cut an arc of measure 3.3 cm on EA such that AX = 3.3 cm.
4. Through X, draw a line QP which is parallel to AB which cuts AS at D.
5. Through B draw an arc taking radius 3.6 cm at C on PQ.
6. Join CB.

Thus, ABCD is the required trapezium.



Q. 4.

(a) We can see that ΔABC is a right-angles triangle.

$\Rightarrow AB^2 + BC^2 = AC^2$ ....[By Pythagoras theorem]

$\Rightarrow 15^2 + BC^2 = 25^2$

$\Rightarrow BC^2 = 400$

$\Rightarrow BC = 20$ cm

Now BC = DB + CD

$\Rightarrow 20 = DB + 7$

$\Rightarrow DB = 13$ cm

Again ADB is a right angled triangle.

$\Rightarrow AB^2 + DB^2 = AD^2$ ....[By Pythagoras theorem]

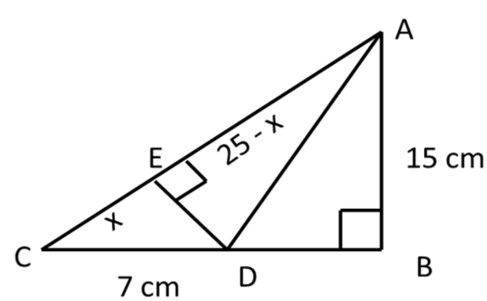
$\Rightarrow 15^2 + 13^2 = 390$

$\Rightarrow BC = 19.8$ cm

In the right-angled ΔCDE

$\Rightarrow ED^2 + CE^2 = CD^2$ ....[By Pythagoras theorem]

$\Rightarrow ED^2 = CD^2 - CE^2 = 7^2 - x^2$



In the right-angled ΔAED

$\Rightarrow ED^2 + AE^2 = AD^2$ ....[By Pythagoras theorem]

$\Rightarrow ED^2 = AD^2 - AE^2 = 19.8^2 - (25 - x)^2$

Since in both the cases length of ED is same

and hence ED^2 is also same in both the cases.

$\Rightarrow 7^2 - x^2 = 19.8^2 - (25 - x)^2$

$\Rightarrow 7^2 - x^2 = 19.8^2 - 625 - x^2 + 50x$

$\Rightarrow 7^2 - 19.8^2 + 625 = 50x$

$\Rightarrow 281.96 = 50x$

$\Rightarrow x = 5.63$ cm

So,

$ED^2 = 7^2 - 5.6^2 = 17.64$

$\Rightarrow ED = 4.2$ cm $=$ DE

(b)

Each interior angle of a regular pentagon $= \dfrac{(2\times 5 - 4)\times 90}{5}$ [n = 5]

$= \dfrac{6\times 90}{5}$

$= 108°$

Each exterior angle of a regular decagon $= \dfrac{360}{10} = 36°$ [n = 10]

$\therefore$ Each interior angle of a regular pentagon = 3(Exterior angle of a regular decagon)

(c) Given $\tan\theta + \cot\theta = 3$,

Squaring both sides,

$(\tan\theta + \cot\theta)^2 = 3^2$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta\cot\theta = 9$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2\tan\theta \times \dfrac{1}{\tan\theta} = 9 \quad \left[\because \cot\theta = \dfrac{\cos\theta}{\sin\theta}\right]$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = 9$

$\Rightarrow \tan^2\theta + \cot^2\theta = 7$

Section – B (40 Marks)

Q. 5.

(a) Consider equation, $x - 2y = 1$ ....(1)

$$\Rightarrow y = \frac{x-1}{2}$$

x	1	3	5
y	0	1	2

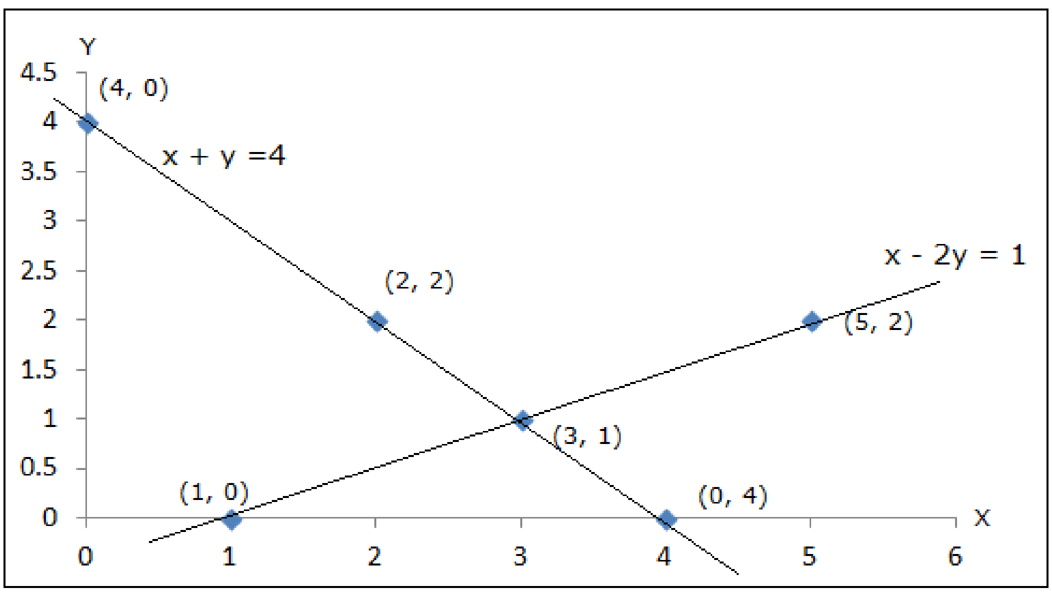
∴ Points are (1, 0), (3, 1) and (5, 2).

Now consider equation $x + y = 4$ ....(2)

x	0	2	4
y	4	2	0

∴ Points are (0, 4), (2, 2) and (4, 0).

Now plotting these points on the graph paper, we get



Since the lines intersect at (3, 1), therefore the solution is x = 3 and y = 1.

(b) In 15 days A and B together can do a piece of work.

Therefore, in 1 day they do $\frac{1}{15}$ work

Let us assume that A takes x days and B takes y days to do the work alone.

So A's one day's work = $\frac{1}{x}$

B's one day's work = $\frac{1}{y}$

$$\frac{1}{x}=\frac{3}{2}.\frac{1}{y}$$
$$\Rightarrow 3x-2y=0$$
$$\Rightarrow 2y=3x$$
$$\Rightarrow y=\frac{3x}{2} \quad ....(i)$$

Also, $\frac{1}{x}+\frac{1}{y}=\frac{1}{15}$

$$\Rightarrow \frac{1}{x}+\frac{2}{3x}=\frac{1}{15}$$
$$\Rightarrow \frac{3+2}{3x}=\frac{1}{15}$$
$$\Rightarrow 3x=75$$
$$\Rightarrow x=25$$
$$\Rightarrow y=\frac{3\times 25}{2}=37.5$$

Hence, A will do the work alone in 25 days and B will do it alone 37 and half days.

(c) Let there be n sides of the polygon. Then, each interior angle is of measure

$$\left(\frac{2n-4}{n}\times 90°\right)$$
$$\therefore \frac{2n-4}{n}\times 90=108$$
$$\Rightarrow (2n-4)\times 90=108n$$
$$\Rightarrow 180n-360=108n$$
$$\Rightarrow 180n-108n=360$$
$$\Rightarrow 72n=360$$
$$\Rightarrow n=5$$

Hence the given polygon has 5 sides.

Q. 6.

(a) (i) Interest for first year = $\frac{5600 \times 14 \times 1}{100} = \text{Rs. } 784$

(ii) Amount at the end of the first year = 5600 + 784 = Rs. 6384

(iii) Interest for the second year =

$\frac{6384 \times 14 \times 1}{100} = \text{ Rs. } 893.76 = \text{ Rs. } 894 \text{ (to the nearest rupee)}$

(b)

(i) Since, the point P lies on the x-axis, its ordinate is 0.

(ii) Since, the point Q lies on the y-axis, its abscissa is 0.

(iii) The co-ordinates of P and Q are (−12, 0) and (0, −16) respectively.

$$PQ = \sqrt{(-12-0)^2 + (0+16)^2}$$
$$= \sqrt{144+256}$$
$$= \sqrt{400}$$
$$= 20$$

(c) Here m∠A +m∠C = 90° as m∠B = 90°

$\Rightarrow 30° + m\angle C = 90°$

$\Rightarrow m\angle C = 60°$

In right-angled ΔABC,

$\tan 30° = \frac{BC}{AB}$

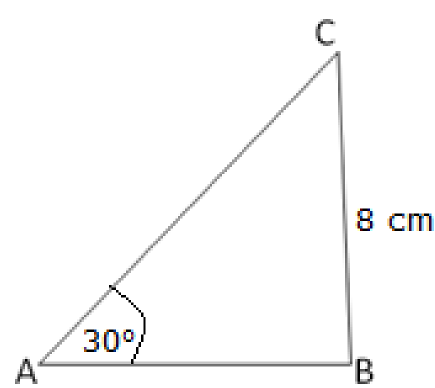
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{8}{AB}$

$\Rightarrow AB = 8\sqrt{3} \text{ cm}$

$\sin 30° = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{8}{AC}$

$\Rightarrow AC = 16 \text{ cm}$



Q. 7.

(a)

$$\frac{3^{n+1}}{3^{n(n-1)}} \div \frac{9^{n+1}}{\left(3^{n+1}\right)^{n-1}}$$

$$= \frac{3^{n+1}}{3^{n(n-1)}} \times \frac{\left(3^{n+1}\right)^{n-1}}{9^{n+1}}$$

$$= \frac{3^{n+1}}{3^{n(n-1)}} \times \frac{3^{(n+1)(n-1)}}{(3 \times 3)^{n+1}}$$

$$= \frac{3^{n+1}}{3^{n^2-n}} \times \frac{3^{\left(n^2-1\right)}}{\left(3^2\right)^{n+1}}$$

$$= \frac{3^{n+1}}{3^{n^2-n}} \times \frac{3^{\left(n^2-1\right)}}{3^{2n+2}}$$

$$= 3^{n+1+n^2-1-(n^2-n)-(2n+2)}$$

$$= 3^{n+1+n^2-1-n^2+n-2n-2}$$

$$= 3^{-2}$$

$$= \frac{1}{3^2}$$

$$= \frac{1}{9}$$

(b)

Area of an isosceles $\Delta = \frac{1}{4} b\sqrt{4a^2 - b^2}$

(where b is the base and a is the length of equal sides)

Given, b = 8 cm and area =12 cm^2

$$\Rightarrow \frac{1}{4} \times 8 \times \sqrt{4a^2 - 8^2} = 12$$

$$\Rightarrow \sqrt{4a^2 - 8^2} = 6$$

$$\Rightarrow 4a^2 - 8^2 = 36$$

$$\Rightarrow 4a^2 = 100$$

$$\Rightarrow a^2 = 25$$

$$\Rightarrow a = 5 \text{ cm}$$

$\therefore$ Perimeter = 2a + b = 2 $\times$ 5 + 8 = 18 cm

(c) Given AC = CD

To prove: BC < CD

Proof: In ΔACD,

m$\angle$ACD = 180° – 70° = 110° [Linear pair]

$\angle CAD = \angle ADC = \dfrac{70°}{2} = 35°$ [Angles opposite to equal sides are equal]

In ΔABC,

m$\angle$BAC = 70° – 35° = 35° [$\angle$BAC = $\angle$BAD – $\angle$CAD]

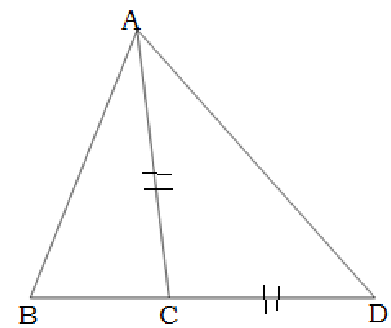
m$\angle$ABC = 180° – (70° + 35°) [Sum of all $\angle$s of a Δ is 180°]

= 75°

$\therefore$ $\angle$BAC < $\angle$ABC

$\therefore$ BC < AC

So, BC < CD [Since AC = CD]



Q. 8.

(a)

$$\cos\theta = \frac{2\sqrt{mn}}{m+n}$$

Now,

$$\sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - \left(\frac{2\sqrt{mn}}{m+n}\right)^2$$

$$= 1 - \left(\frac{4mn}{(m+n)^2}\right)$$

$$= \frac{(m+n)^2 - 4mn}{(m+n)^2}$$

$$= \frac{m^2 + n^2 + 2mn - 4mn}{(m+n)^2}$$

$$= \frac{m^2 + n^2 - 2mn}{(m+n)^2}$$

$$= \frac{(m-n)^2}{(m+n)^2}$$

$$= \left(\frac{m-n}{m+n}\right)^2$$

$$\Rightarrow \sin\theta = \frac{m-n}{m+n}$$

(b) Mean = 20

Number of terms = 5

$\therefore$ Total sum = 20 × 5 = 100

Let the excluded number be x.

Then, $\frac{(100-x)}{4} = 23$

$\Rightarrow$ 100 – x = 23 × 4 = 92

$\Rightarrow$ x = 8

Hence, the excluded number is 8.

(c) Let the side of each of the three equal cubes be 'a' cm.

Surface area of one cube = $6a^2$ cm^2

Therefore, sum of surface areas of the three cubes = 3 × $6a^2$ = $18a^2$ cm^2

Now,

Length of the new cuboid = 3a cm

Breadth of the new cuboid = a cm

Height of the new cuboid = a cm

Total surface area of the new cuboid = 2[(3a × a) + (a × a) + (a × 3a)]

= $2[3a^2 + a^2 + 3a^2]$

= $2[7a^2]$

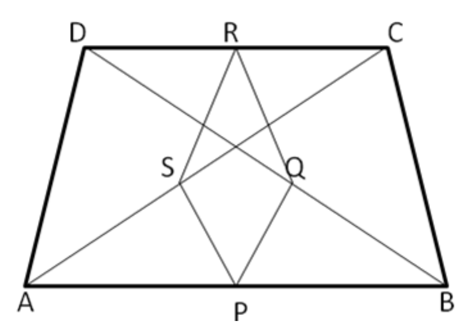
= $14a^2$ cm^2

Thus, the required ratio of T.S.A. of the new cuboid to that of the sum of the S.A. of the 3 cubes = $14a^2 : 18a^2$ = 7 : 9.

Q. 9.

(a) Given: In quadrilateral ABCD; AD = BC. P, Q, R, S are the mid-points of AB, BD, CD and AC respectively.

To Prove: PQRS is a rhombus.



Proof: In ΔACD, RS||AD and $RS = \frac{1}{2}AD$ ....(i)

[Line joining the mid-points of the two sides of triangle is parallel and half of the third side.]

Similarly,

In ΔABD, PQ||AD and $PQ = \frac{1}{2}AD$ ....(ii)

In ΔBCD, QR||BC and $QR = \frac{1}{2}BC$ ....(iii)

In ΔABC, SP||BC and $SP = \frac{1}{2}BC$ ....(iv)

As AD = BC [Given]

RS = PQ = QR = SP and RS||PQ and QR||SP [From (i), (ii), (iii) and (iv)]

Hence PQRS is a rhombus.

(b) Given that we have to construct a grouped frequency distribution table of class size 5.
So, the class intervals will be as 0 – 5, 5 – 10, 10 – 15, 15 –20, and so on.
Required grouped frequency distribution table is as follows:

Distance (in km)	**Tally marks**	**Number of engineers**
0 – 5	𝍸	5
5 – 10	𝍸 𝍸 |	11
10 –15	𝍸 𝍸 |	11
15 – 20	𝍸 ||||	9
20 – 25	|	1
25 – 30	|	1
30 – 35	||	2
Total		40

Only 4 engineers have homes at a distance of more than or equal to 20 km from their work place.

Most of the engineers have their workplace at a distance of upto 15 km from their homes.

(c)

$$\log_x(8x-3)-\log_x 4=2$$

$$\Rightarrow \log_x\left(\frac{8x-3}{4}\right)=2$$

$$\Rightarrow \frac{8x-3}{4}=x^2$$

$$\Rightarrow 8x-3=4x^2$$

$$\Rightarrow 4x^2-8x+3=0$$

$$\Rightarrow 4x^2-6x-2x+3=0$$

$$\Rightarrow 2x(2x-3)-1(2x-3)=0$$

$$\Rightarrow (2x-3)(2x-1)=0$$

$$\Rightarrow 2x-3=0 \text{ or } 2x-1=0$$

$$\Rightarrow x=\frac{3}{2} \text{ or } x=\frac{1}{2}$$

Q. 10.

(a)

Let us assume, on the contrary that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b (b ≠ 0) such that $\sqrt{5} = \frac{a}{b}$

Where a and b are co-prime integers.

$$\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow a = \sqrt{5}b$$

$$\Rightarrow a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 then a is also divisible by 5.

So a = 5k, for some integer k.

$$\text{Now}, a^2 = (5k)^2 = 5(5k^2) = 5b^2$$

$$\Rightarrow b^2 = 5k^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

So our assumption that $\sqrt{5}$ is rational is wrong.

Hence, $\sqrt{5}$ cannot be a rational number. Therefore, $\sqrt{5}$ is irrational.

(b)

$$\tan(\theta_1 + \theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1 \tan\theta_2} = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \times \frac{1}{3}\right)}$$

$$\Rightarrow \tan(\theta_1 + \theta_2) = \frac{\frac{3+2}{6}}{1 - \frac{1}{6}} = \frac{\frac{5}{6}}{\frac{6-1}{6}} = \frac{5}{6} \times \frac{6}{5} = 1$$

$$\Rightarrow \tan(\theta_1 + \theta_2) = 1 = \tan 45^\circ$$

$$\Rightarrow (\theta_1 + \theta_2) = 45^\circ$$

(c) $\frac{x^2+1}{x} = 4$

$\Rightarrow x^2 + 1 = 4x$

$\Rightarrow x^2 - 4x + 1 = 0$ ....(i)

On dividing equation (i) by x, we have

$x - 4 + \frac{1}{x} = 0$

$\Rightarrow x + \frac{1}{x} = 4$ ....(ii)

On cubing equation (ii) both sides, we have

$\left(x + \frac{1}{x}\right)^3 = (4)^3$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 64$

$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 4 = 64$

$\Rightarrow x^3 + \frac{1}{x^3} = 64 - 12$

$\Rightarrow x^3 + \frac{1}{x^3} = 52$

$\therefore \left(2x^3 + \frac{2}{x^3}\right) = 2\left(x^3 + \frac{1}{x^3}\right) = 2 \times 52 = 104$

Q. 11.

(a) $4a^3b - 44a^2b + 112b$

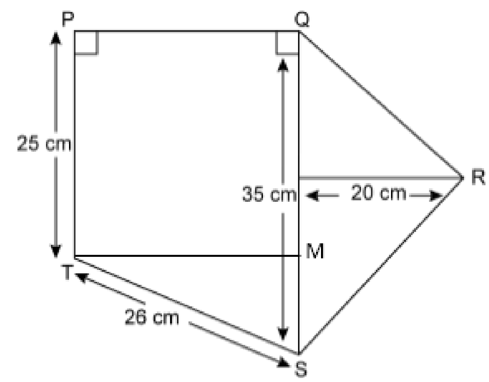
$= 4ab\left[a^2 - 11a + 28\right]$

$= 4ab\left[a^2 - 7a - 4a + 28\right]$

$= 4ab\left[a(a-7) - 4(a-7)\right]$

$= 4ab(a-7)(a-4)$

(b) Construction: Draw TM $\perp$ QS



Area of $\Delta RQS = \frac{1}{2} \times QS \times RN = \frac{1}{2} \times 35 \times 20 = 350 \text{ cm}^2$

Now, $QS = QM + MS$

$\Rightarrow 35 = 25 + MS$

$\Rightarrow MS = 10 \text{ cm}$

In ΔSTM,

$MS^2 + TM^2 = ST^2$

$\Rightarrow TM^2 = ST^2 - MS^2 = (26)^2 - (10)^2 = 676 - 100 = 576$

$\Rightarrow TM = 24 \text{ cm} = PQ$

$\therefore$ Area of trapezium $PQST = \frac{1}{2} \times (PT + QS) \times PQ = \frac{1}{2} \times (25 + 35) \times 24 = 720 \text{ cm}^2$

Thus, area of given figure $=$ Area of $\Delta RQS +$ Area of trapezium PQST

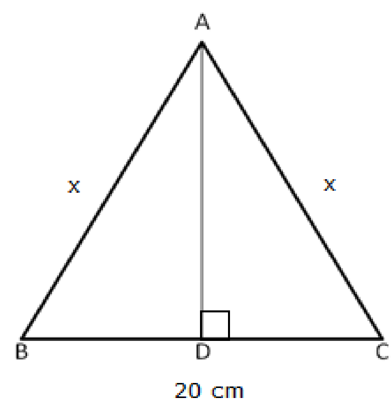
$= 350 \text{ cm}^2 + 720 \text{ cm}^2$

$= 1070 \text{ cm}^2$

(c) Given: In ΔABC, AB = AC = x, BC = 20 cm, Area of ΔABC = 250 cm^2

To find: x

Construction: Draw AD $\perp$ BC



Since ΔABC is an isosceles triangle. AD bisects BC.

BD = DC = 20/2 = 10cm

Area of ΔABC = $\frac{1}{2} \times BC \times AD = 250 \text{cm}^2$[Given]

$\frac{1}{2} \times 20 \times AD = 250 \Rightarrow AD = 25 \text{cm}$

In rt. ΔADC,

$AD^2 + DC^2 = AC^2$

$25^2 + 10^2 = x^2$ [Pythagoras Theorem]

$x^2 = 625 + 100 = 725$

$\Rightarrow x = 5\sqrt{29} \text{ cm}$