1. Without using trigonometric tables, evaluate:

(i) $\frac{\sin 16^{0}}{\cos 74^{0}}$ (ii) $\frac{\sec 11^{0}}{\operatorname{cosec} 79^{0}}$ (iii) $\frac{\tan 27^{0}}{\cot 63^{0}}$

(iv) $\frac{\cos 35^{0}}{\sin 55^{0}}$ (v) $\frac{\operatorname{cosec} 42^{0}}{\sec 48^{0}}$ (vi) $\frac{\cot 38^{0}}{\tan 52^{0}}$

Sol:

(i) $\frac{\sin 16^{0}}{\cos 74^{0}}$

$= \frac{\sin (90^{0} - 74^{0})}{\cos 74^{0}}$

$= \frac{\cos 74^{0}}{\cos 74^{0}}$ [$\because \sin (90 - \theta) = \cos \theta$]

$= 1$

(ii) $\frac{\sec 11^{0}}{\operatorname{cosec} 79^{0}}$

$= \frac{\sec (90^{0} - 79^{0})}{\operatorname{cosec} 79^{0}}$

$= \frac{\operatorname{cosec} 79^{0}}{\operatorname{cosec} 79^{0}}$ [$\because \sec (90 - \theta) = \operatorname{cosec} \theta$]

$= 1$

(iii) $\frac{\tan 27^{0}}{\cot 63^{0}}$

$= \frac{\tan (90^{0} - 63^{0})}{\cot 63^{0}}$

$= \frac{\cos 63^{0}}{\cos 63^{0}}$ [$\because \tan (90 - \theta) = \cot \theta$]

$= 1$

(iv) $\frac{\cos 35^{0}}{\sin 55^{0}}$

$= \frac{\cos (90^{0} - 55^{0})}{\sin 55^{0}}$

$= \frac{\sin 55^{0}}{\sin 55^{0}}$ [$\because \sin (90 - \theta) = \cos \theta$]

$= 1$

(v) $\frac{\operatorname{cosec} 42^{0}}{\sec 48^{0}}$

$= \frac{\operatorname{cosec} (90^{0} - 48^{0})}{\sec 48^{0}}$

$= \frac{\sec 48^{0}}{\sec 48^{0}}$ [$\because \sec (90 - \theta) = \operatorname{cosec} \theta$]

$= 1$

(vi) $\frac{\cot 38^{0}}{\tan 52^{0}}$

$= \frac{\cot (90^{0} - 52^{0})}{\tan 52^{0}}$

$= \frac{\tan 52^0}{\tan 52^0}$ [∵ tan (90 – θ) = cot θ]
= 1

2. Without using trigonometric tables, prove that:
(i) $\cos 81^0 - \sin 9^0 = 0$ (ii) $\tan 71^0 - \cot 19^0 = 0$
(iii) $\text{cosec } 80^0 - \sec 10^0 = 0$ (iv) $\text{cosec}^2 72^0 - \tan^2 18^0 = 1$
(v) $\cos^2 75^0 + \cos^2 15^0 = 1$ (vi) $\tan^2 66^0 - \cot^2 24^0 = 0$
(vii) $\sin^2 48^0 + \sin^2 42^0 = 1$ (viii) $\cos^2 57^0 - \sin^2 33^0 = 0$
(ix) $(\sin 65^0 + \cos 25^0)(\sin 65^0 - \cos 25^0) = 0$

Sol:

(i) LHS = $\cos 81^0 - \sin 9^0$
$= \cos(90^0 - 9^0) - \sin 9^0$
$= \sin 9^0 - \sin 9^0$
$= 0$
= RHS

(ii) LHS = $\tan 71^0 - \cot 19^0$
$= \tan(90^0 - 19^0) - \cot 19^0$
$= \cot 19^0 - \cot 19^0$
$= 0$
= RHS

(iii) LHS = $\text{cosec } 80^0 - \sec 10^0$
$= \text{cosec }(90^0 - 10^0) - \sec 10^0$
$= \sec 10^0 - \sec 10^0$
$= 0$
= RHS

(iv) LHS = $\text{cosec}^2 72^0 - \tan^2 18^0$
$= \text{cosec}^2(90^0 - 18^0) - \tan^2 18^0$
$= \sec^2 18^0 - \tan^2 18^0$
$= 1$
= RHS

(v) LHS = $\cos^2 75^0 + \cos^2 15^0$
$= \cos^2(90^0 - 15^0) + \cos^2 15^0$
$= \sin^2 15^0 + \cos^2 15^0$
$= 1$
= RHS

(vi) LHS = $\tan^2 66^0 - \cot^2 24^0$
$= \tan^2(90^0 - 24^0) - \cot^2 24^0$
$= \cot^2 24^0 - \cot^2 24^0$
$= 0$
= RHS

(vii) LHS = $\sin^2 48^0 + \sin^2 42^0$
$= \sin^2(90^0 - 42^0) + \sin^2 42^0$
$= \cos^2 42^0 + \sin^2 42^0$
$= 1$
= RHS

(viii) LHS = $\cos^2 57^0 - \sin^2 33^0$
$= \cos^2(90^0 - 33^0) - \sin^2 33^0$
$= \sin^2 33^0 - \sin^2 33^0$
$= 0$
= RHS

(ix) LHS = $(\sin 65^0 + \cos 25^0)(\sin 65^0 - \cos 25^0)$
$= \sin^2 65^0 - \cos^2 25^0$
$= \sin^2(90^0 - 25^0) - \cos^2 25^0$
$= \cos^2 25^0 - \cos^2 25^0$
$= 0$
= RHS

3. Without using trigonometric tables, prove that:
(i) $\sin 53^0 \cos 37^0 + \cos 53^0 \sin 37^0 = 1$
(ii) $\cos 54^0 \cos 36^0 - \sin 54^0 \sin 36^0 = 0$
(iii) $\sec 70^0 \sin 20^0 + \cos 20^0 \operatorname{cosec} 70^0 = 2$
(iv) $\sin 35^0 \sin 55^0 - \cos 35^0 \cos 55^0 = 0$
(v) $(\sin 72^0 + \cos 18^0)(\sin 72^0 - \cos 18^0) = 0$
(vi) $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0 = 1$

Sol:
(i) LHS = $\sin 53^0 \cos 37^0 + \cos 53^0 \sin 37^0$
$= \sin(90^0 - 37^0)\cos 37^0 + \cos(90^0 - 37^0)\sin 37^0$
$= \cos 37^0 \cos 37^0 + \sin 37^0 \sin 37^0$
$= \cos^2 37^0 + \sin^2 37^0$
$= 1$
= RHS

(ii) LHS = $\cos 54^0 \cos 36^0 - \sin 54^0 \sin 36^0$
$= \cos(90^0 - 36^0)\cos 36^0 - \sin(90^0 - 36^0)\sin 36^0$
$= \sin 36^0 \cos 36^0 - \cos 36^0 \sin 36^0$
$= 0$
= RHS

(iii) LHS = $\sec 70^0 \sin 20^0 + \cos 20^0 \operatorname{cosec} 70^0$
$= \sec(90^0 - 20^0)\sin 20^0 + \cos 20^0 \operatorname{cosec}(90^0 - 20^0)$
$= \operatorname{cosec} 20^0 \cdot \frac{1}{\operatorname{cosec} 20^0} + \frac{1}{\sec 20^0} \cdot \sec 20^0$
$= 1 + 1$

= 2
= RHS

(iv) LHS = $\sin 35^0 \sin 55^0 - \cos 35^0 \cos 55^0$
= $\sin 35^0 \cos(90^0 - 55^0) - \cos 35^0 \sin(90^0 - 55^0)$
= $\sin 35^0 \cos 35^0 - \cos 35^0 \sin 35^0$
= 0
= RHS

(v) LHS = $(\sin 72^0 + \cos 18^0)(\sin 72^0 - \cos 18^0)$
= $(\sin 72^0 + \cos 18^0)[\cos(90^0 - 72^0) - \cos 18^0]$
= $(\sin 72^0 + \cos 18^0)(\cos 18^0 - \cos 18^0)$
= $(\sin 72^0 + \cos 18^0)(0)$
= RHS

(vi) LHS = $\tan 48^0 \tan 23^0 \tan 42^0 \tan 67^0$
= $\cot(90^0 - 48^0) \cot(90^0 - 23^0) \tan 42^0 \tan 67^0$
= $\cot 42^0 \cot 67^0 \tan 42^0 \tan 67^0$
= $\frac{1}{\tan 42^0} \times \frac{1}{\tan 67^0} \times \tan 42^0 \times \tan 67^0$
= 1
= RHS

4. Without using trigonometric tables, prove that:

(i) $\frac{\sin 70^0}{\cos 20^0} + \frac{\operatorname{cosec} 20^0}{\sec 70^0} - 2\cos 70^0 \operatorname{cosec} 20^0 = 0$

(ii) $\frac{\cos 80^0}{\sin 10^0} + \cos 59^0 \operatorname{cosec} 31^0 = 2$

(iii) $\frac{2\sin 68^0}{\cos 22^0} - \frac{2\cot 15^0}{5\tan 75^0} - \frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5} = 1$

(iv) $\frac{\sin 18^0}{\cos 72^0} + \sqrt{3}(\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0) = 2$

(v) $\frac{7\cos 55^0}{3\sin 35^0} - \frac{4(\cos 70^0 \operatorname{cosec} 20^0)}{3(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)} = 1$

Sol:

(i) LHS = $\frac{\sin 70^0}{\cos 20^0} + \frac{\operatorname{cosec} 20^0}{\sec 70^0} - 2\cos 70^0 \operatorname{cosec} 20^0$
= $\frac{\sin 70^0}{\sin(90^0 - 20^0)} + \frac{\sec(90^0 - 20^0)}{\sec 70^0} - 2\cos 70^0 \sec(90^0 - 20^0)$
= $\frac{\sin 70^0}{\sin 70^0} + \frac{\sec 70^0}{\sec 70^0} - 2\cos 70^0 \sec 70^0$
= $1 + 1 - 2 \times \cos 70^0 \times \frac{1}{\cos 70^0}$
= 2 – 2
= 0
= RHS

(ii) LHS = $\frac{\cos 80^0}{\sin 10^0}$ + $\cos 59^0$ cosec31^0

= $\frac{\cos 80^0}{\sin(90^0 - 10^0)}$ + $\sin(90^0 - 59^0)$ cosec31^0

= $\frac{\cos 80^0}{\cos 80^0}$ + $\sin 31^0$ cosec31^0

= 1 + $\sin 31^0 \times \frac{1}{\sin 31^0}$

= 1 + 1

= 2

= RHS

(iii) LHS = $\frac{2\sin 68^0}{\cos 22^0} - \frac{2\cot 15^0}{5\tan 75^0} - \frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5}$

= $\frac{2\sin 68^0}{\sin(90^0 - 22^0)} - \frac{2\cot 15^0}{5\tan(90^0 - 75^0)} - \frac{3 \times 1 \times \cot(90^0 - 20^0) \times \cot(90^0 - 40^0) \times \tan 50^0 \times \tan 70^0}{5}$

= $\frac{2\sin 68^0}{\sin 68^0)} - \frac{2\cot 15^0}{5\cot 15^0} - \frac{3 \times \cot 70^0\ \cot 50^0 \tan 50^0\ \tan 70^0}{5}$

= 2 - $\frac{2}{5}$ - $\frac{3 \times \frac{1}{\tan 70^0} \times \frac{1}{\tan 50^0} \times \tan 50^0 \times \tan 70^0}{5}$

= 2 - $\frac{2}{5}$ - $\frac{3}{5}$

= $\frac{10-2-3}{5}$

= $\frac{5}{5}$

= 1

= RHS

(iv) LHS = $\frac{\sin 18^0}{\cos 72^0}$ + $\sqrt{3}$ (tan 10^0 tan 30^0 tan 40^0 tan 50^0 tan 80^0)

= $\frac{\sin 18^0}{\sin(90^0 - 72^0)}$ + $\sqrt{3}$ [cot $(90^0 - 10^0) \times \frac{1}{\sqrt{3}} \times$ cot$(90^0 - 40^0) \times \tan 50^0 \times \tan 80^0$]

= $\frac{\sin 18^0}{\sin 18^0}$ + $\sqrt{3}$ $\left(\frac{\cot 80^0 \times \cot 50^0 \times \tan 50^0 \times \tan 80^0}{\sqrt{3}}\right)$

= 1 + $\left(\frac{1}{\tan 80^0} \times \frac{1}{\tan 50^0} \times \tan 50^0 \times \tan 80^0\right)$

= 1 + 1

= 2

= RHS

(v) LHS = $\frac{7\cos 55^0}{3\sin 35^0} - \frac{4(\cos 70^0 cosec\ 20^0)}{3(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)}$

= $\frac{7\cos 55^0}{3\cos(90^0 - 35^0)} - \frac{4\,[\sin(90^0 - 70^0) cosec\ 20^0]}{3\,[\cot(90^0 - 5^0) \times \cot(90^0 - 25^0) \times 1 \times \tan 65^0 \times \tan 85^0}$

= $\frac{7\cos 55^0}{3\cos 55^0} - \frac{4\,(\sin 20^0\ cosec\ 20^0)}{3\,(\cot 85^0 \cot 65^0 \tan 65^0\ \tan 85^0)}$

= $\frac{7}{3} - \frac{4\left(\sin 20^0 \times \frac{1}{\sin 20^0}\right)}{3\left(\frac{1}{\tan 85^0} \times \frac{1}{\tan 65^0} \times \tan 65^0 \times \tan 85^0\right)}$

= $\frac{7}{3}$ - $\frac{4}{3}$

$= \frac{3}{3}$

$= 1$

= RHS

5. Prove that:

(i) $\sin\theta \cos(90^0 - \theta) + \sin(90^0 - \theta) \cos\theta = 1$

(ii) $\frac{\sin\theta}{\cos(90^0 - \theta)} + \frac{\cos\theta}{\sin(90^0 - \theta)} = 2$

(iii) $\frac{\sin\theta \cos(90^0 - \theta)\cos\theta}{\sin(90^0 - \theta)} + \frac{\cos\theta \sin(90^0 - \theta)\sin\theta}{\cos(90^0 - \theta)} = 1$

(iv) $\frac{\cos(90^0 - \theta)\sec(90^0 - \theta)\tan\theta}{\operatorname{cosec}(90^0 - \theta)\sin(90^0 - \theta)\cot(90^0 - \theta)} + \frac{\tan(90^0 - \theta)}{\cot\theta} = 2$

(v) $\frac{\cos(90^0 - \theta)}{1+\sin(90^0 - \theta)} + \frac{1+\sin(90^0 - \theta)}{\cos(90^0 - \theta)} = 2\operatorname{cosec}\theta$

(vi) $\frac{\sec(90^0 - \theta)\operatorname{cosec}\theta - \tan(90^0 - \theta)\cot\theta + \cos^2 25^0 + \cos^2 65^0}{3\tan 27^0 \tan 63^0} = \frac{2}{3}$

(vii) $\cot\theta \tan(90^0 - \theta) - \sec(90^0 - \theta)\operatorname{cosec}\theta + \sqrt{3}\tan 12^0 \tan 60^0 \tan 78^0 = 2$

Sol:

(i) LHS $= \sin\theta \cos(90^0 - \theta) + \sin(90^0 - \theta)\cos\theta$

$= \sin\theta \sin\theta + \cos\theta \cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= 1$

= RHS

Hence proved.

(ii) LHS $= \frac{\sin\theta}{\cos(90^0 - \theta)} + \frac{\cos\theta}{\sin(90^0 - \theta)}$

$= \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\cos\theta}$

$= 1 + 1$

$= 2$

= RHS

Hence proved.

(iii) LHS $= \frac{\sin\theta \cos(90^0 - \theta)\cos\theta}{\sin(90^0 - \theta)} + \frac{\cos\theta \sin(90^0 - \theta)\sin\theta}{\cos(90^0 - \theta)}$

$= \frac{\sin\theta \sin\theta \cos\theta}{\cos\theta} + \frac{\cos\theta \cos\theta \sin\theta}{\sin\theta}$

$= \sin^2\theta + \cos^2\theta$

$= 1$

= RHS

Hence proved.

(iv) LHS = $\frac{\cos(90^0-\theta)\sec(90^0-\theta)\tan\theta}{cosec\,(90^0-\theta)\sin\,(90^0-\theta)\cot\,(90^0-\theta)} + \frac{\tan(90^0-\theta)}{\cot\theta}$

$= \frac{\sin\theta\, cosec\,\theta\tan\theta}{\sec\theta\cos\theta\tan\theta} + \frac{\cot\theta}{\cot\theta}$

= 1 + 1

= 2

= RHS

Hence proved.

(v) LHS = $\frac{\cos(90^0-\theta)}{1+\sin(90^0-\theta)} + \frac{1+\sin(90^0-\theta)}{\cos(90^0-\theta)}$

$= \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta+(1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta+1+\cos^2\theta+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{1+1+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2+2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

$= 2\,\frac{1}{\sin\theta}$

= 2 cosec θ

= RHS

Hence proved.

(vi) LHS = $\frac{\sec(90^0-\theta)cosec\,\theta-\tan(90^0-\theta)\cot\theta+\cos^2 25^0+\cos^2 65^0}{3\tan 27^0\tan 63^0}$

$= \frac{cosec\,\theta\, cosec\,\theta-\cot\theta\cot\theta+\sin^2(90^0-25^0)+\cos^2 65^0}{3\tan 27^0\cot(90^0-63^0)}$

$= \frac{cosec^2\theta-\cot^2\theta+\sin^2 65^0+\cos^2 65^0}{3\tan 27^0\cot 27^0}$

$= \frac{1+1}{3\times\tan 27^0\times\frac{1}{\tan 27^0}}$

$= \frac{2}{3}$

= RHS

(vii) LHS = cot θ tan $(90^0 - \theta)$ – sec $(90^0 - \theta)$ cosec θ + $\sqrt{3}$ tan 12^0 tan 60^0 tan 78^0

= cot θ cot θ - cosec θ cosec θ + $\sqrt{3}$ tan 12^0 × $\sqrt{3}$ × cot $(90^0 - 78^0)$

= $\cot^2\theta$ – $cosec^2\theta$ + 3 tan 12^0 cot 12^0

= -1 + 3 × tan 12^0 × $\frac{1}{\tan 12^0}$

= -1 + 3

= 2

= RHS

6. Without using trigonometric tables, prove that:

(i) $\tan 5^0 \tan 25^0 \tan 30^0 \tan 65^0 \tan 85^0 = 1$

(ii) $\cot 12^0 \cot 38^0 \cot 52^0 \cot 60^0 \cot 78^0 = \frac{1}{\sqrt{3}}$

(iii) $\cos 15^0 \cos 35^0 \operatorname{cosec} 55^0 \cos 60^0 \operatorname{cosec} 75^0 = \frac{1}{2}$

(iv) $\cos 1^0 \cos 2^0 \cos 3^0 \ldots\ldots \cos 180^0 = 0$

(v) $\left(\frac{\sin 49^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\sin 49^0}\right) = 2$

Sol:

(i) LHS = $\tan 5^0 \tan 25^0 \tan 30^0 \tan 65^0 \tan 85^0$

$= \tan(90^0 - 85^0) \tan(90^0 - 65^0) \times \frac{1}{\sqrt{3}} \times \frac{1}{\cot 60^0} \frac{1}{\cot 85^0}$

$= \cot 85^0 \cot 65^0 \frac{1}{\sqrt{3}} \frac{1}{\cot 60^0} \frac{1}{\cot 85^0}$

$= \frac{1}{\sqrt{3}}$ = RHS

(ii) LHS = $\cot 12^0 \cot 38^0 \cot 52^0 \cot 60^0 \cot 78^0$

$= \tan(90^0 - 12^0) \times \tan(90^0 - 38^0) \times \cot 52^0 \times \frac{1}{\sqrt{3}} \times \cot 78^0$

$= \frac{1}{\sqrt{3}} \times \tan 78^0 \times \tan 52^0 \times \cot 52^0 \times \cot 78^0$

$= \frac{1}{\sqrt{3}} \times \tan 78^0 \times \tan 52^0 \times \frac{1}{\tan 52^0} \times \frac{1}{\tan 78^0}$

$= \frac{1}{\sqrt{3}}$

= RHS

(iii) LHS = $\cos 15^0 \cos 35^0 \operatorname{cosec} 55^0 \cos 60^0 \operatorname{cosec} 75^0$

$= \cos(90^0 - 75^0) \cos(90^0 - 55^0) \frac{1}{\sin 55^0} \times \frac{1}{2} \times \frac{1}{\sin 75^0}$

$= \sin 75^0 \sin 55^0 \quad \frac{1}{\sin 55^0} \times \frac{1}{2} \times \frac{1}{\sin 75^0}$

$= \frac{1}{2}$ = RHS

(iv) LHS = $\cos 1^0 \cos 2^0 \cos 3^0 \ldots\ldots \cos 180^0$

$= \cos 1^0 \times \cos 2^0 \times \cos 3^0 \times \ldots\ldots \times \cos 90^0 \times \ldots \times \cos 180^0$

$= \cos 1^0 \times \cos 2^0 \times \cos 3^0 \times \ldots\ldots \times 0 \times \ldots \times \cos 180^0$

= 0

= RHS

(v) LHS = $\left(\frac{\sin 49^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\sin 49^0}\right)$

$= \left(\frac{\cos(90^0 - 49^0)}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\cos(90^0 - 49^0)^0}\right)^2$

$= \left(\frac{\cos 41^0}{\cos 41^0}\right)^2 + \left(\frac{\cos 41^0}{\cos 41^0}\right)^2$

$= 1^2 + 1^2$

= 1 + 1
= 2
= RHS
Disclaimer: The RHS of (v) given in textbook is incorrect. There should be 2 instead 1. The same has been corrected in the solution here.

7. Prove that:

(i) $\sin(70^0 + \theta) - \cos(20^0 - \theta) = 0$

(ii) $\tan(55^0 - \theta) - \cot(35^0 + \theta) = 0$

(iii) $\text{cosec}(67^0 + \theta) - \sec(20^0 - \theta) = 0$

(iv) $\text{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta) = 0$

(v) $\sin(50^0 + \theta) - \cos(40^0 - \theta) + \tan 1^0 \tan 10^0 \tan 80^0 \tan 89^0 = 1$

Sol:

(i) LHS = $\sin(70^0 + \theta) - \cos(20^0 - \theta)$

$= \sin\{90^0 - (20^0 - \theta)\} - \cos(20^0 - \theta)$

$= \cos(20^0 - \theta) - \cos(20^0 - \theta)$

= 0

= RHS

(ii) LHS = $\tan(55^0 - \theta) - \cot(35^0 + \theta)$

$= \tan\{90^0 - (35^0 + \theta)\} - \cot(35^0 + \theta)$

$= \cot(35^0 + \theta) - \cot(35^0 + \theta)$

= 0

= RHS

(iii) LHS = $\text{cosec}(67^0 + \theta) - \sec(23^0 - \theta)$

$= \text{cosec}\{90^0 - (23^0 - \theta)\} - \sec(23^0 - \theta)$

$= \sec(23^0 - \theta) - \sec(23^0 - \theta)$

= 0

= RHS

(iv) LHS = $\text{cosec}(65^0 + \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot(35^0 + \theta)$

$= \text{cosec}\{90^0 - (25^0 - \theta)\} - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \cot\{90^0 - (55^0 - \theta)\}$

$= \sec(25^0 - \theta) - \sec(25^0 - \theta) - \tan(55^0 - \theta) + \tan(55^0 - \theta)$

= 0

= RHS

(v) LHS = $\sin(50^0 + \theta) - \cos(40^0 - \theta) + \tan 1^0 \tan 10^0 \tan 80^0 \tan 89^0$

$= \sin\{90^0 - (40^0 - \theta)\} - \cos(40^0 - \theta) + \{\tan 1^0 \tan(90^0 - 1^0)\}\{\tan 10^0 \tan(90^0 - 10)\}$

$= \cos(40^0 - \theta) - \cos(40^0 - \theta) + (\tan 1^0 \cot 1^0)(\tan 10^0 \cot 10^0)$

$= \left(\frac{1}{\cot 1^0} \times \cot 1^0\right)\left(\tan 10^0 \times \frac{1}{\tan 10^0}\right)$

$= 1 \times 1$

$= 1$

= RHS

8. Express each of the following in terms of trigonometric ratios of angles lying between 0^0 and 45^0:

(i) $\sin 67^0 + \cos 75^0$

(ii) $\cot 65^0 + \tan 49^0$

(iii) $\sec 78^0 + \operatorname{cosec} 56^0$

(iv) $\operatorname{cosec} 54^0 + \sin 72^0$

Sol:

(i) $\sin 67^0 + \cos 75^0$

$= \cos (90^0 - 67^0) + \sin (90^0 - 75^0)$

$= \cos 23^0 + \sin 15^0$

(ii) $\cot 65^0 + \tan 49^0$

$= \cos (90^0 - 65^0) + \cot (90^0 - 49^0)$

$= \cos 25^0 + \cot 41^0$

(iii) $\sec 78^0 + \operatorname{cosec} 56^0$

$= \sec (90^0 - 12^0) + \operatorname{cosec} (90^0 - 34^0)$

$= \operatorname{cosec} 12^0 + \sec 34^0$

(iv) $\operatorname{cosec} 54^0 + \sin 72^0$

$= \sec (90^0 - 54^0) + \cos (90^0 - 72^0)$

$= \sec 36^0 + \cos 18^0$

9. If A, B, C are the angles of a ΔABC, prove that $\tan\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$.

Sol:

In ΔABC,

$A + B + C = 180^0$

$\Rightarrow A + C = 180^0 - B$ ....(i)

Now,

LHS $= \tan\left(\frac{C+A}{2}\right)$

$= \tan\left(\frac{180^0 - B}{2}\right)$ [Using (i)]

$= \tan\left(90^0 - \frac{B}{2}\right)$

$= \cot\frac{B}{2}$

= RHS

10. If $\cos 2\theta = \sin 4\theta$ and 2θ is acute, then find the value of θ.

Sol:

We have,

$\cos 2\theta = \sin 4\theta$

$\Rightarrow \sin(90^0 - 2\theta) = \sin 4\theta$

Comparing both sides, we get

$90^0 - 2\theta = 4\theta$

$\Rightarrow 2\theta + 4\theta = 90^0$

$\Rightarrow 6\theta = 90^0$

$\Rightarrow \theta = \frac{90^0}{6}$

$\therefore \theta = 15^0$

Hence, the value of θ is 15^0.

11. If $\sec 2A = \operatorname{cosec}(A - 42^0)$, where 2A is an acute angle, find the value of A.

Sol:

We have,

$\sec 2A = \operatorname{cosec}(A - 42^0)$

$\Rightarrow \operatorname{cosec}(90^0 - 2A) = \operatorname{cosec}(A - 42^0)$

Comparing both sides, we get

$90^0 - 2A = A - 42^0$

$\Rightarrow 2A + A = 90^0 + 42^0$

$\Rightarrow 3A = 132^0$

$\Rightarrow A = \frac{132^0}{3}$

$\therefore A = 44^0$

Hence, the value of A is 44^0.

12. If $\sin 3A = \cos(A - 26^0)$, where 3A is an acute angle, find the value of A.

Sol:

$\sin 3A = \cos(A - 26^0)$

$\Rightarrow \cos(90^0 - 3A) = \cos(A - 26^0)$ $[\because \sin\theta = \cos(90^0 - \theta)]$

$\Rightarrow 90^0 - 3A = A - 26^0$

$\Rightarrow 116^0 = 4A$

$\Rightarrow A = \frac{116^0}{4} = 29^0$

13. If tan2A = cot (A – 12^0), where 2A is an acute angle, find the value of A.

Sol:

tan2A = cot (A – 12^0)

⇒ cot (90^0 – 2A) = cot (A – 12^0) [∵ tan θ = cot (90^0 – θ)]

⇒ (90^0 – 2A) = (A – 12^0)

⇒ 102^0 = 3A

⇒ A = $\frac{102^0}{3}$ = 34^0

14. If sec 4A = cosec (A – 15^0), where 4A is an acute angle, find the value of A.

Sol:

sec4A = cosec (A – 15^0)

⇒ cosec (90^0 – 4A) = cosec (A – 15^0) [∵ sec θ = cosec (90^0 – θ)]

⇒ (90^0 – 4A) = (A – 15^0)

⇒ 105^0 = 5A

⇒ A = $\frac{105^0}{5}$ = 21^0

15. Without using trigonometric tables, evaluate the following:

Sol:

$\frac{2}{3}$ cosec$^2 58^0$ $-$ $\frac{2}{3}$ cot 58^0 tan 32^0 $-$ $\frac{5}{3}$ tan 13^0 tan 37^0 tan 45^0 tan 53^0 tan 77^0

= $\frac{2}{3}(cosec^2\ 58^0\ -\ cot\ 58^0\ tan\ 32^0)$ $-$ $\frac{5}{3}$ tan 13^0 tan (90^0 – 13^0) tan37^0 tan(90^0 – 37^0) (tan 45^0)

= $\frac{2}{3}\{cosec^2\ 58^0\ -\ cot\ 58^0\ tan(90^0\ -\ 58^0)\}$ $-$ $\frac{5}{3}$ tan 13^0 cot 13^0 tan 37^0 cot 37^0 (1)

= $\frac{2}{3}(cosec^2\ 58^0\ -\ cot\ 58^0\ tan\ 58^0)$ $-$ $\frac{5}{3}$ tan 13^0 $\frac{1}{\tan 13^0}$ tan 37^0 $\frac{1}{\tan 37^0}$

= $\frac{2}{3}(cosec^2\ 58^0\ -\ cot^2 58^0)$ $-$ $\frac{5}{3}$

= $\frac{2}{3}$ $-$ $\frac{5}{3}$

= $-$ 1

Hence proved.