1. Evaluate:
    $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$
    Sol:
    On substituting the values of various T-ratios, we get:
    $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$
    $= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} \right) = \left(\frac{3}{4} + \frac{1}{4} \right) = \frac{4}{4} = 1$

2. Evaluate:
    $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$
    Sol:
    On substituting the values of various T-ratios, we get:
    $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$
    $= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} \right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} \right) = 0$

3. Evaluate:
    $\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$
    Sol:
    On substituting the values of various T-ratios, we get:
    $\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$
    $= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right) = \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \right) = \left(\frac{\sqrt{3}+1}{2\sqrt{2}} \right)$

4. Evaluate:
    $\frac{\sin 30^0}{\cos 45^0} + \frac{\cot 45^0}{\sec 60^0} - \frac{\sin 60^0}{\tan 45^0} + \frac{\cos 30^0}{\sin 90^0}$
    Sol:
    $\frac{\sin 30^0}{\cos 45^0} + \frac{\cot 45^0}{\sec 60^0} - \frac{\sin 60^0}{\tan 45^0} + \frac{\cos 30^0}{\sin 90^0}$
    $= \frac{\left(\frac{1}{2} \right)}{\left(\frac{1}{\sqrt{2}} \right)} + \frac{1}{2} - \frac{\left(\frac{\sqrt{3}}{2} \right)}{1} + \frac{\left(\frac{\sqrt{3}}{2} \right)}{1}$
    $= \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$
    $= \frac{\sqrt{2} + 1}{2}$

5. Evaluate:
    $\frac{5 \cos^2 60^0 + 4 \sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$
    Sol:
    $\frac{5 \cos^2 60^0 + 4 \sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{\left(\frac{5}{4} + \frac{4\times4}{3} - 1\right)}{\left(\frac{1}{4} + \frac{3}{4}\right)}$$

$$= \frac{\left(\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right)}{\left(\frac{4}{4}\right)}$$

$$= \frac{\left(\frac{15+64-12}{12}\right)}{\left(\frac{4}{4}\right)}$$

$$= \frac{\left(\frac{67}{12}\right)}{(1)}$$

$$= \frac{67}{12}$$

6. Evaluate:

   $2\cos^2 60^0 + 3\sin^2 45^0 - 3\sin^2 30^0 + 2\cos^2 90^0$

   Sol:

   On substituting the values of various T-ratios, we get:

   $2\cos^2 60^0 + 3\sin^2 45^0 - 3\sin^2 30^0 + 2\cos^2 90^0$

   $= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2 \times (0)^2$

   $= 2 \times \frac{1}{4} + 3 \times \frac{1}{2} - 3 \times \frac{1}{4} + 0$

   $= \left(\frac{1}{2} + \frac{3}{2} - \frac{3}{4}\right) = \left(\frac{2+6-3}{4}\right) = \frac{5}{4}$

7. Evaluate:

   $\cot^2 30^0 - 2\cos^2 30^0 - \frac{3}{4}\sec^2 45^0 + \frac{1}{4}\csc^2 30^0$

   Sol:

   On substituting the values of various T-ratios, we get:

   $\cot^2 30^0 - 2\cos^2 30^0 - \frac{3}{4}\sec^2 45^0 + \frac{1}{4}\csc^2 30^0$

   $= \left(\sqrt{3}\right)^2 - 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4} \times \left(\sqrt{2}\right)^2 + \frac{1}{4} \times (2)^2$

   $= 3 - 2 \times \frac{3}{4} - \frac{3}{4} \times 2 + \frac{1}{4} \times 4$

   $= 3 - \frac{3}{2} - \frac{3}{2} + 1$

   $= 4 - \left(\frac{3}{2} + \frac{3}{2}\right)$

   $= 4 - 3 = 1$

8. Evaluate:

$(\sin^2 30^0 + 4 \cot^2 45^0 - \sec^2 60^0)(\text{cosec}^2 45^0 \sec^2 30^0)$

Sol:

On substituting the values of various T-ratios, we get:

$(\sin^2 30^0 + 4 \cot^2 45^0 - \sec^2 60^0)(\text{cosec}^2 45^0 \sec^2 30^0)$

$= \left[\left(\frac{1}{2} \right)^2 + 4 \times (1)^2 - (2)^2 \right] \left[\left(\sqrt{2} \right)^2 \left(\frac{2}{\sqrt{3}} \right)^2 \right]$

$= \left(\frac{1}{4} + 4 - 4 \right) \left(2 \times \frac{4}{3} \right)$

$= \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}$

9. Evaluate:

$\frac{4}{cot^2 30^0} + \frac{1}{sin^2 30^0} - 2 \cos^2 45^0 - \sin^2 0^0$

Sol:

On substituting the values of various T-ratios, we get:

$\frac{4}{cot^2 30^0} + \frac{1}{sin^2 30^0} - 2 \cos^2 45^0 - \sin^2 0^0$

$= \frac{4}{\left(\sqrt{3} \right)^2} + \frac{1}{\left(\frac{1}{2} \right)^2} - 2 \times \left(\frac{1}{\sqrt{2}} \right)^2 - (0)^2$

$= \frac{4}{3} + \frac{1}{\frac{1}{4}} - 2 \times \frac{1}{2} - 0$

$= \frac{4}{3} + 4 - 1$

$= \frac{4}{3} + 3 = \frac{4+9}{3} = \frac{13}{3}$

10. Show that:

(i) $\frac{1-\sin 60^0}{\cos 60^0} = \frac{\tan 60^0 - 1}{\tan 60^0 + 1}$

(ii) $\frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \cos 30^0$

Sol:

(i)

$\text{LHS} = \frac{1 - \sin 60^0}{\cos 60^0} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\left(\frac{2 - \sqrt{3}}{2} \right)}{\frac{1}{2}} = \left(\frac{2 - \sqrt{3}}{2} \right) \times 2 = 2 - \sqrt{3}$

$\text{RHS} = \frac{\tan 60^0 - 1}{\tan 60^0 + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1} = \frac{\left(\sqrt{3} - 1 \right)^2}{\left(\sqrt{3} \right)^2 - 1^2} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$

Hence, LHS = RHS

$\therefore \frac{1 - \sin 60^0}{\cos 60^0} = \frac{\tan 60^0 - 1}{\tan 60^0 + 1}$

(ii)

$\text{LHS} = \frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \frac{\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right)}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\frac{\sqrt{3} + \sqrt{3}}{2}}{\frac{2+1+1}{2}} = \frac{\sqrt{3}}{2}$

Also, RHS = $\cos 30^0 = \frac{\sqrt{3}}{2}$

Hence, LHS = RHS

$\therefore \frac{\cos 30^0 + \sin 60^0}{1 + \sin 30^0 + \cos 60^0} = \cos 30^0$

11. Verify each of the following:
(i) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$
(ii) $\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$
(iii) $2 \sin 30^0 \cos 30^0$
(iv) $2 \sin 45^0 \cos 45^0$

Sol:

(i) $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

Also, $\sin 30^0 = \frac{1}{2}$

$\therefore \sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0 = \sin 30^0$

(ii) $\cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2}\right) \times \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right) \times \left(\frac{1}{2}\right) = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Also, $\cos 30^0 = \frac{\sqrt{3}}{2}$

$\therefore \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0 = \cos 30^0$

(iii) $2 \sin 30^0 \cos 30^0$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Also, $\sin 60^0 = \frac{\sqrt{3}}{2}$

$\therefore 2 \sin 30^0 \cos 30^0 = \sin 60^0$

(iv) $2 \sin 45^0 \cos 45^0$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

Also, $\sin 90^0 = 1$

$\therefore 2 \sin 45^0 \cos 45^0 = \sin 90^0$

12. If $A = 45^0$, verify that:
(i) $\sin 2A = 2 \sin A \cos A$
(ii) $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

Sol:

$A = 45^0$

$\Rightarrow 2A = 2 \times 45^0 = 90^0$

(i) $\sin 2A = \sin 90^0 = 1$

$2 \sin A \cos A = 2 \sin 45^0 \cos 45^0 = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

$\therefore \sin 2A = 2 \sin A \cos A$

(ii) $\cos 2A = \cos 90^0 = 0$

$2 \cos^2 A - 1 = 2 \cos^2 45^0 - 1 = 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$

Now, $1 - 2 \sin^2 A = 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 1 = 1 - 2 \times \frac{1}{2} = 1 - 1 = 0$

$\therefore \cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

13. If $A = 30^0$, verify that:

(i) $\sin 2A = \frac{2 \tan A}{1 + tan^2 A}$

(ii) $\cos 2A = \frac{1 - tan^2 A}{1 + tan^2 A}$

(iii) $\tan 2A = \frac{2 \tan A}{1 - tan^2 A}$

Sol:

$A = 30^0$

$\Rightarrow 2A = 2 \times 30^0 = 60^0$

(i) $\sin 2A = \sin 60^0 = \frac{\sqrt{3}}{2}$

$\frac{2 \tan A}{1 + tan^2 A} = \frac{2 \tan 30^0}{1 + tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{4}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{4} = \frac{\sqrt{3}}{2}$

$\therefore \sin 2A = \frac{2 \tan A}{1 + tan^2 A}$

(ii) $\cos 2A = \cos 60^0 = \frac{1}{2}$

$\frac{1 - tan^2 A}{1 + tan^2 A} = \frac{1 - tan^2 30^0}{1 + tan^2 30^0} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(1 - \frac{1}{3}\right)}{1 + \frac{1}{3}} = \frac{\left(\frac{2}{3}\right)}{\frac{4}{3}} = \left(\frac{2}{3}\right) \times \frac{3}{4} = \frac{1}{2}$

$\therefore \cos 2A = \frac{1 - tan^2 A}{1 + tan^2 A}$

(iii) $\tan 2A = \tan 60^0 = \sqrt{3}$

$\frac{2 \tan A}{1 - tan^2 A} = \frac{2 \tan 30^0}{1 - tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$

$\therefore \tan 2A = \frac{2 \tan A}{1 - tan^2 A}$

14. If $A = 60^0$ and $B = 30^0$, verify that:

(i) $\sin (A + B) = \sin A \cos B + \cos A \sin B$

(ii) $\cos (A + B) = \cos A \cos B - \sin A \sin B$

Sol:

$A = 60^0$ and $B = 30^0$

Now, $A + B = 60^0 + 30^0 = 90^0$

Also, $A - B = 60^0 - 30^0 = 30^0$

(i) $\sin (A + B) = \sin 90^0 = 1$

$\sin A \cos B + \cos A \sin B = \sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} + \frac{1}{4}\right) = 1$

$\therefore \sin (A + B) = \sin A \cos B + \cos A \sin B$

(ii) $\cos (A + B) = \cos 90^0 = 0$

$\cos A \cos B - \sin A \sin B = \cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right) = 0$

$\therefore \cos (A + B) = \cos A \cos B - \sin A \sin B$

15. If $A = 60^0$ and $B = 30^0$, verify that:

(i) $\sin (A - B) = \sin A \cos B - \cos A \sin B$

(ii) $\cos (A - B) = \cos A \cos B + \sin A \sin B$

(iii) $\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Sol:

(i) $\sin (A - B) = \sin 30^0 = \frac{1}{2}$

$\sin A \cos B - \cos A \sin B = \sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{2}{4} = \frac{1}{2}$

$\therefore \sin (A - B) = \sin A \cos B - \cos A \sin B$

(ii) $\cos (A - B) = \cos 30^0 = \frac{\sqrt{3}}{2}$

$\cos A \cos B + \sin A \sin B = \cos 60^0 \cos 30^0 + \sin 60^0 \sin 30^0$

$= \left(\frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right) = \left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right) = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\therefore \cos (A - B) = \cos A \cos B + \sin A \sin B$

(iii) $\tan (A - B) = \tan 60^0 = \frac{1}{\sqrt{3}}$

$\frac{\tan A - \tan B}{1 + \tan A \tan B} = \frac{\tan 60^0 - \tan 30^0}{1 + \tan 60^0 \tan 30^0} = \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\sqrt{3} \times \frac{1}{\sqrt{3}}\right)} = \frac{1}{2} \times \frac{3-1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$

$\therefore \tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

16. If A and B are acute angles such that $\tan A = \frac{1}{3}$, $\tan B = \frac{1}{2}$ and $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$, show that $A + B = 45^0$.

Sol:

Given:

$\tan A = \frac{1}{3}$ and $\tan B = \frac{1}{2}$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

On substituting these values in RHS of the expression, we get:

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\left(\frac{1}{3} + \frac{1}{2}\right)}{\left(1 - \frac{1}{3} \times \frac{1}{3}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(1 - \frac{1}{6}\right)} = \frac{\left(\frac{5}{6}\right)}{\left(\frac{5}{6}\right)} = 1$

$\Rightarrow \tan(A + B) = 1 = \tan 45^0$ [$\because \tan 450 = 1$]

$\therefore A + B = 45^0$

17. Using the formula, $\tan 2A = \frac{2 \tan A}{1 - tan^2 A}$, find the value of $\tan 60^0$, it being given that $\tan 30^0 = \frac{1}{\sqrt{3}}$.

Sol:

$A = 30^0$

$\Rightarrow 2A = 2 \times 30^0 = 60^0$

By substituting the value of the given T-ratio, we get:

$\tan 2A = \frac{2 \tan A}{1 - tan^2 A}$

$\Rightarrow \tan 60^0 = \frac{2 \tan 30^0}{1 - tan^2 30^0} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{1 - \frac{1}{3}} = \frac{\left(\frac{2}{\sqrt{3}}\right)}{\frac{2}{3}} = \left(\frac{2}{\sqrt{3}}\right) \times \frac{3}{2} = \sqrt{3}$

$\therefore \tan 60^0 = \sqrt{3}$

18. Using the formula, $\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$, find the value of $\cos 30^0$, it being given that $\cos 60^0 = \frac{1}{2}$.

Sol:

$A = 30^0$

$\Rightarrow 2A = 2 \times 30^0 = 60^0$

By substituting the value of the given T-ratio, we get:

$\cos A = \sqrt{\frac{1 + \cos 2A}{2}}$

$\cos 30^0 = \sqrt{\frac{1 + \cos 60^0}{2}} = \sqrt{\frac{1 + \frac{1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\therefore \cos A = \frac{\sqrt{3}}{2}$

19. Using the formula, sin A = $\sqrt{\frac{1-\cos 2A}{2}}$, find the value of sin 30^0, it being given that cos 60^0 = $\frac{1}{2}$.

Sol:

A = 30^0

$\Rightarrow$ 2A = 2 $\times$ 30^0 = 60^0

By substituting the value of the given T-ratio, we get:

sin A = $\sqrt{\frac{1-\cos 2A}{2}}$

sin 30^0 = $\sqrt{\frac{1-\cos 60^0}{2}}$ = $\sqrt{\frac{1-\frac{1}{2}}{2}}$ = $\sqrt{\frac{\frac{1}{2}}{2}}$ = $\sqrt{\frac{1}{4}}$ = $\frac{1}{2}$

$\therefore$ sin 30^0 = $\frac{1}{2}$

20. In the adjoining figure, ΔABC is a right-angled triangle in which $\angle$B = 90^0, $\angle 30^0$ and AC = 20cm.

Find (i) BC, (ii) AB.

Sol:

From the given right-angled triangle, we have:

$\frac{BC}{AC}$ = sin 30^0

$\Rightarrow \frac{BC}{20}$ = $\frac{1}{2}$

$\Rightarrow$ BC = $\frac{20}{2}$ = 10cm

Also, $\frac{AB}{AC}$ = cos 30^0

$\Rightarrow \frac{AB}{20}$ = $\frac{\sqrt{3}}{2}$

$\Rightarrow$ AB = $\left(20 \times \frac{\sqrt{3}}{2}\right)$ = 10$\sqrt{3}$ cm

$\therefore$ BC = 10cm and AB = 10$\sqrt{3}$ cm

21. In the adjoining figure, ΔABC is right-angled at B and $\angle$A = 30^0. If BC = 6cm, find (i) AB, (ii) AC.

Sol:

From the given right-angled triangle, we have:

$\frac{BC}{AB}$ = tan 30^0

$\Rightarrow \frac{6}{AB}$ = $\frac{1}{\sqrt{3}}$
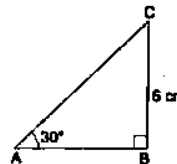
$\Rightarrow$ AB = 6$\sqrt{3}$cm

Also, $\frac{BC}{AC}$ = sin 30^0

$\Rightarrow \frac{6}{AC}$ = $\frac{1}{2}$

$\Rightarrow$ AC = (2 $\times$ 6) = 12 cm

$\therefore$ AB = 6$\sqrt{3}$ cm and AC = 12 cm

22. In the adjoining figure, ∆ABC is right-angled at B and ∠A = 45^0. If AC = $3\sqrt{2}$cm, find (i) BC, (ii) AB.

Sol:

From the right-angled ∆ABC, we have:

$\frac{BC}{AC}$ = sin 45^0

⇒ $\frac{BC}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$ ⇒ BC = 3cm

Also, $\frac{AB}{AC}$ = cos 45^0

⇒ $\frac{AB}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$ ⇒ AB = 3 cm

∴ BC = 3 cm and AB = 3 cm

23. If sin (A + B) = 1 and cos (A – B) = 1, $0^0 \le$ (A + B) $\le 90^0$ and A > B, then find A and B.

Sol:

Here, sin (A + B) = 1

⇒ sin (A + B) = 90^0 [∵ sin 90^0 = 1]

⇒ (A + B) = 90^0 ……(i)

Also, cos (A – B) = 1

⇒ cos (A – B) = 0^0 [∵ cos 0^0 = 1]

⇒ A – B = 0^0 ….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 45^0

24. If sin (A – B) = $\frac{1}{2}$ and cos (A + B) = $\frac{1}{2}$, $0^0 \le$ (A + B) $\le 90^0$ and A > B, then find A and B.

Sol:

Here, sin (A – B) = $\frac{1}{2}$

⇒ sin (A – B) = 30^0 [∵ sin 30^0 = $\frac{1}{2}$]

⇒ (A – B) = 30^0 ……(i)

Also, cos (A + B) = $\frac{1}{2}$

⇒ cos (A + B) = cos 60^0 [∵ cos 60^0 = $\frac{1}{2}$]

⇒ A + B = 60^0 ….(ii)

Solving (i) and (ii), we get:

A = 45^0 and B = 15^0

25. If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, $0^0 \le$ (A + B) $\le 90^0$ and A > B, then find A and B.

Sol:

Here, tan (A – B) = $\frac{1}{\sqrt{3}}$

⇒ tan (A – B) = tan 30^0 [∵ tan 30^0 = $\frac{1}{\sqrt{3}}$]

⇒ (A – B) = 30^0 ……(i)

Also, tan (A + B) = $\sqrt{3}$

$\Rightarrow \tan (A + B) = \tan 60^0$ $[\because \tan 60^0 = \sqrt{3}]$

$\Rightarrow A + B = 60^0$ …….(ii)

Solving (i) and (ii), we get:

$A = 45^0$ and $B = 15^0$

26. If $3x = \text{cosec}\,\theta$ = and $\dfrac{3}{x} = \cot \theta$ find the value of $3\left(x^2 - \dfrac{1}{x^2} \right)$

Sol:

$3\left(x^2 - \dfrac{1}{x^2} \right)$

$= \dfrac{9}{3}\left(x^2 - \dfrac{1}{x^2} \right)$

$= \dfrac{1}{3}\left(9x^2 - \dfrac{9}{x^2} \right)$

$= \dfrac{1}{3}\left[(3x^2) - \left(\dfrac{3}{x}\right)^2 \right]$

$= \dfrac{1}{3}[(\text{cosec}\,\theta)^2 - (\cot \theta)^2]$

$= \dfrac{1}{3}(\text{cosec}^2\theta - \cot^2\theta)$

$= \dfrac{1}{3}(1) = \dfrac{1}{3}$

27. If $\sin (A+B) = \sin A \cos B + \cos A \sin B$ and $\cos (A-B) = \cos A \cos B + \sin A \sin B$

(i) $\sin (75^0)$

(ii) $\cos (15^0)$

Sol:

Let $A = 45^0$ and $B = 30^0$

(i) As, $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\Rightarrow \sin (45^0 + 30^0) = \sin 45^0 \cos 30^0 + \cos 45^0 \sin 30^0$

$\Rightarrow \sin (75^0) = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$\Rightarrow \sin (75^0) = \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$

$\therefore \sin (75^0) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) As, $\cos (A - B) = \cos A \cos B + \sin A \sin B$

$\Rightarrow \cos (45^0 - 30^0) = \cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

$\Rightarrow \cos (15^0) = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2}$

$\Rightarrow \cos (15^0) = \dfrac{\sqrt{3}}{2\sqrt{2}} + \dfrac{1}{2\sqrt{2}}$

$\therefore \cos (15^0) = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

Disclaimer: $\cos 15^0$ can also be written by taking $A = 60^0$ and $B = 45^0$.