Exercise – 14.1

1. A tower stands vertically on the ground. From a point on the ground which is 20 m away from the foot of the tower, the angle of elevation of its top is found to be 60°. Find the height of the tower. [Take $\sqrt{3} = 1.732$]

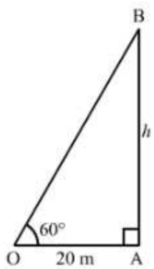
Sol:

Let AB be the tower standing vertically on the ground and O be the position of the obsrever we now have:

$OA = 20m, \angle OAB = 90^\circ \ and \ \angle AOB = 60^\circ$

Let

$AB = hm$



Now, in the right ΔOAB, we have:

$$\frac{AB}{OA} - \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{h}{20} = \sqrt{3}$$

$$\Rightarrow h = 20\sqrt{3} = (20 \times 1.732) = 36.64$$

Hence, the height of the pole is $34.64 \ m$.

2. A kite is flying at a height of 75 in from the level ground, attached to a string inclined at 60°. to the horizontal. Find the length of the string, assuming that there is no slack in it. [Take $\sqrt{3} = 1.732$]

Sol:

Let OX be the horizontal ground and A be the position of the kite.

Also, let O be the position of the observer and OA be the thread.

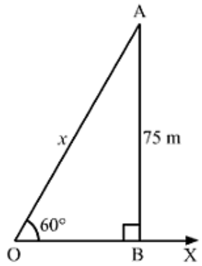
Now, draw $AB \perp OX$.

We have:

$\angle BOA = 60^\circ, OA = 75m \ and \ \angle OBA = 90^\circ$

Height of the kite from the ground $= AB = 75 \ m$

Length of the string $OA = x m$



In the right ΔOBA, we have:

$$\frac{AB}{OA} = \sin 60° = \frac{\sqrt{3}}{2}$$

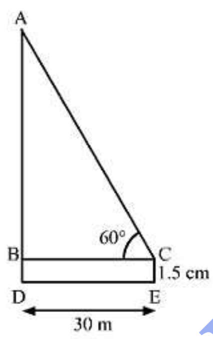
$$\Rightarrow \frac{75}{x} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = \frac{75 \times 2}{\sqrt{3}} = \frac{150}{1.732} = 86.6\,m$$

Hence, the length of the string is $86.6\,m$

3. An observer 1.5m tall is 30 away from a chimney. The angle of elevation of the top of the chimney from his eye is 60°. Find the height of the chimney.

Sol:



Let CE and AD be the heights of the observer and the chimney, respectively.

We have,

$BD = CE = 1.5\ m,\ BC = DE = 30\ m$ and $\angle ACB = 60°$

In ΔABC

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AD - BD}{30}$$

$$\Rightarrow AD - 1.5 = 30\sqrt{3}$$

$$\Rightarrow AD = 30\sqrt{3} + 1.5$$

$$\Rightarrow AD = 30 \times 1.732 + 1.5$$

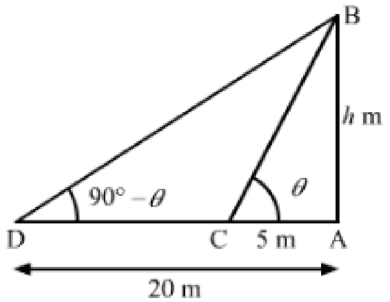
$$\Rightarrow AD = 51.96 + 1.5$$

$\Rightarrow AD = 53.46\ m$

So, the height of the chimney is 53.46 m (approx).

4. The angles of elevation of the top of a tower from two points at distance of 5 metres and 20 metres from the base of the tower and is the same straight line with it, are complementary. Find the height of the tower.

Sol:



Let the height of the tower be AB.
We have.

$AC = 5m, AD = 20m$

Let the angle of elevation of the top of the tower $\left(i.e.\ \angle ACB\right)$ from point C be θ.

Then,

the angle of elevation of the top of the tower (i.e. Z ADB) from point D

$= \left(90° - \theta\right)$

Now, in ΔABC

$$\tan\theta = \frac{AB}{AC}$$

$$\Rightarrow \tan\theta = \frac{AB}{5} \qquad ....(i)$$

Also, in ΔABD,

$$\cot\left(90° - \theta\right) = \frac{AD}{AB}$$

$$\Rightarrow \tan\theta = \frac{20}{AB} \qquad ....(ii)$$

From (i) and (ii), we get

$$\frac{AB}{5} = \frac{20}{AB}$$

$$\Rightarrow AB^2 = 100$$

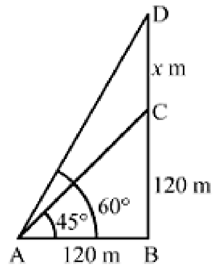
$$\Rightarrow AB = \sqrt{100}$$

$$\therefore AB = 10m$$

So, the height of the tower is 10 m.

5. The angle of elevation of the top of a tower at a distance of 120 m from a point A on the ground is 45°. If the angle of elevation of the top of a flagstaff fixed at the top of the tower, at A is 60°, then find the height of the flagstaff $[Use\ \sqrt{3}=1.732]$

Sol:



Let BC and CD be the heights of the tower and the flagstaff, respectively.

We have,

$AB=120m, \angle BAC=45°, \angle BAD=60°$

Let $CD=x$

In ΔABC,

$$\tan 45° = \frac{BC}{AB}$$

$$\Rightarrow 1 = \frac{BC}{120}$$

$$\Rightarrow BC = 120m$$

Now, in ΔABD,

$$\tan 60° = \frac{BD}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{BC+CD}{120}$$

$$\Rightarrow BC+CD = 120\sqrt{3}$$

$$\Rightarrow 120+x = 120\sqrt{3}$$

$$\Rightarrow x = 120\sqrt{3}-120$$

$$\Rightarrow x = 120\left(\sqrt{3}-1\right)$$

$$\Rightarrow x = 120(1.732-1)$$

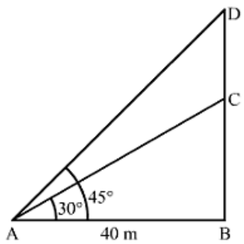
$$\Rightarrow x = 120(0.732)$$

$$\Rightarrow x = 87.84 \approx 87.8m$$

So, the height of the flagstaff is 87. 8 m.

6. From a point on the ground 40m away from the foot of a tower, the angle of elevation of the top of the tower is 30°. The angle of elevation of the top of a water tank (on the top of the tower) is 45°, Find (i) the height of the tower, (ii) the depth of the tank.

Sol:



Let BC be the tower and CD be the water tank.

We have,

$AB = 40m, \angle BAC = 30^\circ$ *and* $\angle BAD = 45^\circ$

In ΔABD,

$$\tan 45^\circ = \frac{BD}{AB}$$

$$\Rightarrow 1 = \frac{BD}{40}$$

$$\Rightarrow BD = 40m$$

Now, in ΔABC,

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{40}$$

$$\Rightarrow BC = \frac{40}{\sqrt{3}}$$

$$\Rightarrow BC = \frac{40}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow BC = \frac{40\sqrt{3}}{3} m$$

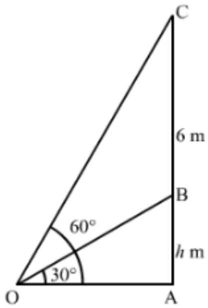
(i) The height of the tower, $BC = \frac{40\sqrt{3}}{3} = \frac{40 \times 1.73}{3} = 23.067 \approx 23.1m$

(ii) The depth of the tank, $CD = (BD - BC) = (40 - 23.1) = 16.9m$

7. The vertical tower stands on a horizontal plane and is surmounted by a vertical flagstaff of height 6m. At a point on the plane, the angle of elevation of the bottom of the flagstaff is 30° and that of the top of the flagstaff 60°. Find the height of the tower

[Use $\sqrt{3} = 1.732$]

Sol:



Let AB be the tower and BC be the flagstaff,

We have,

$BC = 6m, \angle AOB = 30^\circ \text{ and } \angle AOC - 60^\circ$

Let $AB = h$

In ΔAOB,

$$\tan 30^\circ = \frac{AB}{OA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{OA}$$

$$\Rightarrow OA = h\sqrt{3} \qquad \text{..........}(i)$$

Now, in ΔAOC,

$$\tan 60^\circ = \frac{AC}{OA}$$

$$\Rightarrow \sqrt{3} = \frac{AB + BC}{h\sqrt{3}} \qquad \text{[Using (i)]}$$

$$\Rightarrow 3h = h + 6$$

$$\Rightarrow 3h - h = 6$$

$$\Rightarrow 2h = 6$$

$$\Rightarrow h = \frac{6}{2}$$

$$\Rightarrow h = 3\ m$$

So, the height of the tower is 3 m.

8. A statue 1.46m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the status is 60° and from the same point, the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Sol:

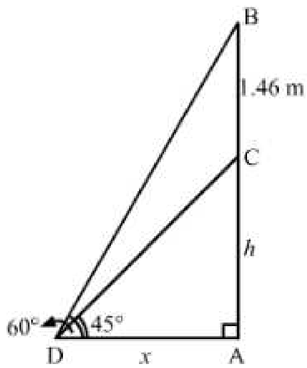
Let AC be the pedestal and BC be the statue such that $BC = 1.46m$.

We have:

$\angle ADC = 45^\circ$ *and* $\angle ADB = 60^\circ$

Let:

$AC = hm$ *and* $AD = xm$



In the right ΔADC, we have:

$$\frac{AC}{AD} = \tan 45^\circ = 1$$

$$\Rightarrow \frac{h}{x} = 1$$

$$\Rightarrow h = x$$

Or,

$$x = h$$

Now, in the right ΔADB, we have:

$$\frac{AB}{AD} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \frac{h + 1.46}{x} = \sqrt{3}$$

On putting $x = h$ in the above equation, we get

$$\frac{h + 1.46}{h} = \sqrt{3}$$

$$\Rightarrow h + 1.46 = \sqrt{3}h$$

$$\Rightarrow h\left(\sqrt{3} - 1\right) = 1.46$$

$$\Rightarrow h = \frac{1.46}{\left(\sqrt{3} - 1\right)} = \frac{1.46}{0.73} = 2m$$

Hence, the height of the pedestal is 2 m.

9. The angle of elevation of the top of an unfinished tower at a distance of 75m from its base is $30°$. How much higher must the tower be raised so that the angle of elevation of its top at the same point may be 60°.

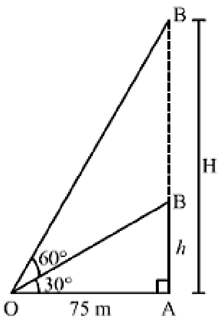
Sol:

Let AB be the unfinished tower, AC be the raised tower and O be the point of observation

We have:

$OA = 75m, \angle AOB = 30°$ *and* $\angle AOC = 60°$

Let $AC = H$ m such that $BC = (H - h)m.$



In ΔAOB, we have:

$$\frac{AB}{OA} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{75} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{75}{\sqrt{3}}m = \frac{75 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 25\sqrt{3}\,m$$

In ΔAOC, we have:

$$\frac{AC}{OA} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{H}{75} = \sqrt{3}$$

$$\Rightarrow H = 75\sqrt{3}m$$

$\therefore$ Required height $= (H - h) = \left(75\sqrt{3} - 25\sqrt{3}\right) = 50\sqrt{3}m = 86.6\,m$

10. On a horizonal plane there is a vertical tower with a flagpole on the top of the tower. At a point, 9 meters away from the foot of the tower, the angle of elevation of the top and bottom of the flagpole are 60° and 30° respectively. Find the height of the tower and the flagpole mounted on it.

Sol:

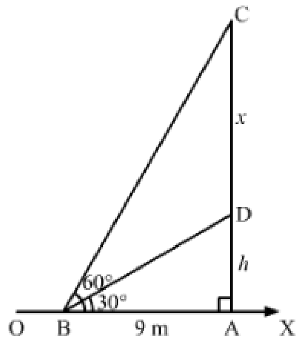
Let OX be the horizontal plane, AD be the tower and CD be the vertical flagpole

We have:

$AB = 9m, \angle DBA = 30°$ and $\angle CBA = 60°$

Let:

$AD = hm$ and $CD = xm$



In the right ΔABD, we have:

$$\frac{AD}{AB} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{9} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{9}{\sqrt{3}} = 5.19\ m$$

Now, in the right ΔABC, we have

$$\frac{AC}{BA} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{h+x}{9} = \sqrt{3}$$

$$\Rightarrow h + x = 9\sqrt{3}$$

By putting $h = \frac{9}{\sqrt{3}}$ in the above equation, we get:

$$\frac{9}{\sqrt{3}} + x = 9\sqrt{3}$$

$$\Rightarrow x = 9\sqrt{3} - \frac{9}{\sqrt{3}}$$

$$\Rightarrow x = \frac{27-9}{\sqrt{3}} = \frac{18}{\sqrt{3}} = \frac{18}{1.73} = 10.4$$

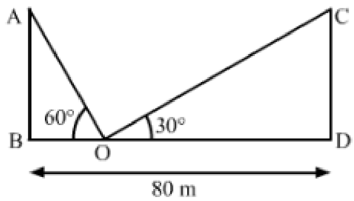
Thus, we have:

Height of the flagpole = 10. 4 m

Height of the tower = 5. 19 m

11. Two poles of equal heights are standing opposite to each other on either side of the road which is 80m wide, From a point P between them on the road, the angle of elevation of the top of one pole is 60° and the angle of depression from the top of another pole at P is 30°. Find the height of each pole and distance of the point P from the poles.

Sol:



Let AB and CD be the equal poles; and BD be the width of the road.

We have,

$\angle AOB = 60^\circ$ *and* $\angle COD = 60^\circ$

In ΔAOB,

$$\tan 60^\circ = \frac{AB}{BO}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{BO}$$

$$\Rightarrow BO = \frac{AB}{\sqrt{3}}$$

Also, in ΔCOD,

$$\tan 30^\circ = \frac{CD}{DO}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{DO}$$

$$\Rightarrow DO = \sqrt{3}CD$$

As, $BD = 80$

$$\Rightarrow BO + DO = 80$$

$$\Rightarrow \frac{AB}{\sqrt{3}} + \sqrt{3}CD = 80$$

$$\Rightarrow \frac{AB}{\sqrt{3}} + \sqrt{3}AB = 80 \qquad \text{(Given: AB = CD)}$$

$$\Rightarrow AB\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = 80$$

$$\Rightarrow AB\left(\frac{1+3}{\sqrt{3}}\right) = 80$$

$$\Rightarrow AB\left(\frac{4}{\sqrt{3}}\right) = 80$$

$\Rightarrow AB = \dfrac{80+\sqrt{3}}{4}$

$\Rightarrow AB = 20\sqrt{3}m$

Also, $BO = \dfrac{AB}{\sqrt{3}} = \dfrac{20\sqrt{3}}{\sqrt{3}} = 20\,m$

So, $DO = 80 - 20 = 60\,m$

Hence, the height of each pole is $20\sqrt{3}\,m$ and point P is at a distance of 20 m from left pole ad 60 m from right pole.

12. Two men are on opposite side of tower. They measure the angles of elevation of the top of the tower as 30° and 45° respectively. If the height of the tower is 50 meters, find the distance between the two men.

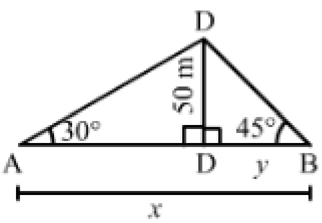
Sol:

Let CD be the tower and A and B be the positions of the two men standing on the opposite sides.

Thus, we have:

$\angle DAC = 30^\circ, \angle DBC = 45^\circ$ *and* $CD = 50\ m$

Let $AB = x\,m$ and $BC = y\,m$ such that $AC = (x-y)\,m.$



In the right ΔDBC, we have:

$\dfrac{CD}{BC} = \tan 45^\circ = 1$

$\Rightarrow \dfrac{50}{y} = 1$

$\Rightarrow y = 50\,m$

In the right ΔACD, we have:

$\dfrac{CD}{AC} = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{50}{(x-y)} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow x - y = 50\sqrt{3}$

On putting $y = 50$ in the above equation, we get:

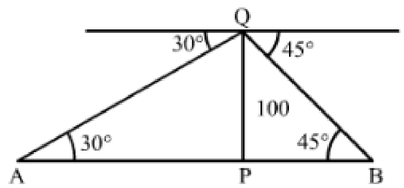
$x - 50 = 50\sqrt{3}$

$\Rightarrow x = 50 + 50\sqrt{3} = 50\left(\sqrt{3}+1\right) = 136.6\,m$

$\therefore$ Distance between the two men $= AB = x = 136.6\,m$

13. From the point of a tower 100m high, a man observe two cars on the opposite sides to the tower with angles of depression 30° and 45° respectively. Find the distance between the cars

Sol:



Let PQ be the tower

We have,

$PQ = 100m, \angle PQR = 30^\circ \text{ and } \angle PBQ = 45^\circ$

In ΔAPQ,

$\tan 30^\circ = \dfrac{PQ}{AP}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{100}{AP}$

$\Rightarrow AP = 100\sqrt{3}\ m$

Also, in ΔBPQ,

$\tan 45^\circ = \dfrac{PQ}{BP}$

$\Rightarrow 1 = \dfrac{100}{BP}$

$\Rightarrow BP = 100\,m$

Now, $AB = AP + BP$

$= 100\sqrt{3} + 100$

$= 100\left(\sqrt{3}+1\right)$

$= 100 \times (1.73 + 1)$

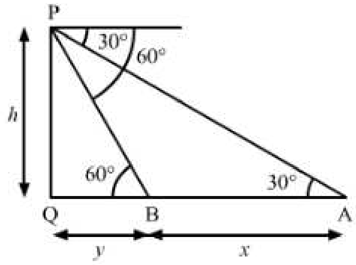
$= 100 \times 2.73$

$= 273\,m$

So, the distance between the cars is $273\,m$.

14. A straight highway leads to the foot of a tower, A man standing on the top of a the tower observe c car at an angle of depression of 30° which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower form this point.

Sol:



Let PQ be the tower.

We have,

$\angle PBQ = 60^\circ$ *and* $\angle PAQ = 30^\circ$

Let $PQ = h, AB = x$ *and* $BQ = y$

In ΔAPQ,

$$\tan 30^\circ = \frac{PQ}{AQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow x + y = h\sqrt{3} \qquad \text{........}(i)$$

Also, in ΔBPQ,

$$\tan 60^\circ = \frac{PQ}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow h = y\sqrt{3} \qquad \text{..........}(ii)$$

Substituting $h = y\sqrt{3}$ in (i), we get

$$x + y = \sqrt{3}\left(y\sqrt{3}\right)$$

$$\Rightarrow x + y = 3y$$

$$\Rightarrow 3y - y = x$$

$$\Rightarrow 2y = x$$

$$\Rightarrow y = \frac{x}{2}$$

As, speed of the car from A to B $= \frac{AB}{6} = \frac{x}{6}$ *units* / sec

So, the time taken to reach the foot of the tower i.e. Q from B $= \frac{BQ}{speed}$

$$= \frac{y}{\left(\frac{x}{6}\right)}$$

$$= \frac{\left(\frac{x}{2}\right)}{\left(\frac{x}{6}\right)}$$

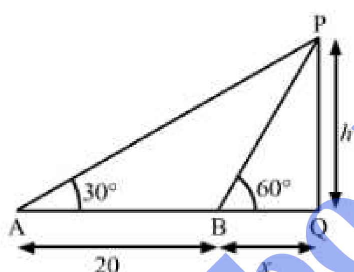
$$= \frac{6}{2}$$

$= 3$ sec

So, the time taken to reach the foot of the tower from the given point is 3 seconds.

15. A TV tower stands vertically on a bank of canal. Form a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60° . From another point 20m away from the point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.

Sol:



Let PQ=h m be the height of the TV tower and BQ= x m be the width of the canal.

We have,

$AB = 20\ m, \angle PAQ = 30°, \angle BQ = x\ and\ PQ = h$

In ΔPBQ,

$$\tan 60° = \frac{PQ}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x\sqrt{3} \qquad ....(i)$$

Again in ΔAPQ,

$$\tan 30° = \frac{PQ}{AQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AB+BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{20+3} \qquad \text{[Using (i)]}$$

$$\Rightarrow 3x = 20 + x$$

$$\Rightarrow 3x - x = 20$$

$$\Rightarrow 2x = 20$$

$$\Rightarrow x = \frac{20}{2}$$

$$\Rightarrow x = 10m$$

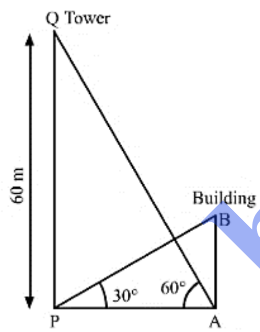
Substituting $x = 10$ in (i), we get

$$h = 10\sqrt{3}m$$

So, the height of the TV tower is $10\sqrt{3}\,m$ and the width of the canal is 10 m.

16. The angle of elevation on the top of a building from the foot of a tower is 30°. The angle of elevation of the top of the tower when seen from the top of the second water is 60°. If the tower is 60m high, find the height of the building.

Sol:



Let AB be thee building and PQ be the tower.

We have,

$$PQ = 60m, \angle APB = 30°, \angle PAQ = 60°$$

In ΔAPQ,

$$\tan 60° = \frac{PQ}{AP}$$

$$\Rightarrow \sqrt{3} = \frac{60}{AP}$$

$\Rightarrow AP = \frac{60}{\sqrt{3}}$

$\Rightarrow AP = \frac{60\sqrt{3}}{3}$

$\Rightarrow AP = 20\sqrt{3}\,m$

Now, in ΔABP,

$\tan 30° = \frac{AB}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}}$

$\Rightarrow AB = \frac{20\sqrt{3}}{\sqrt{3}}$

$\therefore AB = 20\,m$

So, the height of the building is 20 m

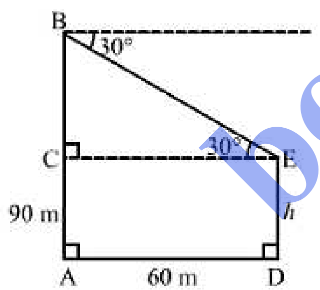
17. The horizontal distance between two towers is 60 meters. The angle of depression of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 90 meters. Find the height of the first tower.

Sol:

Let DE be the first tower and AB be the second tower.

Now, AB = 90 m and AD = 60 m such that CE = 60 m and $\angle BEC = 30°$.

Let DE = h m such that AC = h m and BC = $(90-h)m$.



In the right ΔBCE, we have:

$\frac{BC}{CE} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{(90-h)}{60} = \frac{1}{\sqrt{3}}$

$\Rightarrow (90-h)\sqrt{3} = 60$

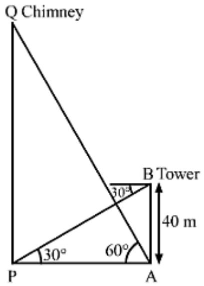
$\Rightarrow h\sqrt{3} = 90\sqrt{3} - 60$

$\Rightarrow h = 90 - \frac{60}{\sqrt{3}} = 90 - 34.64 = 55.36\ m$

$\therefore$ Height of the first tower $= DE = h = 55.36 m$

18. The angle of elevation of the top of a chimney form the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 meters. Find the height of the chimney.

Sol:



Let PQ be the chimney and AB be the tower.

We have,

$AB = 40m, \angle APB = 30° \ and \ \angle PAQ = 60°$

In ΔABP,

$\tan 30° = \frac{AB}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{40}{AP}$

$\Rightarrow AP = 40\sqrt{3}\, m$

Now, in ΔAPQ,

$\tan 60° = \frac{PQ}{AP}$

$\Rightarrow \sqrt{3} = \frac{PQ}{40\sqrt{3}}$

$\therefore PQ = 120 m$

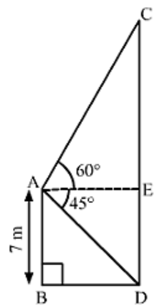
So, the height of the chimney is 120 m.

Hence, the height of the chimney meets the pollution norms.

In this question, management of air pollution has been shown

19. From the top of a 7 meter high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol:



Let AB be the 7-m high building and CD be the cable tower,

We have,

$AB = 7\,m, \angle CAE = 60°, \angle DAE = \angle ADB = 45°$

Also, $DE = AB = 7\,m$

In ΔABD,

$\tan 45° = \dfrac{AB}{BD}$

$\Rightarrow 1 = \dfrac{7}{BD}$

$\Rightarrow BD = 7\,m$

So, $AE = BD = 7\,m$

Also, in ΔACE,

$\tan 60° = \dfrac{CE}{AE}$

$\Rightarrow \sqrt{3} = \dfrac{CE}{7}$

$\Rightarrow CE = 7\sqrt{3}m$

Now, $CD = CE + DE$

$= 7\sqrt{3} + 7$

$= 7\left(\sqrt{3} + 1\right)m$

$= 7(1.732 + 1)$

$= 7(2.732)$

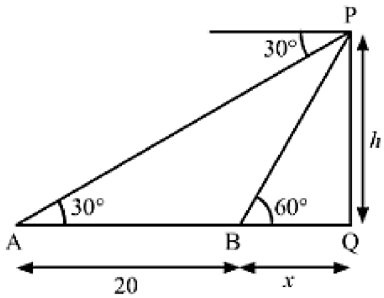
$= 19.124$

$\approx 19.12\,m$

So, the height of the tower is $19.12\,m.$

20. The angle of depression form the top of a tower of a point A on the ground is $30°$. On moving a distance of 20 meters from the point A towards the foot of the tower to a point B, the angle of elevation of the top of the tower to from the point B is $60°$. Find the height of the tower and its distance from the point A.

Sol:



Let PQ be the tower.

We have,

$AB = 20m, \angle PAQ = 30^\circ \text{ and } \angle PBQ = 60^\circ$

Let $BQ = x \text{ and } PQ = h$

In ΔPBQ,

$$\tan 60^\circ = \frac{PQ}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow h = x\sqrt{3} \qquad ....(i)$$

Also, in ΔAPQ,

$$\tan 30^\circ = \frac{PQ}{AQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AB + BQ}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x\sqrt{3}}{20 + x} \qquad \text{[Using (i)]}$$

$$\Rightarrow 20 + x = 3x$$

$$\Rightarrow 3x - x = 20$$

$$\Rightarrow 2x = 20$$

$$\Rightarrow x = \frac{20}{2}$$

$$\Rightarrow x = 10m$$

From (i),

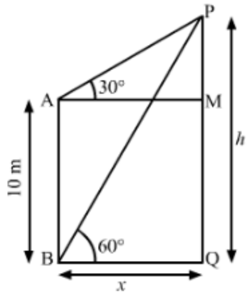
$h = 10\sqrt{3} = 10 \times 1.732 = 17.32m$

Also, $AQ = AB + BQ = 20 + 10 = 30m$

So, the height of the tower is 17. 32 m and its distance from the point A is 30 m.

21. The angle of elevation of the top of a vertical tower from a point on the ground is $60°$. From another point 10 m vertically above the first, its angle of elevation is $30°$. Find the height of the tower.

Sol:



Let PQ be the tower

We have,

$AB = 10m, \angle MAP = 30° \text{ and } \angle PBQ = 60°$

Also, $MQ = AB = 10m$

Let $BQ = x \text{ and } PQ = h$

So, $AM = BQ = x \text{ and } PM = PQ - MQ = h - 10$

In ΔBPQ,

$$\tan 60° = \frac{PQ}{BQ}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} \quad ....(i)$$

Now, in ΔAMP,

$$\tan 30° = \frac{PM}{AM}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-10}{x}$$

$$\Rightarrow h\sqrt{3} - 10\sqrt{3} = x$$

$$\Rightarrow h\sqrt{3} - 10\sqrt{3} = \frac{h}{\sqrt{3}} \quad \text{[Using (i)]}$$

$$\Rightarrow 3h - 30 = h$$

$$\Rightarrow 3h - h = 30$$

$$\Rightarrow 2h = 30$$

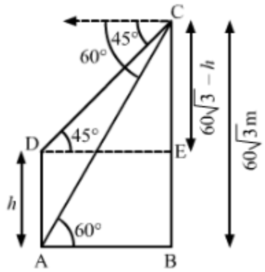
$$\Rightarrow h = \frac{30}{2}$$

$$\therefore h = 15m$$

So, the height of the tower is 15 m.

22. The angles of depression of the top and bottom of a tower as seen from the top of a $60\sqrt{3}\ m$ high cliff are $45°$ and $60°$ respectively. Find the height of the tower.

Sol:



Let AD be the tower and BC be the cliff.

We have,

$BC = 60\sqrt{3}, \angle CDE = 45°\ and\ \angle BAC = 60°$

Let $AD = h$

$\Rightarrow BE = AD = h$

$\Rightarrow CE = BC - BE = 60\sqrt{3} - h$

In ΔCDE,

$$\tan 45° = \frac{CE}{DE}$$

$$\Rightarrow 1 = \frac{60\sqrt{3}-h}{DE}$$

$$\Rightarrow DE = 60\sqrt{3} - h$$

$$\Rightarrow AB = DE = 60\sqrt{3} - h \quad ....(i)$$

Now, in ΔABC,

$$\tan 60° = \frac{BC}{AB}$$

$$\Rightarrow \sqrt{3} = \frac{60\sqrt{3}}{60\sqrt{3}-h} \quad \text{[Using (i)]}$$

$$\Rightarrow 180 - h\sqrt{3} = 60\sqrt{3}$$

$$\Rightarrow h\sqrt{3} = 180 - 60\sqrt{3}$$

$$\Rightarrow h = \frac{180-60\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow h = \frac{180\sqrt{3}-180}{3}$$

$$\Rightarrow h = \frac{180\left(\sqrt{3}-1\right)}{3}$$

$\therefore h = 60\left(\sqrt{3}-1\right)$

$= 60(1.732-1)$

$= 60(0.732)$

Also, $h = 43.92\,m$

So, the height of the tower is 43. 92 m.

23. A man on the deck of a ship, 16m above water level, observe that that angle of elevation and depression respectively of the top and bottom of a cliff are $60°$ and $30°$. Calculate the distance of the cliff from the ship and height of the cliff.

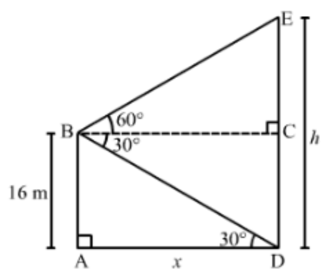
Sol:

Let AB be the deck of the ship above the water level and DE be the cliff.

Now,

$AB = 16\,m$ such that $CD = 16\,m$ and $\angle BDA = 30°$ and $\angle EBC = 60°$

If $AD = x\,m$ and $DE = h\,m$, then $CE = (h-16)\,m.$



In the right ΔBAD, we have

$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{16}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x = 16\sqrt{3} = 27.68\,m$$

In the right ΔEBC, we have:

$$\frac{EC}{BC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{(h-16)}{x} = \sqrt{3}$$

$$\Rightarrow h-16 = x\sqrt{3}$$

$$\Rightarrow h-16 = 16\sqrt{3}\times\sqrt{3} = 48 \qquad \left[\because x = 16\sqrt{3}\right]$$

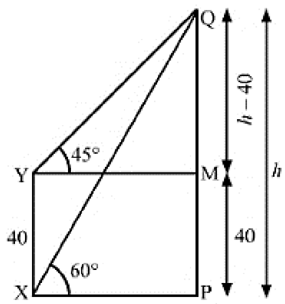
$$\Rightarrow h = 48+16 = 64\,m$$

$\therefore$ Distance of the cliff from the deck of the ship $= AD = x = 27.68\,m$

And,
Height of the cliff $= DE = h = 64m$

24. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is $60°$. At a point Y, 40m vertically above X, the angle of elevation is $45°$. Find the height of tower PQ.

Sol:



We have

$XY = 40m, \angle PXQ = 60° \text{ and } \angle MYQ = 45°$

Let $PQ = h$

Also, $MP = XY = 40m, MQ = PQ - MP = h - 40$

In ΔMYQ,

$$\tan 45° = \frac{MQ}{MY}$$

$$\Rightarrow 1 = \frac{h-40}{MY}$$

$$\Rightarrow MY = h - 40$$

$$\Rightarrow PX = MY = h - 40 \qquad ....(i)$$

Now, in ΔMXQ,

$$\tan 60° = \frac{PQ}{PX}$$

$$\Rightarrow \sqrt{3} = \frac{h}{h-40} \qquad \text{[From (i)]}$$

$$\Rightarrow h\sqrt{3} - 40\sqrt{3} = h$$

$$\Rightarrow h\sqrt{3} - h = 40\sqrt{3}$$

$$\Rightarrow h\left(\sqrt{3} - 1\right) = 40\sqrt{3}$$

$$\Rightarrow h = \frac{40\sqrt{3}}{\left(\sqrt{3} - 1\right)}$$

$$\Rightarrow h = \frac{40\sqrt{3}}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$\Rightarrow h = \frac{40\sqrt{3}\left(\sqrt{3}+1\right)}{\left(3-1\right)}$$

$$\Rightarrow h = \frac{40\sqrt{3}\left(\sqrt{3}+1\right)}{2}$$

$$\Rightarrow h = 20\sqrt{3}\left(\sqrt{3}+1\right)$$

$$\Rightarrow h = 60 + 20\sqrt{3}$$

$$\Rightarrow h = 60 + 20 \times 1.73$$

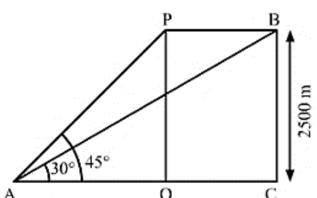
$$\Rightarrow h = 60 + 34.6$$

$$\therefore h = 94.6\,m$$

So, the height of the tower PQ is 94. 6 m.

25. The angle of elevation of an aeroplane from a point on the ground is $45°$ after flying for 15seconds, the elevation changes to $30°$. If the aeroplane is flying at a height of 2500 meters, find the speed of the areoplane.

Sol:



Let the height of flying of the aero-plane be PQ = BC and point A be the point of observation.

We have,

$$PQ = BC = 2500\,m, \angle PAQ = 45° \text{ and } \angle BAC = 30°$$

In ΔPAQ,

$$\tan 45° = \frac{PQ}{AQ}$$

$$\Rightarrow 1 = \frac{2500}{AQ}$$

$$\Rightarrow AQ = 2500\ m$$

Also, in ΔABC,

$$\tan 30° = \frac{BC}{AC}$$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{2500}{AC}$

$\Rightarrow AC = 2500\sqrt{3}\, m$

Now, $QC = AC - AQ$

$= 2500\sqrt{3} - 2500$

$= 2500\left(\sqrt{3} - 1\right) m$

$= 2500(1.732 - 1)$

$= 2500(0.732)$

$= 1830\, m$

$\Rightarrow PB = QC = 1830\, m$

So, the speed of the aero-plane $= \frac{PB}{15}$

$= \frac{1830}{15}$

$= 122\, m/s$

$= 122 \times \frac{3600}{1000}\ km/h$

$= 439.2\ km/h$

So, the speed of the aero-plane is $122\, m/s$ or $439.2\ km/h$.

26. The angle of elevation of the top of a tower from ta point on the same level as the foot of the tower is $30°$. On advancing 150 m towards foot of the tower, the angle of elevation becomes $60°$ Show that the height of the tower is 129.9 metres.

Sol:

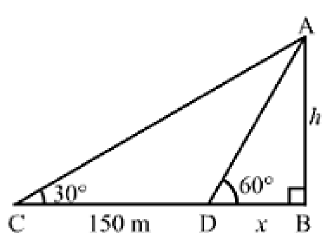
Let AB be the tower

We have:

$CD = 150\, m, \angle ACB = 30°\ and\ \angle ADB = 60°$

Let:

$AB = h\, m\ and\ BD = x\, m$



In the right ΔABD, we have:

$$\frac{AB}{AD} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}}$$

Now, in the right ΔACB, we have:

$$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{x+150} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h = x + 150$$

On putting $x = \frac{h}{\sqrt{3}}$ in the above equation, we get:

$$\sqrt{3}h = \frac{h}{\sqrt{3}} + 150$$

$$\Rightarrow 3h = h + 150\sqrt{3}$$

$$\Rightarrow 2h = 150\sqrt{3}$$

$$\Rightarrow h = \frac{150\sqrt{3}}{2} = 75\sqrt{3} = 75 \times 1.732 = 129.9\,m$$

Hence, the height of the tower is 129.9 m

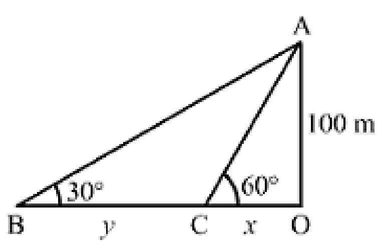
27. As observed form the top of a lighthouse, 100m above sea level, the angle of depression of a ship, sailing directly towards it, changes from $30°$ and $60°$. Determine the distance travelled by the ship during the period of observation.

Sol:

Let OA be the lighthouse and B and C be the positions of the ship.

Thus, we have:

$OA = 100\,m, \angle OBA = 30°$ *and* $\angle OCA = 60°$



Let

$OC = x\,m$ *and* $BC = y\,m$

In the right ΔOAC, we have

$$\frac{OA}{OC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \frac{100}{x} = \sqrt{3}$$

$$\Rightarrow x = \frac{100}{\sqrt{3}} m$$

Now, in the right ΔOBA, we have:

$$\frac{OA}{OB} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{100}{x+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow x + y = 100\sqrt{3}$$

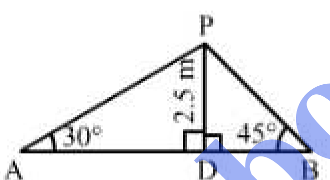
On putting $x = \frac{100}{\sqrt{3}}$ in the above equation, we get:

$$y = 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{300-100}{\sqrt{3}} = \frac{200}{\sqrt{3}} = 115.47\, m$$

$\therefore$ Distance travelled by the ship during the period of observation $= B = y = 115.47\, m$

28. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are $30°$ and $45°$ respectively. If the bridge is at a height of 2.5m from the banks, find the width of the river.

Sol:



Let A and B be two points on the banks on the opposite side of the river and P be the point on the bridge at a height of 2.5 m.

Thus, we have:

$$DP = 2.5, \angle PAD = 30° \text{ and } \angle PBD = 45°$$

In the right ΔAPD, we have:

$$\frac{DP}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{2.5}{AD} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AD = 2.5\sqrt{3} m$$

In the right ΔPDB, we have:

$$\frac{DP}{BD} = \tan 45° = 1$$

$$\Rightarrow \frac{2.5}{BD} = 1$$

$$\Rightarrow BD = 2.5m$$

$$\therefore \text{Width of the river} = AB = (AD + BD) = \left(2.5\sqrt{3} + 2.5\right) = 6.83m$$

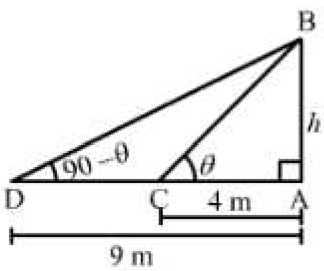
29. The angle of elevation of the top of a tower from to points at distances of 4m and 9m from the base of the tower and in the same straight line with it are complementary. Show that the height of the tower is 6 meters.

Sol:

Let AB be the tower and C and D be two points such that $AC = 4m$ *and* $AD = 9m$.

Let:

$AB = hm, \angle BCA = \theta$ *and* $\angle BDA = 90° - \theta$



In the right ΔBCA, we have:

$$\tan\theta = \frac{AB}{AC}$$

$$\Rightarrow \tan\theta = \frac{h}{4} \qquad ....(1)$$

In the right ΔBDA, we have:

$$\tan(90° - \theta) = \frac{AB}{AD}$$

$$\Rightarrow \cot\theta = \frac{h}{9} \qquad [\tan(90° - \theta) = \cot\theta]$$

$$\Rightarrow \frac{1}{\tan\theta} = \frac{h}{9} \qquad ....(2) \qquad \left[\cot\theta = \frac{1}{\tan\theta}\right]$$

Multiplying equations (1) and (2), we get

$$\tan\theta \times \frac{1}{\tan\theta} = \frac{h}{4} \times \frac{h}{9}$$

$$\Rightarrow 1 = \frac{h^2}{36}$$

$\Rightarrow 36 = h^2$

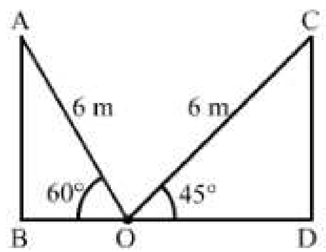
$\Rightarrow h = \pm 6$

Height of a tower cannot be negative

$\therefore$ Height of the tower = 6 m

30. A ladder of length 6meters makes an angle of $45°$ with the floor while leaning against one wall of a room. If the fort of the ladder is kept fixed on the floor and it is made to lean against the opposite wall of the room, it makes an angle of $60°$ with the floor. Find the distance between two walls of the room.

Sol:



Let AB and CD be the two opposite walls of the room and the foot of the ladder be fixed at the point O on the ground.

We have,

$AO = CO = 6m, \angle AOB = 60° \text{ and } \angle COD = 45°$

In ΔABO,

$$\cos 60° = \frac{BO}{AO}$$

$$\Rightarrow \frac{1}{2} = \frac{BO}{6}$$

$$\Rightarrow BO = \frac{6}{2}$$

$$\Rightarrow BO = 3m$$

Also, in ΔCDO,

$$\cos 45° = \frac{DO}{CO}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{DO}{6}$$

$$\Rightarrow DO = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow DO = \frac{6\sqrt{2}}{2}$$

$\Rightarrow DO = 3\sqrt{2}\,m$

Now, the distance between two walls of the room = BD

$= BO + DO$

$= 3 + 3\sqrt{2}$

$= 3\left(1+\sqrt{2}\right)$

$= 3(1+1.414)$

$= 3(2.414)$

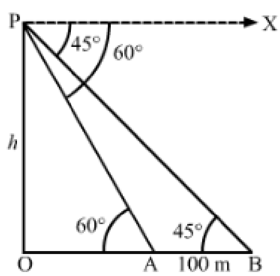
$= 7.242$

$\approx 7.24\,m$

So, the distant between two walls of the room is 7. 24 m.

31. From the top of a vertical tower, the angles depression of two cars in the same straight line with the base of the tower, at an instant are found to be $45°$ and $60°$. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower.

Sol:



Let OP be the tower and points A and B be the positions of the cars.

We have,

$AB = 100\,m, \angle OAP = 60°$ *and* $\angle OBP = 45°$

Let $OP = h$

In ΔAOP,

$$\tan 60° = \frac{OP}{OA}$$

$$\Rightarrow \sqrt{3} = \frac{h}{OA}$$

$$\Rightarrow OA = \frac{h}{\sqrt{3}}$$

Also, in ΔBOP,

$$\tan 45° = \frac{OP}{OB}$$

$$\Rightarrow 1=\frac{h}{OB}$$

$$\Rightarrow OB=h$$

Now, $OB-OA=100$

$$\Rightarrow h-\frac{h}{\sqrt{3}}=100$$

$$\Rightarrow \frac{h\sqrt{3}-h}{\sqrt{3}}=100$$

$$\Rightarrow \frac{h\left(\sqrt{3}-1\right)}{\sqrt{3}}=100$$

$$\Rightarrow h=\frac{100\sqrt{3}}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$

$$\Rightarrow h=\frac{100\sqrt{3}\left(\sqrt{3}+1\right)}{(3-1)}$$

$$\Rightarrow h=\frac{100\left(3+\sqrt{3}\right)}{2}$$

$$\Rightarrow h=50(3+1.732)$$

$$\Rightarrow h=50(4.732)$$

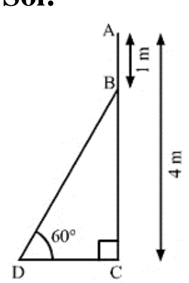
$$\therefore h=236.6\,m$$

So, the height of the tower is 236.6 m.

Disclaimer. The answer given in the textbook is incorrect. The same has been rectified above.

32. An electrician has to repair an electric fault on a pole of height 4 meters. He needs to reach a point 1 meter below the top of the pole to undertake the repair work. What should be the length of the ladder that he should use, which when inclined at an angle of $60°$ to the horizontal would enable him to reach the required position?

Sol:



Let AC be the pole and BD be the ladder

We have,

$AC = 4m, AB = 1m \; and \; \angle BDC = 60°$

And, $BC = AC - AB = 4 - 1 = 3m$

In ΔBDC,

$$\sin 60° = \frac{BC}{BD}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{BD}$$

$$\Rightarrow BD = \frac{3 \times 2}{\sqrt{3}}$$

$$\Rightarrow BD = 2\sqrt{3}$$

$$\Rightarrow BD = 2 \times 1.73$$

$$\therefore BD = 3.46m$$

So, he should use 3.46 m long ladder to reach the required position.

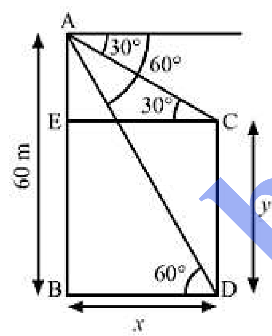
33. From the top of a building AB, 60m high, the angles of depression of the top and bottom of a vertical lamp post CD are observed to the $30°$ and $60°$ respectively. Find

(i) The horizontal distance between AB and CD,

(ii) the height of the lamp post,

(iii) the difference between the heights of the building and the lamp post.

Sol:



We have,

$AB = 60m, \angle ACE = 30° \; and \; \angle ADB = 60°$

Let $BD = CE = x \; and \; CD = BE = y$

$\Rightarrow AE = AB - BE = 60 - y$

In ΔACE,

$$\tan 30° = \frac{AE}{CE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{60-y}{x}$$

$$\Rightarrow x = 60\sqrt{3} - y\sqrt{3} \qquad ....(i)$$

Also, in ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow x = \frac{60-\sqrt{3}}{3}$$

$$\Rightarrow x = 20\sqrt{3}$$

Substituting $x = 20\sqrt{3}$ in (i), we get

$$20\sqrt{3} = 60\sqrt{3} - y\sqrt{3}$$

$$\Rightarrow y\sqrt{3} = 60\sqrt{3} - 20\sqrt{3}$$

$$\Rightarrow y\sqrt{3} = 40\sqrt{3}$$

$$\Rightarrow y = \frac{40\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow y = 40m$$

(i) The horizontal distance between AB and CD = BD = x

$= 20\sqrt{3}$

$= 20 \times 1.732$

$= 34.64m$

(ii) The height of the lamp post $= CD = y = 40m$

(iii) the difference between the heights of the building and the lamp post

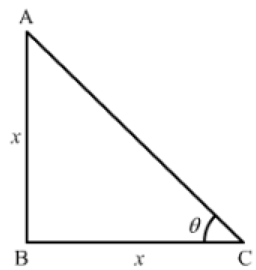
$= AB - CD = 60 - 40 = 20m$

Exercise – Multiple Choice Question

1. If the height of a vertical pole is equal to the length of its shadow on the ground, the angle of elevation of the sun is

(a) $0°$ (b) $30°$ (c) $45°$ (d) $60°$

Sol:



Let AB represents the vertical pole and BC represents the shadow on the ground and θ represents angle of elevation the sun.

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{x}{x} \quad \text{(As, the height of the pole, } AB = \text{the length of the shadow, } BC = x\text{)}$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \tan\theta = \tan 45°$$

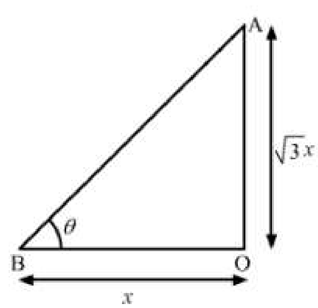
$$\therefore \theta = 45°$$

Hence, the correct answer is option (c).

2. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground the angle of elevation of the sun at that time is

(a) $30°$ (b) $45°$ (c) $60°$ (d) $75°$

Sol:



Here, AO be the pole; BO be its shadow and θ be the angle of elevation of the sun.

Let $BO = x$

Then, $AO = x\sqrt{3}$

In ΔAOB,

$$\tan\theta = \frac{AO}{BO}$$

$$\Rightarrow \tan\theta = \frac{x\sqrt{3}}{x}$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 60°$$

$$\therefore \theta = 60°$$

Hence, the correct answer is option (c).

3. If the length of the shadow of a tower is $\sqrt{3}$ times its height then the angle of elevation of the sun is

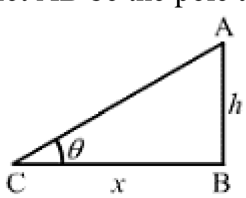
(a) $45°$ (b) $30°$ (c) $60°$

(d) $90°$

Ans: (b)

Sol:

Let AB be the pole and BC be its shadow.



Let $AB = h$ and $BC = x$ such that $x = \sqrt{3}h$ (given) and θ be the angle of elevation.

From ΔABC, we have

$$\frac{AB}{BC} = \tan\theta$$

$$\Rightarrow \frac{h}{x} = \frac{h}{\sqrt{3}h} = \tan\theta$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

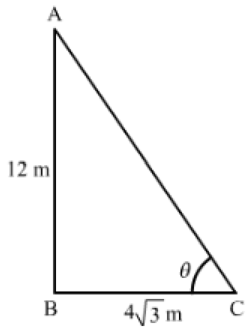
$$\Rightarrow \theta = 30°$$

Hence, the angle of elevation is $30°$.

4. If a pole of 12 m high casts a shadow $4\sqrt{3}$ long on the ground then the sun's elevation is

(a) $60°$ (b) $45°$ (c) $30°$ (d) $90°$

Sol:



Let AB be the pole, BC be its shadow and θ be the sun's elevation.

We have,

$AB = 12\,m \text{ and } BC = 4\sqrt{3}\,m$

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{12}{4\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{3}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{3\sqrt{3}}{3}$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan 60°$$

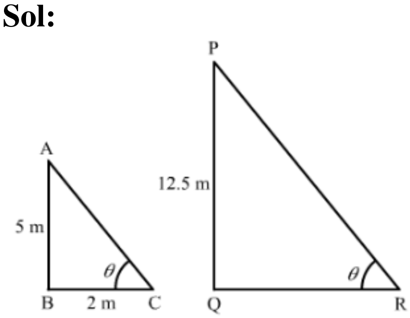
$$\therefore \theta = 60°$$

Hence, the correct answer is option (a).

5. The shadow of a 5-m-long stick is 2m long. At the same time, the length of the shadow of a 12.5-m-high tree is

(a) 3m (b) 3.5m (c) 4.5m (d) 5.

Sol:

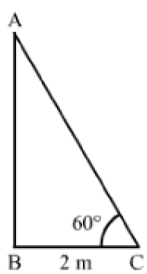


Let AB be a stick and BC be its shadow, and PQ be the tree and QR be its shadow.
We have,
$AB = 5m, BC = 2m, PQ = 12.5\ m$
In ΔABC,
$\tan\theta = \frac{AB}{BC}$
$\Rightarrow \tan\theta = \frac{5}{2}$ ....(i)
Now, in ΔPQR,
$\tan\theta = \frac{PQ}{QR}$
$\Rightarrow \frac{5}{2} = \frac{12.5}{QR}$ [Using (i)]
$\Rightarrow QR = \frac{125 \times 2}{5} = \frac{25}{5}$
$\therefore QR = 5m$
Hence, the correct answer is option (d).

6. A ladder makes an angle of $60°$ with the ground when placed against a wall. If the foot of the ladder is 2m away from the wall, the length of the ladder is

(a) $\frac{4}{\sqrt{3}}$ (b) $4\sqrt{3}$ (c) $2\sqrt{2}m$ (d) 4m

Sol:



Let AB be the wall and AC be the ladder.
We have,
$BC = 2m \text{ and } \angle ACB = 60°$
In ΔABC,
$\cos 60° = \frac{BC}{AC}$
$\Rightarrow \frac{1}{2} = \frac{2}{AC}$

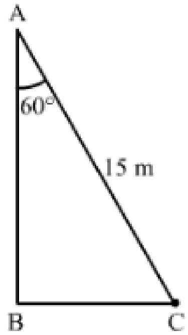
$\therefore AC = 4m$

Hence, the correct answer is option (d).

7. A ladder 15m long just reaches the top of a vertical wall. If the ladder makes an angle of $60°$ with the wall then the height of the wall is

(a) $15\sqrt{3}$ (b) $\frac{15\sqrt{3}}{2}$ (c) $\frac{15}{2}$ (d) 15 m

Sol:



Let AB be the wall and AC be the ladder

We have,

$AC = 15m \ and \ \angle BAC = 60°$

$\cos 60° = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{AB}{15}$

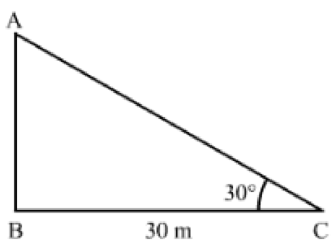
$\therefore AB = \frac{15}{2} \ m$

Hence, the correct answer is option (c).

8. From a point on the ground, 30m away from the foot of a tower, the angle of elevation of the top of the tower is $30°$. The height of the tower is

(a) 30m (b) $10\sqrt{3}$ (c) 10 m (d) $30\sqrt{3}$

Sol:



Let AB be the tower and point C be the point of observation on the ground.

We have,

$BC = 30m \text{ and } \angle ACB = 30°$

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$\Rightarrow AB = \frac{30}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$\Rightarrow AB = \frac{30\sqrt{3}}{3}$$

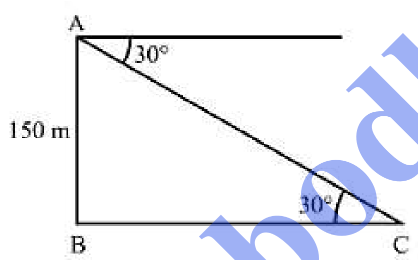
$$\therefore AB = 10\sqrt{3}\ m$$

Hence, the correct answer is option (b).

9. The angle of depression of a car parked on the road from the top of a 150-m. high tower is $30°$. The distance of the car from the tower is

(a) $50\sqrt{3}$ (b) $150\sqrt{3}$ (c) $150\sqrt{2}$ (d) 75 m

Sol:



Let AB be the tower and point C be the position of the car.

We have,

$AB = 150m \text{ and } \angle ACB = 30°$

In ΔABC,

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150}{BC}$$

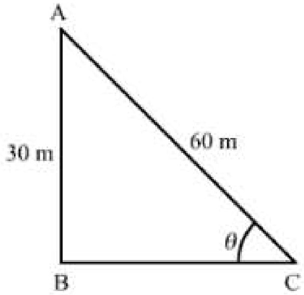
$$\therefore BC = 150\sqrt{3}\ m$$

Hence, the correct answer is option (b).

10. A kite is flying at a height of 30 m from the ground. The length of string from the kite to the ground is 60m. Assuming that there is no slack in the string, the angle of elevation of the kite at the ground is

(a) 45° (b) 30° (c) 60° (d) 90°

Sol:



Let point A be the position of the kite and AC be its string

We have,

$AB = 30m \ and \ AC = 60m$

Let $\angle ACB = \theta$

In ΔABC,

$$\sin\theta = \frac{AB}{AC}$$

$$\Rightarrow \sin\theta = \frac{30}{60}$$

$$\Rightarrow \sin\theta = \frac{1}{2}$$

$$\Rightarrow \sin\theta = \sin 30°$$

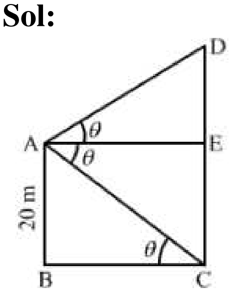
$$\therefore \theta = 30°$$

Hence, the correct answer is option (b).

11. From the top of a cliff 20m high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower, The height of the tower is

(a) 20m (b) 40m (c) 60m (d) 80m

Sol:



Let AB be the cliff and CD be the tower.

We have,

$AB = 20m$

Also, $CE = AB = 20m$

Let $\angle ACB = \angle CAE = \angle DAE = \theta$

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{20}{BC}$$

$$\Rightarrow \tan\theta = \frac{20}{AE} \qquad (As,\ BC = AE)$$

$$\Rightarrow AE = \frac{20}{\tan\theta} \qquad ....(i)$$

Also, in ΔADE,

$$\tan\theta = \frac{DE}{AE}$$

$$\Rightarrow \tan\theta = \frac{DE}{\left(\dfrac{20}{\tan\theta}\right)} \qquad \text{[Using (i)]}$$

$$\Rightarrow \tan\theta = \frac{DE \times \tan\theta}{20}$$

$$\Rightarrow DE = \frac{20 \times \tan\theta}{\tan\theta}$$

$$\Rightarrow DE = 20m$$

Now, $CD = DE + CE$

$= 20 + 20$

$\therefore CD = 40m$

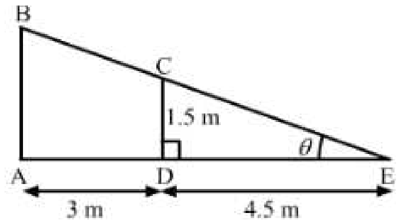
Hence, the correct answer is option (b).

Disclaimer. The answer given in the textbook is incorrect. The same has been rectified above.

12. If a 1.5 m tall girl stands at a distance of 3m from a lamp post and casts a shadow of length 4.5m on the ground, then the height of the lamp post is

(a) 1.5m (b) 2m (c) 2.5m (d) 2.8m

Sol:



Let AB be the lamp post; CD be the girl and DE be her shadow.

We have,

$CD = 1.5\,m, AD = 3\,m, DE = 4.5\,m$

Let $\angle E = \theta$

In ΔCDE,

$$\tan\theta = \frac{CD}{DE}$$

$$\Rightarrow \tan\theta = \frac{1.5}{4.5}$$

$$\Rightarrow \tan\theta = \frac{1}{3} \qquad ....(i)$$

Now, in ΔABE,

$$\tan\theta = \frac{AB}{AE}$$

$$\Rightarrow \frac{1}{3} = \frac{AB}{AD + DE} \qquad \text{[Using (i)]}$$

$$\Rightarrow \frac{1}{3} = \frac{AB}{3 + 4.5}$$

$$\Rightarrow AB = \frac{7.5}{3}$$

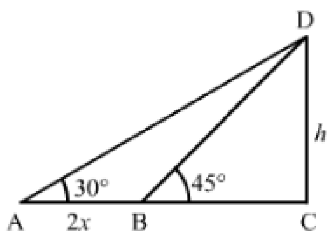
$$\Rightarrow \therefore AB = 2.5\,m$$

Hence, the correct answer is option (c).

13. The length of the shadow of a tower standing on level ground is found to be 2x meter longer when the sun's elevation is $30°$ than when it was $45°$. The height of the tower is

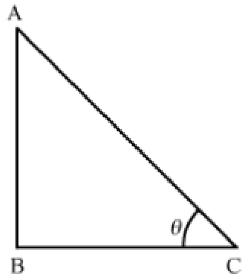
(a) $\left(2\sqrt{3}x\right)m$ (b) $\left(3\sqrt{2}x\right)m$ (c) $\left(\sqrt{3}-1\right)x\ m$ (d) $\left(\sqrt{3}+1\right)x\ m$

Sol:



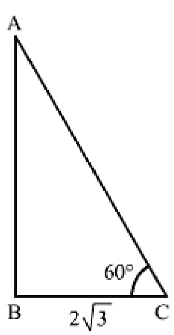
Let CD = h be the height of the tower.
We have,
$AB = 2x, \angle DAC = 30° \text{ and } \angle DBC = 45°$
In ΔBCD,
$$\tan 45° = \frac{CD}{BC}$$
$$\Rightarrow 1 = \frac{h}{BC}$$
$$\Rightarrow BC = h$$
Now, in ΔACD,
$$\tan 30° = \frac{CD}{AC}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{AB+BC}$$
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{2x+h}$$
$$\Rightarrow 2x + h = h\sqrt{3}$$
$$\Rightarrow h\sqrt{3} - h = 2x$$
$$\Rightarrow h\left(\sqrt{3}-1\right) = 2x$$
$$\Rightarrow h = \frac{2x}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$$
$$\Rightarrow h = \frac{2x\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}$$
$$\Rightarrow h = \frac{2x\left(\sqrt{3}+1\right)}{2}$$
$$\therefore h = x\left(\sqrt{3}+1\right)m$$
Hence, the correct answer is option (d).

14. The length of a vertical rod and its shadow are in the ratio $1:\sqrt{3}$. The angle of elevation of the sun is
(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$
Sol:



Let AB be the rod and BC be its shadow; and θ be the angle of elevation of the sun.
We have,
$AB : BC = 1 : \sqrt{3}$
Let $AB = x$
Then, $BC = x\sqrt{3}$
In ΔABC,
$$\tan\theta = \frac{AB}{BC}$$
$$\Rightarrow \tan\theta = \frac{x}{x\sqrt{3}}$$
$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \tan\theta = \tan 30°$$
$$\therefore \theta = 30°$$
Hence, the correct answer is option (a).

15. A pole casts a shadow of length $2\sqrt{3}$ m on the ground when the sun's elevation is $60°$. The height of the pole is
(a) $4\sqrt{3}$ (b) 6m (c) 12m (d) 3m
Sol:



Let AB be the pole and BC be its shadow.
We have,
$BC = 2\sqrt{3}m \text{ and } \angle ACB = 60°$

In ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

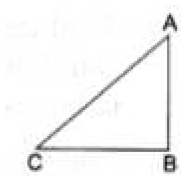
$$\Rightarrow \sqrt{3} = \frac{AB}{2\sqrt{3}}$$

$$\therefore AB = 6m$$

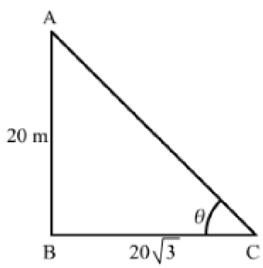
Hence, the correct answer is option (b).

16. In the given figure, a tower AB is 20m high and BC, its shadow on the ground is $20\sqrt{3}$ m long. The sun's altitude is

(a) $30°$ (b) $45°$ (c) $60°$ (d) None of these



Sol:



Let the sun's altitude be θ.

We have,

$$AB = 20m \text{ and } BC = 20\sqrt{3}\ m$$

In ΔABC,

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{20}{20\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \tan 30°$$

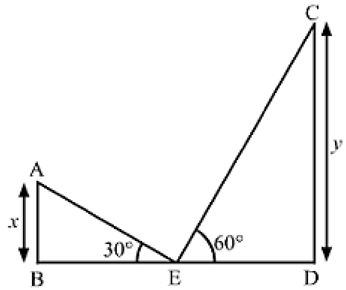
$$\therefore \theta = 30°$$

Hence, the correct answer is option (a).

17. The tops of two towers of heights x and y, standing on a level ground subtend angle of $30°$ and $60°$ respectively at the centre of the line joining their feet. Then x:y is

(a) 1:2 (b) 2:1 (c) 1:3 (d) 3:1

Sol:



Let AB and CD be the two towers such that $AB = x \text{ and } CD = y$.

We have,

$\angle AEB = 30°, \angle CED = 60° \text{ and } BE = DE$

In ΔABE,

$$\tan 30° = \frac{AB}{BE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{BE}$$

$$\Rightarrow BE = x\sqrt{3}$$

Also, in ΔCDE,

$$\tan 60° = \frac{CD}{DE}$$

$$\Rightarrow \sqrt{3} = \frac{y}{DE}$$

$$\Rightarrow DE = \frac{y}{\sqrt{3}}$$

As, $BE = DE$

$$\Rightarrow x\sqrt{3} = \frac{y}{\sqrt{3}}$$

$$\Rightarrow \frac{x}{y} = \frac{1}{\sqrt{3} \times \sqrt{3}}$$

$$\Rightarrow \frac{x}{y} = \frac{1}{3}$$

$\therefore x : y = 1:3$

Hence, the correct answer is option (c).

18. The angle of elevation of the top of a tower from the a point on the ground 30m away from the foot of the tower is $30°$. The height of the tower is

(a) 30m (b) $10\sqrt{3}$ (c) 20m (d) $10\sqrt{2}$

Ans: (b)

Sol:

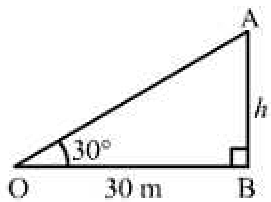
Let AB be the tower and O be the point of observation.

Also,

$\angle AOB = 30°$ *and* $OB = 30m$

Let:

$AB = hm$



In ΔAOB, we have:

$$\frac{AB}{OB} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{30} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}m.$$

Hence, the height of the tower is $10\sqrt{3}\,m$.

19. The string of a kite is 100 m long and it makes an angle of $60°$ with the horizontal. If there is no slack in the string, the height of the kite from the ground is

(a) $50\sqrt{3}$ (b) $100\sqrt{3}$ (c) $50\sqrt{2}$ (d) 100m

Ans: (a)

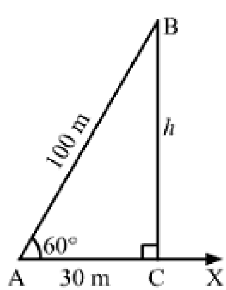
Sol:

Let AB be the string of the kite and AX be the horizontal line.

If $BC \perp AX$, then $AB = 100m$ *and* $\angle BAC = 60°$

Let:

$BC = hm$



In the right ΔACB, we have:

$$\frac{BC}{AB} = \sin 60° = \frac{\sqrt{3}}{2}$$
$$\Rightarrow \frac{h}{100} = \frac{\sqrt{3}}{2}$$
$$\Rightarrow h = \frac{100\sqrt{3}}{2} = 50\sqrt{3}\,m$$

Hence, the height of the kite is $50\sqrt{3}\,m$.

20. If the angles of elevations of the top of a tower from two points at distances a and b from the base and in the same straight line with it are complementary then the height of the tower is

(a) $\sqrt{\frac{a}{b}}$ (b) $\sqrt{ab}$ (c) $\sqrt{a+b}$ (d) $\sqrt{a-b}$

Ans: (b)

Sol:

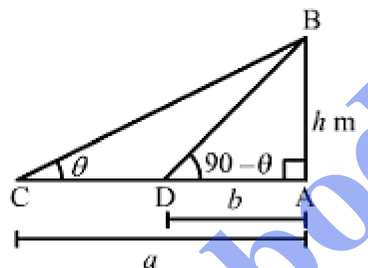
Let AB be the tower and C and D bee the points of observation on AC.

Let:

$\angle ACB = \theta, \angle ADB = 90 - \theta$ *and* $AB = h\,m$

Thus, we have:

$AC = a,\ AD = b$ *and* $CD = a - b$



Now, in the right ΔABC, we have:

$$\tan\theta = \frac{AB}{AC} \Rightarrow \frac{h}{a} = \tan\theta \qquad ....(i)$$

In the right ΔABD, we have:

$$\tan(90-\theta) = \frac{AB}{AD} \Rightarrow \cot\theta = \frac{h}{b} \qquad ....(ii)$$

On multiplying (i) and (ii), we have:

$$\tan\theta \times \cot\theta = \frac{h}{a} \times \frac{h}{b}$$
$$\Rightarrow \frac{h}{a} \times \frac{h}{b} = 1 \qquad \left[\because \tan\theta = \frac{1}{\cot\theta}\right]$$

$\Rightarrow h^2 = ab$

$\Rightarrow h = \sqrt{ab}\ m$

Hence, the height of the tower is $\sqrt{ab}\ m.$

21. On the level ground, the angle of elevations of a tower is $30°$. On moving 20m nearer, the angle of elevation is $60°$. The height of the tower is

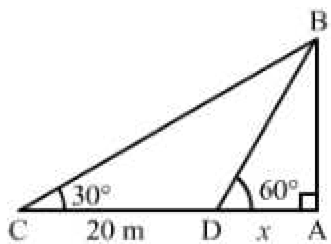
(a) 10m (b) $10\sqrt{3}$ (c) 15m (d) 20m

Ans: (b)

Sol:

Let AB be the tower and C and D be the points of observation such that

$\angle BCD = 30°, \angle BDA = 60°, CD = 20m \text{ and } AD = x m.$



Now, in ΔADB, we have:

$\frac{AB}{AD} = \tan 60° = \sqrt{3}$

$\Rightarrow \frac{AB}{x} = \sqrt{3}$

$\Rightarrow AB = \sqrt{3}x$

In ΔACB, we have:

$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}}$

$\frac{AB}{20+x} = \frac{1}{\sqrt{3}} \Rightarrow AB = \frac{20+x}{\sqrt{3}}$

$\therefore \sqrt{3}x = \frac{20+x}{\sqrt{3}}$

$\Rightarrow 3x = 20 + x$

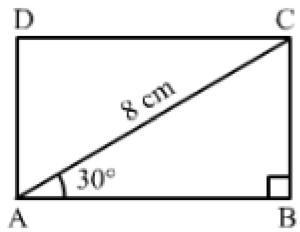
$\Rightarrow 2x = 20 \Rightarrow x = 10$

$\therefore$ Height of the tower $AB = \sqrt{3}x = 10\sqrt{3}\,m$

22. In a rectangle, the angle between a diagonal and a side of $30°$ and the lengths of this diagonal is 8cm. The area the rectangle is

(a) $16cm^2$ (b) $\frac{16}{\sqrt{3}}cm^2$ (c) $16\sqrt{3}cm^2$ (d) $8\sqrt{3}cm^2$

Ans: (c)
Sol:
Let $ABCD$ be the rectangle in which $\angle BAC = 30°$ and $AC = 8$ cm.



In ΔBAC, we have:

$$\frac{AB}{AC} = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{AB}{8} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow AB = 8\frac{\sqrt{3}}{2} = 4\sqrt{3}m$$

Again,

$$\frac{BC}{AC} = \sin 30° = \frac{1}{2}$$

$$\Rightarrow \frac{BC}{8} = \frac{1}{2}$$

$$\Rightarrow BC = \frac{8}{2} = 4\ m$$

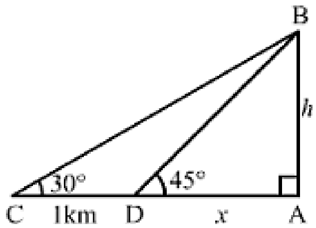
$\therefore$ Area of the rectangle $= (AB \times BC) = (4\sqrt{3} \times 4) = 16\sqrt{3}\ cm^2$

23. From the top of a hill, the angles of depression of two consecutive km stones due east are found to be $30°$ and $45°$. The height of the hill is

(a) $(\sqrt{3}+1)km$ (b) $(3+\sqrt{3})km$ (c) $\frac{1}{2}(\sqrt{3}+1)km$ (d) $\frac{1}{2}(3+\sqrt{3})km$

Ans: (b) $\frac{1}{2}(\sqrt{3}+1)km$

Sol:
Let AB be the hill making angles of depression at points C and D such that
$\angle ADB = 45°, \angle ACB = 30°$ and $CD = 1$ km.
Let:
$AB = h$ km and $AD = x$ km



In ΔADB, we have:

$$\frac{AB}{AD}=\tan 45°=1$$

$$\Rightarrow \frac{h}{x}=1 \Rightarrow h=x \qquad \text{..........}(i)$$

In ΔACB, we have:

$$\frac{AB}{AC}=\tan 30°=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{h}{x+1}=\frac{1}{\sqrt{3}} \qquad \text{..........}(ii)$$

On putting the value of h taken from (i) in (ii), we get:

$$\frac{h}{h+1}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \sqrt{3}h=h+1$$

$$\Rightarrow \left(\sqrt{3}-1\right)h=1$$

$$\Rightarrow h=\frac{1}{\left(\sqrt{3}-1\right)}$$

On multiplying the numerator and denominator of the above equation by $\left(\sqrt{3}+1\right)$, we get:

$$h=\frac{1}{\left(\sqrt{3}-1\right)}\times\frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}=\frac{\left(\sqrt{3}+1\right)}{3-1}=\frac{\left(\sqrt{3}+1\right)}{2}=\frac{1}{2}\left(\sqrt{3}+1\right)km$$

Hence, the height of the hill is $\frac{1}{2}\left(\sqrt{3}+1\right)km.$

24. If the elevation of the sun changes from $30°$ and $60°$ then the difference between the lengths of shadows of a pole 15m high, is

(a) 7.5m (b) 15m (c) $10\sqrt{3}m$ (d) $5\sqrt{3}m$

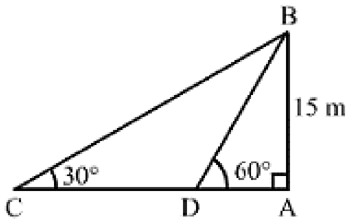
Ans: (c)

Sol:

Let AB be the pole and AC and AD be its shadows.

We have:

$\angle ACB = 30°, \angle ADB = 60°$ *and* $AB = 15m$



In ΔACB, we have

$$\frac{AC}{AB} = \cot 30° = \sqrt{3}$$

$$\Rightarrow \frac{AC}{15} = \sqrt{3} \Rightarrow AC = 15\sqrt{3}m$$

Now, in ΔADB, we have:

$$\frac{AD}{AB} = \cot 60° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{AD}{15} = \frac{1}{\sqrt{3}} \Rightarrow AD = \frac{15}{\sqrt{3}} = \frac{15 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}\ m.$$

$\therefore$ Difference between the lengths of the shadows $= AC - AD = 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3}\ m$

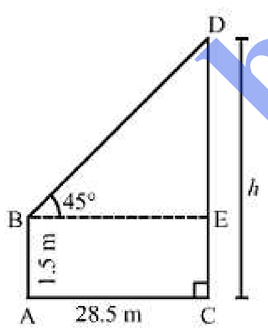
25. An observer 1.5m tall is 28.5 away from a tower and the angle of elevation of the top of the tower from the eye of the observer is $45°$. The height of the tower is

(a) 27m (b) 30m (c) 28.5m (d) None of these

Ans: (b)

Sol:

Let AB be the observer and CD be the tower.



Draw $BE \perp CD$, let $CD = h$ meters. Then,

$AB = 1.5m, BE = AC = 28.5m$ *and* $\angle EBD = 45°$

$DE = (CD - EC) = (CD - AB) = (h - 1.5)m.$

In right ΔBED, we have:

$$\frac{DE}{BE} = \tan 45° = 1$$

$$\Rightarrow \frac{(h-1.5)}{28.5} = 1$$

$$\Rightarrow h = 1.5 = 28.5$$

$$\Rightarrow h = 28.5 + 1.5 = 30m$$

Hence, the height of the tower is 30 m.