Exercise – 11A

1. Show that each of the progressions given below is an AP. Find the first term, common difference and next term of each.

(i) 9, 15, 21, 27,…………

(ii) 11, 6, 1, – 4,……..

(iii) $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ...$

(iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

(v) $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, ...$

Sol:

(i) The given progression 9, 15, 21, 27,…………

Clearly, 15 – 9 = 21 – 15 = 27 – 21 = 6 (Constant)

Thus, each term differs from its preceding term by 6. So, the given progression is an AP.

First term = 9

Common difference =6

Next term of the AP = 27 + 6 = 33

(ii) The given progression 11, 6, 1, – 4,……..

Clearly,6 – 11 = 1 – 6 = –4 – 1 = –5 (Constant)

Thus, each term differs from its preceding term by 6. So, the given progression is an AP.

First term = 11

Common difference = –5

Next term of the $AP = -4 + (-5) = -9$

(iii) The given progression $-1, \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, ...$

Clearly, $\frac{-5}{6} - (-1) = \frac{-2}{3} - \left(\frac{-5}{6}\right) = \frac{-1}{2} - \left(\frac{-2}{3}\right) = \frac{1}{6}$ (Constant)

Thus, each term differs from its preceding term by $\frac{1}{6}$. So, the given progression is an AP.

First term = –1

Common difference = $\frac{1}{6}$

Next tern of the $AP = \frac{-1}{2} + \frac{1}{6} = \frac{-2}{6} = \frac{-1}{3}$

(iv) The given progression $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots\ldots\ldots.$

This sequence can be written as $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, \ldots\ldots..$

Clearly, $2\sqrt{2}-\sqrt{2}=3\sqrt{2}-2\sqrt{2}=4\sqrt{2}-3\sqrt{2}=\sqrt{2}$ (Constant)

Thus, each term differs from its preceding term by $\sqrt{2}$, So, the given progression is an AP.

First term $=\sqrt{2}$

Common difference $=\sqrt{2}$

Next tern of the $AP=4\sqrt{2}+\sqrt{2}=5\sqrt{2}=\sqrt{50}$

(v) This given progression $\sqrt{20}, \sqrt{45}, \sqrt{80}, \sqrt{125}, \ldots\ldots\ldots$

This sequence can be re-written as $2\sqrt{5}, 3\sqrt{5}, 4\sqrt{5}, 5\sqrt{5}, \ldots\ldots\ldots..$

Clearly, $3\sqrt{5}-2\sqrt{5}=4\sqrt{5}-3\sqrt{5}=5\sqrt{5}-4\sqrt{5}=\sqrt{5}$ (Constant)

Thus, each term differs from its preceding term by $\sqrt{5}$. So, the given progression is an AP.

First term $=2\sqrt{5}=\sqrt{20}$

Common difference $=\sqrt{5}$

Next term of the $AP=5\sqrt{5}+\sqrt{5}=6\sqrt{5}=\sqrt{180}$

2. Find:

(i) the 20th term of the AP $9, 13, 17, 21, \ldots\ldots\ldots.$

(ii) the 35th term of AP $20, 17, 14, 11, \ldots\ldots\ldots.$

(iii) the 18th term of the AP $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots\ldots\ldots..$

(iv) the 9th term of the AP $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots\ldots\ldots$

(v) the 15the term of the AP $-40, -15, 10, 35, \ldots\ldots\ldots$

Sol:

(i) The given AP is $9, 13, 17, 21, \ldots\ldots\ldots.$

First term, $a=9$

Common difference, $d=13-9=4$

n^{th} term of the AP, $a_n=a+(n-1)d=9+(n-1)\times 4$

$\therefore$ $20th$ term of the AP, $a_{20}=9+(20-1)\times 4=9+76=85$

(ii) The given AP is $20, 17, 14, 11, \ldots\ldots\ldots.$

First term, $a=20$

Common difference, $d=17-20=-3$

n^{th} term of the AP, $a_n=a+(n-1)d=20+(n-1)\times(-3)$

$\therefore 35th$ term of the AP, $a_{35} = 20+(35-1)\times(-3) = 20-102 = -82$

(iii) The given AP is $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, ...$

This can be re-written as $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, ...$

First term, $a = \sqrt{2}$

Common difference, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

n^{th} term of the AP, $a_n = a+(n-1)d = \sqrt{2}+(n-1)\times 2\sqrt{2}$

$\therefore 18th$ term of the AP, $a_{18} = \sqrt{2}+(18-1)\times 2\sqrt{2} = \sqrt{2}+34\sqrt{2} = 35\sqrt{2} = \sqrt{2450}$

(iv) The given AP is $\frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, ...$

First term, $a = \frac{3}{4}$

Common difference, $d = \frac{5}{4} - \frac{3}{4} = \frac{2}{4} = \frac{1}{2}$

n^{th} term of the AP, $a_n = a+(n-1)d = \frac{3}{4}+(n-1)\times\left(\frac{1}{2}\right)$

$\therefore 9^{th}$ *term* of the AP, $a_9 = \frac{3}{4}+(9-1)\times\frac{1}{2} = \frac{3}{4}+4 = \frac{19}{4}$

(v) The given AP is $-40, -15, 10, 35, ...$

First term, $a = -40$

Common difference, $d = -15-(-40) = 25$

n^{th} term of the AP, $a_n = a+(n-1)d = -40+(n-1)\times 25$

$\therefore 15th$ term of the AP, $a_{15} = -40+(15-1)\times 25 = -40+350 = 310$

3. Find the 37th term of the AP $6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, ...$

Sol:

The given AP is $6, 7\frac{3}{4}, 9\frac{1}{2}, 11\frac{1}{4}, ...$

First term, $a = 6$ and common difference, $d = 7\frac{3}{4} - 6 \Rightarrow \frac{31}{4} - 6 \Rightarrow \frac{31-24}{4} = \frac{7}{4}$

Now, $T_{37} = a+(37-1)d = a+36d$

$= 6+36\times\frac{7}{4} = 6+63 = 69$

$\therefore 37^{th}$ term = 69

4. Find the 25th term of the AP $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, ...$

Sol:

The given AP is $5, 4\frac{1}{2}, 4, 3\frac{1}{2}, 3, ...$

First term = 5

Common difference $= 4\frac{1}{2} - 5 \Rightarrow \frac{9}{2} - 5 \Rightarrow \frac{9-10}{2} = -\frac{1}{2}$

$\therefore a = 5 \text{ and } d = -\frac{1}{2}$

Now, $T_{25} = a + (25-1)d = a + 24d$

$= 5 + 24 \times \left(-\frac{1}{2}\right) = 5 - 12 = -7$

$\therefore 25^{th}$ term $= -7$

5. Find the nth term of each of the following Aps:
(i) 5, 11, 17, 23 …
(ii) 16, 9, 2, -5, …..
Sol:
(i) (6n – 1)
(ii) (23 – 7n)

6. If the nth term of a progression is (4n – 10) show that it is an AP. Find its
(i) first term, (ii) common difference (iii) 16 the term.
Sol:

$T_n = (4_n - 10)$ [Given]

$T_1 = (4 \times 1 - 10) = -6$

$T_2 = (4 \times 2 - 10) = -2$

$T_3 = (4 \times 3 - 10) = 2$

$T_4 = (4 \times 4 - 10) = 6$

Clearly, $[-2-(-6)] = [2-(-2)] = [6-2] = 4$ (Constant)

So, the terms $-6, -2, 2, 6, ...$ forms an AP.

Thus we have

(i) First term $= -6$

(ii) Common difference = 4

(iii) $T_{16} = a + (n-1)d = a + 15d = -6 + 15 \times 4 = 54$

7. How many terms are there in the AP 6,1 0, 14, 18, ….., 174?

Sol:

In the given AP, $a = 6$ and $d = (10-6) = 4$

Suppose that there are n terms in the given AP.

Then, $T_n = 174$

$\Rightarrow a + (n-1)d = 174$

$\Rightarrow 6 + (n-1) \times 4 = 174$

$\Rightarrow 2 + 4n = 174$

$\Rightarrow 4n = 172$

$\Rightarrow n = 43$

Hence, there are 43 terms in the given AP.

8. How many terms are there in the AP 41, 38, 35, ….,8?

Sol:

In the given AP, $a = 41$ *and* $d = (38-41) = -3$

Suppose that there are n terms in the given AP.

Then $T_n = 8$

$\Rightarrow a + (n-1)d = 8$

$\Rightarrow 41 + (n-1) \times (-3) = 8$

$\Rightarrow 44 - 3n = 8$

$\Rightarrow 3n = 36$

$\Rightarrow n = 12$

Hence, there are 12 terms in the given AP.

9. How many terms are there in the AP $18, 15\frac{1}{2}, 13, \ldots\ldots, -47.$?

Sol:

The given AP is $18, 15\frac{1}{2}, 13, \ldots\ldots, -47.$

First term, $a = 18$

Common difference, $d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{31-36}{2} = -\frac{5}{2}$

Suppose there re n terms in the given AP. Then,

$a_n = -47$

$\Rightarrow 18 + (n-1) \times \left(-\frac{5}{2}\right) = -47 \qquad [a_n = a + (n-1)d]$

$\Rightarrow -\frac{5}{2}(n-1) = -47 - 18 = -65$

$\Rightarrow n - 1 = -65 \times \left(-\frac{2}{5}\right) = 26$

$\Rightarrow n = 26 + 1 = 27$

Hence, there are 27 terms in the given AP.

10. Which term of term of the AP 3,8,13,18,….. is 88?

Sol:

In the given AP, first term, $a = 3$ and common difference, $a = (8-3) = 5.$

Let's its n^{th} term be 88

Then, $T_n = 88$

$\Rightarrow a + (n-1)d = 88$

$\Rightarrow 3 + (n-1) \times 5 = 88$

$\Rightarrow 5n - 2 = 88$

$\Rightarrow 5n = 90$

$\Rightarrow n = 18$

Hence, the 18^{th} term of the given AP is 88.

11. Which term of AP 72,68,64,60,… is 0?

Sol:

In the given AP, first term, $a = 72$ and common difference, $d = (68-72) = -4.$

Let its n^{th} term be 0.

Then, $T_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 72 + (n-1) \times (-4) = 0$

$\Rightarrow 76 - 4n = 0$

$\Rightarrow 4n = 76$

$\Rightarrow n = 19$

Hence, the 19^{th} term of the given AP is 0.

12. Which term of the AP $\frac{5}{6},1,1\frac{1}{6},1\frac{1}{3},.....$ is 3?

Sol:

In the given AP, first term $=\frac{5}{6}$ and common difference, $d=\left(1-\frac{5}{6}=\frac{1}{6}\right)$

Let its n^{th} term be 3.

Now, $T_n=3$

$\Rightarrow a+(n-1)d=3$

$\Rightarrow \frac{5}{6}+(n-1)\times\frac{1}{6}=3$

$\Rightarrow \frac{2}{3}+\frac{n}{6}=3$

$\Rightarrow \frac{n}{6}=\frac{7}{3}$

$\Rightarrow n=14$

Hence, the 14^{th} term of the given AP is 3.

13. Which term of the AP 21, 18, 15, …… is -81?

Sol:

The given AP is $21,18,15,.......$

First term, $a=21$

Common difference, $d=18-21=-3$

Suppose n^{th} term of the given AP is -81. then,

$a_n=-81$

$\Rightarrow 21+(n-1)\times(-3)=-81 \quad [a_n=a+(n-1)d]$

$\Rightarrow -3(n-1)=-81-21=-102$

$\Rightarrow n-1=\frac{102}{3}=34$

$\Rightarrow n=34+1=35$

Hence, the $35th$ term of the given AP is -81.

14. Which term of the AP 3,8, 13,18,…. Will be 55 more than its 20^{th} term?

Sol:

Here, $a=3$ and $d=(8-3)=5$

The 20^{th} term is given by

$T_{20}=a+(20-1)d=a+19d=3+19\times5=98$

$\therefore$ Required term $=(98+55)=153$

Let this be the n^{th} term.

Then $T_n = 153$

$\Rightarrow 3 + (n-1) \times 5 = 153$

$\Rightarrow 5n = 155$

$\Rightarrow n = 31$

Hence, the 31^{st} term will be 55 more than 20^{th} term.

15. Which term of the AP 5, 15, 25, ….. will be 130 more than its 31^{st} term?

Sol:

Here, $a = 5$ and $d = (15-5) = 10$

The 31^{st} term is given by

$T_{31} = a + (31-1)d = a + 30d = 5 + 30 \times 10 = 305$

$\therefore$ Required term $= (305 + 130) = 435$

Let this be the n^{th} term.

Then, $T_n = 435$

$\Rightarrow 5 + (n-1) \times 10 = 435$

$\Rightarrow 10n = 440$

$\Rightarrow n = 44$

Hence, the 44^{th} term will be 130 more than its 31^{st} term.

16. If the 10^{th} term of an AP is 52 and 17^{th} term is 20 more than its 13^{th} term, find the AP

Sol:

In the given AP, let the first term be a and the common difference be d.

Then, $T_n = a + (n-1)d$

Now, we have:

$T_{10} = a + (10-1)d$

$\Rightarrow a + 9d = 52$ ....(1)

$T_{13} = a + (13-1)d = a + 12d$ ....(2)

$T_{17} = a + (17-1)d = a + 16d$ ....(3)

But, it is given that $T_{17} = 20 + T_{13}$

i.e., $a + 16d = 20 + a + 12d$

$\Rightarrow 4d = 20$

$\Rightarrow d = 5$

On substituting $d = 5$ in (1), we get:

$a+9\times5=52$

$\Rightarrow a=7$

Thus, $a=7$ *and* $d=5$

$\therefore$ The terms of the AP are $7,12,17,22,.........$

17. Find the middle term of the AP 6, 13, 20, …., 216.

Sol:

The given AP is $6,13,20,........,216.$

First term, $a=6$

Common difference, $d=13-6=7$

Suppose these are n terms in the given AP. Then,

$a_n=216$

$\Rightarrow 6+(n-1)\times7=216$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 7(n-1)=216-6=210$

$\Rightarrow n-1=\frac{210}{7}=30$

$\Rightarrow n=30+1=31$

Thus, the given AP contains 31 terms,

$\therefore$ Middle term of the given AP

$=\left(\frac{31+1}{2}\right)th$ term

$=16th$ term

$=6+(16-1)\times7$

$=6+105$

$=111$

Hence, the middle term of the given AP is 111.

18. Find the middle term of the AP 10, 7, 4, ……., (-62).

Sol:

The given AP is $10,7,4,.....,-62.$

First term, $a=10$

Common difference, $d=7-10=-3$

Suppose these are n terms in the given AP. Then,

$a_n = -62$

$\Rightarrow 10+(n-1)\times(-3) = -62$ $\quad [a_n = a+(n-1)d]$

$\Rightarrow -3(n-1) = -62-10 = -72$

$\Rightarrow n-1 = \frac{72}{3} = 24$

$\Rightarrow n = 24+1 = 25$

Thus, the given AP contains 25 terms.

$\therefore$ Middle term of the given AP

$= \left(\frac{25+1}{2}\right)th$ term

$= 13th$ term

$= 10+(13-1)\times(-3)$

$= 10-36$

$= -26$

Hence, the middle term of the given AP is -26.

19. Find the sum of two middle most terms of the AP $-\frac{4}{3}, -1, \frac{-2}{3}, ..., 4\frac{1}{3}$.

Sol:

The given AP is $-\frac{4}{3}, -1, \frac{-2}{3}, ..., 4\frac{1}{3}$.

First term, $a = -\frac{4}{3}$

Common difference, $d = -1-\left(-\frac{4}{3}\right) = -1+\frac{4}{3} = \frac{1}{3}$

Suppose there are n terms in the given AP. Then,

$a_n = 4\frac{1}{3}$

$\Rightarrow -\frac{4}{3}+(n-1)\times\left(\frac{1}{3}\right) = \frac{13}{3}$ $\quad [a_n = a+(n-1)d]$

$\Rightarrow \frac{1}{3}(n-1) = \frac{13}{3}+\frac{4}{3} = \frac{17}{3}$

$\Rightarrow n-1 = 17$

$\Rightarrow n = 17+1 = 18$

Thus, the given AP contains 18 terms. So, there are two middle terms in the given AP.

The middle terms of the given AP are $\left(\frac{18}{2}\right)th$ terms and $\left(\frac{18}{2}+1\right)th$ term i.e. 9th term and 10th term.

$\therefore$ Sum of the middle most terms of the given AP

= 9th term + 10th term

$=\left[-\frac{4}{3}+(9-1)\times\frac{1}{3}\right]+\left[-\frac{4}{3}+(10-1)\times\frac{1}{3}\right]$

$=-\frac{4}{3}+\frac{8}{3}-\frac{4}{3}+3$

$=3$

Hence, the sum of the middle most terms of the given AP is 3.

20. Find the 8th term from the end of the AP 7, 10, 13, ……, 184.

Sol:

Here, $a=7$ and $d=(10-7)=3, l=184$ and $n=8^{th}$ form the end.

Now, nth term from the end $=\left[l-(n-1)d\right]$

8th term from the end $=\left[184-(8-1)\times 3\right]$

$=\left[184-(7\times 3)\right]=(184-21)=163$

Hence, the 8th term from the end is 163.

21. Find the 6th term form the end of the AP 17, 14, 11, ……, (-40).

Sol:

Here, $a=7$ and $d=(14-17)=-3, l=(-40)$ *and* $n=6$

Now, nth term from the end $=\left[l-(n-1)d\right]$

6th term from the end $=\left[(-40)-(6-1)\times(-3)\right]$

$=\left[-40+(5\times 3)\right]=(-40+15)=-25$

Hence, the 6th term from the end is –25.

22. Is 184 a term of the AP 3, 7, 11, 15, ….?

Sol:

The given AP is $3,7,11,15,......$

Here, $a=3$ and $d=7-3=4$

Let the nth term of the given AP be 184. Then,

$a_n=184$

$\Rightarrow 3+(n-1)\times 4=184 \qquad \left[a_n=a+(n-1)d\right]$

$\Rightarrow 4n-1=184$

$\Rightarrow 4n = 185$

$\Rightarrow n = \frac{185}{4} = 46\frac{1}{4}$

But, the number of terms cannot be a fraction.
Hence, 184 is not a term of the given AP.

23. Is -150 a term of the AP 11, 8, 5, 2, ……?
Sol:
The given AP is 11, 8, 5, 2, ……
Here, $a = 11$ and $d = 8 - 11 = -3$
Let the nth term of the given AP be -150. Then,

$a_n = -150$

$\Rightarrow 11 + (n-1) \times (-3) = -150 \qquad [a_n = a + (n-1)d]$

$\Rightarrow -3n + 14 = -150$

$\Rightarrow -3n = -164$

$\Rightarrow n = \frac{164}{3} = 54\frac{2}{3}$

But, the number of terms cannot be a fraction.
Hence, -150 is not a term of the given AP.

24. Which term of the AP 121, 117, 113, …. is its first negative term?
Sol:
The given AP is 121, 117, 113, …….
Here, $a = 121$ and $d = 117 - 121 = -4$
Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 121 + (n-1) \times (-4) < 0 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 125 - 4n < 0$

$\Rightarrow -4n < -125$

$\Rightarrow n > \frac{125}{4} = 31\frac{1}{4}$

$\therefore n = 32$

Hence, the 32^{nd} term is the first negative term of the given AP.

25. Which term of the AP $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots..$ is the first negative term?

Sol:

The given AP is $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots..$

Here, $a = 20$ and $d = 19\frac{1}{4} - 20 = \frac{77}{4} - 20 = \frac{77-80}{4} = -\frac{3}{4}$

Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 20 + (n-1) \times \left(-\frac{3}{4}\right) < 0 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 20 + \frac{3}{4} - \frac{3}{4}n < 0$

$\Rightarrow \frac{83}{4} - \frac{3}{4}n < 0$

$\Rightarrow -\frac{3}{4}n < -\frac{83}{4}$

$\Rightarrow n > \frac{83}{3} = 27\frac{2}{3}$

$\therefore n = 28$

Hence, the 28th term is the first negative term of the given AP.

26. The 7th term of the an AP is -4 and its 13th term is -16. Find the AP.

Sol:

We have

$T_7 = a + (n-1)d$

$\Rightarrow a + 6d = -4 \qquad \ldots\ldots..(1)$

$T_{13} = a + (n-1)d$

$\Rightarrow a + 12d = -16 \qquad \ldots\ldots.(2)$

On solving (1) and (2), we get

$a = 8 \text{ and } d = -2$

Thus, first term = 8 and common difference $= -2$

$\therefore$ The term of the AP are $8, 6, 4, 2, \ldots\ldots...$

27. The 4th term of an AP is zero. Prove that its 25th term is triple its 11th term.

Sol:

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a + (n-1)d$

Now, $T_4 = a + (4-1)d$

$\Rightarrow a + 3d = 0 \qquad ......(1)$

$\Rightarrow a = -3d$

Again, $T_{11} = a + (11-1)d = a + 10d$

$= -3d + 10d = 7d \qquad$ [Using (1)]

Also, $T_{25} = a + (25-1)d = a + 24d = -3d + 24d = 21d \qquad$ [Using (1)]

i.e., $T_{25} = 3 \times 7d = (3 \times T_{11})$

Hence, 25th term is triple its 11th term.

28. The 8th term of an AP is zero. Prove that its 38th term is triple its 18th term.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_8 = 0 \qquad [a_n = a + (n-1)d]$

$\Rightarrow a + (8-1)d = 0$

$\Rightarrow a + 7d = 0$

$\Rightarrow a = -7d \qquad ....(1)$

Now,

$$\frac{a_{38}}{a_{18}} = \frac{a + (38-1)d}{a + (18-1)d}$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{-7d + 37d}{-7d + 17d} \qquad [From\ (1)]$$

$$\Rightarrow \frac{a_{38}}{a_{18}} = \frac{30d}{10d} = 3$$

$\Rightarrow a_{38} = 3 \times a_{18}$

Hence, the 38th term of the AP id triple its 18th term.

29. The 4th term of an AP is 11. The sum of the 5th and 7th terms of this AP is 34. Find its common difference

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_4 = 11$

$\Rightarrow a + (4-1)d = 11 \qquad [a_n = a + (n-1)d]$

$\Rightarrow a + 3d = 11 \qquad ....(1)$

Now,

$a_5 + a_7 = 34 \qquad$ (Given)

$\Rightarrow (a+4d)+(a+6d)=34$

$\Rightarrow 2a+10d=34$

$\Rightarrow a+5d=17 \qquad ......(2)$

From (1) and (2), we get

$11-3d+5d=17$

$\Rightarrow 2d=17-11=6$

$\Rightarrow d=3$

Hence, the common difference of the AP is 3.

30. The 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94. Find the common difference of the AP.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_9=-32$

$\Rightarrow a+(9-1)d=-32 \qquad [a_n=a+(n-1)d]$

$\Rightarrow a+8d=-32 \qquad ....(1)$

Now,

$a_{11}+a_{13}=-94 \qquad$ (Given)

$\Rightarrow (a+10d)+(a+12d)=-94$

$\Rightarrow 2a+22d=-94$

$\Rightarrow a+11d=-47 \qquad ....(2)$

From (1) and (2), we get

$-32-8d+11d=-47$

$\Rightarrow 3d=-47+32=-15$

$\Rightarrow d=-5$

Hence, the common difference of the AP is -5.

31. Determine the nth term of the AP whose 7th term is -1 and 16th term is 17.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_7=-1$

$\Rightarrow a+(7-1)d=-1 \qquad [a_n=a+(n-1)d]$

$\Rightarrow a+6d=-1 \qquad ....(1)$

Also,

$a_{16}=17$

$\Rightarrow a+15d=17 \qquad ....(2)$

From (1) and (2), we get

$-1-6d+15d=17$

$\Rightarrow 9d=17+1=18$

$\Rightarrow d=2$

Putting $d=2$ in (1), we get

$a+6\times 2=-1$

$\Rightarrow a=-1-12=-13$

$\therefore a_n=a+(n-1)d$

$=-13+(n-1)\times 2$

$=2n-15$

Hence, the nth term of the AP is $(2n-15)$.

32. If 4 times the 4th term of an AP is equal to 18 times its 18th term then find its 22nd term.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$4\times a_4=18\times a_{18}$ (Given)

$\Rightarrow 4(a+3d)=18(a+17d)$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2(a+3d)=9(a+17d)$

$\Rightarrow 2a+6d=9a+153d$

$\Rightarrow 7a=-147d$

$\Rightarrow a=-21d$

$\Rightarrow a+21d=0$

$\Rightarrow a+(22-1)d=0$

$\Rightarrow a_{22}=0$

Hence, the 22nd term of the AP is 0.

33. If 10 times the 10th term of an AP is equal to 15 times the 15th term, show that its 25th term is zero.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$10\times a_{10}=15\times a_{15}$ (Given)

$\Rightarrow 10(a+9d)=15(a+14d)$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2(a+9d)=3(a+14d)$

$\Rightarrow 2a+18d=3a+42d$

$\Rightarrow a=-24d$

$\Rightarrow a+24d=0$

$\Rightarrow a+(25-1)d=0$

$\Rightarrow a_{25}=0$

Hence, the 25th term of the AP is 0.

34. Find the common difference of an AP whose first term is 5 and the sum of its first four terms is half the sum of the next four terms.

Sol:

Let the common difference of the AP be d.

First term, $a=5$

Now,

$a_1+a_2+a_3+a_4=\frac{1}{2}(a_5+a_6+a_7+a_8)$ (Given)

$\Rightarrow a+(a+d)+(a+2d)+(a+3d)=\frac{1}{2}[(a+4d)+(a+5d)+(a+6d)+(a+7d)]$

$[a_n=a+(n-1)d]$

$\Rightarrow 4a+6d=\frac{1}{2}(4a+22d)$

$\Rightarrow 8a+12d=4a+22d$

$\Rightarrow 22d-12d=8a-4a$

$\Rightarrow 10d=4a$

$\Rightarrow d=\frac{2}{5}a$

$\Rightarrow d=\frac{2}{5}\times 5=2$ $(a=5)$

Hence, the common difference of the AP is 2.

35. The sum of the 2nd and 7th terms of an AP is 30. If its 15th term is 1 less than twice its 8th term, find AP.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_2+a_7=30$ (Given)

$\therefore (a+d)+(a+6d)=30$ $[a_n=a+(n-1)d]$

$\Rightarrow 2a+7d=30$ ....(1)

Also,

$a_{15}=2a_8-1$ (Given)

$\Rightarrow a+14d=2(a+7d)-1$

$\Rightarrow a+14d=2a+14d-1$

$\Rightarrow -a=-1$

$\Rightarrow a=1$

Putting $a = 1$in (1), we get

$2\times1+7d=30$

$\Rightarrow 7d=30-2=28$

$\Rightarrow d=4$

So,

$a_2=a+d=1+4=5$

$a_3=a+2d=1+2\times4=9$..........

Hence, the AP is 1, 5, 9, 13, …….

36. For what value of n, the nth terms of the arithmetic progressions 63, 65, 67, … and 3, 10, 17, … are equal?

Sol:

Let the term of the given progressions be t_n and T_n, respectively.

The first AP is 63, 65, 67,...

Let its first term be a and common difference be *d*.

Then a = 63 and d = (65 - 63) = 2

So, its nth term is given by

$t_n=a+(n-1)d$

$\Rightarrow 63+(n-1)\times2$

$\Rightarrow 61+2n$

The second AP is 3, 10, 17,...

Let its first term be A and common difference be D.

Then A = 3 and D = (10 - 3) = 7

So, its nth term is given by

$T_n=A+(n-1)D$

$\Rightarrow 3+(n-1)\times7$

$\Rightarrow 7n-4$

Now, $t_n=T_n$

$\Rightarrow 61+2n=7n-4$

$\Rightarrow 65=5n$

$\Rightarrow n=13$

Hence, the l3 terms of the Al’s are the same.

37. The 17th term of AP is 5 more than twice its 8th term. If the 11th term of the AP is 43, find its nth term.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{17} = 2a_8 + 5$ (Given)

$\therefore a + 16d = 2(a + 7d) + 5 \quad [a_n = a + (n-1)d]$

$\Rightarrow a + 16d = 2a + 14d + 5$

$\Rightarrow a - 2d = -5 \quad ....(1)$

Also,

$a_{11} = 43$ (Given)

$\Rightarrow a + 10d = 43 \quad ....(2)$

From (1) and (2), we get

$-5 + 2d + 10d = 43$

$\Rightarrow 12d = 43 + 5 = 48$

$\Rightarrow d = 4$

Putting $d = 4$ in (1), we get

$a - 2\times 4 = -5$

$\Rightarrow a = -5 + 8 = 3$

$\therefore a_n = a + (n-1)d$

$= 3 + (n-1)\times 4$

$= 4n - 1$

Hence, the nth term of the AP is $(4n-1)$.

38. The 24th term of an AP is twice its 10th term. Show that its 72nd term is 4 times its 15th term.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{24} = 2a_{10}$ (Given)

$\Rightarrow a + 23d = 2(a + 9d) \quad [a_n = a + (n-1)d]$

$\Rightarrow a + 23d = 2a + 18d$

$\Rightarrow 2a - a = 23d - 18d$

$\Rightarrow a = 5d \quad ....(1)$

Now,

$$\frac{a_{72}}{a_{15}} = \frac{a + 71d}{a + 14d}$$

$\Rightarrow \frac{a_{72}}{a_{15}} = \frac{5d+71d}{5d+14d}$ $\quad [From(1)]$

$\Rightarrow \frac{a_{72}}{a_{15}} = \frac{76d}{19d} = 4$

$\Rightarrow a_{72} = 4 \times a_{15}$

Hence, the 72nd term of the AP is 4 times its 15th term.

39. The 19th term of an AP is equal to 3 times its 6th term. If its 9th term is 19, find the AP.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{19} = 3a_6$ (Given)

$\Rightarrow a+18d = 3(a+5d)$ $\quad [a_n = a+(n-1)d]$

$\Rightarrow a+18d = 3a+15d$

$\Rightarrow 3a-a = 18d-15d$

$\Rightarrow 2a = 3d$ ....(1)

Also,

$a_9 = 19$ (Given)

$\Rightarrow a+8d = 19$ ......(2)

From (1) and (2), we get

$\frac{3d}{2} + 8d = 19$

$\Rightarrow \frac{3d+16d}{2} = 19$

$\Rightarrow 19d = 38$

$\Rightarrow d = 2$

Putting $d = 2$ in (1), we get

$2a = 3 \times 2 = 6$

$\Rightarrow a = 3$

So,

$a_2 = a+d = 3+2 = 5$

$a_3 = a+2d = 3+2 \times 2 = 7, ...$

Hence, the AP is $3, 5, 7, 9, ...$

40. If the pth term of an AP is q and its qth term is p then show that its (p + q)th term is zero.

Sol:

In the given AP, let the first be a and the common difference be d.

Then, $T_n = a+(n-1)d$

$\Rightarrow Tp = a+(p-1)d = q \qquad ....(i)$

$\Rightarrow T_q = a+(q-1)d = p \qquad ....(ii)$

On subtracting (i) from (ii), we get:

$(q-p)d = (p-q)$

$\Rightarrow d = -1$

Putting $d=-1$ in (i), we get:

$a = (p+q-1)$

Thus, $a = (p+q-1)$ *and* $d = -1$

Now, $T_{p+q} = a+(p+q-1)d$

$= (p+q-1)+(p+q-1)(-1)$

$= (p+q-1)-(p+q-1) = 0$

Hence, the $(p+q)^{th}$ term is 0 (zero).

41. The first and last terms of an AP are a and l respectively. Show that the sum of the nth term from the beginning and the nth term form the end is (a + l).

Sol:

In the given AP, first term = *a* and last term = *l*.

Let the common difference be *d*.

Then, nth term from the beginning is given by

$T_n = a+(n-1)d \qquad .....(1)$

Similarly, nth term from the end is given by

$T_n = \{l-(n-1)d\} \qquad ......(2)$

Adding (1) and (2), we get

$a+(n-1)d+\{l-(n-1)d\}$

$= a+(n-1)d+l-(n-1)d$

$= a+l$

Hence, the sum of the nth term from the beginning and the nth term from the end $(a+l)$.

42. How many two-digit number are divisible by 6?

Sol:

The two digit numbers divisible by 6 are 12, 18, 24,……, 96

Clearly, these number are in AP.

Here, $a = 12$ and $d = 18-12 = 6$

Let this AP contains n terms. Then,

$a_n = 96$

$\Rightarrow 12 + (n-1) \times 6 = 96 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 6n + 6 = 96$

$\Rightarrow 6n = 96 - 6 = 90$

$\Rightarrow n = 15$

Hence, these are 15 two-digit numbers divisible by 6.

43. How many two-digits numbers are divisible by 3?

Sol:

The two-digit numbers divisible by 3 are 12, 15, 18, ..., 99.

Clearly, these number are in AP.

Here, $a = 12$ and $d = 15 - 12 = 3$

Let this AP contains n terms. Then,

$a_n = 99$

$\Rightarrow 12 + (n-1) \times 3 = 99 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 3n + 9 = 99$

$\Rightarrow 3n = 99 - 9 = 90$

$\Rightarrow n = 30$

Hence, there are 30 two-digit numbers divisible by 3.

44. How many three-digit numbers are divisible by 9?

Sol:

The three-digit numbers divisible by 9 are 108, 117, 126,...., 999.

Clearly, these number are in AP.

Here. a = 108 and d = 117 – 108 = 9

Let this AP contains n terms. Then.

$a_n = 999$

$\Rightarrow 108 + (n-1) \times 9 = 999 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 99 = 999$

$\Rightarrow 9n = 999 - 99 = 900$

$\Rightarrow n = 100$

Hence: there are 100 three-digit numbers divisible by 9.

45. Hoe many numbers are there between 101 and 999, which are divisible by both 2 and 5?

Sol:

The numbers which are divisible by both 2 and 5 are divisible by 10 also.

Now, the numbers between 101 and 999 which are divisible 10 are 110, 120, 130, .., 990.

Clearly, these number are in AP

Here, $a = 110$ and $d = 120 - 110 = 10$
Let this AP contains n terms. Then,
$a_n = 990$
$\Rightarrow 110 + (n-1) \times 10 = 990$ $\quad [a_n = a + (n-1)d]$
$\Rightarrow 10n + 100 = 990$
$\Rightarrow 10n = 990 - 100 = 890$
$\Rightarrow n = 89$
Hence, there are 89 numbers between 101 and 999 which are divisible by both 2 and 5.

46. In a flower bed, there are 43 rose plants in the first row, 41 in second, 39 in the third, and so on. There are 11 rose plants in the last row. How many rows are there in the flower bed?
Sol:
The numbers of rose plants in consecutive rows are 43, 41, 39,..., 11.
Difference of rose plants between two consecutive rows = (41 – 43) = (39 – 41) = –2 [Constant]
So, the given progression is an AP
Here, first term = 43
Common difference = –2
Last term 11
Let n be the last term, then we have:
$T_n = a + (n-1)d$
$\Rightarrow 11 = 43 + (n-1)(-2)$
$\Rightarrow 11 = 45 - 2n$
$\Rightarrow 34 = 2n$
$\Rightarrow n = 17$
Hence, the 17^{th} term is 11 or there are 17 rows in the flower bed.

47. A sum of ₹2800 is to be used to award four prizes. If each prize after the first is ₹200 less than the preceding prize, find the value of each of the prizes.
Sol:
Let the amount of the first prize be ₹a
Since each prize after the first is ₹200 less than the preceding prize, so the amounts of the four prizes are in AP.
Amount of the second prize $=$₹ $(a - 200)$
Amount of the third prize $=$₹ $(a - 2 \times 200) = (a - 400)$
Amount of the fourth prize $=$₹$(a - 3 \times 200) = (a - 600)$
Now,

Total sum of the four prizes = 2,800

$\therefore$ ₹a+₹$(a-200)+$₹$(a-400)+$₹$(a-600)=$₹2,800

$\Rightarrow 4a-1200=2800$

$\Rightarrow 4a=2800+1200=4000$

$\Rightarrow a=1000$

$\therefore$ Amount of the first prize = ₹1,000

Amount of the second prize $=$₹$(1000-200)$=₹800

Amount of the third prize $=$₹$(1000=400)=$₹600

Amount of the fourth prize $=$₹$(1000-600)$ = ₹400

Hence, the value of each of the prizes is ₹1,000, ₹800, ₹600 and ₹400.

Exercise - 11B

1. Determine k so that (3k -2), (4k – 6) and (k +2) are three consecutive terms of an AP.

Sol:

It is given that $(3k-2),(4k-6)$ and $(k+2)$ are three consecutive terms of an AP.

$\therefore (4k-6)-(3k-2)=(k+2)-(4k-6)$

$\Rightarrow 4k-6-3k+2=k+2-4k+6$

$\Rightarrow k-4=-3k+8$

$\Rightarrow k+3k=8+4$

$\Rightarrow 4k=12$

$\Rightarrow k=3$

Hence, the value of k is 3.

2. Find the value of x for which the numbers (5x + 2), (4x - 1) and (x + 2) are in AP.

Sol:

It is given that $(5x+2),(4x-1)$ and $(x+2)$ are in AP.

$\therefore (4x-1)-(5x+2)=(x+2)-(4x-1)$

$\Rightarrow 4x-1-5x-2=x+2-4x+1$

$\Rightarrow -x-3=-3x+3$

$\Rightarrow 3x-x=3+3$

$\Rightarrow 2x=6$

$\Rightarrow x=3$

Hence, the value of x is 3.

3. If (3y – 1), (3y + 5) and (5y + 1) are three consecutive terms of an AP then find the value of y.

Sol:

It is given that $(3y-1),(3y+5)$ and $(5y+1)$ are three consecutive terms of an AP.

$\therefore (3y+5)-(3y-1)=(5y+1)-(3y+5)$

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow 6=2y-4$

$\Rightarrow 2y=6+4=10$

$\Rightarrow y=5$

Hence, the value of y is 5.

4. Find the value of x for which (x + 2), 2x, ()2x + 3) are three consecutive terms of an AP.

Sol:

Since $(x+2), 2x$ and $(2x+3)$ are in AP, we have

$2x-(x+2)=(2x+3)-2x$

$\Rightarrow x-2=3$

$\Rightarrow x=5$

$\therefore x=5$

5. Show that $(a-b)^2, (a^2+b^2)$ and (a^2+b^2) are in AP.

Sol:

The given numbers are $(a-b)^2, (a^2+b^2)$ and $(a+b)^2$.

Now,

$(a^2+b^2)-(a-b)^2=a^2+b^2-(a^2-2ab+b^2)=a^2+b^2-a^2+2ab-b^2=2ab$

$(a+b)^2-(a^2+b^2)=a^2+2ab+b^2-a^2-b^2=2ab$

So, $(a^2+b^2)-(a-b)^2=(a+b)^2-(a^2+b^2)=2ab$ (Constant)

Since each term differs from its preceding term by a constant, therefore, the given numbers are in AP.

6. Find the three numbers in AP whose sum is 15 and product is 80.

Sol:

Let the required numbers be $(a-d), a$ and $(a+d)$.

Then $(a-d)+a+(a+d)=15$

$\Rightarrow 3a = 15$

$\Rightarrow a = 5$

Also, $(a-d).a.(a+d) = 80$

$\Rightarrow a(a^2 - d^2) = 80$

$\Rightarrow 5(25 - d^2) = 80$

$\Rightarrow d^2 = 25 - 16 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 5$ and $d = \pm 3$

Hence, the required numbers are $(2, 5 \ and \ 8) \ or \ (8, 5 \ and \ 2)$.

7. The sum of three numbers in AP is 3 and their product is -35. Find the numbers.

Sol:

Let the required numbers be $(a-d), a$ and $(a+d)$.

Then $(a-d) + a + (a+d) = 3$

$\Rightarrow 3a = 3$

$\Rightarrow a = 1$

Also, $(a-d).a.(a+d) = -35$

$\Rightarrow a(a^2 - d^2) = -35$

$\Rightarrow 1.(1 - d^2) = -35$

$\Rightarrow d^2 = 36$

$\Rightarrow d = \pm 6$

Thus, $a = 1$ and $d = \pm 6$

Hence, the required numbers are $(-5, 1 \ and \ 7) \ or \ (7, 1 \ and \ -5)$.

8. Divide 24 in three parts such that they are in AP and their product is 440.

Sol:

Let the required parts of 24 be $(a-d), a$ and $(a+d)$ such that they are in AP.

Then $(a-d) + a + (a+d) = 24$

$\Rightarrow 3a = 24$

$\Rightarrow a = 8$

Also, $(a-d).a.(a+d) = 440$

$\Rightarrow a(a^2 - d^2) = 440$

$\Rightarrow 8(64 - d^2) = 440$

$\Rightarrow d^2 = 64 - 55 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 8$ and $d = \pm 3$

Hence, the required parts of 24 are $(5, 8, 11)$ *or* $(11, 8, 5)$.

9. The sum of three consecutive terms of an AP is 21 and the sum of the squares of these terms is 165. Find these terms

Sol:

Let the required terms be $(a-d), a$ and $(a+d)$.

Then $(a-d)+a+(a+d)=21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

Also, $(a-d)^2 + a^2 + (a+d)^2 = 165$

$\Rightarrow 3a^2 + 2d^2 = 165$

$\Rightarrow (3\times 49 + 2d^2) = 165$

$\Rightarrow 2d^2 = 165 - 147 = 18$

$\Rightarrow d^2 = 9$

$\Rightarrow d = \pm 3$

Thus, $a = 7$ and $d = \pm 3$

Hence, the required terms are $(4, 7, 10)$ *or* $(10, 7, 4)$.

10. The angles of quadrilateral are in whose AP common difference is $10°$. Find the angles.

Sol:

Let the required angles be $(a-15)°, (a-5)°, (a+5)°$ *and* $(a+15)°$, as the common difference is 10 (given).

Then $(a-15)° + (a-5)° + (a+5)° + (a+15)° = 360°$

$\Rightarrow 4a = 360$

$\Rightarrow a = 90$

Hence, the required angles of a quadrilateral are

$(90-15)°, (90-5)°, (90+5)°$ *and* $(90+15)°$; *or* $75°, 85°, 95°$ *and* $105°$.

11. Find four numbers in AP whose sum is 8 and the sum of whose squares is 216.

Sol:

(4, 6, 8, 10) or (10, 8, 6, 4)

12. Divide 32 into four parts which are the four terms of an AP such that the product of the first and fourth terms is to product of the second and the third terms as 7:15.

Sol:

Let the four parts in AP be $(a-3d),(a-d),(a+d)$ *and* $(a+3d)$. Then,

$(a-3d)+(a-d)+(a+d)+(a+3d)=32$

$\Rightarrow 4a=32$

$\Rightarrow a=8$ ....(1)

Also,

$(a-3d)(a+3d):(a-d)(a+d)=7:15$

$\Rightarrow \frac{(8-3d)(8+3d)}{(8-d)(8+d)}=\frac{7}{15}$ $[From\ (1)]$

$\Rightarrow \frac{64-9d^2}{64-d^2}=\frac{7}{15}$

$\Rightarrow 15(64-9d^2)=7(64-d^2)$

$\Rightarrow 960-135d^2=448-7d^2$

$\Rightarrow 135d^2-7d^2=960-448$

$\Rightarrow 128d^2=512$

$\Rightarrow d^2=4$

$\Rightarrow d=\pm 2$

When $a=8$ and $d=2$,

$a-3d=8-3\times 2=8-6=2$

$a-d=8-2=6$

$a+d=8+2=10$

$a+3d=8+3\times 2=8+6=14$

When $a=8$ and $d=-2$,

$a-3d=8-3\times(-2)=8+6=14$

$a-d=8-(-2)=8+2=10$

$a+d=8-2=6$

$a+3d=8+3\times(-2)=8-6=2$

Hence, the four parts are 2, 6, 10 and 14.

13. The sum of first three terms of an AP is 48. If the product of first and second terms exceeds 4 times the third term by 12. Find the AP.

Sol:

Let the first three terms of the AP be $(a-d), a$ *and* $(a+d)$. Then,

$(a-d)+a+(a+d)=48$
$\Rightarrow 3a=48$
$\Rightarrow a=16$
Now,
$(a-d)\times a=4(a+d)+12$ (Given)
$\Rightarrow (16-d)\times 16=4(16+d)+12$
$\Rightarrow 256-16d=64+4d+12$
$\Rightarrow 16d+4d=256-76$
$\Rightarrow 20d=180$
$\Rightarrow d=9$
When $a=16$ and $d=9$,
$a-d=16-9=7$
$a+d=16+9=25$
Hence, the first three terms of the AP are 7, 16, and 25.

Exercise - 11C

1. The first three terms of an AP are respectively (3y – 1), (3y + 5) and (5y + 1), find the value of y.
Sol:
The terms $(3y-1),(3y+5)$ and $(5y+1)$ are in AP.
$\therefore (3y+5)-(3y-1)=(5y+1)-(3y+5)$
$\Rightarrow 3y+5-3y+1=5y+1-3y-5$
$\Rightarrow 6=2y-4$
$\Rightarrow 2y=10$
$\Rightarrow y=5$
Hence, the value of y is 5.

2. If $k,(2k-1)$ and $(2k+1)$ are the three successive terms of an AP, find the value of k.
Sol:
It is given that $k,(2k-1)$ and $(2k+1)$ are the three successive terms of an AP.
$\therefore (2k-1)-k=(2k+1)-(2k-1)$
$\Rightarrow k-1=2$
$\Rightarrow k=3$
Hence, the value of k is 3.

3. If 18, a, (b - 3) are in AP, then find the value of (2a – b)

Sol:

It is given that 18, $a,(b-3)$ are in AP.

$\therefore a-18=(b-3)-a$

$\Rightarrow a+a-b=18-3$

$\Rightarrow 2a-b=15$

Hence, the required value is 15.

4. If the numbers a, 9, b, 25 from an AP, find a and b.

Sol:

It is given that the numbers $a,9,b,25$ from an AP.

$\therefore 9-a=b-9=25-b$

So,

$b-9=25-b$

$\Rightarrow 2b=34$

$\Rightarrow b=17$

Also,

$9-a=b-9$

$\Rightarrow a=18-b$

$\Rightarrow a=18-17 \qquad (b=17)$

$\Rightarrow a=1$

Hence, the required values of a and b are 1 and 17, respectively.

5. If the numbers (2n – 1), (3n+2) and (6n -1) are in AP, find the value of n and the numbers

Sol:

It is given that the numbers $(2n-1),(3n+2)$ and $(6n-1)$ are in AP.

$\therefore (3n+2)-(2n-1)=(6n-1)-(3n+2)$

$\Rightarrow 3n+2-2n+1=6n-1-3n-2$

$\Rightarrow n+3=3n-3$

$\Rightarrow 2n=6$

$\Rightarrow n=3$

When, $n=3$

$2n-1=2\times3-1=6-1=5$

$3n+2=3\times3+2=9+2=11$

$6n-1=6\times3-1=18-1=17$

Hence, the required value of n is 3 and the numbers are 5, 11 and 17.

6. How many three-digit natural numbers are divisible by 7?

Sol:

The three digit natural numbers divisible by 7 are $105, 112, 119......, 994$

Clearly, these number are in AP.

Here, $a = 105$ *and* $d = 112 - 105 = 7$

Let this AP contains n terms. Then,

$a_n = 994$

$\Rightarrow 105 + (n-1) \times 7 = 994 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 7n + 98 = 994$

$\Rightarrow 7n = 994 - 98 = 986$

$\Rightarrow n = 128$

Hence, there are 128 three digit numbers divisible by 7.

7. How many three-digit natural numbers are divisible by 9?

Sol:

The three-digit natural numbers divisible by 9 are 108, 117, 126 ,..... 999.

Clearly, these number are in AP.

Here. a = 108 and d = 117 – 108 = 9

Let this AP contains n terms. Then,

$a_n = 999$

$\Rightarrow 108 + (n-1) \times 9 = 999 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 99 = 999$

$\Rightarrow 9n = 999 - 99 = 900$

$\Rightarrow n = 100$

Hence, there are 100 three-digit numbers divisible by 9.

8. If the sum of first m terms of an AP is $(2m^2 + 3m)$ then what is its second term?

Sol:

Let S_m denotes the sum of first m terms of the AP.

$\therefore S_m = 2m^2 + 3m$

$\Rightarrow S_{m-1} = 2(m-1)^2 + 3(m-1) = 2(m^2 - 2m + 1) + 3(m-1) = 2m^2 - m - 1$

Now,

m^{th} term of AP, $a_m = S_m - S_{m-1}$

$\therefore a_m = (2m^2 + 3m) - (2m^2 - m - 1) = 4m + 1$

Putting $m = 2$, we get

$a_2 = 4 \times 2 + 1 = 9$

Hence, the second term of the AP is 9.

9. What is the sum of first n terms of the AP a, 3a, 5a, …..

Sol:

The given AP is $3a, 5a, \ldots\ldots..$

Here,

First term, A = a

Common difference, $D = 3a - a = 2a$

$\therefore$ Sum of the n terms, S_n

$= \frac{n}{2}\left[2 \times a + (n-1) \times 2a\right]$ $\left\{S_n = \frac{n}{2}\left[2A + (n-1)D\right]\right\}$

$= \frac{n}{2}(2a + 2an - 2a)$

$= \frac{n}{2} \times 2an$

$= an^2$

Hence, the required sum is an^2.

10. What is the 5th term form the end of the AP 2, 7, 12, …., 47?

Sol:

The given AP is 2, 7, 12, ..., 47.

Let us re-write the given AP in reverse order i.e. 47, 42, .., 12, 7, 2.

Now, the 5th term from the end of the given AP is equal to the 5th term from beginning of the AP 47, 42,.... ,12, 7, 2.

Consider the AP 47, 42,..., 12, 7, 2.

Here, $a = 47$ and $d = 42 - 47 = -5$

5th term of this AP

$= 47 + (5 - 1) \times (-5)$

$= 47 - 20$

$= 27$

Hence, the 5th term from the end of the given AP is 27.

11. If a_n denotes the nth term of the AP 2, 7, 12, 17, … find the value of $(a_{30} - a_{20})$.

Sol:

The given AP is 2, 7, 12, 17,………

Here, $a = 2$ and $d = 7 - 2 = 5$

$\therefore a_{30} - a_{20}$

$= [2 + (30-1)\times 5] - [2 + (20-1)\times 5] \qquad [a_n = a + (n-1)d]$

$= 147 - 97$

$= 50$

Hence, the required value is 50.

12. The nth term of an AP is (3n +5). Find its common difference.

Sol:

We have

$T_n = (3n+5)$

Common difference $= T_2 - T_1$

$T_1 = 3\times 1 + 5 = 8$

$T_2 = 3\times 2 + 5 = 11$

$d = 11 - 8 - 3$

Hence, the common difference is 3.

13. The nth term of an AP is (7 – 4n). Find its common difference.

Sol:

We have

$T_n = (7-4n)$

Common difference $= T_2 - T_1$

$T_1 = 7 - 4\times 1 = 3$

$T_2 = 7 - 4\times 2 = -1$

$d = -1 - 3 = -4$

Hence, the common difference is -4.

14. Write the next term for the AP $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

Sol:

The given AP is $\sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

On simplifying the terms, we get:

$2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, ...$

Here, $a = 2\sqrt{2}$ and $d = (3\sqrt{2} - 2\sqrt{2}) = \sqrt{2}$

$\therefore$ Next term, $T_4 = a + 3d = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2} = \sqrt{50}$

15. Write the next term of the AP $\sqrt{2},\sqrt{8},\sqrt{18},.........$

Sol:

The given AP is $\sqrt{2},\sqrt{8},\sqrt{18},.........$

On simplifying the terms, we get:

$\sqrt{2},2\sqrt{2},3\sqrt{2},......$

Here, $a=\sqrt{2}$ and $d=\left(2\sqrt{2}-\sqrt{2}\right)=\sqrt{2}$

$\therefore$ Next term, $T_4=a+3d=\sqrt{2}+3\sqrt{2}=4\sqrt{2}=\sqrt{32}$

16. Which term of the AP 21, 18, 15, … is zero?

Sol:

In the given AP, first term, $a=21$ and common difference, $d=\left(18-21\right)=-3$

Let's its n^{th} term be 0.

Then, $T_n=0$

$\Rightarrow a+\left(n-1\right)d=0$

$\Rightarrow 21+\left(n-1\right)\times\left(-3\right)=0$

$\Rightarrow 24-3n=0$

$\Rightarrow 3n=24$

$\Rightarrow n=8$

Hence, the 8^{th} term of the given AP is 0.

17. Find the sum of the first n natural numbers.

Sol:

The first n natural numbers are 1, 2, 3, 4, 5,……..,n

Here, a = 1 and d = (2 – 1) = 1

Sum of n terms of an AP is given by

$$S_n=\frac{n}{2}\left[2a+\left(n-1\right)d\right]$$

$$=\left(\frac{n}{2}\right)\times\left[2\times1+\left(n-1\right)\times1\right]$$

$$=\left(\frac{n}{2}\right)\times\left[2+n-1\right]=\left(\frac{n}{2}\right)\times\left(n+1\right)=\frac{n\left(n+1\right)}{2}$$

18. Find the sum of first n even natural numbers.

Sol:

The first n even natural numbers are 2, 4, 6, 8, 10, …., n.

Here, $a = 2$ and $d = (4-2) = 2$

Sum of n terms of an AP is given by

$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \left(\frac{n}{2}\right)\times\left[2\times 2+(n-1)\times 2\right]$

$= \left(\frac{n}{2}\right)\times\left[4+2n-2\right] = \left(\frac{n}{2}\right)\times(2n+2) = n(n+1)$

Hence, the required sum is $n(n+1)$.

19. The first term of an AP is p and its common difference is q. Find its 10th term.

Sol:

Here, $a = p$ and $d = q$

Now, $T_n = a+(n-1)d$

$\Rightarrow T_n = p+(n-1)q$

$\therefore T_{10} = p+9q$

20. If $\frac{4}{5}, a, 2$ are in AP, find the value of a.

Sol:

If 45, a and 2 are three consecutive terms of an AP, then we have:

$a - 45 = 2 - a$

$\Rightarrow 2a = 2+45$

$\Rightarrow 2a = 145$

$\Rightarrow a = 75$

21. If (2p +1), 13, (5p -3) are in AP, find the value of p.

Sol:

Let $(2p+1), 13, (5p-3)$ be three consecutive terms of an AP.

Then $13-(2p+1) = (5p-3)-13$

$\Rightarrow 7p = 28$

$\Rightarrow p = 4$

$\therefore$ When $p = 4, (2p+1), 13$ and $(5p-3)$ from three consecutive terms of an AP.

22. If (2p – 1), 7, 3p are in AP, find the value of p.

Sol:

Let $(2p-1), 7$ and $3p$ be three consecutive terms of an AP.

Then $7-(2p-1)=3p-7$

$\Rightarrow 5p=15$

$\Rightarrow p=3$

$\therefore$ When $p=3, (2p-1), 7$ and $3p$ form three consecutive terms of an AP.

23. If the sum of first p terms of an AP is (ap^2+bp), find its common difference.

Sol:

Let S_p denotes the sum of first p terms of the AP.

$\therefore S_p = ap^2+bp$

$\Rightarrow S_{p-1} = a(p-1)^2+b(p-1)$

$= a(p^2-2p+1)+b(p-1)$

$= ap^2-(2a-b)p+(a-b)$

Now,

p^{th} term of $AP, a_p = S_p - S_{p-1}$

$= (ap^2+bp)-\left[ap^2-(2a-b)p+(a-b)\right]$

$= ap^2+bp-ap^2+(2a-b)p-(a-b)$

$= 2ap-(a-b)$

Let d be the common difference of the AP.

$\therefore d = a_p - a_{p-1}$

$= \left[2ap-(a-b)\right]-\left[2a(p-1)-(a-b)\right]$

$= 2ap-(a-b)-2a(p-1)+(a-b)$

$= 2a$

Hence, the common difference of the AP is 2a.

24. If the sum of first n terms is $(3n^2+5n)$, find its common difference.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2+5n$

$\Rightarrow S_{n-1} = 3(n-1)^2+5(n-1)$

$=3\left(n^2-2n+1\right)+5(n-1)$

$=3n^2-n-2$

Now,

n^{th} term of $AP, a_n = S_n - S_{n-1}$

$=\left(3n^2+5n\right)-\left(3n^2-n-2\right)$

$=6n+2$

Let d be the common difference of the AP.

$\therefore d = a_n - a_{n-1}$

$=(6n+2)-[6(n-1)+2]$

$=6n+2-6(n-1)-2$

$=6$

Hence, the common difference of the AP is 6.

25. Find an AP whose 4^{th} term is 9 and the sum of its 6^{th} and 13^{th} terms is 40.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_4 = 9$

$\Rightarrow a+(4-1)d=9$ $\quad [a_n = a+(n-1)d]$

$\Rightarrow a+3d=9$ ....(1)

Now,

$a_6 + a_{13} = 40$ (Given)

$\Rightarrow (a+5d)+(a+12d)=40$

$\Rightarrow 2a+17d=40$ ....(2)

From (1) and (2), we get

$2(9-3d)+17d=40$

$\Rightarrow 18-6d+17d=40$

$\Rightarrow 11d = 40-18=22$

$\Rightarrow d=2$

Putting $d=2$ in (1), we get

$a+3\times 2=9$

$\Rightarrow a=9-6=3$

Hence, the AP is 3, 5, 7, 9, 11,…….

Exercise – 11D

1. Find the sum of each of the following Aps:

(i) 2, 7, 12, 17, ……. to 19 terms.

(ii) 9, 7, 5, 3 … to 14 terms

(iii) -37, -33, -29, … to 12 terms.

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$ to 11 terms.

(v) 0.6, 1.7, 2.8, …. to 100 terms

Sol:

(i) The given AP is 2, 7, 12, 17,………

Here, $a = 2$ and $d = 7 - 2 = 5$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$$S_{19} = \frac{19}{2}\left[2 \times 2 + (19-1) \times 5\right]$$

$$= \frac{19}{2} \times (4 + 90)$$

$$= \frac{19}{2} \times 94$$

$$= 893$$

(ii) The given AP is 9, 7, 5, 3,……….

Here, $a = 9$ and $d = 7 - 9 = -2$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$$S_{14} = \frac{14}{2}\left[2 \times 9 + (14-1) \times (-2)\right]$$

$$= 7 \times (18 - 26)$$

$$= 7 \times (-8)$$

$$= -56$$

(iii) The given AP is -37, -33, -29,……..

Here, $a = -37$ and $d = -33 - (-37) = -33 + 37 = 4$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$$S_{12} = \frac{12}{2}\left[2 \times (-37) + (12-1) \times 4\right]$$

$$= 6 \times (-74 + 44)$$

$$= 6 \times (-30)$$

$= -180$

(iv) The given AP is $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots\ldots$

Here, $a = \frac{1}{15}$ and $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{11} = \frac{11}{2}\left[2 \times \left(\frac{1}{15}\right) + (11-1) \times \frac{1}{60}\right]$

$= \frac{11}{2} \times \left(\frac{2}{15} + \frac{10}{60}\right)$

$= \frac{11}{2} \times \left(\frac{18}{60}\right)$

$= \frac{33}{20}$

(v) The given AP is $0.6, 1.7, 2.8, \ldots\ldots\ldots\ldots$

Here, $a = 0.6$ and $d = 1.7 - 0.6 = 1.1$

Using formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we have

$S_{100} = \frac{100}{2}\left[2 \times 0.6 + (100-1) \times 1.1\right]$

$= 50 \times (1.2 + 108.9)$

$= 50 \times 110.1$

$= 5505$

2. Find the sum of each of the following arithmetic series:

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) $34 + 32 + 30 + \ldots + 10$

(iii) $(-5) + (-8) + (-11) + \ldots + (-230)$

Sol:

(i) The given arithmetic series is $7 + 10\frac{1}{2} + 14 + \ldots\ldots + 84.$

Here, $a = 7, d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{21-4}{2} = \frac{7}{2}$ and $l = 84.$

Let the given series contains n terms. Then,

$a_n = 84$

$\Rightarrow 7 + (n-1) \times \frac{7}{2} = 84$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow \frac{7}{2}n + \frac{7}{2} = 84$

$\Rightarrow \frac{7}{2}n = 84 - \frac{7}{2} = \frac{161}{2}$

$\Rightarrow n = \frac{161}{7} = 23$

$\therefore$ Required sum $= \frac{23}{2} \times (7 + 84)$ $\quad \left[S_n = \frac{n}{2}(a + l)\right]$

$= \frac{23}{2} \times 91$

$= \frac{2030}{2}$

$1046\frac{1}{2}$

(ii) The given arithmetic series is $34 + 32 + 30 + ... + 10$.
Here, $a = 34, d = 32 - 34 = -2$ and $l = 10$.
Let the given series contain *n* terms. Then,

$a_n = 10$

$\Rightarrow 34 + (n-1) \times (-2) = 10$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow -2n + 36 = 10$

$\Rightarrow -2n = 10 - 36 = -26$

$\Rightarrow n = 13$

$\therefore$ Required sum $= \frac{13}{2} \times (34 + 10)$ $\quad \left[S_n = \frac{n}{2}(a + l)\right]$

$= \frac{13}{2} \times 44$

$= 286$

(iii) The given arithmetic series is $(-5) + (-8) + (-11) + ... + (-230)$.

Here, $a = -5, d = -8 - (-5) = -8 + 5 = -3$ and $l = 230$.

Let the given series contain *n* terms. Then,

$a_n = -230$

$\Rightarrow -5 + (n-1)\times(-3) = -230 \qquad [a_n = a + (n-1)d]$

$\Rightarrow -3n - 2 = -230$

$\Rightarrow -3n = -230 + 2 = -228$

$\Rightarrow n = 76$

$\therefore$ Required sum $= \frac{76}{2}\times[(-5)+(-230)] \qquad \left[S_n = \frac{n}{2}(a+l)\right]$

$= \frac{76}{2}\times(-235)$

$= -8930$

3. Find the sum of first n terms of an AP whose nth term is (5 - 6n). Hence, find the sum of its first 20 terms.

Sol:

Let a_n be the nth term of the AP.

$\therefore a_n = 5 - 6n$

Putting $n = 1$, we get

First term, $a = a_1 = 5 - 6\times 1 = -1$

Putting $n = 2$, we get

$a_2 = 5 - 6\times 2 = -7$

Let d be the common difference of the AP.

$\therefore d = a_2 - a_1 = -7 - (-1) = -7 + 1 = -6$

Sum of first n tern of the AP, S_n

$= \frac{n}{2}[2\times(-1) + (n-1)\times(-6)] \qquad \left\{S_n = \frac{n}{2}[2a + (n-1)d]\right\}$

$= \frac{n}{2}(-2 - 6n + 6)$

$= n(2 - 3n)$

$= 2n - 3n^2$

Putting $n = 20$, we get

$S_{20} = 2\times 20 - 3\times 20^2 = 40 - 1200 = -1160$

4. The sum of the first n terms of an AP is $(3n^2 + 6n)$. Find the nth term and the 15th term of this AP.

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2 + 6n$

$\Rightarrow S_{n-1} = 3(n-1)^2 + 6(n-1)$

$= 3(n^2 - 2n + 1) + 6(n-1)$

$= 3n^2 - 3$

$\therefore n^{th}$ term of the AP, a_n

$= S_n - S_{n-1}$

$= (3n^2 + 6n) - (3n^2 - 3)$

$= 6n + 3$

Putting $n = 15$, we get

$a_{15} = 6 \times 15 + 3 = 90 + 3 = 93$

Hence, the n^{th} term is $(6n+3)$ and 15th term is 93.

5. The sum of the first n terms of an AP is given by $S_n = (3n^2 - n)$. Find its

(i) nth term,

(ii) first term and

(iii) common difference.

Sol:

Given: $S_n = (3n^2 - n)$ ......(i)

Replacing n by $(n-1)$ in (i), we get:

$S_{n-1} = 3(n-1)^2 - (n-1)$

$= 3(n^2 - 2n + 1) - n + 1$

$= 3n^2 - 7n + 4$

(i) Now, $T_n = (S_n - S_{n-1})$

$= (3n^2 - n) - (3n^2 - 7n + 4) = 6n - 4$

$\therefore n^{th}$ term, $T_n = (6n - 4)$ ....(ii)

(ii) Putting $n = 1$ in (ii), we get:

$T_1 = (6 \times 1) - 4 = 2$

(iii) Putting $n = 2$ in (ii), we get:

$T_2 = (6 \times 2) - 4 = 8$

$\therefore$ Common difference, $d = T_2 - T_1 = 8 - 2 = 6$

6. The sum of the first n terms of an AP is $\left(\frac{5n^2}{2}+\frac{3n}{2}\right)$. Find its nth term and the 20th term of this AP.

Sol:

$$S_n=\left(\frac{5n^2}{2}+\frac{3n}{2}\right)=\frac{1}{2}\left(5n^2+3n\right) \qquad ....(i)$$

Replacing n by $(n-1)$ in (i), we get:

$$S_{n-1}=\frac{1}{2}\times\left[5(n-1)^2+3(n-1)\right]$$

$$=\frac{1}{2}\times\left[5n^2-10n+5+3n-3\right]=\frac{1}{2}\times\left[5n^2-7n+2\right]$$

$$\therefore T_n=S_n-S_{n-1}$$

$$=\frac{1}{2}\left(5n^2+3n\right)-\frac{1}{2}\times\left[5n^2-7n+2\right]$$

$$=\frac{1}{2}(10n-2)=5n-1 \qquad ....(ii)$$

Putting $n=20$ in (ii), we get

$$T_{20}=(5\times 20)-1=99$$

Hence, the 20th term is 99.

7. The sum of the first n term sofa an AP is $\left(\frac{3n^2}{2}+\frac{5n}{2}\right)$. Find its nth term and the 25th term

Sol:

Let S_n denotes the sum of first n terms of the AP.

$$\therefore S_n=\frac{3n^2}{2}+\frac{5n}{2}$$

$$\Rightarrow S_{n-1}=\frac{3(n-1)^2}{2}+\frac{5(n-1)}{2}$$

$$=\frac{3(n^2-2n+1)}{2}+\frac{5(n-1)}{2}$$

$$=\frac{3n^2-n-2}{2}$$

$\therefore n^{th}$ term of the AP, a_n

$$=S_n-S_{n-1}$$

$=\left(\frac{3n^2+5n}{2}\right)-\left(\frac{3n^2-n-2}{2}\right)$

$=\frac{6n+2}{2}$

$=3n+2$

Putting $n=25$, we get

$a_{25}=3\times25+1=75+1=76$

Hence, the nth term is $(3n+1)$ and 25th term is 76.

8. How many terms of the AP 21, 18, 15, … must be added to get the sum 0?

Sol:

Thee given AP is 21, 18, 15,……….

Here, $a=21$ and $d=18-21=-3$

Let the required number of terms be n. Then,

$S_n=0$

$\Rightarrow\frac{n}{2}\left[2\times21+(n-1)\times(-3)\right]=0 \qquad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow\frac{n}{2}(42-3n+3)=0$

$\Rightarrow n(45-3n)=0$

$\Rightarrow n=0 \text{ or } 45-3n=0$

$\Rightarrow n=0 \text{ or } n=15$

$\therefore n=15$ (Number of terms cannot be zero)

Hence, the required number of terms is 15.

9. How many terms of the AP 9, 17, 25, … must be taken so that their sum is 636?

Sol:

The given AP is 9, 17, 25,……

Here, $a=9$ and $d=17-9=8$

Let the required number of terms be n. Then,

$S_n=636$

$\Rightarrow\frac{n}{2}\left[2\times9+(n-1)\times8\right]=636 \qquad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow\frac{n}{2}(18+8n-8)=636$

$\Rightarrow\frac{n}{2}(10+8n)=636$

$\Rightarrow n(5+4n)=636$

$\Rightarrow 4n^2+5n-636=0$

$\Rightarrow 4n^2-48n+53n-636=0$

$\Rightarrow 4n(n-12)+53(n-12)=0$

$\Rightarrow (n-12)(4n+53)=0$

$\Rightarrow n-12=0 \text{ or } 4n+53=0$

$\Rightarrow n=12 \text{ or } n=-\frac{53}{4}$

$\therefore n=12$ (Number of terms cannot negative)

Hence, the required number of terms is 12.

10. How many terms of the AP 63, 60, 57, 54, ….. must be taken so that their sum is 693? Explain the double answer.

Sol:

The given AP is 63, 60, 57, 54,………..

Here, $a=63$ and $d=60-63=-3$

Let the required number of terms be *n*. Then,

$S_n=693$

$\Rightarrow \frac{n}{2}\left[2\times 63+(n-1)\times(-3)\right]=693$ $\left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow \frac{n}{2}(126-3n+3)=693$

$\Rightarrow n(129-3n)=1386$

$\Rightarrow 3n^2-129n+1386=0$

$\Rightarrow 3n^2-66n-63n+1386=0$

$\Rightarrow 3n(n-22)-63(n-22)=0$

$\Rightarrow (n-22)(3n-63)=0$

$\Rightarrow n-22=0 \text{ or } 3n-63=0$

$\Rightarrow n=22 \text{ or } n=21$

So, the sum of 21 terms as well as that of 22 terms is 693. This is because the 22nd term of the AP is 0.

$a_{22}=63+(22-1)\times(-3)=63-63=0$

Hence, the required number of terms is 21 or 22.

11. How many terms of the AP $20, 19\frac{1}{3}, 18\frac{2}{3}, \ldots$ must be taken so that their sum is 300? Explain the double answer.

Sol:

The given AP is 20, $19\frac{1}{3}, 18\frac{2}{3}, \ldots\ldots$

Here, $a = 20$ and $d = 19\frac{1}{3} - 20 = \frac{58}{3} - 20 = \frac{58-60}{3} = -\frac{2}{3}$

Let the required number of terms be n. Then,

$S_n = 300$

$\Rightarrow \frac{n}{2}\left[2\times 20 + (n-1)\times\left(-\frac{2}{3}\right)\right] = 300 \qquad \left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow \frac{n}{2}\left(40 - \frac{2}{3}n + \frac{2}{3}\right) = 300$

$\Rightarrow \frac{n}{2}\times\frac{(122-2n)}{3} = 300$

$\Rightarrow 122n - 2n^2 = 1800$

$\Rightarrow 2n^2 - 122n + 1800 = 0$

$\Rightarrow 2n^2 - 50n - 72n + 1800 = 0$

$\Rightarrow 2n(n-25) - 72(n-25) = 0$

$\Rightarrow (n-25)(2n-72) = 0$

$\Rightarrow n - 25 = 0 \ or \ 2n - 72 = 0$

$\Rightarrow n = 25 \ or \ n = 36$

So, the sum of first 25 terms as well as that of first 36 terms is 300. This is because the sum of all terms from 26^{th} to 36^{th} is 0.

12. Find the sum of all odd numbers between 0 and 50.

Sol:

All odd numbers between 0 and 50 are 1, 3, 5, 7,……… 49.

This is an AP in which $a = 1, d = (3-1) = 2$ and $l = 49$.

Let the number of terms be n.

Then, $T_n = 49$

$\Rightarrow a + (n-1)d = 49$

$\Rightarrow 1 + (n-1)\times 2 = 49$

$\Rightarrow 2n = 50$

$\Rightarrow n = 25$

$\therefore$ Required sum $= \frac{n}{2}(a+l)$

$= \frac{25}{2}[1+49] = 25 \times 25 = 625$

Hence, the required sum is 625.

13. Find the sum of all natural numbers between 200 and 400 which are divisible by 7.

Sol:

Natural numbers between 200 and 400 which are divisible by 7 are 203, 210 ,.... 399.

This is an AP with a = 203, d = 7 and l = 399.

Suppose there are n terms in the AP. Then,

$a_n = 399$

$\Rightarrow 203 + (n-1) \times 7 = 399 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 7n + 196 = 399$

$\Rightarrow 7n = 399 - 196 = 203$

$\Rightarrow n = 29$

$\therefore$ Required sum $= \frac{29}{2}(203+399) \qquad \left[S_n = \frac{n}{2}(a+l)\right]$

$= \frac{29}{2} \times 602$

$= 8729$

Hence, the required sum is 8729.

14. Find the sum of first forty positive integers divisible by 6.

Sol:

The positive integers divisible by 6 are 6, 12, 18,…..

This is an AP with $a = 6$ and $d = 6$.

Also, $n = 40$ (Given)

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{40} = \frac{40}{2}[2 \times 6 + (40-1) \times 6]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Hence, the required sum is 4920.

15. Find the sum of first 15 multiples of 8.

Sol:

The first 15 multiples of 8 are 8, 16, 24, 32,…….

This is an AP in which $a=8, d=(16-8)=8$ and $n=15$.

Thus, we have:

$l=a+(n-1)d$

$=8+(15-1)8$

$=120$

$\therefore$ Required sum $=\frac{n}{2}(a+l)$

$=\frac{15}{2}[8+120]=15\times 64=960$

Hence, the required sum is 960.

16. Find the sum of all multiples of 9 lying between 300 and 700.

Sol:

The multiples of 9 lying between 300 and 700 are 306, 315,………, 693.

This is an AP with $a=306, d=9$ and $l=693$.

Suppose these are n terms in the AP. Then,

$a_n=693$

$\Rightarrow 306+(n-1)\times 9=693$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 9n+297=693$

$\Rightarrow 9n=693-297=396$

$\Rightarrow n=44$

$\therefore$ Required sum $=\frac{44}{2}(306=693)$ $\quad \left[S_n=\frac{n}{2}(a+l)\right]$

$=22\times 999$

$=21978$

Hence, the required sum is 21978.

17. Find the sum of all three-digits natural numbers which are divisible by 13.

Sol:

All three-digit numbers which are divisible by 13 are 104, 117, 130, 143,……. 938.

This is an AP in which a = 104, d = (117 – 104) = 13 and l = 938

Let the number of terms be n

Then $T_n=938$

$\Rightarrow a+(n-1)d=988$

$\Rightarrow 104+(n-1)\times 13=988$

$\Rightarrow 13n=897$

$\Rightarrow n=69$

$\therefore$ Required sum $=\frac{n}{2}(a+l)$

$=\frac{69}{2}[104+988]=69\times 546=37674$

Hence, the required sum is 37674.

18. Find the sum of first 100 even number which are divisible by 5.

Sol:

The first few even natural numbers which are divisible by 5 are 10, 20, 30, 40, ...

This is an AP in which a = 10, d = (20 – 10) = 10 and n = 100

The sum of n terms of an AP is given by

$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$=\left(\frac{100}{2}\right)\times\left[2\times 10+(100-1)\times 10\right]$ $\left[\because a=10, d=10 \text{ and } n=100\right]$

$=50\times[20+990]=50\times 1010=50500$

Hence, the sum of the first hundred even natural numbers which are divisible by 5 is 50500.

19. Find the sum of the following.

$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots$ up to n terms.

Sol:

On simplifying the given series, we get:

$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots n \text{ terms}$

$=(1+1+1+\ldots\ldots n \text{ terms})-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots\ldots+\frac{n}{n}\right)$

$=n-\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots\ldots+\frac{n}{n}\right)$

Here, $\left(\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots\ldots+\frac{n}{n}\right)$ is an AP whose first term is $\frac{1}{n}$ and the common difference is $\left(\frac{2}{n}-\frac{1}{n}\right)=\frac{1}{n}$.

The sum of terms of an AP is given by

$$S_n=\frac{n}{2}\left[2a+(n-1)d\right]$$

$$=n-\left[\frac{n}{2}\left\{2\times\left(\frac{1}{n}\right)+(n-1)\times\left(\frac{1}{n}\right)\right\}\right]$$

$$=n-\left[\frac{n}{2}\left[\left(\frac{2}{n}\right)+\left(\frac{n-1}{n}\right)\right]\right]=n-\left\{\frac{n}{2}\left(\frac{n+1}{n}\right)\right\}$$

$$=n-\left(\frac{n+1}{2}\right)=\frac{n-1}{2}$$

20. In an AP. It is given that $S_5+S_7=167$ and $S_{10}=235$, then find the AP, where S_n denotes the sum of its first n terms.

Sol:

Let a be the first term and d be the common difference of thee AP. Then,

$$S_5+S_7=167$$

$$\Rightarrow \frac{5}{2}(2a+4d)+\frac{7}{2}(2a+6d)=167 \qquad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$$

$$\Rightarrow 5a+10d+7a+21d=167$$

$$\Rightarrow 12a+31d=167 \qquad \ldots\ldots\ldots\ldots(1)$$

Also,

$$S_{10}=235$$

$$\Rightarrow \frac{10}{2}(2a+9d)=235$$

$$\Rightarrow 5(2a+9d)=235$$

$$\Rightarrow 2a+9d=47$$

Multiplying both sides by 6, we get

$$12a+54d=282 \qquad \ldots\ldots\ldots\ldots(2)$$

Subtracting (1) from (2), we get

$$12a+54d-12a-31d=282-167$$

$$\Rightarrow 23d=115$$

$$\Rightarrow d=5$$

Putting $d=5$ in (1), we get

$12a + 31 \times 5 = 167$

$\Rightarrow 12a + 155 = 167$

$\Rightarrow 12a = 167 - 155 = 12$

$\Rightarrow a = 1$

Hence, the AP is 1, 6, 11, 16,…….

21. In an AP, the first term is 2, the last term is 29 and the sum of all the terms is 155. Find the common difference.

Sol:

Here, $a = 2, l = 29 \text{ and } S_n = 155$

Let d be the common difference of the given AP and n be the total number of terms.

Then, $T_n = 29$

$\Rightarrow a + (n-1)d = 29$

$\Rightarrow 2 + (n-1)d = 29 \qquad ....(i)$

The sum of n terms of an AP is given by

$S_n = \frac{n}{2}[a + l] = 155$

$\Rightarrow \frac{n}{2}[2 + 29] = \left(\frac{n}{2}\right) \times 31 = 155$

$\Rightarrow n = 10$

Putting the value of n in (i), we get:

$\Rightarrow 2 + 9d = 29$

$\Rightarrow 9d = 27$

$\Rightarrow d = 3$

Thus, the common difference of the given AP is 3.

22. In an AP, the first term is -4, the last term is 29 and the sum of all its terms is 150. Find its common difference.

Sol:

Suppose there are n terms in the AP.

Here, $a = -4, l = 29$ and $S_n = 150$

$S_n = 150$

$\Rightarrow \frac{n}{2}(-4 + 29) = 150 \qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow n = \frac{150 \times 2}{25} = 12$

Thus, the AP contains 12 terms.

Let d be the common difference of the AP.

$\therefore a_{12} = 29$

$\Rightarrow -4 + (12-1) \times d = 29 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 11d = 29 + 4 = 33$

$\Rightarrow d = 3$

Hence, the common difference of the AP is 3.

23. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol:

Suppose there are n terms in the AP.

Here, a = 17, d = 9 and $l = 350$

$\therefore a_n = 350$

$\Rightarrow 17 + (n-1) \times 9 = 350 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 9n + 8 = 350$

$\Rightarrow 9n = 350 - 8 = 342$

$\Rightarrow n = 38$

Thus, there are 38 terms in the AP.

$\therefore S_{38} = \frac{28}{2}(17 + 350) \qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$= 19 \times 367$

$= 6973$

Hence, the required sum is 6973.

24. The first and last terms of an AP are 5 and 45 respectively. If the sum of all its terms is 400, find the common difference and the number of terms.

Sol:

Suppose there are n term in the AP.

Here, $a = 5, l = 45$ and $S_n = 400$

$S_n = 400$

$\Rightarrow \frac{n}{2}(5 + 45) = 400 \qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$\Rightarrow \frac{n}{2} \times 50 = 400$

$\Rightarrow n = \frac{400 \times 2}{50} = 16$

Thus, there are 16 terms in the AP.

Let d be the common difference of the AP.

$\therefore a_{16} = 45$

$\Rightarrow 5 + (16-1) \times d = 45 \qquad [a_n = a + (n-1)d]$

$\Rightarrow 15d = 45 - 5 = 40$

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

Hence, the common difference of the AP is $\frac{8}{3}$.

25. In an AP, the first term is 22, nth terms is -11 and sum of first n terms is 66. Find the n and hence find the4 common difference.

Sol:

Here, $a = 22, T_n = -11$ and $S_n = 66$

Let d be the common difference of the given AP.

Then, $T_n = -11$

$\Rightarrow a + (n-1)dd = 22 + (n-1)d = -11$

$\Rightarrow (n-1)d = -33 \qquad ....(i)$

The sum of n terms of an AP is given by

$S_n = \frac{n}{2}[2a + (n-1)d] = 66 \qquad$ [Substituting the value off $(n-1)d$ from (i)]

$\Rightarrow \frac{n}{2}[2 \times 22 + (-33)] = \left(\frac{n}{2}\right) \times 11 = 66$

$\Rightarrow n = 12$

Putting the value of n in (i), we get:

$11d = -33$

$\Rightarrow d = -3$

Thus, $n = 12$ and $d = -3$

26. The 12th term of an AP is -13 and the sum of its first four terms is 24. Find the sum of its first 10 terms.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{12} = -13$

$\Rightarrow a + 11d = -13 \qquad ....(1) \qquad [a_n = a + (n-1)d]$

Also,

$S_4 = 24$

$\Rightarrow \frac{4}{2}(2a+3d)=24 \qquad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 2a+3d=12 \qquad ....(2)$

Solving (1) and (2), we get

$2(-13-11d)+3d=12$

$\Rightarrow -26-22d+3d=12$

$\Rightarrow -19d=12+26=38$

$\Rightarrow d=-2$

Putting $d=-2$ in (1), we get

$a+11\times(-2)=-13$

$\Rightarrow a=-13+22=9$

$\therefore$ Sum of its first 10 terms, S_{10}

$=\frac{10}{2}\left[2\times 9+(10-1)\times(-2)\right]$

$=5\times(18-18)$

$=5\times 0$

$=0$

Hence, the required sum is 0.

27. The sum of the first 7 terms of an AP is 182. If its 4th and 17th terms are in the ratio 1:5, find the AP.

Sol:

Let a be the first term and d be the common difference of the AP.

$\therefore S_7=182$

$\Rightarrow \frac{7}{2}(2a+6d)=182 \qquad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow a+3d=26 \qquad ....(1)$

Also,

$a_4 : a_{17}=1:5 \qquad$ (Given)

$\Rightarrow \frac{a+3d}{a+16d}=\frac{1}{5} \qquad \left[a_n=a+(n-1)d\right]$

$\Rightarrow 5a+15d=a+16d$

$\Rightarrow d=4a \qquad ....(2)$

Solving (1) and (2), we get

$a + 3\times 4a = 26$
$\Rightarrow 13a = 26$
$\Rightarrow a = 2$
Putting $a = 2$ in (2), we get
$d = 4\times 2 = 8$
Hence, the required AP is 2, 10, 18, 26,………

28. The sum of the first 9 terms of an AP is 81 and that of its first 20 terms is 400. Find the first term and common difference of the AP.
Sol:
Here, $a = 4, d = 7 \; and \; l = 81$
Let the nth term be 81.
Then $T_n = 81$
$\Rightarrow a + (n-1)d = 4 + (n-1)7 = 81$
$\Rightarrow (n-1)7 = 77$
$\Rightarrow (n-1) = 11$
$\Rightarrow n = 12$
Thus, there are 12 terms in the AP.
The sum of n terms of an AP is given by
$$S_n = \frac{n}{2}[a + l]$$
$$\therefore S_{12} = \frac{12}{2}[4 + 81] = 6\times 85 = 510$$
Thus, the required sum is 510.

29. The sum of the first 7 terms of an AP is 49 and the sum of its first 17 term is 289. Find the sum of its first n terms.
Sol:
Let a be the first term and d be the common difference of the given AP.
Then, we have:
$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_7 = \frac{7}{2}[2a + 6d] = 7[a + 3d]$$
$$S_{17} = \frac{17}{2}[2a + 16d] = 17[a + 8d]$$
However, $S_7 = 49$ and $S_{17} = 289$
Now, $7(a + 3d) = 49$

$\Rightarrow a+3d=7 \quad ....(i)$

Also, $17[a+8d]=289$

$\Rightarrow a+8d=17 \quad ....(ii)$

Subtracting (i) from (ii), we get:

$5d=10$

$\Rightarrow d=2$

Putting $d=2$ in (i), we get

$a+6=7$

$\Rightarrow a=1$

Thus, $a=1$ and $d=2$

$\therefore$ Sum of n terms of AP = $\frac{n}{2}\left[2\times1+(n-1)\times2\right]=n\left[1+(n-1)\right]=n^2$

30. Two Aps have the same common difference. If the fist terms of these Aps be 3 and 8 respectively. Find the difference between the sums of their first 50 terms.

Sol:

Let a_1 and a_2 be the first terms of the two APs.

Here, $a_1=8$ *and* $a_2=3$

Suppose d be the common difference of the two Aps

Let S_{50} and $S'_{50}=\frac{50}{2}\left[2a_1+(50-1)d\right]-\frac{50}{2}\left[2a_2+(50-1)d\right]$

$=25(2\times8\times49d)-25(2\times3+49d)$

$=25\times(16-6)$

$=250$

Hence, the required difference between the two sums is 250.

31. The sum first 10 terms of an AP is -150 and the sum of its next 10 terms is -550 . Find the AP.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$S_{10}=-150$ (Given)

$\Rightarrow \frac{10}{2}(2a+9d)=-150 \qquad \left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 5(2a+9d)=-150$

$\Rightarrow 2a+9d=-30 \quad ....(1)$

It is given that the sum of its next 10 terms is -550.

Now,

S_{20} = Sum of first 20 terms = Sum of first 10 terms + Sum of the next 10 terms =

$-150+(-550)=-700$

$\therefore S_{20}=-700$

$\Rightarrow \frac{20}{2}(2a+19d)=-700$

$\Rightarrow 10(2a+19d)=-700$

$\Rightarrow 2a+19d=-70 \qquad ....(2)$

Subtracting (1) from (2), we get

$(2a+19d)-(2a+9d)=-70-(-30)$

$\Rightarrow 10d=-40$

$\Rightarrow d=-4$

Putting $d=-4$ in (1), we get

$2a+9\times(-4)=-30$

$\Rightarrow 2a=-30+36=6$

$\Rightarrow a=3$

Hence, the required AP is $3,-1,-5,-9,$............

32. The 13th terms of an AP is 4 times its 3rd term. If its 5th term is 16, Find the sum of its first 10 terms.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{13}=4\times a_3 \qquad$ (Given)

$\Rightarrow a+12d=4(a+2d) \qquad [a_n=a+(n-1)d]$

$\Rightarrow a+12d=4a+8d$

$\Rightarrow 3a=4d \qquad ....(1)$

Also,

$a_5=16 \qquad$ (Given)

$\Rightarrow a+4d=16 \qquad ....(2)$

Solving (1) and (2), we get

$a+3a=16$

$\Rightarrow 4a=16$

$\Rightarrow a=4$

Putting $a=4$ in (1), we get

$4d=3\times4=12$

$\Rightarrow d=3$

Using the formula, $S_4 = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$S_{10} = \frac{10}{2}\left[2\times4+(10-1)\times3\right]$

$=5\times(8+27)$

$=5\times35$

$=175$

Hence, the required sum is 175.

33. The 16th term of an AP is 5 times its 3rd term. If its 10th term is 41, find the sum of its first 15 terms.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$a_{16} = 5\times a_3$ (Given)

$\Rightarrow a+15d = 5(a+2d)$ $\left[a_n = a+(n-1)d\right]$

$\Rightarrow a+15d = 5a+10d$

$\Rightarrow 4a = 5d$

Also,

$a_{10} = 41$ (Given)

$\Rightarrow a+9d-41$ ....(2)

Solving (1) and (2), we get

$a+9\times\frac{4a}{5} = 41$

$\Rightarrow \frac{5a+36a}{5} = 41$

$\Rightarrow \frac{41a}{5} = 41$

$\Rightarrow a = 5$

Putting $a=5$ in (1), we get

$5d = 4\times5 = 20$

$\Rightarrow d = 4$

Using the formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$S_{15} = \frac{15}{2}\left[2\times5+(15-1)\times4\right]$

$= \frac{15}{2}\times(10+56)$

$= \frac{15}{2}\times66$

$= 495$

Hence, the required sum is 495.

34. An AP 5, 12, 19, ... has 50 term. Find its last term. Hence, find the sum of its last 15 terms.

Sol:

The given AP is $5, 12, 19, ...$

Here, $a = 5, d = 12-5 = 7$ and $n = 50.$

Since there are 50 terms in the AP, so the last term of the AP is a_{50}.

$l = a_{50} = 5+(50-1)\times7$ $\quad\left[a_n = a+(n-1)d\right]$

$= 5+343$

$= 348$

Thus, the last term of the AP is 348.

Now,

Sum of the last 15 terms of the AP

$= S_{50} - S_{35}$

$= \frac{50}{2}\left[2\times5+(50-1)\times7\right] - \frac{35}{2}\left[2\times5+(35-1)\times7\right]$

$\left\{S_n = \frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$= \frac{50}{2}\times(10+343) - \frac{35}{2}\times(10+238)$

$= \frac{50}{2}\times353 - \frac{35}{2}\times248$

$= \frac{17650-8680}{2}$

$= \frac{8970}{2}$

$= 4485$

Hence, the require sum is 4485.

35. An AP 8, 10, 12, … has 60 terms. Find its last term. Hence, find the sum of its last 10 terms.

Sol:

The given AP is 8, 10, 12,……

Here, $a=8, d=10-8=2$ and $n=60$

Since there are 60 terms in the AP, so the last term of the AP is a_{60}.

$l=a_{60}=8+(60-1)\times 2$ $\quad [a_n=a+(n-1)d]$

$=8+118$

$=126$

Thus, the last term of the AP is 126.

Now,

Sum of the last 10 terms of the AP

$=S_{60}-S_{50}$

$=\frac{60}{2}[2\times 8+(60-1)\times 2]-\frac{50}{2}[2\times 8+(50-1)\times 2]$

$\left\{S_n=\frac{n}{2}[2a+(n-1)d]\right\}$

$=30\times(16+118)-25\times(16+98)$

$=30\times 134-25\times 114$

$=4020-2850$

$=1170$

Hence, the required sum is 1170.

36. The sum of the 4th and 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44. Find the sum of its first 10 terms.

Sol:

Let *a* be the first and *d* be the common difference of the AP.

$\therefore a_4+a_8=24$ $\quad$ (Given)

$\Rightarrow (a+3d)+(a+7d)=24$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2a+10d=24$

$\Rightarrow a+5d=12$ $\quad$ ......(1)

Also,

$\therefore a_6+a_{10}=44$ $\quad$ (Given)

$\Rightarrow (a+5d)+(a+9d)=44$ $\quad [a_n=a+(n-1)d]$

$\Rightarrow 2a+14d=44$

$\Rightarrow a+7d=22$ $\quad$ ......(2)

Subtracting (1) from (2), we get

$(a+7d)-(a+5d)=22-12$

$\Rightarrow 2d=10$

$\Rightarrow d=5$

Putting $d = 5$ in (1), we get

$a + 5 \times 5 = 12$

$\Rightarrow a = 12 - 25 = -13$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$S_{10} = \frac{10}{2}\left[2 \times (-13) + (10-1) \times 5\right]$

$= 5 \times (-26 + 45)$

$= 5 \times 19$

$= 95$

Hence, the required sum is 95.

37. The sum of fist m terms of an AP is ($4m^2 - m$). If its nth term is 107, find the value of n. Also, Find the 21st term of this AP.

Sol:

Let S_m denotes the sum of the first m terms of the AP. Then,

$S_m = 4m^2 - m$

$\Rightarrow S_{m-1} = 4(m-1)^2 - (m-1)$

$= 4\left(m^2 - 2m + 1\right) - (m-1)$

$= 4m^2 - 9m + 5$

Suppose a_m denote the m^{th} term of the AP.

$\therefore a_m = S_m - S_{m-1}$

$= \left(4m^2 - m\right) - \left(4m^2 - 9m + 5\right)$

$= 8m - 5 \qquad ....(1)$

Now,

$a_n = 107 \qquad$ (Given)

$\Rightarrow 8n - 5 = 107 \qquad$ [From (1)]

$\Rightarrow 8n = 107 + 5 = 112$

$\Rightarrow n = 14$

Thus, the value of n is 14.

Putting $m = 21$ in (1), we get

$a_{21} = 8 \times 21 - 5 = 168 - 5 = 163$

Hence, the 21st term of the AP is 163.

38. The sum of first q terms of an AP is $(63q-3q^2)$. If its pth term is -60, find the value of p. Also, find the 11th term of its AP.

Sol:

Let S_q denote the sum of the first q terms of the AP. Then,

$S_q = 63q - 3q^2$

$\Rightarrow S_{q-1} = 63(q-1) - 3(q-1)^2$

$= 63q - 63 - 3(q^2 - 2q + 1)$

$= -3q^2 + 69q - 66$

Suppose a_q denote the q^{th} term of the AP.

$\therefore a_q = S_q - S_{q-1}$

$= (63q - 3q^2) - (-3q^2 + 69q - 66)$

$= -6q + 66$ ....(1)

Now,

$a_p = -60$ (Given)

$\Rightarrow -6p + 66 = -60$ [From (1)]

$\Rightarrow -6p = -60 - 66 = -126$

$\Rightarrow p = 21$

Thus, the value of p is 21.

Putting $q = 11$ in (1), we get

$a_{11} = -6 \times 11 + 66 = -66 + 66 = 0$

Hence, the 11th term of the AP is 0.

39. Find the number of terms of the AP -12, -9, -6, .., 21. If 1 is added to each term of this AP then the sum of all terms of the AP thus obtained.

Sol:

The given AP is $-12, -9, -6, \ldots\ldots, 21$.

Here, $a = -12, d = -9 - (-12) = -9 + 12 = 3 \text{ and } l = 21$

Suppose there are n terms in the AP.

$\therefore l = a_n = 21$

$\Rightarrow -12 + (n-1) \times 3 = 21$ $[a_n = a + (n-1)d]$

$\Rightarrow 3n - 15 = 21$

$\Rightarrow 3n = 21 + 15 = 36$

$\Rightarrow n = 12$

Thus, there are 12 terms in the AP.

If 1 is added to each term of the AP, then the new AP so obtained is $-11,-8,-5,........,22$.

Here, first term, $A=-11$; last term, $L=22$ *and* $n=12$

$\therefore$ Sum of the terms of this AP

$=\frac{12}{2}(-11+22)$ $\left[S_n=\frac{n}{2}(a+l)\right]$

$=6\times 11$

$=66$

Hence, the required sum is 66.

40. Sum of the first 14 terms of and AP is 1505 and its first term is 10. Find its 25th term.

Sol:

Let d be the common difference of the AP.

Here, $a=10$ and $n=14$

Now,

$S_{14}=1505$ (Given)

$\Rightarrow \frac{14}{2}\left[2\times 10+(14-1)\times d\right]=1505$ $\left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 7(20+13d)=1505$

$\Rightarrow 20+13d=215$

$\Rightarrow 13d=215-20=195$

$\Rightarrow d=15$

$\therefore 25^{th}$ term of the AP, a_{25}

$=10+(25-1)\times 15$ $\left[a_n=a+(n-1)d\right]$

$=10+360$

$=370$

Hence, the required term is 370.

41. Find the sum of fist 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol:

Let a be the first term and d be the common difference of the AP. Then,

$d=a_3-a_2=18-14=4$

Now,

$a_2=14$ (Given)

$\Rightarrow a+d=14$ $\left[a_n=a+(n-1)d\right]$

$\Rightarrow a+4=14$

$\Rightarrow a=14-4=10$

Using the formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_{51} = \frac{51}{2}\left[2\times10+(51-1)\times4\right]$$
$$= \frac{51}{2}(20+200)$$
$$= \frac{51}{2}\times220$$
$$= 5610$$

Hence, the required sum is 5610.

42. In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees that each section of each class will plant will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two section, find how many trees were planted by student. Which value is shown in the question?

Sol:

Number of trees planted by the students of each section of class 1 = 2

There are two sections of class 1.

$\therefore$ Number of trees planted by the students of class $1 = 2\times2 = 4$

Number of trees planted by the students of each section of class 2 = 4

There are two sections of class 2.

$\therefore$ Number of trees planted by the students of class $2 = 2\times4 = 8$

Similarly,

Number of trees planted by the students of class $3 = 2\times6 = 12$

So, the number of trees planted by the students of different classes are 4, 8, 12, ...

$\therefore$ Total number of trees planted by the students $= 4+8+12+......$ up to 12 terms

This series is an arithmetic series.

Here, $a = 4, d = 8-4 = 4$ and $n = 12$

Using the formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_{12} = \frac{12}{2}\left[2\times4+(12-1)\times4\right]$$
$$= 6\times(8+44)$$
$$= 6\times52$$
$$= 312$$

Hence, the total number of trees planted by the students is 312.

The values shown in the question are social responsibility and awareness for conserving nature.

43. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3m apart in a straight line. There are 10 potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and he continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Sol:

Distance covered by the competitor to pick and drop the first potato $= 2\times 5m = 10m$

Distance covered by the competitor to pick and drop the second potato

$= 2\times(5+3)m = 2\times 8m = 16\ m$

Distance covered by the competitor to pick and drop the third potato

$= 2\times(5+3+3)m = 2\times 11m = 22$ m and so on.

$\therefore$ Total distance covered by the competitor $= 10m + 16m + 22m + \ldots\ldots$ up to 10 terms

This is an arithmetic series.

Here, $a = 10, d = 16 - 10 = 6$ and $n = 10$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$S_{10} = \frac{10}{2}\left[2\times 10 + (10-1)\times 6\right]$

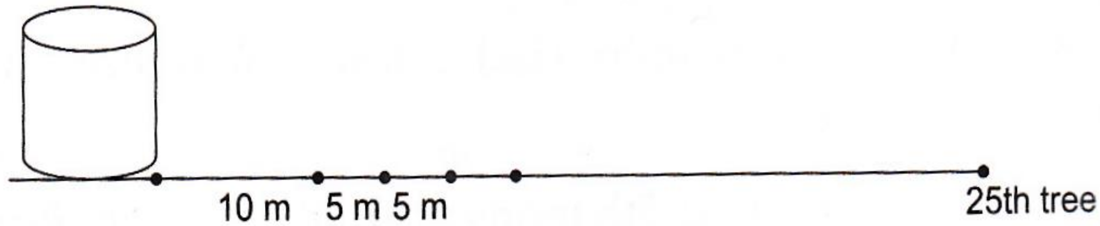
$= 5\times(20+54)$

$= 5\times 74$

$= 370$

Hence, the total distance the competitor has to run is 370 m.

44. There are 25 trees at equal distance of 5 m in a line with a water tank, the distance of the water tank from the nearest tree being 10 m. A gardener waters all the trees separately, starting from the water tank and returning back to the water tank after watering each tree to get water for the next. Find the total distance covered by the gardener in order to water all the trees.



Sol:

Distance covered by the gardener to water the first tree and return to the water tank
$= 10m + 10m = 20m$

Distance covered by the gardener to water the second tree and return to the water tank
$= 15m + 15m = 30m$

Distance covered by the gardener to water the third tree and return to the water tank
$= 20m + 20m = 40m$ and so on.

$\therefore$ Total distance covered by the gardener to water all the trees $= 20m + 30m + 40m + ...$ up to 25 terms

This series is an arithmetic series.

Here, $a = 20, d = 30 - 20 = 10$ *and* $n = 25$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d,\right]$ we get

$$S_{25} = \frac{25}{2}\left[2 \times 20 + (25-1) \times 10\right]$$
$$= \frac{25}{2}(40 + 240)$$
$$= \frac{25}{2} = 280$$
$$= 3500$$

Hence, the total distance covered by the gardener to water all the trees 3500 m.

45. A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each prize.

Sol:

Let the value of the first prize be *a*.

Since the value of each prize is 20 less than its preceding prize, so the values of the prizes are in AP with common difference – ₹20.

$$\Rightarrow \frac{40}{2}\left[2a+(40-1)d\right]=36000$$

$\therefore d = -$₹$\Rightarrow 20(2a+39d)=36000$

$$\Rightarrow 2a+39d=1800 \qquad ....(2)$$

Number of cash prizes to be given to the students, n = 7

Total sum of the prizes, $S_7 =$ ₹700

Using the formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_7 = \frac{7}{2}\left[2a+(7-1)\times(-20)\right]=700$$

$$\Rightarrow \frac{7}{2}(2a-120)=700$$

$$\Rightarrow 7a-420=700$$

$$\Rightarrow 7a=700+420=1120$$

$$\Rightarrow a=160$$

Thus, the value of the first prize is ₹160.

Hence, the value of each prize is ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 and ₹40.

46. A man saved ₹33000 in 10 months. In each month after the first, he saved ₹100 more than he did in the preceding month. How much did he save in the first month?

Sol:

Let the money saved by the man in the first month be ₹a

It is given that in each month after the first, he saved ₹100 more than he did in the preceding month. So, the money saved by the man every month is in AP with common difference ₹100.

$\therefore d =$ ₹100

Number of months, $n = 10$

Sum of money saved in 10 months, $S_{10} =$ ₹ 33,000

Using the formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$, we get

$$S_{10} = \frac{10}{2}\left[2a+(10-1)\times 100\right]=33000$$

$$\Rightarrow 5(2a+900)=33000$$

$$\Rightarrow 2a+900=6600$$

$$\Rightarrow 2a=6600-900=5700$$

$$\Rightarrow a=2850$$

Hence, the money saved by the man in the first month is ₹2,850.

47. A man arranges to pay off debt of ₹36000 by 40 monthly instalments which form an arithmetic series. When 30 of the installments are paid, he dies leaving on-third of the debt unpaid. Find the value of the first instalment.

Sol:

Let the value of the first installment be ₹a.

Since the monthly installments form an arithmetic series, so let us suppose the man increases the value of each installment by ₹d every month.

$\therefore$ Common difference of the arithmetic series = ₹d

Amount paid in 30 installments = ₹$36{,}000-\frac{1}{3}\times$ ₹ $36{,}000=$ ₹ $36{,}000-$ ₹ $12{,}000=$ ₹ $24{,}000$

Let S_n denote the total amount of money paid in the n installments. Then,

$S_{30}=24{,}000$

$\Rightarrow \frac{30}{2}\left[2a+(30-1)d\right]=24000$ $\quad\left\{S_n=\frac{n}{2}\left[2a+(n-1)d\right]\right\}$

$\Rightarrow 15(2a+29d)=24000$

$\Rightarrow 2a+29d=1600$ ....(1)

Also,

$S_{40}=$ ₹$36{,}000$

$\Rightarrow \frac{40}{2}\left[2a+(40-1)d\right]=36000$

$\Rightarrow 20(2a+39d)=36000$

$\Rightarrow 2a+39d=1800$ ....(2)

Subtracting (1) from (2), we get

$(2a+39d)-(2a+29d)=1800-1600$

$\Rightarrow 10d=200$

$\Rightarrow d=20$

Putting $d=20$ in (1), we get

$2a+29\times 20=1600$

$\Rightarrow 2a+580=1600$

$\Rightarrow 2a=1600-580=1020$

$\Rightarrow a=510$

Thus, the value of the first installment is ₹510.

48. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹200 for the first day, ₹ 250 for the second day, ₹300 for the third day, etc. the penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol:

It is given that the penalty for each succeeding day is 50 more than for the preceding day, so the amount of penalties are in AP with common difference ₹50

Number of days in the delay of the work = 30

The amount of penalties are ₹200, ₹250, ₹300,... up to 30 terms.

$\therefore$ Total amount of money paid by the contractor as penalty,

$S_{30} = ₹\ 200 + ₹\ 250 + ₹\ 300 + \ldots\ldots.$ up to 30 terms

Here, $a = ₹\ 200, d = ₹\ 50$ *and* $n = 30$

Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get

$$S_{30} = \frac{30}{2}\left[2 \times 200 + (30-1) \times 50\right]$$
$$= 15(400 - 1450)$$
$$= 15 \times 1850$$
$$= 27750$$

Hence, the contractor has to pay ₹27,750 as penalty

Exercise - Multiple Choice Questions

1. The common difference of the AP $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{2}, \ldots\ldots.$ is

(a) p (b) –p (c) -1 (d) 1

Answer: (c) -1

Sol:

The given AP is $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{2}, \ldots\ldots.$

$\therefore$ Common difference, $d = \frac{1-p}{p} - \frac{1}{p} = \frac{1-p-1}{p} = \frac{-p}{p} = -1$

2. The common difference of the AP $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots\ldots$ is

(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) b (d) –b

Answer: (d) –b

Sol:

The given AP is $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots\ldots$

$\therefore$ Common difference, $d = \frac{1-3b}{3} - \frac{1}{3} = \frac{1-3b-1}{3} = \frac{-3b}{3} = -b$

3. The next term of the AP $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots$ is

(a) $\sqrt{70}$ (b) $\sqrt{84}$ (c) $\sqrt{98}$ (d) $\sqrt{112}$

Answer: (d) $\sqrt{112}$

Sol:

The given terms of the AP can be written as $\sqrt{7}, \sqrt{4 \times 7}, \sqrt{9 \times 7}, \ldots\ldots\ldots i.e. \sqrt{7}, 2\sqrt{7}, 3\sqrt{7}, \ldots\ldots$

$\therefore$ Next term $= 4\sqrt{7} = \sqrt{16 \times 7} = \sqrt{112}$

4. If $4, x_1, x_2, x_3, 28$ are in AP then $x_{3,} = ?$

(a) 19 (b) 23 (c) 22 (d) cannot be determined

Answer: (c) 22

Sol:

Here, $a = 4, l = 28$ *and* $n = 5$

Then, $T_5 = 28$

$\Rightarrow a + (n-1)d = 28$

$\Rightarrow 4 + (5-1)d = 28$

$\Rightarrow 4d = 24$

$\Rightarrow d = 6$

Hence, $x_3 = 28 - 6 = 22$

5. If the nth term of an AP is (2n + 1) then the sum of its first three terms is

(a) 6n+3 (b) 15 (c) 12 (d) 21

Answer: (b) 15

Sol:

nth term of the AP, $a_n = 2n + 1$ (Given)

$\therefore$ First term, $a_1 = 2 \times 1 + 1 = 2 + 1 = 3$

Second term, $a_2 = 2 \times 2 + 1 = 4 + 1 = 5$

Third term, $a_3 = 2 \times 3 + 1 = 6 + 1 = 7$

$\therefore$ Sum of the first three terms $a_1 + a_2 + a_3 = 3 + 5 + 7 = 15$

6. The sum of first n terms of an AP is $(3n^2+6n)$. The common difference of the AP is

(a) 6 (b) 9 (c) 15 (d) -3

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 3n^2+6n$

$\Rightarrow S_{n-1} = 3(n-1)^2+6(n-1)$

$= 3(n^2-2n+1)+6(n-1)$

$= 3n^2-3$

So,

n^{th} term of the AP, $a_n = S_n - S_{n-1}$

$= (3n^2+6n)-(3n^2-3)$

$= 6n+3$

Let d be the common difference of the AP.

$\therefore d = a_n - a_{n-1}$

$= (6n+3)-[6(n-1)+3]$

$= 6n+3-6(n-1)-3$

$= 6$

Thus, the common difference of the AP is 6.

7. The sum of first n terms of an AP is $(5n-n^2)$. The nth term of the AP is

(a) (5 - 2n) (b) (6 – 2n) (c) (2n – 5) (d) (2n – 6)

Answer: (b) (6 – 2n)

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 5n-n^2$

$\Rightarrow S_{n-1} = 5(n-1)-(n-1)^2$

$= 5n-5-n^2+2n-1$

$= 7n-n^2-6$

$\therefore n^{th}$ term of the AP, $a_n = S_n - S_{n-1}$

$= (5n-n^2)-(7n-n^2-6)$

$= 6-2n$

Thus, the nth term of the AP is $(6-2n)$.

8. The sum of the first n terms of an AP is $(4n^2+2n)$. The nth term of this AP is

(a) (6n - 2) (b) (7n – 3) (c) (8n – 2) (d) (8n + 2)

Answer: (c) (8n – 2)

Sol:

Let S_n denotes the sum of first n terms of the AP.

$\therefore S_n = 4n^2 + 2n$

$\Rightarrow S_{n-1} = 4(n-1)^2 + 2(n-1)$

$= 4(n^2 - 2n + 1) + 2(n-1)$

$= 4n^2 - 6n + 2$

$\therefore n^{th}$ term of the AP, $a_n = S_n - S_{n-1}$

$= (4n^2 + 2n) - (4n^2 - 6n + 2)$

$= 8n - 2$

Thus, the n^{th} term of thee AP is $(8n-2)$

9. The 7[th] term of an AP is -1 and its 16[th] term is 17. The nth term of the AP is

(a) (3n + 8) (b) (4n – 7) (c) (15 – 2n) (d) (2n – 15)

Answer: (d) (2n – 15)

Sol:

Let a be the first term and d be the common difference of the AP. Then,

nth term of the AP, $a_n = a + (n-1)d$

Now,

$a_7 = -1$ (Given)

$\Rightarrow a + 6d = -1$ ....(1)

Also,

$a_{16} = 17$ (Given)

$\Rightarrow a + 15d = 17$ ....(2)

Subtracting (1) from (2), we get

$(a+15d) - (a+6d) = 17 - (-1)$

$\Rightarrow 9d = 18$

$\Rightarrow d = 2$

Putting $d = 2$ in (1), we get

$a + 6 \times 2 = -1$

$\Rightarrow a = -1 - 12 = -13$

$\therefore$ nth term of the AP, $a_n = -13 + (n-1) \times 2 = 2n - 15$

10. The 5th term of an AP is -3 and its common difference is -4. The sum of the first 10 terms is
(a) 50 (b) -50 (c) 30 (d) -30
Answer: (b) -50
Sol:
Let a be the first term of the AP.
Here. $d=-4$
$a_5=-3$ (Given)
$\Rightarrow a+(5-1)\times(-4)=-3$ $\left[a_n=a+(n-1)d\right]$
$\Rightarrow a-16=-3$
$\Rightarrow a=16-3=13$
Using the formula, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$, we get
$S_{10}=\frac{10}{2}\left[2\times13+(10-1)\times(-4)\right]$
$=5\times(26-36)$
$=5\times(-10)$
$=-50$
Thus, the sum of its first 10 terms is -50.

11. The 5th term of an AP is 20 and the sum of its 7th and 11th terms is 64. The common difference of the AP is
(a) 4 (b) 5 (c) 3 (d) 2
Answer: (c) 3
Sol:
Let a be the first term and d be the common difference of the AP. Then,
$a_5=20$
$\Rightarrow a+(5-1)d=20$ $\left[a_n=a+(n-1)d\right]$
$\Rightarrow a+4d=20$ ....(1)
Now,
$a_7+a_{11}=64$ (Given)
$\Rightarrow (a+6d)+(a+10d)=64$
$\Rightarrow 2a+16d=64$
$\Rightarrow a+8d=32$ ....(2)
From (1) and (2), we get
$20-4d+8d=32$
$\Rightarrow 4d=32-20=12$
$\Rightarrow d=3$

Thus, the common difference of the AP is 3.

12. The 13th term of an AP is 4 times its 3rd term. If its 5th term is 16 then the sum of its first ten terms is
(a) 150 (b) 175 (c) 160 (d) 135
Answer: (b) 175
Sol:
Let a be the first term and d be the common difference of the AP. Then,
$a_{13} = 4 \times a_3$ (Given)
$\Rightarrow a + 12d = 4(a + 2d)$ $\left[a_n = a + (n-1)d\right]$
$\Rightarrow a + 12d = 4a + 8d$
$\Rightarrow 3a = 4d$ ....(1)
Also
$a_5 = 16$ (Given)
$\Rightarrow a + 4d = 16$ ....(2)
Solving (1) and (2), we get
$a + 3a = 16$
$\Rightarrow 4a = 16$
$\Rightarrow a = 4$
Putting $a = 4$ in (1), we get
$4d = 3 \times 4 = 12$
$\Rightarrow d = 3$
Using the formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$, we get
$S_{10} = \frac{10}{2}\left[2 \times 4 + (10-1) \times 3\right]$
$= 5 \times (8 + 27)$
$= 5 \times 35$
$= 175$
Thus, the sum of its first 10 terms is 175.

13. An AP 5, 12, 9, …. has 50 terms. Its last term is
(a) 343 (b) 353 (c) 348 (d) 362
Answer: (c) 348
Sol:
The given AP is 5, 12, 19,……
Here, $a = 5, d = 12 - 5 = 7$ and $n = 50$

Since there are 50 terms in the AP, so the last term of the AP is a_{50}.

$a_{50} = 5 + (50 - 1) \times 7$ $\qquad [a_n = a + (n-1)d]$

$= 5 + 343$

$= 348$

Thus, the last term of the AP is 348.

14. The sum of the first 20 odd natural numbers is

(a) 100 (b) 210 (c) 400 (d) 420

Answer: (c) 400

Sol:

The first 20 odd natural numbers are 1, 3, 5,.., 39.

These numbers are in AP.

Here. a = 1, l = 39 and n = 20

$\therefore$ Sum of first 20 odd natural numbers

$= \frac{20}{2}(1 + 39)$ $\qquad \left[S_n = \frac{n}{2}(a + l)\right]$

$= 10 \times 40$

$= 400$

15. The sum of first 40 positive integers divisible by 6 is

(a) 2460 (b) 3640 (c) 4920 (d) 4860

Answer: (c) 4920

Sol:

The positive integers divisible by 6 are 6, 12, 18,....

This is an AP with a = 6 and d = 6.

Also, n = 40 (Given)

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{40} = \frac{40}{2}[2 \times 6 + (40 - 1) \times 6]$

$= 20(12 + 234)$

$= 20 \times 246$

$= 4920$

Thus, the required sum is 4920

16. How many two digit numbers are divisible by 3?

(a) 25 (b) 30 (c) 32 (d) 36

Answer: (b) 30

Sol:

The two-digit numbers divisible by 3 are 12, 15, 18,…… 99.
Clearly, these number are in AP.
Here, a = 12 and d = $15-12=3$
Let this AP contains n terms. Then.

$a_n = 99$

$\Rightarrow 12+(n-1)\times 3 = 99 \qquad [a_n = a+(n-1)d]$

$\Rightarrow 3n+9 = 99$

$\Rightarrow 3n = 99-9 = 90$

$\Rightarrow n = 30$

Thus, there are 30 two-digit numbers divisible by 3.

17. How many three-digit number are divisible by 9?
(a) 86 (b) 90 (c) 96 (d) 100
Answer: (d) 100
Sol:
The three-digit numbers divisible by 9 are 108, 117, 126 ,…. 999.
Clearly, these numbers are in AP.
Here, $a = 108$ *and* $d = 117-108 = 9$
Let this AP contains n terms. Then,

$a_n = 999$

$\Rightarrow 108+(n-1)\times 9 = 999 \qquad [a_n = a+(n-1)d]$

$\Rightarrow 9n+99 = 999$

$\Rightarrow 9n = 999-99 = 900$

$\Rightarrow n = 100$

Thus, there are 100 three-digit numbers divisible by 9.

18. What is the common difference of an AP in which $a_{18}-a_{14} = 32$?
(a) 8 (b) -8 (c) 4 (d) -4
Answer: (a) 8
Sol:
Let a be the first term and d be the common difference of the AP. Then.

$a_{18}-a_{14} = 32$

$\Rightarrow (a+17d)-(a+13d) = 32 \qquad [a_n = a+(n-1)d]$

$\Rightarrow 4d = 32$

$\Rightarrow d = 8$

Thus, the common difference of the AP is 8.

19. If a_n denotes the nth term of the AP 3, 8, 13, 18, … then what is the value of $(a_{30} - a_{20})$?

(a) 40 (b) 36 (c) 50 (d) 56

Answer: (c) 50

Sol:

The given AP is 3, 8,13,18....

Here, $a = 3$ and $d = 8 - 3 = 5$

$\therefore a_{30} - a_{20}$

$= [3 + (30-1)\times 5] - [3 + (20-1)\times 5]$ $\quad [a_n = a + (n-1)d]$

$= 148 - 98$

$= 50$

Thus, the required value is 50.

20. Which term of the AP 72, 63, 54, …. is 0 ?

(a) 8th (b) 9th (c) 10th (d) 11th

Answer: (b) 9th

Sol:

The given AP is 72, 63, 54,.....

Here, $a = 72$ and $d = 63 - 72 = -9$

Suppose nth term of the given AP is 0. Then.

$a_n = 0$

$\Rightarrow 72 + (n-1)\times(-9) = 0$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow -9n + 81 = 0$

$\Rightarrow n = \frac{81}{9} = 9$

Thus, the 9th term of the given AP is 0.

21. Which term of the AP 25, 20, 15, … is the first negative term?

(a) 10th (b) 9th (c) 8th (d) 7th

Answer: (d) 7th

Sol:

The given AP is 25, 20, 15,……

Here, $a = 25$ and $d = 20 - 25 = -5$

Let the nth term of the given AP be the first negative term. Then,

$a_n < 0$

$\Rightarrow 25 + (n-1)\times(-5) < 0$ $\quad [a_n = a + (n-1)d]$

$\Rightarrow 30 - 5n < 0$

$\Rightarrow -5n < -30$

$\Rightarrow n > \frac{30}{5} = 6$

$\therefore n = 7$

Thus, the 7th term is the first negative term of the given AP.

22. Which term of the AP 21, 42, 63, 84, … is 210?
(a) 9^{th} (b) 10^{th} (c) 11^{th} (d) 12^{th}
Answer: (b) 10^{th}
Sol:
Here, $a = 21$ and $d = (42 - 21) = 21$
Let 210 be the nth term of the given AP.
Then, $T_n = 210$

$\Rightarrow a + (n-1)d = 210$

$\Rightarrow 21 + (n-1) \times 21 = 210$

$\Rightarrow 21n = 210$

$\Rightarrow n = 10$

Hence, 210 is the 10^{th} term of the AP.

23. What is 20^{th} term form the end of the AP 3, 8, 13, …., 253?
(a) 163 (b) 158 (c) 153 (d) 148
Answer: (b) 158
Sol:
The given AP is 3, 8, 13,...., 253.
Let us re-write the given AP in reverse order i.e. 253, 248, .., 13, 8, 3.
Now, the 20th term from the end of the given AP is equal to the 20th term from beginning of the AP 253, 248,., 13, 8, 3.
Consider the AP 253, 248,….. 13, 8, 3.
Here, $a = 253$ and $d = 248 - 253 = -5$

$\therefore$ 20th term of this AP

$= 253 + (20-1) \times (-5)$

$= 253 - 95$

$= 158$

Thus, the 20th term from the end of the given AP is 158.

24. (5 + 13 + 21 + … + 181) = ?
(a) 2476 (b) 2337 (c) 2219 (d) 2139
Answer: (d) 2139
Sol:

Here, $a = 5, d = (13-5) = 8$ and $l = 181$

Let the number of terms be n.

Then, $T_n = 181$

$\Rightarrow a + (n-1)d = 181$

$\Rightarrow 5 + (n-1) \times 8 = 181$

$\Rightarrow 8n = 184$

$\Rightarrow n = 23$

$\therefore$ Required sum $= \frac{n}{2}(a+l)$

$= \frac{23}{2}(5+181) = 23 \times 93 = 2139$

Hence, the required sum is 2139.

25. The sum of first 16 terms of the AP 10, 6, 2, …. is

(a) 320 (b) -320 (c) -352 (d) -400

Answer: (b) -320

Sol:

Here, $a = 10, d = (6-10) = -4$ *and* $n = 16$

Using the formula, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{16} = \frac{16}{2}[2 \times 10 + (16-1) \times (-4)]$

$[\because a = 10, d = -4 \text{ and } n = 16]$

$= 8 \times [20 - 60] = 8 \times (-40) = -320$

Hence, the sum of the first 16 terms of the given AP is -320.

26. How many terms of the AP 3, 7, 11, 15, … will make the sum 406?

(a) 10 (b) 12 (c) 14 (d) 20

Answer: (c) 14

Sol:

Here, $a = 3$ and $d = (7-3) = 4$

Let the sum of n terms be 406.

Then, we have:

$S_n = \frac{n}{2}[2a + (n-1)d] = 406$

$\Rightarrow \frac{n}{2}[2 \times 3 + (n-1) \times 4] = 406$

$\Rightarrow n\left[3+2n-2\right]=406$

$\Rightarrow 2n^2-28n+29n-406$

$\Rightarrow 2n^2+n-406=0$

$\Rightarrow 2n^2-28n+29n-406=0$

$\Rightarrow 2n\left(n-14\right)+29\left(n-14\right)=0$

$\Rightarrow \left(2n+29\right)\left(n-14\right)=0$

$\Rightarrow n=14$ ($\because n$ can't be a fraction)

Hence, 14 terms will make the sum 406.

27. The 2nd term of an AP is 13 and 5th term is 25. What is its 17 term?
(a) 69 (b) 73 (c) 77 (d) 81
Answer: (b) 73
Sol:

$T_2=a+d=13$ ....(i)

$T_5=a+4d=25$ ....(ii)

On subtracting (i) from (ii), we get:

$\Rightarrow 3d=12$

$\Rightarrow d=4$

On putting the value of d in (i), we get:

$\Rightarrow a+4=13$

$\Rightarrow a=9$

Now, $T_{17}=a+16d=9+16\times4=73$

Hence, the 17th term is 73.

28. The 17th term of an AP exceeds its 10th term by 21. The common difference of the AP is
(a) 3 (b) 2 (c) -3 (d) -2
Answer: (a) 3
Sol:

$T_{10}=a+9d$

$T_{17}=a+16d$

Also, $a+16d=21+T_{10}$

$\Rightarrow a+16d=21+9d$

$\Rightarrow 7d=21$

$\Rightarrow d=3$

Hence, the common difference of the AP is 3.

29. The 8th term of an AP is 17 and its 14th term is 29. The common difference of the AP is
(a) 3 (b) 2 (c) 5 (d) -2
Answer: (b) 2
Sol:
$T_8 = a + 7d = 17 \qquad ......(i)$
$T_{14} = a + 13d = 29 \qquad ......(ii)$
On subtracting (i) from (ii), we get:
$\Rightarrow 6d = 12$
$\Rightarrow d = 12$
Hence, the common difference is 2.

30. The 7th term of an AP is 4 and its common difference is -4. What is its first term?
(a) 16 (b) 20 (c) 24 (d) 28
Answer: (d) 28
Sol:
$T_7 = a + 6d$
$\Rightarrow a + 6 \times (-4) = 4$
$\Rightarrow a = 4 + 24 = 28$
Hence, the first term us 28.