Exercise - 1A

1. What do you mean by Euclid's division algorithm?

Sol:

Euclid's division algorithm states that for any two positive integers a and b, there exit unique integers q and r, such that $a = bq + r$. where $0 \leq r \leq b$.

2. A number when divided by 61 gives 27 as quotient and 32 as remainder. Find the number.

Sol:

We know, Dividend = Divisor × Quotient + Remainder

Given: Divisor = 61, Quotient = 27, Remainder = 32

Let the Dividend be x.

$\therefore x = 61 \times 27 + 32$

$= 1679$

Hence, the required number is 1679.

3. By what number should be 1365 be divided to get 31 as quotient and 32 as remainder?

Sol:

Given: Dividend = 1365, Quotient = 31, Remainder = 32

Let the divisor be x.

Dividend = Divisor × Quotient + Remainder

$1365 = x \times 31 + 32$

$\Rightarrow 1365 - 32 = 31x$

$\Rightarrow 1333 = 31x$

$\Rightarrow x = \frac{1333}{31} = 43$

Hence, 1365 should be divided by 43 to get 31 as quotient and 32 as remainder.

4. Using Euclid's algorithm, find the HCF of

(i) 405 and 2520

(ii) 504 and 1188

(iii) 960 and 1575

Sol:

(i)

```
405)2520(6
  - 2430
    90)405(4
     - 360
      45)90(2
       - 90
          0
```

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On applying Euclid's algorithm, i.e. dividing 2520 by 405, we get:
Quotient = 6, Remainder = 90
$\therefore 2520 = 405 \times 6 + 90$
Again on applying Euclid's algorithm, i.e. dividing 405 by 90, we get:
Quotient = 4, Remainder = 45
$\therefore 405 = 90 \times 4 + 45$
Again on applying Euclid's algorithm, i.e. dividing 90 by 45, we get:
$\therefore 90 = 45 \times 2 + 0$
Hence, the HCF of 2520 and 405 is 45.

(ii)

```
504 ) 1188 ( 2
    - 1008
    ------
       180 ) 504 ( 2
           - 360
           -----
             144 ) 180 ( 1
                 - 144
                 -----
                    36 ) 144 ( 4
                       -  144
                       ------
                            0
```

On applying Euclid's algorithm, i.e. dividing 1188 by 504, we get:
Quotient = 2, Remainder = 180
$\therefore 1188 = 504 \times 2 + 180$
Again on applying Euclid's algorithm, i.e. dividing 504 by 180, we get:
Quotient = 2, Remainder = 144
$\therefore 504 = 180 \times 2 + 144$
Again on applying Euclid's algorithm, i.e. dividing 180 by 144, we get:
Quotient = 1, Remainder = 36
$\therefore 180 = 144 \times 1 + 36$
Again on applying Euclid's algorithm, i.e. dividing 144 by 36, we get:
$\therefore 144 = 36 \times 4 + 0$
Hence, the HCF of 1188 and 504 is 36.

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(iii)

$$\begin{array}{l}
960\ )\ 1575\ (1 \\
\quad \underline{-\ 960} \\
\qquad 615)\ 960\ (1 \\
\qquad\quad \underline{-\ 615} \\
\qquad\qquad 345)\ 615\ (1 \\
\qquad\qquad\quad \underline{-\ 345} \\
\qquad\qquad\qquad 270\ )\ 345\ (1 \\
\qquad\qquad\qquad\quad \underline{-\ 270} \\
\qquad\qquad\qquad\qquad 75\ )\ 270\ (3 \\
\qquad\qquad\qquad\qquad\quad \underline{-\ 225} \\
\qquad\qquad\qquad\qquad\qquad 45\ )\ 75\ (1 \\
\qquad\qquad\qquad\qquad\qquad\quad \underline{-\ 45} \\
\qquad\qquad\qquad\qquad\qquad\qquad 30)\ 45\ (1 \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{-\ 30} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad 15\ )\ 30\ (2 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{-\ 30} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

On applying Euclid's algorithm, i.e. dividing 1575 by 960, we get:

Quotient = 1, Remainder = 615

$\therefore 1575 = 960 \times 1 + 615$

Again on applying Euclid's algorithm, i.e. dividing 960 by 615, we get:

Quotient = 1, Remainder = 345

$\therefore 960 = 615 \times 1 + 345$

Again on applying Euclid's algorithm, i.e. dividing 615 by 345, we get:

Quotient = 1, Remainder = 270

$\therefore 615 = 345 \times 1 + 270$

Again on applying Euclid's algorithm, i.e. dividing 345 by 270, we get:

Quotient = 1, Remainder = 75

$\therefore 345 = 270 \times 1 + 75$

Again on applying Euclid's algorithm, i.e. dividing 270 by 75, we get:

Quotient = 3, Remainder = 45

$\therefore 270 = 75 \times 3 + 45$

Again on applying Euclid's algorithm, i.e. dividing 75 by 45, we get:

Quotient = 1, Remainder = 30

$\therefore 75 = 45 \times 1 + 30$

Again on applying Euclid's algorithm, i.e. dividing 45 by 30, we get:

Quotient = 1, Remainder = 15

$\therefore 45 = 30 \times 1 + 15$

Again on applying Euclid's algorithm, i.e. dividing 30 by 15, we get:

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Quotient = 2, Remainder = 0
$\therefore 30 = 15 \times 2 + 0$
Hence, the HCF of 960 and 1575 is 15.

5. Show that every positive integer is either even or odd?
Sol:
Let us assume that there exist a smallest positive integer that is neither odd nor even, say n.
Since n is least positive integer which is neither even nor odd, $n - 1$ must be either odd or even.
Case 1: If $n - 1$ is even, $n - 1 = 2k$ for some k.
But this implies $n = 2k + 1$
this implies n is odd.
Case 2: If $n - 1$ is odd, $n - 1 = 2k + 1$ for some k.
But this implies $n = 2k + 2\ (k+1)$
this implies n is even.
In both ways we have a contradiction.
Thus, every positive integer is either even or odd.

6. Show that every positive even integer is of the form (6m+1) or (6m+3) or (6m+5)where m is some integer.
Sol:
Let n be any arbitrary positive odd integer.
On dividing n by 6, let m be the quotient and r be the remainder. So, by Euclid's division lemma, we have
$n = 6m + r$, where $0 \le r < 6$.
As $0 \le r < 6$ and r is an integer, r can take values 0, 1, 2, 3, 4, 5.
$\Rightarrow n = 6m$ or $n = 6m + 1$ or $n = 6m + 2$ or $n = 6m + 3$ or $n = 6m + 4$ or $n = 6m + 5$
But $n \ne 6m$ or $n \ne 6m + 2$ or $n \ne 6m + 4$ ($\because 6m, 6m + 2, 6m + 4$ are multiples of 2, so an even integer whereas n is an odd integer)
$\Rightarrow n = 6m + 1$ or $n = 6m + 3$ or $n = 6m + 5$
Thus, any positive odd integer is of the form $(6m + 1)$ or $(6m + 3)$ or $(6m + 5)$, where m is some integer.

7. Show that every positive even integer is of the form 4m and that every positive odd integer is of the form 4m + 1 for some integer m.
Sol:
Let n be any arbitrary positive odd integer.
On dividing n by 4, let m be the quotient and r be the remainder. So, by Euclid's division lemma, we have
$n = 4m + r$, where $0 \le r < 4$.

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As $0 \le r < 4$ and r is an integer, r can take values 0, 1, 2, 3.
$\Rightarrow n = 4m$ or $n = 4m + 1$ or $n = 4m + 2$ or $n = 4m + 3$
But $n \ne 4m$ or $n \ne 4m + 2$ ($\because$ $4m$, $4m + 2$ are multiples of 2, so an even integer whereas n is an odd integer)
$\Rightarrow n = 4m + 1$ or $n = 4m + 3$
Thus, any positive odd integer is of the form $(4m + 1)$ or $(4m + 3)$, where m is some integer.

Exercise - 1B

1. Using prime factorization, find the HCF and LCM of
(i) 36, 84
(ii) 23, 31
(iii) 96, 404
(iv) 144, 198
(v) 396, 1080
(vi) 1152, 1664
In each case verify that HCF × LCM = product of given numbers.
Sol:
(i) Prime factorization:
$36 = 2^2 \times 3$
$84 = 2^2 \times 3 \times 7$
HCF = product of smallest power of each common prime factor in the numbers $= 2^2 \times 3 = 12$
LCM = product of greatest power of each prime factor involved in the numbers $= 2^2 \times 3^2 \times 7 = 252$
(ii) Prime factorization:
$23 = 23$
$31 = 31$
HCF = product of smallest power of each common prime factor in the numbers = 1
LCM = product of greatest power of each prime factor involved in the numbers $= 23 \times 31 = 713$
(iii) Prime factorization:
$96 = 2^5 \times 3$
$404 = 2^2 \times 101$
HCF = product of smallest power of each common prime factor in the numbers $= 2^2 = 4$
LCM = product of greatest power of each prime factor involved in the numbers $= 2^5 \times 3 \times 101 = 9696$
(iv) Prime factorization:

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$144 = 2^4 \times 3^2$
$198 = 2 \times 3^2 \times 11$
HCF = product of smallest power of each common prime factor in the numbers
$= 2 \times 3^2 = 18$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^4 \times 3^2 \times 11 = 1584$

(v) Prime factorization:
$396 = 2^2 \times 3^2 \times 11$
$1080 = 2^3 \times 3^3 \times 5$
HCF = product of smallest power of each common prime factor in the numbers
$= 2^2 \times 3^2 = 36$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^3 \times 3^3 \times 5 \times 11 = 11880$

(vi) Prime factorization:
$1152 = 2^7 \times 3^2$
$1664 = 2^7 \times 13$
HCF = product of smallest power of each common prime factor in the numbers
$= 2^7 = 128$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^7 \times 3^2 \times 13 = 14976$

2. Using prime factorization, find the HCF and LCM of
(i) 8, 9, 25
(ii) 12,15, 21
(iii) 17,23,29
(iv) 24, 36, 40
(v) 30, 72, 432
(vi) 21, 28, 36, 45

Sol:
(i) $8 = 2 \times 2 \times 2 = 2^3$
$9 = 3 \times 3 = 3^2$
$25 = 5 \times 5 = 5^2$
HCF = product of smallest power of each common prime factor in the numbers = 1
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^3 \times 3^2 \times 5^2 = 1800$

(ii) $12 = 2 \times 2 \times 3 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$
HCF = product of smallest power of each common prime factor in the numbers = 3
LCM = product of greatest power of each prime factor involved in the numbers

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$= 2^2 \times 3 \times 5 \times 7 = 420$

(iii) $17 = 17$
$23 = 23$
$29 = 29$
HCF = product of smallest power of each common prime factor in the numbers = 1
LCM = product of greatest power of each prime factor involved in the numbers
$= 17 \times 23 \times 29 = 11339$

(iv) $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$
HCF = product of smallest power of each common prime factor in the numbers
$= 2^2 = 4$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^3 \times 3^2 \times 5 = 360$

(v) $30 = 2 \times 3 \times 5$
$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
$432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$
HCF = product of smallest power of each common prime factor in the numbers
$= 2 \times 3 = 6$
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^4 \times 3^3 \times 5 = 2160$

(vi) $21 = 3 \times 7$
$28 = 2 \times 2 \times 7 = 2^2 \times 7$
$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
$45 = 5 \times 3 \times 3 = 5 \times 3^2$
HCF = product of smallest power of each common prime factor in the numbers = 1
LCM = product of greatest power of each prime factor involved in the numbers
$= 2^2 \times 3^2 \times 5 \times 7 = 1260$

3. The HCF of two numbers is 23 and their LCM is 1449. If one of the numbers is 161, find the other.

Sol:

Let the two numbers be a and b.
Let the value of a be 161.
Given: HCF = 23 and LCM = 1449
We know, $a \times b = \text{HCF} \times \text{LCM}$
$\Rightarrow 161 \times b = 23 \times 1449$
$\Rightarrow \quad \therefore b = \frac{23 \times 1449}{161} = \frac{33327}{161} = 207$
Hence, the other number b is 207.

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4. The HCF of two numbers is 145 and their LCM is 2175. If one of the numbers is 725, find the other.

Sol:

HCF of two numbers = 145

LCM of two numbers = 2175

Let one of the two numbers be 725 and other be x.

Using the formula, product of two numbers = HCF × LCM

we conclude that

$725 \times x = 145 \times 2175$

$x = \frac{145 \times 2175}{725}$

$= 435$

Hence, the other number is 435.

5. The HCF of two numbers is 18 and their product is 12960. Find their LCM.

Sol:

HCF of two numbers = 18

Product of two numbers = 12960

Let their LCM be x.

Using the formula, product of two numbers = HCF × LCM

we conclude that

$12960 = 18 \times x$

$x = \frac{12960}{18}$

$= 720$

Hence, their LCM is 720.

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6. Is it possible to have two numbers whose HCF is 18 and LCM is 760?

Give reason.

Sol:

No, it is not possible to have two numbers whose HCF is 18 and LCM is 760.

Since, HCF must be a factor of LCM, but 18 is not factor of 760.

7. Find the simplest form of

(i) $\frac{69}{92}$ (ii) $\frac{473}{645}$ (iii) $\frac{1095}{1168}$ (iv) $\frac{368}{496}$

Sol:

(i) Prime factorization of 69 and 92 is:

$69 = 3 \times 23$

$92 = 2^2 \times 23$

Therefore, $\frac{69}{92} = \frac{3 \times 23}{2^2 \times 23} = \frac{3}{2^2} = \frac{3}{4}$

Thus, simplest form of $\frac{69}{92}$ is $\frac{3}{4}$.

(ii) Prime factorization of 473 and 645 is:

$473 = 11 \times 43$

$645 = 3 \times 5 \times 43$

Therefore, $\frac{473}{645} = \frac{11 \times 43}{3 \times 5 \times 43} = \frac{11}{15}$

Thus, simplest form of $\frac{473}{645}$ is $\frac{11}{15}$.

(iii) Prime factorization of 1095 and 1168 is:

$1095 = 3 \times 5 \times 73$

$1168 = 2^4 \times 73$

Therefore, $\frac{1095}{1168} = \frac{3 \times 5 \times 73}{2^4 \times 73} = \frac{15}{16}$

Thus, simplest form of $\frac{1095}{1168}$ is $\frac{15}{16}$.

(iv) Prime factorization of 368 and 496 is:

$368 = 2^4 \times 23$

$496 = 2^4 \times 31$

Therefore, $\frac{368}{496} = \frac{2^4 \times 23}{2^4 \times 31} = \frac{23}{31}$

Thus, simplest form of $\frac{368}{496}$ is $\frac{23}{31}$.

8. Find the largest number which divides 438 and 606 leaving remainder 6 in each case.

Answer:

Largest number which divides 438 and 606, leaving remainder 6 is actually the largest number which divides 438 – 6 = 432 and 606 – 6 = 600, leaving remainder 0.

Therefore, HCF of 432 and 600 gives the largest number.

Now, prime factors of 432 and 600 are:

$432 = 2^4 \times 3^3$

$600 = 2^3 \times 3 \times 5^2$

HCF = product of smallest power of each common prime factor in the numbers = $2^3 \times 3$ = 24

Thus, the largest number which divides 438 and 606, leaving remainder 6 is 24.

9. Find the largest number which divides 320 and 457 leaving remainders 5 and 7 respectively.

Answer:

We know that the required number divides 315 (320 – 5) and 450 (457 – 7).

∴ Required number = HCF (315, 450)

On applying Euclid's lemma, we get:

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$$
\begin{array}{l}
315 \overline{)\,450\,}(\,1 \\
\quad -\underline{315} \\
\qquad 135 \overline{)\,315\,}(\,2 \\
\qquad\quad -\underline{270} \\
\qquad\qquad 45 \overline{)\,135\,}(\,3 \\
\qquad\qquad\quad -\underline{135} \\
\qquad\qquad\qquad 0
\end{array}
$$

Therefore, the HCF of 315 and 450 is 45.
Hence, the required number is 45.

10. Find the least number which when divides 35, 56 and 91 leaves the same remainder 7 in each case.

Answer:

Least number which can be divided by 35, 56 and 91 is LCM of 35, 56 and 91.
Prime factorization of 35, 56 and 91 is:
$35 = 5 \times 7$
$56 = 2^3 \times 7$
$91 = 7 \times 13$
LCM = product of greatest power of each prime factor involved in the numbers = $2^3 \times 5 \times 7 \times 13 = 3640$
Least number which can be divided by 35, 56 and 91 is 3640.
Least number which when divided by 35, 56 and 91 leaves the same remainder 7 is 3640 + 7 = 3647.
Thus, the required number is 3647.

11. Find the smallest number which when divides 28 and 32, leaving remainders 8 and 12 respectively.

Answer:

Let the required number be x.
Using Euclid's lemma,
$x = 28p + 8$ and $x = 32q + 12$, where p and q are the quotients
$\Rightarrow 28p + 8 = 32q + 12$
$\Rightarrow 28p = 32q + 4$
$\Rightarrow 7p = 8q + 1$….. (1)
Here $p = 8n - 1$ and $q = 7n - 1$ satisfies (1), where n is a natural number
On putting n = 1, we get
$p = 8 - 1 = 7$ and $q = 7 - 1 = 6$
Thus, $x = 28p + 8$
$= 28 \times 7 + 8$

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= 204

Hence, the smallest number which when divided by 28 and 32 leaves remainders 8 and 12 is 204.

12. Find the smallest number which when increased by 17 is exactly divisible by both 468 and 520.

Answer:

The smallest number which when increased by 17 is exactly divisible by both 468 and 520 is obtained by subtracting 17 from the LCM of 468 and 520.

Prime factorization of 468 and 520 is:

$468 = 2^2 \times 3^2 \times 13$

$520 = 2^3 \times 5 \times 13$

LCM = product of greatest power of each prime factor involved in the numbers = $2^2 \times 3^2 \times 5 \times 13 = 4680$

The required number is 4680 – 17 = 4663.

Hence, the smallest number which when increased by 17 is exactly divisible by both 468 and 520 is 4663.

13. Find the greatest number of four digits which is exactly divisible by 15, 24 and 36.

Answer:

Prime factorization:

$15 = 3 \times 5$

$24 = 2^3 \times 3$

$36 = 2^2 \times 3^2$

LCM = product of greatest power of each prime factor involved in the numbers = $2^3 \times 3^2 \times 5 = 360$

Now, the greatest four digit number is 9999.

On dividing 9999 by 360 we get 279 as remainder.

Thus, 9999 – 279 = 9720 is exactly divisible by 360.

Hence, the greatest number of four digits which is exactly divisible by 15, 24 and 36 is 9720.

14. In a seminar, the number of participants in Hindi, English and mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required, if in each room, the same number of participants are to be seated and all of them being in the same subject.

Answer:

Minimum number of rooms required = $\frac{Total\ number\ of\ participants}{HCF\ (60,84,108)}$

Prime factorization of 60, 84 and 108 is:

$60 = 2^2 \times 3 \times 5$

$84 = 2^2 \times 3 \times 7$

$108 = 2^2 \times 3^3$

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HCF = product of smallest power of each common prime factor in the numbers = $2^2 \times 3$ = 12

Total number of participants = 60 + 84 + 108 = 252

Therefore, minimum number of rooms required = $\frac{252}{12}$ = 21

Thus, minimum number of rooms required is 21.

15. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subject wise and the height of each stack is the same. How many stacks will be there?

Answer:

Total number of English books = 336

Total number of mathematics books = 240

Total number of science books = 96

∴ Number of books stored in each stack = HCF (336, 240, 96)

Prime factorization:

$336 = 2^4 \times 3 \times 7$

$240 = 2^4 \times 3 \times 5$

$96 = 2^5 \times 3$

∴ HCF = Product of the smallest power of each common prime factor involved in the numbers = $2^4 \times 3 = 48$

Hence, we made stacks of 48 books each.

∴ Number of stacks = $\frac{336}{48} + \frac{240}{48} + \frac{96}{48}$ = (7+5+2) = 14

16. Three pieces of timber 42m, 49m and 63m long have to be divided into planks of the same length. What is the greatest possible length of each plank? How many planks are formed?

Answer:

The lengths of three pieces of timber are 42m, 49m and 63m respectively.

We have to divide the timber into equal length of planks.

∴ Greatest possible length of each plank = HCF (42, 49, 63)

Prime factorization:

$42 = 2 \times 3 \times 7$

$49 = 7 \times 7$

$63 = 3 \times 3 \times 7$

∴HCF = Product of the smallest power of each common prime factor involved in the numbers = 7

Hence, the greatest possible length of each plank is 7m.

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17. Find the greatest possible length which can be used to measure exactly the lengths 7m, 3m 85cm and 12m 95cm.

Answer:

The three given lengths are 7m (700cm), 3m 85cm (385cm) and 12m 95m (1295cm). ($\because$1m = 100cm).

$\therefore$ Required length = HCF (700, 385, 1295)

Prime factorization:

$700 = 2 \times 2 \times 5 \times 5 \times 7 = 2^2 \times 5^2 \times 7$

$385 = 5 \times 7 \times 11$

$1295 = 5 \times 7 \times 37$

$\therefore$HCF = $5 \times 7 = 35$

Hence, the greatest possible length is 35cm.

18. Find the maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and the same number of pencils.

Answer:

Total number of pens = 1001

Total number pencils = 910

$\therefore$ Maximum number of students who get the same number of pens and pencils = HCF (1001, 910)

Prime factorization:

$1001 = 11 \times 91$

$910 = 10 \times 91$

$\therefore$ HCF = 91

Hence, 91 students receive same number of pens and pencils.

19. Find the least number of square tiles required to pave the ceiling of a room 15m 17cm long and 9m 2cm broad.

Answer:

It is given that:

Length of a tile = 15m 17m = 1517cm [$\because$ 1m = 100cm]

Breadth of a tile = 9m 2m = 902cm

$\therefore$ Side of each square tile = HCF (1517, 902)

Prime factorization:

$1517 = 37 \times 41$

$902 = 22 \times 41$

$\therefore$ HCF = product of smallest power of each common prime factor in the numbers = 41

$\therefore$ Required number of tiles = $\frac{Area\ of\ ceiling}{Area\ of\ one\ tile} = \frac{1517 \times 902}{41 \times 41} = 37 \times 22 = 814$

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20. Three measuring rods are 64 cm, 80 cm and 96 cm in length. Find the least length of cloth that can be measured an exact number of times, using any of the rods.

Answer:

Length of the three measuring rods are 64cm, 80cm and 96cm, respectively.

∴ Length of cloth that can be measured an exact number of times = LCM (64, 80, 96)

Prime factorization:

$64 = 2^6$

$80 = 2^4 \times 5$

$96 = 2^5 \times 3$

∴ LCM = product of greatest power of each prime factor involved in the numbers = $2^6 \times 3 \times 5 = 960$cm = 9.6m

Hence, the required length of cloth is 9.6m.

21. An electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 a.m. At what time will they beep together at the earliest?

Answer:

Beep duration of first device = 60 seconds

Beep duration of second device = 62 seconds

∴ Interval of beeping together = LCM (60, 62)

Prime factorization:

$60 = 2^2 \times 3 \times 5$

$62 = 2 \times 31$

∴ LCM = $2^2 \times 3 \times 5 \times 31 = 1860$ seconds = $\frac{1860}{60}$ = 31min

Hence, they will beep together again at 10 : 31 a.m.

22. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, 12 minutes respectively. In 30 hours, how many times do they toll together?

Answer:

Six bells toll together at intervals of 2,4, 6, 8, 10 and 12 minutes, respectively.

Prime factorization:

$2 = 2$

$4 = 2 \times 2$

$6 = 2 \times 3$

$8 = 2 \times 2 \times 2$

$10 = 2 \times 5$

$12 = 2 \times 2 \times 3$

∴ LCM (2, 4, 6, 8, 10, 12) = $2^3 \times 3 \times 5 = 120$

Hence, after every 120minutes (i.e. 2 hours), they will toll together.

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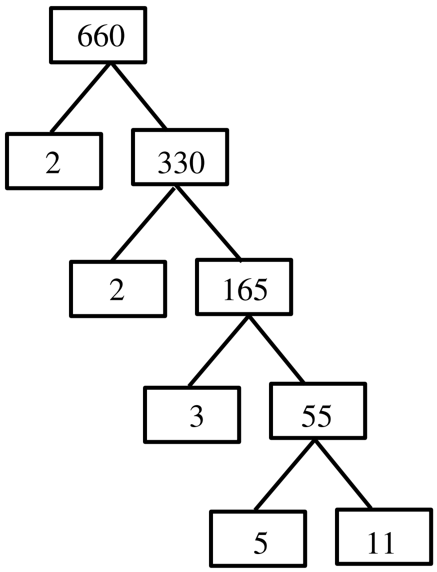
∴ Required number of times = $\left(\frac{30}{2} + 1\right)$ = 16

23. Find the missing numbers in the following factorization:

Answer:

660 = 2 × 2 × 3 × 5 × 11



Exercise - 1C

1. Without actual division, show that each of the following rational numbers is a terminating decimal. Express each in decimal form.

(i) $\frac{23}{2^3 \times 5^2}$ (ii) $\frac{24}{125}$ (iii) $\frac{171}{800}$ (iv) $\frac{15}{1600}$ (v) $\frac{17}{320}$

(vi) $\frac{19}{3125}$

Answer:

(i) $\frac{23}{2^3 \times 5^2} = \frac{23 \times 5}{2^3 \times 5^3} = \frac{115}{1000} = 0.115$

We know either 2 or 5 is not a factor of 23, so it is in its simplest form

Moreover, it is in the form of $(2^m \times 5^n)$.

Hence, the given rational is terminating.

(ii) $\frac{24}{125} = \frac{24}{5^3} = \frac{24 \times 2^3}{5^3 \times 2^3} = \frac{192}{1000} = 0.192$

We know 5 is not a factor of 23, so it is in its simplest form.

Moreover, it is in the form of $(2^m \times 5^n)$.

Hence, the given rational is terminating.

(iii) $\frac{171}{800} = \frac{171}{2^5 \times 5^2} = \frac{171 \times 5^3}{2^5 \times 5^5} = \frac{21375}{100000} = 0.21375$

We know either 2 or 5 is not a factor of 171, so it is in its simplest form.

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Moreover, it is in the form of ($2^m \times 5^n$).

Hence, the given rational is terminating.

(iv) $\frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^6} = \frac{9375}{1000000} = 0.009375$

We know either 2 or 5 is not a factor of 15, so it is in its simplest form.

Moreover, it is in the form of ($2^m \times 5^n$).

Hence, the given rational is terminating.

(v) $\frac{17}{320} = \frac{17}{2^6 \times 5} = \frac{17 \times 5^5}{2^6 \times 5^6} = \frac{53125}{1000000} = 0.053125$

We know either 2 or 5 is not a factor of 17, so it is in its simplest form.

Moreover, it is in the form of ($2^m \times 5^n$).

Hence, the given rational is terminating.

(vi) $\frac{19}{3125} = \frac{19}{5^5} = \frac{19 \times 2^5}{5^5 \times 2^5} = \frac{608}{100000} = 0.00608$

We know either 2 or 5 is not a factor of 19, so it is in its simplest form.

Moreover, it is in the form of ($2^m \times 5^n$).

Hence, the given rational is terminating.

2. Without actual division show that each of the following rational numbers is a non-terminating repeating decimal.

(i) $\frac{11}{2^3 \times 3}$ (ii) $\frac{73}{2^3 \times 3^3 \times 5}$ (iii) $\frac{129}{2^2 \times 5^7 \times 7^5}$ (iv) $\frac{9}{35}$

(v) $\frac{77}{210}$ (vi) $\frac{32}{147}$ (vii) $\frac{29}{343}$ (viii) $\frac{64}{455}$

Answer:

(i) $\frac{11}{2^3 \times 3}$

We know either 2 or 3 is not a factor of 11, so it is in its simplest form.

Moreover, $(2^3 \times 3) \neq (2^m \times 5^n)$

Hence, the given rational is non – terminating repeating decimal.

(ii) $\frac{73}{2^3 \times 3^3 \times 5}$

We know 2, 3 or 5 is not a factor of 73, so it is in its simplest form.

Moreover, $(2^2 \times 3^3 \times 5) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(iii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

We know 2, 5 or 7 is not a factor of 129, so it is in its simplest form.

Moreover, $(2^2 \times 5^7 \times 7^5) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(iv) $\frac{9}{35} = \frac{9}{5 \times 7}$

We know either 5 or 7 is not a factor of 9, so it is in its simplest form.

Moreover, $(5 \times 7) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

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(v) $\frac{77}{210} = \frac{77 \div 7}{210 \div 7} = \frac{11}{30} = \frac{11}{2 \times 3 \times 5}$

We know 2, 3 or 5 is not a factor of 11, so $\frac{11}{30}$ is in its simplest form.

Moreover, $(2 \times 3 \times 7) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(vi) $\frac{32}{147} = \frac{32}{3 \times 7^2}$

We know either 3 or 7 is not a factor of 32, so it is in its simplest form.

Moreover, $(3 \times 7^2) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(vii) $\frac{29}{343} = \frac{29}{7^3}$

We know 7 is not a factor of 29, so it is in its simplest form.

Moreover, $7^3 \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

(viii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

We know 5, 7 or 13 is not a factor of 64, so it is in its simplest form.

Moreover, $(5 \times 7 \times 13) \neq (2^m \times 5^n)$

Hence, the given rational is non-terminating repeating decimal.

3. Express each of the following as a rational number in its simplest form:

(i) $0.\overline{8}$ (ii) $2.\overline{4}$ (iii) $0.\overline{24}$ (iv) $0.\overline{12}$ (v) $2.\overline{24}$ (vi) $0.\overline{365}$

Answer:

(i) Let x = $\overline{0.8}$

$\therefore$ x = 0.888 ...(1)

10x = 8.888 ...(2)

On subtracting equation (1) from (2), we get

$9x = 8 \Rightarrow x = \frac{8}{9}$

$\therefore 0.8 = \frac{\overline{8}}{9}$

(ii) Let x = $\overline{2.4}$

$\therefore$ x = 2.444 ...(1)

10x = 24.444 ...(2)

On subtracting equation (1) from (2), we get

$9x = 22 \Rightarrow x = \frac{22}{9}$

$\therefore 2.4 = \frac{\overline{22}}{9}$

(iii) Let x = $\overline{0.24}$

$\therefore$ x = 0.2424 ...(1)

100x = 24.2424 ...(2)

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On subtracting equation (1) from (2), we get

$99x = 24 \Rightarrow x = \frac{8}{33}$

$\therefore 0.\overline{24} = \frac{8}{33}$

(iv) Let $x = \overline{0.12}$

$\therefore x = 0.1212 \quad \ldots(1)$

$100x = 12.1212 \quad \ldots(2)$

On subtracting equation (1) from (2), we get

$99x = 12 \Rightarrow x = \frac{4}{33}$

$\therefore 0.\overline{12} = \frac{4}{33}$

(v) Let $x = \overline{2.24}$

$\therefore x = 2.2444 \quad \ldots(1)$

$10x = 22.444 \quad \ldots(2)$

$100x = 224.444 \quad \ldots(3)$

On subtracting equation (2) from (3), we get

$90x = 202 \Rightarrow x = \frac{202}{90} = \frac{101}{45}$

$\therefore \overline{2.24} = \frac{101}{45}$

(vi) Let $x = \overline{0.365}$

$\therefore x = 0.3656565 \quad \ldots(1)$

$10x = 3.656565 \quad \ldots(2)$

$1000x = 365.656565 \quad \ldots(3)$

On subtracting equation (2) from (3), we get

$990x = 362 \Rightarrow x = \frac{362}{990} = \frac{181}{495}$

$\therefore \overline{0.365} = \frac{181}{495}$

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Exercise 1D

1. Define (i) rational numbers (ii) irrational numbers (iii) real numbers

Answer:

Rational numbers: The numbers of the form $\frac{p}{q}$ where p, q are integers and $q \neq 0$ are called rational numbers.

Example: $\frac{2}{3}$

Irrational numbers: The numbers which when expressed in decimal form are expressible as non-terminating and non-repeating decimals are called irrational numbers.

Example: $\sqrt{2}$

Real numbers: The numbers which are positive or negative, whole numbers or decimal numbers and rational numbers or irrational number are called real numbers.

Example: 2, $\frac{1}{3}$,$\sqrt{2}$, -3 etc.

2. Classify the following numbers as rational or irrational:

(i) $\frac{22}{7}$ (ii) 3.1416 (iii) π (iv) 3. $\overline{142857}$ (v) 5.636363
(vi) 2.040040004 (vii) 1.535335333 (viii) 3.121221222
(ix) $\sqrt{21}$ (x) $\sqrt[3]{3}$

Answer:

(i) $\frac{22}{7}$ is a rational number because it is of the form of $\frac{p}{q}$, q ≠ 0.
(ii) 3.1416 is a rational number because it is a terminating decimal.
(iii) π is an irrational number because it is a non-repeating and non-terminating decimal.
(iv) 3. $\overline{142857}$ is a rational number because it is a repeating decimal.
(v) 5.636363… is a rational number because it is a non-terminating and non-repeating decimal.
(vi) 2.040040004… is an irrational number because it is a non-terminating and non-repeating decimal.
(vii) 1.535335333… is an irrational number because it is a non-terminating and non-repeating decimal.
(viii) 3.121221222… is an irrational number because it is a non-terminating and non-repeating decimal.
(ix) $\sqrt{21} = \sqrt{3} \times \sqrt{7}$ is an irrational number because $\sqrt{3}$ and $\sqrt{7}$ are irrational and prime numbers.
(x) $\sqrt[3]{3}$ is an irrational number because 3 is a prime number. So, $\sqrt{3}$ is an irrational number.

3. Prove that each of the following numbers is irrational:

(i) $\sqrt{6}$ (ii) 2 - $\sqrt{3}$ (iii) 3 + $\sqrt{2}$ (iv) 2 + $\sqrt{5}$
(v) 5 + 3$\sqrt{2}$ (vi) 3$\sqrt{7}$ (vii) $\frac{3}{\sqrt{5}}$ (viii) 2 - 3$\sqrt{5}$
(ix) $\sqrt{3}$ + $\sqrt{5}$

Answer:

(i) Let $\sqrt{6} = \sqrt{2} \times \sqrt{3}$ be rational.
Hence, $\sqrt{2}$, $\sqrt{3}$ are both rational.
This contradicts the fact that$\sqrt{2}$, $\sqrt{3}$ are irrational.
The contradiction arises by assuming $\sqrt{6}$ is rational.
Hence, $\sqrt{6}$ is irrational.

(ii) Let 2 - $\sqrt{3}$ be rational.
Hence, 2 and 2 - $\sqrt{3}$ are rational.

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$\therefore (2 - 2 + \sqrt{3}) = \sqrt{3}$ = rational [$\because$ Difference of two rational is rational]

This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction arises by assuming $2 - \sqrt{3}$ is rational.

Hence, $2 - \sqrt{3}$ is irrational.

(iii) Let $3 + \sqrt{2}$ be rational.

Hence, 3 and $3 + \sqrt{2}$ are rational.

$\therefore 3 + \sqrt{2} - 3 = \sqrt{2}$ = rational [$\because$Difference of two rational is rational]

This contradicts the fact that $\sqrt{2}$ is irrational.

The contradiction arises by assuming $3 + \sqrt{2}$ is rational.

Hence, $3 + \sqrt{2}$ is irrational.

(iv) Let $2 + \sqrt{5}$ be rational.

Hence, $2 + \sqrt{5}$ and $\sqrt{5}$ are rational.

$\therefore (2 + \sqrt{5}) - 2 = 2 + \sqrt{5} - 2 = \sqrt{5}$ = rational [$\because$Difference of two rational is rational]

This contradicts the fact that $\sqrt{5}$ is irrational.

The contradiction arises by assuming $2 - \sqrt{5}$ is rational.

Hence, $2 - \sqrt{5}$ is irrational.

(v) Let, $5 + 3\sqrt{2}$ be rational.

Hence, 5 and $5 + 3\sqrt{2}$ are rational.

$\therefore (5 + 3\sqrt{2} - 5) = 3\sqrt{2}$ = rational [$\because$Difference of two rational is rational]

$\therefore \frac{1}{3} \times 3\sqrt{2} = \sqrt{2}$ = rational [$\because$Product of two rational is rational]

This contradicts the fact that $\sqrt{2}$ is irrational.

The contradiction arises by assuming $5 + 3\sqrt{2}$ is rational.

Hence, $5 + 3\sqrt{2}$ is irrational.

(vi) Let $3\sqrt{7}$ be rational.

$\frac{1}{3} \times 3\sqrt{7} = \sqrt{7}$ = rational [$\because$Product of two rational is rational]

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction arises by assuming $3\sqrt{7}$ is rational.

Hence, $3\sqrt{7}$ is irrational.

(vii) Let $\frac{3}{\sqrt{5}}$ be rational.

$\therefore \frac{1}{3} \times \frac{3}{\sqrt{5}} = \frac{1}{\sqrt{5}}$ = rational [$\because$Product of two rational is rational]

This contradicts the fact that $\frac{1}{\sqrt{5}}$ is irrational.

$\therefore \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{1}{5}\sqrt{5}$

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So, if $\frac{1}{\sqrt{5}}$ is irrational, then $\frac{1}{5}\sqrt{5}$ is rational

$\therefore 5 \times \frac{1}{5}\sqrt{5} = \sqrt{5} =$ rational [$\because$ Product of two rational is rational]

Hence $\frac{1}{\sqrt{5}}$ is irratonal

The contradiction arises by assuming $\frac{3}{\sqrt{5}}$ is rational.

Hence, $\frac{3}{\sqrt{5}}$ is irrational.

(viii) Let 2 - $3\sqrt{5}$ be rational.

Hence 2 and 2 - $3\sqrt{5}$ are rational.

$\therefore 2 - (2 - 3\sqrt{5}) = 2 - 2 + 3\sqrt{5} = 3\sqrt{5} =$ rational [$\because$Difference of two rational is rational]

$\therefore \quad \frac{1}{3} \times 3\sqrt{5} = \sqrt{5} =$ rational [$\because$Product of two rational is rational]

This contradicts the fact that $\sqrt{5}$ is irrational.

The contradiction arises by assuming 2 - $3\sqrt{5}$ is rational.

Hence, 2 - $3\sqrt{5}$ is irrational.

(ix) Let $\sqrt{3} + \sqrt{5}$ be rational.

$\therefore \sqrt{3} + \sqrt{5} = a$, where a is rational.

$\therefore \sqrt{3} = a - \sqrt{5}$ ….(1)

On squaring both sides of equation (1), we get

$$3 = (a - \sqrt{5})^2 = a^2 + 5 - 2\sqrt{5}a$$

$$\Rightarrow \quad \sqrt{5} = \frac{a^2+2}{2a}$$

This is impossible because right-hand side is rational, whereas the left-hand side is irrational.

This is a contradiction.

Hence, $\sqrt{3} + \sqrt{5}$ is irrational.

4. Prove that $\frac{1}{\sqrt{3}}$ is irrational.

Answer:

Let $\frac{1}{\sqrt{3}}$ be rational.

$\therefore \frac{1}{\sqrt{3}} = \frac{a}{b}$, where a, b are positive integers having no common factor other than 1

$\therefore \sqrt{3} = \frac{b}{a}$ ….(1)

Since a, b are non-zero integers, $\frac{b}{a}$ is rational.

Thus, equation (1) shows that $\sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is rational.

The contradiction arises by assuming $\sqrt{3}$ is rational.

Hence, $\frac{1}{\sqrt{3}}$ is irrational.

5. (i) Give an example of two irrationals whose sum is rational.
(ii) Give an example of two irrationals whose product is rational.

Answer:

(i) Let $(2 + \sqrt{3})$, $(2 - \sqrt{3})$ be two irrationals.

$\therefore (2 + \sqrt{3}) + (2 - \sqrt{3}) = 4$ = rational number

(ii) Let $2\sqrt{3}$, $3\sqrt{3}$ be two irrationals.

$\therefore 2\sqrt{3} \times 3\sqrt{3} = 18$ = rational number.

6. State whether the given statement is true or false:

(i) The sum of two rationals is always rational
(ii) The product of two rationals is always rational
(iii) The sum of two irrationals is an irrational
(iv) The product of two irrationals is an irrational
(v) The sum of a rational and an irrational is irrational
(vi) The product of a rational and an irrational is irrational

Answer:

(i) The sum of two rationals is always rational - True
(ii) The product of two rationals is always rational - True
(iii) The sum of two irrationals is an irrational - False

Counter example: $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are two irrational numbers. But their sum is 4, which is a rational number.

(iv) The product of two irrationals is an irrational – False

Counter example:

$2\sqrt{3}$ and $4\sqrt{3}$ are two irrational numbers. But their product is 24, which is a rational number.

(v) The sum of a rational and an irrational is irrational - True
(vi) The product of a rational and an irrational is irrational - True

7. Prove that $(2\sqrt{3} - 1)$ is irrational.

Answer:

Let $x = 2\sqrt{3} - 1$ be a rational number.

$x = 2\sqrt{3} - 1$

$\Rightarrow x^2 = (2\sqrt{3} - 1)^2$

$\Rightarrow x^2 = (2\sqrt{3})^2 + (1)^2 - 2(2\sqrt{3})(1)$

$\Rightarrow x^2 = 12 + 1 - 4\sqrt{3}$

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$\Rightarrow x^2 - 13 = -4\sqrt{3}$

$\Rightarrow \frac{13 - x^2}{4} = \sqrt{3}$

Since x is rational number, x^2 is also a rational number.

$\Rightarrow$ 13 - x^2 is a rational number

$\Rightarrow \frac{13 - x^2}{4}$ is a rational number

$\Rightarrow \sqrt{3}$ is a rational number

But $\sqrt{3}$ is an irrational number, which is a contradiction.

Hence, our assumption is wrong.

Thus, $(2\sqrt{3} - 1)$ is an irrational number.

8. Prove that $(4 - 5\sqrt{2})$ is irrational.

Answer:

Let $x = 4 - 5\sqrt{2}$ be a rational number.

$x = 4 - 5\sqrt{2}$

$\Rightarrow x^2 = (4 - 5\sqrt{2})^2$

$\Rightarrow x^2 = 4^2 + (5\sqrt{2})^2 - 2(4)(5\sqrt{2})$

$\Rightarrow x^2 = 16 + 50 - 40\sqrt{2}$

$\Rightarrow x^2 - 66 = -40\sqrt{2}$

$\Rightarrow \frac{66 - x^2}{40} = \sqrt{2}$

Since x is a rational number, x^2 is also a rational number.

$\Rightarrow$ 66 - x^2 is a rational number

$\Rightarrow \frac{66 - x^2}{40}$ is a rational number

$\Rightarrow \sqrt{2}$ is a rational number

But $\sqrt{2}$ is an irrational number, which is a contradiction.

Hence, our assumption is wrong.

Thus, $(4 - 5\sqrt{2})$ is an irrational number.

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9. Show that $(5 - 2\sqrt{3})$ is irrational.

Answer:

Let $x = 5 - 2\sqrt{3}$ be a rational number.

$x = 5 - 2\sqrt{3}$

$\Rightarrow x^2 = (5 - 2\sqrt{3})^2$

$\Rightarrow x^2 = 5^2 + (2\sqrt{3})^2 - 2(5)(2\sqrt{3})$

$\Rightarrow x^2 = 25 + 12 - 20\sqrt{3}$

$\Rightarrow x^2 - 37 = -20\sqrt{3}$

$\Rightarrow \frac{37 - x^2}{20} = \sqrt{3}$

Since x is a rational number, x^2 is also a rational number.

$\Rightarrow$ 37 - x^2 is a rational number

$\Rightarrow \frac{37 - x^2}{20}$ is a rational number

$\Rightarrow \sqrt{3}$ is a rational number

But $\sqrt{3}$ is an irrational number, which is a contradiction.

Hence, our assumption is wrong.

Thus, (5 - $2\sqrt{3}$) is an irrational number.

10. Prove that $5\sqrt{2}$ is irrational.

Answer:

Let $5\sqrt{2}$ is a rational number.

$\therefore 5\sqrt{2} = \frac{p}{q}$, where p and q are some integers and HCF(p, q) = 1 ….(1)

$\Rightarrow 5\sqrt{2}q = p$

$\Rightarrow (5\sqrt{2}q)^2 = p^2$

$\Rightarrow 2(25q^2) = p^2$

$\Rightarrow p^2$ is divisible by 2

$\Rightarrow$ p is divisible by 2 ….(2)

Let p = 2m, where m is some integer.

$\therefore 5\sqrt{2}q = 2m$

$\Rightarrow (5\sqrt{2}q)^2 = (2m)^2$

$\Rightarrow 2(25q^2) = 4m^2$

$\Rightarrow 25q^2 = 2m^2$

$\Rightarrow q^2$ is divisible by 2

$\Rightarrow$ q is divisible by 2 ….(3)

From (2) and (3) is a common factor of both p and q, which contradicts (1).

Hence, our assumption is wrong.

Thus, $5\sqrt{2}$ is irrational.

11. Show that $\frac{2}{\sqrt{7}}$ is irrational.

Answer:

$\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2}{7}\sqrt{7}$

Let $\frac{2}{7}\sqrt{7}$ is a rational number.

$\therefore \frac{2}{7}\sqrt{7} = \frac{p}{q}$, where p and q are some integers and HCF(p,q) = 1 ….(1)

$\Rightarrow 2\sqrt{7}q = 7p$

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$\Rightarrow(2\sqrt{7}q)^2 = (7p)^2$
$\Rightarrow 7(4q^2) = 49p^2$
$\Rightarrow 4q^2 = 7p^2$
$\Rightarrow$ q^2 is divisible by 7
$\Rightarrow$ q is divisible by 7 …..(2)
Let q = 7m, where m is some integer.
$\therefore 2\sqrt{7}q = 7p$
$\Rightarrow [2\sqrt{7}\,(7m)]^2 = (7p)^2$
$\Rightarrow 343(4m^2) = 49p^2$
$\Rightarrow 7(4m^2) = p^2$
$\Rightarrow$ p^2 is divisible by 7
$\Rightarrow$ p is divisible by 7 ….(3)
From (2) and (3), 7 is a common factor of both p and q, which contradicts (1).
Hence, our assumption is wrong.
Thus, $\frac{2}{\sqrt{7}}$ is irrational.

Exercise 1E

1. What do you mean by Euclid's division algorithm.
Answer:
Euclid's division lemma, states that for any two positive integers *a* and *b*, there exist unique whole numbers *q* and *r*, such that
$a = b \times q + r$ where $0 \le r < b$

2. State fundamental theorem of arithmetic?
Answer:
The fundamental theorem of arithmetic, states that every integer greater than 1 either is prime itself or is the product of prime numbers, and this product is unique.

3. Express 360 as product of its prime factors
Answer:
Prime factorization:
$360 = 2^3 \times 3^2 \times 5$

4. If a and b are two prime numbers then find the HCF(a, b)
Answer:
Prime factorization:
a = a
b = b
HCF = product of smallest power of each common factor in the numbers = 1
Thus, HCF(a,b) = 1

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5. If a and b are two prime numbers then find the HCF(a, b)

Answer:

Prime factorization:

a = a

b = b

LCM = product of greatest power of each prime factor involved in the numbers = a × b

Thus, LCM (a, b) = ab

6. The product of two numbers is 1050 and their HCF is 25. Find their LCM.

Answer:

HCF of two numbers = 25

Product of two numbers = 1050

Let their LCM be x.

Using the formula, Product of two numbers = HCF × LCM

We conclude that,

1050 = 25 × x

$x = \frac{1050}{25}$

$= 42$

Hence, their LCM is 42.

7. What is a composite number?

Answer:

A composite number is a positive integer which is not prime (i.e. which has factors other than 1 and itself).

8. If a and b are relatively prime then what is their HCF?

Answer:

If two numbers are relatively prime then their greatest common factor will be 1.

Thus, HCF (a, b) = 1.

9. If the rational number $\frac{a}{b}$ has a terminating decimal expansion, what is the condition to be satisfied by b?

Answer:

Let x be a rational number whose decimal expansion terminates.

Then, we can express x in the form $\frac{a}{b}$, where a and b are coprime, and prime factorization of b is of the form $(2^m \times 5^n)$, where m and n are non-negative integers.

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10. Find the simplest form of $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$

Answer:

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{3\times3\times5}+3\sqrt{2\times2\times5}}{2\sqrt{5}}$$

$$= \frac{2\times3\sqrt{5}+3\times2\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{12\sqrt{5}}{2\sqrt{5}}$$

$$= 6$$

Thus, simplified form of $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ is 6.

11. Write the decimal expansion of $\frac{73}{(2^4\times5^3)}$

Answer:

Decimal expansion:

$$\frac{73}{(2^4\times5^3)} = \frac{73\times5}{2^4\times5^4}$$

$$= \frac{365}{(2\times5)^4}$$

$$= \frac{365}{(10)^4}$$

$$= \frac{365}{10000}$$

$$= 0.0365$$

Thus, the decimal expansion of $\frac{73}{(2^4\times5^3)}$ is 0.0365.

12. Show that there is no value of n for which $\left(2^n\times5^n\right)$ ends in 5.

Answer:

We can write:

$(2^n\times5^n) = (2\times5)^n$

$= 10^n$

For any value of n, we get 0 in the end.

Thus, there is no value of n for which $(2^n\times5^n)$ ends in 5.

13. Is it possible to have two numbers whose HCF if 25 and LCM is 520?

Answer:

No, it is not possible to have two numbers whose HCF is 25 and LCM is 520.

Since, HCF must be a factor of LCM, but 25 is not a factor of 520.

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14. Give an example of two irrationals whose sum is rational.

Answer:

Let the two irrationals be $4 - \sqrt{5}$ and $4 + \sqrt{5}$

$(4 - \sqrt{5}) + (4 + \sqrt{5}) = 8$

Thus, sum (i.e., 8) is a rational number.

15. Give an example of two irrationals whose product is rational.

Answer:

Let the two irrationals be $4\sqrt{5}$ and $3\sqrt{5}$

$(4\sqrt{5}) \times (3\sqrt{5}) = 60$

Thus, product (i.e., 60) is a rational number.

16. If a and b are relatively prime, what is their LCM?

Answer:

If two numbers are relatively prime then their greatest common factor will be 1.

$\therefore$ HCF (a,b) = 1

Using the formula, Product of two numbers = HCF × LCM

we conclude that,

$a \times b = 1 \times LCM$

$\therefore$ LCM = ab

Thus, LCM (a,b) is ab.

17. The LCM of two numbers is 1200, show that the HCF of these numbers cannot be 500. Why ?

Answer:

If the LCM of two numbers is 1200 then, it is not possible to have their HCF equals to 500. Since, HCF must be a factor of LCM, but 500 is not a factor of 1200.

Short answer Questions

18. Express $0.\overline{4}$ as a rational number simplest form.

Answer:

Let x be $0.\overline{4}$

$x = 0.\overline{4}$ ….(1)

Multiplying both sides by 10, we get

$10x = 4.\overline{4}$ ….(2)

Subtracting (1) from (2), we get

$10x - x = 4.\overline{4} - 0.\overline{4}$

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$\Rightarrow 9x = 4$

$\Rightarrow x = \frac{4}{9}$

Thus, simplest form of $0.\overline{4}$ as a rational number is $\frac{4}{9}$.

19. Express $0.\overline{23}$ as a rational number in simplest form.

Answer:

Let x be $0.\overline{23}$

$x = 0.\overline{23}$ ….(1)

Multiplying both sides by 100, we get

$100x = 23.\overline{23}$ ….(2)

Subtracting (1) from (2), we get

$100x - x = 23.\overline{23} - 0.\overline{23}$

$\Rightarrow 99x = 23$

$\Rightarrow x = \frac{23}{99}$

Thus, simplest form of $0.\overline{23}$ as a rational number is $\frac{23}{99}$.

20. Explain why 0.15015001500015……. is an irrational form.

Answer:

Irrational numbers are non-terminating non-recurring decimals.

Thus, 0.15015001500015…. is an irrational number.

21. Show that $\frac{\sqrt{2}}{3}$ is irrational.

Answer:

Let $\frac{\sqrt{2}}{3}$ is a rational number.

$\therefore \frac{\sqrt{2}}{3} = \frac{p}{q}$ where p and q are some integers and HCF(p,q) = 1 …..(1)

$\Rightarrow \sqrt{2}q = 3p$

$\Rightarrow (\sqrt{2}q)^2 = (3p)^2$

$\Rightarrow 2q^2 = 9p^2$

$\Rightarrow p^2$ is divisible by 2

$\Rightarrow$p is divisible by 2 ….(2)

Let p = 2m, where m is some integer.

$\therefore \sqrt{2}q = 3p$

$\Rightarrow \sqrt{2}q = 3(2m)$

$\Rightarrow (\sqrt{2}q)^2 = [\ 3(2m)\ ]^2$

$\Rightarrow 2q^2 = 4\ (9p^2)$

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$\Rightarrow q^2 = 2\ (9p^2)$
$\Rightarrow q^2$ is divisible by 2
$\Rightarrow$ q is divisible by 2 …(3)
From (2) and (3), 2 is a common factor of both p and q, which contradicts (1).
Hence, our assumption is wrong.
Thus, $\frac{\sqrt{2}}{3}$ is irrational.

22. Write a rational number between $\sqrt{3}$ and 2

Answer:
Since, $\sqrt{3}$ = 1.732….
So, we may take 1.8 as the required rational number between $\sqrt{3}$ and 2.
Thus, the required rational number is 1.8.

23. Explain why $3.\overline{1416}$ is a rational number ?

Answer:
Since, $3.\overline{1416}$ is a non-terminating repeating decimal.
Hence, is a rational number.

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Exercise MCQ

1. Which of the following rational numbers is expressible as a non-terminating decimal?

(a) $\frac{1351}{1250}$ (b) $\frac{2017}{250}$ (c) $\frac{3219}{1800}$

Answer:
$\frac{1351}{1250} = \frac{1351}{5^4 \times 2}$
We know 2 and 5 are not the factors of 1351.
So, the given rational is in its simplest form.
And it is of the form $(2^m \times 5^n)$ for some integers m, n.
So, the given number is a terminating decimal.
$\therefore \frac{1351}{5^4 \times 2} = \frac{1351 \times 2}{5^4 \times 2^4} = \frac{10808}{10000} = 1.0808$

$\frac{2017}{250} = \frac{2017}{5^3 \times 2}$
We know 2 and 5 are not the factors of 2017.
So, the given rational is in its simplest form.
And it is of the form $(2^m \times 5^n)$ for some integers m, n.
So, the given rational number is a terminating decimal.
$\therefore \frac{2017}{5^3 \times 2} = \frac{2017 \times 2^2}{5^3 \times 2^3} = \frac{8068}{1000} = 8.068$

$\frac{3219}{1800} = \frac{3219}{2^3 \times 5^2 \times 3^2}$

We know 2, 3 and 5 are not the factors of 3219.

So, the given rational is in its simplest form.

$\therefore (2^3 \times 5^2 \times 3^2) \neq (2^m \times 5^n)$ for some integers m, n.

Hence, $\frac{3219}{1800}$ is not a terminating decimal.

$\frac{3219}{1800} = 1.78833333\ldots.$

Thus, it is a repeating decimal.

$\frac{1723}{625} = \frac{1723}{5^4}$

We know 5 is not a factor of 1723.

So, the given rational number is in its simplest form.

And it is not of the form $(2^m \times 5^n)$

Hence, $\frac{1723}{625}$ is not a terminating decimal.

2. If a= a = $(2^2 \times 3^3 \times 5^4)$ and b = $(2^3 \times 3^2 \times 5)$, then HCF (a, b) = ?

(a) 90 (b) 180 (c) 360 (d) 540

Answer:

(b) 180

It is given that:

a = $(2^2 \times 3^3 \times 5^4)$ and b = $(2^3 \times 3^2 \times 5)$

$\therefore$HCF (a,b) = Product of smallest power of each common prime factor in the numbers.

$= 2^2 \times 3^2 \times 5$

$= 180$

3. HCF of $(2^3 \times 3^2 \times 5)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3 \times 5^3 \times 7)$ is

(a) 30 (b) 48 (c) 60 (d) 105

Answer:

(c) 60

HCF = $(2^3 \times 3^2 \times 5, 2^2 \times 3^3 \times 5^2, 2^4 \times 3 \times 5^3 \times 7)$

HCF = Product of smallest power of each common prime factor in the numbers

$= 2^2 \times 3 \times 5$

$= 60$

4. LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is

(a) 40 (b) 560 (c) 1120 (d) 1680

Answer:

(c) 1680

LCM = $(2^3 \times 3 \times 5, 2^4 \times 5 \times 7)$

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$\therefore$ LCM = Product of greatest power of each prime factor involved in the numbers

$= 2^4 \times 3 \times 5 \times 7$

$= 16 \times 3 \times 5 \times 7$

$= 1680$

5. The HCF of two numbers is 27 and their LCM is 162. If one of the numbers is 54, what is the other number?

(a) 36 (b) 45 (c) 9 (d) 81

Answer:

(d) 81

Let the two numbers be x and y.

It is given that:

x = 54

HCF = 27

LCM = 162

We know,

$x \times y = \text{HCF} \times \text{LCM}$

$\Rightarrow 54 \times y = 27 \times 162$

$\Rightarrow 54\,y = 4374$

$\Rightarrow \therefore y = \frac{4374}{54} = 81$

6. The product of two numbers is 1600 and their HCF is 5. The LCM of the numbers is

(a) 8000 (b) 1600 (c) 320 (d) 1605

Answer:

(c) 320

Let the two numbers be x and y.

It is given that:

$x \times y = 1600$

HCF = 5

We know,

$\text{HCF} \times \text{LCM} = x \times y$

$\Rightarrow 5 \times \text{LCM} = 1600$

$\Rightarrow \therefore \text{LCM} = \frac{1600}{5} = 320$

7. What is the largest number that divided each one of the 1152 and 1664 exactly?

(a) 32 (b) 64 (c) 128 (d) 256

Answer:

(c) 128

Largest number that divides each one of 1152 and 1664 = HCF (1152, 1664)

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We know,

$$1152 = 2^7 \times 3^2$$
$$1664 = 2^7 \times 13$$

$\therefore$ HCF $= 2^7 = 128$

8. What is the largest number that divides 70 and 125, leaving remainders 5 and 8 respectively?

(a) 13 (b) 9 (c) 3 (d) 585

Answer:

(a) 13

We know the required number divides 65 (70 – 5) and 117 (125 – 8)

$\therefore$ Required number = HCF (65, 117)

We know,

$$65 = 13 \times 5$$
$$117 = 13 \times 3 \times 3$$

$\therefore$ HCF = 13

9. What is the largest number that divides 245 and 1029, leaving remainder 5 in each case?

(a) 15 (b) 16 (c) 9 (d) 5

Answer:

(b) 16

We know that the required number divides 240 (245 – 5) and 1024 (1029 – 5).

$\therefore$ Required number = HCF (240, 1024)

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$
$$1024 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$\therefore$ HCF $= 2 \times 2 \times 2 \times 2 = 16$

10. The simplest form of $\frac{1095}{1168}$

(a) $\frac{17}{26}$ (b) $\frac{25}{26}$ (c) $\frac{13}{16}$ (d) $\frac{15}{16}$

Answer:

(d) $\frac{15}{16}$

$$\frac{1095}{1168} = \frac{1095 \div 73}{1168 \div 73} = \frac{15}{16}$$

Hence, HCF of 1095 and 1168 is 73.

11. Euclid's division lemma states that for any positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy

(a) $1 < r < v$ (b) $0 < r \le b$ (c) $0 \le r < b$ (d) $0 < r < b$

Answer:

(c) $0 \le r < b$

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Euclid's division lemma, states that for any positive integers *a* and *b*, there exist unique integers *q* and *r*, such that $a = bq + r$
where *r* must satisfy $0 \le r < b$

12. A number when divided by 143 leaves 31 as remainder. What will be the remainder when the same number is divided by 13?
(a) 0 (b) 1 (c) 3 (d) 5
Answer:
(d) 5
We know,
Dividend = Divisor × Quotient + Remainder.
It is given that:
Divisor = 143
Remainder = 13
So, the given number is in the form of 143x + 31, where x is the quotient.
$\therefore$ 143x + 31 = 13 (11x) + (13 × 2) + 5 = 13 (11x + 2) + 5
Thus, the remainder will be 5 when the same number is divided by 13.

13. Which of the following is an irrational number?
(a) $\frac{22}{7}$ (b) 3.1416
(c) $3.\overline{1416}$ (d) 3.141141114…..
Answer:
(d) 3.141141114…..
3.141141114 is an irrational number because it is a non-repeating and non-terminating decimal.

14. π is
(a) an integer (b) a rational number
(c) an irrational number (d) none of these
Answer:
(c) π is an irrational number
π is an irrational number because it is a non-repeating and non-terminating decimal.

15. $2.\overline{35}$ is
(a) an integer (b) a rational number
(c) an irrational number (d) none of these
Answer:
(b) $2.\overline{35}$ is a rational number
$2.\overline{35}$ is a rational number because it is a repeating decimal.

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16. 2.13113111311113…… is

(a) an integer (b) a rational number

(c) an irrational number (d) none of these

Answer:

(c) an irrational number

It is an irrational number because it is a non-terminating and non-repeating decimal.

17. $1.23\overline{48}$ is

(a) an integer (b) a rational number

(c) an irrational number (d) none of these

Answer:

(b) $1.23\overline{48}$ a rational number

It is a rational number because it is a repeating decimal.

18. Which of the following rational numbers is expressible as a terminating decimal?

(a) $\frac{124}{165}$ (b) $\frac{131}{30}$ (c) $\frac{2027}{625}$ (d) $\frac{1625}{462}$

Answer:

(c) $\frac{2027}{625}$

$\frac{124}{165} = \frac{124}{5 \times 33}$; we know 5 and 33 are not the factors of 124. It is in its simplest form and it cannot be expressed as the product of ($2^m \times 5^n$) for some non-negative integers m, n.

So, it cannot be expressed as a terminating decimal.

$\frac{131}{30} = \frac{131}{5 \times 6}$; we know 5 and 6 are not the factors of 131. It is in its simplest form and it cannot be expressed as the product of ($2^m \times 5^n$) for some non-negative integers m, n.

So, it cannot be expressed as a terminating decimal.

$\frac{2027}{625} = \frac{2027 \times 2^4}{5^4 \times 2^4} = \frac{32432}{10000} = 3.2432$; as it is of the form ($2^m \times 5^n$), where m, n are non-negative integers.

So, it is a terminating decimal.

$\frac{1625}{462} = \frac{1625}{2 \times 7 \times 33}$; we know 2, 7 and 33 are not the factors of 1625. It is in its simplest form and it cannot be expressed as the product of ($2^m \times 5^n$) for some non-negative integers m, n.

So, it cannot be expressed as a terminating decimal.

19. The decimal expansion of the rational number $\frac{37}{2^5 \times 5}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) four decimal places

Answer:

(b) two decimal places.

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$\frac{37}{2^5 \times 5} = \frac{37 \times 5}{2^2 \times 5^2} = \frac{185}{100} = 1.85$

So, the decimal expansion of the rational number terminates after two decimal places.

20. The decimal expansion of the number $\frac{14753}{1250}$ will terminate after

(a) one decimal place (b) two decimal places

(c) three decimal places (d) four decimal places

Answer:

(d) four decimal places

$\frac{14753}{1250} = \frac{14753}{5^4 \times 2} = \frac{14753 \times 2^3}{5^4 \times 2^4} = \frac{118024}{1000} = 11.8024$

So, the decimal expansion of the number will terminate after four decimal places.

21. The number 1.732 is

(a) an integer (b) a rational number

(c) an irrational number (d) none of these

Answer:

Clearly, 1.732 is a terminating decimal.

Hence, a rational number.

Hence, the correct answer is option (b).

22. If a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then, the least prime factor of (a + b) is

(a) 2 (b) 3 (c) 5 (d) 8

Answer:

(a) 2

Since 5 + 3 = 8, the least prime factor of a + b has to be 2, unless a + b is a prime number greater than 2.

23. $\sqrt{2}$ is

(a) an integer (b) an irrational number

(c) a rational number (d) none of these

Answer:

Let $\sqrt{2}$ is a rational number.

$\therefore \sqrt{2} = \frac{p}{q}$, where p and q are some integers and HCF(p,q) = 1 ….(1)

$\Rightarrow \sqrt{2}q = p$

$\Rightarrow (\sqrt{2}q)^2 = p^2$

$\Rightarrow 2q^2 = p^2$

$\Rightarrow p^2$ is divisible by 2

$\Rightarrow$ p is divisible by 2 …..(2)

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Let p = 2m, where m is some integer.

∴ $\sqrt{2}q$ = p

⇒ $\sqrt{2}q$ = 2m

⇒ $(\sqrt{2}q)^2 = (2m)^2$

⇒$2q^2 = 4m^2$

⇒ $q^2 = 2m^2$

⇒ q^2 is divisible by 2

⇒ q is divisible by 2 …..(3)

From (2) and (3), 2 is a common factor of both p and q, which contradicts (1).

Hence, our assumption is wrong.

Thus, $\sqrt{2}$ is an irrational number.

Hence, the correct answer is option (b).

24. $\frac{1}{\sqrt{2}}$ is

(a) a fraction (b) a rational number

(c) an irrational number (d) none of these

Answer:

(c) an irrational number.

$\frac{1}{\sqrt{2}}$ is an irrational number.

25. (2 + $\sqrt{2}$) is

(a) an integer (b) a rational number

(c) an irrational number (d) none of these

Answer:

(c) an irrational number

2 + $\sqrt{2}$ is an irrational number.

if it is rational, then the difference of two rational is rational.

∴ (2 + $\sqrt{2}$) – 2 = $\sqrt{2}$ = irrational.

26. What is the least number that is divisible by all the natural numbers from 1 to 10 (both inclusive)?

Answer:

(c) 2520

We have to find the least number that is divisible by all numbers from 1 to 10.

∴ LCM (1 to 10) = $2^3 \times 3^2 \times 5 \times 7 = 2520$

Thus, 2520 is the least number that is divisible by every element and is equal to the least common multiple.

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Exercise - Formative assessment

1. The decimal representation of $\frac{71}{150}$ is

(a) a terminating decimal
(b) a non-terminating, repeating decimal
(c) a non-terminating and non-repeating decimal
(d) none of these

Answer:

(b) a non-terminating, repeating decimal

$\frac{71}{150} = \frac{71}{2 \times 3 \times 5^2}$

We know that 2, 3 or 5 are not factors of 71.

So, it is in its simplest form.

And, $(2 \times 3 \times 5^2) \neq (2^m \times 5^n)$

$\therefore \frac{71}{150} = 0.47\overline{3}$

Hence, it is a non-terminating, repeating decimal.

2. Which of the following has a terminating decimal expansion?

(a) $\frac{32}{91}$ (b) $\frac{19}{80}$ (c) $\frac{23}{45}$ (d) $\frac{25}{42}$

Answer:

(b) $\frac{19}{80}$

$\frac{19}{80} = \frac{19}{2^4 \times 5}$

We know 2 and 5 are not factors of 19, so it is in its simplest form.

And $(2^4 \times 5) = (2^m \times 5^n)$

Hence, $\frac{19}{80}$ is a terminating decimal.

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3. On dividing a positive integer n by 9, we get 7 as a remainder. What will be the remainder if (3n -1) is divided by 9?

(a) 1 (b) 2 (c) 3 (d) 4

Answer:

(b) 2

Let q be the quotient.

It is given that:

Remainder = 7

On applying Euclid's algorithm, i.e. dividing n by 9, we have

$n = 9q + 7$

$\Rightarrow 3n = 27q + 21$

$\Rightarrow 3n - 1 = 27q + 20$

$\Rightarrow 3n - 1 = 9 \times 3q + 9 \times 2 + 2$

$\Rightarrow 3n - 1 = 9 \times (3q + 2) + 2$

So, when (3n-1) is divided by 9, we get the remainder 2.

4. $0.\overline{68} + 0.\overline{73} = ?$

(a) $1.\overline{41}$ (b) $1.\overline{42}$ (c) $0.\overline{141}$ (d) none of these

Answer:

(b) $1.\overline{42}$

Short answer Question: (2 marks)

5. Show that any number of the form 4^n, $n \in N$ can never end with the digit 0.

Answer:

If 4^n ends with 0, then it must have 5 as a factor.

But we know the only prime factor of 4^n is 2.

Also we know from the fundamental theorem of arithmetic that prime factorization of each number is unique.

Hence, 4^n can never end with the digit 0.

6. The HCF of two numbers is 27 and their LCM is 162. If one of the number is 81, find the other.

Answer:

Let the two numbers be x and y

It is given that:

x = 81

HCF = 27 and LCM = 162

We know, Product of two numbers = HCF × LCM

$$\Rightarrow x \times y = 27 \times 162$$

$$\Rightarrow 81 \times y = 4374$$

$$\Rightarrow y = \frac{4374}{81} = 54$$

Hence, the other number is y is 54.

7. Examine whether $\frac{17}{30}$ is a terminating decimal.

Answer:

$\frac{17}{30} = \frac{17}{2 \times 3 \times 5}$

We know that 2,3 and 5 are not the factors of 17.

So, $\frac{17}{30}$ is in its simplest form.

Also, $30 = 2 \times 3 \times 5 \neq (2^m \times 5^n)$

Hence, $\frac{17}{30}$ is a non-terminating decimal.

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8. Find the simplest form of $\frac{148}{185}$.

Answer:

$\frac{148}{185} = \frac{148 \div 37}{185 \div 37} = \frac{4}{5}$ ($\because$HCF of 148 and 185 is 37)

Hence, the simplest form is $\frac{4}{5}$.

9. Which of the following numbers are irrational?

(a) $\sqrt{2}$ (b) $\sqrt[3]{6}$ (c) 3.142857 (d) $2.\overline{3}$

(e) π (f) $\frac{22}{7}$ (g) 0.232332333…. (h) $5.2\overline{741}$

Answer:

(a) $\sqrt{2}$ is irrational ($\because$if p is prime, then $\sqrt{p}$ is irrational).

(b) $\sqrt[3]{6} = \sqrt[3]{2} \times \sqrt[3]{3}$ is irrational.

(c) 3.142857 is rational because it is a terminating decimal.

(d) $2.\overline{3}$ is rational because it is a non-terminating, repeating decimal.

(e) π is irrational because it is a non-repeating, non –terminating decimal.

(f) $\frac{22}{7}$ is rational because it is in the form of $\frac{p}{q}$, q $\neq$ 0.

(g) 0.232332333…. is irrational because it is a non-terminating, non-repeating decimal.

(h) $5.2\overline{741}$ is rational because it is a non-terminating, repeating decimal.

10. Prove that $(2 + \sqrt{3})$ is irrational.

Answer:

Let $(2 + \sqrt{3})$ be rational.

Then, both $(2 + \sqrt{3})$ and 2 are rational.

$\therefore$ { $(2 + \sqrt{3}) - 2$ } is rational [$\because$ Difference of two rational is rational]

$\Rightarrow \sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction arises by assuming $(2 + \sqrt{3})$ is rational.

Hence, $(2 + \sqrt{3})$ is irrational.

Short answer Question: (2 marks)

11. Find the HCF and LCM of 12, 15, 18, 27.

Answer:

Prime factorization:

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

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$18 = 2 \times 3 \times 3 = 2 \times 3^2$
$27 = 3 \times 3 \times 3 = 3^3$
Now,
HCF = Product of smallest power of each common prime factor in the number
$= 3$
LCM = Product of greatest power of each prime factor involved in the number
$= 2^2 \times 3^3 \times 5 = 540$

12. Give an example of two irrationals whose sum is rational.
Answer:
Let $(2 + \sqrt{2})$ and $(2 - \sqrt{2})$ be two irrational numbers.
Sum $= (2 + \sqrt{2}) + (2 - \sqrt{2}) = 2 + \sqrt{2} + 2 - \sqrt{2} = 4$, which is a rational number.

13. Give prime factorization of 4620.
Answer:
Prime factorization:
$4620 = 2 \times 2 \times 3 \times 5 \times 7 \times 11 = 2^2 \times 3 \times 5 \times 7 \times 11$

14. Find the HCF of 1008 and 1080 by prime factorization method.
Answer:
Prime factorization:
$1008 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^4 \times 3^2 \times 7$
$1080 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^3 \times 3^3 \times 5$
HCF = Product of smallest power of each common prime factor in the number.
$= 2^3 \times 3^2 = 72$

15. Find the HCF and LCM of $\frac{8}{9}, \frac{10}{27}$ and $\frac{16}{81}$.
Answer:
HCF of fractions $= \frac{HCF\ of\ numerators}{LCM\ of\ denominators}$
LCM of fractions $= \frac{LCM\ of\ numerators}{HCF\ of\ denominators}$
Prime factorization of the numbers given in the numerators are as follows:
$8 = 2 \times 2 \times 2$
$10 = 2 \times 5$
$16 = 2 \times 2 \times 2 \times 2$
HCF of numerators = 2
LCM of numerators $= 2^4 \times 5 = 80$
Prime factorization of numbers given in the denominators are as follows:
$9 = 3 \times 3$

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$27 = 3 \times 3 \times 3$
$81 = 3 \times 3 \times 3 \times 3$
HCF of denominators = $3 \times 3 = 9$
LCM of denominators = $3^4 = 81$
$\therefore$ HCF of fractions = $\frac{HCF\ of\ numerators}{LCM\ of\ denominators} = \frac{2}{81}$
$\therefore$ LCM of fractions = $\frac{LCM\ of\ numerators}{HCF\ of\ denominators} = \frac{80}{9}$

16. Find the largest number which divides 546 and 764, leaving remainders 6 and 8 respectively.
Answer:
We know the required number divides 540 (546 – 6) and 756 (764 – 8), respectively.
$\therefore$ Required largest number = HCF (540, 756)
Prime factorization:
$540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$
$756 = 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 2^2 \times 3^2 \times 7$
$\therefore$ HCF = $2^2 \times 3^3 = 108$
Hence, the largest number is 108.

17. Prove that $\sqrt{3}$ is an irrational number.
Answer:
Let $\sqrt{3}$ be rational and its simplest form be $\frac{a}{b}$.
Then, a, b are integers with no common factors other than 1 and $b \neq 0$.
Now, $\sqrt{3} = \frac{a}{b} \Rightarrow 3 = \frac{a^2}{b^2}$ [on squaring both sides]
$\Rightarrow 3b^2 = a^2$ ….(1)
$\Rightarrow$ 3 divides a^2 [since 3 divides $3b^2$]
$\Rightarrow$ 3 divides a [since 3 is prime, 3 divides $a^2 \Rightarrow$ 3 divides a]
Let a = 3c for some integer c.
Putting a = 3c in equation (1), we get
$3b^2 = 9c^2 \Rightarrow b = 3c^2$
$\Rightarrow$ 3 divides b^2 [since 3 divides $3c^2$]
$\Rightarrow$ 3 divides b [since 3 is prime, 3 divides $b^2 \Rightarrow$ 3 divides b]
Thus, 3 is a common factor of both a, b.
But this contradicts the fact that a, b have no common factor other than 1.
The contradiction arises by assuming $\sqrt{3}$ is rational.
Hence, $\sqrt{3}$ is rational.

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18. Show that every positive odd integer is of the form (4q+1) or (4q+2) for some integer q.

Answer:

Let a be the given positive odd integer.

On dividing a by 4, let q be the quotient and r the remainder.

Therefore, by Euclid's algorithm we have

$a = 4q + r \qquad 0 \le r < 4$

$\Rightarrow \quad a = 4q + r \qquad r = 0, 1, 2, 3$

$\Rightarrow \quad a = 4q, a = 4q + 1, a = 4q + 2, a = 4q + 3$

But, 4q and 4q + 2 = 2 (2q + 1) = even

Thus, when a is odd, it is of the form (4q + 1) or (4q + 3) for some integer q.

19. Show that one and only one out of n, (n+2) and (n+4) is divisible by 3, where n is any positive integer.

Answer:

Let q be quotient and r be the remainder.

On applying Euclid's algorithm, i.e. dividing n by 3, we have

$n = 3q + r \qquad 0 \le r < 3$

$\Rightarrow n = 3q + r \qquad r = 0, 1 \text{ or } 2$

$\Rightarrow n = 3q \text{ or } n = (3q + 1) \text{ or } n = (3q + 2)$

Case 1: If n = 3q, then n is divisible by 3.

Case 2: If n = (3q+1), then (n+2) = 3q + 3 = 3(q + 1), which is clearly divisible by 3.

In this case, (n+2) is divisible by 3.

Case 3: If n = (3q+2), then (n+4) = 3q + 6 = 3(q + 2), which is clearly divisible by 3.

In this case, (n+4) is divisible by 3.

Hence, one and only one out of n, (n+2) and (n+4) is divisible by 3.

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20. Show that $(4 + 3\sqrt{2})$ is irrational.

Answer:

Let $(4+3\sqrt{2})$ be a rational number.

Then both $(4+3\sqrt{2})$ and 4 are rational.

$\Rightarrow (4+3\sqrt{2} - 4) = 3\sqrt{2}$ = rational [∵Difference of two rational numbers is rational]

$\Rightarrow 3\sqrt{2}$ is rational.

$\Rightarrow \frac{1}{3}(3\sqrt{2})$ is rational. [∵ Product of two rational numbers is rational]

$\Rightarrow \sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational (when 2 is prime, $\sqrt{2}$ is irrational)

Hence, $(4 + 3\sqrt{2})$ is irrational.