9. Quadrilaterals and Parallelograms

Exercise 9A

1. Question

Three angles of a quadrilateral measure 56°, 115° and 84°. Find the measure of the fourth angle.

Answer

Let the measure of the fourth angle be xo.

Since the sum of the angles of a quadrilateral is 360°, we have:

∴ 56° + 115° + 84° + x° = 360°

∴ 255° + x° = 360°

∴ x° = 105°

Hence, the measure of the fourth angle is 105°.

2. Question

The angles of a quadrilateral are in the ratio 2:4:5:7. Find the angles.

Answer

Our given ratio of angles is 2:4:5:7. Let common multiplying factor be x°.

Hence, ∠A = 2x°, ∠B = 4x°, ∠C = 5x° and ∠D = 7x°

Since the sum of the angles of a quadrilateral is 360°, we have:

∴ 2x + 4x + 5x + 7x = 360°

∴ 18 x = 360°

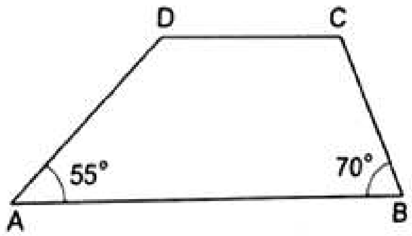
∴ x = 20°

∴ ∠A = 40°; ∠B = 80°; ∠C = 100°; ∠D = 140°

Hence, the measure of the angles are 40°, 80°, 100° and 140°

3. Question

In the adjoining figure, $ABCD$ is a trapezium in which $AB \parallel DC$. If $\angle A = 55^\circ$ and $\angle B = 70^\circ$, find $\angle C$ and $\angle D$.



Answer

Here given that ABCD is trapezium where AB || DC.

We observe that ∠ A and ∠ D are the interior angles on the same side of transversal line AD, whereas ∠ B and ∠ C are the interior angles on the same side of transversal line BC.

As ∠A and ∠D are interior angles, we have,

∠A + ∠D = 180°

∴ ∠D = 180° − ∠A

∴ ∠ D = 180° − 55° = 125°

Similarly for ∠ B and ∠C,

∠ B + ∠C = 180°

∴ ∠C = 180°− ∠B

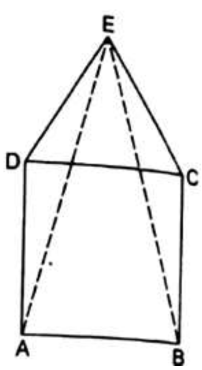
∴ ∠ C = 180° − 70° = 110°

Hence, measure of ∠ D and ∠ C are 125°and 110° respectively.

4. Question

In the adjoining figure, *ABCD* is a square and ΔEDC is an equilateral triangle. Prove that

(i) AE = BE (ii) ∠DAE = 15°



Answer

(i) Here it is given that in ABCD is a square and ΔEDC is an equilateral triangle.

Hence, we say that AB = BC = CD = DA and ED = EC = DC

Now in Δ*ADE* and Δ*BCE,* we have,

AD = BC … given

DE = EC … given

∠ADE = ∠BCE …

as both angles are sum of 60° and 90°

∴ *ΔADE ≅ ΔBCE*

Now by cpct,

AE = BE …(1)

(ii) Here ∠ADE = 90°+ 60° = 150°

DA = DC … given

DC = DE … given

∴ DA = DE

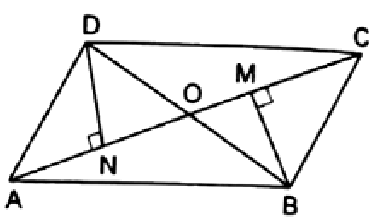
This means that sides of square and triangles are equal.

∴ Δ*ADE* and Δ*BCE* are isosceles triangles.

Hence, ∠DAE = ∠DEA = $\frac{1}{2}$(180° − 150°) = 30°/2 =15°

5. Question

In the adjoining figure, BM⊥AC and DN⊥AC. If BM = DN, prove that AC bisects BD.



Answer

Given: In ABCD, in which BM ⊥ AC and DN ⊥ AC and BM = DN.

To prove: *AC* bisects BD ie. DO = BO

Proof:

Now, in ΔOND and ΔOMB, we have,

∠OND = ∠OMB …90° each

∠DON = ∠ BOM …Vertically opposite angles

Also, DN = BM …Given Hence, by AAS congruence rule,

ΔOND ≅ ΔOMB

∴ OD = OB …CPCT

Hence, AC bisects BD.

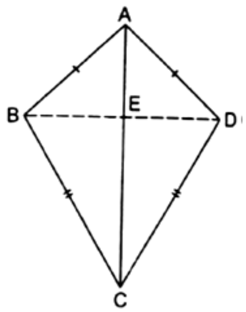
6. Question

In the given figure, $ABCD$ is a quadrilateral in which $AB = AD$ and $BC = DC$. Prove that

(i) AC bisects $\angle A$ and $\angle C$,

(ii) $BE = DE$,

(iii) $\angle ABC = \angle ADC$.



Answer

Given: In ABCD, $AB = AD$ and $BC = DC$.

To prove: (i) AC bisects $\angle A$ and $\angle C$,

(ii) $BE = DE$,

(iii) $\angle ABC = \angle ADC$.

Proof:

(i) In ΔABC and ΔADC, we have,

AB = AD …given

BC = DC …given

AC = AC … common side

Hence, by SSS congruence rule,

ΔABC ≅ ΔADC

∴ ∠BAC = ∠DAC and ∠BCA = ∠DCA …By cpct

Thus, AC bisects ∠A and ∠ C.

(ii) Now, in ΔABE and ΔADE, we have,

AB = AD …given

∠BAE = ∠DAE …from i

AE = AE …common side

Hence, by SAS congruence rule,

ΔABE ≅ ΔADE

∴ BE = DE …by cpct

(iii) ΔABC ≅ ΔADC from ii

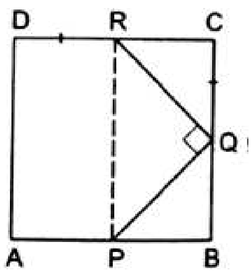
∴ ∠ABC = ∠ADC …by cpct

7. Question

In the given figure, ABCD is a square and $\angle PQR = 90^\circ$. If PB = QC = DR, prove that

(i) $QB = RC$, (ii) $PQ = QR$,

(iii) $\angle QPR = 45^\circ$.



Answer

Given: ABCD is where $\angle PQR = 90^\circ$. and $PB = QC = DR$,

To prove: (i) $QB = RC$, (ii) $PQ = QR$,

(iii) $\angle QPR = 45^\circ$.

Proof:

(i) Here,

BC = CD …Sides of square

CQ = DR …Given

BC = BQ + CQ

∴ CQ = BC − BQ

∴ DR = BC – BQ ...(1)

Also,

CD = RC+ DR

∴ DR = CD − RC = BC − RC ...(2)

From (1) and (2), we have,

BC − BQ = BC − RC

∴ BQ = RC

(ii) Now in ΔRCQ and ΔQBP, we have,

PB = QC …Given

BQ = RC …from (i)

∠RCQ = ∠QBP …90° each

Hence by SAS congruence rule,

ΔRCQ ≅ ΔQBP

∴ QR = PQ …by cpct

(iii) ΔRCQ ≅ ΔQBP and QR = PQ … from (ii)

∴ In ΔRPQ,

∠QPR = ∠QRP = $\frac{1}{2}$ (180° − 90°) = $\frac{90^\circ}{2}$ = 45°

∴ ∠QPR = 45°

8. Question

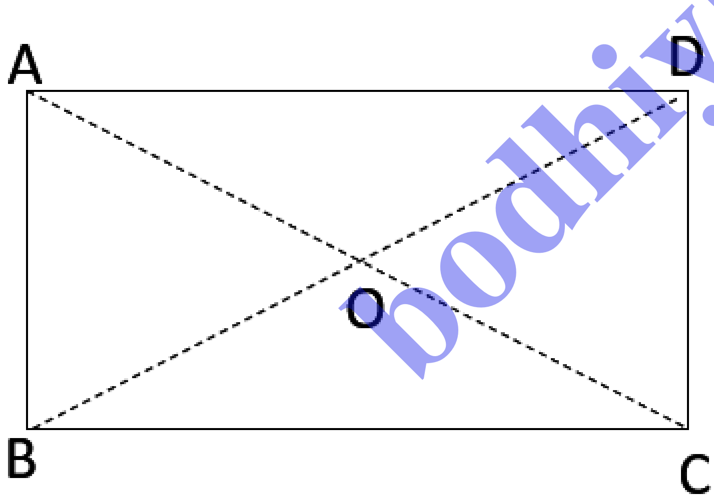
If is a point within a quadrilateral $ABCD$, show that $OA + OB + OC + OD > AC + BD$.

Answer

Given: In *ABCD*, *O* is any point within the quadrilateral.

To prove: $OA + OB + OC + OD > AC + BD$.

Proof:



We know that the sum of any two sides of a triangle is greater than the third side.So, in Δ*AOC*,

OA + *OC* > *AC* …(1)

Also, in Δ *BOD*,

OB + *OD* > *BD* …(2)

Adding 1 and 2, we get,

(*OA* + *OC*) + (*OB* + *OD*) > (*AC* + *BD*)

∴ *OA* + *OB* + *OC* + *OD* > *AC* + *BD*

Hence proved.

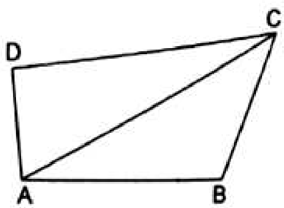
9. Question

In the adjoining figure, $ABCD$ is a quadrilateral and AC is one of its diagonals. Prove that:

(i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD > DA$

(iii) $AB + BC + CD + DA > AC + BD$



Answer

Given: In ABCD, AC is one of diagonals.

To prove:

(i) $AB + BC + CD + DA > 2AC$

(ii) $AB + BC + CD > DA$

(iii) $AB + BC + CD + DA > AC + BD$

Proof:

(i) We know that the sum of any two sides of a triangle is greater than the third side.In ΔABC,

$AB + BC > AC$ ...(1)

In ΔACD,

$CD + DA > AC$ ...(2)

Adding (1) and (2), we get,

$AB + BC + CD + DA > 2AC$

(ii) In ΔABC, we have,

$AB + BC >$ AC ...(1)

We also know that the length of each side of a triangle is greater than the positive difference of the length of the other two sides.

In ΔACD, we have:

AC $> DA - CD$ ...(2)

From (1) and (2), we have,

$AB + BC > DA - CD$

∴ $AB + BC + CD > DA$

(ii) In ΔABC,

$AB + BC > AC$ ...(1)

In ΔACD,

$CD + DA > AC$ ...(2)

In ΔBCD,

$BC + CD > BD$ ...(3)

In ΔABD,

$DA + AB > BD$ ...(4)

Adding 1, 2, 3 and 4, we get,

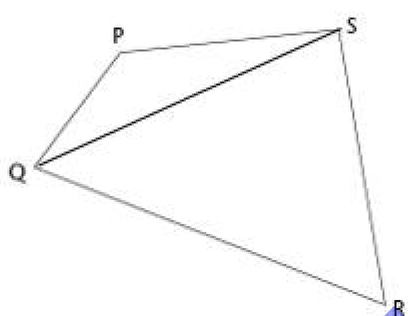
$2(AB + BC + CD + DA) > 2(AC + BD)$

∴ $AB + BC + CD + DA > AC + BD$

10. Question

Prove that the sum of all the angles of a quadrilateral is 360°.

Answer



Given: Consider a PQRS where QS is diagonal.

To prove: $\angle P + \angle Q + \angle R + \angle S = 360°$

Proof:

For ΔPQS, we have,

$\angle P + \angle PQS + \angle PSQ = 180°$ ... (1) ...Using Angle sum property of Triangle

Similarly, in ΔQRS, we have,

∴ $\angle SQR + \angle R + \angle QSR = 180°$ ... (2) ...Using Angle sum property of Triangle

On adding (1) and (2), we get

$\angle P + \angle PQS + \angle PSQ + \angle SQR + \angle R + \angle QSR = 180° + 180°$

∴ $\angle P + \angle PQS + \angle SQR + \angle R + \angle QSR + \angle PSQ = 360°$

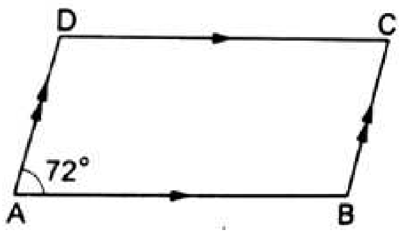
∴ ∠P + ∠Q + ∠R + ∠S = 360°

∴ The sum of all the angles of a quadrilateral is 360°.

Exercise 9B

1. Question

In the adjoining figure, $ABCD$ is a parallelogram in which $\angle A = 72^\circ$. Calculate $\angle B, \angle C$ and $\angle D$.



Answer

In ABCD, $\angle A$ = 72°

We know that opposite angles of a parallelogram are equal.

Hence, $\angle A = \angle C$ and $\angle B = \angle D$

∴ ∠C = 72°

$\angle A$ and ∠B are adjacent angles.

∴ $\angle A$ + ∠B = 180°

∠B = 180°° − $\angle A$

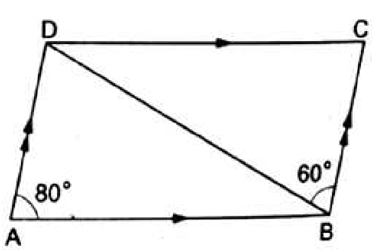
∠B = 180° − 72° = 108°

∴ ∠B = $\angle D$ = 108°

Hence, ∠B = $\angle D$ = 108° and ∠C = 72°

2. Question

In the adjoining figure, $ABCD$ is a parallelogram in which $\angle DAB = 80^\circ$ and $\angle DBC = 60^\circ$. Calculate $\angle CDB$ and $\angle ADB$.



Answer

It is given that ABCD is parallelogram and $\angle DAB$ = 80° and $\angle DBC$ = 60°

We need to find measure of $\angle CDB$ and $\angle ADB$

In *ABCD, AD || BC, BD as transversal,*

$\angle DBC = \angle ADB = 60°$ …Alternate interior angles ...(i)

As $\angle DAB$ and $\angle ADC$ are adjacent angles,

$\angle DAB + \angle ADC = 180°$

$\therefore \angle ADC = 180°° - \angle DAB$

$\angle ADC = 180° - 80° = 100°$

Also,

$\angle ADC = \angle ADB + \angle CDB$

$\therefore \angle ADC = 100°$

$\angle ADB + \angle CDB = 100°$ ...(ii)

From (i) and (ii), we get:

$60° + \angle CDB = 100°$

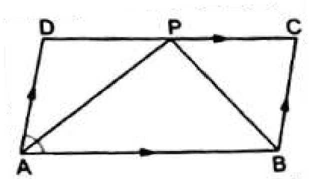
$\Rightarrow \angle CDB = 100° - 60° = 40°$

Hence, $\angle CDB = 40°$ and $\angle ADB = 60°$

3. Question

In the adjoining figure, $ABCD$ is a parallelogram in which $\angle A = 60°$. If the bisectors of $\angle A$ and $\angle B$ meet DC at P, prove that

(i) $\angle APB = 90°$, (ii) $AD = DP$ and $PB = PC = BC$, (iii) $DC = 2AD$.



Answer

Given: ABCD is a parallelogram. The bisectors of $\angle A$ and $\angle B$ meet DC at P,.

To prove: (i) $\angle APB = 90°$, (ii) $AD = DP$ and $PB = PC = BC$, (iii) $DC = 2AD$.

Proof:

$\therefore \angle A = \angle C$ and $\angle B = \angle D$ … Opposite angles

And $\angle A + \angle B = 180°$ … Adjacent angles

$\therefore \angle B = 180° - \angle A$

$180° - 60° = 120°$ … as $\angle A = 60°$

$\therefore \angle A = \angle C = 60°$ and $\angle B = \angle D = 120°$

(i) In Δ *APB,*

$\angle PAB = \frac{60°}{2} = 30°$and $\angle PBA = \frac{120°}{2} = 60°$

$\therefore \angle APB = 180° - (30° + 60°) = 90°$

(ii) In Δ *ADP,* $\angle PAD = 30°$ and $\angle ADP = 120°$

$\therefore \angle APB = 180° - (30° + 120°) = 30°$

Hence,

$\angle PAD = \angle APB = 30°$

Hence, ΔADP is an isosceles triangle and $AD = DP$.In Δ *PBC,*

$\angle PBC = 60°$

$\angle BPC = 180° - (90° + 30°) = 60°and\angle BCP = 60°$ …Opposite angle of $\angle A$

$\therefore \angle PBC = \angle BPC = \angle BCP$

Hence, ΔPBC is an equilateral triangle and, therefore, $PB = PC = BC$.

(iii) $DC = DP + PC$

From (ii), we have

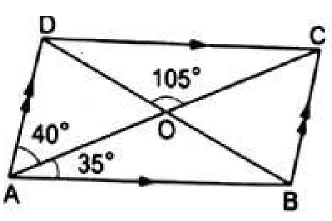
$DC = AD + BC$ …$AD = BC$ $DC = AD + AD$

$DC = 2\,AD$

4. Question

In the adjoining figure, $ABCD$ is a parallelogram in which $\angle BAO = 35°$, $\angle DAO = 40°$ and $\angle COD = 105°$.

Calculate (i) $\angle ABO$, (ii) $\angle ODC$, (iii) $\angle ODC$, (iv) $\angle CBD$.



Answer

In ABCD, $\angle BAO = 35°$, $\angle DAO = 40°$ and $\angle COD = 105°$.

(i) In ΔAOB,

$\angle BAO = 35°$

$\angle AOB = \angle COD = 105°$ …Vertically opposite angels

$\therefore \angle ABO = 180° - (35° + 105°) = 40°$ … Using Angle sum property of Triangle

(ii) $\angle ODC$ and $\angle ABO$ are alternate angles for transversal BD

$\therefore \angle ODC = \angle ABO = 40°$

(iii) $\angle ACB = \angle CAD = 40°°$ ...Alternate angles for transversal AC

(iv) $\angle CBD = \angle ABC - \angle ABD \ldots(1)$

$\angle ABC = 180° - \angle BAD$ ...Adjacent angles are supplementary

$\angle ABC = 180° - 75° = 105°$

$\angle CBD = 105° - \angle ABD \ldots$ *as* $\angle ABD = \angle ABO$

$\angle CBD = 105° - 40° = 65°$

5. Question

In a||gm $ABCD$, if $\angle A = (2x + 25)^\circ$ and $\angle B = (3x - 5)^\circ$, find the value of x and the measure of each angle of the parallelogram.

Answer

It is given that in ABCD, $\angle A = (2x + 25)^\circ$ and $\angle B = (3x - 5)^\circ$,

We know that opposite angles of parallelogram are equal.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

Also,

$\angle A + \angle B = 180°$ ...Adjacent angles of parallelogram are supplementary

$\therefore (2x + 25)° + (3x - 5)° = 180°$

$5x° + 20° = 180°$

$5x° = 160°$

$x° = 32°$

$\therefore \angle A = 2 \times 32 + 25 = 89°$

$\therefore \angle B = 3 \times 32 - 5 = 91°$

Hence, $x = 32°$, $\angle A = \angle C = 89°$ and $\angle B = \angle D = 91°$

6. Question

If an angle of a parallelogram is four-fifths of its adjacent angle, find the angles of the parallelogram.

Answer

Let ABCD be the parallelogram.

We know that opposite angles of parallelogram are equal.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$By given conditions, Let $\angle A = x°$ and $\angle B = \frac{4x°}{5}$

Also, adjacent angles of parallelogram are supplementary,

$\therefore x° + \frac{4x°}{5} = 180°$

$\frac{9x°}{5} = 180°$

$\therefore x = 100°$

Hence, $\angle A = 100°$ and $\angle B = \frac{4 \times 100°}{5} = 80°$

Hence, $\angle A = \angle C = 100°$; $\angle B = \angle D = 80°$

7. Question

Find the measure of each angle of a parallelogram, if one of its angles is 30° less than twice the smallest angle.

Answer

Let ABCD be the parallelogram.

We know that opposite angles of parallelogram are equal.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

Let $\angle A$ be the smallest angle whose measure is $x°$.

$\therefore \angle B = (2x - 30)°$

We know that adjacent angles of parallelogram are supplementary,

$\angle A + \angle B = 180°$

$x + 2x - 30° = 180°$

$3x = 210°$

$x = 70°$

$\therefore \angle B = 2 \times 70° - 30° = 110°$

Hence, $\angle A = \angle C = 70°$ and $\angle B = \angle D = 110°$

8. Question

$ABCD$ is a parallelogram in which $AB = 9.5$ cm and its perimeter is 30 cm. Find the length of each side of the parallelogram.

Answer

Here ABCD is parallelogram.

We know that the opposite sides of a parallelogram are parallel and equal.

Hence, $AB = DC = 9.5$ cm

Also let $BC = AD = x\ cm$

Now,

Perimeter of $ABCD = 30$ cm …(given)

$\therefore AB + BC + CD + DA = 30$ cm

$\therefore 9.5 + x + 9.5 + x = 30$

$\therefore 19 + 2x = 30$

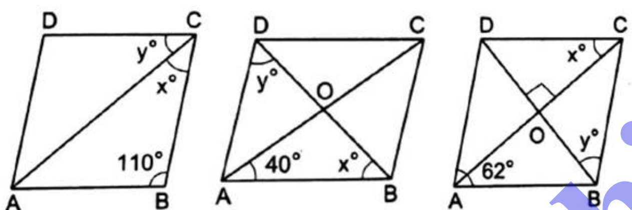
$\therefore 2x = 11$

$\therefore x = 5.5$ cm

Hence, length of each side is AB = DC = 9.5 cm and $BC = DA = 5.5$ cm

9. Question

In each of the figures given below, $ABCD$ is a rhombus. Find the value of x and y in each case.



(i) (ii) (iii)

Answer

(i) ABCD is a rhombus.

We know that rhombus is type of parallelogram whose all sides are equal.

In ΔABC, $\angle BAC = \angle BCA = \frac{1}{2}(180° - 110°) = 35°$

Hence x = 35°

But AB || DC …opposite sides of rhombus are parallel

$\angle BAC$ = ∠DCA …for transversal AC

$\therefore \angle BAC$ = ∠DCA = 35°

Hence, x = y = 35°

(ii) ABCD is a rhombus.

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

∴ in ΔAOB,

$\angle OAB = 40°$, $\angle AOB = 90°$

∴ $\angle ABO = 180° - (40° + 90°) = 50°$

Hence x = 50°

Now in ΔDAB,

AB = AD … as rhombus has all sides equal.

ie. Δ*AOB is isosceles triangle.*

Also base angles of *isosceles triangle are equal.*

Hence, $x = y = 50°$

(iii) ABCD is a rhombus.

We know that rhombus is type of parallelogram whose all sides are equal.

So in ΔDCB,

DC = BC

∴ $\angle CDB = \angle CBD = y°$ base angles of *isosceles triangle are equal.*

Now, x = $\angle CAB$ …alternate angles with transversal AC

∴ $x = \frac{1}{2} \angle BAD$

∴ $x = \frac{1}{2} \times 62°$

∴ $x = 31°$

In ΔDOC,

We know sum of angles of triangle is 180°

∠CDO + ∠DOC + ∠OCD = 180°

∴ ∠CDO + 90° + 31° = 180°

∴ ∠CDO = 59°

∴ y = 59°

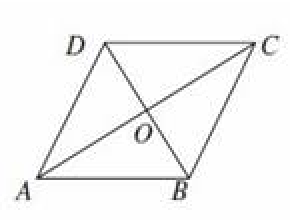
Hence, $x = 31°$ *and* y = 59°

10. Question

The lengths of the diagonals of a rhombus are 24 cm and 18 cm respectively. Find the length of each side of the rhombus.

Answer

Let ABCD be rhombus.



Here, AC and BD are the diagonals of $ABCD$, where AC = 24 cm and BD = 18 cm.

Let the diagonals intersect each other at O.

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

∴ ΔAOB is a right angle triangle in which $OA = \frac{24}{2} = 12$ cm and $OB = \frac{18}{2} = 9$ cm.

Now, $AB^2 = OA^2 + OB^2$ ...Pythagoras theorem

∴ $AB^2 = (12)^2 + (9)^2$

∴ $AB^2 = 144 + 81 = 225$

∴ $AB = 15$ cm

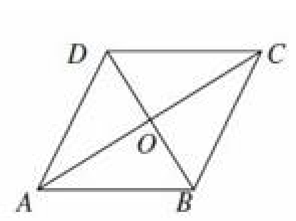
Hence, the side of the rhombus is 15 cm

11. Question

Each side of a rhombus is 10 cm long and one of its diagonals measures 16 cm. Find the length of the other diagonal and hence find the area of the rhombus.

Answer

Let ABCD be rhombus.



We know that rhombus is type of parallelogram whose all sides are equal.

∴ $AB = BC = CD = DA = 10$ cm

Let the diagonals AC and BD intersect each other at O, where AC = 16 cm and let BD = x

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

∴ ΔAOB is a right angle triangle, in which $OB = BD \div 2 = x \div 2$ and $OA = AC \div 2 = 16 \div 2 = 8$ cm.

Now, $AB = OA^2 + OB^2$...by pythagoras theorem∴ $10^2 = (\frac{x}{2})^2 + 8^2$

ie. $100 - 64 = \frac{x^2}{4}$

$36 \times 4 = x^2$

∴ $x^2 = 144$

∴ $x = 12$ cm

Hence, the length of the other diagonal is 12 cm

We know that area of rhombus is,

Area of rhombus = $\frac{1}{2}$ × (Diagonal1) × (Diagonal2)

Hence,

Area of ABCD = $\frac{1}{2}$ × AC × BD

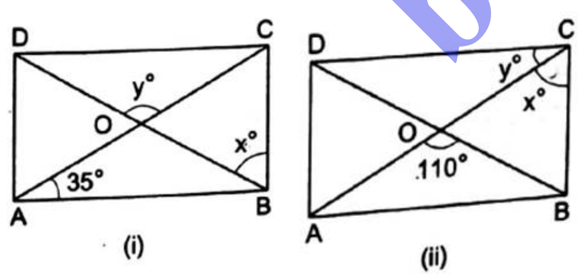
= $\frac{1}{2} \times 16 \times 12$

= 96 cm^2

Hence, the area of rhombus is 96 cm^2

12. Question

In each of the figures given below, $ABCD$ is a rectangle. Find the values of x and y in each case.



Answer

(i) Here, ABCD is rectangle.

We know that the diagonals of a rectangle are congruent and bisect each other.

∴ In Δ AOB, we have $OA = OB$

This means that Δ AOB is isosceles triangle.

We know that base angles of isosceles triangle are equal.

$\therefore \angle OAB = \angle OBA = 35°$

$\therefore \therefore x = 90° - 35° = 55°$

Also, $\angle AOB = 180° - (35° + 35°) = 110°$

$\therefore y = \angle AOB = 110°$ …Vertically opposite angles

Hence, $x = 55°$ and $y = 110°$

(ii) Here, ABCD is rectangle.

We know that the diagonals of a rectangle are congruent and bisect each other.

$\therefore$ In Δ AOB, we have $OA = OB$

This means that Δ AOB is isosceles triangle.

We know that base angles of isosceles triangle are equal.

$\therefore \angle OAB = \angle OBA = \frac{1}{2} \times (180° - 110°) = 35°$

$\therefore y = \angle BAC = 35°$ … alternate angles with transversal AC

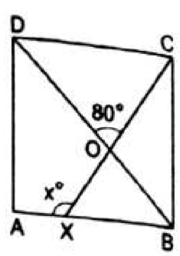
Also, $x = 90° - y$ … $\because \angle C = 90° = x + y$

$\therefore x = 90° - 35° = 55°$

Hence, $x = 55°$ and $y = 35°$

13. Question

In the adjoining figures, $ABCD$ is a square. A line segment CX cuts AB at X and the diagonal BD at O such that $\angle COD = 80°$ and $\angle OXA = x°$. Find the value of x.



Answer

Here, ABCD is square.

Here AC and BD are diagonals.

We know that the angles of a square are bisected by the diagonals.

$\therefore \angle OBX = 45°$ $\because \angle ABC = 90°$ and BD bisects $\angle ABC$

And $\angle BOX = \angle COD = 80°$ … Vertically opposite angles

$\therefore$ In ΔBOX, we have:

$\angle AXO = \angle OBX + \angle BOX$ … Exterior angle theorem

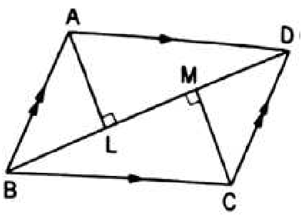
⇒ ∠AXO = 45° + 80° = 125°

∴ x =125°

14. Question

In the adjoining figures, AL and CM are perpendiculars to the diagonal BD of a ||gm $ABCD$. Prove that

(i) $\Delta ALD \cong \Delta CMB$, (ii) $AL = CM$.



Answer

Here, ABCD is parallelogram.

Hence, AD || BC and AD = BC

(i) In ΔALD and ΔCMB, we have,AD = BC

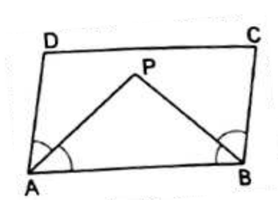
∠ALD = ∠CMB (90^0 each)

∠ADL = ∠CBM (Alternate interior angle)∴ ΔALD ≅ ΔCMB

(ii) As ΔALD ≅ ΔCMB ...from 1∴ AL = CM ...by cpct

15. Question

In the adjoining figures, $ABCD$ is a parallelogram in which the bisectors of $\angle A$ and $\angle B$ intersect at a point P. Prove that $\angle APB = 90°$.



Answer

ABCD is parallelogram.

We know that the sum of the adjacent angles in parallelogram is 180°

∴ ∠ A + ∠B = 180°

∴ $\frac{\angle A}{2} + \frac{\angle B}{2} = \frac{180°}{2}$ = 90°

In Δ APB, we have:

∠PAB = ∠A /2

∠*PBA* = ∠*B* /2

∴ ∠APB = 180 − (∠*PAB* + ∠*PBA*) ...Angle sum property of triangle

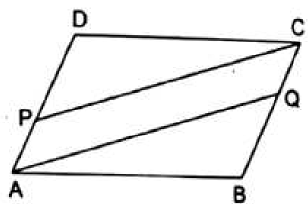
∴ ∠*APB* = 180 − ($\frac{\angle A}{2}$ + $\frac{\angle B}{2}$)

∴ ∠*APB* = 180 − 90 = 90°

Hence, proved.

16. Question

In the adjoining figures, $ABCD$ is a parallelogram. If P and Q are points on AD and BC respectively such that $AP = \frac{1}{3}AD$ and $CQ = \frac{1}{3}BC$, prove that $AQCP$ is a parallelogram.



Answer

ABCD is parallelogram

We know that opposite sides and angles of parallelogram *are equal.*

∴ ∠*B* = ∠*D and AD* = *BC* and *AB* = *DC*

Also, AD || *BC* and *AB*|| *DC*

It is given that $AP = \frac{1}{3}AD$ and $CQ = \frac{1}{3}BC$,

Hence, *AP* = *CQ* ... ∵ *AD* = *BC*

In Δ*DPC* and Δ*BQA*, we have,

AB = *CD*

∠*B* = ∠*D*

DP = *QB* ... *as* $AP = \frac{1}{3}AD$ and $CQ = \frac{1}{3}BC$,

Hence, by SAS test for congruency,

Δ*DPC* ≅ Δ*BQA*

∴ *PC* = *QA* ... *by cpct*

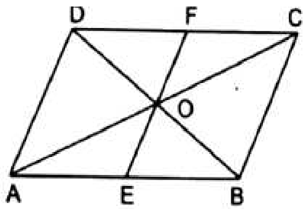
Hence, from above, in AQCP, we have,

AP = CQ and PC = QA

∴ AQCP is a parallelogram.

17. Question

In the adjoining figures, $ABCD$ is a parallelogram whose diagonals intersect each other at O. A line segment EOF is drawn to meet AB at E and DC at F. Prove that $OE = OF$.



Answer

ABCD is parallelogram.

∴ in ΔODF and ΔOBE, we have:

$OD = OB$ ... Diagonals bisects each other

$\angle DOF = \angle BOE$ ... Vertically opposite angles

$\angle FDO = \angle OBE$ ... Alternate interior angles

Hence, by SAA *test for congruency,*

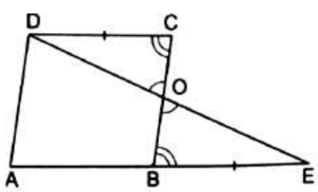
$\Delta ODF \cong \Delta OBE$

∴ $OF = OE$ ...by cpct

Hence, proved.

18. Question

In the adjoining figures, $ABCD$ is a parallelogram in which AB is produced to E so that $BE = AB$. Prove that ED bisects BC.



Answer

ABCD is parallelogram.

In ΔODC and ΔOEB, we have,

$DC = BE$ ...as $DC = AB$

$\angle COD = \angle BOE$ ... Vertically opposite angles are equal

$\angle OCD = \angle OBE$ ... Alternate angles with transversal BC

Hence, by SAA *test for congruency, we get,*

$\Delta ODC \cong \Delta OEB$

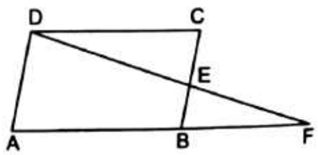
∴ $OC = OB$ *...by cpct*

We know that $BC = OC + OB$.

∴ ED bisects BC.

19. Question

In the adjoining figures, $ABCD$ is a parallelogram and E is the midpoint of side BC. If DE and AB when produced meet at F, prove that $AF = 2AB$.



Answer

ABCD is parallelogram.

Also given that BE = CE

In ABCD, AB || DC

∠DCE = ∠EBF … Alternate angles with transversal DF

In ΔDCE and ΔBFE, we have,

∠DCE = ∠EBF …from above

∠DEC = ∠BEF … Vertically opposite angles

Also, BE = CE … givenHence, by ASA congruence rule,

ΔDCE ≅ ΔBFE

∴ DC = BF … by cpct

But DC = AB, as ABCD is a parallelogram.

∴ DC = AB = BF

Now, AF = AB + BF

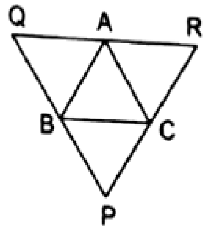
From above, we get,

AF = AB + AB = 2AB

Hence, proved.

20. Question

A ΔABC is given. If lines are drawn through $A, B, C,$ parallel respectively to the sides BC, CA and AB, forming $\Delta PQR,$ as shown in the adjoining figure, show that $BC = \frac{1}{2} QR$.



Answer

Here given that *BC* || *QA* and *CA* || *QB which means that BCQA* is a parallelogram.

∴ BC = QA ...(1)

Similarly, *BC* || *AR* and *AB* || *CR, which means BCRA* is a parallelogram.

∴ *BC = AR ...(2)*

But *QR = QA + AR*

From (1) and (2), we get,

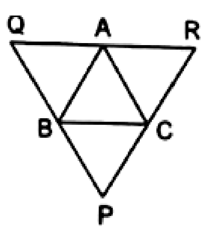
QR = BC + BC

∴ *QR = 2BC*

Hence, BC = $\frac{1}{2}$ QR

21. Question

In the adjoining figure, ΔABC is a triangle and through A, B, C lines are drawn, parallel respectively to BC, CA and $AB,$ intersecting at P, Q and $R.$ Prove that the perimeter of ΔPQR is double the perimeter of ΔABC.



Answer

Here, Perimeter of Δ*ABC = AB + BC + CA*

And Perimeter of Δ*PQR = PQ + QR + PR*

Given that *BC* || *QA* and *CA* || *QB which means BCQA* is a parallelogram.

∴ BC = QA ...(1)

Similarly, *BC* || *AR* and *AB* || *CR*, which means *BCRA* is a parallelogram.

∴ *BC = AR ...(2)*

But, QR = QA + AR

From 1 and 2,

$QR = BC + BC$

$\therefore QR = 2BC$

$\therefore$ BC = $\frac{1}{2}$ QR

Similarly, CA = $\frac{1}{2}$ PQ and AB = $\frac{1}{2}$ PR

Now,

Perimeter of $\Delta ABC = AB + BC + CA$

= $\frac{1}{2}$ QR + $\frac{1}{2}$ PQ + $\frac{1}{2}$ PR

= $\frac{1}{2}$ (PR + QR + PQ)

This states that,

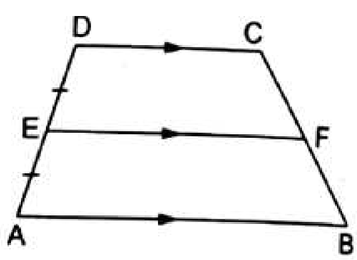
Perimeter of ΔABC = $\frac{1}{2}$ (Perimeter of ΔPQR)

$\therefore$ Perimeter of ΔPQR = 2 × Perimeter of ΔABC

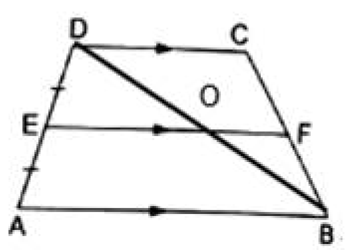
Exercise 9C

1. Question

In the adjoining figure, $ABCD$ is a trapezium in which $AB \parallel DC$ and E is the midpoint of AD. A line segment $EF \parallel AB$ meets BC at F. Show that F is the midpoint of BC.



Answer



Here, ABCD is trapezium.

Join BD to cut EF at O.

It is given that, in ΔDAB, E is the mid point of AD and $EO \parallel AB$.

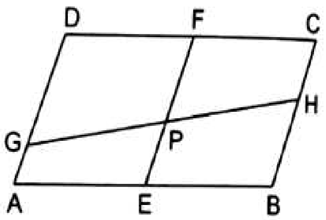
$\therefore$ O is the midpoint of BD …By converse of mid point theorem

Now in ΔBDC, O is the mid point of BD and OF || DC.

∴ F is the midpoint of BC … By converse of mid point theorem

2. Question

In the adjoining figure, ABCD is a || gm in which E and F are the midpoints of AB and CD respectively. If GH is a line segment that cuts AD, EF and BC at G, P and H respectively, prove that GP = PH.



Answer

Here, ABCD is parallelogram.

By the properties of parallelogram,

AD || *BC* and *AB* || *DC*

AD = *BC* and *AB* = *DC*

Also,

AB = *AE* + *BE* and *DC* = *DF* + *FC*

This means that,

AE = *BE* = *DF* = *FC*

Now, *DF* = *AE* and DF || AE, that is *AEFD* is a parallelogram.

Hence, AD || *EF*

Similarly, BEFC is also a parallelogram.

Hence, *EF* || *BC*

∴ *AD* || *EF* || *BC*

Thus, *AD, EF* and *BC* are three parallel lines cut by the transversal line *DC* at *D*, *F* and *C,* respectively such that *DF* = FC.

Also, the lines *AD, EF* and *BC* are also cut by the transversal *AB* at *A, E* and *B*, respectively such that *AE* = *BE*.

Similarly, they are also cut by *GH*.

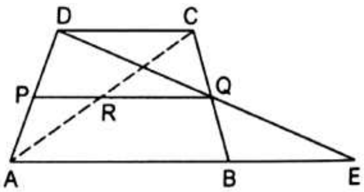
Hence by intercept theorem,

∴ GP = PH

Hence proved.

3. Question

In the adjoining figure, $ABCD$ is a trapezium in which $AB \| DC$ and P, Q are the midpoints of AD and BC respectively. DQ and AB when produced meet at E Also, AC and PQ intersect at R. Prove that (i) $DQ = QE$, (ii) $PR \| AB$, (iii) $AR = RC$.



Answer

Here, ABCD is trapezium.

Hence, *AB* || *DC*

Also given that AP = *PD* and *BQ* = *CQ*

(i) In Δ*QCD* and Δ*QBE*, we have,

∠*DQC* = ∠*BQE* ...Vertically opposite angles

∠*DCQ* = ∠*EBQ* ...Alternate angles with transversal BC

BQ = *CQ* ... *P* is the midpoint

Hence, by AAS test of congruency,

Δ*QCD* ≅ Δ*QBE*

Hence, *DQ* = *QE* ...by cpct

(ii) Also, in Δ*ADE*, *P* and *Q* are the midpoints of *AD* and *DE* respectively

∴ *PQ* || *AE*

Hence, PQ || *AB* || *DC*

ie. *AB* || *PR* || *DC*

(iii) *PQ, AB* and *DC* are cut by transversal *AD* at *P* such that *AP* = *PD*.

Also they are cut by transversal *BC* at *Q* such that *BQ* = *QC*.

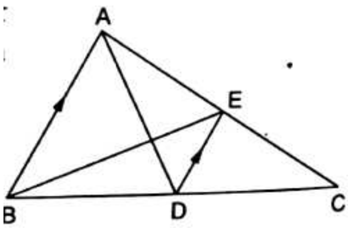
Similarly, lines *PQ, AB* and *DC* are also cut by *AC* at *R*.

Hence, by intercept theorem,

∴ *AR* = *RC*

4. Question

In the adjoining figure, AD is a median of ΔABC and $DE \| BA$. Show that BE is also a median of ΔABC.



Answer

In ΔABC, *AD is median.*

$\therefore BD = DC$

We know that the line drawn through the midpoint of one side of a triangle and parallel to another side bisects the third side.

So, in ΔABC, *D* is the mid point of *BC* and $DE \parallel BA$.

Hence, DE bisects *AC.*

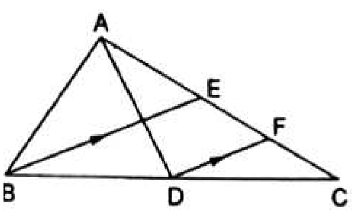
$\therefore AE = EC$

This means that E is the midpoint of AC.

$\therefore$ *BE is median of* ΔABC.

5. Question

In the adjoining figure, AD and BE are the medians of ΔABC and $DF \parallel BE$. Show that $CF = \frac{1}{4}AC$.



Answer

Here in ΔABC AD and BE are medians.

Hence, in ΔABC, we have: $AC = AE + EC$

But $AE = EC$ … *as E is midpoint of AC*

$\therefore AC = 2EC$ …(1)

Now in ΔBEC,

$DF \parallel BE$

Also, $EF = CF$ … *by* midpoint theorem, as *D* is the midpoint of *BC*

But,

$EC = EF + CF$

$\therefore EC = 2\ CF$ …(2)

From 1 and 2, we get,

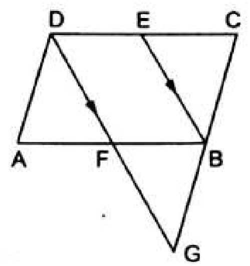
AC = 4 CF

$\therefore CF = \frac{1}{4} AC.$

6. Question

In the adjoining figure, $ABCD$ is a parallelogram. E is the midpoint of DC and through D, a line segment is drawn parallel to EB to meet CB produced at G and it cuts AB at F. Prove that

(i) $AD = \frac{1}{2} GC$, (ii) $DG = 2EB$.



Answer

ABCD is parallelogram.

(i) In Δ *DCG,* we have:

DG || *EB*

DE = *EC* … *E* is the midpoint of *DC*)

Also, *GB* = *BC* … by midpoint theorem

∴ *B* is the midpoint of *GC*.

Also, *GC* = *GB* + *BC*

GC = 2*BC*

GC = 2 *AD* …*as AD* = *BC*

$\therefore AD = \frac{1}{2} GC$

(ii) Now, in Δ *DCG, DG* || *EB* and *E* is the midpoint of *DC* and *B* is the midpoint of *GC*.

$\therefore EB = \frac{1}{2} DG$ … by midpoint theorem

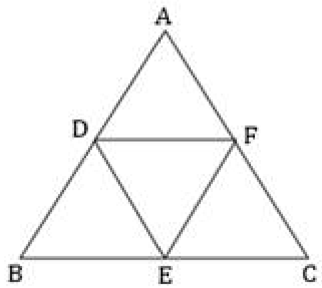
∴ DG = 2 EB

7. Question

Prove that the line segments joining the middle points of the sides of a triangle divide it into four congruent triangles.

Answer

Let triangle be ΔABC. *D, E* and *F* are the midpoints of sides *AB, BC* and *CA*, respectively.



By midpoint theorem, for D and E as midpoints of sides AB and BC,

DE // AC

Similarly, DF // BC and *EF // AB*.

∴ *ADEF, BDFE* and *DFCE* are all parallelograms.

But, DE is the diagonal of the *BDFE*.

∴ $\Delta BDE \cong \Delta FED$ *…(1)*

Similarly, DF is the diagonal of the parallelogram ADEF.

∴ $\Delta DAF \cong \Delta FED$ *…(2)*

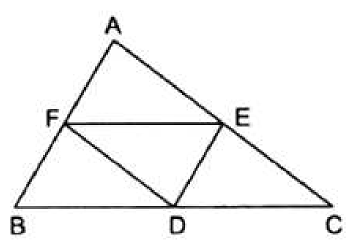
And, EF is the diagonal of the parallelogram DFCE.

∴ $\Delta EFC \cong \Delta FED$ …(3)

Hence, all the four triangles are congruent.

8. Question

In the adjoining figure, D, E, F are the midpoints of the sides BC, CA and AB respectively, of ΔABC. Show that $\angle EDF = \angle A, \angle DEF = \angle B$ and $\angle DFE = \angle C$.



Answer

Here, in $\Delta ABC.$, D, E, F are the midpoints of the sides BC, CA and AB respectively.

By mid point theorem, as *F* and *E* are the mid points of sides *AB* and *AC,*

FE || *BC*

Similarly, DE || *FB* and *FD* || *AC*.

Therefore, *AFDE, BDEF* and *DCEF* are all parallelograms.

We know that opposite angles in parallelogram are equal.

∴ In AFDE, we have,

∠*A* = ∠EDF

In *BDEF*, we have,

∠*B* = ∠*DEF*

In DCEF, we have,

∠ C = ∠ DFE

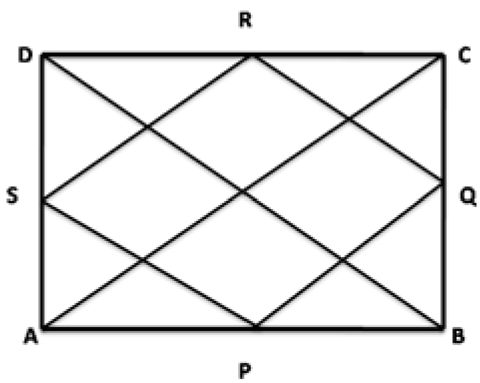
Hence proved.

9. Question

Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rectangle is a rhombus.

Answer

Let *ABCD* be the rectangle and *P, Q, R* and *S* be the midpoints of *AB, BC, CD* and *DA*, respectively.



Join diagonals of the rectangle.

In Δ *ABC,* we have, by midpoint theorem,∴ *PQ* || *AC* and *PQ* = $\frac{1}{2}$ *AC*

Similarly, SR || *AC* and *SR* =$\frac{1}{2}$ *AC.*

As, PQ || *AC* and *SR* || *AC, then also PQ* || *SR*

Also, PQ = SR, each equal to $\frac{1}{2}$ *AC …(1)*

So, PQRS is a parallelogram

Now, in Δ*SAP* and Δ*QBP*, we have,

AS = *BQ* ∠*A* = ∠*B* = 90°*AP* = *BP*

∴ *By SAS test of congruency,*

Δ*SAP* ≅ ΔQBP

Hence, PS = PQ …by cpct …(2)

Similarly, $\Delta SDR \cong \Delta QCR$

∴ SR = RQ … by cpct …(3)

Hence, from 1, 2 and 3 we have,

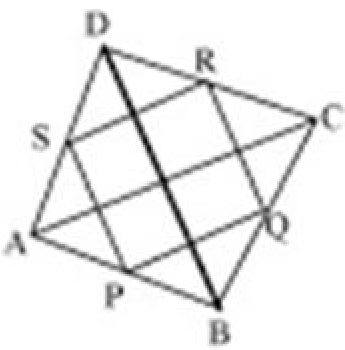
*PQ = PQ = SR = RQ*Hence, *PQRS* is a rhombus.

Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rectangle is a rhombus.

10. Question

Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rhombus is a rectangle.

Answer



In ΔABC, P and Q are mid points of AB and BC respectively.

∴ PQ|| AC and PQ = 1/2AC … (1) …Mid point theorem

Similarly in ΔACD, R and S are mid points of sides CD and AD respectively.

∴ SR||AC and SR = 1/2AC …(2) …Mid point theorem

From (1) and (2), we get

PQ||SR and PQ = SR

Hence, PQRS is parallelogram (pair of opposite sides is parallel and equal)

Now, RS || AC and QR || BD.

Also, AC ⊥ BD … as diagonals of rhombus are perpendicular bisectors of each other.

∴RS ⊥ QR.

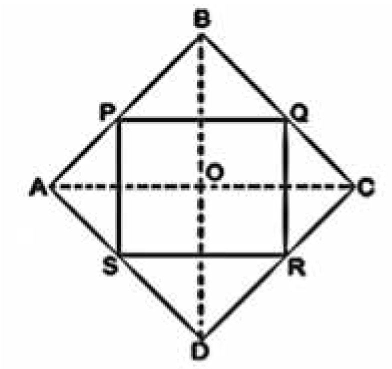
Thus, PQRS is a rectangle.

Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a rhombus is a rectangle.

11. Question

Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a square is a square.

Answer



Let *ABCD* be the square and *P, Q, R* and *S* be the midpoints of *AB, BC, CD* and *DA*, respectively.

Join diagonals of the square.

In Δ *ABC,* we have, by midpoint theorem,

∴ $PQ \parallel AC$ and $PQ = \frac{1}{2} AC$

Similarly, $SR \parallel AC$ and $SR = \frac{1}{2} AC$.

As, $PQ \parallel AC$ and $SR \parallel AC$, *then also* $PQ \parallel SR$

Also, $PQ = SR$, *each equal to* $\frac{1}{2} AC$ *…(1)*

So, PQRS is a parallelogram

Now, in Δ*SAP* and Δ*QBP*, we have,

$AS = BQ$

$\angle A = \angle B = 90°$

$AP = BP$

∴ *By SAS test of congruency,*

$\Delta SAP \cong \Delta QBP$

Hence, $PS = PQ$ *…by cpct …(2)*

Similarly, $\Delta SDR \cong \Delta QCR$

∴ $SR = RQ$ *… by cpct …(3)*

Hence, from 1, 2 and 3 we have,

$PQ = PQ = SR = RQ$

We know that the diagonals of a square bisect each other at right angles.

∴ $\angle EOF = 90^o$

Now, $RQ \parallel DB$

⇒$RE \parallel FO$

Also, $SR \parallel AC$

⇒$FR \parallel OE$

∴ *OERF* is a parallelogram.

So, $\angle FRE = \angle EOF = 90^{0}$ (Opposite angles are equal)

Thus, *PQRS* is a parallelogram with $\angle R = 90^{0}$ and $PQ = PS = SR = RQ$.

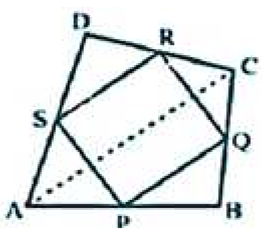
This means that PQRS is square.

Hence, the quadrilateral formed by joining the midpoints of the pairs of adjacent sides of a square is a square.

12. Question

Prove that the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other.

Answer



In ΔADC, S and R are the midpoints of AD and DC respectively.

By midpoint theorem,

Hence SR || AC and SR = $\frac{1}{2}$ AC ... (1)

Similarly, in ΔABC, P and Q are midpoints of AB and BC respectively.

PQ || AC and PQ = $\frac{1}{2}$ AC ...(2) ...By midpoint theorem

From equations (1) and (2), we get

PQ || SR and PQ = SR ...(3)

Here, one pair of opposite sides of quadrilateral PQRS is equal and parallel.

Hence PQRS is a parallelogram

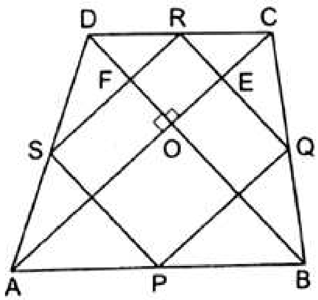
Hence the diagonals of parallelogram PQRS bisect each other.

Thus PR and QS bisect each other.

Hence, the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other.

13. Question

In the given figure, *ABCD* is a quadrilateral whose diagonals intersect at right angles. Show that the quadrilateral formed by joining the midpoints of the pairs of adjacent sides is a rectangle.



Answer

Here, in ABCD, diagonals intersect at 90°

Also, in *ABCD*, *P*, *Q*, *R* and *S* be the midpoints of *AB, BC, CD* and *DA,* respectively.

In Δ *ABC,* we have,

∴ *PQ* || *AC* and *PQ* = $\frac{1}{2}$ *AC ...by midpoint theorem*

Similarly, in Δ*DAC,*

SR || *AC* and *SR* = $\frac{1}{2}$ *AC ...by midpoint theorem*

Now, PQ || *AC* and *SR* || *AC*

∴ *PQ* || *SR*

Also, *PQ* = *SR* = $\frac{1}{2}$ *AC*

Hence, PQRS is parallelogram.

We know that the diagonals of the given quadrilateral bisect each other at right angles.

∴ ∠ *EOF* = 90°

Also, *RQ* || *DB*

∴ *RE* || *FO*

Also, *SR* || *AC*

∴ *FR* || *OE*

∴ *OERF* is a parallelogram.

So, ∠*FRE* = ∠*EOF* = 90° ...Opposite angles of parallelogram are equal

Thus, *PQRS* is a parallelogram with ∠*R* = 90°.

∴ *PQRS* is a rectangle.

CCE Questions

1. Question

Three angles of a quadrilateral are 80^o, 95^o and 112^o. Its fourth angle is

A. 78^o

B. 73^o

C. 85^o

D. 100^o

Answer

Let the fourth angle be x

$80^o + 95^o + 112^o + x^o = 360^o$ (Sum of angles of quadrilateral)

$287^o + x^o = 360^o$

$x = 360^o - 287^o$

$= 73^o$

Hence, option (B) is correct

2. Question

Three angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. The smallest of these angles is

A. 45^o

B. 60^o

C. 36^o

D. 48^o

Answer

Let the angles be 3x, 4x, 5x and 6x

$3x + 4x + 5x + 6x = 360^o$ (Sum of angles of a quadrilateral)

$18x = 360^o$

$x = \frac{360}{18}$

$x = 20^o$

∴ Angles of the quadrilateral are:

$3x = 3 \times 20^o = 60^o$

$4x = 4 \times 20^o = 80^o$

$5x = 5 \times 20^o = 100^o$

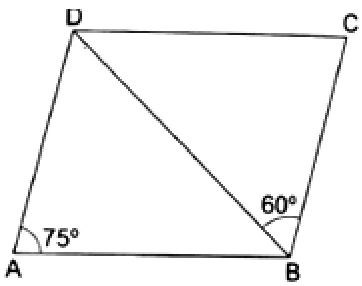
$6x = 6 \times 20^o = 120^o$

Hence, the smallest angle is 60^o

∴ Option (B) is correct

3. Question

In the given figure, ABCD is a parallelogram in which ∠BAD = 75° and ∠CBD = 60°. Then, ∠BDC = ?



A. 60^o

B. 75^o

C. 45^o

D. 50^o

Answer

It is given in the question that,

In parallelogram ABCD: ∠ BAD = 75^o, ∠ CBD = 60^o

Now, ∠ DAB = ∠ DCB = 75^o (Opposite angles)

Also, in triangle DBC we know that sum of angles of a triangle is 180^o

∠ DBC + ∠ BDC + ∠ DCB = 180^o

60^o + ∠ BDC + 75^o = 180^o

135^o + ∠ BDC = 180^o

∠ BDC = $180^o - 135^o$

∠ BDC = 45^o

Hence, option (C) is correct

4. Question

In which of the following figures are the diagonals equal?

A. Parallelogram

B. Rhombus

C. Trapezium

D. Rectangle

Answer

As we know that from all the quadrilaterals given below, diagonals of a rectangle are equal

Hence, option (D) is correct

5. Question

If the diagonals of a quadrilateral bisect each other at right angles, then the figure is a

A. Trapezium

B. Parallelogram

C. Rectangle

D. Rhombus

Answer

As we know that from all the quadrilaterals given below the diagonals of rhombus bisect each other at right angles

Hence, option (D) is correct

6. Question

The lengths of the diagonals of a rhombus are 16 cm and 12 cm. The length of each side of the rhombus is

A. 10 cm

B. 12 cm

C. 9 cm

D. 8 cm

Answer

Let us assume a rhombus ABCD where,

AB = BC = CD = DA

Now, in triangle OBC by using Pythagoras theorem we get:

$BC^2 = OB^2 + OC^2$

$BC^2 = 6^2 + 8^2$

$BC^2 = 36 + 64$

$BC^2 = 100$

$BC = \sqrt{100}$

BC = 10 cm

∴ AB = BC = CD = DA = 10 cm

Hence, option (A) is correct

7. Question

The length of each side of a rhombus is 10cm and one of its diagonals is of length 16 cm. The length of the other diagonal is

A. 13 cm

B. 12 cm

C. $2\sqrt{39}$ cm

D. 6 cm

Answer

It is given in the question that,

ABCD is rhombus where, AB = BC = CD = DA

Now, by using Pythagoras theorem in triangle BOC we have:

$BC^2 = OB^2 + OC^2$

$(10)^2 = OB^2 + (8)^2$

$100 = OB^2 + 64$

$OB^2 = 100 - 64$

$OB^2 = 36$

OB = 6 cm

∴ Length of diagonal, BC = OB + OD

BC = 6 + 6

BC = 12 cm

Hence, option (B) is correct

8. Question

If ABCD is a parallelogram with two adjacent angles ∠A = ∠ B, then the parallelogram is a

A. rhombus

B. trapezium

C. rectangle

D. none of these

Answer

It is given in the question that,

ABCD is a parallelogram where two adjacent angles ∠ A = ∠ B

We know that, sum of adjacent angles is 180^o

∴∠ A + ∠ B = 180^o

2∠ A = 180^o

∠ A = 180/2

∠ A = 90^o

As, ∠ A = ∠ B = ∠ C = ∠ D = 90^o

∴ ABCD is a rectangle as all the angles are equal to 90^o

Hence, option (C) is correct

9. Question

In a quadrilateral ABCD, if AO and BO are the bisectors of ∠A and ∠B respectively, ∠C = 70° and ∠D = 30°. Then, ∠AOB = ?

A. 40^o

B. 50^o

C. 80^o

D. 100^o

Answer

It is given in the question that, ABCD is a quadrilateral where AO and BO are the bisectors of ∠ A and ∠ B

We know that, sum of all angles of a quadrilateral is equal to 360^o

∴∠ A + ∠ B + ∠ C + ∠ D = 360^o

∠ A + ∠ B + 70^o + 30^o = 360^o

∠ A + ∠ B = 360^o – 100^o

∠ A + ∠ B = 260^o

1/2 (∠A + ∠B) = 1/2 × 260°

1/2 (∠A + ∠B = 130°

Now, in triangle AOB

$1/2\ (\angle A + \angle B) + \angle AOB = 180°$

$130^{o} + \angle AOB = 180^{o}$

$\angle AOB = 180^{o} - 130^{o}$

$\angle AOB = 50^{o}$

Hence, option (B) is correct

10. Question

The bisectors of any two adjacent angles of a parallelogram intersect at

A. 30^{o}

B. 45^{o}

C. 60^{o}

D. 90^{o}

Answer

We know that,

Sum of two adjacent angles = 180^{o}

Also, sum of bisector of adjacent angles = $180/2 = 90^{o}$

As sum of angles of a triangle = 180^{o}

∴ Sum of 2 adjacent angles + Intersection angle = 180^{o}

90^{o} + Intersection angle = 180^{o}

∴ Intersection angle = $180^{o} - 90^{o}$

= 90^{o}

Hence, option (D) is correct

11. Question

The bisectors of the angles of a parallelogram enclose a

A. Rhombus

B. Square

C. Rectangle

D. Parallelogram

Answer

From all the given quadrilateral we know that the bisectors of the angles of a parallelogram enclose a rectangle

Hence, option (C) is correct

12. Question

The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a

A. Rhombus

B. Square

C. Rectangle

D. Parallelogram

Answer

We know that, the figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a parallelogram

Hence, option (D) is correct

13. Question

The figure formed by joining the mid-points of the adjacent sides of a square is a

A. Rhombus

B. Square

C. Rectangle

D. Parallelogram

Answer

We know that, the figure formed by joining the mid-points of the adjacent sides of a square is a square

Hence, option (B) is correct

14. Question

The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a

A. rhombus

B. square

C. rectangle

D. parallelogram

Answer

We know that, the figure formed by joining the mid-points of the adjacent sides of a parallelogram is parallelogram

Hence, option (D) is correct

15. Question

The figure formed by joining the mid-points of the adjacent sides of a rectangle is a

A. rhombus

B. square

C. rectangle

D. parallelogram

Answer

We know that, the figure formed by joining the mid-points of the adjacent sides of a rectangle is a rhombus

Hence, option (A) is correct

16. Question

The figure formed by joining the mid-points of the adjacent sides of a rhombus is a

A. rhombus

B. square

C. rectangle

D. parallelogram

Answer

We know that, the figure formed by joining the mid-points of the adjacent sides of a rhombus is a rectangle

Hence, option (C) is correct

17. Question

If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is

A. 108^o

B. 54^o

C. 72^o

D. 81^o

Answer

We know that,

Sum of two adjacent angles is equal to 180^o

$\therefore \angle A + \angle B = 180^o$

According to the condition given in the question, we have

∠ A = x° then ∠ B = 2/3 x°

∴ x° + 2x/3 ° = 180°

5x/3 ° = 180°

⇒ $x = \frac{180 \times 3}{5}$

⇒ x = 540°/5

⇒ x = 108°

∴∠ A = 108° and,

∠ B = 2/3 × 108°

∠ B = 2× 36° = 72°

Thus, the smallest angle = ∠ B = 72°

Hence, option (C) is correct

18. Question

If one angle of a parallelogram is 24° less than twice the smallest angle, then the largest angle of the parallelogram is

A. 68°

B. 102°

C. 112°

D. 136°

Answer

As per the question,

Let the smallest angle be x° and the largest angle be (2x – 24)°

Since, the sum of adjacent angles of a parallelogram is 180°

∴ x + (2x – 24) = 180°

3x – 24 = 180°

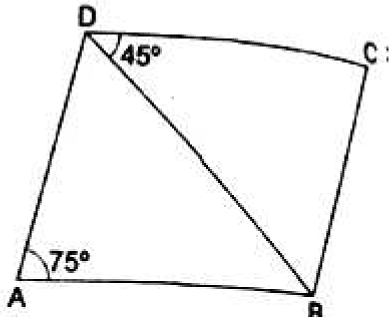
x = 68°

Hence, the largest angle is: 2x – 24 = 2(68) – 24 = 136 – 24 = 112

∴Option A is correct

19. Question

In the given figure, ABCD is a parallelogram in which ∠BDC = 45° and ∠BAD = 75°. Then, ∠CBD = ?



A. 45^o

B. 55^o

C. 60^o

D. 75^o

Answer

As per the question,

∠ BAD = ∠ BCD = 75° (opposite angles of parallelogram)

Now, in ΔBCD,

∠BCD + ∠CBD + ∠BCD = 180°

45 + ∠CBD + 75 = 180°

∠CBD = 60°

∴ Option C is correct

20. Question

If area of a ||gm with sides a and b is A and that of a rectangle with sides a and b is B, then

A. A > B

B. A = B

C. A < B

D. A ≥ B

Answer

Let the height of the parallelogram be 'h'

Now, h < b (Since, perpendicular distance is the shortest)

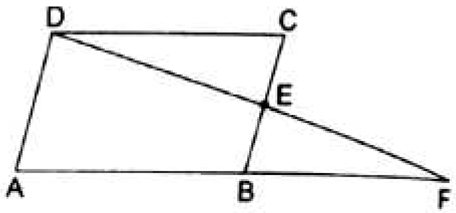
∴ a × h < a × b

A < B

∴Option C is correct

21. Question

In the given figure, ABCD is a ||gm and E is the mid-point of BC. Also, DE and AB when produced meet at F. Then,



A. $AF = \frac{3}{2}AB$

B. AF = 2AB

C. AF = 3AB

D. $AF^2 = 2AB^2$

Answer

According to the condition given in the question, we have

In triangle DCE and FBE

BE = EC (E is the mid-point of BC)

∠ CED = ∠ BEF (Vertically opposite angles)

∠ CDE = ∠ EFB (Alternate interior angles)

∴ΔDCE ≅ ΔFBE (By AAS congruence rule)

DC = BF (BY CPCT)

As AB is parallel to DC, then AB = DC

∴ AB = DC = BF

AF = AB + BF

AF = AB + AB

AF = 2AB

Hence, option (B) is correct

22. Question

The parallel sides of a trapezium are a and b respectively. The line joining the mid-points of its non-parallel sides will be

A. $\frac{1}{2}(a - b)$

B. $\frac{1}{2}(a + b)$

C. $\frac{2ab}{(a + b)}$

D. $\sqrt{ab}$

Answer

It is given in the question that,

ABCD is a trapezium

Draw EF parallel to AB and DC, and join BD intersecting EF at point M.

Now, E is the midpoint of AD and EM ∥ AB. Hence, using midpoint theorem,

EM = 1/2 AB

⇒ EM = 1/2 b

Similarly, FM = 1/2

⇒ DC = 1/2 a

EF = EM + FM

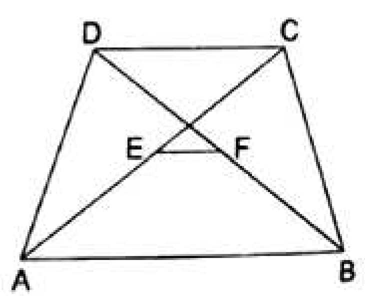
EF = 1/2 a + 1/2 b

EF = 1/2 (a + b)

∴Option B is correct

23. Question

In a trapezium ABCD, if E and F be the mid-point of the diagonals AC and BD respectively. Then, EF = ?



A. $\frac{1}{2}AB$

B. $\frac{1}{2}CD$

C. $\frac{1}{2}(AB + CD)$

D. $\frac{1}{2}(AB - CD)$

Answer

Construction: Join CF and extent it to cut AB at point M

Firstly, in triangle MFB and triangle DFC

DF = FB (As F is the mid-point of DB)

∠DFC = ∠MFB (Vertically opposite angle)

∠DFC = ∠FBM (Alternate interior angle)

∴ By ASA congruence rule

ΔMFB ≅ DFC

Now, in triangle CAM

E and F are the mid-points of AC and CM respectively

∴ EF = 1/2 (AM)

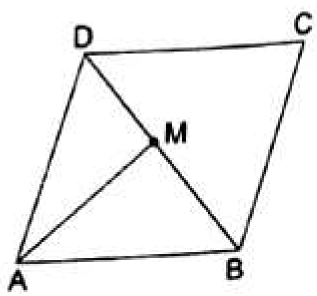
EF = 1/2 (AB – MB)

EF = 1/2 (AB-CD)

Hence, option D is correct

24. Question

In the given figure, ABCD is a parallelogram, M is the mid-point of BD and BD bisects ∠B as well as ∠D. Then, ∠AMB = ?



A. 45^{o}

B. 60^{o}

C. 90^{o}

D. 30^{o}

Answer

Since, ABCD is a parallelogram,

∴∠B = ∠D (opposite angle)

1/2 ∠B = 1/2 ∠D

∠ADB = ∠ABD

∴ ADB is an isosceles triangle.

Since, M is the midpoint of BD

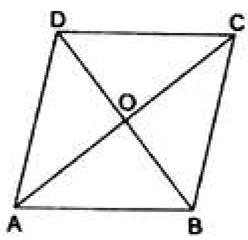
∴ AM is a median of ΔADB.

Now, ∠AMB = 90° (AM is perpendicular to BD)

∴Option C is correct

25. Question

In the given figure, ABCD is a rhombus. Then,



A. $AC^2 + BD^2 = AB^2$

B. $AC^2 + BD^2 = 2AB^2$

C. $AC^2 + BD^2 = 4AB^2$

D. $2(AC^2 + BD^2) = 3AB^2$

Answer

Since, we know that the diagonals of a rhombus bisect each other at 90°.

Hence, $OA = \frac{1}{2} AC$, $OB = \frac{1}{2} BD$ and ∠AOB = 90°

$AB^2 = OA^2 + OB^2$

$AB^2 = (\frac{1}{2} AC)^2 + (\frac{1}{2} BD)^2$

$= \frac{1}{4}(AC)^2 + \frac{1}{4}(BD)^2$

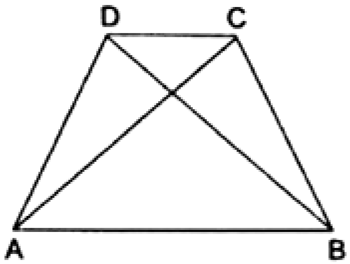
$AB^2 = \frac{1}{4}(AC^2 + BD^2)$

$4AB^2 = (AC^2 + BD^2)$

∴ Option C is correct

26. Question

In a trapezium ABCD, if AB || CD, then $(AC^2 + BD^2)$ = ?



A. $BC^2 + AD^2 + 2BC \cdot AD$

B. $AB^2 + CD^2 + 2AB \cdot CD$

C. $AB^2 + CD^2 + 2AD \cdot BC$

D. $BC^2 + AD^2 + 2AB \cdot CD$

Answer

Draw perpendicular from D on AB meeting it on E and from C on AB meeting AB at F

∴ DEFC will be a parallelogram and thus, EF = CD

Now, In ΔABC

Since, ∠B is acute

∴ $AC^2 = BC^2 + AB^2 - 2AB \times AE$ (i)

Similarly, In ΔABD,

Since ∠A is acute

∴ $BD^2 = AD^2 + AB^2 - 2AB \times AF$ (ii)

Adding (i) and (ii),

$AC^2 + BD^2 = (BC^2 + AD^2) + (AB^2 + AB^2) - 2AB (AE + BF)$

$= (BC^2 + AD^2) + 2AB (AB - AE - BF)$ [Since, AB = AE + EF + FB and AB - AE = BE]

$= (BC^2 + AD^2) + 2AB (BE - BF)$

$= (BC^2 + AD^2) + 2AB.EF$

Now, we know that CD = EF

Thus, $AC^2 + BD^2 = (BC^2 + AD^2) + 2AB.CD$

∴ Option D is correct

27. Question

Two parallelograms stand on equal bases and between the same parallels. The ratio of their areas is

A. 1:2

B. 2:1

C. 1:3

D. 1:1

Answer

We know that,

Area of a parallelogram = base × height

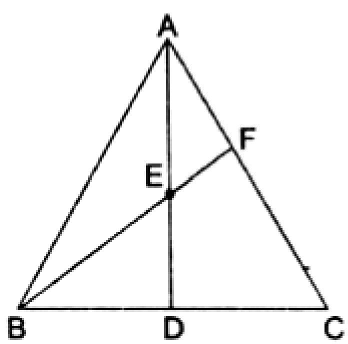
Now, if both parallelograms are on the same base and between the same parallels, then their heights will be equal.

Hence, their areas will also be equal

∴ Option D is correct

28. Question

In the given figure, AD is a median of ΔABC and E is the mid-point of AD. If BE is joined and produced to meet AC in F, then AF = ?



A. $\frac{1}{2}AC$

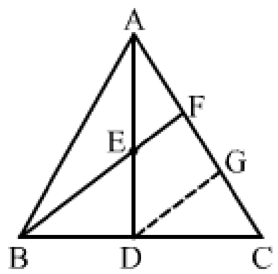
B. $\frac{1}{3}AC$

C. $\frac{2}{3}AC$

D. $\frac{3}{4}AC$

Answer

Let G be the mid-point of FC and join DG



In ΔBCF,

G is the mid-point of FC and D is the mid-point of BC

Thus, DG|| BF

DG || EF

Now, In Δ ADG,

E is the mid-point of AD and EF is parallel to DG.

Thus, F is the mid-point of AG.

AF = FG = GC [G is the mid-point of FC]

Hence, AF $=\frac{1}{3}$ AC

∴ Option B is correct

29. Question

If ∠A, ∠B, ∠C and ∠D of a quadrilateral ABCD taken in order, are in the ratio 3 : 7 : 6: 4, then ABCD is a

A. Rhombus

B. Kite

C. Trapezium

D. Parallelogram

Answer

Let the required angles be 3x, 7x, 6x and 4x

3x + 7x + 6x + 4x = 360° (Sum of angles of quadrilateral)

20x = 360°

x = 18°

Hence, angles are:

3x = 3 ×18° = 54°

7x = 7 × 18° = 126°

6x = 6 × 18° = 108°

$4x = 4 \times 18° = 72°$

Now we can observe that, 54° + 126° = 180° and 72° + 108° = 180°

Thus, ABCD is a trapezium.

Hence option C is correct.

30. Question

Which of the following is not true for a parallelogram?

A. Opposite sides are equal.

B. Opposite angles are equal.

C. Opposite angles are bisected by the diagonals.

D. Diagonals bisect each other.

Answer

We know that,

In any parallelogram, opposite angles are bisected by the diagonals

∴ Option C is correct

31. Question

If APB and CQD are two parallel lines, then the bisectors of ∠APQ, ∠BPQ, ∠CQP and ∠PQD enclose a

A. square

B. rhombus

C. rectangle

D. kite

Answer

It is given in the question that,

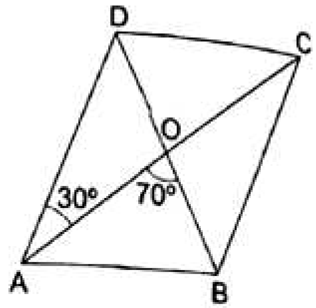
APB and CQD are two parallel lines,

Thus, the bisectors of ∠CQP, ∠APQ, ∠BPQ and ∠PQD enclose a rectangle.

Hence, option C is correct.

32. Question

The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O such that ∠DAC = 30° and ∠AOB = 70°. Then, ∠DBC = ?



A. 40°

B. 35°

C. 45°

D. 50°

Answer

In the given figure,

∠OAD = ∠OCB (Alternate interior angle)

∠OCB = 30°

∠AOB + ∠BOC = 180° (Linear pair)

70° + ∠BOC = 180°

∠BOC = 110°

Now, In ΔBOC,

∠OBC + ∠BOC + ∠OCB = 180°

∠OBC + 110° + 30° = 180°

∠OBC = 40°

∴ ∠DBC = 40°

Hence, Option A is correct.

33. Question

Three statements are given below:

I. In a ||gm, the angle bisectors of two adjacent angles enclose a right angle.

II. The angle bisectors of a ||gm form a rectangle.

III. The triangle formed by joining the mid-points of the sides of an isosceles triangle is not necessarily an isosceles triangle.

Which is true?

A. I only

B. II only

C. I and II

D. II and III

Answer

We can clearly observe that statement I and statement II are correct. Whereas Statement III is not correct because the triangle formed by joining the midpoints of the sides of an isosceles triangle is always an isosceles triangle

Therefore, Option C is correct

34. Question

Three statements are given below:

I. In a rectangle ABCD, the diagonal AC bisects ∠A as well as ∠C.

II. In a square ABCD, the diagonal AC bisects ∠A as well as ∠C.

III. In a rhombus ABCD, the diagonal AC bisects ∠A as well as ∠C.

Which is true?

A. I only

B. II and III

C. I and III

D. I and II

Answer

We can clearly observe that statement II and statement III are correct and Statement I is wrong because the diagonals of a rectangle does not bisect ∠A and ∠C. And this is so because the adjacent sides are unequal in a rectangle.

∴ Option B is correct

35. Question

In each of the questions one question is followed by two statements I and II. Choose the correct option.

Is quadrilateral ABCD a ||gm?

I. Diagonals AC and BD bisect each other.

II. Diagonals AC and BD are equal.

A. if the question can be answered by one of the given statements alone and not by the other;

B. if the question can be answered by either statement alone;

C. if the question can be answered by both the statements together but not by any one of the two;

D. if the question cannot be answered by using both the statements together.

Answer

Here, as we know that if the diagonals of a quadrilateral bisects each other, then it is a parallelogram.

But as per II, if the diagonals of a quadrilateral are equal, then it is not necessarily a parallelogram which is not true. Thus, II does not give the answer.

Therefore Option A is correct.

36. Question

In each of the questions one question is followed by two statements I and II. Choose the correct option.

Is quadrilateral ABCD a rhombus?

I. Quad. ABCD is a ||gm.

II. Diagonals AC and BD are perpendicular to each other.

A. if the question can be answered by one of the given statements alone and not by the other;

B. if the question can be answered by either statement alone;

C. if the question can be answered by both the statements together but not by any one of the two;

D. if the question cannot be answered by using both the statements together.

Answer

Here, we can observe that neither I not II can alone justify the answer to the given question. But if we consider both I and II together then they completely satisfies the answer.

∴ Option C is correct.

37. Question

In each of the questions one question is followed by two statements I and II. Choose the correct option.

Is ||gm ABCD a square?

I. Diagonals of ||gm ABCD are equal.

II. Diagonals of ||gm ABCD intersect at right angles.

A. if the question can be answered by one of the given statements alone and not by the other;

B. if the question can be answered by either statement alone;

C. if the question can be answered by both the statements together but not by any one of the two;

D. if the question cannot be answered by using both the statements together.

Answer

We know that when the diagonals of a parallelogram are equal, it might be a square or a rectangle. But if the diagonals of that parallelogram intersect at a right angle, then it is definitely a square. Thus, it can be concluded that both I and II together will give the answer.

Therefore, Option C is correct.

38. Question

In each of the questions one question is followed by two statements I and II. Choose the correct option.

Is quad. ABCD a parallelogram?

I. Its opposite sides are equal.

II. Its opposite angles are equal.

A. if the question can be answered by one of the given statements alone and not by the other;

B. if the question can be answered by either statement alone;

C. if the question can be answered by both the statements together but not by any one of the two;

D. if the question cannot be answered by using both the statements together.

Answer

We know that a quadrilateral is a parallelogram when either I or II holds true.

Hence, the correct answer is (b)

39. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct option.

Assertion (A)	**Reason (R)**
If three angles of a quadrilateral are 130^{o}, 70^{o}, and 60^{o}, then the fourth angle is 100^{o}.	The sum of all the angle of a quadrilateral is 360^{o}.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Let the fourth angle be x,

$130^o + 70^o + 60° + x° = 360°$ (angle sum of quadrilateral)

$x° = 360^o - (130^o + 70^o + 60^o)$

$x° = 100^o$

Thus, it can be observed that reason and assertion both are true and the reason explains the assertion.

Therefore Option A is correct.

40. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct option.

Assertion (A)	**Reason (R)**
ABCD is a quadrilateral in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Then, PQRS is a parallelogram.	The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

It is given that, ABCD is a quadrilateral in which P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Then, PQRS is a parallelogram

Also, the line segment joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.

Hence, both assertion and reason are true and reason is correct explanation of the assertion

∴ Option (a) is correct

41. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct option.

Assertion (A)	**Reason (R)**
In a rhombus ABCD, the diagonal AC bisects ∠A as well as ∠C.	The diagonals of a rhombus bisect each other at right angles.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

It is given that,

In a rhombus ABCD, the diagonal AC bisects ∠A as well as ∠ C which is true

And we know that, the diagonals of a rhombus bisect each other at right angles.

Hence, both assertion and reason are true but reason is not the correct explanation of assertion

∴ Option (b) is correct

42. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct option.

Assertion (A)	**Reason (R)**
Every parallelogram is a rectangle.	The angle bisectors of a parallelogram form a rectangle.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

The statement given in assertion is not true as every parallelogram is not a rectangle whereas, statement given in the reason is true as the angle bisectors of a parallelogram form a rectangle

Hence, assertion is false whereas reason is true

∴ Option (d) is correct

43. Question

Each question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct option.

Assertion (A)	**Reason (R)**
The diagonals of a ||gm bisect each other.	If the diagonals of a ||gm are equal and intersect at right angles, then the parallelogram is a square.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

We know that,

The diagonals of a ||gm bisect each other

Also we know that, if the diagonals of a ||gm are equal and intersect at right angles, then the parallelogram is a square

Hence, both assertion and reason are true but reason is not the correct explanation of the assertion

Hence, option (b) is correct

44. Question

Match the following columns:

Column I	**Column II**
(a) Angle bisectors of a parallelogram form a	(p) parallelogram
(b) The quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a square is a	(q) rectangle
(c) The quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a rectangle is a	(r) square
(d) The figure formed by joining the mid-points of the pairs of adjacent sides of a quadrilateral is a	(s) rhombus

The correct answer is:

(a) -……, (b) -……..,

(c) -……, (d) -………,

Answer

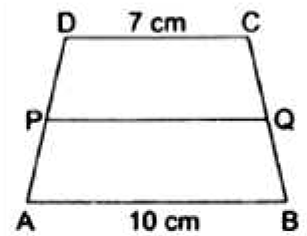
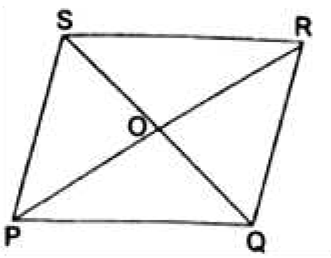
The correct match for the above given table is as follows:

Column I	**Column II**
(a) Angle bisectors of a parallelogram form a	(q) Rectangle
(b) The quadrilateral formed by joining the mid-points of the pairs of adjacent sides of a square is a	(r) Square
(c) The quadrilateral formed by joining the mid-points	(s) Rhombus
(d) The figure formed by joining the mid-points of the pairs of adjacent sides of a quadrilateral is a	(p) Parallelogram

45. Question

Match the following columns:

Column I	**Column II**
(a) In the given figure, ABCD is a trapezium in which AB = 10 cm and CD = 7 cm. If P and Q are the mid-points of AD and BC respectively, then PQ =	(p) equal

	
(b) In the given figure, PQRS is a ||gm whose diagonals intersect at O. If PR = 13 cm, then OR = 	(q) at right angles
(c) The diagonals of a square are	(r) 8.5 cm
(d) The diagonals of a rhombus bisect each other	(s) 6.5 cm

The correct answer is:

(a) -……, (b) -……..,

(c) -……, (d) -………,

Answer

a) PQ = $\frac{1}{2}$ (AB + CD)

PQ = $\frac{1}{2}$ (17)

PQ = 8.5 cm

(b) OR = $\frac{1}{2}$ (PR)

OR = $\frac{1}{2}$ (13)

OR = 6.5 cm

(c) We know that,

The diagonals of a square are equal

(d) We also know that,

The diagonals of a rhombus bisect each other at right angles

∴ The correct match is as follows:

(a) - (r)

(b)- (s)

(c) - (p)

(d) - (q)

Formative Assessment (Unit Test)

1. Question

Which is false?

A. In a ||gm, the diagonals are equal.

B. In a ||gm, the diagonals bisect each other.

C. If a pair of opposite sides of a quadrilateral is equal, then it is a ||gm.

D. If the diagonals of a ||gm are perpendicular to each other, then it is a rhombus.

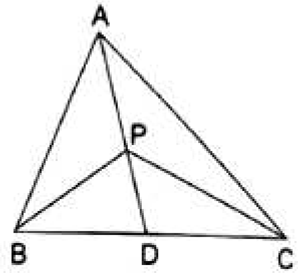
Answer

from the above given four statements option A is false as we know that in any parallelogram the diagonals are not equal

Hence, option A is correct

2. Question

If P is a point on the median AD of a ΔABC, then ar (ΔABP) = ar(ΔACP).



A. True

B. False

Answer

In ΔABC,

Since, AD is the median

Thus, BD = DC

Let the height of ΔABC be h

ar(ΔABD) = ar(ΔABD)

1/2 × h × BD = 1/2 × h × BD

1/2 × h × BD = 1/2 × h × CD

∴ ar (ΔABD) = ar (ΔADC)

Let H be the height of ΔBPD and ΔPDC

∴ ar (ΔBPD) = ar (ΔPDC)

Now, ar(ΔABD) = ar (ΔABP) + ar (ΔBPD)

And, ar(ΔACD) = ar(ΔACP) + ar(ΔPDC)

Thus, ar(ΔABP) = ar(ΔACP)

∴ Option A is correct

3. Question

The angles of a quadrilateral are in the ratio 1:3:5:6. Find its greatest angle.

Answer

Let the angles be x, 3x, 5x and 6x.

x + 3x + 5x + 6x = 360° (sum of angles of quadrilateral)

$15x° = 360^{0}$

$x° = 24^{0}$

Therefore, angles are as follows:

$x° = 24^o$

$3x° = 24^o \times 3 = 72^o$

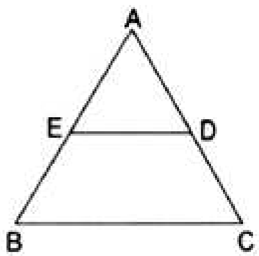
$5x° = 24^o \times 5 = 120^o$

$6x° = 24^o \times 6 = 144^o$

Hence, 144° is the greatest angle.

4. Question

In a ΔABC, D and E are the mid-points of AB and AC respectively and DE = 5.6 cm. Find the length of BC.



Answer

We know that in ΔABC, D and E are the midpoints of AB and AC, respectively.

Now using mid-point theorem,

$DE = \frac{1}{2}(BC)$

BC= 2 × DE

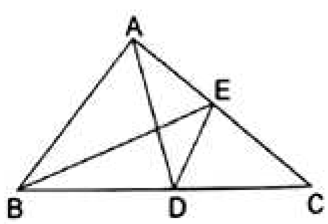
BC= 2 × 5.6

= 11.2 cm

Thus, BC = 11.2 cm

5. Question

In the given figure, AD is the median and DE || AB. Prove that BE is the median.



Answer

In ΔABC, using mid point theorem

We know that D is the mid-point of BC and DE|| AB.

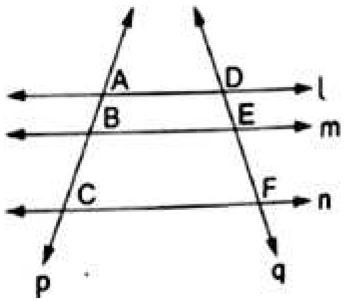
Thus, AE = EC and $DE = \frac{1}{2}(AB)$

Now, E is the mid point of AC

Thus, BE is the median

6. Question

In the given figure, lines l, m and n are parallel lines and the lines p and q are transversals. If AB = 5 cm, BC = 15 cm, then DE : EF = ?



Answer

Here, we have:

l || m || n

And p and q are the transversal lines

Thus, AB : BC = 5 : 15

AB : BC = 1 : 3

∴ Using intercept theorem,

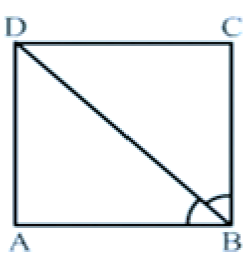
DE : EF = 1 : 3

7. Question

ABCD is a rectangle in which diagonal BD bisects ∠B. Show that ABCD is a square.

Answer

Let there be a rectangle ABCD with AB = CD and BC = AD and ∠A = ∠B = ∠C= ∠D = 90^{o}



Since, BD bisects ∠B

∠ABD = ∠DBC (i)

And, ∠ADB = ∠DBC [Alternate interior angles]

∠ABD = ∠ADB. [From (i)]

AB = DA. (Sides opposite to equal angles)

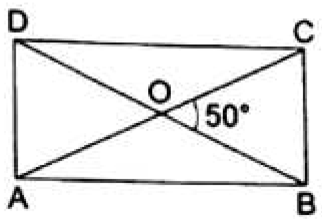
∴ AB = CD = DA = BC

Since, all the sides are equal and all the angles are equal to 90^{o}, thus the quadrilateral is a square.

Hence, ABCD is a square.

8. Question

The diagonals of a rectangle ABCD intersect at the point O. If ∠BOC = 50°, then ∠OAD = ?



A. 50^{o}

B. 55^{o}

C. 65^{o}

D. 75^{o}

Answer

∠BOC = ∠AOD (Vertically opposite angles)

Angle AOD = 50°

In Δ AOD, Since, the diagonals are equal, thus the bisectors will also be equal)

Thus, OA = OD

∴ ∠OAD = ∠ODA

$= \frac{1}{2}(180° - 50°)$

$= \frac{1}{2}(130°)$

= 65°

∴ Option C is correct

9. Question

Match the following column:

Column I	Column II
(a) Sum of all the angles of a quadrilateral is	(p) Right angles
(b) In a ||gm, the angle bisectors of two adjacent angles intersect at	(q) Rectangle
(c) Angle bisectors of a ||gm form a	(r) 90°
(d) The diagonals of a square are equal and bisect each other at an angle of	(s) 4 right angles

The correct answer is:

(a) -……, (b) -……..,

(c) -……, (d) -………,

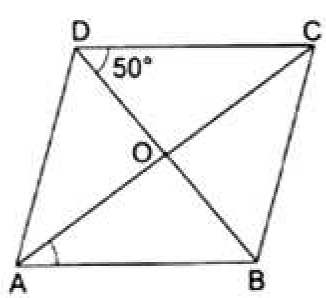
Answer

The correct match for the above given table is as follows:

Column I	Column II
(a) Sum of all the angles of a quadrilateral is	(s) 4 right angles
(b) In a ‖gm, the angle bisectors of two adjacent angles intersect at	(p) Right angles
(c) Angle bisectors of a ‖gm form a	(q) Rectangle
(d) The diagonals of a square are equal and bisect each other at an angle of	(r) 90^{o}

10. Question

The diagonals of a rhombus, ABCD intersect at the point O. If ∠BDC = 50°, then ∠OAB = ?



A. 50^{o}

B. 40^{o}

C. 25^{o}

D. 20^{o}

Answer

$\angle BDC = \angle ABD$ (Alternate interior angles)

$\angle ABD = 50^{o}$

Now, In ΔAOB,

$\angle DBA = 50^{o}$ and $\angle AOB = 90^{o}$

Thus, $\angle OAB = 180^{o} - (90^{o} + 50^{o})$

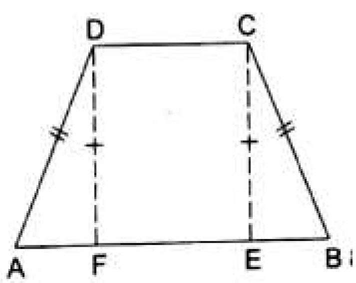
$\angle OAB = 180° - 140°$

$\angle OAB = 40^{o}$

∴ Option B is correct.

11. Question

ABCD is a trapezium in which AB || CD and AD = BC, then $\angle A = \angle B$ is



A. true

B. false

Answer

Construction: Draw perpendicular line from D and C to AB such that it cuts AB at F and E, respectively.

Now, In ΔADF and ΔBCE,

AD = BC (Given)

$\angle AFD = \angle BEC$ (90^{o} each)

DF = CE (Perpendicular distance between the same parallels)

∴ By SSA axiom

$\Delta ADF \cong \Delta BCE$

$\angle A = \angle B$ (by c.p.c.t.)

Therefore Option A is correct.

12. Question

Look at the statements given below:

I. If AD, BE and CF be the altitudes of a ΔABC such that AD = BE = CF, then ΔABC is an equilateral triangle.

II. If D is the mid-point of hypotenuse AC of a right ΔABC, then BD = AC.

III. In an isosceles ΔABC in which AB = AC, the altitude AD bisects BC.

Which is true?

A. I only B. II only

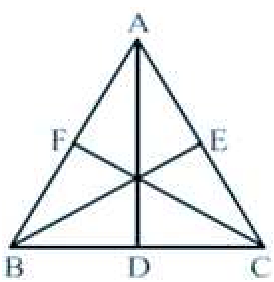
C. I and III D. II and III

Answer

We can clearly observe that statement I and statement III are correct.

We can prove the statement as follows:

In ΔABC, altitudes AD, BE and CF are equal



Now, In ΔABE and ΔACF,

BE = CF (Given)

∠A = ∠A (common)

∠AEB = ∠AFC (Each 90°)

Therefore, by AAS axiom,

ΔABE ≅ Δ ACF

AB = AC (by cpct)

In the same way, ΔBCF ≅ ΔBAD

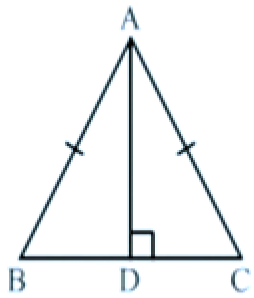
thus, BC = AB (by cpct)

Therefore AB = AC = BC

Thus, ΔABC is an equilateral triangle.

We can prove the IIIrd statement as follows:

Let ΔABC be an isosceles triangle with AD as an altitude



Now, In ΔABD and ΔADC,

AB = AC (Given)

$\angle B = \angle C$ (Angles opposite to equal sides)

$\angle BDA = \angle CDA$ (each 90°)

Therefore by AAS axiom,

$\Delta ABD \cong \Delta ADC$

BD = DC (by congruent parts of congruent triangles)

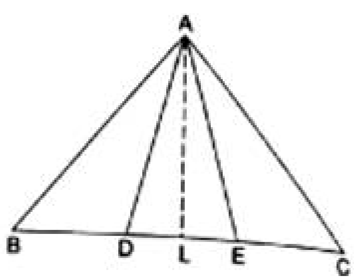
∴ D is the mid-point of BC and hence AD bisects BC.

13. Question

In the given figure, D and E are two points on side BC of ΔABC such that BD = DE = EC.

Prove that

ar (ΔABD) = ar (ΔADE) = ar (ΔAEC).



Answer

Area of a triangle = 1/2 (Base × Height)

Now, draw AL perpendicular to BC and h be the height of ΔABC i.e. AL

Thus, Height of ΔABD = Height of ΔADE = Height of ΔAEC

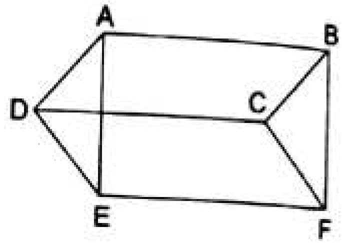
It is given that the bases BD, DE and EC of ΔABD, ΔADE and ΔAEC respectively are equal.

Now, since base and height both are equal of all the triangles therefore,

ar(ΔABD) = ar(ΔADE) = ar(ΔAEC)

14. Question

In the given figure ABCD, DCFE and ABFE are parallelograms. Show that ar(ΔADE) = ar(ΔBCF).



Answer

Now, here in ΔADE and ΔBCF,

AD = BC (Opposite sides of parallelogram ABCD

DE = CF (Opposite sides of parallelogram DCEF)

AE = BF (Opposite sides of parallelogram ABFE)

∴ By SSS axiom,

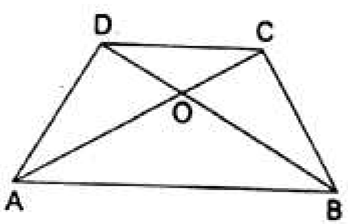
ΔADE ≅ ΔBCF

And,

ar(ΔADE) = ar(ΔBCF) (By cpct)

15. Question

In the given figure, ABCD is a trapezium in which AB || DC and diagonals AC and BD intersect at O. Prove that ar(ΔAOD) = ar(ΔBOC).



Answer

Here, in trapezium ABCD,

AB || DC and AC and BD are the diagonals intersecting at O.

Now, since ΔACD and ΔBCD lie on the same base and between the same parallels.

Thus, ar(ΔACD) = ar(ΔBCD)

Subtracting ar(ΔCOD) from both the sides, we get:

ar(ΔACD) − ar(ΔCOD) = ar(ΔBCD)− ar(ΔCOD)

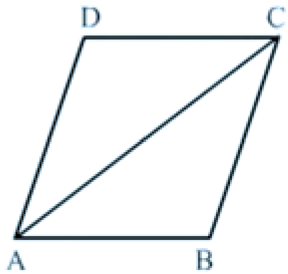
∴ ar(ΔAOD) = ar(ΔBOC)

16. Question

Show that a diagonal divides a parallelogram into two triangles of equal area.

Answer

Let there be a parallelogram ABCD and with one of its diagonal as AC.



Now, In ΔCDA and ΔABC,

DA = BC (Opposite sides of parallelogram ABCD)

AC = AC (Common)

CD = AB (Opposite sides of parallelogram ABCD)

∴ By SSS axiom

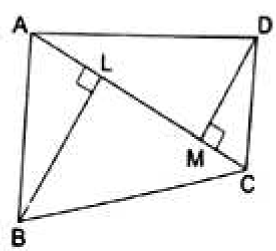
ΔCDA ≅ ΔABC

ar(ΔCDA) = ar(ΔABC) (by cpct)

Thus, we can say that the diagonal of a parallelogram divides it into two triangles of equal area.

17. Question

In the given figure, AC is a diagonal of quad. ABCD in which BL ⊥ AC and DM ⊥ AC. Prove that or (quad. ABCD) $= \frac{1}{2} \times AC \times (BL + DM)$.



Answer

Here we have ABCD as a quadrilateral with one of its diagonal as AC and BL and DM are perpendicular to AC

Thus, ar(ABCD) = ar(ΔADC) + ar(ΔABC)

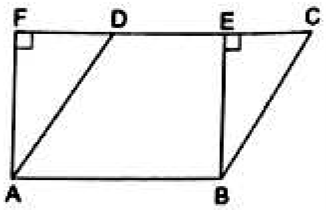
Since, (BL ⊥ AC) and (DM ⊥ AC)

∴ Area of ABCD = ($\frac{1}{2}$ × AC × BL) + ($\frac{1}{2}$ × AC × DM)

= $\frac{1}{2}$× AC × (BL + DM)

18. Question

||gm ABCD and rectangle ABEF have the same base AB and are equal in areas. Show that the perimeter of the ||gm is greater than that of the rectangle.



Answer

Here we know that parallelogram ABCD and rectangle ABEF are on the same base AB and between the same parallels such that:

AB = CD and AB = EF

So, CD = FE

Now, adding AB on both sides

AB + CD = AB + FE (i)

Since we know that hypotenuse is the longest side of a triangle

∴ AD > AF (ii)

And, BC > BE (iii)

Adding (ii) and (iii),

AD + BC > AF + BE (iv)

Now, Perimeter of ABCD = AB + BC + CD + AD

And, Perimeter of ABEF = AB + BE + FE + AF

Adding (i) and (iv),

AB + CD + AD + BC > AB + FE + AF + BE

Thus, we can say that the perimeter of parallelogram ABCD is greater than that of rectangle ABEF.

19. Question

In the adjoining figure, ABCD is a ||gm and E is the mid-point of side BC. If DE and AB when produced meet at F, prove that AF = 2AB.

Answer

Here we have parallelogram ABCD with AB || DC

Thus, DC || BF

Now, in ΔDEC and ΔFEB,

∠DCF = ∠EBF (Alternate interior angle)

CE = BE (E is the mid-point of BC

∠CED = ∠BEF (Vertically opposite angle)

Therefore, by ASA axiom,

ΔDEC ≅ ΔFEB

CD = BF (by cpct)

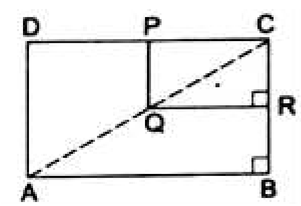
And CD = AB (Opposite sides of a parallelogram ABCD)

So, AF = AB + BF = AB + AB = 2AB

20. Question

In the adjoining figure, ABCD and PQRC are rectangles, where Q is the mid-point of AC.

Prove that (i) DP = PC (ii) $PR = \frac{1}{2}AC.$



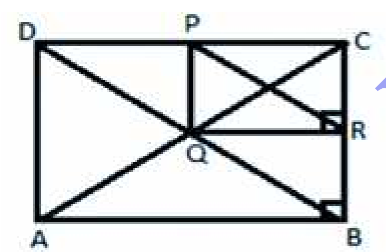
Answer

(i) Here, we have

∠CRQ = ∠CBA = 90^{0}

Thus, RQ || AB

Now, In ΔABC,



Q is the mid-point of AC and QR || AB.

Thus, R is the mid-point of BC.

In the same way, P is the midpoint of DC.

Hence, DP = PC

(ii) Here, let us join B to D.

Now, In ΔCDB,

P and R are the mid points of DC and BC respectively.

Since, AC = BD

Thus, PR || DB and PR = $\frac{1}{2}$ DB = $\frac{1}{2}$ AC