29. Probability

Exercise 29A

1. Question

Let A and B be the events such that

$$P(A)=\frac{7}{13}, P(B)=\frac{9}{13} \text{ and } P(A\cap B)=\frac{4}{13}.$$

Find

(i) P(A / B)

(ii) P(B / A)

(iii) P(A ∪ B)

(iv) $P(\overline{B}/\overline{A})$

Answer

Given - A and B be the events such that $P(A)=\frac{7}{13}$, $P(B)=\frac{9}{13}$ and

$$P(A\cap B)=\frac{4}{13}$$

To find – (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A\cup B)$ (iv) $P(\overline{B}/\overline{A})$

Formula to be used – By conditional probability, $P(A/B)=\frac{P(A\cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i) $P(A/B)$

$$=\frac{P(A\cap B)}{P(B)}$$

$$=\frac{4}{13}\div\frac{9}{13}$$

$$=\frac{4}{9}$$

(ii) $P(B/A)$

$$=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{4}{13}\div\frac{7}{13}$$

$$=\frac{4}{7}$$

(iii) $P(A\cup B)$

$$=P(A)+P(B)-P(A\cap B)$$

$$=\frac{7}{13}+\frac{9}{13}-\frac{4}{13}$$

$$=\frac{12}{13}$$

(iv) $P(\bar{B}/\bar{A}) = \frac{P(\bar{A}\cap\bar{B})}{P(\bar{A})}$

Now, by De-Morgan's Law, $(A \cup B)^C = A^C \cap B^C$

$\therefore P(\bar{A} \cap \bar{B}) = P\overline{(A \cup B)}$

$\therefore \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})}$

$= \frac{P\overline{(A \cup B)}}{P(\bar{A})}$

$= \frac{1 - P(A \cup B)}{1 - P(A)}$

$= \frac{1 - \frac{12}{13}}{1 - \frac{7}{13}}$

$= \frac{1}{6}$

2. Question

Let A and B be the events such that

$P(A) = \frac{5}{11}, P(B) = \frac{6}{11}$ and $P(A \cup B) = \frac{7}{11}$.

Find

(i) P(A ∩ B)

(ii) P(A / B)

(iii) P(B / A)

(iv) $P(\bar{A} / \bar{B})$

Answer

Given - A and B be the events such that $P(A) = \frac{5}{11}$, $P(B) = \frac{6}{11}$ and

$P(A \cup B) = \frac{7}{11}$

To find – (i) $P(A \cap B)$ (ii) $P(A/B)$ (iii)$P(B/A)$ (iv) $P(\bar{A}/\bar{B})$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A\cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{5}{11} + \frac{6}{11} - \frac{7}{11}$

$= \frac{4}{11}$

(ii) $P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{4}{11} \div \frac{6}{11}$

$= \frac{4}{6}$

$= \frac{2}{3}$

(iii) $P(B/A)$

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{4}{11} \div \frac{5}{11}$

$= \frac{4}{5}$

(iv) $P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

Now, by De-Morgan's Law, $(A \cup B)^C = A^C \cap B^C$

$\therefore P(\bar{A} \cap \bar{B}) = P\overline{(A \cup B)}$

$\therefore \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})}$

$= \frac{P\overline{(A \cup B)}}{P(\bar{A})}$

$= \frac{1 - P(A \cup B)}{1 - P(B)}$

$= \frac{1 - \frac{7}{11}}{1 - \frac{6}{11}}$

$= \frac{4}{5}$

3. Question

Let A and B be the events such that

$P(A) = \frac{3}{10}, P(B) = \frac{1}{2}$ and $P(B/A) = \frac{2}{5}$.

Find

(i) $P(A \cap B)$

(ii) $P(A \cup B)$

(iii) $P(A / B)$

Answer

Given - A and B be the events such that $P(A) = \frac{3}{10}$, $P(B) = \frac{1}{2}$ and

$P(B/A) = \frac{2}{5}$

To find – (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A/B)$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(B/A) = \frac{P(A \cap B)}{P(A)}$

⇨ $P(A \cap B) = P(A)P(B/A)$

$= \frac{3}{10} \times \frac{2}{5}$

$= \frac{3}{25}$

(ii) $P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{3}{10} + \frac{1}{2} - \frac{3}{25}$

$= \frac{15 + 25 - 6}{50}$

$= \frac{34}{50}$

$= \frac{17}{50}$

(iii) $P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{3}{25} \div \frac{1}{2}$

$= \frac{6}{25}$

4. Question

Let A and B be the events such that

$2P(A) = P(B) = \frac{5}{13}$ and $P(A/B) = \frac{2}{5}$.

Find

(i) P(A ∩ B)

(ii) P(A ∪ B).

Answer

Given - A and B be the events such that $2P(A) = P(B) = \frac{5}{13}$ and

$P(A/B) = \frac{2}{5}$

To find – (i) $P(A \cap B)$ (ii) $P(A \cup B)$

Formula to be used – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

(i)$P(A/B)=\frac{P(A\cap B)}{P(B)}$

$\Rightarrow P(A\cap B)=P(B)P(A/B)$

$=\frac{5}{13}\times\frac{2}{5}$

$=\frac{2}{13}$

(ii) $P(A\cup B)$

$=P(A)+P(B)-P(A\cap B)$

$=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}$

$=\frac{5+10-4}{26}$

$=\frac{11}{26}$

5. Question

A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

Answer

A die has 6 faces and its sample space S={1,2,3,4,5,6}.

The total number of outcomes = 6.

Let P(A) be the probability of getting an even number.

The sample space of A = {2,4,6}

$\therefore P(A)=\frac{3}{6}=\frac{1}{2}$

Let P(B) be the probability of getting a number whose value is greater than 2.

The sample space of B = {3,4,5,6}

$\therefore (A\cap B)=\{4,6\}$

$\therefore P(A\cap B)=\frac{2}{6}=\frac{1}{3}$

Tip – By conditional probability, $P(A/B)=\frac{P(A\cap B)}{P(B)}$ where $P(A/B)$ is the probability of occurrence of the event A given that B has already occurred.

The probability of getting a number greater than 2 given that the outcome is even is given by:

$P(B/A)$

$=\frac{P(A\cap B)}{P(A)}$

$=\frac{1/3}{1/2}$

$=\frac{2}{3}$

6. Question

A coin is tossed twice. If the outcome is at most one tail, what is the probability that both head and tail have appeared?

Answer

A coin has 2 sides and its sample space S={H,T}

The total number of outcomes = 2.

A coin is tossed twice.

Let P(A) be the probability of getting at most 1 tail.

The sample space of A = {(H,H),(H,T),(T,H)}

Let P(B) be the probability of getting a head.

The sample space of B = {H}

$\therefore \mathrm{P(B)} = \frac{1}{2}$

The probability of getting at most one tail and a head

$\mathrm{i.e.} (\mathrm{A} \cap \mathrm{B}) = \{(\mathrm{H}, \mathrm{H})\}$

$\therefore \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \frac{1}{3}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both head and tail have appeared:

$\mathrm{P(A/B)}$

$= \frac{\mathrm{P}(\mathrm{A} \cap \mathrm{B})}{\mathrm{P(B)}}$

$= \frac{1/3}{1/2}$

$= \frac{2}{3}$

7. Question

Three coins are tossed simultaneously. Find the probability that all coins show heads if at least one of the coins shows a head.

Answer

When three coins are tossed simultaneously, the total number of outcomes = 2^3 = 8, and the sample space is given by S= {(H,H,H),(H,H,T),(H,T,T),(H,T,H),(T,H,T),(T,T,H),(T,H,H),(T,T,T)}

Let P(A) be the probability of getting 3 heads.

The sample space of A = {(H,H,H)}

$\therefore \mathrm{P(A)} = \frac{1}{8}$

Let P(B) be the probability of getting at least head.

Probability of one head = 1 – probability of no heads = 1-1/8 = 7/8

$\therefore \mathrm{P(B)} = \frac{7}{8}$

The probability that the throw is either all heads or at least one head i.e. $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \frac{7}{8}$

Now,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B)$

$= P(A) + P(B) - P(A \cup B)$

$= \frac{1}{8} + \frac{7}{8} - \frac{7}{8}$

$= \frac{1}{8}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that all coins show heads if at least one of the coins

showed a head:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/8}{7/8}$

$= \frac{1}{7}$

8. Question

Two unbiased dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.

Answer

Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2=36

Let P(A) be the probability of getting a sum greater than 8.

Let P(B) be the probability of getting 4 on the first die.

The sample space of B = {(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)}

$\therefore P(B) = \frac{6}{36} = \frac{1}{6}$

Let $P(A \cap B)$ be the probability of getting 4 on the first die and the sum greater than or equal to 8

The sample space of $(A \cap B) = \{(4,4),(4,5),(4,6)\}$

$\therefore P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that sum of the numbers is greater than or equal to 8 given that 4 was thrown first:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/12}{1/6}$

$= \frac{1}{2}$

9. Question

A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?

Answer

A die thrown twice will have a total outcome of 6^2=36.

Let P(A) be the probability of getting the number 5 at least once.

Let P(B) be the probability of getting sum = 8.

The sample space of B = {(2,6),(3,5),(4,4),(5,3),(6,2)}

$\therefore P(B) = \frac{5}{36}$

Let $P(A \cap B)$ be the probability of getting the number 5 at least once and the sum equal to 8

The sample space of $(A \cap B) = \{(3,5),(5,3)\}$

$\therefore P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that the number 5 have appeared at least once given that the sum = 8:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/18}{5/36}$

$= \frac{2}{5}$

10. Question

Two dice were thrown and it is known that the numbers which come up were different. Find the probability that the sum of the two numbers was 5.

Answer

Two die having 6 faces each when tossed simultaneously will have a total outcome of 6^2=36

Let P(A) be the probability of getting a sum equal to 5.

Let P(B) be the probability of getting 2 different numbers.

Probability of getting 2 different numbers

= 1 – probability of getting same numbers

= 1 – 1/6

=5/6

$\therefore P(B) = \frac{5}{6}$

Let $P(A \cap B)$ be the probability of getting a sum = 5 and two different numbers at the same time.

The sample space of $(A \cap B) = \{(1,4), (2,3), (3,2), (4,1)\}$

$\therefore P(A \cap B) = \frac{4}{36} = \frac{1}{9}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that the sum = 5 given that two different numbers were thrown:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/9}{5/6}$

$= \frac{2}{15}$

11. Question

A coin is tossed and then a die is thrown. Find the probability of obtaining a 6, given that a head came up.

Answer

A coin is tossed and a die thrown.

A coin having two sides have a total outcome of 2 viz. {H,T}

A die has 6 faces and will have a total outcome of 6 i.e. {1, 2,3,4,5,6}

Let P(A) be the probability of getting the number 6.

$\therefore P(A) = \frac{1}{6}$

Let P(B) be the probability of getting a head.

The sample space of B ={H}

$\therefore P(B) = \frac{1}{2}$

Let $P(A \cap B)$ be the probability of getting the number 6 and a head.

$\therefore P(A \cap B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that 6 came up given that head came up:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/12}{1/2}$

$= \frac{1}{6}$

12. Question

A couple has 2 children. Find the probability that both are boys if it is known that (i) one of the children is a boy, and (ii) the elder child is a boy.

Answer

A couple has two children.

The sample space S = {(B,B),(B,G),(G,B),(G,G)}

Let P(A) be the probability of both being boys.

(i) Let P(B) be the probability of one of them being a boy.

The sample space of B ={(B,B),(B,G),(G,B)}

$\therefore \mathrm{P(B)} = \frac{3}{4}$

Let $\mathrm{P(A \cap B)}$ be the probability of one of them being a boy and both being boys.

$\therefore \mathrm{(A \cap B)} = \{\mathrm{(B,B)}\}$

$\therefore \mathrm{P(A \cap B)} = \frac{1}{4}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both are boys given that one of them is a boy:

$\mathrm{P(A/B)}$

$= \frac{\mathrm{P(A \cap B)}}{\mathrm{P(B)}}$

$= \frac{1/4}{3/4}$

$= \frac{1}{3}$

(ii) Let P(B) be the probability of the elder being a boy.

The sample space of B ={(B,B),(B,G)}

$\therefore \mathrm{P(B)} = \frac{1}{2}$

Let $\mathrm{P(A \cap B)}$ be the probability of the elder being a boy and both being boys.

$\therefore \mathrm{(A \cap B)} = \{\mathrm{(B,B)}\}$

$\therefore \mathrm{P(A \cap B)} = \frac{1}{4}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that both are boys given that the elder is a boy:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/4}{1/2}$

$= \frac{1}{2}$

13. Question

In a class, 40% students study mathematics; 25% study biology and 15% study both mathematics and biology. One student is selected at random. Find the probability that

(i) he studies mathematics if it is known that he studies biology

(ii) he studies biology if it is known that he studies mathematics.

Answer

Let P(A) be the probability of students studying mathematics.

$\therefore P(A)=0.40$

Let P(B) be the probability of students studying biology.

$\therefore P(B)=0.25$

Let $P(A \cap B)$ be the probability of students studying both mathematics and biology.

$\therefore P(A \cap B)=0.15$

One student is selected at random.

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

(i)The probability that he studies mathematics given that he studies biology:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{0.15}{0.25}$

$= \frac{3}{5}$

(ii)The probability that he studies biology given that he studies mathematics:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{0.15}{0.40}$

$= \frac{3}{8}$

14. Question

The probability that a student selected at random from a class will pass in Hindi is $\frac{4}{5}$ and the probability that he passes in Hindi and English is $\frac{1}{2}$. What is the probability that he will pass in English if it is known that he has passed in Hindi?

Answer

One student is selected at random.

Let P(A) be the probability of students passing in English.

Let P(B) be the probability of students passing in Hindi.

$\therefore P(B) = \frac{4}{5}$

Let $P(A \cap B)$ be the probability of students passing in both English and Hindi.

$\therefore P(A \cap B) = \frac{1}{2}$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that he will pass in English given that he passes in Hindi:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{1/2}{4/5}$

$= \frac{5}{8}$

15. Question

The probability that a certain person will buy a shirt is 0.2, the probability that he will buy a coat is 0.3 and the probability that he will buy a shirt given that he buys a coat is 0.4. Find the probability that he will buy both a shirt and a coat.

Answer

Let P(A) be the probability of a certain person buying a shirt.

$\therefore P(A)=0.2$

Let P(B) be the probability of him buying a coat.

$\therefore P(B)=0.3$

Let $P(A \cap B)$ be the probability that he buys both a shirt and a coat.

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

The probability that he will buy a shirt given that he buys a coat:

$P(A/B) = \frac{P(A \cap B)}{P(B)} = 0.4$

$\Rightarrow P(A \cap B)=P(B)P(A/B)$

$=0.3 \times 0.4$

$= 0.12$

16. Question

In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.

(i) Find the probability that he reads neither Hindi nor English news paper.

(ii) If he reads Hindi newspaper, what is the probability that he reads English newspaper?

(iii) If he reads English newspaper, what is the probability that he reads Hindi newspaper?

Answer

Let P(A) be the probability of students reading Hindi newspaper.

$\therefore P(A)=0.60$

Let P(B) be the probability of them reading English newspaper.

$\therefore P(B)=0.40$

Let $P(A \cap B)$ be the probability them reading both.

$\therefore P(A \cap B) = 0.20$

Let $P(A \cup B)$ be the probability them reading either one of them.

$\therefore P(A \cup B)$

$=P(A)+P(B)-P(A \cap B)$

$=0.60+0.40-0.20$

$=0.80$

(i)The probability that none of them reads either of them

$= 1 - 0.8$

$= 0.2$

$=1/5$

Tip – By conditional probability, $P(A/B) = \frac{P(A \cap B)}{P(B)}$ *where* $P(A/B)$ *is the probability of occurrence of the event A given that B has already occurred.*

(ii)The probability that he reads the English one given that he reads the Hindi one:

$P(A/B)$

$= \frac{P(A \cap B)}{P(A)}$

$= \frac{0.20}{0.60}$

$= \frac{1}{3}$

(iii)The probability that he reads the Hindi one given that he reads the English one:

$P(A/B)$

$= \frac{P(A \cap B)}{P(B)}$

$= \frac{0.20}{0.40}$

$=\frac{1}{2}$

17. Question

Two integers are selected at random from integers 1 through 11. If the sum is even, find the probability that both the numbers selected are odd.

Answer

Two integers are selected at random.

The first choice has 11 options from the 11 integers, and the second choice has 10 options from the remaining 10 integers.

Let P(A) be the probability of choosing both numbers odd.

Let P(B) be the probability of choosing the numbers to yield an even number.

Sample space of B = {(1,3),(1,5),(1,7),(1,9),(1,11),(3,5),(3,7),(3,9),(3,11),(5,7),(5,9),(5,11),(7,9),(7,11),(9,11),(2,4),(2,6),(2,8),(2,10),(4,6),(4,8),(4,10),(6,8),(6,10),(8,10)}

$\therefore P(B)=\frac{25}{11\times 10}=\frac{25}{110}$

Let $P(A\cap B)$ be the probability of getting both odd numbers giving an even sum.

∴(A∩B)={(1,3),(1,5),(1,7),(1,9),(1,11),(3,5),(3,7),(3,9),(3,11),(5,7),

(5,9),(5,11),(7,9),(7,11),(9,11),}

$\therefore P(A\cap B)=\frac{15}{110}$

The probability of getting both numbers odd given that sum is even:

$P(A/B)$

$=\frac{P(A\cap B)}{P(B)}$

$=\frac{15/110}{25/110}$

$=\frac{15}{25}$

$=\frac{3}{5}$

Exercise 29B

1. Question

A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn, and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.

Answer

Given: A bag contains 17 tickets , numbered 1 to 17, and each trial is independent of the other.

Hence the sample space is given by S = {1,2,3,......,17}

To find: the probability that both the tickets are drawn show even numbers.

Let , success : ticket drawn is even.i.e $\frac{8}{17}$

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{8}{17}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{7}{16}$

Hence , the probability that both the tickets show even numbers with each trial being independent is given by

$P_1 \times P_2 = \frac{8}{17} \times \frac{7}{16} = \frac{7}{34}$

2. Question

Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black if the first marble is not replaced before the second draw.

Answer

Given: A box containing 3 black and 4 white marbles .Each trail is independent of the other trial.

Hence the sample space is given by S = {1B,2B,3B,1W,2W,3W,4W}

To find: the probability that both the marbles are drawn are black.

Let , success : marble drawn is black.i.e $\frac{3}{7}$

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{3}{7}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{2}{6}$

Hence , the probability that both the marbles are drawn are black ,with each trial being independent is given by

$P_1 \times P_2 = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7}$

3. Question

A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade.

Answer

Given: a well shuffled deck of 52 cards. Each draw is independent of the other.

To find: the probability that the first card is drawn is a club and the second card is a spade.

Let , success for the first trail be getting a club.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{13}{52}$

let , success for the second trail be getting a spade.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{13}{51}$

Hence , the probability that the first card is drawn is a club and the second card is a spade ,with each trial being independent is given by

$P_1 \times P_2 = \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$

4. Question

There is a box containing 30 bulbs, of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs are chosen are defective?

Answer

Given: A box containing 30 bulbs of which 5 are defective.Each trail is independent of the other trial.

To find: the probability that both the bulbs are chosen are defective.

Let , success :bulb chosen is defective .i.e $\frac{5}{30}$

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{5}{30}$

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{4}{29}$

Hence , the probability that both the bulbs are chosen are defective,with each trial being independent is given by

$P_1 \times P_2 = \frac{5}{30} \times \frac{4}{29} = \frac{2}{87}$

5. Question

A bag contains 1.0 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black?

Answer

Given: A bag containing 10 white and 15 black balls .Each trial is independent of the other trial.

To find: the probability that the first ball is drawn is white and the second ball drawn is black.

Let , success in the first draw be getting a white ball.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{10}{25}$

Let success in the second draw be getting a black ball.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{15}{24}$

Hence , the probability that the first ball is drawn is white and the second ball drawn is black,with each trial being independent is given by

$P_1 \times P_2 = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$

6. Question

An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.

Answer

Given: An urn containing 5 white and 8 black balls .Each trial is independent of the other trial.

To find: the probability that the first draws gives 3 white and the second draw gives 3 black balls.

Let , success in the first draw be getting 3 white balls.

Now , the Probability of success in the first trial is

$P_1(\text{success}) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

Let success in the second draw be getting 3 black balls.

Probability of success in the second trial without replacement of the first draw is given by

$P_2(\text{success}) = \frac{{}^8C_3}{{}^{10}C_3} = \frac{56}{120} = \frac{7}{15}$

Hence , the probability that the first draws gives 3 white and the second draw gives 3 black balls,with each trial being independent is given by

$P_1 \times P_2 = \frac{5}{143} \times \frac{7}{15} = \frac{7}{429}$

7. Question

Let E_1 and E_2 be the events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$.

Find:

(i)$P(E_1 \cup E_2)$, when E_1 and E_2 are mutually exclusive.

(ii)$P(E_1 \cap E_2)$, when E_1 and E_2 are independent

Answer

Given: E_1 and E_2 are two events such that $P(E_1) = \frac{1}{3}$ and $P(E_2) = \frac{3}{5}$

To Find: i)$P(E_1 \cup E_2)$ when E_1 and E_2 are mutually exclusive.

We know that,

When two events are mutually exclusive $P(E_1 \cap E_2) = 0$

Hence, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$= \frac{1}{3} + \frac{3}{5}$

$= \frac{14}{15}$

Therefore , $P(E_1 \cup E_2) = \frac{14}{15}$ when E_1 and E_2 are mutually exclusive.

ii) $P(E_1 \cap E_2)$ when E_1 and E_2 are independent.

We know that when E_1 and E_2 are independent ,

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{3} \times \frac{3}{5}$

$= \frac{1}{5}$

Therefore, $P(E_1 \cap E_2) = \frac{1}{5}$ when E_1 and E_2 are independent.

8. Question

If E_1 and E_2 are the two events such that $P(E_1) = \frac{1}{4}, P(E_2) = \frac{1}{3}$ and $P(E_1 \cup E_2) = \frac{1}{2}$, show that E_1 and E_2 are independent events.

Answer

Given: E_1 and E_2 are two events such that $P(E_1) = \frac{1}{4}$ and $P(E_2) = \frac{1}{3}$ and

$P(E_1 \cup E_2) = \frac{1}{2}$

To show: E_1 and E_2 are independent events.

We know that,

Hence, $P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$

$= \frac{1}{4} + \frac{1}{3} - \frac{1}{2}$

$= \frac{1}{12}$ Equation 1

Since The condition for two events to be independent is

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$ Equation 2

Since, Equation 1 = Equation 2

$\Rightarrow$ E_1 and E_2 are independent events.

Hence proved.

9. Question

If E_1 and E_2 are independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$, find

(i) $P(E_1 \cap E_2)$

(ii) $P(E_1 \cap E_2)$

(iii) $P(\overline{E}_1 \cap \overline{E}_2)$

(iv) $P(\overline{E}_1 \cap E_2)$

Answer

Given: E_1 and E_2 are two independent events such that $P(E_1) = 0.3$ and $P(E_2) = 0.4$

To Find: i)$P(E_1 \cap E_2)$

We know that,

when E_1 and E_2 are independent ,

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= 0.3 \times 0.4$

$= 0.12$

Therefore, $P(E_1 \cap E_2) = 0.12$ when E_1 and E_2 are independent.

ii) P($E_1 \cup E_2$) when E_1 and E_2 are independent.

We know that,

Hence, P($E_1 \cup E_2$) = P(E_1) + P(E_2) - P($E_1 \cap E_2$)

= 0.3 + 0.4 – (0.3 $\times$0.4)

= 0.58

Therefore , P($E_1 \cup E_2$) = 0.58 when E_1 and E_2 are Independent.

iii) P($\overline{E_1} \cap \overline{E_2}$) = P($\overline{E_1}$) $\times$ P($\overline{E_2}$)

since , P(E_1) = 0.3 and P(E_2) = 0.4

$\Rightarrow$ P($\overline{E_1}$) = 1 - P(E_1) = 0.7 and P($\overline{E_2}$) = 1 - P(E_2) = 0.6

Since, E_1 and E_2 are two independent events

$\Rightarrow$ $\overline{E_1}$ and $\overline{E_2}$ are also independent events.

Therefore, P($\overline{E_1} \cap \overline{E_2}$) = 0.7 $\times$ 0.6 = 0.42

iv) P($\overline{E_1} \cap E_2$) = P($\overline{E_1}$) $\times$ P(E_2)

= 0.7 $\times$ 0.4

= 0.28

Therefore , P($\overline{E_1} \cap E_2$) = 0.28

10. Question

Let A and B be the events such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and P(not A or not B) $=\frac{1}{4}$.

State whether A and B are

(i) mutually exclusive

(ii) independent

Answer

Given: A and B are the events such that P(A) =$\frac{1}{2}$ and P(B) = $\frac{7}{12}$ and

P(not A or not B) =$\frac{1}{4}$

To Find: i)If A and B are mutually exclusive

Since P(not A or not B) =$\frac{1}{4}$ i.e., P($\bar{A} \cup \bar{B}$) = $\frac{1}{4}$

we know that , P($\bar{A} \cup \bar{B}$) = P(A $\cap$ B)’= 1- P(A $\cap$ B) =$\frac{1}{4}$

$\Rightarrow$ P(A $\cap$ B) = 1- $\frac{1}{4}$ = $\frac{3}{4}$ Equation 1

Since for two mutually exclusive events P(A $\cap$ B) = 0

But here P(A $\cap$ B) $\neq$ 0

Therefore , A and B are not mutually exclusive.

ii)If A and B are independent

The condition for two events to be independent is given by

$P(E_1 \cap E_2) = P(E_1) \times P(E_2)$

$= \frac{1}{2} \times \frac{7}{12}$

$= \frac{7}{24}$ Equation 2

Since Equation 1 $\neq$ Equation 2

$\Rightarrow$ A and B are not independent

11. Question

Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal's selection is 1/3, and that of Vimal's selection is 3. Find the probability that only one of them will be selected.

Answer

Given: let A denote the event 'kamal is selected' and let B denote the event 'vimal is selected'.

Therefore , P(A) $= \frac{1}{3}$ and P(B) $= \frac{1}{5}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find:The probability that only one of them will be selected.

Now ,

P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

$= P(A \cap \bar{B}) + P(B \cap \bar{A})$

$= P(A) \times P(\bar{B}) + P(B) \times P(\bar{A})$

$= P(A) \times [1-P(B)] + P(B) \times [1-P(A)]$

$= \frac{1}{3}[1-\frac{1}{5}] + \frac{1}{5}[1-\frac{1}{3}]$

$= \frac{4}{15} + \frac{2}{15}$

$= \frac{2}{5}$

Therefore , The probability that only one of them will be selected is $\frac{2}{5}$

12. Question

Arun and Ved appeared for an interview for two vacancies. The probability of Arun's selection is 1/4, and that of Ved's rejection is 2/3. Find the probability that at least one of them will be selected.

Answer

Given : let A denote the event 'Arun is selected' and let B denote the event 'ved is selected'.

Therefore , P(A) $= \frac{1}{4}$ and $P(\bar{B}) = \frac{2}{3} \Rightarrow P(B) = \frac{1}{3}$ and $P(\bar{A}) = \frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: The probability that atleast one of them will get selected.

Now,

P(atleast one of them getting selected) = P(selecting only Arun) + P(selecting only ved) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$) + P(A $\cap$ B)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$) + P(A) $\times$P(B)

= $(\frac{1}{4} \times \frac{2}{3}) + (\frac{1}{3} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{1}{3})$

= $\frac{2}{12} + \frac{3}{12} + \frac{1}{12}$

= $\frac{1}{2}$

Therefore , The probability that atleast one of them will get selected is $\frac{1}{2}$

13. Question

A and B appear for an interview for two vacancies in the same post. The probability of A's selection is 1/6 and that of B's selection is 1/4. Find the probability that

(i) both of them are selected

(ii) only one of them is selected

(iii) none is selected

(iv) at least one of them is selected.

Answer

Given : A and B appear for an interview ,then P(A) =$\frac{1}{6}$ and P(B) = $\frac{1}{4}$ $\Rightarrow$ P($\bar{A}$) = $\frac{5}{6}$ and P($\bar{B}$) = $\frac{3}{4}$

Also, A and B are independent .A and not B are independent, not A and B are independent.

To Find: i) The probability that both of them are selected.

We know that, P(both of them are selected) = P(A $\cap$ B) = P(A) $\times$P(B)

= $\frac{1}{6} \times \frac{1}{4}$

= $\frac{1}{24}$

Therefore , The probability that both of them are selected is $\frac{1}{24}$

ii) P(only one of them is selected) = P(A and not B or B and not A)

= P(A and not B) + (B and not A)

= P(A $\cap$ $\bar{B}$) + P(B $\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= $(\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6})$

= $\frac{3}{24} + \frac{5}{24}$

= $\frac{1}{3}$

Therefore, the probability that only one of them Is selected is $\frac{1}{3}$

iii)none is selected

we know that P(none is selected) = P($\bar{A}$ $\cap$ $\bar{B}$)

$= P(\bar{A}) \times P(\bar{B})$

$= \frac{5}{6} \times \frac{3}{4}$

$= \frac{5}{8}$

Therefore , the probability that none is selected is $\frac{5}{8}$

iv) atleast one of them is selected

Now, P(atleast one of them is selected) = P(selecting only A) + P(selecting only B) + P(selecting both)

= P(A and not B) +P (B and not A) +P (A and B)

$= P(A \cap \bar{B}) + P(B \cap \bar{A}) + P(A \cap B)$

$= P(A) \times P(\bar{B}) + P(B) \times P(\bar{A}) + P(A) \times P(B)$

$= (\frac{1}{6} \times \frac{3}{4}) + (\frac{1}{4} \times \frac{5}{6}) + (\frac{1}{6} \times \frac{1}{4})$

$= \frac{3}{24} + \frac{5}{24} + \frac{1}{24}$

$= \frac{3}{8}$

Therefore, the probability that atleast one of them is selected is $\frac{3}{8}$

14. Question

Given the probability that A can solve a problem is 2/3, and the probability that B can solve the same problem is %, find the probability that

(i)at least one of A and B will solve the problem

(ii)none of the two will solve the problem

Answer

Given : Here probability of A and B that can solve the same problem is given , i.e., $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}$ and $P(\bar{B}) = \frac{2}{5}$

Also, A and B are independent . not A and not B are independent.

To Find: i) atleast one of A and B will solve the problem

Now , P(atleast one of them will solve the problem) = 1 – P(both are unable to solve)

$= 1 - P(\bar{A} \cap \bar{B})$

$= 1 - P(\bar{A}) \times P(\bar{B})$

$= 1 - (\frac{1}{3} \times \frac{2}{5})$

$= \frac{13}{15}$

Therefore , atleast one of A and B will solve the problem is $\frac{13}{15}$

ii) none of the two will solve the problem

Now, P(none of the two will solve the problem) = $P(\bar{A} \cap \bar{B})$

$= P(\bar{A}) \times P(\bar{B})$

$= \frac{1}{3} \times \frac{2}{5}$

$= \frac{2}{15}$

Therefore , none of the two will solve the problem is $\frac{2}{15}$

15. Question

A problem is given to three students whose chances of solving it are 1/4, 1/5 and 1/6, respectively. Find the probability that the problem is solved.

Answer

Given : let A , B and C be three students whose chances of solving a problem is given i.e , P(A) $= \frac{1}{4}$, P(B) $= \frac{1}{5}$ and P(C) $= \frac{1}{6}$.

$\Rightarrow$ P($\bar{A}$) $= \frac{3}{4}$, P($\bar{B}$) $= \frac{4}{5}$ and P($\bar{C}$)$= \frac{5}{6}$

To Find: The probability that the problem is solved .

Here , P(the problem is solved) = 1 - P(the problem is not solved)

= 1- P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= 1 – [P($\bar{A}$) × P($\bar{B}$) × P($\bar{C}$)]

$= 1 - [\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}]$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$

Therefore , The probability that the problem is solved is $\frac{1}{2}$.

16. Question

The probabilities of A, B, C solving a problem are 1/3, 1/4 and 1/6, respectively. If all the three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

Answer

Given : let A , B and C be three students whose chances of solving a problem is given i.e , P(A) $= \frac{1}{3}$, P(B) $= \frac{1}{4}$ and P(C) $= \frac{1}{6}$.

$\Rightarrow$ P($\bar{A}$) $= \frac{2}{3}$, P($\bar{B}$) $= \frac{3}{4}$ and P($\bar{C}$)$= \frac{5}{6}$

To Find: The probability that excatly one of them will solve it .

Now, P(excatly one of them will solve it) = P(A and not B and not c) +P (B and not A and not C) +P (C and not A and not B)

= P(A $\cap \bar{B} \cap \bar{C}$) + P(B $\cap \bar{A} \cap \bar{C}$) + P(C $\cap \bar{A} \cap \bar{B}$)

= P(A) ×P($\bar{B}$) × P($\bar{C}$) + P(B) × P($\bar{A}$) × P($\bar{C}$) + P(C) ×P($\bar{B}$) × P($\bar{A}$)

$= [\frac{1}{3} \times \frac{3}{4} \times \frac{5}{6}] + [\frac{1}{4} \times \frac{2}{3} \times \frac{5}{6}] + [\frac{1}{6} \times \frac{3}{4} \times \frac{2}{3}]$

$= \frac{15}{72} + \frac{10}{72} + \frac{6}{72}$

$= \frac{31}{72}$

Therefore , The probability that excatly one of them will solve the problem is $\frac{31}{72}$

17. Question

A can hit a target 4 times in 5 shots, B can hit 3 times in 4 shots, and C can hit 2 times in 3 shots. Calculate the probability that

(i) A, B and C all hit the target

(ii) B and C hit and A does not hit the target.

Answer

Given : let A , B and C chances of hitting a target is given i.e , P(A) $= \frac{4}{5}$, P(B) $= \frac{3}{4}$ and P(C) $= \frac{2}{3}$.

$\Rightarrow$ P($\bar{A}$) $= \frac{1}{5}$, P($\bar{B}$) $= \frac{1}{4}$ and P($\bar{C}$)$= \frac{1}{3}$

To Find: i)The probability that A , B and C all hit the target.

Now ,P(all hitting the target) = P($A \cap$ B $\cap$ C)

= P(A) $\times$P(B) $\times$ P(C)

$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3}$

$= \frac{2}{5}$

Hence , The probability that A , B and C all hit the target is $\frac{2}{5}$

ii) B and C hit and A does not hit the target

Here, P(B and C hit and not A) = P(B $\cap$ C $\cap$ $\bar{A}$)

= P(B) $\times$P(C) $\times$ P($\bar{A}$)

$= \frac{3}{4} \times \frac{2}{3} \times \frac{1}{5}$

$= \frac{1}{10}$

Hence , the probability that B and C hit and A does not hit the target is $\frac{1}{10}$

18. Question

Neelam has offered physics, chemistry and mathematics in Class XII. She estimates that her probabilities of receiving a grade A in these courses are 0.2, 0.3 and 0.9 respectively. Find the probabilities that Neelam receives

(i) all A grades

(ii) no A grade

(iii) exactly 2 A grades.

Answer

Given : let A , B and C represent the subjects physics,chemistry and mathematics respectively ,the probability of neelam getting A grade in these three subjects is given i.e , P(A) = 0.2 , P(B) = 0.3 and P(C) =0.9

$\Rightarrow$ P($\bar{A}$) = 0.8 , P($\bar{B}$) = 0.7 and P($\bar{C}$)= 0.1

To Find: i)The probability that neelam gets all A grades

Here, P(getting all A grades) = P($A \cap B \cap C$)

= P(A) $\times$P(B) $\times$ P(C)

=0.2 $\times$ 0.3 $\times$ 0.9

= 0.054

Therefore, The probability that neelam gets all A grades is 0.054.

ii)no A grade

Here , P(getting no A grade) = P($\bar{A} \cap \bar{B} \cap \bar{C}$)

= P($\bar{A}$) $\times$ P($\bar{B}$) $\times$ P($\bar{C}$)

= 0.8 $\times$ 0.7 $\times$0.1

= 0.056

Therefore, The probability that neelam gets no A grade is 0.056.

iii)excatly 2 a grades

P(getting excatly 2 A grades) = P(A and B and not c) +P (B and C and not A) +P (C and A and not B)

= P($A \cap B \cap \bar{C}$) + P($B \cap C \cap \bar{A}$) + P($C \cap A \cap \bar{B}$)

= P(A) $\times$P(B) $\times$ P($\bar{C}$) + P(B) $\times$ P(C) $\times$ P($\bar{A}$) + P(C) $\times$P(A) $\times$ P($\bar{B}$)

= [0.2 $\times$ 0.3 $\times$0.1] + [0.3 $\times$ 0.9 $\times$ 0.8] + [0.9 $\times$ 0.2 $\times$0.7]

= 0.006 + 0.216 +0.126

= 0.348

Therefore, The probability that neelam gets excatly 2 A grades is 0.348.

19. Question

An article manufactured by a company consists of two parts X and Y. In the process of manufacture of part X. 8 out of 100 parts may be defective. Similarly, 5 out of 100 parts of Y may be defective. Calculate the probability that the assembled product will not be defective.

Answer

Given: X and Y are the two parts of a company that manufactures an article.

Here the probability of the parts being defective is given i.e, P(X) =$\frac{8}{100}$ and P(Y) = $\frac{5}{100}$ $\Rightarrow$ P($\bar{X}$) = $\frac{92}{100}$ and P($\bar{Y}$) = $\frac{95}{100}$

To Find: the probability that the assembled product will not be defective.

Here,

P(product assembled will not be defective) = 1 – P(product assembled to be defective)

=1 – [P(X and not Y) + P(Y and not X) +P(both)]

= 1- [P($X \cap \bar{Y}$) + P($Y \cap \bar{X}$) + P($X \cap Y$)]

= 1- [P(X) $\times$P($\bar{Y}$) + P(Y) $\times$ P($\bar{X}$) + P(X) $\times$P(Y)]

= 1-[($\frac{8}{100} \times \frac{95}{100}$) + ($\frac{5}{100} \times \frac{92}{100}$)+ ($\frac{8}{100} \times \frac{5}{100}$)]

=1- [$\frac{760}{10000} + \frac{460}{10000} + \frac{40}{10000}$]

$= \frac{437}{500}$

Therefore, The probability that the assembled product will not be defective is $\frac{437}{500}$.

20. Question

A town has two fire-extinguishing engines, functioning independently. The probability of availability of each engine when needed is 0.95. What is the probability that

(i) neither of them is available when needed?

(ii) an engine is available when needed?

Answer

Given: Let A and B be two fire extinguishing engines . The probability of availability of each of the two fire extinguishing engines is given i.e., P(A) = 0.95 and P(B) = 0.95 $\Rightarrow$ P($\bar{A}$) = 0.05 and P($\bar{B}$) = 0.05

To Find: i) The probability that neither of them is available when needed

Here, P(not A and not B) = P($\bar{A} \cap \bar{B}$)

= P($\bar{A}$) $\times$P($\bar{B}$)

=0.05 $\times$ 0.05

= 0.0025 = $\frac{1}{400}$

Therefore, The probability that neither of them is available when needed is $\frac{1}{400}$

ii)an engine is available when needed

Here, P(A and not B or B and not A) = P(A$\cap$ $\bar{B}$) + P(B$\cap$ $\bar{A}$)

= P(A) $\times$P($\bar{B}$) + P(B) $\times$ P($\bar{A}$)

= (0.95 $\times$0.05) + (0.95 $\times$ 0.05)

= 0.0475 + 0.0475

= 0.095

$= \frac{19}{200}$

Therefore, The probability that an engine is available when needed is $\frac{19}{200}$

21. Question

A machine operates only when all of its three components function. The probabilities of the failures of the first, second and third components are 0.14, 0.10 and 0.05, respectively. What is the probability that the machine will fail?

Answer

Given: let A ,B and C be the three components of a machine which works only if all its three compenents function.the probabilities of the failures of A,B

and C respectively is given i.e, P(A) = 0.14 ,P(B) = 0.10 and P(C) = 0.05

$\Rightarrow$ P($\bar{A}$) = 0.86 and P($\bar{B}$) = 0.90 and P($\bar{C}$) = 0.95

To Find: The probability that the machine will fail.

Here, P(the machine will fail) = 1 – P(the machine will function)

= 1 – P(all three components are working)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C})$

$= 1 – [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})]$

$= 1 - [0.86 \times 0.90 \times 0.95]$

$= 1 – 0.7353$

$= 0.2647$

Therefore, The probability that the machine will fail is 0.2647.

22. Question

An anti-aircraft gun can take a maximum of 4 shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shots are 0.4, 0.3, 0.2 and 0.1 respectively. What is the probability that at least one shot hits the plane?

Answer

Given:Let A ,B , C and Dbe first second third and fourth shots whose probability of hitting the plane is given i.e, P(A) = 0.4 ,P(B) = 0.3 , P(C) = 0.2 and P(D) = 0.1 respectively

$\Rightarrow P(\bar{A}) = 0.6$ and $P(\bar{B}) = 0.7$ and $P(\bar{C}) = 0.8$ and $P(\bar{D}) = 0.9$

To Find: The probability that atleast one shot hits the plane .

Here , P(atleast one shot hits the plane) = 1 – P(none of the shots hit the plane)

$= 1 - P(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D})$

$= 1 – [P(\bar{A}) \times P(\bar{B}) \times P(\bar{C}) \times P(\bar{D})]$

$= 1 - [0.6 \times 0.7 \times 0.8 \times 0.9]$

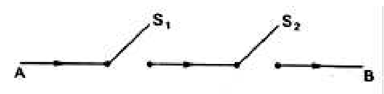
$= 1 – 0.3024$

$= 0.6976$

Therefore, The probability that atleast one shot hits the plane is 0.6976.

23. Question

Let S_1 and S_2 be the two switches and let their probabilities of working be given by $P(S_1) = 4/5$ and $P(S_2) = 9/10$. Find the probability that the current flows from the terminal A to terminal B when S_1 and S_2 are installed in series, shown as follows:



Answer

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{4}{5}$ and $P(S_2) = \frac{9}{10}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in series.

Now, since the current in series flows from end to end

$\Rightarrow$ the flow of current from terminal A to terminal B is given by

$P(S_1 \cap S_2) = P(S_1) \times P(S_2)$

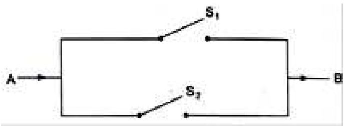
$= \frac{4}{5} \times \frac{9}{10}$

$= \frac{18}{25}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in series is $\frac{18}{25}$

24. Question

Let S_1 and S_2 be two the switches and let their probabilities of working be given by $P(S_1) = 2/3$ and $P(S_2) = 3/_4$. Find the probability that the current flows from terminal A to terminal B, when S_1 and S_2 are installed in parallel, as shown below:



Answer

Given: S_1 and S_2 are two swiches whose probabilities of working be given by

$P(S_1) = \frac{2}{3}$ and $P(S_2) = \frac{3}{4}$

To Find: the probability that the current flows from terminal A to terminal B when

S_1 and S_2 are connected in parallel.

Now, since current in parallel flows in two or more paths and hence the sum of currents through each path is equal to total current that flows from the source.

⇒ the flow of current from terminal A to terminal B in a parallel circuit is given by

$P(S_1 \cup S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2)$

$= P(S_1) + P(S_2) - [\,P(S_1) \times P(S_2)]$

$= \frac{2}{3} + \frac{3}{4} - \frac{1}{2}$

$= \frac{11}{12}$

Therefore, The probability that the current flows from terminal A to terminal B when S_1 and S_2 are connected in parallel is $\frac{11}{12}$

25. Question

A coin is tossed. If a head comes up, a die is thrown, but if a tail comes up, the coin is tossed again. Find the probability of obtaining

(i) two tails

(ii) a head and the number 6

(iii) a head and an even number.

Answer

Given : let H be head, and T be tails where as 1,2,3,4,5,6 be the numbers on the dice which are thrown when a head comes up or else coin is tossed again if its tail.

According to the question ,sample space S = {(TH),(TT) ,(H1),(H2),(H3),(H4),(H5),(H6)}

To Find: i)the probability of obtaining two tails

From sample space, it is clear that the probability of obtaining two tails is $\frac{1}{8}$

i.e., {TT} with total no of elements in sample space as 8.

ii) the probability of obtaining a head and the number 6

From sample space, it is clear that the probability of obtaining a head and the number 6 is $\frac{1}{8}$

i.e., {H6} with total no of elements in sample space as 8.

iii) the probability of obtaining a head and an even number

From sample space, it is clear that the probability of obtaining a head and an even number is $\frac{3}{8}$

i.e, {H2,H4,H6} with total no of elements in sample space as 8.