24. Cross, or Vector, Product of Vectors

Exercise 24

1 A. Question

Find $\left(\vec{a}\times\vec{b}\right)$ and $\left|\vec{a}\times\vec{b}\right|$, when

$\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}=2\hat{i}+3\hat{j}-4\hat{k}$

Answer

$\vec{a}\times\vec{b}=(a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(a_1b_2-b_1a_2)k$

Here,

We

have

$\vec{a}=i-j+2k$ and $\vec{b}=2i+3j-4k$

$\Rightarrow a_1=1, a_2=-1, a_3=2$ and $b_1=2, b_2=3, b_3=-4$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b}=((-1\times-4)-3\times2)i+(2\times2-(-4)\times1)j+(1\times3-2\times(-1))k$

$\Rightarrow |a\times b|=\sqrt{(-2)^2+8^2+5^2}$

$\vec{a}\times\vec{b}=\left(-2\hat{i}+8\hat{j}+5\hat{k}\right)$ and $\left|\vec{a}\times\vec{b}\right|=\sqrt{93}$

1 B. Question

Find $\left(\vec{a}\times\vec{b}\right)$ and $\left|\vec{a}\times\vec{b}\right|$, when

$\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$

Answer

$\vec{a}\times\vec{b}=(a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(a_1b_2-b_1a_2)k$

Here,

We

have $\vec{a}=2i-j+3k$ and $\vec{b}=3i+5j-2k$

$\Rightarrow a_1=2, a_2=-1, a_3=3$ and $b_1=3, b_2=5, b_3=-2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b}=((-1\times-2)-5\times3)i+(3\times3-(-2)\times2)j+(2\times5-3\times(-1))k$

$\Rightarrow |a\times b|=\sqrt{(-17)^2+13^2+7^2}=13\sqrt{3}$

$\Rightarrow \vec{a}\times\vec{b}=(-17)i+(13)j+(7)k$

1 C. Question

Find $\left(\vec{a}\times\vec{b}\right)$ and $\left|\vec{a}\times\vec{b}\right|$, when

$\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$

Answer

$\vec{a}\times\vec{b}=(a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(a_1b_2-b_1a_2)k$

Here,

We

have $\vec{a}=i-7j+7k$ and $\vec{b}=3i-2j+2k$

$\Rightarrow a_1=1, a_2=-7, a_3=7$ and $b_1=3, b_2=-2, b_3=2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b}=((-7\times 2)-(-2)\times 7)i+(7\times 3-1\times 2)j+((-2)\times 1-3\times(-7))k$

$\Rightarrow |a\times b|=\sqrt{(0)^2+19^2+19^2}=19\sqrt{2}$

$\Rightarrow \vec{a}\times\vec{b}=(0)i+(19)j+(19)k$

1 D. Question

Find $\left(\vec{a}\times\vec{b}\right)$ and $\left|\vec{a}\times\vec{b}\right|$, when

$\vec{a}=4\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=3\hat{i}+\hat{k}$

Answer

$\vec{a}\times\vec{b}=(a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(a_1b_2-b_1a_2)k$

Here,

We

have $\vec{a}=4i+j-2k$ and $\vec{b}=3i+0j+k$

$\Rightarrow a_1=4, a_2=1, a_3=-2$ and $b_1=3, b_2=0, b_3=1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b}=(1\times 1-(0)\times -2)i+(-2\times 3-1\times 4)j+(4\times 0-3\times 1)k$

$\Rightarrow |a\times b|=\sqrt{1^2+(-10)^2+(-3)^2}=\sqrt{110}$

$\Rightarrow \vec{a}\times\vec{b}=i-10j-3k$

1 E. Question

Find $\left(\vec{a}\times\vec{b}\right)$ and $\left|\vec{a}\times\vec{b}\right|$, when

$\vec{a}=3\hat{i}+4\hat{j}$ and $\vec{b}=\hat{i}+\hat{j}+\hat{k}$

Answer

$\vec{a}\times\vec{b}=(a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(a_1b_2-b_1a_2)k$

Here,

We

have $\vec{a} = 3i + 4j + 0k$ and $\vec{b} = i + j + k$

⇒ $a_1 = 3, a_2 = 4, a_3 = 0$ and $b_1 = 1, b_2 = 1, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (4 \times 1 - 1 \times 0)i + (0 \times 1 - 1 \times 3)j + (3 \times 1 - 1 \times 4)k$

⇒ $|a \times b| = \sqrt{4^2 + (-3)^2 + (-1)^2} = \sqrt{26}$

⇒ $\vec{a} \times \vec{b} = 4i - 3j - k$

2. Question

Find λ if $\left(2\hat{i} + 6\hat{j} + 14\hat{k}\right) \times \left(\hat{i} - \lambda\hat{j} + 7\hat{k}\right) = \vec{0}$.

Answer

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i + 6j + 14k$ and $\vec{b} = i - \lambda j + 7k$

⇒ $a_1 = 2, a_2 = 6, a_3 = 14$ and $b_1 = 1, b_2 = \lambda, b_3 = 7$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (6 \times 7 - (-\lambda) \times 14)i + (14 \times 1 - 2 \times 7)j + (2 \times (-\lambda) - 1 \times 6)k$

⇒ $\vec{a} \times \vec{b} = 0i + 0j + 0k$

⇒ $42 + 14\lambda = 0,$

⇒ $\lambda = -3$

3. Question

If $\vec{a} = \left(-3\hat{i} + 4\hat{j} - 7\hat{k}\right)$ and $\vec{b} = \left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$, find $\left(\vec{a} \times \vec{b}\right)$.

Verify that (i) $\vec{a}$ and $\left(\vec{a} \times \vec{b}\right)$ are perpendicular to each other

and (ii) $\vec{b}$ and $\left(\vec{a} \times \vec{b}\right)$ are perpendicular to each other.

Answer

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = -3i + 4j - 7k$ and $\vec{b} = 6i + 2j - 3k$

⇒ $a_1 = -3, a_2 = 4, a_3 = -7$ and $b_1 = 6, b_2 = 2, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3, in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (4 \times (-3) - 2 \times (-7))i + ((-7) \times 6 - (-3) \times (-3))j + ((-3) \times 2 - 6 \times 4)k$

$\Rightarrow \vec{a} \times \vec{b} = 2i - 51j - 30k$

If $\vec{a}$ and $\vec{a} \times \vec{b}$ are perpendicular to each other then,

$\Rightarrow \vec{a}.(\vec{a} \times \vec{b}) = 0$

i.e.,

$\vec{a}.(\vec{a} \times \vec{b}) = (-6) - (204) + (210) = 0$

And in the similar way, we have,

$\vec{b}.(\vec{a} \times \vec{b}) = (12) - (102) + (90) = 0$

Hence proved.

4. Question

Find the value of:

i. $(\hat{i} \times \hat{j}) \cdot \hat{k} + \hat{i} \cdot \hat{j}$ ii. $(\hat{j} \times \hat{k}) \cdot \hat{i} + \hat{j} \cdot \hat{k}$ iii. $\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$

Answer

i.

The value of $(i \times j).k + i.j$ is, $\ldots \text{As } i \times j = k \text{ and } i.j = 0$

$\Rightarrow (k).k + 0 = 1$

ii.

The value of $(j \times k).i + j.k$ is, … $\ldots \text{As } j \times k = i \text{ and } j.k = 0$

$\Rightarrow (i).i + 0 = 1$

iii.

The value of $i \times (j + k) + j \times (k + i) + k \times (i + j)$ is, … $\ldots \text{As } i \times k = -j, i \times j = k, j \times k = i, j \times i = -k, k \times i = j, k \times j = -i$

$\Rightarrow k - j + i - k + j - i = 0$

5 A. Question

Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

Answer

Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ …where k is a scalor

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + j - 2k$ and $\vec{b} = 2i + 3j - k$

$\Rightarrow a_1 = 3, a_2 = 1, a_3 = -2$ and $b_1 = 2, b_2 = 3, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (1 \times -1 - 3 \times -2)i + (-2 \times 2 - (-1) \times 3)j + (3 \times 3 - 2 \times 1)k$

$\Rightarrow |a \times b| = \sqrt{(5)^2 + (-1)^2 + (7)^2} = 5\sqrt{3}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{5i - 1j + 7k}{5\sqrt{3}}$

$\Rightarrow \vec{r} = \pm \frac{5i - 1j + 7k}{5\sqrt{3}}$

5 B. Question

Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$

Answer

Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i - 2j + 3k$ and $\vec{b} = i + 2j - k$

$\Rightarrow a_1 = 1, a_2 = -2, a_3 = 3$ and $b_1 = 1, b_2 = 2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-2 \times -1 - 2 \times 3)i + (3 \times 1 - (-1) \times 1)j + (1 \times 2 - (-2) \times 1)k$

$\Rightarrow |a \times b| = \sqrt{(-4)^2 + (4)^2 + (4)^2} = 4\sqrt{3}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{-4i + 4j + 4k}{4\sqrt{3}}$

$\Rightarrow \vec{r} = \pm \frac{-i + j + k}{\sqrt{3}}$

5 C. Question

Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{k}$

Answer

Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i + 3j - 2k$ and $\vec{b} = -i + 0j + 3k$

$\Rightarrow a_1 = 1, a_2 = 3, a_3 = -2$ and $b_1 = -1, b_2 = 0, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (9 - 0)i + (2 - 3)j + (0 - (-3))k$

$\Rightarrow |a \times b| = \sqrt{(9)^2 + (-1)^2 + (3)^2} = \sqrt{91}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{9i-j+3k}{\sqrt{91}}$

$\Rightarrow \vec{r} = \pm\frac{9i-j+3k}{\sqrt{91}}$

5 D. Question

Find the unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ when

$\vec{a} = 4\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - \hat{k}$

Answer

Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i + 2j - k$ and $\vec{b} = i + 4j - k$

$\Rightarrow a_1 = 4, a_2 = 2, a_3 = -1$ and $b_1 = 1, b_2 = 4, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (2 \times -1 - (-1) \times 4)i + (-1 \times 1 - (-1) \times 4)j + (4 \times 4 - 1 \times 2)k$

$\Rightarrow |a \times b| = \sqrt{(2)^2 + (3)^2 + (14)^2} = \sqrt{209}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{2i+3j+14k}{\sqrt{209}}$

$\Rightarrow \vec{r} = \pm \frac{2i+3j+14k}{\sqrt{209}}$

6. Question

Find the unit vectors perpendicular to the plane of the vectors

$\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$

Answer

Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\vec{a} \times \vec{b})$ ...where k is a scalar

Thus, we have r is a unit vector,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i - 6j - 3k$ and $\vec{b} = 4i + 3j - k$

$\Rightarrow a_1 = 2, a_2 = -6, a_3 = -3$ and $b_1 = 4, b_2 = 3, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-6 \times (-1) - 3 \times (-3))i + (-3 \times 4 - (-1) \times 2)j + (2 \times 3 - 4 \times (-6))k$

$\Rightarrow |a \times b| = \sqrt{(15)^2 + (-10)^2 + (30)^2} = \sqrt{1225}$

$\Rightarrow \vec{a} \times \vec{b} = \frac{3i-2j+6k}{7}$

$\vec{r} = \pm \frac{3i - 2j + 6k}{7}$

7. Question

Find a vector of magnitude 6 which is perpendicular to both the vectors

$\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$.

Answer

Let $\vec{r}$ be the vector which is perpendicular to $\vec{a}$ & $\vec{b}$ then we have,

$\vec{r} = k.(\hat{a} \times \hat{b})$ ...where k is a scalar

Thus, we have r is vector of magnitude 6,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i - j + 3k$ and $\vec{b} = -2i + j - 2k$

$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 3$ and $b_1 = -2, b_2 = 1, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-1 \times (-2) - 1 \times (3))i + (3 \times (-2) - (-2) \times 4)j + (4 \times 1 - (-2) \times (-1))k$

$\Rightarrow |a \times b| = \sqrt{(-1)^2 + (2)^2 + (2)^2} = 3$

$\Rightarrow \hat{a} \times \hat{b} = \frac{-i+2j+2k}{3}$

$$\vec{r} = \pm k.\frac{-i + 2j + 2k}{3}$$

Here, as r is of magnitude 6 thus,

k = 6,

Thus, $\vec{r} = \pm 2(-i + 2j + 2k)$

8. Question

Find a vector of magnitude 5 units, perpendicular to each of the vectors

$\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$, where $\vec{a} = \left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\vec{b} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

Answer

$\vec{a} + \vec{b} = 2i + 3j + 4k = \vec{l}$

$\vec{a} - \vec{b} = 0i - i - 2k = \vec{m}$

Let $\vec{r}$ be the vector which is perpendicular to $\vec{l}$ & $\vec{m}$ then we have,

$\vec{r} = k.(\hat{l} \times \hat{m})$ ...where k is a scalar

Thus, we have r is vector of magnitude 5,

So,

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{l} = 2i + 3j + 4k$ and $\vec{m} = 0i - j - 2k$

$\Rightarrow a_1 = 2, a_2 = 3, a_3 = 4$ and $b_1 = 0, b_2 = -1, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{l} \times \vec{m} = (-2)i + (4)j + (-2)k$

$\Rightarrow |a \times b| = \sqrt{(-2)^2 + (4)^2 + (-2)^2} = \sqrt{24}$

$\Rightarrow \hat{a} \times \hat{b} = \frac{-i+2j-k}{\sqrt{6}}$

$\vec{r} = \pm k.\frac{-i + 2j - k}{\sqrt{6}}$

Here, as r is of magnitude 5 thus,

k = 5,

Thus, $\vec{r} = \pm 5(\frac{-i + 2j - k}{\sqrt{6}})$

9. Question

Find an angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and $\left|\vec{a} \times \vec{b}\right| = \sqrt{3}$.

Answer

We are given that $\overrightarrow{|a|} = 1$ and $\overrightarrow{|b|} = 2$.

And $|\vec{a} \times \vec{b}| = \sqrt{3}$,

So we have,

$|\vec{a} \times \vec{b}| = \overrightarrow{|a|}.\overrightarrow{|b|}\sin\theta = \sqrt{3}$

$\Rightarrow \overrightarrow{|a|}.\overrightarrow{|b|}\sin\theta = 1 \times 2 \times \sin\theta$

$\Rightarrow 2\sin\theta = \sqrt{3}$

$\Rightarrow \theta = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

10. Question

If $\vec{a} = \left(\hat{i} - \hat{j}\right)$, $\vec{b} = \left(3\,\hat{j} - \hat{k}\right)$ and $\vec{c} = \left(7\,\hat{i} - \hat{k}\right)$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d} = 1$.

Answer

Given that

Let $\vec{d}$ be the vector which is perpendicular to a & $\vec{b}$ then we have,

$\vec{d} = k.(\hat{a} \times \hat{b})$ ...where k is a scalar

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i - j$ and $\vec{b} = 0i + 3j - k$

$\Rightarrow a_1 = 1, a_2 = -1, a_3 = 0$ and $b_1 = 0, b_2 = 3, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (1)i + (1)j + (3)k$

$\Rightarrow |a \times b| = \sqrt{(1)^2 + (1)^2 + (3)^2} = \sqrt{11}$

$\Rightarrow \hat{a} \times \hat{b} = \frac{i+j+3k}{\sqrt{11}}$

$\vec{d} = \pm k.\frac{i+j+3k}{\sqrt{11}}$

Given that $\vec{c}.\vec{d} = 1$

$\vec{c} = 7i - k$

$\Rightarrow \vec{c}.\vec{d} = \frac{7k-3k}{\sqrt{11}} = 1,$

$\Rightarrow k = \frac{\sqrt{11}}{4}$

$\Rightarrow \vec{d} = \frac{i+j+3k}{4}$

11. Question

If $\vec{a} = \left(4\hat{i} + 5\hat{j} - \hat{k}\right)$, $\vec{b} = \left(\hat{i} - 4\hat{j} + \hat{k}\right)$ and $\vec{c} = \left(3\hat{i} + \hat{j} - \hat{k}\right)$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and for which $\vec{c} \cdot \vec{d} = 21$.

Answer

Given that

Let $\vec{d}$ be the vector which is perpendicular to $a \,\&\, \vec{b}$ then we have,

$\vec{d} = k.(\hat{a} \times \hat{b})$ ...where k is a scalar

We have,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i + 5j - k$ and $\vec{b} = i - 4j + k$

$\Rightarrow a_1 = 4, a_2 = 5, a_3 = -1$ and $b_1 = 1, b_2 = -4, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (1)i + (-5)j + (-21)k$

$\Rightarrow |a \times b| = \sqrt{(1)^2 + (-5)^2 + (-21)^2} = \sqrt{467}$

$\Rightarrow \hat{a} \times \hat{b} = \frac{i-5j-21k}{\sqrt{467}}$

$\vec{d} = \pm k.\frac{i-5j-21k}{\sqrt{467}}$

Given that $\vec{c}.\vec{d} = 21$

$\vec{c} = 3i + j - k$

$\Rightarrow \vec{c}.\vec{d} = \frac{19k}{\sqrt{467}} = 21,$

$\Rightarrow k = \frac{\sqrt{467}}{19 \times 21}$

$\vec{d} = \frac{i-5j-21k}{319} \times \sqrt{467}$

12. Question

Prove that $\left|\vec{a}\times\vec{b}\right| = \left(\vec{a}\cdot\vec{b}\right)\tan\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

Answer

We know that $|\vec{a}.\vec{b}| = ||\vec{a}||\vec{b}|\cos\theta|$

And $|\vec{a}\times\vec{b}| = ||\vec{a}||\vec{b}|\sin\theta|$

So,

$\tan\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}.\vec{b}|}$

Hence, proved.

13. Question

Write the value of p for which $\vec{a} = \left(3\hat{i}+2\hat{j}+9\hat{k}\right)$ and $\vec{b} = \left(\hat{i}+p\hat{j}+3\hat{k}\right)$ are parallel vectors.

Answer

As the vectors are parallel vectors so, $\vec{a}\times\vec{b} = 0$

Thus,

We have,

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + 2j + 9k$ and $\vec{b} = i + pj + 3k$

$\Rightarrow a_1 = 3, a_2 = 2, a_3 = 9$ and $b_1 = 1, b_2 = p, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times\vec{b} = (6-9p)i + (0)j + (3p-2)k = 0$

$\Rightarrow 6 - 9p = 0$

⇒ Thus, p $= \frac{2}{3}$.

14 A. Question

Verify that $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \left(\vec{a}+\vec{b}\right)+\left(\vec{a}\times\vec{c}\right)$, when

$\vec{a} = \hat{i}-\hat{j}-3\hat{k}$, $\vec{b} = 4\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c} = 2\hat{i}-\hat{j}+2\hat{k}$

Answer

To verify $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})\times(\vec{a}+\vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

Given, $\vec{a} = \hat{i}-\hat{j}-3\hat{k}$ $\vec{b} = 4\hat{i}-3\hat{j}+\hat{k}$ $\vec{c} = 2\hat{i}-\hat{j}+2\hat{k}$

$\vec{a}\times(\vec{b}+\vec{c}) = (i - j - 3k)\times(6i - 4j + 3k)$

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i - j - 3k$ and $\vec{b} + \vec{c} = 6i - 4j + 3k$

$\Rightarrow a_1 = 1, a_2 = -1, a_3 = -3$ and $b_1 = 6, b_2 = -4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times(\overrightarrow{b+c}) = (-3-12)i + (3+18)j + (-4+6)k$

$\Rightarrow (-15)i + (21)j + (2)k$

RHS is

$(\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c}) = (-10i + 13j + k) + (-5i + 8j + k)$

$\Rightarrow (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c}) = (-15)i + (21)j + (2)k$

Thus, LHS = RHS.

14 B. Question

Verify that $\vec{a}\times\left(\vec{b}+\vec{c}\right) = \left(\vec{a}+\vec{b}\right) + \left(\vec{a}\times\vec{c}\right)$, when

$\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$.

Answer

To verify $\vec{a}\times(\vec{b}+\vec{c}) = (\vec{a}+\vec{b})\times(\vec{a}+\vec{c})$

We need to prove L.H.S = R.H.S

L.H.S we have,

Given, $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a}\times(\vec{b}+\vec{c}) = (4i - j + k)\times(2i + 0j + 2k)$

$\vec{a}\times\vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 4i - j + k$ and $\vec{b} + \vec{c} = 2i + 0j + 2k$

$\Rightarrow a_1 = 4, a_2 = -1, a_3 = 1$ and $b_1 = 2, b_2 = 0, b_3 = 2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a}\times(\overrightarrow{b+c}) = (-2)i + (-2)j + (2)k$

$\Rightarrow (-2)i + (-2)j + (2)k$

RHS is

$(\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c}) = (-2i - 3j + 5k) + (0i + j - 3k)$

$\Rightarrow (\vec{a}\times\vec{b}) + (\vec{a}\times\vec{c}) = (-2)i + (-2)j + (2)k$

Thus, LHS = RHS.

15 A. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = -3\hat{i} - 2\hat{j} + \hat{k}$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = i + 2j + 3k$ and $\vec{b} = -3i - 2j + k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = -3, b_2 = -2, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (8)i + (-10)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(8)^2 + (-10)^2 + (4)^2} = \sqrt{180}$

$\Rightarrow$ area $= 6\sqrt{5}$ sq units

15 B. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = \left(3\hat{i} + \hat{j} + 4\hat{k}\right)$ and $\vec{b} = \left(\hat{i} - \hat{j} + \hat{k}\right)$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + j + 4k$ and $\vec{b} = i - j + k$

$\Rightarrow a_1 = 3, a_2 = 1, a_3 = 4$ and $b_1 = 1, b_2 = -1, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (5)i + (-1)j + (-4)k$

$\Rightarrow |a \times b| = \sqrt{(5)^2 + (-1)^2 + (-4)^2} = \sqrt{42}$

$\Rightarrow$ area $= \sqrt{42}$ sq units

15 C. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} - \hat{j}$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i + j + 3k$ and $\vec{b} = i - j + 0k$

⇒ $a_1 = 2, a_2 = 1, a_3 = 3$ and $b_1 = 1, b_2 = -1, b_3 = 0$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (3)i + (3)j + (-3)k$

⇒ $|a \times b| = \sqrt{(3)^2 + (3)^2 + (-3)^2} = 3\sqrt{3}$

⇒ $\text{area} = 3\sqrt{3}$ sq units

15 D. Question

Find the area of the parallelogram whose adjacent sides are represented by the vectors:

$\vec{a} = 2\hat{i}$ and $\vec{b} = 3\hat{j}$

Answer

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 2i + 0j + 0k$ and $\vec{b} = 0i + 3j + 0k$

⇒ $a_1 = 2, a_2 = 0, a_3 = 0$ and $b_1 = 0, b_2 = 3, b_3 = 0$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (6)k$

⇒ $|a \times b| = 6$

⇒ $\text{area} = 6$ sq units

16 A. Question

Find the area of the parallelogram whose diagonal are represented by the vectors

$\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$

Answer

The diagonals are $\vec{a}+\vec{b}=3i+j-2k$ & $\vec{a}-\vec{b}=i-3j+4k$

Thus, $\vec{a}=2i-j+k, \vec{b}=i+2j-3k$

The area of the parallelogram = $|\vec{a}\times\vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a}\times\vec{b}|$

$\vec{a}\times\vec{b}=\ (a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(\ a_1b_2-b_1a_2)k$

Here,

We

have $\vec{a}=2i-j+k$ and $\vec{b}=i+2j-3k$

⇒ $a_1=2, a_2\ =\ -1, a_3=1$ and $b_1\ =1, b_2\ =2, b_3\ =-3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a}\times\vec{b}=(3-2)i+7j+(5)k$

⇒ $|a\times b|=\sqrt{(1)^2+(7)^2+(5)^2}=5\sqrt{3}$

⇒

⇒ $\text{area}=5\sqrt{3}$ sq units

16 B. Question

Find the area of the parallelogram whose diagonal are represented by the vectors

$\vec{d}_1=2\,\hat{i}-\hat{j}+\hat{k}$ and $\vec{d}_2=3\,\hat{i}+4\,\hat{j}-\hat{k}$

Answer

The diagonals are $\vec{a}+\vec{b}=2i-j+k$ & $\vec{a}-\vec{b}=3i+4j-k$

Thus, $\vec{a}=\frac{5}{2}i+\frac{3}{2}j, \vec{b}=-\frac{1}{2}i-\frac{5}{2}j+k$

The area of the parallelogram = $|\vec{a}\times\vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a}\times\vec{b}|$

$\vec{a}\times\vec{b}=\ (a_2b_3-b_2a_3)i+(a_3b_1-b_3a_1)j+(\ a_1b_2-b_1a_2)k$

Here,

We

have, $\vec{a}=\frac{5}{2}i+\frac{3}{2}j, \vec{b}=-\frac{1}{2}i-\frac{5}{2}j+k$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a}\times\vec{b}=\left(\frac{3}{2}\right)i-\frac{5}{2}j+\left(-\frac{11}{2}\right)k$

⇒ $|a\times b|=\sqrt{\left(\frac{3}{2}\right)^2+\left(-\frac{5}{2}\right)^2+\left(-\frac{11}{2}\right)^2}=\frac{1}{2}\sqrt{155}$

⇒

$\Rightarrow \text{area} = \frac{1}{2}\sqrt{155}$ sq units

16 C. Question

Find the area of the parallelogram whose diagonal are represented by the vectors

$\vec{d}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{d}_2 = -\hat{i} + 2\hat{j}$.

Answer

The diagonals are $\vec{a} + \vec{b} = i - 3j + 2k$ & $\vec{a} - \vec{b} = -i + 2j + 0k$

Thus, $\vec{a} = 0i - \frac{1}{2}j + k, \vec{b} = i - \frac{5}{2}j + k$

The area of the parallelogram = $|\vec{a} \times \vec{b}|$, where a and b are vectors of it's adjacent sides.

Area = $|\vec{a} \times \vec{b}|$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 0i - \frac{1}{2}j + k$ and $\vec{b} = i - \frac{5}{2}j + k$

$\Rightarrow a_1 = 0, a_2 = -\frac{1}{2}, a_3 = 1$ and $b_1 = 1, b_2 = -\frac{5}{2}, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (2)i + 1j + \left(\frac{1}{2}\right)k$

$\Rightarrow |a \times b| = \sqrt{(2)^2 + (1)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{21}$

$\Rightarrow$

$\Rightarrow \text{area} = \frac{\sqrt{21}}{2}$ sq units

17 A. Question

Find the area of the triangle whose two adjacent sides are determined by the vectors

$\vec{a} = -2\hat{i} - 5\hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} - \hat{k}$

Answer

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = -2i + 0j - 5k$ and $\vec{b} = i - 2j - k$

$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5$ and $b_1 = 1, b_2 = -2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (8)i + (-10)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$

$\Rightarrow$ area $= \frac{\sqrt{165}}{2}$ sq units

17 B. Question

Find the area of the triangle whose two adjacent sides are determined by the vectors

$\vec{a} = 3\hat{i} + 4\hat{j}$ and $\vec{b} = -5\hat{i} + 7\hat{j}$.

Answer

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\vec{a} = 3i + 4j + 0k$ and $\vec{b} = -5i + 7j + 0k$

$\Rightarrow a_1 = 3, a_2 = 4, a_3 = 0$ and $b_1 = -5, b_2 = 7, b_3 = 0$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (41)k$

$\Rightarrow |a \times b| = 41$

$\Rightarrow$ area $= \frac{41}{2}$ sq units

18 A. Question

Using vectors, find the area of ΔABC whose vertices are

A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5)

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = 0, b_2 = 4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-6)i + (-3)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(-6)^2 + (-3)^2 + (4)^2} = \sqrt{61}$

$\Rightarrow$ area $= \frac{\sqrt{61}}{2}$ sq units

18 B. Question

Using vectors, find the area of ΔABC whose vertices are

A(1, 2, 3), B(2, −1, 4) and C(4, 5, Δ1) ((considering Δ1 as 1))

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = i - 3j + 1k$ and $\overrightarrow{AC} = 3i + 3j - 2k$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i - 3j + k$ and $\overrightarrow{AC} = 3i + 3j - 2k$

$\Rightarrow a_1 = 1, a_2 = -3, a_3 = 1$ and $b_1 = 3, b_2 = 3, b_3 = -2$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (3)i + (5)j + (12)k$

$\Rightarrow |a \times b| = \sqrt{(3)^2 + (5)^2 + (12)^2} = \sqrt{178}$

$\Rightarrow$ area $= \frac{\sqrt{178}}{2}$ sq units

18 C. Question

Using vectors, find the area of ΔABC whose vertices are

A(3, −1, 2), B(1, −1, −3) and C(4, −3, 1)

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -2i + 0j - 5k$ and $\overrightarrow{AC} = i - 2j - k$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it's adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -2i - 5k$ and $\overrightarrow{AC} = i - 2j - k$

$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5$ and $b_1 = 1, b_2 = -2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-10)i + (-7)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$

$\Rightarrow \text{area} = \frac{\sqrt{165}}{2}$ sq units

18 D. Question

Using vectors, find the area of ΔABC whose vertices are

A(1, −1, 2), B(2, 1, −1) and C(3, −1, 2).

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = i + 2j - 3k$ and $\overrightarrow{AC} = 2i$

The area of the triangle = $\frac{|\vec{a} \times \vec{b}|}{2}$, where a and b are it’s adjacent sides vectors.

Area = $\frac{|\vec{a} \times \vec{b}|}{2}$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i + 2j - 3k$ and $\overrightarrow{AC} = 2i$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3$ and $b_1 = 0, b_2 = 4, b_3 = 3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-6) + (-4)k$

$\Rightarrow |a \times b| = \sqrt{(-6)^2 + (-4)^2} = \sqrt{52}$

$\Rightarrow \text{area} = \frac{\sqrt{52}}{2}$ sq units

19 A. Question

Using vector method, show that the given points A, B, C are collinear:

A(3, −5, 1), B(−1, 0, 8) and C(7, −10, −6)

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -4i + 5j + 7k$ and $\overrightarrow{AC} = 4i - 5j - 7k$

To prove that A, B, C are collinear we need to prove that

$\vec{a} \times \vec{b} = 0$.

So,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = i + 2j + 3k$ and $\overrightarrow{AC} = 4j + 3k$

$\Rightarrow a_1 = -4, a_2 = 5, a_3 = 7$ and $b_1 = 4, b_2 = -5, b_3 = -7$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$

$\Rightarrow |a \times b| = 0$

19 B. Question

Using vector method, show that the given points A, B, C are collinear:

A(6, −7, −1), B(2, −3, 1) and C(4, −5, 0).

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -4i + 4j + 2k$ and $\overrightarrow{AC} = -2i + 2j + k$

To prove that A, B, C are collinear we need to prove that

$\vec{a} \times \vec{b} = 0$.

So,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -4i + 4j + 2k$ and $\overrightarrow{AC} = -2i + 2j + k$

$\Rightarrow a_1 = -4, a_2 = 4, a_3 = 2$ and $b_1 = -2, b_2 = 2, b_3 = 1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (0)i + (0)j + (0)k$

$\Rightarrow |a \times b| = 0$

Thus, A, B and C are collinear.

20. Question

Show that the point A, B, C with position vectors $\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$, $\left(\hat{i} + \hat{j} + \hat{k}\right)$ and $\left(-\hat{i} + 4\hat{j} - 2\hat{k}\right)$ respectively are collinear.

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -2i + 3j - 3k$ and $\overrightarrow{AC} = -4i + 6j - 6k$

To prove that A, B, C are collinear we need to prove that

$\vec{a} \times \vec{b} = 0$·

So,

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -2i + 3j - 3k$ and $\overrightarrow{AC} = -4i + 6j - 6k$

⇒ $a_1 = -2, a_2 = 3, a_3 = -3$ and $b_1 = -4, b_2 = 6, b_3 = -6$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\vec{a} \times \vec{b} = (0)i + (0)j + (0)k$

⇒ $|a \times b| = 0$

Thus, A, B and C are collinear.

21. Question

Show that the points having position vectors $\vec{a}, \vec{b}, \left(\vec{c} = 3\vec{a} - 2\vec{b}\right)$ are collinear, whatever be $\vec{a}, \vec{b}, \vec{c}$.

Answer

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = \vec{b} - \vec{a}$ and $\overrightarrow{AC} = \vec{c} - \vec{a} = 2\vec{a} + 2\vec{b}$

To prove that A, B, C are collinear we need to prove that

$\overrightarrow{AB} \times \overrightarrow{AC} = 0$·

So,

Here,

We

have $\overrightarrow{AB} = \vec{b} - \vec{a}$ and $\overrightarrow{AC} = 2\vec{a} + 2\vec{b}$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

⇒ $\overrightarrow{AB} \times \overrightarrow{AC} = (\vec{b} - \vec{a}) \times (2\vec{a} + 2\vec{b})$

⇒ $\overrightarrow{AB} \times \overrightarrow{AC} = \vec{b} \times \overrightarrow{2a} + 0 - 0 - \vec{a} \times \overrightarrow{2b} = 0$

Thus, A, B and C are collinear.

22. Question

Show that the points having position vector $\left(-2\vec{a} + 3\vec{b} + 5\vec{c}\right)$, $\left(\vec{a} + 2\vec{b} + 3\vec{c}\right)$ and $\left(7\vec{a} - \vec{c}\right)$ are collinear, whatever be $\vec{a}, \vec{b}, \vec{c}$.

Answer

We have, $A = -2\vec{a} + 3\vec{b} + 5\vec{c}, B = \vec{a} + 2\vec{b} + 3\vec{c}, C = 7\vec{a} - \vec{c}$

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = 3\vec{a} - \vec{b} - 2\vec{c}$ and $\overrightarrow{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$

To prove that A, B, C are collinear we need to prove that

$\overrightarrow{AB} \times \overrightarrow{AC} = 0 \cdot$

So,

Here,

We

have

$\overrightarrow{AB} = 3\vec{a} - \vec{b} - 2\vec{c}$ and $\overrightarrow{AC} = 9\vec{a} - 3\vec{b} - 6\vec{c}$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = (3\vec{a} - \vec{b} - 2\vec{c}) \times (9\vec{a} - 3\vec{b} - 6\vec{c})$

$\Rightarrow \overrightarrow{AB} \times \overrightarrow{AC} = 0$

Thus, A, B and C are collinear.

23. Question

Find a unit vector perpendicular to the plane ABC, where the points A, B, C, are $(3,-1,2)$, $(1,-1,-3)$ and $(4,-3,1)$ respectively.

Answer

A unit vector perpendicular to the plane ABC will be,

$\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

Through the vertices we get the adjacent vectors as,

$\overrightarrow{AB} = -2i + 0j - 5k$ and $\overrightarrow{AC} = i - 2j - k$

$\vec{a} \times \vec{b} = (a_2b_3 - b_2a_3)i + (a_3b_1 - b_3a_1)j + (a_1b_2 - b_1a_2)k$

Here,

We

have $\overrightarrow{AB} = -2i + 0j - 5k$ and $\overrightarrow{AC} = i - 2j - k$

$\Rightarrow a_1 = -2, a_2 = 0, a_3 = -5$ and $b_1 = 1, b_2 = -2, b_3 = -1$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-10)i + (-7)j + (4)k$

$\Rightarrow |a \times b| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{165}$

$\Rightarrow \text{unit vector} = \frac{-10i - 7j + 4k}{\sqrt{165}}$

24. Question

If $\vec{a} = (\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{b} = (\hat{i} - 3\hat{k})$ then find $|\vec{b} \times 2\vec{a}|$.

Answer

$\vec{a} = i + 2j + 3k$ and $\vec{b} = i - 3k$

Then, $|\vec{b} \times \overrightarrow{2a}|$,

We have, $\vec{b} \times \vec{a} = (-2a_2.b_3 + 2b_2.a_3)i - (a_3.2b_1 - 2b_{3.}a_1)j - (a_1.2b_2 - 2b_1a_2)k$

Here,

We

have $\vec{a} = i + 2j + 3k$ and $\vec{b} = i - 3k$

$\Rightarrow a_1 = 1, a_2 = 2, a_3 = 3 \text{ and } b_1 = 1, b_2 = 0, b_3 = -3$

Thus, substituting the values of a_1, a_2, a_3 and b_1, b_2 and b_3,

in equation (i) we get

$\Rightarrow \vec{a} \times \vec{b} = (-12)i + (12)j + (-4)k$

$\Rightarrow |a \times b| = \sqrt{(-12)^2 + (12)^2 + (-4)^2} = 4\sqrt{19}$

25. Question

If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, find $\vec{a} \cdot \vec{b}$.

Answer

We have, $|\vec{a}|^2|\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2$

So, $|\vec{a}.\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a} \times \vec{b}|^2$

$\Rightarrow |\vec{a}.\vec{b}|^2 = 10^2 - 8^2 = 6^2$

$\Rightarrow |\vec{a}.\vec{b}| = 6$

26. Question

If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $(\vec{a} \times \vec{b}) = (3\hat{i} + 2\hat{j} + 6\hat{k})$, find the angle between $\vec{a}$ and $\vec{b}$.

Answer

We have, $|\vec{a}|^2|\vec{b}|^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a}.\vec{b}|^2$

$\Rightarrow \vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{3^2 + 2^2 + 6^2} = 7$

$\Rightarrow 7 = 7 \times 2\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$