20. Homogeneous Differential Equations

Exercise 20

1. Question

In each of the following differential equation show that it is homogeneous and solve it.

xdy = (x + y)dx

Answer

Xdy = (x + y)dx

$$\frac{dy}{dx} = \frac{x + y}{x}$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = 1 + \frac{vx}{x}$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v - v$$

$$\Rightarrow x\frac{dv}{dx} = 1$$

$$\Rightarrow dv = \frac{dx}{x}$$

Integrating both the sides we get:

$$\int dv = \int \frac{dx}{x} + c$$

$$v = \ln|x| + c$$

Resubstituting the value of y = vx we get

$$\frac{y}{x} = \ln|x| + c$$

$$y = x\ln|x| + cx$$

Ans: $y = x\ln|x| + cx$

2. Question

In each of the following differential equation show that it is homogeneous and solve it.

$(x^2 - y^2)dx + 2xydy = 0$

Answer

$(x^2 - y^2)dx + 2xydy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} = \frac{y}{2x} - (\frac{2y}{x})^{-1}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{vx}{2x} - (\frac{2vx}{x})^{-1}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v}{2} - (2v)^{-1}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{2} - \frac{1}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{v}{2} - \frac{1}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = -\left(\frac{2v^2 + 2}{4v}\right)$$

$$\Rightarrow \frac{2v}{v^2 + 1} = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\int \frac{2v}{v^2 + 1}dv = -\int \frac{dx}{x} + c$$

$$\Rightarrow \ln|v^2 + 1| = -\ln|x| + \ln c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^2 + 1\right| + \ln|x| = \ln c$$

$$\Rightarrow \left(\left(\frac{y}{x}\right)^2 + 1\right)(x) = c$$

$$\Rightarrow x^2 + y^2 = cx$$

Ans: $x^2 + y^2 = cx$

3. Question

In each of the following differential equation show that it is homogeneous and solve it.

$x^2dy + y(x + y)dx = 0$

Answer

$x^2dy + y(x + y)dx = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{y(x + y)}{x^2} = -(\frac{y}{x} + \frac{y^2}{x^2})$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\left(\frac{vx}{x} + \frac{(vx)^2}{x^2}\right)$$

$$\Rightarrow x\frac{dv}{dx} = -v - v^2 - v = -2v - v^2$$

$$\Rightarrow \frac{dv}{2v + v^2} = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\int \frac{dv}{2v + v^2} = -\int \frac{dx}{x} + c$$

$$\Rightarrow \int \frac{dv}{1 + 2v + v^2 - 1} = -\ln|x| + \ln|c|$$

$$\Rightarrow \int \frac{dv}{(v + 1)^2 - 1^2} + \ln|x| = \ln|c|$$

$$\Rightarrow \frac{1}{2}\ln\left|\frac{v + 1 - 1}{v + 1 + 1}\right| + \ln|x| = \ln|c|$$

$$\Rightarrow \ln\left|\frac{v + 1 - 1}{v + 1 + 1}\right| + 2\ln|x| = 2\ln|c|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \ln\left|\frac{\frac{y}{x}}{\frac{y}{x} + 2}\right| + \ln x^2 = \ln|c|^2$$

$$\Rightarrow \ln\left|\frac{y}{y + 2x}\right| + \ln x^2 = \ln|c|^2$$

$\Rightarrow x^2y = c^2(y + 2x)$

Ans: $x^2y = c^2(y + 2x)$

4. Question

In each of the following differential equation show that it is homogeneous and solve it.

(x - y)dy - (x + y)dx = 0

Answer

(x - y)dy - (x + y)dx = 0

$$\Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y} \Rightarrow \frac{dy}{dx} = \frac{1 + \frac{y}{x}}{1 - \frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1 + \frac{vx}{x}}{1 - \frac{vx}{x}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1 + v}{1 - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + v}{1 - v} - v = \frac{1 + v - v + v^2}{1 - v} = \frac{1 + v^2}{1 - v}$$

Integrating both the sides we get:

$$\int \frac{1 - v}{1 + v^2} dv = \int \frac{dx}{x} + c$$

$$\Rightarrow \int \frac{1}{1 + v^2} dv - \int \frac{v}{1 + v^2} dv = \ln|x| + c$$

$$\Rightarrow \tan^{-1} v - \frac{\ln|1 + v^2|}{2} = \ln|x| + c$$

Resubstituting the value of y = vx we get

$$\tan^{-1}\frac{y}{x} - \frac{\ln\left|1 + \left(\frac{y}{x}\right)^2\right|}{2} = \ln|x| + c$$

$$\Rightarrow \tan^{-1}\frac{y}{x} = \frac{\ln|y^2 + x^2|}{2} + c$$

Ans: $\tan^{-1}\frac{y}{x} = \frac{\ln|y^2 + x^2|}{2} + c$

5. Question

In each of the following differential equation show that it is homogeneous and solve it.

(x + y)dy + (y - 2x)dx = 0

Answer

(x + y)dy + (y - 2x)dx = 0

$$\Rightarrow \frac{dy}{dx} = \frac{2x - y}{x + y} = \frac{2 - \frac{y}{x}}{1 + \frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2 - \frac{vx}{x}}{1 + \frac{vx}{x}} = \frac{2 - v}{1 + v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2 - v}{1 + v} - v = \frac{2 - v - v - v^2}{1 + v} = \frac{2 - 2v - v^2}{1 + v}$$

$\Rightarrow x\frac{dv}{dx} = \frac{2 - 2v - v^2}{1 + v}$

$\Rightarrow \frac{1 + v}{2 - 2v - v^2}dv = \frac{dx}{x}$

Integrating both the sides we get:

$\int \frac{1 + v}{2 - 2v - v^2}dv = \int \frac{dx}{x} + c$

$\Rightarrow -\frac{\ln|-2 + 2v + v^2|}{2} = \ln|x| + \ln|c|$

Resubstituting the value of y = vx we get

$\Rightarrow -\frac{\ln\left|-2 + 2\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2\right|}{2} = \ln|x| + \ln|c|$

$\Rightarrow -\frac{\ln\frac{|-2x + 2y + y^2|}{x}}{2} = \ln|x| + \ln|c|$

$\Rightarrow y^2 + 2xy - 2x^2 = c$

Ans: $y^2 + 2xy - 2x^2 = c$

6. Question

In each of the following differential equation show that it is homogeneous and solve it.

$(x^2 + 3xy + y^2)dx - x^2dy = 0$

Answer

$(x^2 + 3xy + y^2)dx - x^2dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2} = 1 + 3\frac{y}{x} + \frac{y^2}{x^2}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = 1 + 3\frac{vx}{x} + \frac{(vx)^2}{x^2}$

$\Rightarrow x\frac{dv}{dx} = 1 + 3v + v^2 - v = 1 + 2v + v^2$

$\Rightarrow \frac{dv}{1 + 2v + v^2} = \frac{dx}{x}$

Integrating both the sides we get:

$\int \frac{dv}{1 + 2v + v^2} = \int \frac{dx}{x} + c'$

$\Rightarrow \int \frac{dv}{(v + 1)^2} = \int \frac{dx}{x} + c'$

$$\Rightarrow \frac{(v+1)^{-2+1}}{-2+1} = \ln|x| + c'$$

$$\Rightarrow \frac{-1}{v+1} = \ln|x| + c'$$

$$\Rightarrow \frac{1}{v+1} + \ln|x| = c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{1}{\frac{y}{x}+1} + \ln|x| = c$$

$$\Rightarrow \frac{x}{y+x} + \ln|x| = c$$

7. Question

In each of the following differential equation show that it is homogeneous and solve it.

$2xydx + (x^2 + 2y^2)dy = 0$

Answer

$2xydx + (x^2 + 2y^2)dy = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{2xy}{x^2 + 2y^2} = -\frac{2}{(\frac{y}{x})^{-1} + 2\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{2}{\left(\frac{vx}{x}\right)^{-1} + 2\frac{vx}{x}} = -\frac{2v}{1+2v^2}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{2v}{1+2v^2} - v = -\frac{2v + v + 2v^3}{1+2v^2} = -\frac{3v+2v^3}{1+2v^2}$$

$$\Rightarrow \frac{1+2v^2}{3v+2v^3}dv = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\int \frac{1+2v^2}{3v+2v^3}dv = \int \frac{dx}{x} + c'$$

$$\Rightarrow \frac{\ln|3v+2v^3|}{3} = \ln|x| + c'$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{\ln\left|3\frac{y}{x} + 2(\frac{y}{x})^3\right|}{3} = \ln|x| + c'$$

$\Rightarrow 3x^2y + 2y^3 = C$

Ans: $3x^2y + 2y^3 = C$

8. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$\frac{dy}{dx} + \frac{x - 2y}{2x - y} = 0$$

Answer

$$\Rightarrow \frac{dy}{dx} = -\frac{x - 2y}{2x - y} = -\frac{1 - 2\frac{y}{x}}{2 - \frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{1 - 2\frac{vx}{x}}{2 - \frac{vx}{x}}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\frac{1 - 2v}{2 - v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1 - 2v}{2 - v} - v = -\frac{1 - 2v + 2v - v^2}{2 - v} = -\frac{1 - v^2}{2 - v}$$

$$\Rightarrow \frac{2 - v}{v^2 - 1}dv = \frac{dx}{x}$$

$$\Rightarrow \frac{v - 2}{v^2 - 1}dv = -\frac{dx}{x}$$

$$\Rightarrow \frac{v}{v^2 - 1}dv - \frac{2}{v^2 - 1}dv = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{v}{v^2 - 1}dv - \int \frac{2}{v^2 - 1}dv = -\int \frac{dx}{x} + c$$

$$\Rightarrow \frac{\ln|v^2 - 1|}{2} - 2 \times \frac{1}{2}\ln\left|\frac{v - 1}{v + 1}\right| = -\ln|x| + \ln|c|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{\ln\left|\left(\frac{y}{x}\right)^2 - 1\right|}{2} - 2 \times \frac{1}{2}\ln\left|\frac{\left(\frac{y}{x}\right) - 1}{\left(\frac{y}{x}\right) + 1}\right| = -\ln|x| + \ln|c|$$

⇒ $(y - x) = C(y + x)^3$

Ans: $(y - x) = C(y + x)^3$

9. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$\frac{dy}{dx}+\frac{x^2-y^2}{3xy}=0$$

Answer

$$\frac{dy}{dx} = -\frac{x^2-y^2}{3xy}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{3x}\right)^{-1} + \left(\frac{y}{3x}\right)$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\left(\frac{vx}{3x}\right)^{-1} + \left(\frac{vx}{3x}\right)$$

$$\Rightarrow x\frac{dv}{dx} == -\left(\frac{v}{3}\right)^{-1} + \left(\frac{v}{3}\right)$$

$$\Rightarrow x\frac{dv}{dx} == -\frac{3}{v} + \left(\frac{v}{3}\right) = \frac{-9 + v^2}{3v}$$

$$\Rightarrow \frac{3v}{v^2-9}dv = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{3v}{v^2-9}dv = \int \frac{dx}{x} + c$$

$$\Rightarrow \frac{3}{2}\ln|v^2-9| = \ln|x| + \ln|c|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{3}{2}\ln\left|\left(\frac{y}{x}\right)^2 - 9\right| = \ln|x| + \ln|c|$$

⇒ $(x^2 + 2y^2)^3 = Cx^2$

Ans: $(x^2 + 2y^2)^3 = Cx^2$

10. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$\frac{dy}{dx}=\frac{x^2+y^2}{2xy}$$

Answer

$$\frac{dy}{dx} = -\frac{x^2 + y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{2x}\right)^{-1} - \left(\frac{y}{2x}\right)$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\left(\frac{vx}{2x}\right)^{-1} - \left(\frac{vx}{2x}\right)$$

$$\Rightarrow x\frac{dv}{dx} = = -\left(\frac{v}{2}\right)^{-1} + \left(\frac{v}{2}\right)$$

$$\Rightarrow x\frac{dv}{dx} = = -\frac{2}{v} + \left(\frac{v}{2}\right) = \frac{-4 + v^2}{2v}$$

$$\Rightarrow \frac{2v}{v^2 - 4}dv = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{2v}{v^2 - 4}dv = \int \frac{dx}{x} + c$$

$$\Rightarrow \frac{2}{2}\ln|v^2 - 4| = \ln|x| + \ln|c|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^2 - 4\right| = \ln|x| + \ln|c|$$

⇒ $(x^2 - y^2) = cx$

Ans: $(x^2 - y^2) = cx$

11. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$\frac{dy}{dx} = \frac{2xy}{(x^2 - y^2)}$$

Answer

$$\Rightarrow \frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\left(\frac{y}{x}\right)^{-1} - \left(\frac{y}{x}\right)}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2}{\left(\frac{vx}{x}\right)^{-1} - \left(\frac{vx}{x}\right)} = \frac{2}{(v)^{-1} - (v)}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{2v}{(v)^2 - 1}$$

$$\Rightarrow \frac{2v}{(v)^2 - 1}dv = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{2v}{(v)^2 - 1}dv = -\int \frac{dx}{x} + c$$

$$\Rightarrow \ln|(v)^2 - 1| = \ln|x| + \ln|c|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^2 - 1\right| = \ln|x| + \ln|c|$$

$\Rightarrow y = C(y^2 + x^2)$

Ans: $y = C(y^2 + x^2)$

12. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$x^2\frac{dy}{dx} = 2xy + y^2$$

Answer

$$\Rightarrow x^2\frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = 2\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = 2\left(\frac{vx}{x}\right) + \left(\frac{vx}{x}\right)^2 = 2(v) + (v)^2$$

$$\Rightarrow x\frac{dv}{dx} = 2v - v + (v)^2$$

$$\Rightarrow x\frac{dv}{dx} = v + (v)^2$$

$$\Rightarrow \frac{dv}{v + (v)^2} = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{v + (v)^2} = \int \frac{dx}{x} + c$$

$$\Rightarrow \int \frac{dv}{\frac{1}{4} + v + (v)^2 - \frac{1}{4}} = \ln|x| + \ln|c|$$

$$\Rightarrow \int \frac{dv}{\left(v + \frac{1}{2}\right)^2 - \frac{1}{2}^2} = \ln|x| + \ln|c|$$

$$\Rightarrow \frac{1}{2 \times \frac{1}{2}} \ln\left|\frac{v + \frac{1}{2} - \frac{1}{2}}{v + \frac{1}{2} + \frac{1}{2}}\right| = \ln|xc|$$

$$\Rightarrow \ln\left|\frac{v}{v + 1}\right| = \ln|xc|$$

$$\Rightarrow \frac{v}{v + 1} = xc$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{\frac{y}{x}}{\frac{y}{x} + 1} = xc$$

$$\Rightarrow y = x(y + x)c$$

Ans: $y = x(y + x)c$

13. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

Answer

$$\Rightarrow x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = 1 + \frac{vx}{x} + \left(\frac{vx}{x}\right)^2 = 1 + v + (v)^2$$

$$\Rightarrow x\frac{dv}{dx} = 1 + v + (v)^2 - v = 1 + (v)^2$$

$$\Rightarrow \frac{dv}{1 + (v)^2} = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{1 + (v)^2} = \int \frac{dx}{x} + c$$

⇒ tan $^{-}$ v = ln|x| + c

Resubstituting the value of y = vx we get

⇒ tan $^{-}$ (y/x) = ln|x| + c

Ans: tan $^{-}$ (y/x) = ln|x| + c

14. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$y^2 + (x^2 - xy)\frac{dy}{dx} = 0$$

Answer

$$\frac{dx}{dy} = \frac{xy - x^2}{y^2} = \frac{x}{y} - \left(\frac{x}{y}\right)^2$$

$$\Rightarrow \frac{dx}{dy} = f\left(\frac{x}{y}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put x = vy

$$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\Rightarrow v + y\frac{dv}{dy} = \frac{vy}{y} - \left(\frac{vy}{y}\right)^2$$

$$\Rightarrow y\frac{dv}{dy} = v - v^2 - v$$

$$\Rightarrow y\frac{dv}{dy} = -v^2$$

$$\Rightarrow \frac{dv}{v^2} = -\frac{dy}{y}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{v^2} = -\int \frac{dy}{y} + c'$$

$$\Rightarrow \frac{-1}{\frac{x}{y}} = -\ln|y| + c'$$

$$\Rightarrow \frac{y}{x} = \ln|y| + c$$

⇒ y = x(ln|y| + c)

Ans: y = x(ln|y| + c)

15. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$x\frac{dy}{dx}-y=2\sqrt{y^2-x^2}$$

Answer

$$\Rightarrow x\frac{dy}{dx}-y = 2\sqrt{y^2-x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y + 2\sqrt{y^2-x^2}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + 2\sqrt{\left(\frac{y}{x}\right)^2 - 1}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x} + 2\sqrt{\left(\frac{vx}{x}\right)^2 - 1}$$

$$\Rightarrow x\frac{dv}{dx} = v - v + 2\sqrt{(v)^2 - 1}$$

$$\Rightarrow x\frac{dv}{dx} = 2\sqrt{(v)^2 - 1}$$

$$\Rightarrow \frac{dv}{\sqrt{(v)^2-1}} = 2\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{\sqrt{(v)^2-1}} = 2\int \frac{dx}{x} + c'$$

$$\Rightarrow \ln\left|v + \sqrt{(v)^2-1}\right| = 2\ln|x| + \ln|c'|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \ln\left|\left(\frac{y}{x}\right) + \sqrt{\left(\frac{y}{x}\right)^2 - 1}\right| = 2\ln|x| + \ln|c'|$$

$\Rightarrow y + \sqrt{y^2 - x^2} = C|x|^3$

Ans: $y + \sqrt{y^2 - x^2} = C|x|^3$

16. Question

In each of the following differential equation show that it is homogeneous and solve it.

$y^2dx + (x^2 + xy + y^2)dy = 0$

Answer

$\Rightarrow y^2dx + (x^2 + xy + y^2)dy = 0$

$\Rightarrow \frac{dx}{dy} = -\frac{x^2 + xy + y^2}{x^2}$

$\Rightarrow \frac{dx}{dy} = -(1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2)$

$\Rightarrow \frac{dx}{dy} = f\left(\frac{x}{y}\right)$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put x = vy

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\Rightarrow v + y\frac{dv}{dy} = -(1 + \frac{y}{vy} + \left(\frac{y}{vy}\right)^2)$

$\Rightarrow v + y\frac{dv}{dy} = -\left(1 + \frac{1}{v} + \left(\frac{1}{v}\right)^2\right) = -\left(\frac{1 + v + v^2}{v^2}\right)$

$\Rightarrow y\frac{dv}{dy} = -\left(\frac{1 + v + v^2}{v^2}\right) - v = -(\frac{1 + v + v^2 + v^3}{v^2})$

$\Rightarrow \frac{v^2dv}{1 + v + v^2 + v^3} = -\frac{dy}{y}$

Integrating both the sides we get:

$\Rightarrow \int \frac{v^2dv}{1 + v + v^2 + v^3} = -\int \frac{dy}{y} + c$

Resubstituting the value of x = vy we get

$\Rightarrow \log\left|\frac{y}{y + x}\right| + \log|x| + \frac{x}{(y + x)} = C$

Ans: $\log\left|\frac{y}{y + x}\right| + \log|x| + \frac{x}{(y + x)} = C$

17. Question

In each of the following differential equation show that it is homogeneous and solve it.

$(x - y)\frac{dy}{dx} = x + 3y$

Answer

$\Rightarrow \frac{dy}{dx} = \frac{x + 3y}{x - y}$

$$\Rightarrow \frac{dy}{dx} = \frac{1 + 3\frac{y}{x}}{1 - \frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1 + 3\frac{vx}{x}}{1 - \frac{vx}{x}}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 + 3v}{1 - v} - v = \frac{1 + 3v - v + v^2}{1 - v} = \frac{1 + 2v + v^2}{1 - v}$$

$$\Rightarrow \frac{1 - v}{1 + 2v + v^2}dv = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{1 - v}{1 + 2v + v^2}dv = \int \frac{dx}{x} + c$$

$$\Rightarrow \int \frac{v - 1}{1 + 2v + v^2}dv = -\int \frac{dx}{x} + c$$

$$\Rightarrow \frac{\ln|1 + 2v + v^2|}{2} = -\ln|x| + \ln c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{\ln\left|1 + 2\frac{y}{x} + \left(\frac{y}{x}\right)^2\right|}{2} = -\ln|x| + \ln c$$

$$\Rightarrow \log|x + y| + \frac{2x}{(x+y)} = C$$

Ans: $\log|x + y| + \frac{2x}{(x+y)} = C$

18. Question

In each of the following differential equation show that it is homogeneous and solve it.

$(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$

Answer

$$\Rightarrow (x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(x^3 + 3xy^2)}{(y^3 + 3x^2y)} = -\frac{3xy^2\left(\frac{x^3}{3xy^2} + 1\right)}{3x^2y\left(\frac{y^3}{3x^2y} + 1\right)} = -\frac{y}{x}\frac{\left(\frac{x^2}{3y^2} + 1\right)}{\left(\frac{y^2}{3x^2} + 1\right)}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\frac{vx}{x}\frac{\left(\frac{x^2}{3(vx)^2} + 1\right)}{\left(\frac{(vx)^2}{3x^2} + 1\right)} = -v\frac{\left(\frac{1}{3(v)^2} + 1\right)}{\left(\frac{(v)^2}{3} + 1\right)} = -\frac{1 + 3(v)^2}{3 + (v)^2} \times \frac{1}{v}$$

$$= -\frac{1 + 3(v)^2}{3v + (v)^3}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{1 + 3(v)^2}{3v + (v)^3} - v = -\frac{1 + 3(v)^2 + 3(v)^2 + (v)^4}{3v + (v)^3}$$

$$= \frac{1 + 6(v)^2 + (v)^4}{3v + (v)^3}$$

$$\Rightarrow \frac{3v + (v)^3}{1 + 6(v)^2 + (v)^4}dv = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{3v + (v)^3}{1 + 6(v)^2 + (v)^4}dv = -\int \frac{dx}{x} + c$$

$$\Rightarrow \frac{\ln|1 + 6(v)^2 + (v)^4|}{4} + \ln|x| = \ln|c|$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{\ln\left|1 + 6(\frac{y}{x})^2 + (\frac{y}{x})^4\right|}{4} + \ln|x| = \ln|c|$$

$\Rightarrow y^4 + 6x^2y^2 + x^4 = C$

Ans: $y^4 + 6x^2y^2 + x^4 = C$

19. Question

In each of the following differential equation show that it is homogeneous and solve it.

$(x - \sqrt{xy})\,dy = ydx$

Answer

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x - \sqrt{xy}} = \frac{1}{\frac{x}{y} - \sqrt{\frac{x}{y}}} = \frac{1}{\left(\frac{y}{x}\right)^{-1} - \sqrt{\left(\frac{y}{x}\right)^{-1}}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1}{\left(\frac{vx}{x}\right)^{-1} - \sqrt{\left(\frac{vx}{x}\right)^{-1}}} = \frac{1}{\frac{1}{v} - \frac{1}{\sqrt{v}}} = \frac{v\sqrt{v}}{\sqrt{v} - v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v\sqrt{v}}{\sqrt{v}-v} - v = \frac{v\sqrt{v} - v\sqrt{v} + v^2}{\sqrt{v}-v} = \frac{v^2}{\sqrt{v}-v}$$

$$\Rightarrow \frac{\sqrt{v}-v}{v^2}dv = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{v^{\frac{3}{2}}}dv - \frac{1}{v}dv = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{1}{v^{\frac{3}{2}}}dv - \int \frac{1}{v}dv = \int \frac{dx}{x} + c$$

$$\Rightarrow \frac{-1}{\sqrt{v}} - \ln|v| = \ln|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{-1}{\sqrt{\left(\frac{y}{x}\right)}} - \ln\left(\frac{y}{x}\right) = \ln|x| + c$$

$$\Rightarrow 2\sqrt{\frac{x}{y}} + \log|y| = C$$

Ans: $2\sqrt{\frac{x}{y}} + \log|y| = C$

20. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$x^2\frac{dy}{dx} + y^2 = xy$$

Answer

$$\Rightarrow \frac{dy}{dx} = \frac{xy - y^2}{x^2} = \frac{y}{x} - (\frac{y}{x})^2$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \left(\frac{vx}{x}\right)^2 = v - v^2$$

$$\Rightarrow x\frac{dv}{dx} = -v^2$$

$$\Rightarrow \frac{dv}{-v^2} = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{-v^2} = \int \frac{dx}{x} + c$$

$$\Rightarrow \frac{1}{v} = \ln|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{1}{\frac{y}{x}} = \ln|x| + c$$

$$\Rightarrow \frac{x}{y} = \ln|x| + \ln|c|$$

$$\Rightarrow \frac{x}{y} = \ln|xc|$$

$$\Rightarrow e^{\frac{x}{y}} = xc$$

Ans: $e^{\frac{x}{y}} = xc$

21. Question

In each of the following differential equation show that it is homogeneous and solve it.

$x\frac{dy}{dx} = y$ (log y - log x + 1)

Answer

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x}\left(\log\left(\frac{vx}{x}\right) + 1\right) = v(\log(v) + 1)$$

$$\Rightarrow x\frac{dv}{dx} = v\log v$$

$$\Rightarrow \frac{dv}{v\log v} = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{v\log v} = \int \frac{dx}{x} + c$$

$$\Rightarrow \log|\log v| = \log|xc|$$

$$\Rightarrow \log|v| = xc$$

$$\Rightarrow v = e^{xc}$$

Resubstituting the value of y = vx we get

$\Rightarrow y = xe^{xc}$

Ans: $y = xe^{xc}$

22. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$x\frac{dy}{dx} - y + x\sin\frac{y}{x} = 0$$

Answer

$$\Rightarrow x\frac{dy}{dx} - y + x\sin\frac{y}{x} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x\sin\frac{y}{x}}{x} = \frac{y}{x} - \sin\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\frac{vx}{x} = v - \sin v$$

$$\Rightarrow x\frac{dv}{dx} = -\sin v$$

$$\Rightarrow \frac{dv}{\sin v} = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{\sin v} = -\int \frac{dx}{x} + c$$

$$\Rightarrow \log\tan\left(\frac{v}{2}\right) = -\log|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \log\tan\left(\frac{y}{2x}\right) = -\log|x| + \log c$$

$$\Rightarrow X\tan\left(\frac{y}{2x}\right) = C$$

Ans: $X\tan\left(\frac{y}{2x}\right) = C$

23. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$x\frac{dy}{dx} = y - x\cos^2\left(\frac{y}{x}\right)$$

Answer

$$\Rightarrow \frac{dy}{dx} = \frac{y - x\cos^2\left(\frac{y}{x}\right)}{x} = \left(\frac{y}{x}\right) - \cos^2\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \left(\frac{vx}{x}\right) - \cos^2\left(\frac{vx}{x}\right) = v - \cos^2 v$$

$$\Rightarrow x\frac{dv}{dx} = -\cos^2 v$$

$$\Rightarrow \frac{dv}{\cos^2 v} = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{\cos^2 v} = -\int \frac{dx}{x} + c$$

$$\Rightarrow \tan v = -\ln|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \tan\left(\frac{y}{x}\right) + \ln|x| = c$$

Ans: $\tan\left(\frac{y}{x}\right) + \ln|x| = c$

24. Question

In each of the following differential equation show that it is homogeneous and solve it.

$$\left(x\cos\frac{y}{x}\right)\frac{dy}{dx} = \left(y\cos\frac{y}{x}\right) + x$$

Answer

$$\Rightarrow \left(x\cos\frac{y}{x}\right)\frac{dy}{dx} = y\cos\frac{y}{x} + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\cos\frac{y}{x} + x}{x\cos\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx}{x} + \sec\frac{vx}{x} = v + \sec v$

$\Rightarrow x\frac{dv}{dx} = \sec v$

$\Rightarrow \frac{dv}{\sec v} = \frac{dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int \frac{dv}{\sec v} = \int \frac{dx}{x} + c$

⇒ sinv = ln|x| + c

Resubstituting the value of y = vx we get

$\Rightarrow \sin\left(\frac{y}{x}\right) = \ln|x| + c$

Ans: $\sin\left(\frac{y}{x}\right) = \ln|x| + c$

25. Question

Find the particular solution of the different equation.$2xy + y^2 - 2x^2\frac{dy}{dx} = 0$, it being given that y = 2 when x = 1

Answer

$2xy + y^2 - 2x^2\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} = \frac{y}{x} + \frac{y^2}{2x^2}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx}{x} + \frac{(vx)^2}{2x^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2}{2}$

$\Rightarrow \frac{dv}{v^2} = \frac{2dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int \frac{dv}{v^2} = 2\int \frac{dx}{x} + c$

$\Rightarrow \frac{-1}{v} = 2\ln|x| + c$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{-x}{y} = 2\ln|x| + c$$

Now,

y = 2 when x = 1

$$\Rightarrow \frac{-1}{2} = 2\ln|1| + c$$

$$\Rightarrow c = \left(-\frac{1}{2}\right) \Rightarrow y = \frac{2x}{(1-\log|x|)}$$

Ans: $y = \frac{2x}{(1-\log|x|)}$

26. Question

Find the particular solution of the differential equation $\left\{x\sin^2\frac{y}{x} - y\right\}$dx + xdy = 0, it being given that y = $\frac{\pi}{4}$ when x = 1.

Answer

$$\Rightarrow \frac{dy}{dx} = \frac{y - x\sin^2\left(\frac{y}{x}\right)}{x} = \left(\frac{y}{x}\right) - \sin^2\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \left(\frac{y}{x}\right) - \sin^2\left(\frac{y}{x}\right) = v - \sin^2 v$$

$$\Rightarrow x\frac{dv}{dx} = -\sin^2 v$$

$$\Rightarrow \frac{dv}{\sin^2 v} = -\frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{\sin^2 v} = -\int \frac{dx}{x} + c$$

$$\Rightarrow \cot v = \ln|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \cot\left(\frac{y}{x}\right) = \ln|x| + c$$

y = $\frac{\pi}{4}$ when x = 1

$\Rightarrow \cot\left(\frac{\frac{\pi}{4}}{1}\right) = \ln|1| + c$

⇒ c = 1

Ans: $\cot\left(\frac{y}{x}\right) = \ln|x| + 1$

27. Question

Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{y(2y-x)}{x(2y+x)}$ given that y = 1 when x = 1.

Answer

$\Rightarrow \frac{dy}{dx} = \frac{y(2y-x)}{x(2y+x)}$

$\Rightarrow \frac{dy}{dx} = \frac{y\left(2\frac{y}{x}-1\right)}{x\left(2\frac{y}{x}+1\right)}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx\left(2\frac{vx}{x}-1\right)}{x\left(2\frac{vx}{x}+1\right)} = v\left(\frac{2v-1}{2v+1}\right)$

$\Rightarrow x\frac{dv}{dx} = v\left(\frac{2v-1}{2v+1}\right) - v$

$\Rightarrow x\frac{dv}{dx} = v\left(\frac{2v-1-2v-1}{2v+1}\right) \Rightarrow x\frac{dv}{dx} = \frac{-2v}{2v+1}$

$\Rightarrow \frac{2v+1}{2v}dv = \frac{-dx}{x}$

$\Rightarrow dv + \left(\frac{1}{2v}\right)dv = \frac{-dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int\left(dv + \left(\frac{1}{2v}\right)dv\right) = -\int\frac{dx}{x} + c$

$\Rightarrow v + \frac{\ln|v|}{2} = -\ln|x| + c$

Resubstituting the value of y = vx we get

$\Rightarrow \frac{y}{x} + \frac{\ln\left|\frac{y}{x}\right|}{2} = -\ln|x| + c$

y = 1 when x = 1

1 + 0 = - 0 + c

$\Rightarrow c = 1$

$\Rightarrow \frac{y}{x} + \frac{1}{2}\log|xy| = 1$

Ans: $\frac{y}{x} + \frac{1}{2}\log|xy| = 1$

28. Question

Find the particular solution of the differential equation $xe^{y/x} - y + x\frac{dy}{dx} = 0$, given that $y(1) = 0$.

Answer

$\Rightarrow xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$

$\Rightarrow x\frac{dy}{dx} = y - xe^{\frac{y}{x}}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right) - e^{\frac{y}{x}}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \left(\frac{vx}{x}\right) - e^{\frac{vx}{x}}$

$\Rightarrow x\frac{dv}{dx} = -e^{v}$

$\Rightarrow \frac{dv}{e^{v}} = \frac{-dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int \frac{dv}{e^{v}} = -\int \frac{dx}{x} + c$

$\Rightarrow -e^{-v} = -\ln|x| + c$

Resubstituting the value of y = vx we get

$\Rightarrow -e^{-\left(\frac{y}{x}\right)} = -\ln|x| + c$

Now,y(1) = 0

$\Rightarrow -e^{-(0)} = -\ln|1| + c$

$\Rightarrow c = -1$

$\Rightarrow \log|x| + e^{-y/x} = 1$

Ans: $\log|x| + e^{-y/x} = 1$

29. Question

Find the particular solution of the differential equation $xe^{y/x} - y + x\frac{dy}{dx} = 0$, given that y(e) = 0.

Answer

$$\Rightarrow xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$$

$$\Rightarrow x\frac{dy}{dx} = y - xe^{\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right) - e^{\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \left(\frac{vx}{x}\right) - e^{\frac{vx}{x}}$$

$$\Rightarrow x\frac{dv}{dx} = -e^{v}$$

$$\Rightarrow \frac{dv}{e^{v}} = \frac{-dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int \frac{dv}{e^{v}} = -\int \frac{dx}{x} + c$$

$$\Rightarrow -e^{-v} = -\ln|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow -e^{-\left(\frac{y}{x}\right)} = -\ln|x| + c$$

Now,y(e) = 0

$$\Rightarrow -e^{-(0)} = -\ln|e| + c$$

$$\Rightarrow c = 0$$

⇒ y = - xlog(log|x|)

Ans: y = - xlog(log|x|)

30. Question

The slope of the tangent to a curve at any point (x,y) on it is given by $\frac{y}{x} - \left(\cot\frac{y}{x}\right)\left(\cos\frac{y}{x}\right)$, where x>0 and y>0. If the curve passes through the point $\left(1, \frac{\pi}{4}\right)$, find the equation of the curve.

Answer

It is given that:

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \cot\frac{y}{x}\cos\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

⇒ the given differential equation is a homogenous equation.

The solution of the given differential equation is :

Put y = vx

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \cot\frac{vx}{x}\cos\frac{vx}{x}$$

$$\Rightarrow x\frac{dv}{dx} = -\cot v\cos v$$

$$\Rightarrow \frac{dv}{-\cot v\cos v} = \frac{dx}{x}$$

Integrating both the sides we get:

$$\Rightarrow \int\frac{dv}{-\cot v\cos v} = \int\frac{dx}{x} + c$$

$$\Rightarrow \frac{-1}{\cos v} = \ln|x| + c$$

Resubstituting the value of y = vx we get

$$\Rightarrow \frac{-1}{\cos\frac{y}{x}} = \ln|x| + c$$

the curve passes through the point $\left(1, \frac{\pi}{4}\right)$

$$\Rightarrow \frac{-1}{\cos\frac{\frac{\pi}{x}}{1}} = \ln|1| + c$$

$$\Rightarrow c = -\sqrt{2}$$

$$\Rightarrow \sec\frac{y}{x} + \log|x| = \sqrt{2}$$

Ans:The equation of the curve is: $\sec\frac{y}{x} + \log|x| = \sqrt{2}$