2. Functions

Exercise 2A

1. Question

Define a function. What do you mean by the domain and range of a function? Give examples.

Answer

Definition:A relation R from a set A to a set B is called a function if each element of A has a unique image in B.

It is denoted by the symbol f:A→B which reads ‘f’ is a function from A to B ‘f’ maps A to B.

Let f:A→B,then the set A is known as the domain of f & the set B is known as co - domain of f .The set of images of all the elements of A is known as the range of f.

Thus, Domain of f = $\{a | a \in A, (a, f(a)) \in f\}$

Range of f = $\{f(a) \mid a \in A, f(a) \in B\}$

Example: The domain of $y = \sin x$ is all values of x i.e. R , since there are no restrictions on the values for x. The range of y *is* betweeen −1 and 1. We could write this as $-1 \le y \le 1$.

2. Question

Define each of the following:

(i) injective function

(ii) surjective function

(iii) bijective function

(iv) many - one function

(v) into function

Give an example of each type of functions.

Answer

1)injective function

Definition: A function f: A → B is said to be a one - one function or injective mapping if different elements of A have different f images in B.

A function *f* is injective if and only if whenever *f(x) = f(y), x = y*.

Example: f(x) = x + 9 from the set of real number R to R is an injective function. When x = 3,then :f(x) = 12,when f(y) = 8,the value of y can only be 3,so x = y.

(ii) surjective function

Definition: If the function f:A→B is such that each element in B (co - domain) is the ‘f’ image of atleast one element in A , then we say that f is a function of A ‘onto’ B .Thus f: A→B is surjective if, for all b∈B, there are some a∈A such that f(a) = b.

Example: The function f(x) = 2x from the set of natural numbers N to the set of non negative even numbers is a surjective function.

(iii) bijective function

Definition: A function f (from set A to B) is bijective if, for every y in B, there is exactly one x in A such that f(x) = y.Alternatively, f is bijective if it is a one - to - one correspondence between those sets, in other words, both injective and surjective.

Example: If f(x) = x^2,from the set of positive real numbers to positive real numbers is both injective and surjective.Thus it is a bijective function.

(iv)many - one function

Defintion : A function f: A→B is said to be a many one functions if two or more elements of A have the same f image in B.

trigonometric functions such as sinx are many - to - one since sinx = sin(2π + x) = sin(4π + x) and so one...

(v) into function

Definition: If f:A→B is such that there exists atleast one element in co - domain , which is not the image of any element in the domain , then f(x) is into.

Let f(x) = y = x – 1000

⇒ x = y + 1000 = g(y) (say)

Here g(y) is defined for each y∈ I , but g(y) ∉ N for y ≤ − 1000. Hence,f is into.

3. Question

Give an example of a function which is

(i) one - one but not onto

(ii) one - one and onto

(iii) neither one - one nor onto

(iv) onto but not one - one.

Answer

(i) one - one but not onto

f(x) = 6x

For One - One

$f(x_1) = 6x_1$

$f(x_2) = 6x_2$

put $f(x_1) = f(x_2)$ we get

$6x_1 = 6x_2$

Hence, if $f(x_1) = f(x_2)$, $x_1 = x_2$

Function f is one - one

For Onto

f(x) = 6x

let f(x) = y ,such that y∈N

6x = y

$\Rightarrow x = \frac{y}{6}$

If y = 1

$x = \frac{1}{6} = 0.166667$

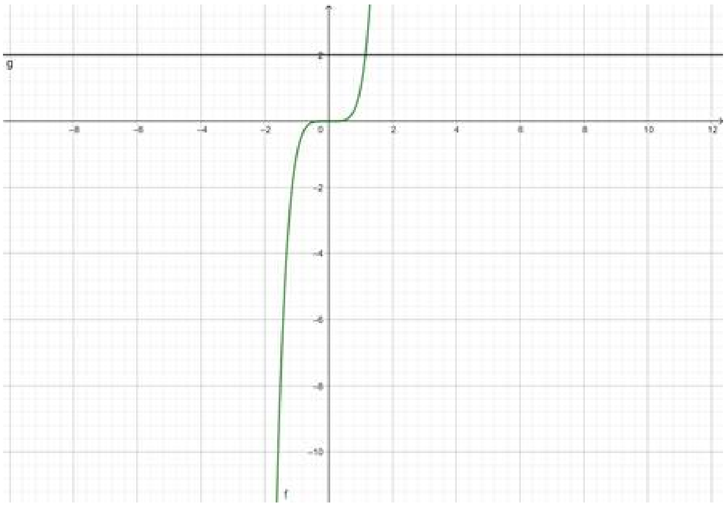
which is not possible as x∈N

Hence, f is not onto.

(ii) one - one and onto

$f(x) = x^5$

$\Rightarrow y = x^5$



Since the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) is one - one.

The range of f(x) = (- ∞,∞) = R(Codomain)

∴f(x) is onto

∴f(x) is one - one and onto.

(iii) neither one - one nor onto

$f(x) = x^2$

for one one:

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

$f(x_1) = f(x_2)$

$\Rightarrow (x_1)^2 = (x_2)^2$

$\Rightarrow x_1 = x_2$ or $x_1 = - x_2$

Since x_1 does not have a unique image it is not one - one

For onto

f(x) = y

such that y∈R

$x^2 = y$

$\Rightarrow x = \pm\sqrt{y}$

If y is negative under root of a negative number is not real

Hence,f(x) is not onto.

∴f(x) is neither onto nor one - one

(iv) onto but not one - one.

Consider a function f:Z→N such that f(x) = |x|.

Since the Z maps to every single element in N twice, this function is onto but not one - one.

Z - integers

N - natural numbers.

4. Question

Let f : R → R be defined by

$$f(x) = \begin{cases} 2x + 3, & \text{when} \quad x < -2 \\ x^2 - 2, & \text{when} \quad -2 \le x \le 3 \\ 3x - 1, & \text{when} \quad x > 3 \end{cases}$$

Find (i) f(2) (ii) f(4) (iii) f(- 1) (iv) f(- 3).

Answer

i)f(2)

Since f(x) = x^2 - 2 , when x = 2

∴ f(2) = $(2)^2$ - 2 = 4 - 2 = 2

∴f(2) = 2

ii)f(4)

Since f(x) = 3x - 1 , when x = 4

∴f(4) = (3×4) - 1 = 12 - 1 = 11

∴f(4) = 11

iii)f(- 1)

Since f(x) = x^2 - 2 , when x = - 1

∴ f(- 1) = $(- 1)^2$ - 2 = 1 - 2 = - 1

∴f(- 1) = - 1

iv)f(- 3)

Since f(x) = 2x + 3 , when x = - 3

∴f(- 3) = 2×(- 3) + 3 = - 6 + 3 = - 3

∴f(- 3) = - 3

5. Question

Show that the function f: R → R : f(x) = 1 + x^2 is many - one into.

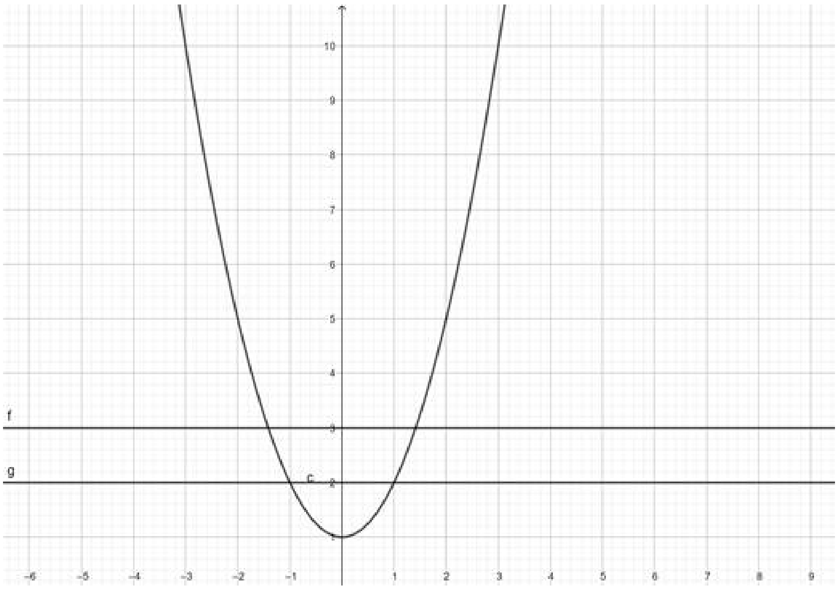
Answer

To show: f: R → R : f(x) = 1 + x^2 is many - one into.

Proof:

f(x) = 1 + x^2

⇒y = 1 + x^2



Since the lines cut the curve in 2 equal valued points of y therefore the function f(x) is many one.

The range of f(x) = $[1,\infty) \neq R$(Codomain)

$\therefore$f(x) is not onto

$\Rightarrow$f(x) is into

Hence, showed that f: R → R : f(x) = 1 + x^2 is many - one into.

6. Question

Show that the function f : R → R : f(x) = x^4 is many - one and into.

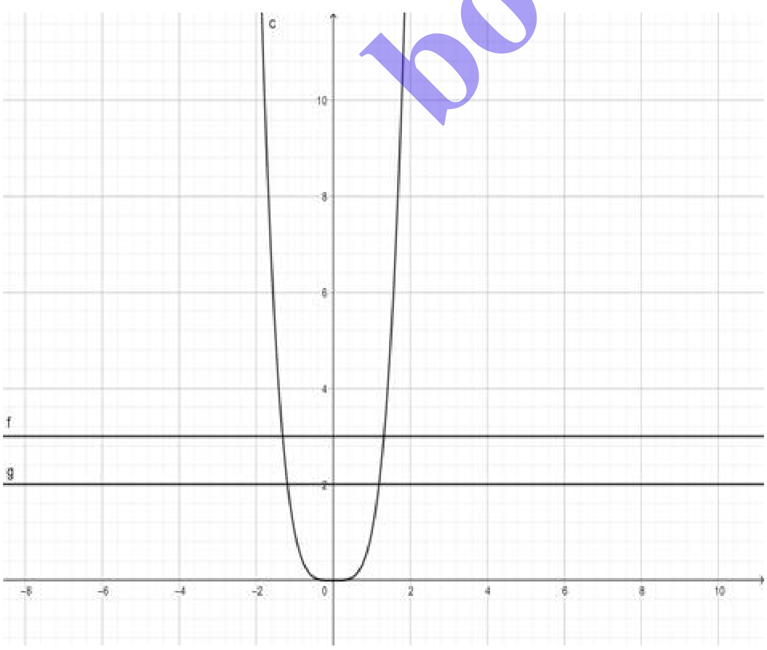
Answer

To show: f: R → R : f(x) = x^4 is many - one into.

Proof:

f(x) = x^4

$\Rightarrow$y = x^4



Since the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) is many ones.

The range of f(x) = $[0,\infty) \neq R$(Codomain)

∴f(x) is not onto

⇒f(x) is into

Hence, showed that f: R → R : f(x) = x^4 is many - one into.

7. Question

Show that the function f: R → R : f(x) = x^5 is one - one and onto.

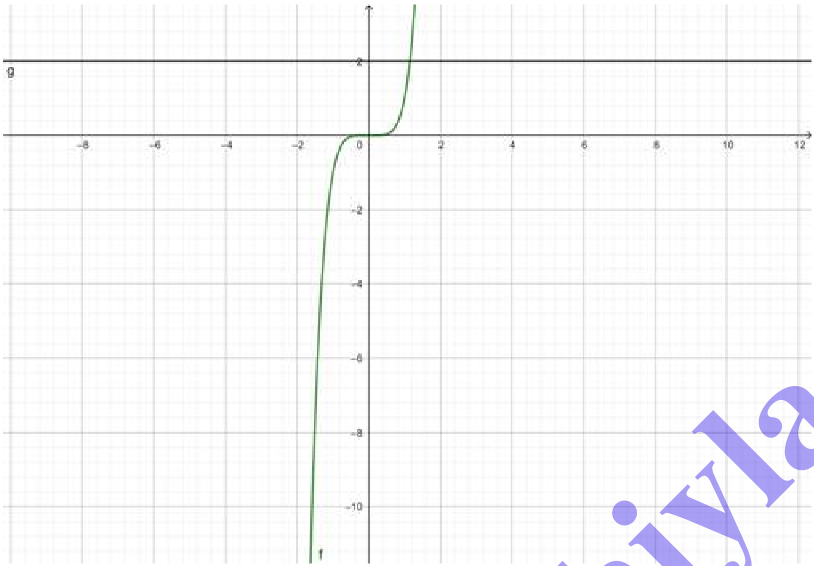
Answer

To show: f: R → R : : f(x) = x^5 is one - one and onto.

Proof:

f(x) = x^5

⇒y = x^5



Since the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) is one - one.

The range of f(x) = (- ∞,∞) = R(Codomain)

∴f(x) is onto

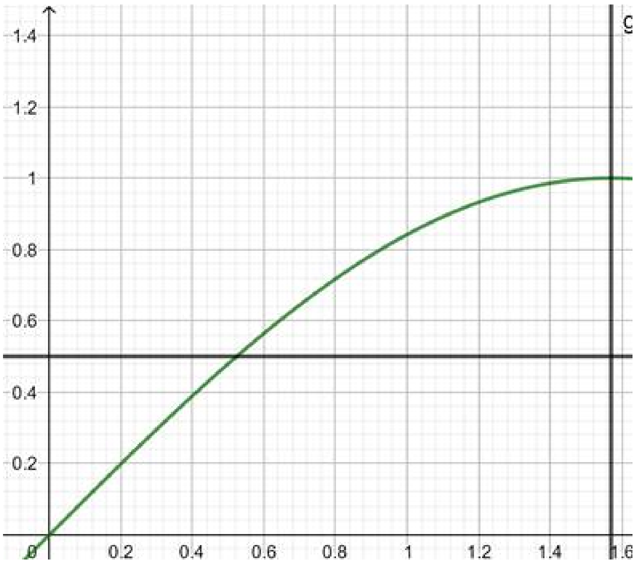
Hence, showed f: R → R : f(x) = x^5 is one - one and onto.

8. Question

Let $f:\left[0,\frac{\pi}{2}\right]\to R:f(x)=\sin x$ and $g:\left[0,\frac{\pi}{2}\right]\to R:g(x)=\cos x.$ Show that each one of f and g is one - one but (f + g) is not one - one.

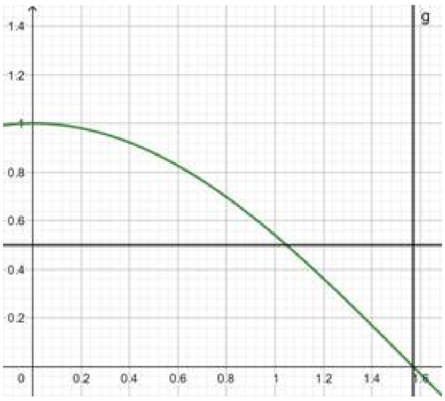
Answer

$$f:\left[0,\frac{\pi}{2}\right]\to R:f(x)=\sin x$$



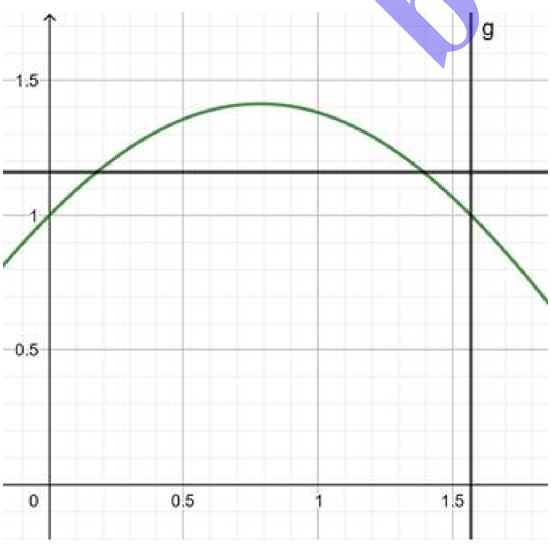
Here in this range, the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) = sinx is one - one.

$$g:\left[0,\frac{\pi}{2}\right]\rightarrow R:g(x)=\cos x.$$



in this range, the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) = cosx is also one - one.

(f + g):[0,$\frac{\pi}{2}$] →R = sinx + cosx



in this range the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) = cosx + sinx is not one - one.

Hence,showed that each one of f and g is one - one but (f + g) is not one - one.

9. Question

Show that the function

(i) f : N → N : f(x) = x^2 is one - one into.

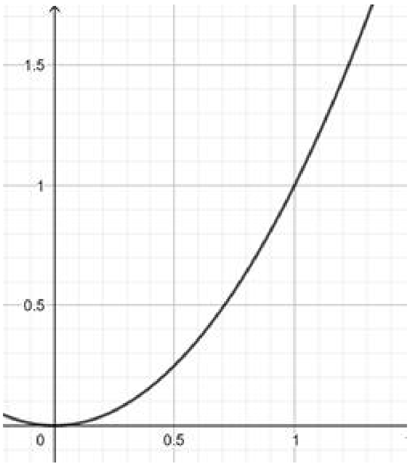
(ii) f : Z → Z : f(x) = x^2 is many - one into

Answer

(i) f : N → N : f(x) = x^2 is one - one into.

f(x) = x^2

⇒y = x^2



Since the function f(x) is monotonically increasing from the domain N → N

∴f(x) is one –one

Range of f(x) = (0,∞)≠N(codomain)

∴f(x) is into

∴f : N → N : f(x) = x^2 is one - one into.

(ii) f : Z → Z : f(x) = x^2 is many - one into

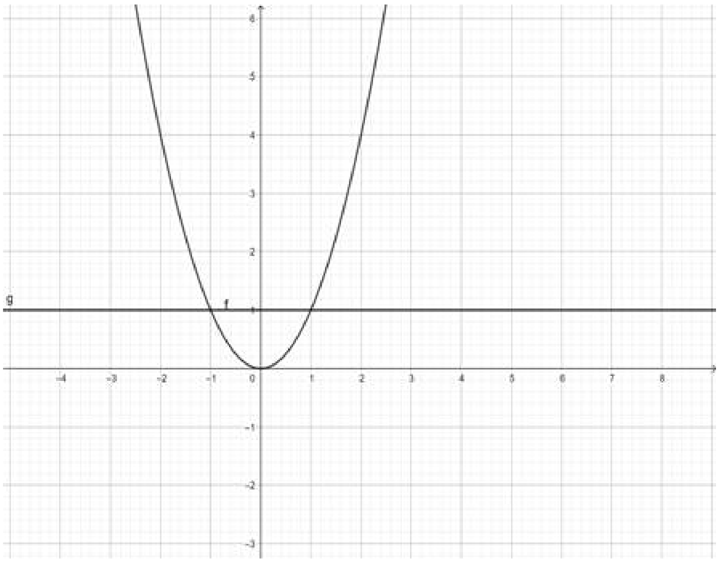
f(x) = x^2

⇒y = x^2

in this range the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) = x^2 is many - one .

Range of f(x) = (0,∞)≠Z(codomain)

∴f(x) is into



∴ f : Z → Z : f(x) = x^2 is many - one into

10. Question

Show that the function

(i) f : N → N : f(x) = x^3 is one - one into

(ii) f : Z → Z : f(x) = x^3 is one - one into

Answer

(i) f : N → N : f(x) = x^3 is one - one into.

f(x) = x^3

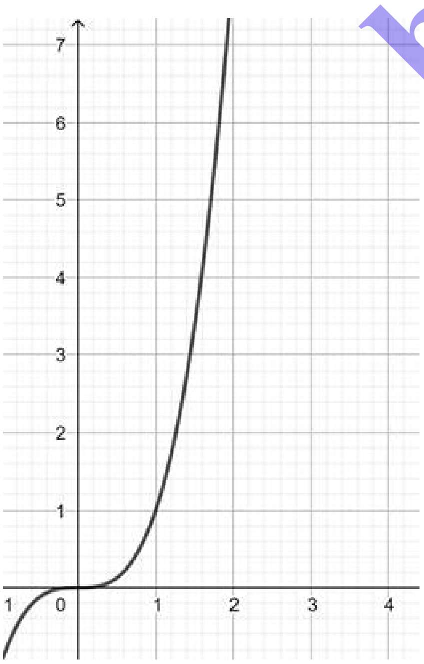
Since the function f(x) is monotonically increasing from the domain N → N

∴f(x) is one –one

Range of f(x) = (- ∞,∞)≠N(codomain)

∴f(x) is into

∴f : N → N : f(x) = x^2 is one - one into.



(ii) f : Z → Z : f(x) = x^3 is one - one into

$f(x) = x^3$

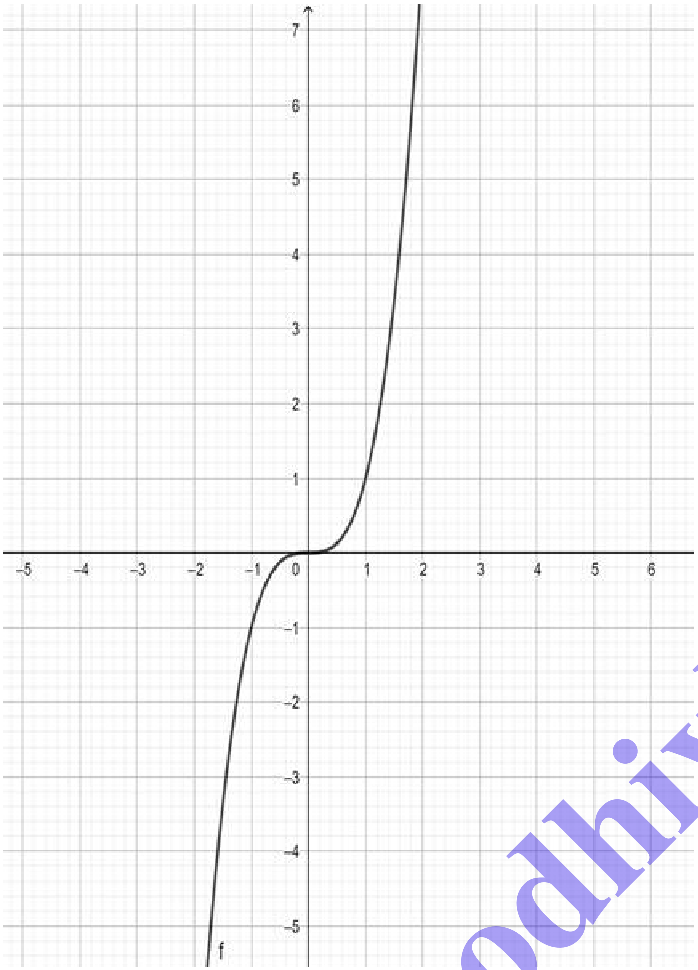
Since the function f(x) is monotonically increasing from the domain Z → Z

∴f(x) is one –one

Range of f(x) = (- ∞,∞)≠Z(codomain)

∴f(x) is into

∴ f : Z → Z : $f(x) = x^3$ is one - one into.



11. Question

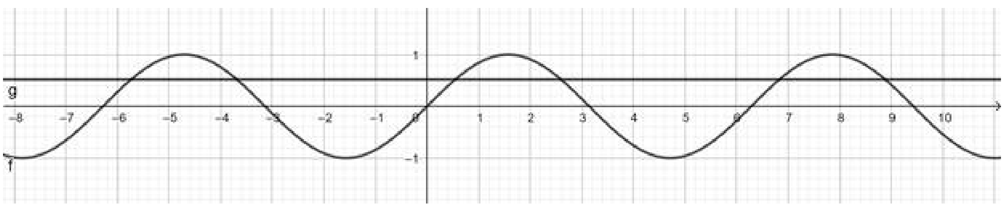
Show that the function f : R → R : f(x) = sin x is neither one - one nor onto.

Answer

f(x) = sinx

y = sinx

Here in this range, the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) = sinx is not one - one.



Range of f(x) = [- 1,1]≠R(codomain)

∴f(x) is not onto.

Hence, showed that the function f : R → R : f(x) = sin x is neither one - one nor onto.

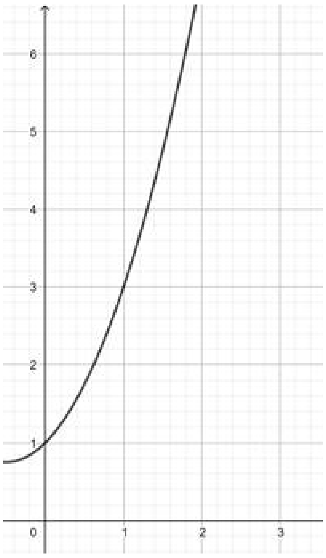
12. Question

Prove that the function f : N → N : f(n) = (n^2 + n + 1) is one - one but not onto.

Answer

In the given range of N f(x) is monotonically increasing.

∴f(n) = n^2 + n + 1 is one one.



But Range of f(n) = [0.75,∞)≠N(codomain)

Hence,f(n) is not onto.

Hence, proved that the function f : N → N : f(n) = (n^2 + n + 1) is one - one but not onto.

13. Question

Show that the function f: N → Z, defined by

$$f(n)=\begin{cases}\frac{1}{2}(n-1), \text{ when n is odd}\\ -\frac{1}{2}n, \text{ when n is even}\end{cases}$$

is both one - one and onto.

Answer

$$f(n)=\begin{cases}\frac{1}{2}(n-1), \text{ when n is odd}\\ -\frac{1}{2}n, \text{ when n is even}\end{cases}$$

f(1) = 0

f(2) = - 1

f(3) = 1

f(4) = - 2

f(5) = 2

f(6) = - 3

Since at no different values of x we get same value of y∴f(n) is one –one

And range of f(n) = Z = Z(codomain)

∴ the function f: N → Z, defined by

$$f(n)=\begin{cases}\frac{1}{2}(n-1), \text{ when n is odd} \\ -\frac{1}{2}n, \text{ when n is even}\end{cases}$$

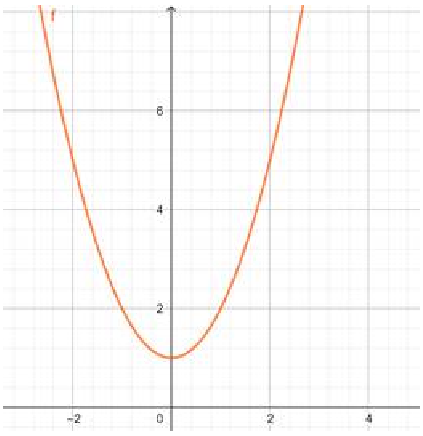
is both one - one and onto.

14. Question

Find the domain and range of the function

F : R → R : f(x) = x^2 + 1.

Answer



Since the function f(x) can accept any values as per the given domain R, therefore, the domain of the function f(x) = x^2 + 1 is R.

The minimum value of f(x) = 1

⇒Range of f(x) = [- 1,∞]

i.e range (f) = {y ∈ R : y ≥ 1}

Ans: dom (f) = R and range (f) = {y ∈ R : y ≥ 1}

15. Question

Which of the following relations are functions? Give reasons. In case of a function, find its domain and range.

(i) f = {(- 1, 2), (1, 8), (2, 11), (3, 14)}

(ii) g = {(1, 1), (1, - 1), (4, 2), (9, 3),(16, 4)}

(iii) h = {(a, b), (b, c), (c, b), (d, c)}

Answer

For a relation to be a function each element of 1st set should have different image in the second set(Range)

i) (i) f = {(- 1, 2), (1, 8), (2, 11), (3, 14)}

Here, each of the first set element has different image in second set.

∴f is a function whose domain = { - 1, 1, 2, 3} and range (f) = {2, 8, 11, 14}

(ii) g = {(1, 1), (1, - 1), (4, 2), (9, 3),(16, 4)}

Here, some of the first set element has same image in second set.

∴ g is not a function.

(iii) h = {(a, b), (b, c), (c, b), (d, c)}

Here, each of the first set element has different image in second set.

∴h is a function whose domain = {a, b, c, d} and range (h) = {b, c}

(range is the intersection set of the elements of the second set elements.)

16. Question

Find the domain and range of the real function, defined by $f(x) = \frac{x^2}{(1+x^2)}$. Show that f is many - one.

Answer

For domain $(1 + x^2) \neq 0$

$\Rightarrow x^2 \neq -1$

⇒dom(f) = R

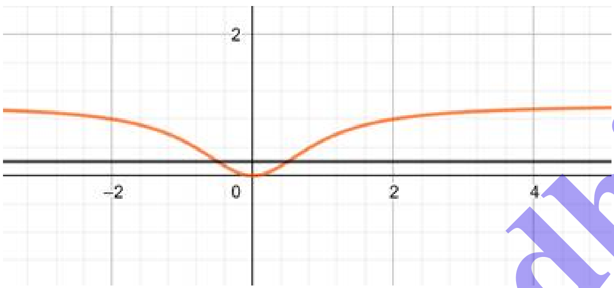
For the range of x:

$\Rightarrow y = \frac{x^2+1-1}{x^2+1} = 1 - \frac{1}{x^2+1}$

$y_{min} = 0$ (when x = 0)

$y_{max} = 1$ (when x = ∞)

∴range of f(x) = [0,1)



For many one the lines cut the curve in 2 equal valued points of y therefore the function f(x) $= \frac{x^2}{x^2+1}$ is many - one.

Ans:

dom(f) = R

range(f) = [0,1)

function f(x) $= \frac{x^2}{x^2+1}$ is many - one.

17. Question

Show that the function

$$f : R \to R : f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is irrational} \end{cases}$$

is many - one into.

Find (i) $f\left(\frac{1}{2}\right)$ (ii) $f\left(\sqrt{2}\right)$ (iii) $f(\pi)$

(iv) $f\left(2+\sqrt{3}\right)$.

Answer

(i) $f\left(\frac{1}{2}\right)$

Here, x = 1/2,which is rational

∴f(1/2) = 1

(ii) $f\left(\sqrt{2}\right)$

Here, x = √2,which is irrational

∴f(√2) = - 1

(iii) $f\left(\pi\right)$

Here, x = ∏, which is irrational

$f\left(\pi\right)$ = - 1

(iv) $f\left(2+\sqrt{3}\right)$.

Here,x = 2 + √3, which is irrational

∴f(2 + √3) = - 1

Ans. (i) 1 (ii) - 1 (iii) - 1 (iv) - 1

Exercise 2B

1. Question

Let A = {1, 2, 3, 4}. Let f : A→ A and g : A→ A,

defined by f = {(1, 4), (2, 1), (3,3),(4, 2)} and g = {(1, 3), (2, 1), (3, 2), (4, 4)}.

Find (i) g of (ii) f o g (iii) f o f.

Answer

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)} and g = {(1, 3), (2, 1),

(3, 2), (4, 4)}

Solution: We have,

gof(1) = g(f(1)) = g(4) = 4

gof(2) = g(f(2)) = g(1) = 3

gof(3) = g(f(3)) = g(3) = 2

gof(4) = g(f(4)) = g(2) = 1

Ans) g o f = {(1, 4), (2, 3), (3, 2), (4, 1)}

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)} and g = {(1, 3), (2, 1), (3, 2), (4, 4)}

Solution: We have,

fog(1) = f(g(1)) = f(3) = 3

fog(2) = f(g(2)) = f(1) = 4

fog(3) = f(g(3)) = f(2) = 1

fog(4) = f(g(4)) = f(4) = 2

Ans) f o g = {(1, 3), (2, 4), (3, 1), (4, 2)}

(iii) f o f

To find: f o f

Formula used: f o f = f(f(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)}

Solution: We have,

fof(1) = f(f(1)) = f(4) = 2

fof(2) = f(f(2)) = f(1) = 4

fof(3) = f(f(3)) = f(3) = 3

fof(4) = f(f(4)) = f(2) = 1

Ans) f o f = {(1, 2), (2, 4), (3, 3), (4, 1)}

2. Question

Let f : {3, 9, 12} → {1, 3, 4} and g : {1, 3, 4, 5} → {3, 9} be defined as f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3), (4, 9), (5, 9)}.

Find (i) (g o f) (ii) (f o g).

Answer

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3),(4, 9), (5, 9)}

Solution: We have,

gof(3) = g(f(3)) = g(1) = 3

gof(9) = g(f(9)) = g(3) = 3

gof(12) = g(f(12)) = g(4) = 9

Ans) g o f = {(3, 3), (9, 3), (12, 9)}

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3),(4, 9), (5, 9)}

Solution: We have,

fog(1) = f(g(1)) = f(3) = 1

fog(3) = f(g(3)) = f(3) = 1

fog(4) = f(g(4)) = f(9) = 3

fog(5) = f(g(5)) = f(9) = 3

Ans) f o g = {(1, 1), (3, 1), (4, 3), (5, 3)}

3. Question

Let f : R → R : f(x) = x^2 and g : R → R : g(x) = (x + 1).

Show that (g o f) ≠ (f o g).

Answer

To prove: (g o f) ≠ (f o g)

Formula used: (i) g o f = g(f(x))

(ii) f o g = f(g(x))

Given: (i) f : R → R : f(x) = x^2

(ii) g : R → R : g(x) = (x + 1)

Proof: We have,

g o f = g(f(x)) = g(x^2) = (x^2 + 1)

f o g = f(g(x)) = g(x+1) = [$(x+1)^2$ + 1] = x^2 + 2x + 2

From the above two equation we can say that (g o f) ≠ (f o g)

Hence Proved

4. Question

Let f : R → R : f(x) = (2x + 1) and g : R → R : g(x) = (x^2 - 2).

Write down the formulae for

(i) (g o f) (ii) (f o g)

(iii) (f o f) (iv) (g o g)

Answer

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f : R → R : f(x) = (2x + 1)

(ii) g : R → R : g(x) = (x^2 - 2)

Solution: We have,

g o f = g(f(x)) = g(2x + 1) = [$(2x + 1)^2$ – 2]

⇒ $4x^2$ + 4x + 1 – 2

⇒ $4x^2$ + 4x – 1

Ans). g o f (x) = $4x^2 + 4x - 1$

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: (i) f : R → R : f(x) = (2x + 1)

(ii) g : R → R : g(x) = $(x^2 - 2)$

Solution: We have,

f o g = f(g(x)) = f$(x^2 - 2)$ = [$2(x^2 - 2) + 1$]

⇒ $2x^2 - 4 + 1$

⇒ $2x^2 - 3$

Ans). f o g (x) = $2x^2 - 3$

(iii) f o f

To find: f o f

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = (2x + 1)

Solution: We have,

f o f = f(f(x)) = f(2x + 1) = [2(2x + 1) + 1]

⇒ 4x + 2 + 1

⇒ 4x + 3

Ans). f o f (x) = 4x+ 3

(iv) g o g

To find: g o g

Formula used: g o g = g(g(x))

Given: (i) g : R → R : g(x) = $(x^2 - 2)$

Solution: We have,

g o g = g(g(x)) = g$(x^2 - 2)$ = [$(x^2 - 2)^2 - 2$]

⇒ $x^4 - 4x^2 + 4 - 2$

⇒ $x^4 - 4x^2 + 2$

Ans). g o g (x) = $x^4 - 4x^2 + 2$

5. Question

Let f : R → R : f(x) = $(x^2 + 3x + 1)$ and g: R → R : g(x) = (2x - 3). Write down the formulae for

(i) g o f

(ii) f o g

(iii) g o g

Answer

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f : R → R : f(x) = (x^2 + 3x + 1)

(ii) g: R → R : g(x) = (2x - 3)

Solution: We have,

g o f = g(f(x)) = g(x^2 + 3x + 1) = [2(x^2 + 3x + 1) – 3]

⇒ $2x^2 + 6x + 2 – 3$

⇒ $2x^2 + 6x – 1$

Ans). g o f (x) = $2x^2 + 6x – 1$

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: (i) f : R → R : f(x) = (x^2 + 3x + 1)

(ii) g: R → R : g(x) = (2x - 3)

Solution: We have,

f o g = f(g(x)) = f(2x - 3) = [$(2x - 3)^2$ + 3(2x – 3) + 1]

⇒ $4x^2 - 12x + 9 + 6x – 9 + 1$

⇒ $4x^2 - 6x + 1$

Ans). f o g (x) = $4x^2 - 6x + 1$

(iii) g o g

To find: g o g

Formula used: g o g = g(g(x))

Given: (i) g: R → R : g(x) = (2x - 3)

Solution: We have,

g o g = g(g(x)) = g(2x - 3) = [2(2x – 3) - 3]

⇒ 4x – 6 - 3

⇒ 4x - 9

Ans). g o g (x) = 4x – 9

6. Question

Let f : R → R : f(x) = |x|, prove that f o f = f.

Answer

To prove: f o f = f

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = |x|

Solution: We have,

f o f = f(f(x)) = f(|x|) = ||x|| = |x| = f(x)

Clearly f o f = f.

Hence Proved.

7. Question

Let f : R → R : f(x) = x^2, g : R → R : g(x) = tan x

and h : R → R : h(x) = log x.

Find a formula for h o (g o f).

Show that [h o (g o f)] $\sqrt{\frac{\pi}{4}} = 0.$

Answer

I

To find: formula for h o (g o f)

To prove: Show that [h o (g o f)] $\sqrt{\frac{\pi}{4}} = 0$

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = x^2

(ii) g : R → R : g(x) = tan x

(iii) h : R → R : h(x) = log x

Solution: We have,

h o (g o f) = h o g(f(x)) = h o g(x^2)

= h(g(x^2)) = h (tan x^2)

= log (tan x^2)

h o (g o f) = log (tan x^2)

For, [h o (g o f)] $\sqrt{\frac{\pi}{4}}$

$= \log \left[\tan \left(\sqrt{\frac{\pi}{4}}\right)^2\right]$

$= \log \left[\tan \frac{\pi}{4}\right]$

= log 1

= 0

Hence Proved.

8. Question

Let f : R → R : f(x) (2x - 3) and $g : R \to R : g(x) = \frac{1}{2}(x + 3).$

Show that (f o g) = I_R = (g o f).

Answer

To prove: (f o g) = I_R = (g o f).

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = (2x - 3)

(ii) $g : R \rightarrow R : g(x) = \frac{1}{2}(x+3)$

Solution: We have,

f o g = f(g(x))

$= f\left(\frac{1}{2}(x+3)\right)$

$= \left[2\left(\frac{1}{2}(x+3)\right) - 3\right]$

= x + 3 – 3

= x

$= I_R$

g o f = g(f(x))

$= g(2x - 3)$

$= \frac{1}{2}(2x-3+3)$

$= \frac{1}{2}(2x)$

= x

$= I_R$

Clearly we can see that (f o g) = I_R = (g o f) = x

Hence Proved.

9. Question

Let f : Z → Z : f(x) = 2x. Find g : Z → Z : g o f = I_Z.

Answer

To find: g : Z → Z : g o f = I_Z

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) g : Z → Z : g o f = I_Z

Solution: We have,

f(x) = 2x

Let f(x) = y

⇒ y = 2x

$\Rightarrow x = \frac{y}{2}$

$\Rightarrow x = \frac{y}{2}$

Let $g(y) = \frac{y}{2}$

Where g: Z → Z

For g o f,

⇒ g(f(x))

⇒ g(2x)

⇒ $\frac{2x}{2}$

⇒ x = I_Z

Clearly we can see that (g o f) = x = I_Z

The required function is $g(x) = \frac{x}{2}$

10. Question

Let f : N → N : f(x) = 2x, g : N → N : g(y) = 3y + 4 and h : N → N : h(z) = sin z. Show that h o (g o f) = (h o g) o f.

Answer

To show: h o (g o f) = (h o g) o f

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : N → N : f(x) = 2x

(ii) g : N → N : g(y) = 3y + 4

(iii) h : N → N : h(z) = sin z

Solution: We have,

LHS = h o (g o f)

⇒ h o (g(f(x))

⇒ h(g(2x))

⇒ h(3(2x) + 4)

⇒ h(6x +4)

⇒ sin(6x + 4)

RHS = (h o g) o f

⇒ (h(g(x))) o f

⇒ (h(3x + 4)) o f

⇒ sin(3x+4) o f

Now let sin(3x+4) be a function u

RHS = u o f

⇒ u(f(x))

⇒ u(2x)

⇒ sin(3(2x) + 4)

⇒ sin(6x + 4) = LHS

Hence Proved.

11. Question

If f be a greatest integer function and g be an absolute value function, find the value of $(f \circ g)\left(\frac{-3}{2}\right)+(g \circ f)\left(\frac{4}{3}\right)$.

Answer

To find: $(fog)\left(\frac{-3}{2}\right)+(gof)\left(\frac{4}{3}\right)$

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f is a greatest integer function

(ii) g is an absolute value function

f(x) = [x] (greatest integer function)

g(x) = |x| (absolute value function)

$f\left(\frac{4}{3}\right)= \left[\frac{4}{3}\right]=1$ ... (i)

$g\left(\frac{-3}{2}\right)= \left|\frac{-3}{2}\right| = 1.5$ ... (ii)

Now, for $(fog)\left(\frac{-3}{2}\right)+(gof)\left(\frac{4}{3}\right)$

$\Rightarrow f\left(g\left(\frac{-3}{2}\right)\right)+g\left(f\left(\frac{4}{3}\right)\right)$

Substituting values from (i) and (ii)

$\Rightarrow f(1.5) + g(1)$

⇒ [1.5] + |1|

⇒ 1 + 1 = 2

Ans) 2

12. Question

Let f : R → R : f(x) = x^2 + 2 and $g: R \rightarrow R: g(x)=\frac{x}{x-1}, x \neq 1$. find f o g and g o f and hence find (f o g) (2) and (g o f) (-3).

Answer

To find: f o g, g o f ,(f o g) (2) and (g o f) (-3)

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = x^2 + 2

(ii) $g: R \rightarrow R: g(x) = \frac{x}{x-1}, x \neq 1$

f o g = f(g(x))

$\Rightarrow f\left(\frac{x}{x-1}\right)$

$\Rightarrow \left(\frac{x}{x-1}\right)^2 + 2$

Ans) $\Rightarrow \frac{(x)^2}{(x-1)^2} + 2$

$f \circ g\,(2) = \frac{(2)^2}{(2-1)^2} + 2$

$= \frac{4}{1} + 2$

Ans) = 6

g o f = g(f(x))

$\Rightarrow g(x^2+2)$

$\Rightarrow \frac{x^2+2}{x^2+2-1}$

Ans) $\Rightarrow \frac{x^2+2}{x^2+1}$

$(g \circ f)\,(-3) = \frac{-3^2+2}{-3^2+1}$

$= \frac{9+2}{9+1}$

Ans) $= \frac{11}{10}$

Exercise 2C

1. Question

Prove that the function f: R → R : f(x)= 2x is one-one and onto.

Answer

To prove: function is one-one and onto

Given: f: R → R : f(x)= 2x

We have,

f(x) = 2x

For, $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

∴ f(x) is one-one

f(x) = 2x

Let f(x) = y such that $y \in R$

$\Rightarrow y = 2x$

$\Rightarrow x = \frac{y}{2}$

Since $y \in R$,

$\Rightarrow \frac{y}{2} \in R$

⇒ x will also be a real number, which means that every value of y is associated with some x

∴ f(x) is onto

Hence Proved

2. Question

Prove that the function f: N → N : f (x)=3x is one-one and into.

Answer

To prove: function is one-one and into

Given: f: N → N : f(x)= 3x

We have,

$f(x) = 3x$

For, $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

∴ f(x) is one-one

$f(x) = 3x$

Let f(x) = y such that $y \in N$

$\Rightarrow y = 3x$

$\Rightarrow x = \frac{y}{3}$

If y = 1,

$\Rightarrow x = \frac{1}{3}$

But as per question $x \in N$, hence x can not be $\frac{1}{3}$

Hence f(x) is into

Hence Proved

3. Question

Show that the function f : R → R : f (x) = x^2 is neither one-one nor onto.

Answer

To prove: function is neither one-one nor onto

Given: f : R → R : f (x) = x^2

Solution: We have,

$f(x) = x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$

Since x_1 doesn't has unique image

$\therefore$ f(x) is not one-one

$f(x) = x^2$

Let f(x) = y such that $y \in R$

$\Rightarrow y = x^2$

$\Rightarrow x = \sqrt{y}$

If y = -1, as $y \in R$

Then x will be undefined as we cannot place the negative value under the square root

Hence f(x) is not onto

Hence Proved

4. Question

Show that the function $f : N \to N : f(x) = x^2$ is one-one and into.

Answer

To prove: function is one-one and into

Given: $f : N \to N : f(x) = x^2$

Solution: We have,

$f(x) = x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

Here we can't consider $x_1 = -x_2$ as $x \in N$, we can't have negative values

$\therefore$ f(x) is one-one

$f(x) = x^2$

Let f(x) = y such that $y \in N$

$\Rightarrow y = x^2$

$\Rightarrow x = \sqrt{y}$

If y = 2, as $y \in N$

Then we will get the irrational value of x, but $x \in N$

Hence f(x) is not into

Hence Proved

5. Question

Show that the function $f : R \to R : f(x) = x^4$ is neither one-one nor onto.

Answer

To prove: function is neither one-one nor onto

Given: $f : R \to R : f(x) = x^4$

We have,

$f(x) = x^4$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^4 = x_2^4$

$\Rightarrow (x_1^4 - x_2^4) = 0$

$\Rightarrow (x_1^2 - x_2^2)(x_1^2 + x_2^2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2)(x_1^2 + x_2^2) = 0$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$ or, $x_1^2 = -x_2^2$

We are getting more than one value of x_1 (no unique image)

$\therefore$ f(x) is not one-one

$f(x) = x^4$

Let f(x) = y such that $y \in R$

$\Rightarrow y = x^4$

$\Rightarrow x = \sqrt[4]{y}$

If y = -2, as $y \in R$

Then x will be undefined as we can't place the negative value under the square root

Hence f(x) is not onto

Hence Proved

6. Question

Show that the function $f : Z \to Z : f(x) = x^3$ is one-one and into.

Answer

To prove: function is one-one and into

Given: $f : Z \to Z : f(x) = x^3$

Solution: We have,

$f(x) = x^3$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ f(x) is one-one

$f(x) = x^3$

Let f(x) = y such that $y \in Z$

$\Rightarrow y = x^3$

$\Rightarrow x = \sqrt[3]{y}$

If y = 2, as $y \in Z$

Then we will get an irrational value of x, but $x \in Z$

Hence f(x) is into

Hence Proved

7. Question

Let R_0 be the set of all nonzero real numbers. Then, show that the function $f : R_0 \rightarrow R_0 : f(x) = \frac{1}{x}$ is one-one and onto.

Answer

To prove: function is one-one and onto

Given: $f : R_0 \rightarrow R_0 : f(x) = \frac{1}{x}$

We have,

$f(x) = \frac{1}{x}$

For, $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ f(x) is one-one

$f(x) = \frac{1}{x}$

Let f(x) = y such that $y \in R_0$

$\Rightarrow y = \frac{1}{x}$

$\Rightarrow x = \frac{1}{y}$

Since $y \in R_0$,

$\Rightarrow \frac{1}{y} \in R_0$

$\Rightarrow$ x will also $\in R_0$, which means that every value of y is associated with some x

$\therefore$ f(x) is onto

Hence Proved

8. Question

Show that the function f : R → R : $f(x) = 1 + x^2$ is many-one into.

Answer

To prove: function is many-one into

Given: f : R → R : $f(x) = 1 + x^2$

We have,

$f(x) = 1 + x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$

Clearly x_1 has more than one image

∴ f(x) is many-one

$f(x) = 1 + x^2$

Let f(x) = y such that $y \in R$

$\Rightarrow y = 1 + x^2$

$\Rightarrow x^2 = y - 1$

$\Rightarrow x = \sqrt{y-1}$

If y = 3, as $y \in R$

Then x will be undefined as we can't place the negative value under the square root

Hence f(x) is into

Hence Proved

9. Question

Let $f : R \to R : f(x) = \frac{2x-7}{4}$ be an invertible function. Find f^{-1}.

Answer

To find: f^{-1}

Given: $f: R \to R : f(x) = \frac{2x-7}{4}$

We have,

$f(x) = \frac{2x-7}{4}$

Let f(x) = y such that $y \in R$

$\Rightarrow y = \frac{2x-7}{4}$

$\Rightarrow 4y = 2x - 7$

$\Rightarrow 4y + 7 = 2x$

$\Rightarrow x = \frac{4y+7}{2}$

$\Rightarrow f^{-1} = \frac{4y+7}{2}$

Ans) $f^{-1}(y) = \frac{4y+7}{2}$ for all $y \in R$

10. Question

Let f : R → R : f(x) = 10x + 3. Find f^{-1}.

Answer

To find: f^{-1}

Given: f : R → R : f(x) = 10x + 3

We have,

f(x) = 10x + 3

Let f(x) = y such that $y \in R$

$\Rightarrow y = 10x + 3$

$\Rightarrow y - 3 = 10x$

$\Rightarrow x = \frac{y-3}{10}$

$\Rightarrow f^{-1} = \frac{y-3}{10}$

Ans) $f^{-1}(y) = \frac{y-3}{10}$ for all $y \in R$

11. Question

$$f : R \to R : f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is rational} \end{cases}$$

Show that f is many-one and into.

Answer

To prove: function is many-one and into

Given: $f : R \to R : f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is irrational} \end{cases}$

We have,

f(x) = 1 when x is rational

It means that all rational numbers will have same image i.e. 1

⇒ f(2) = 1 = f (3) , As 2 and 3 are rational numbers

Therefore f(x) is many-one

The range of function is [{-1},{1}] but codomain is set of real numbers.

Therefore f(x) is into

12. Question

Let f (x) = x + 7 and g(x) = x -7, x∈ R. Find (f o g) (7).

Answer

To find: (f o g) (7)

Formula used: f o g = f(g(x))

Given: (i) f (x) = x + 7

(ii) g (x) = x – 7

We have,

f o g = f(g(x)) = f(x – 7) = [(x – 7) + 7]

⇒ x

(f o g) (x) = x

(f o g) (7) = 7

Ans). (f o g) (7) = 7

13. Question

Let f : R → R and g : R → R defined by f(x) = x^2 and g(x) = (x + 1). Show that g o f ≠ f o g.

Answer

To prove: g o f ≠ f o g

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = x^2

(ii) g : R → R :g(x) = (x + 1)

We have,

f o g = f(g(x)) = f(x + 7)

f o g = $(x + 7)^2 = x^2 + 14x + 49$

g o f = g(f(x)) = g(x^2)

g o f = $(x^2 + 1) = x^2 + 1$

Clearly g o f ≠ f o g

Hence Proved

14. Question

Let f : R → R : f(x) = $(3 - x^3)^{1/3}$. Find f o f.

Answer

To find: f o f

Formula used: (i) f o f = f(f(x))

Given: (i) f : R → R : f(x) = $(3 - x^3)^{1/3}$

We have,

f o f = f(f(x)) = $f((3 - x^3)^{1/3})$

f o f = $[3 - \{(3 - x^3)^{1/3}\}^3]^{1/3}$

$= [3 - (3 - x^3)]^{1/3}$

$= [3 - 3 + x^3]^{1/3}$

$= [x^3]^{1/3}$

= x

Ans) f o f (x) = x

15. Question

Let f : R → R : f(x) = 3x + 2, find f{f(x)}.

Answer

To find: f{f(x)}

Formula used: (i) f o f = f(f(x))

Given: (i) f : R → R : f(x) = 3x + 2

We have,

f{f(x)} = f(f(x)) = f(3x + 2)

f o f =3(3x + 2) + 2

= 9x + 6 + 2

= 9x + 8

Ans) f{f(x)} = 9x + 8

16. Question

Let f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down g o f.

Answer

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f = {(1, 2), (3, 5), (4, 1)}

(ii) g = {(1, 3), (2, 3), (5, 1)}

We have,

gof(1) = g(f(1)) = g(2) = 3

gof(3) = g(f(3)) = g(5) = 1

gof(4) = g(f(4)) = g(1) = 3

Ans) g o f = {(1, 3), (3, 1), (4, 3)}

17. Question

Let A = {1, 2, 3, 4} and f = {(1, 4), (2, 1) (3, 3), (4, 2)}. Write down (f o f).

Answer

To find: f o f

Formula used: f o f = f(f(x))

Given: (i) f = {(1, 4), (2, 1) (3, 3), (4, 2)}

We have,

fof(1) = f(f(1)) = f(4) = 2

fof(2) = f(f(2)) = f(1) = 4

fof(3) = f(f(3)) = f(3) = 3

fof(4) = f(f(4)) = f(2) = 1

Ans) f o f = {(1, 2), (2, 4), (3, 3), (4, 1)}

18. Question

Let f(x) = $8x^3$ and g(x) = $x^{1/3}$. Find g o f and f o g.

Answer

To find: g o f and f o g

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f(x) = $8x^3$

(ii) g(x) = $x^{1/3}$

We have,

g o f = g(f(x)) = g($8x^3$)

g o f = $(8x^3)^{\frac{1}{3}}$ = 2x

f o g = f(g(x)) = f($x^{1/3}$)

f o g = $8\left(x^{\frac{1}{3}}\right)^3$ = 8x

Ans) g o f = 2x and f o g = 8x

19. Question

Let f : R → R : f(x) = 10x + 7. Find the function g : R→ R : g o f = f o g = I_g.

Answer

To find: the function g : R → R : g o f = f o g = I_g

Formula used: (i) g o f = g(f(x))

(ii) f o g = f(g(x))

Given: f : R → R : f(x) = 10x + 7

We have,

f(x) = 10x + 7

Let f(x) = y

⇒ y = 10x + 7

⇒ y – 7 = 10x

⇒ $x = \frac{y-7}{10}$

Let $g(y) = \frac{y-7}{10}$ where g: R → R

g o f = g(f(x)) = g(10x + 7) $= \frac{(10x+7)-7}{10}$

= x

$= I_g$

f o g = f(g(x)) = f $\left(\frac{x-7}{10}\right)$

$= 10\left(\frac{x-7}{10}\right)+7$

= x – 7 + 7

= x

Clearly g o f = f o g = I_gAns). $g(x) = \frac{x-7}{10}$

20. Question

Let A = {1, 2, 3), B = {4, 5, 6, 7) and let f = {(1, 4), (2,5), (3, 6)} be a function from A to B. State whether f is one-one.

Answer

To state: Whether f is one-one

Given: f = {(1, 4), (2,5), (3, 6)}

Here the function is defined from A → B

For a function to be one-one if the images of distinct elements of A under f are distinct

i.e. 1,2 and 3 must have a distinct image.

From f = {(1, 4), (2, 5), (3, 6)} we can see that 1, 2 and 3 have distinct image.

Therefore f is one-one

Ans) f is one-one

Exercise 2D

1. Question

Let A = {2, 3, 4, 5} and B = {7, 9, 11, 13}, and

let f = {(2, 7), (3, 9), (4, 11), (5, 13)}.

Show that f is invertible and find f^{-1}.

Answer

To Show: that f is invertible

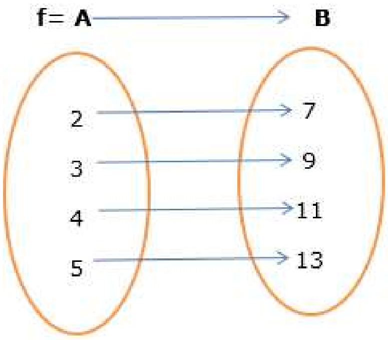
To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_l, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, f(x1) = f(x2) ↔ $x_1 = x_2$ or $x_1 \neq x_2$ ↔ f(x1) $\neq$ f(x2)

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.



As we see that inthe above figure (2 is mapped with 7), (3 is mapped with 9), (4 is mapped with 11),

(5 is mapped with 13)

So it is one-one functions.

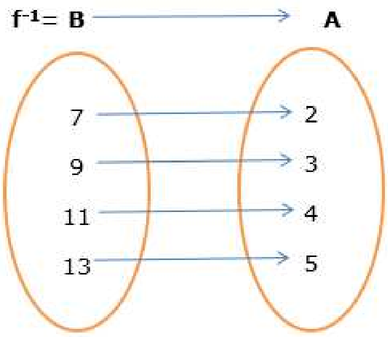
Now elements of B are known as co-domain. Also, a range of a function is also the elements of B(by definition)

So it is onto functions.

Hence Proved that f is invertible.

Now, We know that if f : A → B then f^{-1} : B → A (if it is invertible)

So,



So f^{-1} = {(7, 2), (9, 3), (11, 4), (13, 5)}

2. Question

Show that the function f : R → R : f(x) = 2x + 3 is invertible and find f^{-1}.

Answer

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in R$ and f(x) = 2x+3.So $f(x_1) = f(x_2) \rightarrow 2x_1+3 = 2x_2+3 \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R.

Let y = f(x) = 2x+3 , So x = $\frac{y-3}{2}$ [Range of f(x) = Domain of y]

So Domain of y is R(real no.) = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{y-3}{2}$

3. Question

Let f : Q → Q : f(x) = 3x —4. Show that f is invertible and find f^{-1}.

Answer

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and f(x) = 3x-4.So $f(x_1) = f(x_2) \rightarrow 3x_1 - 4 = 3x_2 - 4 \rightarrow x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is Q.

Let y = f(x) = 3x- 4 , So x = $\frac{y+4}{3}$ [Range of f(x) = Domain of y]

So Domain of y is Q = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = Q

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{y+4}{3}$

4. Question

Let $f : R \rightarrow R : f(x) = \frac{1}{2}(3x+1)$.. Show that f is invertible and find f^{-1}.

Answer

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and $f(x) = \frac{(3x+1)}{2}$.So $f(x_1) = f(x_2) \rightarrow \frac{(3x_1+1)}{2} = \frac{(3x_2+1)}{2} \rightarrow x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R.

Let $y = f(x) = \frac{(3x+1)}{2}$, So $x = \frac{2y-1}{3}$ [Range of f(x) = Domain of y]

So Domain of y is R = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{2y-1}{3}$

5. Question

If $f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$, show that (f o f) (x) = x for all $x \neq \frac{2}{3}$.

Hence, find f^{-1}.

Answer

To Show: that f o f (x) = x

Finding (f o f) (x) = $\frac{(4\frac{(4x+3)}{(6x-4)}+3)}{(6\frac{(4x+3)}{(6x-4)}-4)} = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{35x}{35} = x.$

6. Question

Show that the function f on $A = R - \left\{\frac{2}{3}\right\}$, defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence, find f^{-1}.

Answer

To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in Q$ and f(x) $=\frac{(4x+3)}{(6x-4)}$ .So $f(x_1) = f(x_2) \to \frac{(4x_1+3)}{(6x_1-4)} = \frac{(4x_2+3)}{(6x_2-4)} \to$ on solving we get $x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$, f(x) is one-one

Given co-domain of f(x) is R except 3x-2=0.

Let y = f(x) $= \frac{(4x+3)}{(6x-4)}$So x $= \frac{4y+3}{6y-4}$ [Range of f(x) = Domain of y]

So Domain of y is R (except 3x-2=0) = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R except 3x-2=0

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{4y+3}{6y-4}$.

7. Question

Show that the function f on $A = R - \left\{\frac{-4}{3}\right\}$ into itself, defined by $f(x) = \frac{4x}{(3x+4)}$ is one-one and onto. Hence, find f^{-1}.

Answer

To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in Q$ and f(x) $=\frac{4x}{(3x+4)}$ .So $f(x_1) = f(x_2) \to \frac{(4x_1)}{(3x_1+4)} = \frac{(4x_2)}{(3x_2+4)} \to$ on solving we get $x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1= x_2$, f(x) is one-one

Given co-domain of f(x) is R except 3x+4=0.

Let y = f(x) $= \frac{(4x)}{(3x+4)}$ So x $= \frac{4y}{4-3y}$ [Range of f(x) = Domain of y]

So Domain of y is R = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R except 3x+4=0

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{4y}{4-3y}$.

8. Question

Let R_+ be the set of all positive real numbers. show that the function $f : R_+ \rightarrow [-5, \infty]$: $f(x) = (9x^2 + 6x - 5)$ is invertible. Find f^{-1}.

Answer

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function $f : A \rightarrow B$ is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in R$ and $f(x) = (9x^2 + 6x - 5)$.So $f(x_1) = f(x_2) \rightarrow (9x_1^2 + 6x_1 - 5) = (9x_2^2 + 6x_2 - 5)$ on solving we get $\rightarrow x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is $[-5, \infty]$

Let $y = f(x) = (9x^2 + 6x - 5)$, So $x = \frac{-1+\sqrt{y+6}}{3}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = $[-5, \infty]$

Hence, Range of f(x) = co-domain of f(x) = $[-5, \infty]$

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{-1+\sqrt{y+6}}{3}$.

9. Question

Let $f : N \rightarrow R : f(x) = 4x^2 + 12x + 15$. Show that $f: N \rightarrow$ range (f) is invertible. Find f^{-1}.

Answer

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function $f : A \rightarrow B$ is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in R$ and $f(x) = 4x^2 + 12x + 15$ So $f(x_1) = f(x_2) \rightarrow (4x_1^2 + 12x_1 + 15) = (4x_2^2 + 12x_2 + 15)$, on solving we get $\rightarrow x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is Range(f).

Let y = f(x) = $4x^2 + 12x + 15$, So x = $\frac{-3+\sqrt{y-6}}{2}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = [6, ∞]

Hence, Range of f(x) = co-domain of f(x) = [6, ∞]

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{-3+\sqrt{y-6}}{2}$.

10. Question

Let A = R – {2} and B = R – {1}. If $f : A \rightarrow B : f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

Answer

To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in Q$ and f(x) = $\frac{x-1}{x-2}$.So $f(x_1) = f(x_2) \rightarrow \frac{x_1-1}{x_1-2} = \frac{(x_2-1)}{x_2-2}$, on solving we get → $x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R – {1}

Let y = f(x) = $\frac{x-1}{x-2}$, So x = $\frac{2y-1}{y-1}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = R – {1}

Hence, Range of f(x) = co-domain of f(x) = R – {1}.

So, f(x) is onto function

As it is a bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{2y-1}{y-1}$

11. Question

Let f and g be two functions from R into R, defined by f(x) = |x| + x and g(x) = |x| - x for all x∈ R. Find f o g and g o f.

Answer

To Find: Inverse of f o g and g o f.

Given: f(x) = |x| + x and g(x) = |x| - x for all $x \in R$

f o g (x) = f(g(x)) = |g(x)| + g(x) = ||x| - x | + |x| - x

Case 1) when $x \geq 0$

f(g(x)) = 0 (i.e. |x| - x)

Case 2) when $x < 0$

f(g(x)) = -4x

g o f (x) = g(f(x)) = |f(x)| - f(x) = ||x| + x | - |x| - x

Case 1) when $x \geq 0$

g(f(x)) = 0 (i.e. |x| - x)

Case 2) when $x < 0$

g(f(x)) = 0

Objective Questions

1. Question

Mark (√) against the correct answer in the following:

$f : N \to N : f(x) = 2x$ is

A. one - one and onto

B. one - one and into

C. many - one and onto

D. many - one and into

Answer

f(x) = 2x

For One - One

$f(x_1) = 2x_1$

$f(x_2) = 2x_2$

put $f(x_1) = f(x_2)$ we get

$2x_1 = 2x_2$

Hence, if $f(x_1) = f(x_2)$, $x_1 = x_2$

Function f is one - one

For Onto

f(x) = 2x

let f(x) = y, such that $y \in N$

2x = y

$\Rightarrow x = \frac{y}{2}$

If y = 1

$x = \frac{1}{2} = 0.5$

which is not possible as x∈N

Hence, f is not onto., f is into

Hence, option b is correct

2. Question

Mark (√) against the correct answer in the following:

f : N → N : f(x) = x^2 + x + 1 is

A. one - one and onto

B. one - one and into

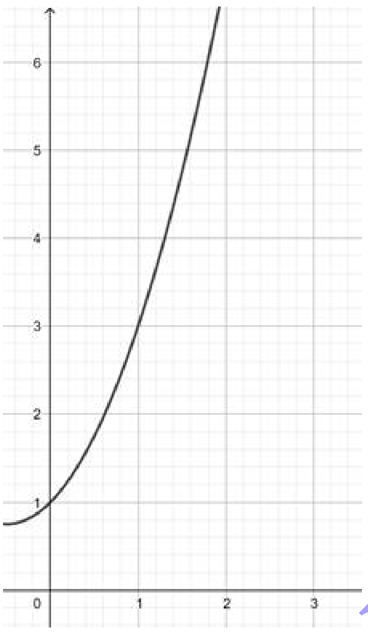
C. many - one and onto

D. many - one and into

Answer

In the given range of N f(x) is monotonically increasing.

∴f(x) = x^2 + x + 1 is one one.



But Range of f(n) = [0.75,∞)≠N(codomain)

Hence, f(x) is not onto.

Hence, the function f : N → N : f(x) = (x^2 + x + 1) is one - one but not onto. i.e. into

3. Question

Mark (√) against the correct answer in the following:

f : R → R : f(x) = x^2 is

A. one - one and onto

B. one - one and into

C. many - one and onto

D. many - one and into

Answer

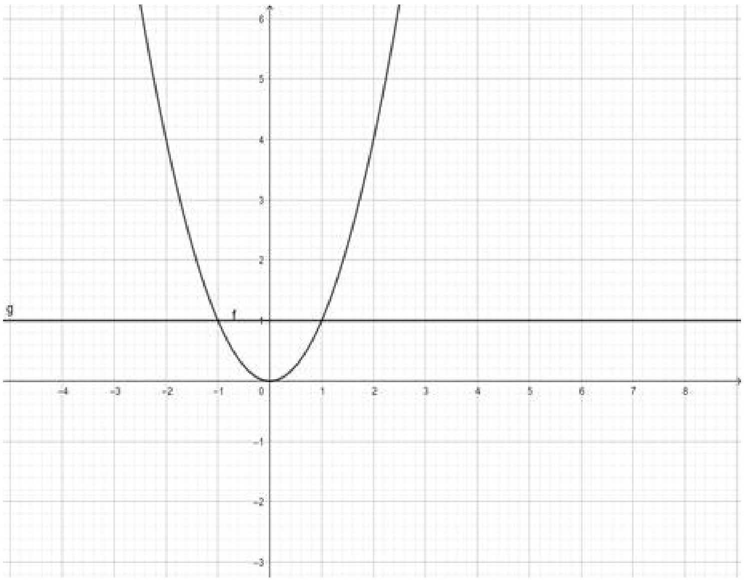
f(x) = x^2

$\Rightarrow y = x^2$

in this range the lines cut the curve in 2 equal valued points of y, therefore, the function $f(x) = x^2$ is many - one .

Range of $f(x) = (0,\infty) \neq R$(codomain)

$\therefore f(x)$ is into



$\therefore$ f : R → R: $f(x) = x^2$ is many - one into

4. Question

Mark (√) against the correct answer in the following:

f : R → R : $f(x) = x^3$ is

A. one - one and onto

B. one - one and into

C. many - one and onto

D. many - one and into

Answer

$f(x) = x^3$

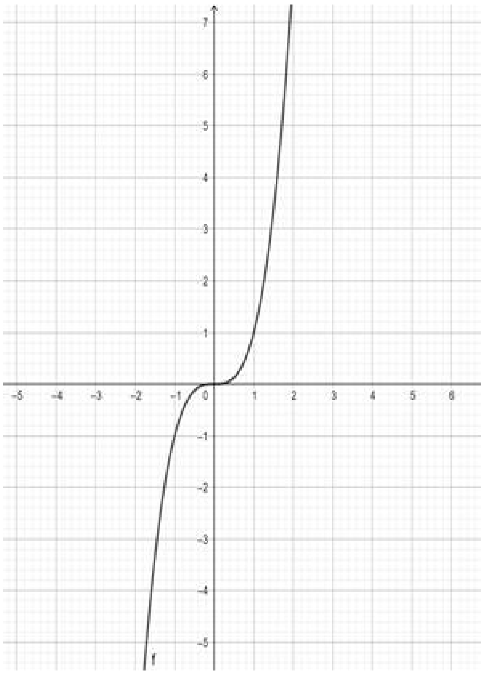
Since the function f(x) is monotonically increasing from the domain R → R

$\therefore f(x)$ is one –one

Range of $f(x) = (- \infty,\infty) \neq R$(codomain)

$\therefore f(x)$ is into

$\therefore$ f : R → R: $f(x) = x^3$ is one - one into.



5. Question

Mark (√) against the correct answer in the following:

f : $R^+ \to R^+$: f(x) = e^x is

A. many - one and into

B. many - one and onto

C. one - one and into

D. one - one and onto

Answer

f(x) = e^x

Since the function f(x) is monotonically increasing from the domain $R^+ \to R^+$

∴f(x) is one –one

Range of f(x) = (1,∞) = R^+ (codomain)

∴f(x) is onto

∴ f : $R^+ \to R^+$: f(x) = e^x is one - one onto.

6. Question

Mark (√) against the correct answer in the following:

$f:\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\to[-1,1]:f(x)=\sin x$ is

A. one - one and into

B. one - one and onto

C. many - one and into

D. many - one and onto

Answer

$$f:\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\to[-1,1]:f(x)=\sin x$$

Here in this range, the function is NOT repeating its value,

Therefore it is one - one.

Range = Codomain

∴Function is onto

Hence, option B is the correct choice.

7. Question

Mark (√) against the correct answer in the following:

f : R → R : f(x) = cos x is

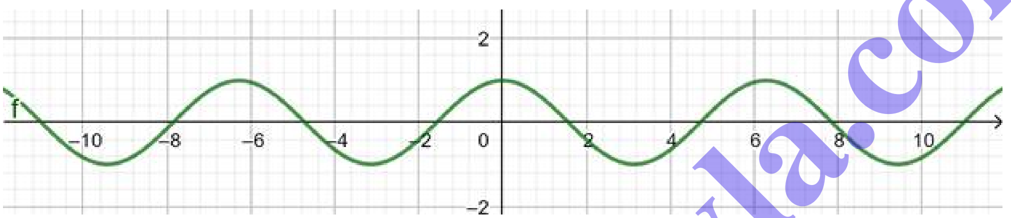
A. one - one and into

B. one - one and onto

C. many - one and into

D. many - one and onto

Answer



f(x) = cosx

y = cosx

Here in this range the lines cut the curve in many equal valued points of y therefore the function f(x) = cosx is not one - one.

⇒f(x) = many one

Range of f(x) = [- 1,1]≠R(codomain)

∴f(x) is not onto.

⇒f(x) = into

Hence,f(x) = cosx is many one and into

Ans: (c) many - one and into

8. Question

Mark (√) against the correct answer in the following:

f : C → R : f(z) = |z| is

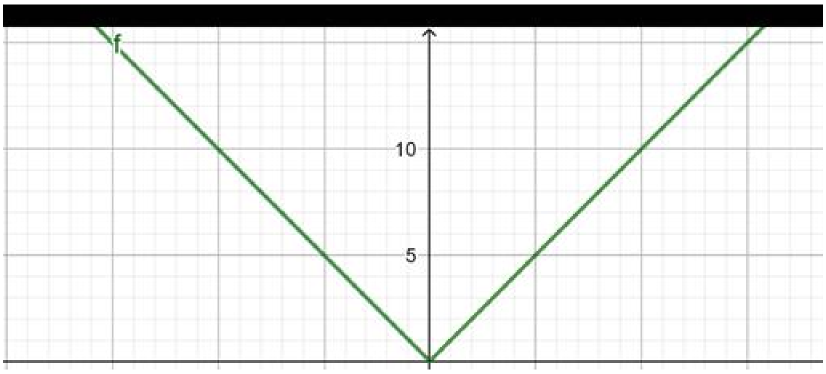
A. one - one and into

B. one - one and onto

C. many - one and into

D. many - one and onto

Answer



Here in this range the lines cut the curve in 2 equal valued points of y therefore the function f(z) = |z| is not one - one

⇒f(z) = many one.

Range of f(z) = [0,∞)≠R(codomain)

∴f(z) is not onto.

⇒f(z) = into

Hence, f(z) = |z| is many one and into

9. Question

Mark (√) against the correct answer in the following:

Let A = R – {3} and B = R – {1}. Then $f : A \to A : f(x) = \frac{(x-2)}{(x-3)}$ is

A. one - one and into

B. one - one and onto

C. many - one and into

D. many - one and onto

Answer

$$f : A \to A : f(x) = \frac{(x-2)}{(x-3)}$$

In this function

x = 3 and y = 1 are the asymptotes of this curve and these are not included in the functions of the domain and range respectively therefore the function f(x) is one one sice there are no different values of x which has same value of y .

and the function has no value at y = 1 here range = codomain

∴ f(x) is onto

101. Question

Mark (√) against the correct answer in the following:

Let $f : N \to N : f(x) = \begin{cases} \frac{1}{2}(n+1), \text{when n is odd} \\ \frac{n}{2}, \text{when n is even.} \end{cases}$

Then, f is

A. one - one and into

B. one - one and onto

C. many - one and into

D. many - one and onto

Answer

f(1) = 1

f(2) = 1

f(3) = 2

f(4) = 2

f(5) = 3

f(6) = 3

Since at different values of x we get same value of y ∴f(n) is many –one

And range of f(n) = N = N(codomain)

∴ the function f: N → Z, defined by

$$f:N \to N:f(x)=\begin{cases}\frac{1}{2}(n+1), \text{when n is odd}\\ \frac{n}{2}, \text{when n is even.}\end{cases}$$ is both many - one and onto.

11. Question

Mark (√) against the correct answer in the following:

Let A and B be two non - empty sets and let

f : (A × B) → (B × A) : f(a, b) = (b, a). Then, f is

A. one - one and into

B. one - one and onto

C. many - one and into

D. many - one and onto

Answer

SINCE, f(a, b) = (b, a).There is no same value of y at different values of x∴function is one one

∴Range(A×B)≠Codomain(B × A)

⇒function is into

12. Question

Mark (√) against the correct answer in the following:

Let f : Q → Q : f(x) = (2x + 3). Then, $f^{-1}(y)$ = ?

A. (2y - 3)

B. $\frac{1}{(2y-3)}$

C. $\frac{1}{2}(y-3)$

D. none of these

Answer

f(x) = 2x + 3

⇒y = 2x + 3

x⟺y

⇒x = 2y + 3

⇒x - 3 = 2y

⇒$\frac{x-3}{2} = y$

x⟺y

⇒$\frac{y-3}{2} = x$

13. Question

Mark (√) against the correct answer in the following:

Let $f: R - \left\{\frac{-4}{3}\right\} \to - \left\{\frac{4}{3}\right\} : f(x) = \frac{4x}{(3x+4)}$. Then $f^{-1}(y)$ = ?

A. $\frac{4y}{(4-3y)}$

B. $\frac{4y}{(4y+3)}$

C. $\frac{4y}{(3y-4)}$

D. None of these

Answer

f(x) = $\frac{4x}{3x+4}$

⇒y = $\frac{4x}{3x+4}$

x⟺y

⇒x = $\frac{4y}{3y+4}$

⇒3yx + 4x = 4y

⇒y(3x - 4) = - 4x

⇒y = $\frac{4x}{4-3x}$

x⟺y

⇒x = $\frac{4y}{4-3y}$

145. Question

Mark (√) against the correct answer in the following:

Let f : N → X : f(x) = $4x^2$ + 12x + 15. Then, $f^{-1}(y)$ = ?

A. $\frac{1}{2}\left(\sqrt{y-4}+3\right)$

B. $\frac{1}{2}\left(\sqrt{y-6}-3\right)$

C. $\frac{1}{2}\left(\sqrt{y-4}+5\right)$

D. None of these

Answer

$f(x) = 4x^2 + 12x + 15$

$\Rightarrow y = 4x^2 + 12x + 15$

$\Rightarrow y = (2x + 3)^2 + 6$

$\Rightarrow \sqrt{(y - 6)} = 2x + 3$

$\Rightarrow \frac{1}{2}\left(\sqrt{y-6}-3\right) = x$

$f^{-1}(y) = \frac{1}{2}\left(\sqrt{y-6}-3\right)$

15. Question

Mark (√) against the correct answer in the following:

If $f(x)=\frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$ then (f o f) (x) = ?

A. x

B. (2x - 3)

C. $\frac{4x-6}{3x+4}$

D. None of these

Answer

$f(x) = \frac{4x + 3}{6x - 4}$

$\Rightarrow f(f(x)) = \frac{4f(x) + 3}{6f(x) - 4} = (f \circ f)(x)$

$\Rightarrow f(f(x)) = \frac{4(\frac{4x + 3}{6x - 4}) + 3}{6(\frac{4x + 3}{6x - 4}) - 4}$

$\Rightarrow f(f(x)) = \frac{16x + 12 + 18x - 12}{24x + 18 - 24x + 16} = \frac{34x}{34} = x$

16. Question

Mark (√) against the correct answer in the following:

If $f(x) = (x^2 - 1)$ and $g(x) = (2x + 3)$ then (g o f) (x) = ?

A. $(2x^2 + 3)$

B. $(3x^2 + 2)$

C. $(2x^2 + 1)$

D. None of these

Answer

$f(x) = (x^2 - 1)$

$g(x) = (2x + 3)$

$\therefore (g \circ f)(x) = g(f(x))$

$$\Rightarrow g\big(f(x)\big) = 2f(x) + 3$$

$$\Rightarrow g\big(f(x)\big) = 2\big((x^2 - 1)\big) + 3 = 2x^2 - 2 + 3 = 2x^2 + 1$$

17. Question

Mark (√) against the correct answer in the following:

If $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ then f(x) = ?

A. x^2

B. $(x^2 - 1)$

C. $(x^2 - 2)$

D. None of these

Answer

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\Rightarrow f(x) = x^2 - 2$$

18. Question

Mark (√) against the correct answer in the following:

If $f(x) = \frac{1}{(1-x)}$ then (f o f o f) (x) = ?

A. $\frac{1}{(1-3x)}$

B. $\frac{x}{(1+3x)}$

C. x

D. None of these

Answer

$$f(x) = \frac{1}{1-x}$$

$$\Rightarrow (f \circ f \circ f)(x) = f\Big(f\big(f(x)\big)\Big)$$

$$\Rightarrow f\big(f(x)\big) = \frac{1}{1 - f(x)} = \frac{1}{1 - \frac{1}{1-x}} = \frac{1-x}{1-x-1} = \frac{x-1}{x} = 1 - \frac{1}{x}$$

$$\Rightarrow f\left(f(f(x))\right) = \frac{1}{1-f(f(x))} = \frac{1}{1-\left(1-\frac{1}{x}\right)} = \frac{1}{\frac{1}{x}} = x$$

19. Question

Mark (√) against the correct answer in the following:

If $f(x) = \sqrt[3]{3-x^3}$ then (f o f) (x) = ?

A. $x^{1/3}$

B. x

C. $\left(1-x^{1/3}\right)$

D. None of these

Answer

$f(x) = \sqrt[3]{3-x^3}$

$\Rightarrow f(f(x)) = \sqrt[3]{3-f(x)^3} = \sqrt[3]{3-\left(\sqrt[3]{3-x^3}\right)^3}$

$\Rightarrow f(f(x)) = \sqrt[3]{3-(3-x^3)}$

$\Rightarrow f(f(x)) = \sqrt[3]{x^3} = x$

20. Question

Mark (√) against the correct answer in the following:

If $f(x) = x^2 - 3x + 2$ then (f o f) (x) = ?

A. x^4

B. $x^4 - 6x^3$

C. $x^4 - 6x^3 + 10x^2$

D. None of these

Answer

$f(x) = x^2 - 3x + 2$

$\Rightarrow f(x) = x^2 - 2x - x + 2 = x(x - 2) - 1(x - 2)$

$\Rightarrow f(x) = (x - 2)(x - 1)$

$\Rightarrow f(x) = (x - 2)(x - 1)$

$\Rightarrow f(f(x)) = (f(x) - 2)(f(x) - 1)$

$\Rightarrow f(f(x)) = ((x - 2)(x - 1) - 2) ((x - 2)(x - 1) - 1)$

$\Rightarrow f(f(x)) = (x^2 - 3x + 2 - 2) (x^2 - 3x + 2 - 1)$

$\Rightarrow f(f(x)) = (x^2 - 3x) (x^2 - 3x + 1)$

$\Rightarrow f(f(x)) = x^4 - 3x^3 + x^2 - 3x^3 + 9x^2 - 3x$

$\Rightarrow f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$

21. Question

Mark (√) against the correct answer in the following:

If f(x) = $8x^3$ and g(x) = $x^{1/3}$ then (g o f) (x) = ?

A. x

B. 2x

C. $\frac{x}{2}$

D. $3x^2$

Answer

f(x) = $8x^3$

g(x) = $x^{1/3}$

$\Rightarrow (g\,o\,f)(x) = (f(x))^{\frac{1}{3}} = (8x^3)^{\frac{1}{3}} = 2x$

22. Question

Mark (√) against the correct answer in the following:

If f(x) = x^2, g(x) = tan x and h(x) = log x then $\{h\ o(g\ o\ f)\}\left(\sqrt{\frac{\pi}{4}}\right) = ?$

A. 0

B. 1

C. $\frac{1}{x}$

D. $\frac{1}{2}\log\frac{\pi}{4}$

Answer

f(x) = x^2, g(x) = tan x and h(x) = log x

$\Rightarrow g(f(x)) = \tan(f(x)) = \tan(x^2)$

$\Rightarrow h(g(f(x))) = \log(g(f(x))) = \log(\tan(x^2))$

$\Rightarrow h\left(g\left(f\left(\sqrt{\frac{\pi}{4}}\right)\right)\right) = \log\left(\tan\left(\sqrt{\frac{\pi}{4}}^{2}\right)\right) = \log\left(\tan\left(\frac{\pi}{4}\right)\right) = \log(1) = 0$

23. Question

Mark (√) against the correct answer in the following:

If f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)} then (g o f) = ?

A. {(3, 1), (1, 3), (3, 4)}

B. {(1, 3), (3, 1), (4, 3)}

C. {(3, 4), (4, 3), (1, 3)}

D. {(2, 5), (5, 2), (1, 5)}

Answer

g = {(2, 3), (5, 1), (1, 3)}

(g o f) = {(dom(f), 3), (dom(f), 1), (dom(f), 3)}

⇒(g o f) = {(1, 3), (3, 1), (4, 3)}

24. Question

Mark (√) against the correct answer in the following:

Let $f(x) = \sqrt{9 - x^2}$. Then, dom (f) = ?

A. [- 3, 3]

B. [- ∞, - 3]

C. [3, ∞)

D. (- ∞, - 3] ∪ (4, ∞)

Answer

F(x) = $\sqrt{9 - x^2}$

$\sqrt{9 - x^2}$ should be ≥ 0

$\Rightarrow 9 - x^2 \geq 0$

$\Rightarrow x^2 \leq 9$

$\Rightarrow -3 \leq x \leq 3$

∴dom(f) = [- 3, 3]

25. Question

Mark (√) against the correct answer in the following:

Let $f(x)\sqrt{\frac{x-1}{x+4}}$. Then, dom (f) - ?

A. [1, 4)

B. [1, 4]

C. (- ∞, 4]

D. (- ∞, 1] ∪ (4, ∞)

Answer

$f(x) = \sqrt{\frac{x-1}{x-4}}$

$\sqrt{\frac{x-1}{x-4}} \geq 0$

$\Rightarrow x - 1 \geq 0$

$\Rightarrow x \geq 1$

And x≠4

$x > 4$ and $x \leq 1$

$\Rightarrow$ dom (f) $= (-\infty, 1] \cup (4, \infty)$

26. Question

Mark (√) against the correct answer in the following:

Let $f(x) = e^{\sqrt{x^2-1}}.\log(x-1).$ Then, dom (f) = ?

A. (- ∞, 1]

B. [- 1, ∞)

C. (1, ∞)

D. (- ∞, - 1] ∪ (1, ∞)

Answer

$f(x) = e^{\sqrt{x^2-1}}\log(x-1)$

$x - 1 > 0$

$\Rightarrow x > 1$

And

$\Rightarrow x^2 - 1 \geq 0$

$\Rightarrow x^2 \geq 1$

$\Rightarrow -1 \leq x \geq 1$

Taking the intersection we get

Dom(f) = (1, ∞)

27. Question

Mark (√) against the correct answer in the following:

Let $f(x) = \frac{x}{(x^2-1)}$. Then, dom (f) = ?

A. R

B. R – {1}

C. R – { - 1}

D. R – { - 1, 1}

Answer

$f(x) = \frac{x}{x^2 - 1}$

X^2 - 1≠0

x≠(1, - 1)

∴ Dom(f) = R - { - 1,1}

28. Question

Mark (√) against the correct answer in the following:

Let $f(x) = \frac{\sin^{-1} x}{x}$. Then, dom (f) = ?

A. (-1, 1)

B. [-1, 1]

C. [-1, 1] – {0}

D. none of these

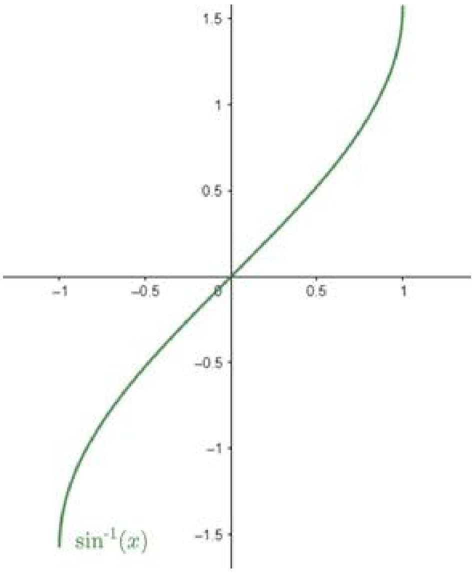
Answer

Given: $f(x) = \frac{\sin^{-1} x}{x}$

From f(x), $x \neq 0$

Now, domain of $\sin^{-1}x$ is [-1, 1] as the values of $\sin^{-1}x$ lies between -1 and 1.

We can see that from this graph:



Domain of f(x) = [-1, 1] – 0

Hence, B is the correct answer.

29. Question

Mark (√) against the correct answer in the following:

Let f(x) = $\cos^{-1} 2x$. Then, dom (f) = ?

A. [- 1, 1]

B. $\left[\frac{-1}{2}, \frac{1}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

Answer

f(x) = $\cos^{-1} 2x$.

domain of $\cos^{-1}x$ = [- 1,1]

on multiplying by an integer the domain decreases by same number

$\Rightarrow$domain of $\cos^{-1}2x$ = [- 1/2,1/2]

30. Question

Mark (√) against the correct answer in the following:

Let f(x) = $\cos^{-1}(3x - 1)$. Then, dom (f) = ?

A. $\left(0, \frac{2}{3}\right)$

B. $\left[0, \frac{2}{3}\right]$

C. $\left[\frac{-2}{3}, \frac{2}{3}\right]$

D. None of these

Answer

f(x) = $\cos^{-1}(3x - 1)$.

domain of $\cos^{-1}x$ = [- 1,1]

on multiplying by an integer the domain decreases by same number

$\Rightarrow$domain of $\cos^{-1}3x$ = [- 1/3,1/3]

$\Rightarrow$domain of $\cos^{-1}(3x - 1)$ = [1/3 - 1/3,1/3 + 1/3] = [0,2/3]

31. Question

Mark (√) against the correct answer in the following:

Let f(x) = √cos x. Then, dom (f) = ?

A. $\left[0, \frac{\pi}{2}\right]$

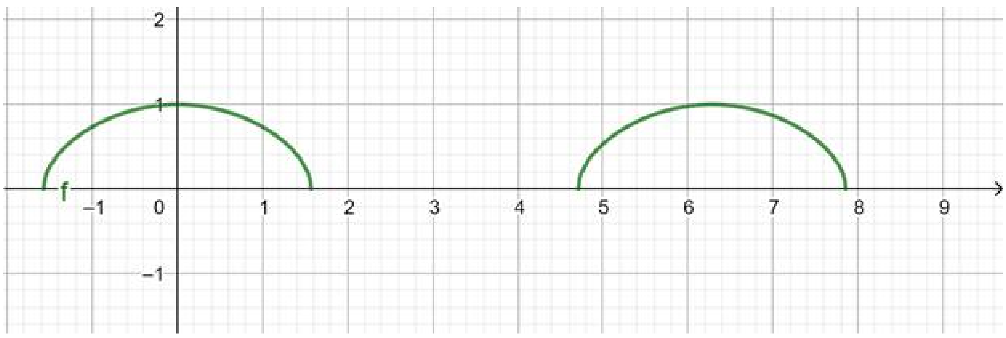
B. $\left[\frac{3\pi}{2}, 2\pi\right]$

C. $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$

D. none of these

Answer

$f(x) = \sqrt{\cos x}$



As per the diagram

We can imply that domain of √cosx

is $\left[0,\frac{\pi}{2}\right]\left[\frac{3\pi}{2},2\pi\right]$

32. Question

Mark (√) against the correct answer in the following:

Let f(x) = √log (2x – x^2). Then, dom (f) = ?

A. (0, 2)

B. [1, 2]

C. (- ∞, 1]

D. None of these

Answer

f(x) = √log (2x – x^2).

$2x - x^2 > 1$

$\Rightarrow x^2 - 2x + 1 < 0$

$\Rightarrow (x-1)^2 < 0$

$\Rightarrow x - 1 < 0$

$\Rightarrow x < 1$

$\log(2x - x^2) > 0$

$\Rightarrow 2x - x^2 > e^0 = 1$

$\Rightarrow x < 1$

Dom(f) = (- ∞, 1)

33. Question

Mark (√) against the correct answer in the following:

Let f(x) = x^2. Then, dom (f) and range (f) are respectively.

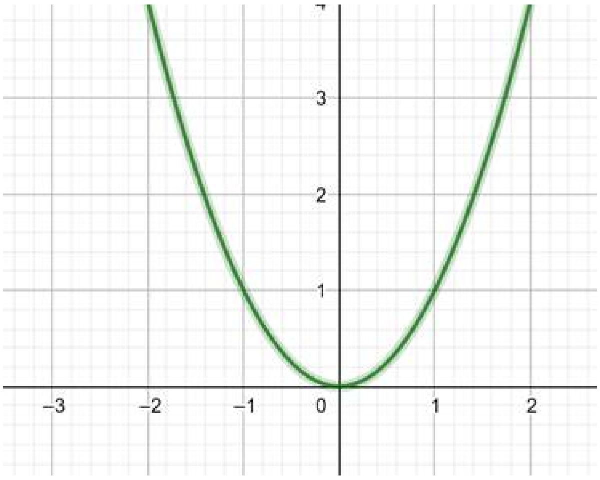
A. R and R

B. R^+ and R^+

C. R and R^+

D. R and R – {0}

Answer



According to sketched graph of x^2

Domain of f(x) = R

And Range of f(x) = R^+

34. Question

Mark (√) against the correct answer in the following:

Let $f(x) = x^3$. Then, dom (f) and range (f) are respectively

A. R and R

B. R^+ and R^+

C. R and R^+

D. R^+ and R

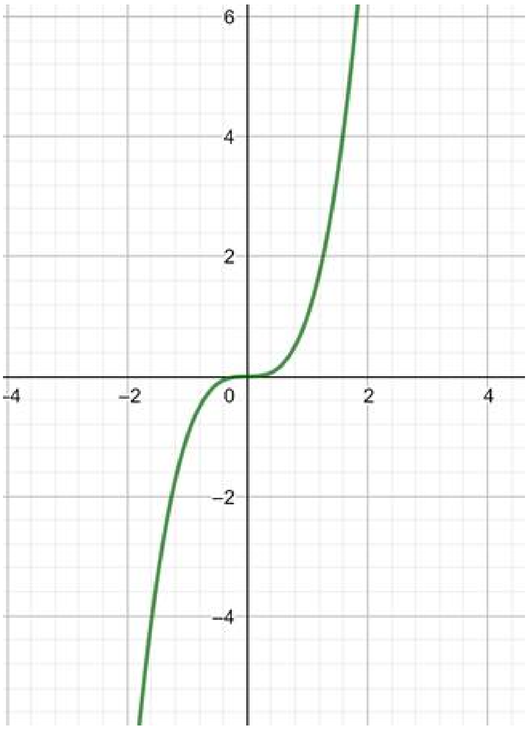
Answer

According to sketched graph of x^3

Domain of f(x) = R

And Range of f(x) = R

Since x^3 is a, monotonically increasing function



35. Question

Mark (√) against the correct answer in the following:

Let f(x) = log (1 – x) + $\sqrt{x^2}$ – 1. Then, dom (f) = ?

A. (1, ∞)

B. (- ∞, - 1]

C. [- 1, 1)

D. (0, 1)

Answer

log (1 – x) + $\sqrt{(x^2 - 1)}$

1 - x>0

$x < 1$

$X^2 - 1 \geq 0$

$X^2 \geq 1$

$\Rightarrow -1 \leq x \geq 1$

Taking intersection of the ranges we get

Dom (f) = (b) (- ∞, - 1]

36. Question

Mark (√) against the correct answer in the following:

Let $f(x) = \frac{1}{(1-x^2)}$. Then, range (f) = ?

A. (- ∞, 1]

B. [1, ∞)

C. [- 1, 1]

D. none of these

Answer

$$f(x) = \frac{1}{1 - x^2}$$

$$\Rightarrow y = \frac{1}{1 - x^2}$$

$$\Rightarrow y - yx^2 = 1$$

$$\Rightarrow y - 1 = yx^2$$

$$\Rightarrow x = \sqrt{\frac{y - 1}{y}}$$

$$\Rightarrow \frac{y - 1}{y} \geq 0$$

$$\Rightarrow y \geq 1$$

∴ range (f) = [1, ∞)

37. Question

Mark (√) against the correct answer in the following:

Let $f(x) = \frac{x^2}{(1 + x^2)}$. Then, range (f) = ?

A. [1, ∞)

B. [0, 1)

C. [- 1, 1]

D. (0, 1]

Answer

$$f(x) = \frac{x^2}{1 + x^2}$$

$$\Rightarrow y = \frac{x^2}{1 + x^2}$$

$$\Rightarrow y + yx^2 = x^2$$

$$\Rightarrow y = x^2(1 - y)$$

$$\Rightarrow x = \sqrt{\frac{y}{1 - y}}$$

$$\frac{y}{1 - y} \geq 0$$

$\Rightarrow y \geq 0$

And

$1 - y > 0$

$\Rightarrow y < 1$

Taking intersection we get

range (f) = [0, 1)

38. Question

Mark (√) against the correct answer in the following:

The range of $f(x) = x + \frac{1}{x}$ is

A. [- 2, 2]

B. [2, ∞)

C. (- ∞, - 2]

D. none of these

Answer

$f(x) = x + \frac{1}{x}$

For this type

Range is

$-2 \le y \ge 2$

39. Question

Mark (√) against the correct answer in the following:

The range of f(x) = a^x, where a > 0 is

A. [- ∞, 0]

B. [- ∞, 0)

C. [0, ∞)

D. (0, ∞)

Answer

f(x) = a^x

when x<0

0<a^x<1

When x≥0

a^x>0

Therefore range of f(x) = a^x = (0, ∞)