

15. Integration Using Partial Fractions

Exercise 15A

1. Question

Evaluate:

$$\int \frac{dx}{x(x+2)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x(x+2)},$$

$$\text{Putting } \frac{1}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2} \dots \dots (1)$$

Which implies $A(x+2) + Bx = 1$, putting $x+2=0$

Therefore $x=-2$,

And $B = -0.5$

Now put $x=0$, $A = \frac{1}{2}$,

From equation (1), we get

$$\frac{1}{x(x+2)} = \frac{1}{2} \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x+2}$$

$$\int \frac{1}{x(x+2)} dx = \frac{1}{2} \int \frac{1}{x} dx - \frac{1}{2} \int \frac{1}{x+2} dx$$

$$= \frac{1}{2} \log|x| - \frac{1}{2} \log|x+2| + c$$

$$= \frac{1}{2} [\log|x| - \log|x+2|] + c$$

$$= \frac{1}{2} \log \left| \frac{x}{x+2} \right| + c$$

2. Question

Evaluate:

$$\int \frac{(2x+1)}{(x+2)(x+3)} dx$$

Answer

$$\text{Let } I = \int \frac{(2x+1)}{(x+2)(x+3)} dx,$$

$$\text{Putting } \frac{2x+1}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} \dots \dots (1)$$

Which implies $2x+1 = A(x+3) + B(x+2)$

Now put $x+3=0$, $x=-3$

$$2 \times (-3) + 1 = A(0) + B(-3+2)$$

$$\text{So } B = \frac{7}{5}$$

Now put $x+2=0$, $x=-2$

$$-4+1 = A(-2-3) + B(0)$$

So $A = \frac{3}{5}$

From equation (1), we get ,

$$\frac{2x + 1}{(x + 2)(x - 3)} = \frac{3}{5} \times \frac{1}{x + 2} + \frac{7}{5} \times \frac{1}{x - 3}$$

$$\int \frac{2x + 1}{(x + 2)(x - 3)} dx = \frac{3}{5} \int \frac{1}{x + 2} dx + \frac{7}{5} \int \frac{1}{x - 3} dx$$

$$= \frac{3}{5} \log|x + 2| + \frac{7}{5} \log|x - 3| + c$$

3. Question

Evaluate:

$$\int \frac{x}{(x+2)(3-2x)} dx$$

Answer

Let $I = \int \frac{x}{(x+2)(3-2x)} dx,$

Putting $\frac{x}{(x+2)(3-2x)} = \frac{A}{x+2} + \frac{B}{3-2x} \dots \dots (1)$

Which implies $A(3-2x)+B(x+2)=x$

Now put $3-2x=0$

Therefore, $x = \frac{3}{2}$

$$A(0) + B\left(\frac{3}{2} + 2\right) = \frac{3}{2}$$

$$B\left(\frac{7}{2}\right) = \frac{3}{2}$$

$$B = \frac{3}{7}$$

Now put $x+2=0$

Therefore, $x=-2$

$$A(7)+B(0)=-2$$

$$A = \frac{-2}{7}$$

Now From equation (1) we get

$$\frac{x}{(x + 2)(3 - 2x)} = \frac{-2}{7} \times \frac{1}{x + 2} + \frac{3}{7} \times \frac{1}{3 - 2x}$$

$$\int \frac{x}{(x + 2)(3 - 2x)} dx = \frac{-2}{7} \int \frac{1}{x + 2} dx + \frac{3}{7} \int \frac{1}{3 - 2x} dx$$

$$= \frac{-2}{7} \log|x + 2| + \frac{3}{7} \times \frac{1}{-2} \log|3 - 2x| + c$$

$$= \frac{-2}{7} \log|x + 2| + \frac{3}{7} \times \frac{1}{-2} \log|3 - 2x| + c$$

$$= \frac{-2}{7} \log|x + 2| - \frac{3}{14} \log|3 - 2x| + c$$

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4. Question

Evaluate:

$$\int \frac{dx}{x(x-2)(x-4)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x(x-2)(x-4)},$$

$$\text{Putting } \frac{1}{x(x-2)(x-4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-4} \dots \dots (1)$$

Which implies,

$$A(x-2)(x-4) + Bx(x-4) + Cx(x-2) = 1$$

Now put $x-2=0$

Therefore, $x=2$

$$A(0) + B \times 2(2-4) + C(0) = 1$$

$$B \times 2(-2) = 1$$

$$B = -\frac{1}{4}$$

Now put $x-4=0$

Therefore, $x=4$

$$A(0) + B \times (0) + C \times 4(4-2) = 1$$

$$C \times 4(2) = 1$$

$$C = \frac{1}{8}$$

Now put $x=0$

$$A(0-2)(0-4) + B(0) + C(0) = 1$$

$$A = \frac{1}{8}$$

Now From equation (1) we get

$$\frac{1}{x(x-2)(x-4)} = \frac{1}{8} \times \frac{1}{x} - \frac{1}{4} \times \frac{1}{x-2} + \frac{1}{8} \times \frac{1}{x-4}$$

$$\int \frac{dx}{x(x-2)(x-4)} = \frac{1}{8} \int \frac{1}{x} dx - \frac{1}{4} \int \frac{1}{x-2} dx + \frac{1}{8} \int \frac{1}{x-4} dx$$

$$= \frac{1}{8} \log|x| - \frac{1}{4} \log|x-2| + \frac{1}{8} \log|x-4| + c$$

5. Question

Evaluate:

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$$

Answer

$$\text{Let } I = \int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx$$

$$\text{Putting } \frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{x-3} \dots \dots (1)$$

Which implies,

$$A(x+2)(x-2)+B(x-1)(x-3)+C(x-1)(x+2)=2x-1$$

Now put $x+2=0$

Therefore, $x=-2$

$$A(0)+B(-2-1)(-2-3)+C(0)=2x-2-1$$

$$B(-3)(-5)=-5$$

$$B = -\frac{1}{3}$$

Now put $x-3=0$

Therefore, $x=3$

$$A(0)+B(0)+C(2)(5)=5$$

$$C = \frac{1}{2}$$

Now put $x-1=0$

Therefore, $x=1$

$$A(3)(-2)+B(0)+C(0)=1$$

$$A = -\frac{1}{6}$$

Now From equation (1) we get,

$$\frac{(2x-1)}{(x-1)(x+2)(x-3)} = \frac{-1}{6} \times \frac{1}{x-1} - \frac{1}{3} \times \frac{1}{x+2} + \frac{1}{2} \times \frac{1}{x-3}$$

$$\int \frac{(2x-1)}{(x-1)(x+2)(x-3)} dx = \frac{-1}{6} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{1}{x+2} dx + \frac{1}{2} \int \frac{1}{x-3} dx$$

$$= \frac{-1}{6} \log|x-1| - \frac{1}{3} \log|x+2| + \frac{1}{2} \log|x-3| + c$$

6. Question

Evaluate:

$$\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$$

Answer

$$\text{Let } I = \int \frac{(2x-3)}{(x^2-1)(2x+3)} dx$$

$$\text{Putting } \frac{(2x-3)}{(x-1)(x+1)(2x+3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{2x+3} \dots \dots (1)$$

Which implies,

$$A(x+1)(2x+3)+B(x-1)(2x+3)+C(x-1)(x+1)=2x-3$$

Now put $x+1=0$

Therefore, $x=-1$

$$A(0)+B(-1-1)(-2+3)+C(0)=-2-3$$

$$B = -\frac{5}{2}$$

Now put $x-1=0$

Therefore, $x=1$

$$A(2)(2+3)+B(0)+C(0)=-1$$

$$A = -\frac{1}{10}$$

Now put $2x+3=0$

$$\text{Therefore, } x = -\frac{3}{2}$$

$$A(0) + B(0) + C\left(\frac{-3}{2} - 1\right)\left(\frac{-3}{2} + 1\right) = 2\left(\frac{-3}{2}\right) - 3$$

$$C\left(\frac{-5}{2}\right)\left(\frac{-1}{2}\right) = -3 - 3$$

$$C = -\frac{24}{5}$$

.Now From equation (1) we get,

$$\frac{(2x-3)}{(x^2-1)(2x+3)} = \frac{-1}{10} \times \frac{1}{x-1} + \frac{5}{2} \times \frac{1}{x+1} - \frac{24}{5} \times \frac{1}{2x+3}$$

$$\int \frac{(2x-3)}{(x^2-1)(2x+3)} dx = \frac{-1}{10} \int \frac{1}{x-1} dx + \frac{5}{2} \int \frac{1}{x+1} dx - \frac{24}{5} \int \frac{1}{2x+3} dx$$

$$= \frac{-1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{24 \log|2x+3|}{5 \cdot 2} + c$$

$$= \frac{-1}{10} \log|x-1| + \frac{5}{2} \log|x+1| - \frac{12}{5} \log|2x+3| + c$$

7. Question

Evaluate:

$$\int \frac{(2x+5)}{(x^2-x-2)} dx$$

Answer

$$\text{Let } I = \int \frac{(2x+5)}{(x^2-x-2)} dx = \int \frac{(2x+5)}{(x-2)(x+1)} dx$$

$$\text{Putting } \frac{(2x+5)}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} \dots \dots (1)$$

Which implies,

$$A(x+1)+B(x-2)=2x+5$$

Now put $x+1=0$

Therefore, $x=-1$

$$A(0)+B(-1-2)=3$$

$$B=-1$$

Now put $x-2=0$

Therefore, $x=2$

$$A(2+1)+B(0)=2 \times 2+5=9$$

$$A=3$$

Now From equation (1) we get,

$$\frac{(2x+5)}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{-1}{x+1}$$

$$\int \frac{(2x+5)}{(x-2)(x+1)} dx = \int \frac{3}{x-2} + \int \frac{-1}{x+1}$$

$$= 3 \log|x-2| - \log|x+1| + c$$

8. Question

Evaluate:

$$\int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx$$

Answer

$$\text{Let } I = \int \frac{(x^2+5x+3)}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2+2x+1}{(x^2+3x+2)} dx = \int \frac{x^2+3x+2}{(x^2+3x+2)} dx + \int \frac{2x+1}{(x^2+3x+2)} dx$$

$$\text{Which implies } I = \int dx + \int \frac{2x+1}{(x^2+3x+2)} dx$$

Therefore, $I = x + I_1$

$$\text{Where, } I_1 = \int \frac{2x+1}{(x^2+3x+2)} dx$$

$$\text{Putting } \frac{(2x+1)}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2} \dots \dots (1)$$

Which implies,

$$A(x+2)+B(x+1)=2x+1$$

Now put $x+2=0$

Therefore, $x=-2$

$$A(0)+B(-1)=-4+1$$

$$B=3$$

Now put $x+1=0$

Therefore, $x=-1$

$$A(-1+2)+B(0)=-2+1$$

$$A=-1$$

Now From equation (1) we get,

$$\frac{(2x+1)}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{3}{x+2}$$

$$\int \frac{(2x+1)}{(x+1)(x+2)} dx = - \int \frac{1}{x+1} dx + \int \frac{3}{x+2} dx$$

$$= - \log|x+1| + 3 \log|x+2| + c$$

9. Question

Evaluate:

$$\int \frac{(x^2+1)}{(x^2-1)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2+1}{x^2-1} dx$$

$$I = \int \left(1 + \frac{2}{x^2-1}\right) dx$$

$$I = \int dx + 2 \int \frac{1}{x^2-1} dx$$

$$I = x + 2 \times \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + c$$

$$I = x + \log \left| \frac{x-1}{x+1} \right| + c$$

10. Question

Evaluate:

$$\int \frac{x^3}{(x^2-4)} dx$$

Answer

$$\text{Let } I = \int \frac{x^3}{x^2-4} dx$$

$$I = \int x + \frac{4x}{x^2-4} dx$$

$$I = \int x dx + \int \frac{4x}{x^2-4} dx$$

$$= \frac{x^2}{2} + \int \frac{4x}{(x-2)(x+2)} dx$$

$$\text{Let } I_1 = \int \frac{4x}{(x-2)(x+2)} dx$$

So

$$I = \frac{x^2}{2} + I_1$$

$$\text{Therefore } I_1 = \int \frac{4x}{x^2-4} dx$$

Putting $x^2-4=t$

$$2x dx = dt$$

$$I_1 = 2 \int \frac{dt}{t}$$

$$I_1 = 2 \log |x^2 - 4| + c$$

Putting the value of I_1 in I ,

$$I = \frac{x^2}{2} + 2 \log |x^2 - 4| + c$$

11. Question

Evaluate:

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$$\int \frac{(3+4x-x^2)}{(x+2)(x-1)} dx$$

Answer

$$\text{Let } I = \int \frac{3+4x-x^2}{(x+2)(x-1)} dx$$

$$= \int \left(-1 + \frac{5x+1}{(x+2)(x-1)} \right) dx$$

$$= \int -dx + \int \frac{5x+1}{(x+2)(x-1)} dx$$

$$= -x + I_1$$

$$I_1 = \int \frac{5x+1}{(x+2)(x-1)} dx$$

$$\text{Put } \frac{5x+1}{(x+2)(x-1)} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$$

$$A(x-1)+B(x+2)=5x+1$$

Now put $x-1=0$

Therefore, $x=1$

$$A(0)+B(1+2)=5+1=6$$

$$B=2$$

Now put $x+2=0$

Therefore, $x=-2$

$$A(-2-1)+B(0)=5 \times (-2)+1$$

$$A=3$$

Now From equation (1) we get,

$$\frac{5x+1}{(x+2)(x-1)} = \frac{3}{(x+2)} + \frac{2}{(x-1)}$$

$$\int \frac{5x+1}{(x+2)(x-1)} dx = 3 \int \frac{1}{(x+2)} dx + 2 \int \frac{1}{(x-1)} dx$$

$$3 \log|x+2| + 2 \log|x-1| + c$$

Therefore,

$$I = -x + 3 \log|x+2| + 2 \log|x-1| + c$$

12. Question

Evaluate:

$$\int \frac{x^2}{(x-1)(x-2)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2}{(x-1)(x-2)} dx$$

$$= \int \left\{ (x+3) + \frac{7x-6}{(x-1)(x-2)} \right\} dx$$

$$= \frac{x^2}{2} + 3x + \int \frac{7x-6}{(x-1)(x-2)} dx$$

$$= \frac{x^2}{2} + 3x + I_1 \dots \dots (1)$$

Where,

$$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx$$

$$\text{Putting } \frac{7x-6}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} \dots \dots (2)$$

$$A(x-2)+B(x-1)=7x-6$$

Now put $x-2=0$

Therefore, $x=2$

$$A(0)+B(2-1)=7 \times 2-6$$

$$B=8$$

Now put $x-1=0$

Therefore, $x=1$

$$A(1-2)+B(0)=7-6=1$$

$$A=-1$$

Now From equation (2) we get,

$$\frac{7x-6}{(x-1)(x-2)} = \frac{-1}{x-1} + \frac{8}{x-2}$$

$$I_1 = \int \frac{7x-6}{(x-1)(x-2)} dx = - \int \frac{1}{x-1} dx + 8 \int \frac{1}{x-2} dx$$

$$= -\log|x-1| + 8\log|x-2| + c$$

Now From equation (1) we get,

$$I = \frac{x^2}{2} + 3x - \log|x-1| + 8\log|x-2| + c$$

13. Question

Evaluate:

$$\int \frac{(x^3-x-2)}{(1-x^2)} dx$$

Answer

$$\text{Let } I = \int \frac{(x^3-x-2)}{(1-x^2)} dx$$

$$= \int \left(-x + \frac{-2}{1-x^2} \right) dx$$

$$= \int -x dx + (-2) \int \frac{1}{1-x^2} dx$$

$$= \frac{-x^2}{2} - \log \left| \frac{1+x}{1-x} \right| + c$$

$$= \frac{-x^2}{2} + \log \left| \frac{1-x}{1+x} \right| + c$$

14. Question

Evaluate:

$$\int \frac{(2x+1)}{(4-3x-x^2)} dx$$

Answer

$$\text{Let } I = \int \frac{2x+1}{(4-3x-x^2)} dx$$

$$= \int \frac{2x+1}{(1-x)(4+x)} dx$$

$$\text{Putting } \frac{2x+1}{(1-x)(4+x)} = \frac{A}{1-x} + \frac{B}{4+x} \dots \dots \dots (1)$$

$$A(4+x) + B(1-x) = 2x+1$$

Now put $1-x=0$

Therefore, $x=1$

$$A(5) + B(0) = 3$$

$$A = \frac{3}{5}$$

Now put $4+x=0$

Therefore, $x=-4$

$$A(0) + B(5) = -8+1 = -7$$

$$B = \frac{-7}{5}$$

Now From equation (1) we get,

$$\frac{2x+1}{(1-x)(4+x)} = \frac{3}{5} \times \frac{1}{1-x} + \frac{-7}{5} \times \frac{1}{4+x}$$

$$\int \frac{2x+1}{(1-x)(4+x)} dx = \frac{3}{5} \int \frac{1}{1-x} dx + \frac{-7}{5} \int \frac{1}{4+x} dx$$

$$= \frac{-3}{5} \log|1-x| - \frac{7}{5} \log|4+x| + c$$

$$= -\frac{1}{5} [3\log|1-x| + 7\log|4+x|] + c$$

15. Question

Evaluate:

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx$$

Answer

Put $x^2=t$

$2x dx = dt$

$$\int \frac{dt}{(1+t)(3+t)} = \frac{1}{2} \int \left(\frac{1}{1+t} - \frac{1}{3+t} \right) dt$$

$$\frac{1}{2}[\log|1+t| - \log|3+t|] + c = \frac{1}{2} \log \left| \frac{1+t}{3+t} \right| + c$$

$$= \frac{1}{2} \log \left| \frac{1+x^2}{3+x^2} \right| + c$$

16. Question

Evaluate:

$$\int \frac{\cos x}{(\cos^2 x - \cos x - 2)} dx$$

Answer

$$\text{Let } I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$$

Putting $t = \sin x$

$$dt = \cos x dx$$

$$I = \int \frac{dt}{(1+t)(2+t)}$$

$$\text{Now putting, } \frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \dots \dots \dots (1)$$

$$A(2+t) + B(1+t) = 1$$

$$\text{Now put } t+1=0$$

$$\text{Therefore, } t=-1$$

$$A(2-1) + B(0) = 1$$

$$A=1$$

$$\text{Now put } t+2=0$$

$$\text{Therefore, } t=-2$$

$$A(0) + B(-2+1) = 1$$

$$B=-1$$

Now From equation (1) we get,

$$\frac{1}{(1+t)(2+t)} = \frac{1}{1+t} + \frac{-1}{2+t}$$

$$\int \frac{1}{(1+t)(2+t)} dt = \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$$

$$= \log|1+t| - \log|t+2| + c$$

$$= \log \left| \frac{t+1}{t+2} \right| + c$$

So,

$$I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx = \log \left| \frac{\sin x + 1}{\sin x + 2} \right| + c$$

17. Question

Evaluate:

$$\int \frac{\sec^2 x}{(2+\tan x)(3+\tan x)} dx$$

Answer

$$\text{Let } I = \int \frac{\sec^2 x}{(2+\tan x)(3+\tan x)} dx$$

Putting $t = \tan x$

$$dt = \sec^2 x dx$$

$$I = \int \frac{dt}{(2+t)(3+t)},$$

$$\text{Now putting, } \frac{1}{(3+t)(2+t)} = \frac{A}{2+t} + \frac{B}{3+t} \dots\dots\dots (1)$$

$$A(3+t) + B(2+t) = 1$$

Now put $t+2=0$

Therefore, $t = -2$

$$A(3-2) + B(0) = 1$$

$$A = 1$$

Now put $t+3=0$

Therefore, $t = -3$

$$A(0) + B(2-3) = 1$$

$$B = -1$$

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= \log \left| \frac{t+2}{t+3} \right| + c$$

So,

$$I = \int \frac{\sec^2 x}{(2+\tan x)(3+\tan x)} dx = \log \left| \frac{\tan x + 2}{\tan x + 3} \right| + c$$

18. Question

Evaluate:

$$\int \frac{\sin x \cos x}{(\cos^2 x - \cos x - 2)} dx$$

Answer

$$\text{Let } I = \int \frac{\sin x \cos x}{\cos^2 x - \cos x - 2} dx$$

Putting $t = \cos x$

$$dt = -\sin x dx$$

$$I = \int \frac{(-dt)t}{t^2 - t - 2} = - \int \frac{t dt}{(t+1)(t-2)},$$

Now putting, $\frac{-t}{(t+1)(t-2)} = \frac{A}{t+1} + \frac{B}{t-2} \dots \dots \dots (1)$

$A(t-2)+B(t+1)=-t$

Now put $t-2=0$

Therefore, $t=2$

$A(0)+B(2+1)=-2$

$B = \frac{-2}{3}$

Now put $t+1=0$

Therefore, $t=-1$

$A(-1-2)+B(0)=1$

$A = \frac{-1}{3}$

Now From equation (1) we get,

$\frac{-t}{(t+1)(t-2)} = \frac{-1}{3} \times \frac{1}{t+1} - \frac{2}{3} \times \frac{1}{t-2}$

$\int \frac{-t}{(t+1)(t-2)} dt = \frac{-1}{3} \int \frac{1}{t+1} - \frac{2}{3} \int \frac{1}{t-2}$

$= \frac{-1}{3} \log|t+1| - \frac{2}{3} \log|t-2| + c$

So,

$I = \int \frac{\sin x \cos x}{\cos^2 x - \cos x - 2} dx = \frac{-1}{3} \log|\cos x + 1| - \frac{2}{3} \log|\cos x - 2| + c$

19. Question

Evaluate:

$\int \frac{e^x}{(e^{2x} + 5e^x + 6)} dx$

Answer

Let $I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$

Putting $t=e^x$

$dt=e^x dx$

$I = \int \frac{dt}{(t^2 + 5t + 6)}$

Now putting, $\frac{1}{(t^2+5t+6)} = \frac{A}{2+t} + \frac{B}{3+t} \dots \dots \dots (1)$

$A(3+t)+B(2+t)=1$

Now put $t+2=0$

Therefore, $t=-2$

$A(3-2)+B(0)=1$

$A=1$

Now put $t+3=0$

Therefore, $t=-3$

$$A(0)+B(2-3)=1$$

$$B=-1$$

Now From equation (1) we get,

$$\frac{1}{(2+t)(3+t)} = \frac{1}{2+t} + \frac{-1}{3+t}$$

$$\int \frac{1}{(2+t)(3+t)} dt = \int \frac{1}{2+t} dt - \int \frac{1}{3+t} dt$$

$$= \log|2+t| - \log|t+3| + c$$

$$= \log \left| \frac{t+2}{t+3} \right| + c$$

$$= \log \left| \frac{e^x+2}{e^x+3} \right| + c$$

20. Question

Evaluate:

$$\int \frac{e^x}{(e^{3x}-3e^{2x}-e^x+3)} dx$$

Answer

$$\text{Let } I = \int \frac{e^x}{e^{3x}-3e^{2x}-e^x+3} dx$$

Putting $t=e^x$

$$dt=e^x dx$$

$$I = \int \frac{dt}{(t^3-3t^2-t+3)} = \int \frac{dt}{(t^2)(t-3)-(t-3)} = \int \frac{dt}{(t^2-1)(t-3)}$$

$$\text{Now putting, } \frac{1}{(t-1)(t+1)(t-3)} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{t-3} \dots \dots (1)$$

$$A(t+1)(t-3)+B(t-1)(t-3)+C(t-1)(t+1)=1$$

Now put $t+1=0$

Therefore, $t=-1$

$$A(0)+B(-1-1)(-1-3)+C(0)=1$$

$$B(-2)(-4)=1$$

$$B = \frac{1}{8}$$

Now put $t-1=0$

Therefore, $t=1$

$$A(1+1)(1-3)+B(0)+C(0)=1$$

$$A = \frac{-1}{4}$$

Now put $t-3=0$

Therefore, $t=3$

$$A(0)+B(0)+C(3-1)(3+1)=1$$

$$C = \frac{1}{8}$$

Now From equation (1) we get,

$$\frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \times \frac{1}{t-1} + \frac{1}{8} \times \frac{1}{t+1} + \frac{1}{8} \times \frac{1}{t-3}$$

$$\int \frac{1}{(t-1)(t+1)(t-3)} = \frac{-1}{4} \int \frac{1}{t-1} + \frac{1}{8} \int \frac{1}{t+1} + \frac{1}{8} \int \frac{1}{t-3}$$

$$= \frac{-1}{4} \log|t-1| + \frac{1}{8} \log|t+1| + \frac{1}{8} \log|t-3| + c$$

$$\int \frac{e^x}{e^{3x} - 3e^{2x} - e^x + 3} dx = \frac{-1}{4} \log|e^x - 1| + \frac{1}{8} \log|e^x + 1| + \frac{1}{8} \log|e^x - 3| + c$$

21. Question

Evaluate:

$$\int \frac{2 \log x}{x[2(\log x)^2 - \log x - 3]} dx$$

Answer

$$\text{Let } I = \int \frac{2 \log x}{x[2(\log x)^2 - \log x - 3]} dx$$

Putting $t = \log x$

$$dt = dx/x$$

$$I = \int \frac{2t dt}{(2t^2 - t - 3)}$$

$$\text{Now putting, } \frac{2t}{(2t^2 - t - 3)} = \frac{A}{2t-3} + \frac{B}{t+1} \dots \dots \dots (1)$$

$$A(t+1) + B(2t-3) = 2t$$

Now put $2t-3=0$

$$\text{Therefore, } t = \frac{3}{2}$$

$$A\left(\frac{3}{2} + 1\right) + B(0) = 2 \times \frac{3}{2} = 3$$

$$A = \frac{6}{5}$$

Now put $t+1=0$

Therefore, $t=-1$

$$A(0) + B(-2-3) = -2$$

$$B = \frac{2}{5}$$

Now From equation (1) we get,

$$\frac{2t}{(2t^2 - t - 3)} = \frac{6}{5} \times \frac{1}{2t-3} + \frac{2}{5} \times \frac{1}{t+1}$$

$$\int \frac{2t}{(2t^2 - t - 3)} dt = \frac{6}{5} \int \frac{1}{2t - 3} dt + \frac{2}{5} \int \frac{1}{t + 1} dt$$

$$= \frac{6}{5} \log \left| \frac{6}{5} \times \frac{\log(2t - 3)}{2} \right| + \frac{2}{5} \log|\log x + 1| + c$$

$$\int \frac{2 \log x}{x[2(\log x)^2 - \log x - 3]} dx = \frac{3}{5} \log|2 \log x - 3| + \frac{2}{5} \log|\log x + 1| + c$$

22. Question

Evaluate:

$$\int \frac{\operatorname{cosec}^2 x}{(1 - \cot^2 x)} dx$$

Answer

$$\text{Let } I = \int \frac{\operatorname{cosec}^2 x}{(1 - \cot^2 x)} dx$$

Putting $t = \cot x$

$$dt = -\operatorname{cosec}^2 x dx$$

$$I = \int \frac{-dt}{(1 - t^2)} = - \int \frac{1}{(1 - t^2)} dt$$

$$= \frac{-1}{2} \log \left| \frac{1 + \cot x}{1 - \cot x} \right| + c$$

23. Question

Evaluate:

$$\int \frac{\sec^2 x}{(\tan^3 x + 4 \tan x)} dx$$

Answer

$$\text{Let } I = \int \frac{\sec^2 x}{(\tan^3 x + 4 \tan x)} dx$$

Putting $t = \tan x$

$$dt = \sec^2 x dx$$

$$I = \int \frac{dt}{(t^3 + 4t)} = \int \frac{dt}{t(t^2 + 4)}$$

$$\text{Now putting, } \frac{1}{t(t^2 + 4)} = \frac{A}{t} + \frac{Bt + C}{t^2 + 4} \dots \dots (1)$$

$$A(t^2 + 4) + (Bt + C)t = 1$$

Putting $t = 0$,

$$A(0 + 4) + B(0) = 1$$

$$A = \frac{1}{4}$$

By equating the coefficients of t^2 and constant here,

$$A + B = 0$$

$$\frac{1}{4} + B = 0$$

$$B = -\frac{1}{4}, C = 0$$

Now From equation (1) we get,

$$\begin{aligned} \int \frac{1}{t(t^2+4)} dt &= \frac{1}{4} \int \frac{dt}{t} - \frac{1}{4} \int \frac{t}{t^2+4} dt \\ &= \frac{1}{4} \log t - \frac{1}{4} \times \frac{1}{2} \log(t^2+4) + c \\ &= \frac{1}{4} \log \tan x - \frac{1}{8} \log(\tan^2 x + 4) + c \end{aligned}$$

41. Question

$$\int \frac{dx}{(x^2-1)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x^2-1}$$

$$\text{Put } \frac{1}{x^2-1} = \frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \dots \dots \dots (1)$$

$$A(x^2+x+1) + (Bx+C)(x-1) = 1$$

Now putting $x-1=0$

$$x=1$$

$$A(1+1+1) + 0 = 1$$

$$A = \frac{1}{3}$$

By equating the coefficient of x^2 and constant term, $A+B=0$

$$\frac{1}{3} + B = 0$$

$$B = -\frac{1}{3}$$

$$A-C=1$$

$$\frac{1}{3} - C = 1$$

$$C = \frac{1}{3} - 1$$

$$C = \frac{-2}{3}$$

From the equation(1), we get,

$$\frac{1}{(x-1)(x^2+x+1)} = \frac{1}{3} \times \frac{1}{x-1} + \frac{-\frac{1}{3}x - \frac{2}{3}}{x^2+x+1}$$

$$\begin{aligned} I &= \int \frac{1}{(x-1)(x^2+x+1)} dx \\ &= \frac{1}{3} \int \frac{1}{x-1} dx - \frac{1}{3} \int \frac{x}{x^2+x+1} dx - \frac{2}{3} \int \frac{1}{x^2+x+1} dx \end{aligned}$$

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$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \int \frac{2x+1-1}{x^2+x+1} dx - \frac{2}{3} \int \frac{1}{x^2+x+1} dx$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \int \frac{2x+1}{x^2+x+1} dx + \frac{1}{6} \int \frac{1}{x^2+x+1} dx - \frac{2}{3} \int \frac{1}{x^2+x+1} dx$$

Put $t=x^2+x+1$

$$dt=(2x+1)dx$$

$$I = \frac{1}{3} \log|x-1| - \frac{1}{6} \int \frac{dt}{t} + \left(\frac{1}{6} - \frac{2}{3}\right) \int \frac{dx}{x^2+x+1}$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \log t + \left(\frac{1-4}{6}\right) \int \frac{dx}{x^2 + 2 \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \frac{1}{2} \times \frac{1}{\sqrt{3}/2} \tan^{-1} \frac{x+1/2}{\sqrt{3}/2} + c$$

$$= \frac{1}{3} \log|x-1| - \frac{1}{6} \log|x^2+x+1| - \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x+1}{\sqrt{3}} + c$$

42. Question

$$\int \frac{dx}{(x^2+1)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x^2+1}$$

$$\text{Put } \frac{1}{x^2-1} = \frac{1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \dots \dots \dots (1)$$

$$A(x^2-x+1)+(Bx+C)(x+1)=1$$

Now putting $x+1=0$

$$x=-1$$

$$A(1+1+1)+C(0)=1$$

$$A = \frac{1}{3}$$

By equating the coefficient of x^2 and constant term, $A+B=0$

$$\frac{1}{3} + B = 0$$

$$B = -\frac{1}{3}$$

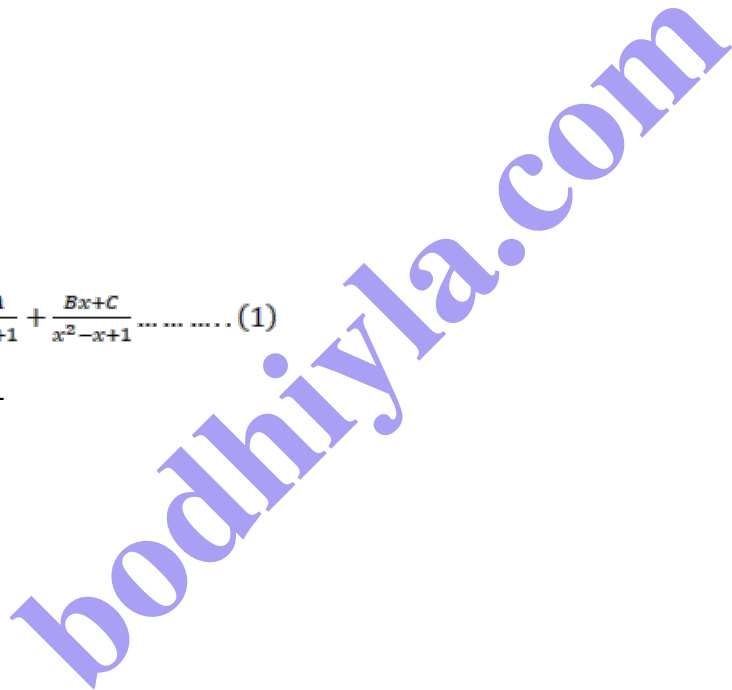
$$A+C=1$$

$$\frac{1}{3} + C = 1$$

$$C = 1 - \frac{1}{3}$$

$$C = \frac{2}{3}$$

From the equation(1), we get,



$$\frac{1}{(x+1)(x^2-x+1)} = \frac{1}{3} \times \frac{1}{x+1} + \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1}$$

$$\begin{aligned} I &= \int \frac{1}{(x+1)(x^2-x+1)} dx \\ &= \frac{1}{3} \int \frac{1}{x+1} dx - \frac{1}{3} \int \frac{x}{x^2-x+1} dx + \frac{2}{3} \int \frac{1}{x^2-x+1} dx \\ &= \frac{1}{3} \log|x+1| - \frac{1}{6} \int \frac{2x-1+1}{x^2-x+1} dx + \frac{2}{3} \int \frac{1}{x^2-x+1} dx \\ &= \frac{1}{3} \log|x+1| - \frac{1}{6} \int \frac{2x-1}{x^2-x+1} dx - \frac{1}{6} \int \frac{1}{x^2-x+1} dx + \frac{2}{3} \int \frac{1}{x^2-x+1} dx \\ &= \frac{1}{3} \log|x+1| - \frac{1}{6} \log|x^2-x+1| - \frac{1}{2} \times \frac{1}{\sqrt{3}/2} \tan^{-1} \frac{x-1/2}{\sqrt{3}/2} + c \\ &= \frac{1}{3} \log|x+1| - \frac{1}{6} \log|x^2-x+1| + \frac{1}{\sqrt{3}} \tan^{-1} \frac{2x-1}{\sqrt{3}} + c \end{aligned}$$

24. Question

Evaluate:

$$\int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$$

Answer

$$\text{Let } I = \int \frac{\sin 2x}{(1+\sin x)(2+\sin x)} dx$$

Putting $t = \sin x$

$$dt = \cos x dx$$

$$I = \int \frac{2t}{(1+t)(2+t)} dt$$

$$\text{Now putting, } \frac{2t}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t} \dots \dots \dots (1)$$

$$A(2+t) + B(1+t) = 2t$$

$$\text{Now put } t+2=0$$

$$\text{Therefore, } t=-2$$

$$A(0) + B(1-2) = -4$$

$$B=4$$

$$\text{Now put } t+1=0$$

$$\text{Therefore, } t=-1$$

$$A(2-)+B(0)=-2$$

$$A=-2$$

Now from equation (1), we get,

$$\frac{2t}{(1+t)(2+t)} = \frac{-2}{1+t} + \frac{4}{2+t}$$

$$\int \frac{2t}{(1+t)(2+t)} dt = -2 \int \frac{1}{1+t} dt + 4 \int \frac{1}{2+t} dt$$

$$= 4 \log|2 + t| - 2 \log|1 + t| + c$$

So,

$$\int \frac{\sin 2x}{(1 + \sin x)(2 + \sin x)} dx = 4 \log|2 + t| - 2 \log|1 + t| + c$$

25. Question

Evaluate:

$$\int \frac{e^x}{e^x(e^x - 1)} dx$$

Answer

$$\text{Let } I = \int \frac{e^x}{e^x(e^x - 1)} dx$$

Putting $t = e^x$

$$dt = e^x dx$$

$$I = \int \frac{dt}{t(t - 1)}$$

$$\text{Now putting, } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \dots \dots (1)$$

$$A(t-1) + Bt = 1$$

Now put $t-1=0$

Therefore, $t=1$

$$A(0) + B(1) = 1$$

$$B = 1$$

Now put $t=0$

$$A(0-1) + B(0) = 1$$

$$A = -1$$

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\int \frac{1}{t(t-1)} dt = - \int \frac{1}{t} dt + \int \frac{1}{t-1} dt$$

$$= - \log t + \log|t - 1| + c$$

$$= \log \left| \frac{t-1}{t} \right| + c$$

$$= \log \left| \frac{e^x - 1}{e^x} \right| + c$$

43. Question

$$\int \frac{dx}{(x+1)^2(x^2+1)}$$

Answer

$$\text{Let } I = \int \frac{dx}{(x^2+1)(x+1)^2}$$

$$\text{Put } \frac{1}{(x^2+1)(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+1} \dots \dots \dots (1)$$

$$A(x+1)(x^2+1)+B(x^2+1)+(Cx+D)(x+1)^2=1$$

$$\text{Put } x+1=0$$

$$x=-1$$

$$A(0)+B(1+1)+0=1$$

$$B = \frac{1}{2}$$

By equating the coefficient of x^2 and constant term, $A+C=0$

$$A+B+2C=0 \dots \dots (2)$$

$$A + 2C = \frac{-1}{2} \dots \dots \dots (3)$$

$$A+B+D=1$$

Solving (2) and (3), we get,

$$\frac{1}{(x^2+1)(x+1)^2} = \frac{1}{2} \times \frac{1}{x+1} + \frac{1}{2} \times \frac{1}{(x+1)^2} + \frac{-\frac{1}{2}x+0}{x^2+1}$$

$$\int \frac{1}{(x^2+1)(x+1)^2} dx = \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{1}{(x+1)^2} dx - \frac{1}{2} \int \frac{x}{x^2+1} dx$$

$$= \frac{1}{2} \log|x+1| - \frac{1}{2} \times \frac{1}{x+1} - \frac{1}{4} \log|x^2+1| + c$$

26. Question

Evaluate:

$$\int \frac{dx}{x(x^4-1)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x(x^4-1)}$$

$$\text{Putting } t=x^4$$

$$dt=4x^3 dx$$

$$I = \int \frac{x^3 dx}{x^4(x^4-1)} = \frac{1}{4} \times \int \frac{dt}{t(t-1)}$$

$$\text{Now putting, } \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} \dots \dots \dots (1)$$

$$A(t-1)+Bt=1$$

$$\text{Now put } t-1=0$$

$$\text{Therefore, } t=1$$

$$A(0)+B(1) = 1$$

$$B=1$$

$$\text{Now put } t=0$$

$$A(0-1)+B(0)=1$$

$$A = -1$$

Now From equation (1) we get,

$$\frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\frac{1}{4} \int \frac{1}{t(t-1)} dt = -\frac{1}{4} \int \frac{1}{t} dt + \frac{1}{4} \int \frac{1}{t-1} dt$$

$$= -\frac{1}{4} \log t + \frac{1}{4} \log |t-1| + c$$

$$= -\frac{1}{4} \log x^4 + \frac{1}{4} \log |x^4 - 1| + c$$

$$= -\log |x| + \frac{1}{4} \log |x^4 - 1| + c$$

44. Question

$$\int \frac{17}{(2x+1)(x^2+4)} dx$$

Answer

$$\text{Let } I = \int \frac{17}{(2x+1)(x^2+4)} dx$$

$$\text{Put } \frac{17}{(2x+1)(x^2+4)} = \frac{A}{2x+1} + \frac{Bx+C}{x^2+4} \dots \dots \dots (1)$$

$$A(x^2+4) + (Bx+C)(2x+1) = 17$$

$$\text{Put } 2x+1=0$$

$$x = -\frac{1}{2}$$

$$A\left(\frac{1}{4} + 4\right) + 0 = 17$$

$$A\left(\frac{17}{4}\right) = 17$$

$$A = 4$$

By equating the coefficient of x^2 and constant term,

$$A + 2B = 0$$

$$4 + 2B = 0$$

$$B = -2$$

$$4A + C = 17$$

$$4 \times 4 + C = 17$$

$$C = 1$$

From the equation(1), we get,

$$\frac{17}{(2x+1)(x^2+4)} = \frac{4}{2x+1} + \frac{-2x+1}{x^2+4}$$

$$\int \frac{17}{(2x+1)(x^2+4)} dx = 4 \int \frac{1}{2x+1} dx - 2 \int \frac{2x}{x^2+4} dx + \int \frac{1}{x^2+2^2} dx$$

$$= \frac{4 \log|2x + 1|}{2} - \log|x^2 + 4| + \frac{1}{2} \tan^{-1} \frac{x}{2} + c$$

$$= 2 \log|2x + 1| - \log|x^2 + 4| + \frac{1}{2} \tan^{-1} \frac{x}{2} + c$$

27. Question

Evaluate:

$$\int \frac{(1-x^2)}{x(1-2x)} dx$$

Answer

$$\text{Let } I = \int \frac{(x^2-1)}{x(2x-1)} dx = \int \left(\frac{1}{2} + \frac{(\frac{1}{2}x-1)}{x(2x-1)} \right) dx = \int \frac{1}{2} dx + \int \frac{x}{x(2x-1)} dx - \int \frac{1}{x(2x-1)} dx$$

$$I = \frac{1}{2}x + \frac{1}{2} \times \frac{\log|2x-1|}{2} - I_1 \dots \dots (1)$$

$$\text{Where } I_1 = \int \frac{1}{x(2x-1)} dx \dots \dots (2)$$

$$\text{Now putting, } \frac{1}{x(2x-1)} = \frac{A}{x} + \frac{B}{2x-1}$$

$$A(2x-1) + Bx = 1$$

$$\text{Putting } 2x-1=0$$

$$x = \frac{1}{2}$$

$$A(0) + B\left(\frac{1}{2}\right) = 1$$

$$B=2$$

$$\text{Putting } x=0,$$

$$A(0-1) + B(0) = 1$$

$$A=-1$$

From equation (2), we get,

$$\frac{1}{x(2x-1)} = -\frac{1}{x} + \frac{2}{2x-1}$$

$$\int \frac{1}{x(2x-1)} dx = -\int \frac{1}{x} dx + 2 \int \frac{1}{2x-1} dx$$

$$= -\log|x| + \frac{2 \log|2x-1|}{2} + c$$

$$= \log|2x-1| - \log|x| + c$$

From equation (1),

$$I = \frac{1}{2}x + \frac{1}{4} \log|2x-1| - \log|2x-1| + \log|x| + c$$

$$= \frac{1}{2}x - \frac{3}{4} \log|1-2x| + \log|x| + c$$

45. Question

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$$\int \frac{dx}{(x^2+2)(x^2+4)}$$

Answer

$$\text{Let } I = \int \frac{dx}{(x^2+2)(x^2+4)}$$

$$\text{Put } \frac{1}{(x^2+2)(x^2+4)} = \frac{1}{(t+2)(t+4)} = \frac{A}{t+2} + \frac{B}{t+4} \dots \dots \dots (1)$$

$$A(t+4)+B(t+2) = 1$$

$$\text{Put } t+4=0$$

$$t=-4$$

$$A(0)+B(-4+2)=1$$

$$B = -\frac{1}{2}$$

$$\text{Put } t+2=0$$

$$t=-2$$

$$A(-2+4)+B(0)=1$$

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{1}{(t+2)(t+4)} = \frac{1}{2} \times \frac{1}{t+2} - \frac{1}{2} \times \frac{1}{t+4}$$

$$\int \frac{1}{(x^2+2)(x^2+4)} dx = \frac{1}{2} \int \frac{1}{x^2+2} dx - \frac{1}{2} \int \frac{1}{x^2+4} dx$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{2} \tan^{-1} \frac{x}{2} + c$$

$$= \frac{1}{4} \tan^{-1} \frac{x}{\sqrt{2}} - \frac{1}{4} \tan^{-1} \frac{x}{2} + c$$

28. Question

Evaluate:

$$\int \frac{(x^2+x+1)}{(x+2)(x+1)^2} dx$$

Answer

$$\text{Let } I = \int \frac{x^2+x+1}{(x+2)(x+1)^2} dx$$

$$\text{Now putting, } \frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} \dots \dots \dots (1)$$

$$A(x+1)^2+B(x+2)(x+1)+C(x+2)=x^2+x+1$$

$$\text{Now put } x+1=0$$

$$\text{Therefore, } x=-1$$

$$A(0)+B(0)+C(-1+2) = 1-1+1=1$$

$$C=1$$

Now put $x+2=0$

Therefore, $x=-2$

$$A(-2+1)^2+B(0)+C(0) = 4-2+1=3$$

$$A=3$$

Equating the coefficient of x^2 , $A+B=1$

$$3+B=1$$

$$B=-2$$

Form equation (1), we get,

$$\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}$$

So,

$$\begin{aligned} \int \frac{x^2+x+1}{(x+2)(x+1)^2} dx &= \int \frac{3}{x+2} dx - \int \frac{2}{x+1} dx + \int \frac{1}{(x+1)^2} dx \\ &= 3 \log|x+2| - 2 \log|x+1| - \frac{1}{1+x} + c \end{aligned}$$

46. Question

$$\frac{x^2+1}{(x^2+4)(x^2+25)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2+1}{(x^2+4)(x^2+25)} dx$$

$$\text{Putting } \frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{t+1}{(t+4)(t+25)} = \frac{A}{t+4} + \frac{B}{t+25} \dots \dots \dots (1)$$

Where $t=x^2$

$$(A+B)t+(25A+4B)=t+1$$

$$A+B=1 \dots \dots \dots (1)$$

$$25A+4B=1 \dots \dots \dots (2)$$

Solving equation (1) and (2), we get,

$$A = \frac{-1}{7} \text{ and } B = \frac{8}{7}$$

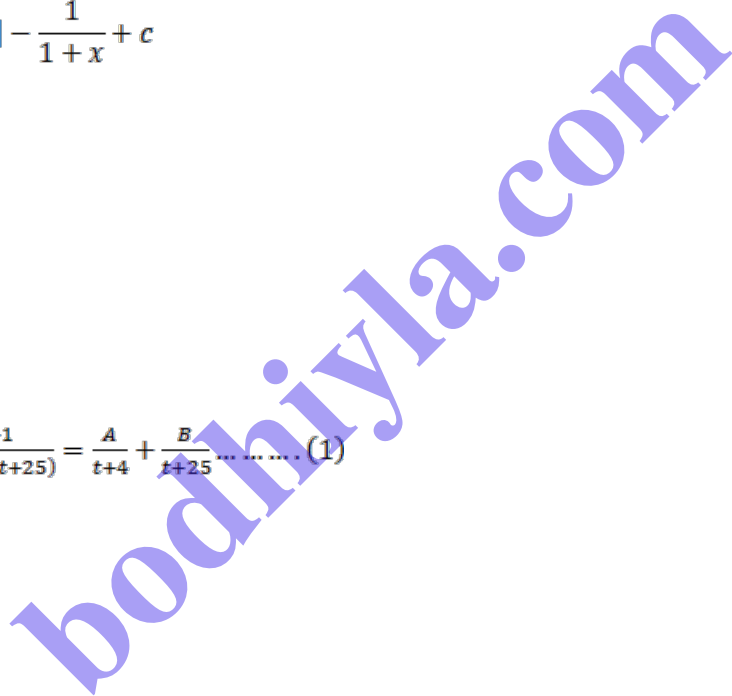
Now,

$$\frac{t+1}{(t+4)(t+25)} = \frac{-1}{7} \times \frac{1}{t+4} + \frac{8}{7} \times \frac{1}{t+25}$$

$$\frac{x^2+1}{(x^2+4)(x^2+25)} = \frac{-1}{7} \times \frac{1}{x^2+4} + \frac{8}{7} \times \frac{1}{x^2+25}$$

$$\int \frac{x^2+1}{(x^2+4)(x^2+25)} dx = \frac{-1}{7} \int \frac{1}{x^2+2^2} dx + \frac{8}{7} \int \frac{1}{x^2+5^2} dx$$

$$= -\frac{1}{7} \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{7} \times \frac{1}{5} \tan^{-1}\left(\frac{x}{5}\right) + c$$



$$= -\frac{1}{14} \tan^{-1}\left(\frac{x}{2}\right) + \frac{8}{35} \tan^{-1}\left(\frac{x}{5}\right) + c$$

29. Question

Evaluate:

$$\int \frac{(2x+9)}{(x+2)(x-3)^2} dx$$

Answer

$$\text{Let } I = \int \frac{2x+9}{(x+2)(x-3)^2} dx$$

$$\text{Now putting, } \frac{2x+9}{(x+2)(x-3)^2} = \frac{A}{(x+2)} + \frac{B}{(x-3)} + \frac{C}{(x-3)^2} \dots \dots (1)$$

$$A(x-3)^2 + B(x+2)(x-3) + C(x+2) = 2x+9$$

Now put $x-3=0$

Therefore, $x=3$

$$A(0) + B(0) + C(3+2) = 6+9=15$$

$$C=3$$

Now put $x+2=0$

Therefore, $x=-2$

$$A(-2-3)^2 + B(0) + C(0) = -4+9=5$$

$$A = \frac{1}{5}$$

Equating the coefficient of x^2 , we get,

$$A+B=0$$

$$\frac{1}{5} + B = 0$$

$$B = -\frac{1}{5}$$

From equation (1), we get,

$$\frac{2x+9}{(x+2)(x-3)^2} = \frac{1}{5} \times \frac{1}{(x+2)} - \frac{1}{5} \times \frac{1}{(x-3)} + \frac{3}{(x-3)^2}$$

$$\int \frac{2x+9}{(x+2)(x-3)^2} dx = \frac{1}{5} \int \frac{1}{(x+2)} dx - \frac{1}{5} \int \frac{1}{(x-3)} dx + 3 \int \frac{1}{(x-3)^2} dx$$

$$= \frac{1}{5} \log|x+2| - \frac{1}{5} \log|x-3| - \frac{3}{x-3} + c$$

47. Question

$$\int \frac{dx}{(e^x-1)^2}$$

Answer

putting $t=e^x-1$

$$e^x=t+1$$

$$dt = e^x dx$$

$$\frac{dt}{e^x} = dx$$

$$\frac{dt}{t+1} = dx$$

$$\text{Put } \frac{1}{(1+t)t^2} = \frac{A}{t+1} + \frac{Bt+C}{t^2} \dots \dots (1)$$

$$A(t^2) + (Bt+C)(t+1) = 1$$

$$\text{Put } t+1=0$$

$$t=-1$$

$$A=1$$

Equating coefficients

$$A+B=0$$

$$1+B=0$$

$$B=-1$$

$$C=1$$

From equation (1), we get,

$$\frac{1}{(1+t)t^2} = \frac{1}{t+1} + \frac{-t+1}{t^2}$$

$$\int \frac{1}{(1+t)t^2} dt = \int \frac{1}{t+1} dt - \int \frac{t}{t^2} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - \int \frac{1}{t} dt + \int \frac{1}{t^2} dt$$

$$= \log|t+1| - \log|t| - \frac{1}{t} + c$$

$$\int \frac{1}{(e^x-1)^2} dx = \log|e^x| - \log|e^x-1| - \frac{1}{e^x-1} + c$$

48. Question

$$\int \frac{dx}{x(x^5+1)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x(x^5+1)}$$

$$\text{Put } t=x^5$$

$$dt=5x^4 dx$$

$$\int \frac{dt}{\frac{(5x^4)}{x(t+1)}} = \frac{1}{5} \int \frac{dt}{x^5(t+1)} = \frac{1}{5} \int \frac{dt}{t(t+1)}$$

$$\text{Putting } \frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \dots \dots (1)$$

$$A(t+1) + Bt = 1$$

$$\text{Now put } t+1=0$$

$$t=-1$$

$$A(0)+B(-1)=1$$

$$B=-1$$

Now put $t=0$

$$A(0+1)+B(0)=1$$

$$A=1$$

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)} dt = \int \frac{1}{t} dt - \int \frac{1}{t+1} dt$$

$$= \log t - \log|t+1| + c$$

$$= \log \left| \frac{t}{t+1} \right| + c$$

$$\int \frac{dx}{x(x^5+1)} = \frac{1}{5} \int \frac{dt}{t(t+1)} = \frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + c$$

$$= \log x - \frac{1}{5} \log|x^5+1| + c$$

30. Question

Evaluate:

$$\int \frac{(x^2+1)}{(x-1)^2(x+3)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2+1}{(x+3)(x-1)^2} dx$$

$$\text{Now putting, } \frac{x^2+1}{(x+3)(x-1)^2} = \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \dots \dots \dots (1)$$

$$A(x-1)^2+B(x+3)(x-1)+C(x+3)=x^2+1$$

Now put $x-1=0$

Therefore, $x=1$

$$A(0)+B(0)+C(4) = 2$$

$$C = \frac{1}{2}$$

Now put $x+3=0$

Therefore, $x=-3$

$$A(-3-1)^2+B(0)+C(0) = 9+1=10$$

$$A = \frac{5}{8}$$

By equating the coefficient of x^2 , we get, $A+B=1$

$$\frac{5}{8} + B = 1$$

$$B = 1 - \frac{5}{8} = \frac{3}{8}$$

From equation (1), we get,

$$\frac{x^2 + 1}{(x + 3)(x - 2)^2} = \frac{5}{8} \times \frac{1}{(x + 3)} + \frac{3}{8} \times \frac{1}{(x - 2)} + \frac{1}{(x - 2)^2}$$

$$\int \frac{x^2 + 1}{(x + 3)(x - 2)^2} dx = \frac{5}{8} \int \frac{1}{(x + 3)} dx + \frac{3}{8} \int \frac{1}{(x - 2)} dx + \int \frac{1}{(x - 2)^2} dx$$
$$= \frac{5}{8} \log|x + 3| + \frac{3}{8} \log|x - 2| - \frac{1}{2(x - 2)} + c$$

31. Question

Evaluate:

$$\int \frac{(x^2 + 1)}{(x + 3)(x - 1)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2 + 1}{(x - 3)(x - 1)^2} dx$$

$$\text{Now putting, } \frac{x^2 + 1}{(x - 3)(x - 1)^2} = \frac{A}{(x - 3)} + \frac{B}{(x - 1)} + \frac{C}{(x - 1)^2} \dots \dots (1)$$

$$A(x - 1)^2 + B(x - 3)(x - 1) + C(x - 3) = x^2 + 1$$

Putting $x - 1 = 0$,

$$x = 1$$

$$A(0) + B(0) + C(1 - 3) = 1 + 1$$

$$C = -1$$

Putting $x - 3 = 0$,

$$x = 3$$

$$A(3 - 1)^2 + B(0) + C(0) = 9 + 1$$

$$A(4) = 10$$

$$A = \frac{5}{2}$$

Equating the coefficient of x^2

$$A + B = 1$$

$$\frac{5}{2} + B = 1$$

$$B = 1 - \frac{5}{2} = \frac{-3}{2}$$

$$\text{From (i) } \int \frac{x^2 + 1}{(x - 3)(x - 1)^2} dx = \frac{5}{2} \int \frac{1}{x - 3} dx - \frac{3}{2} \int \frac{1}{x - 1} dx - \int \frac{1}{(x - 1)^2} dx$$

$$= \frac{5}{2} \log|x - 3| - \frac{3}{2} \log|x - 1| + \frac{1}{x - 1} + C$$

49. Question

$$\int \frac{dx}{x(x^2 + 1)}$$

Answer

$$\text{Let } I = \int \frac{dx}{x(x^6+1)}$$

$$\text{Put } t=x^6$$

$$dt=6x^5 dx$$

$$\int \frac{dt}{\frac{(6x^5)}{x(t+1)}} = \frac{1}{6} \int \frac{dt}{x^6(t+1)} = \frac{1}{6} \int \frac{dt}{t(t+1)}$$

$$\text{Putting } \frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \dots\dots\dots(1)$$

$$A(t+1)+Bt=1$$

$$\text{Now put } t+1=0$$

$$t=-1$$

$$A(0)+B(-1)=1$$

$$B=-1$$

$$\text{Now put } t=0$$

$$A(0+1)+B(0)=1$$

$$A=1$$

$$\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$$

$$\int \frac{1}{t(t+1)} dt = \int \frac{1}{t} dt - \int \frac{1}{t+1} dt$$

$$= \log t - \log|t+1| + c$$

$$= \log \left| \frac{t}{t+1} \right| + c$$

$$\int \frac{dx}{x(x^6+1)} = \frac{1}{6} \int \frac{dt}{t(t+1)} = \frac{1}{6} \log \left| \frac{x^6}{x^6+1} \right| + c$$

$$= \log x - \frac{1}{6} \log|x^6+1| + c$$

32. Question

Evaluate:

$$\int \frac{(x^2+x+1)}{(x+2)(x^2+1)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$$

$$\text{Now putting, } \frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{(x+2)} + \frac{Bx+C}{(x^2+1)}$$

$$A(x^2+1)+(Bx+C)(x+2) = x^2+x+1$$

$$Ax^2+A+Bx^2+Cx+2Bx+2C = x^2+x+1$$

$$(A+B)x^2+(C+2B)x+(A+2C) = x^2+x+1$$

$$\text{Equating coefficients } A+B=1\dots\dots(i)$$

$$A+2C=1$$

$$A=1-2C \dots\dots (ii)$$

$$2B+C=1$$

$$2B=1-C$$

$$B = \frac{1-C}{2} \dots\dots (iii)$$

$$(1-2C) + \frac{1-C}{2} = 1$$

$$2-4C+1-C=2$$

$$3-5C=2$$

$$-5C=-1$$

$$C = \frac{1}{5}$$

$$\text{And } 2B = 1 - \frac{1}{5} = \frac{4}{5}$$

$$B = \frac{2}{5}$$

$$A = 1 - 2 \times \frac{1}{5}$$

$$= 1 - \frac{2}{5}$$

$$= \frac{3}{5}$$

$$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \int \frac{A}{(x+2)} dx + \int \frac{Bx+C}{(x^2+1)} dx$$

$$= \frac{3}{5} \times \int \frac{1}{(x+2)} dx + \frac{1}{5} \times \int \frac{2x+1}{(x^2+1)} dx$$

$$= \frac{3}{5} \log|x+2| + \frac{1}{5} I_1 + C_1$$

$$I_1 = \int \frac{2x+1}{(x^2+1)} dx = \int \frac{2x}{(x^2+1)} dx + \int \frac{1}{(x^2+1)} dx$$

$$= \log|x^2+1| + \tan^{-1}x + C_2$$

$$I = \int \frac{x^2+x+1}{(x+2)(x^2+1)} dx = \frac{3}{5} \log|x+2| + \frac{1}{5} \log|x^2+1| + \frac{1}{5} \tan^{-1}x + C$$

50. Question

$$\int \frac{dx}{\sin x (3+2\cos x)}$$

Answer

$$\text{let } I = \int \frac{dx}{\sin x (3+2\cos x)}$$

$$\text{Put } t = \cos x$$

$$dt = -\sin x dx$$

$$\frac{dt}{-\sin x} = dx$$

$$I = \int \frac{dt}{\frac{-\sin x}{\sin x(3+2t)}} = - \int \frac{dt}{\sin^2 x(3+2t)} = - \int \frac{dt}{(1-\cos^2 x)(3+2t)} = - \int \frac{dt}{(1-t^2)(3+2t)}$$

$$\frac{1}{(1-t^2)(3+2t)} = \frac{1}{(1-t)(1+t)(3+2t)}$$

Putting $\frac{1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t} \dots \dots (1)$

$$A(1+t)(3+2t) + B(1-t)(3+2t) + C(1+t)(1-t) = 1$$

Now Putting $1+t=0$

$$t = -1$$

$$A(0) + B(2)(3-2) + C(0) = 1$$

$$B = \frac{1}{2}$$

Now Putting $1-t=0$

$$t = 1$$

$$A(2)(5) + B(0) + C(0) = 1$$

$$A = \frac{1}{10}$$

Now Putting $3+2t=0$

$$t = -\frac{3}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{9}{4}\right) = 1$$

$$C = \frac{-4}{5}$$

$$\frac{1}{(1-t)(1+t)(3+2t)} = \frac{1}{10} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{1+t} - \frac{4}{5} \times \frac{1}{3+2t}$$

$$\int \frac{1}{(1-t)(1+t)(3+2t)} dt = \frac{1}{10} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt - \frac{4}{5} \int \frac{1}{3+2t} dt$$

$$= -\frac{1}{10} \log|1-t| + \frac{1}{2} \log|1+t| - \frac{4}{5} \times \frac{\log|3+2t|}{2} + c$$

$$= -\frac{1}{10} \log|1-\cos x| + \frac{1}{2} \log|1+\cos x| - \frac{2}{5} \log|3+2\cos x| + c$$

33. Question

Evaluate:

$$\int \frac{2x}{(2x+1)^2} dx$$

Answer

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Let $I = \int \frac{2x}{(2x+1)^2} dx$

Now putting, $\frac{2x}{(2x+1)^2} = \frac{A}{(2x+1)} + \frac{B}{(2x+1)^2} \dots \dots \dots (1)$

$A(2x+1)+B = 2x$

Putting $2x+1=0$,

$x = \frac{-1}{2}$

$A(0)+B=-1$

$B=-1$

By equating the coefficient of x,

$2A=2$

$A=1$

From equation (1),we get,

$\frac{2x}{(2x+1)^2} = \frac{1}{(2x+1)} - \frac{1}{(2x+1)^2}$

$\int \frac{2x}{(2x+1)^2} dx = \int \frac{1}{(2x+1)} dx - \int \frac{1}{(2x+1)^2} dx$

$= \frac{\log|2x+1|}{2} + \frac{1}{2(2x+1)} + c$

$= \frac{1}{2} \left[\log|2x+1| + \frac{1}{2x+1} \right] + c$

51. Question

$\int \frac{dx}{\cos x(5-4 \sin x)}$

Answer

let $I = \int \frac{dx}{\cos x(5-4 \sin x)}$

Put $t = \sin x$

$dt = \cos x dx$

$I = \int \frac{dt}{(1-\sin^2 x)(5-4t)} = \int \frac{dt}{(1-t^2)(5-4t)}$

$\frac{1}{(1-t^2)(5-4t)} = \frac{1}{(1-t)(1+t)(5-4t)}$

Putting $\frac{1}{(1-t)(1+t)(5-4t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{5-4t} \dots \dots (1)$

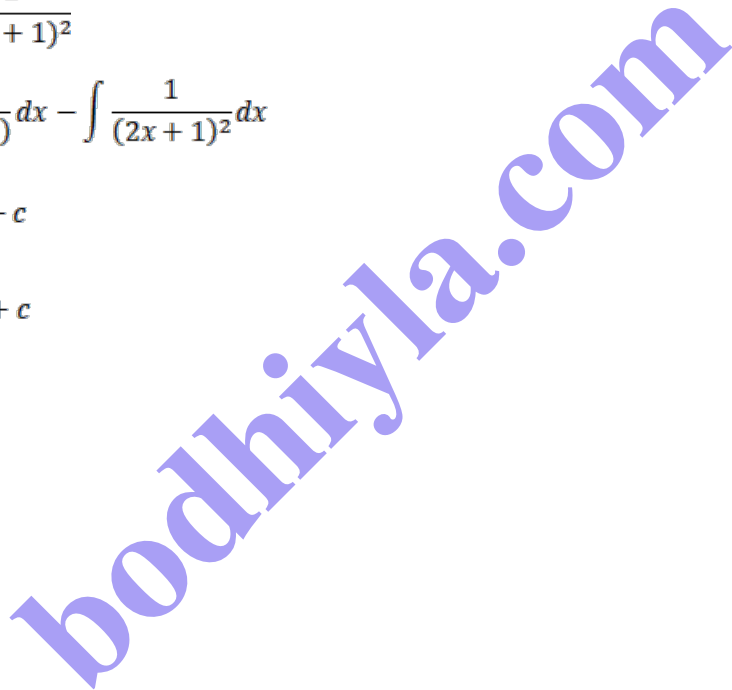
$A(1+t)(5-4t)+B(1-t)(5-4t)+C(1+t)(1-t)=1$

Now Putting $1+t=0$

$t=-1$

$A(0)+B(2)(9)+C(0)=1$

$B = \frac{1}{18}$



Now Putting $1-t=0$

$$t=1$$

$$A(2) + B(0) + C(0) = 1$$

$$A = \frac{1}{2}$$

Now Putting $5-4t=0$

$$t = \frac{5}{4}$$

$$A(0) + B(0) + C\left(1 - \frac{25}{16}\right) = 1$$

$$C = \frac{-16}{9}$$

From equation(1), we get,

$$\frac{1}{(1-t)(1+t)(5-4t)} = \frac{1}{2} \times \frac{1}{1-t} + \frac{1}{18} \times \frac{1}{1+t} - \frac{16}{9} \times \frac{1}{5-4t}$$

$$\int \frac{1}{(1-t)(1+t)(5-4t)} dt = \frac{1}{2} \int \frac{1}{1-t} dt + \frac{1}{18} \int \frac{1}{1+t} dt - \frac{16}{9} \int \frac{1}{5-4t} dt$$

$$= -\frac{1}{2} \log|1-t| + \frac{1}{18} \log|1+t| - \frac{16}{9} \times \frac{\log|5-4t|}{-4} + c$$

$$= -\frac{1}{2} \log|1-\sin x| + \frac{1}{18} \log|1+\sin x| + \frac{4}{9} \log|5-4\sin x| + c$$

34. Question

Evaluate:

$$\int \frac{3x+1}{(x+2)(x-2)^2} dx$$

Answer

$$\text{Let } I = \int \frac{3x+1}{(x+2)(x-2)^2} dx$$

$$\text{Now putting, } \frac{3x+1}{(x+2)(x-2)^2} = \frac{A}{(x+2)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} \dots \dots (1)$$

$$A(x-2)^2 + B(x+2)(x-2) + C(x+2) = 3x+1$$

Putting $x-2=0$,

$$x=2$$

$$A(0) + B(0) + C(2+1) = 3 \times 2 + 1$$

$$C = \frac{7}{4}$$

Putting $x+2=0$,

$$x=-2$$

$$A(-4)^2 + B(0) + C(0) = -6 + 1 = -5$$

$$A = \frac{-5}{16}$$

By equation the coefficient of x^2 , we get, $A+B=0$

$$\frac{-5}{16} + B = 0$$

$$B = \frac{5}{16}$$

$$I = -\frac{5}{16} \log|x+2| + \frac{5}{16} \log|x-2| - \frac{7}{4(x-2)} + c$$

52. Question

$$\int \frac{dx}{\sin x \cos^2 x}$$

Answer

$$\text{Let } I = \int \frac{1}{\sin x \times \cos^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin x \times \cos^2 x} dx = \int \frac{\sin^2 x}{\sin x \times \cos^2 x} dx + \int \frac{\cos^2 x}{\sin x \times \cos^2 x} dx$$

$$= \int \frac{\sin x}{\cos^2 x} dx + \int \frac{1}{\sin x} dx$$

$$= \int (\tan x \sec x + \operatorname{cosec} x) dx$$

$$= \sec x - \frac{1}{2} \log \cot^2 \frac{x}{2} = \sec x - \frac{1}{2} \log \left(\frac{1 + \cos x}{1 - \cos x} \right) + c$$

53. Question

$$\int \frac{\tan x}{(1 - \sin x)} dx$$

Answer

$$\text{let } I = \int \frac{\tan x}{(1 - \sin x)} dx = \int \frac{\sin x}{\cos x(1 - \sin x)} dx$$

Put $t = \sin x$

$$dt = \cos x dx$$

$$I = \int \frac{\sin x \times \cos x}{\cos^2 x (1 - \sin x)} dx = \int \frac{t dt}{(1 - \sin^2 x)(1 - t)} = \int \frac{t dt}{(1 - t^2)(1 - t)}$$

$$\text{Putting } \frac{t}{(1-t)(1+t)(1-t)} = \frac{A}{1+t} + \frac{B}{1-t} + \frac{C}{(1-t)^2} \dots \dots (1)$$

$$A(1+t)^2 + B(1-t)(1+t) + C(1+t) = t$$

Now Putting $1-t=0$

$$t=1$$

$$A(0) + B(0) + C(1+1) = 1$$

$$C = \frac{1}{2}$$

Now Putting $1+t=0$

$$t=-1$$

$$A(2)^2 + B(0) + C(0) = -1$$

$$A = -\frac{1}{4}$$

By equating the coefficient of t^2 , we get, $A-B=0$

$$\frac{-1}{4} - B = 0$$

$$B = -\frac{1}{4}$$

From equation(1),we get,

$$\frac{t}{(1-t)(1+t)(1-t)} = \frac{-1}{4} \times \frac{1}{1+t} - \frac{1}{4} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{(1-t)^2}$$

$$\int \frac{t}{(1-t)(1+t)(1-t)} dt = \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= -\frac{1}{4} \log|1+t| - \frac{1}{4} \log|1-t| - \frac{1}{2} \times \frac{1}{1-t} + c$$

$$= -\frac{1}{4} \log|1+\sin x| - \frac{1}{4} \log|1-\sin x| - \frac{1}{2} \times \frac{1}{1-\sin x} + c$$

35. Question

Evaluate:

$$\int \frac{(5x+8)}{x^2(3x+8)} dx$$

Answer

$$\text{Let } I = \int \frac{5x+8}{x^2(3x+8)} dx$$

$$\text{Now putting, } \frac{5x+8}{x^2(3x+8)} = \frac{A}{(3x+8)} + \frac{Bx+C}{x^2} \dots \dots (1)$$

$$Ax^2+(Bx + C)(3x+8) = 5x+8$$

Putting $3x+8=0$,

$$x = -\frac{8}{3}$$

$$A\left(\frac{64}{9}\right) + B(0) = 5\left(-\frac{8}{3}\right) + 8$$

$$A\left(\frac{64}{9}\right) = \frac{-40 + 24}{3}$$

$$A\left(\frac{64}{9}\right) = \frac{-16}{3}$$

$$A = \frac{-3}{4}$$

By equating the coefficient of x^2 and constant term,

$$A+3B=0$$

$$\frac{-3}{4} + 3B = 0$$

$$3B = \frac{3}{4}$$

$$B = \frac{1}{4}$$

$$8C=8$$

$$C=1$$

From equation (1), we get,

$$\begin{aligned}\int \frac{5x+8}{x^2(3x+8)} dx &= \frac{-3}{4} \times \int \frac{1}{(3x+8)} dx + \frac{1}{4} \times \int \frac{x+1}{x^2} dx \\ &= \frac{-3}{4} \times \frac{\log(3x+8)}{3} + \frac{1}{4} \int \frac{x}{x^2} dx + \int \frac{1}{x^2} dx \\ &= -\frac{1}{4} \log|3x+8| + \frac{1}{4} \log x - \frac{1}{x} + c\end{aligned}$$

Putting $x+2=0$,

$$x=-2$$

$$A(-4)^2+B(0)+C(0)=-6+1=-5$$

$$A = \frac{-5}{16}$$

54. Question

$$\int \frac{dx}{(\sin x + \sin 2x)}$$

Answer

$$\text{let } I = \int \frac{dx}{(\sin x + \sin 2x)} = \int \frac{dx}{(\sin x + 2 \sin x \cos x)}$$

Put $t = \cos x$

$$dt = -\sin x dx$$

$$\frac{-dt}{\sin x} = dx$$

$$I = \int \frac{-dt}{\sin^2 x (1+2t)} = \int \frac{dt}{(1-\cos^2 x)(1+2t)} = \int \frac{dt}{(1-t^2)(1+2t)}$$

$$\text{Putting } \frac{t}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t} \dots \dots (1)$$

$$A(1+t)(1+2t) + B(1-t)(1+2t) + C(1-t^2) = 1$$

Putting $1+t=0$

$$t=-1$$

$$A(0) + B(2)(1-2) + C(0) = 1$$

$$B = -\frac{1}{2}$$

Putting $1-t=0$

$$t=1$$

$$A(2)(3) + B(0) + C(0) = 1$$

$$A = \frac{1}{6}$$

Putting $1+2t=0$

$$t = -\frac{1}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{1}{4}\right) = 1$$

$$C = \frac{4}{3}$$

From equation(1), we get,

$$\frac{1}{(1-t)(1+t)(1+2t)} = \frac{1}{6} \times \frac{1}{1-t} - \frac{1}{2} \times \frac{1}{1+t} + \frac{4}{3} \times \frac{1}{1+2t}$$

$$\int \frac{1}{(1-t)(1+t)(1+2t)} dt = \frac{1}{6} \int \frac{1}{1-t} dt - \frac{1}{2} \int \frac{1}{1+t} dt + \frac{4}{3} \int \frac{1}{1+2t} dt$$

$$= \frac{1}{6} \log|1-t| - \frac{1}{2} \log|1+t| + \frac{2}{3} \log|1+2t| + c$$

$$= \frac{1}{6} \log|1 - \cos x| - \frac{1}{2} \log|1 + \cos x| + \frac{2}{3} \log|1 + 2\cos x| + c$$

36. Question

Evaluate:

$$\int \frac{(5x^2 - 18x + 17)}{(x-1)^2(2x-3)} dx$$

Answer

$$\text{Let } I = \int \frac{5x^2 - 18x + 17}{(x-1)^2(2x-3)} dx$$

$$\text{Now putting, } \frac{5x^2 - 18x + 17}{(x-1)^2(2x-3)} = \frac{A}{(2x-3)} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \dots (1)$$

$$A(x-1)^2 + B(2x-3)(x-1) + C(2x-3) = 5x^2 - 18x + 17$$

Putting $x-1=0$,

$$x=1$$

$$A(0) + B(0) + C(2-3) = 5 - 18 + 17$$

$$C(-1) = 4$$

Putting $2x-3=0$,

$$x = \frac{3}{2}$$

$$A\left(\frac{3}{2} - 1\right)^2 + B(0) + C(0) = 5\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 17$$

$$A\left(\frac{1}{4}\right) + 0 = 5 \times \frac{9}{4} - 27 + 17$$

$$A\left(\frac{1}{4}\right) = \frac{45}{4} - 10 = \frac{5}{4}$$

$$A=5$$

By equating the coefficient of x^2 , we get ,

$$A+2B=5$$

$$5+2B=5$$

$$2B=0$$

$$B=0$$

From equation (1), we get,

$$\frac{5x^2+18x+17}{(x-1)^2(2x-3)} = 5 \times \frac{1}{(2x-3)} + 0 - 4 \times \frac{1}{(x-1)^2}$$

$$\int \frac{5x^2+18x+17}{(x-1)^2(2x-3)} dx = \frac{5}{2} \log(2x-3) + \frac{4}{x-1} + c$$

37. Question

Evaluate:

$$\int \frac{8}{(x+2)(x^2+4)} dx$$

Answer

$$\text{Let } I = \int \frac{8}{(x+2)(x^2+4)} dx$$

$$\text{Now putting, } \frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \dots \dots (1)$$

$$A(x^2+4) + (Bx+C)(x+2) = 8$$

Putting $x+2=0$,

$$x = -2$$

$$A(4+4) + 0 = 8$$

$$A = 1$$

By equating the coefficient of x^2 and constant term, $A+B=0$

$$1+B=0$$

$$B = -1$$

$$4A+2C=8$$

$$4 \times 1 + 2C = 8$$

$$2C = 4$$

$$C = 2$$

From equation (1), we get,

$$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{x^2+4}$$

$$\int \frac{8}{(x+2)(x^2+4)} dx = \int \frac{1}{x+2} dx - \int \frac{x}{x^2+4} dx + 2 \int \frac{1}{x^2+4} dx$$

$$= \log|x+2| - \frac{1}{2} \log(x^2+4) + 2 \times \frac{1}{2} \times \tan^{-1} \frac{x}{2} + c$$

$$= \log|x+2| - \frac{1}{2} \log|x^2+4| + \tan^{-1} \frac{x}{2} + c$$

55. Question

$$\int \frac{x^2}{(x^4 - x^2 - 12)} dx$$

Answer

$$\text{Let } I = \int \frac{x^2}{(x^4 - x^2 - 12)} dx$$

$$\text{Putting } \frac{x^2}{(x^4 - x^2 - 12)} = \frac{t}{t^2 - t - 12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3} \dots \dots \dots (1)$$

Where $t = x^2$

$$A(t+3) + B(t-4) = t$$

Now put $t+3=0$

$$t = -3$$

$$A(0) + B(-7) = -3$$

$$B = \frac{3}{7}$$

Now put $t-4=0$

$$t = 4$$

$$A(4+3) + B(0) = 4$$

$$A = \frac{4}{7}$$

From equation(1)

$$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$$

$$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7} \times \frac{1}{x^2-2^2} + \frac{3}{7} \times \frac{1}{x^2+(\sqrt{3})^2}$$

$$\int \frac{x^2}{(x^2-4)(x^2+3)} dx = \frac{4}{7} \int \frac{1}{x^2-2^2} dx + \frac{3}{7} \int \frac{1}{x^2+(\sqrt{3})^2} dx$$

$$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} \log \left| \frac{x-2}{x+2} \right| + \frac{3}{7} \times \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + c$$

$$= \frac{1}{7} \log \left| \frac{x-2}{x+2} \right| + \frac{\sqrt{3}}{7} \tan^{-1} \frac{x}{\sqrt{3}} + c$$

56. Question

$$\int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)} dx$$

Answer

$$\text{Let } I = \int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)} dx$$

$$\text{Putting } \frac{(x^2)^2}{(x^2+1)(x^2+9)(x^2+16)} = \frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16} \dots \dots \dots (1)$$

Where $t = x^2$

$$t^2 = A(t+9)(t+16) + B(t+1)(t+16) + C(t+1)(t+9)$$

Now put $t+1=0$

$$t=-1$$

$$A(8)(15)+B(0)+C(0)=1$$

$$A = \frac{1}{120}$$

Now put $t+9=0$

$$t=-9$$

$$A(-9+9)(-9+16)+B(-9+1)(-9+16)+C(-9+1)(-9+9)=(-9)^2$$

$$A(0)+B(-56)+C(0)=81$$

$$B = -\frac{81}{56}$$

Now put $t+16=0$

$$t=-16$$

$$A(0)+B(0)+C(-15)(-7)=(-16)^2$$

$$A(0)+B(0)+C(105)=256$$

$$C = \frac{256}{105}$$

From equation(1)

$$\frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16}$$

$$\begin{aligned} \int \frac{t^2}{(t+1)(t+9)(t+16)} dt &= \int \left[\frac{\frac{1}{120}}{t+1} - \frac{\frac{81}{56}}{t+9} + \frac{\frac{256}{105}}{t+16} \right] dt \\ &= \frac{1}{120} \int \frac{1}{t+1} dt - \frac{81}{56} \int \frac{1}{t+9} dt + \frac{256}{105} \int \frac{1}{t+16} dt \\ &= \frac{1}{120} \int \frac{1}{x^2+1} dx - \frac{81}{56} \int \frac{1}{x^2+9} dx + \frac{256}{105} \int \frac{1}{x^2+16} dx \\ &= \frac{1}{120} \int \frac{1}{x^2+1} dx - \frac{81}{56} \int \frac{1}{x^2+(3)^2} dx + \frac{256}{105} \int \frac{1}{x^2+(4)^2} dx \\ &= \frac{1}{120} \tan^{-1} x - \frac{81}{56} \times \frac{1}{3} \tan^{-1} \left(\frac{x}{3} \right) + \frac{256}{105} \times \frac{1}{4} \tan^{-1} \left(\frac{x}{4} \right) + c \\ &= \frac{1}{120} \tan^{-1} x - \frac{27}{56} \tan^{-1} \left(\frac{x}{3} \right) + \frac{64}{105} \tan^{-1} \left(\frac{x}{4} \right) + c \end{aligned}$$

38. Question

Evaluate:

$$\int \frac{(3x+5)}{(x^3-x^2+x-1)} dx$$

Answer

$$\text{Let } I = \int \frac{3x+5}{(x^3-x^2+x-1)} dx$$

$$\text{Now putting, } \frac{3x+5}{(x^3-x^2+x-1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)} \dots \dots (1)$$

$$A(x^2+1)+(Bx+C)(x-1)=3x+5$$

Putting $x-1=0$,

$$X=1$$

$$A(2)+B(0)=3+5=8$$

$$A=4$$

By equating the coefficient of x^2 and constant term, $A+B=0$

$$4+B=0$$

$$B=-4$$

$$A-C=5$$

$$4-C=5$$

$$C=-1$$

From equation (1), we get,

$$\frac{3x+5}{(x-1)(x^2+1)} = \frac{4}{x-1} + \frac{-4x-1}{x^2+1}$$

$$\int \frac{3x+5}{(x-1)(x^2+1)} dx = 4 \int \frac{1}{x-1} dx - 4 \int \frac{1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= 4 \log(x-1) - \frac{4}{2} \log(x^2+1) - \tan^{-1}x + c$$

$$= 4 \log(x-1) - 2 \log(x^2+1) - \tan^{-1}x + c$$

57. Question

$$\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$$

Answer

$$\text{let } I = \int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$$

$$\text{Put } t = \cos 2x$$

$$dt = -2 \sin 2x dx$$

$$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2} \int \frac{dt}{(t-2)(1-t)}$$

$$\text{Putting } \frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t} \dots \dots (1)$$

$$A(1-t) + B(t-2) = 1$$

$$\text{Putting } 1-t=0$$

$$t=1$$

$$A(0) + B(1-2) = 1$$

$$B=-1$$

$$\text{Putting } t-2=0$$

$$t=2$$

$$A(1-2) + B(0) = 1$$

$$A=-1$$

From equation (1), we get,

$$\frac{1}{(t-2)(1-t)} = \frac{-1}{t-2} + \frac{-1}{1-t}$$

$$\int \frac{1}{(t-2)(1-t)} dt = \int \frac{1}{2-t} dt + \int \frac{1}{t-1} dt$$

$$= -\log|2-t| + \log|t-1| + c$$

$$= \log|t-1| - \log|2-t| + c$$

$$= \log|\cos 2x - 1| - \log|2 - \cos 2x| + c$$

39. Question

Evaluate:

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx$$

Answer

$$\text{Let } I = \int \frac{2x}{(x^2+1)(x^2+3)} dx$$

$$\text{Put } t=x^2$$

$$dt=2x dx$$

$$\text{Now putting, } \frac{1}{(t+1)(t+3)} = \frac{A}{t+1} + \frac{B}{t+3} \dots \dots (1)$$

$$A(t+3) + B(t+1) = 1$$

$$\text{Putting } t+3=0,$$

$$X=-3$$

$$A(0) + B(-3+1)=1$$

$$B = -\frac{1}{2}$$

$$\text{Putting } t+1=0,$$

$$X=-1$$

$$A(-1+3)+B(0)=1$$

$$A = \frac{1}{2}$$

From equation(1), we get,

$$\frac{1}{(t+1)(t+3)} = \frac{1}{2} \times \frac{1}{t+1} - \frac{1}{2} \times \frac{1}{t+3}$$

$$\int \frac{1}{(t+1)(t+3)} dt = \frac{1}{2} \int \frac{1}{t+1} dt - \frac{1}{2} \int \frac{1}{t+3} dt$$

$$= \frac{1}{2} \log|t+1| - \frac{1}{2} \log|t+3| + c$$

$$\int \frac{2x}{(x^2+1)(x^2+3)} dx = \frac{1}{2} \log|x^2+1| - \frac{1}{2} \log|x^2+3| + c$$

58. Question

$$\int \frac{2}{(1-x)(1+x^2)} dx$$

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Answer

Let $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Put $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1} \dots \dots \dots (1)$

$A(1+x^2)+Bx(1-x)+C(1-x) = 2$

Put $x=1$

$2=2A+0+0$

$A=1$

Put $x=0$

$2=A+C$

$C=2-A$

$C=2-1=1$

Putting $x=2$

We have $2=5A-2B-C$

$2=5 \times 1 - 2B - 1$

$2B=2$

$B=1$

$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x}{1+x^2} + \frac{1}{1+x^2}$

$\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$

$-\log|1-x| + \frac{1}{2} \log|1+x^2| + \tan^{-1}x + c$

40. Question

Evaluate:

$\int \frac{x^2}{(x^2-1)} dx$

Answer

Let $I = \int \frac{x^2}{(x^2-1)} dx$

Put $t=x^2$

$dt=2x dx$

Now putting, $\frac{x^2}{(x^2-1)} = \frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} \dots \dots \dots (1)$

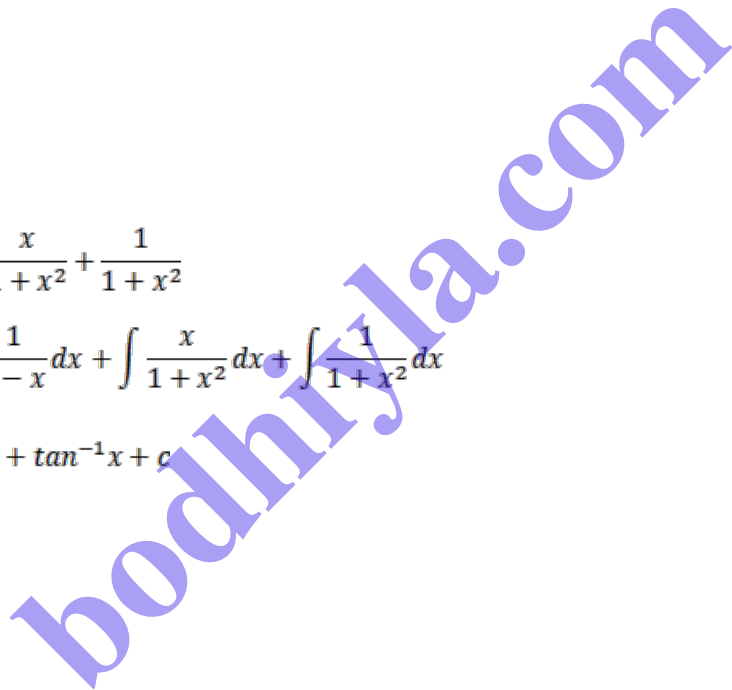
$A(t+1)+B(t-1) = t$

Putting $t+1=0$,

$t=-1$

$A(0)+B(-1-1)=-1$

$B = \frac{1}{2}$



Putting $t-1=0$,

$$t=1$$

$$A(1+1)+B(0)=1$$

$$A = \frac{1}{2}$$

From equation(1),we get,

$$\frac{t}{(t-1)(t+1)} = \frac{1}{2} \times \frac{1}{t-1} + \frac{1}{2} \times \frac{1}{t+1}$$

$$\int \frac{x^2}{(x^4-1)} dt = \frac{1}{2} \int \frac{1}{x^2-1} dt + \frac{1}{2} \int \frac{1}{x^2+1} dt$$

$$= \frac{1}{2} \times \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \tan^{-1}x + c$$

$$= \frac{1}{4} \log \left| \frac{x-1}{x+1} \right| + \frac{1}{2} \tan^{-1}x + c$$

59. Question

$$\int \frac{2x^2+1}{x^2(x^2+4)} dx$$

Answer

$$\text{Let } I = \int \frac{2x^2+1}{x^2(x^2+4)} dx$$

Again let $x^2=t$

$$\frac{2t+1}{t(t+4)} = \frac{A}{t} + \frac{B}{(t+4)} \dots\dots(1)$$

$$2t+1=A(t+4)+B(t)$$

Putting $t=-4$

$$2(-4)+1=A(-4+4)+B(-4)$$

$$-8+1=0-4B$$

$$-7=-4B$$

$$B = \frac{7}{4}$$

Putting $t=0$

$$2(0)+1=A(0+4)+B(0)$$

$$1=4A$$

$$A = \frac{1}{4}$$

$$\frac{2t+1}{t(t+4)} = \frac{1}{4} + \frac{7}{4} \frac{1}{(t+4)}$$

$$\int \frac{2t+1}{t(t+4)} dt = \int \frac{2x^2+1}{x^2(x^2+4)} dx = \frac{1}{4} \int \frac{1}{x^2} dx + \frac{7}{4} \int \frac{1}{(x^2+2^2)} dx$$

$$= \frac{1}{4} \times \frac{(-1)}{x} + \frac{7}{4} \times \frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) + c$$

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$$I = \frac{-1}{4x} + \frac{7}{8} \tan^{-1}\left(\frac{x}{2}\right) + c$$

Exercise 15B

1. Question

Evaluate:

$$\int x^{-6} dx$$

Answer

$$\int x^{-6} dx = \frac{x^{-6+1}}{-6+1} + c$$

$$\because \left\{ \int x^n = \frac{x^{n+1}}{n+1} + c \right\}$$

$$= \frac{x^{-5}}{-5} + c$$

$$\int x^{-6} dx = -\frac{1}{5x^5} + c$$

2. Question

Evaluate:

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

Answer

$$\int (\sqrt{x} + 1/\sqrt{x}) dx = \int (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) dx$$

$$\left\{ \int x^n = \frac{x^{n+1}}{n+1} + c \right\}$$

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} dx$$

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int \frac{2}{3} x^{\frac{3}{2}} + 2\sqrt{x} + c$$

3. Question

Evaluate:

$$\int \sin 3x dx$$

Answer

$$\int \sin 3x dx = \frac{-1}{3} \cos 3x + c$$

$$\left\{ \int \sin ax dx = \frac{-1}{a} \cos ax \right\}$$

4. Question

Evaluate:

$$\int \frac{x^2}{(1+x^3)} dx$$

Answer

$$\text{Let } x^3 + 1 = t$$

$$3x^2 dx = dt$$

$$\frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln t + c$$

$$\int \frac{x^2}{1+x^3} dx = \frac{1}{3} \ln(x^3 + 1) + c$$

5. Question

Evaluate:

$$\int \frac{2 \cos x}{3 \sin^2 x} dx$$

Answer

$$\text{Let } \sin x = t$$

$$\cos x dx = dt$$

$$\int \frac{2 \cos x}{3 \sin^2 x} dx = \int \frac{2}{3} dt = -\frac{2}{3} + c$$

$$\int \frac{2 \cos x}{3 \sin^2 x} dx = -\frac{2}{3} \csc x + c$$

6. Question

Evaluate:

$$\int \frac{(3 \sin \phi - 2) \cos \phi}{(5 - \cos^2 \phi - 4 \sin \phi)} d\phi$$

Answer

$$\frac{(3 \sin \phi - 2) \cos \phi}{(4 + 1 - \cos^2 \phi - 4 \sin \phi)} = \frac{3(\sin \phi - 2) \cos \phi + 4 \cos \phi}{(\sin \phi - 2)^2}$$

$$= \frac{3 \cos \phi}{(\sin \phi - 2)} + \frac{4 \cos \phi}{(\sin \phi - 2)^2}$$

$$\int \left(\frac{3 \cos \phi}{(\sin \phi - 2)} + \frac{4 \cos \phi}{(\sin \phi - 2)^2} \right) d\phi$$

$$\text{Let } (\sin \phi - 2) = t$$

$$\cos \phi d\phi = dt$$

$$\int \frac{3dt}{t} + \frac{4dt}{t^2} = 3 \ln t - \frac{4}{t} + c$$

$$\int \frac{(3 \sin \phi - 2) \cos \phi}{(5 - \cos^2 \phi - 4 \sin \phi)} d\phi = 3 \ln |\sin \phi - 2| - \frac{4}{(\sin \phi - 2)} + c$$

7. Question

Evaluate:

$$\int \sin^2 x \, dx$$

Answer

$$\int \sin^2 x \, dx = \int \frac{1}{2} - \frac{\cos 2x}{2} \, dx$$

$$\{1 - \cos 2x = 2 \sin^2 x\}$$

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\left\{ \int \cos ax \, dx = \frac{1}{a} \sin ax \right\}$$

8. Question

Evaluate:

$$\int \frac{(\log x)^2}{x} \, dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(\log x)^2}{x} dx = \frac{(\log x)^3}{3} + c$$

9. Question

Evaluate:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

$$\int \frac{(x+1)(x+\log x)^2}{x} = \int \left(1 + \frac{1}{x}\right) (x + \log x)^2 dx$$

Let $x + \log x = t$

$$\left(1 + \frac{1}{x}\right) dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(x+1)(x+\log x)^2}{x} = \frac{(x+\log x)^3}{3} + c$$

10. Question

Evaluate:

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$$\int \frac{\sin x}{(1 + \cos x)} dx$$

Answer

$$\text{Let } 1 + \cos x = t$$

$$-\sin x dx = dt$$

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{\sin x}{(1 + \cos x)} dx = -\ln|1 + \cos x| + c$$

11. Question

Evaluate:

$$\int \frac{(1 + \tan x)}{(1 - \tan x)} dx$$

Answer

$$\frac{1 + \tan x}{1 - \tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

$$\text{Let } \cos x - \sin x = t$$

$$-(\sin x + \cos x) dx = dt$$

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{1 + \tan x}{1 - \tan x} dx = -\ln|\cos x - \sin x| + c$$

12. Question

Evaluate:

$$\int \frac{(1 - \cot x)}{(1 + \cot x)} dx$$

Answer

$$\frac{1 - \cot x}{1 + \cot x} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

$$\text{Let } \sin x + \cos x = t$$

$$(\cos x - \sin x) dx = dt$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{-dt}{t} = -\ln|\sin x + \cos x| + c$$

$$\int \frac{1 - \cot x}{1 + \cot x} dx = -\ln|\sin x + \cos x| + c$$

13. Question

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Evaluate:

$$\int \frac{(1 + \cot x)}{(x + \log \sin x)} dx$$

Answer

Let $(x + \log(\sin x)) = t$

$$(1 + \cot x) dx = dt$$

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{(1 + \cot x)}{(x + \log \sin x)} = \ln|x + \log(\sin x)| + c$$

14. Question

Evaluate:

$$\int \frac{(1 - \sin 2x)}{(x + \cos^2 x)} dx$$

Answer

Let $(x + \cos^2 x) = t$

$$(1 - \sin 2x) dx = dt$$

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{1 - \sin 2x}{x + \cos^2 x} = \ln|x + \cos^2 x| + c$$

15. Question

Evaluate:

$$\int \frac{\sec^2(\log x)}{x} dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int \sec^2 t dt = \tan t + c$$

$$\int \frac{\sec^2(\log x)}{x} dx = \tan(\log x) + c$$

16. Question

Evaluate:

$$\int \frac{\sin(2 \tan^{-1} x)}{(1 + x^2)} dx$$

Answer

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Let $\tan^{-1} x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$\int \sin 2t = -\frac{\cos 2t}{2} + c$$

$$\int \frac{\sin(2 \tan^{-1} x)}{(1+x^2)} dx = \frac{-1}{2} \cos(2 \tan^{-1} x) + c$$

17. Question

Evaluate:

$$\int \frac{\tan x \sec^2 x}{(1 - \tan^2 x)} dx$$

Answer

Let $1 - \tan^2 x = t$

$$-2 \tan x \cdot \sec^2 x dx = dt$$

$$\frac{-1}{2} \int \frac{dt}{t} = \frac{-1}{2} \log t + c$$

$$\int \frac{\tan x \sec^2 x}{1 - \tan^2 x} dx = \frac{-1}{2} \log |1 - \tan^2 x| + c$$

18. Question

Evaluate:

$$\int \frac{(x^4 + 1)}{(x^2 + 1)} dx$$

Answer

$$\frac{x^4 + 1}{x^2 + 1} = \frac{x^4 - 1 + 2}{x^2 + 1}$$

$$= x^2 - 1 + \frac{2}{x^2 + 1}$$

$$\int \left(x^2 - 1 + \frac{2}{x^2 + 1} \right) dx = \frac{x^3}{3} - x + 2 \tan^{-1} x + c$$

19. Question

Evaluate:

$$\int \tan^{-1} \sqrt{\frac{1 - \sin x}{1 + \sin x}} dx$$

Answer

$$\sin x = \cos \left(\frac{\pi}{2} - x \right)$$

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$$\tan^{-1} \sqrt{\frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)}} = \tan^{-1} \sqrt{\frac{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}}$$

$$= \tan^{-1} \left(\tan\left(\frac{\pi}{4} - \frac{x}{2}\right) \right)$$

$$\int \left(\frac{\pi}{4} - \frac{x}{2}\right) dx = \frac{\pi}{4}x - \frac{x^2}{4} + c$$

20. Question

Evaluate:

$$\int \log(1+x^2) dx$$

Answer

Using Integration by Parts

$$\int u_1 v_1 dx = u \int v dx - \int u' \int v dx dx + c$$

Here 1 is the first function and $\log(x^2 + 1)$ is second function

$$\int \log(1+x^2) dx = (\log(1+x^2))x - \int \frac{2x}{1+x^2} x dx$$

$$= (\log(1+x^2))x - 2 \int \frac{x^2+1-1}{x^2+1} dx$$

$$= (\log(1+x^2))x - 2x + 2 \int (-1) dx + c$$

21. Question

Evaluate:

$$\int \cos x \cos 3x dx$$

Answer

$$\frac{1}{2} \int 2 \cos x \cos 3x dx$$

$$\{2 \cos A \cos B = \cos(A+B) + \cos(A-B)\}$$

$$\frac{1}{2} \int (\cos 4x + \cos 2x) dx = \frac{\sin 4x}{8} + \frac{\sin 2x}{4} + c$$

22. Question

Evaluate: Evaluate $\int \sin 3x \sin x dx$

Answer

$$\frac{1}{2} \int 2 \sin 3x \sin x dx$$

$$\{2 \sin A \sin B = \cos(A-B) - \cos(A+B)\}$$

$$\frac{1}{2} \int (\cos 2x - \cos 4x) dx = \frac{\sin 2x}{4} - \frac{\sin 4x}{8} + c$$

23. Question

Evaluate:

$$\int \frac{xe^x}{(x+1)^2} dx$$

Answer

$$\frac{e^x(x+1-1)}{(x+1)^2} = e^x \left(\frac{1}{x+1} - \frac{1}{(x+1)^2} \right)$$

$$\int (e^x(f(x) + f'(x))) dx = e^x f(x) + c$$

$$\int \frac{xe^x}{(x+1)^2} dx = \frac{e^x}{x+1} + c$$

24. Question

Evaluate:

$$\int e^x \{ \tan x - \log \cos x \} dx$$

Answer

$$\int (e^x(f(x) + f'(x))) dx = e^x f(x) + c$$

Here $f(x) = -\log \cos x$

$$\int e^x (\tan x - \log \cos x) dx = e^x (\log \cos x) + c$$

25. Question

Evaluate:

$$\int \frac{dx}{(1-\sin x)}$$

Answer

Multiplying Num^r and Den^r with $(1+\sin x)$

$$\int \frac{1 + \sin x}{\cos^2 x} dx = \int \sec^2 x + \sec x \tan x dx$$

$$= \tan x + \sec x + c$$

26. Question

Evaluate:

$$\int x \cos x^2 dx$$

Answer

Let $x^2 = t$

$$2x dx = dt$$

$$\frac{1}{2} \int \cos t dt = \frac{1}{2} \sin t + c$$

$$\int x \cos x^2 dx = \frac{1}{2} \sin x^2 + c$$

27. Question

Evaluate:

$$\int \frac{\cot x}{\sqrt{\sin x}} dx$$

Answer

$$\frac{\cot x}{\sqrt{\sin x}} = \frac{\cos x}{(\sin x)^{3/2}}$$

Let $\sin x = t$

$$\cos x dx = dt$$

$$\int \frac{dt}{t^{3/2}} = \frac{-2}{\sqrt{t}} + c$$

$$\int \frac{\cot x}{\sqrt{\sin x}} dx = \frac{-2}{\sqrt{\sin x}} + c$$

28. Question

Evaluate:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

Answer

$$\frac{\sec^2 x}{\operatorname{cosec}^2 x} = \tan^2 x$$

$$\int \tan^2 x dx = \int (\sec^2 x - 1) dx$$

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx = \tan x - x + c$$

29. Question

Evaluate:

$$\int \sin^{-1}(\cos x) dx$$

Answer

$$\int \sin^{-1}(\cos x) dx = \int \left(\frac{\pi}{2} - \cos^{-1}(\cos x) \right) dx$$

$$\int \left(\frac{\pi}{2} - x \right) dx = \frac{\pi}{2} x - \frac{x^2}{2} + c$$

30. Question

Evaluate:

$$\int \frac{dx}{(\sqrt{x+2} + \sqrt{x+1})}$$

Answer

On rationalizing

$$\int \frac{dx}{(\sqrt{x+2} + \sqrt{x+1})} = \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(\sqrt{x+2} + \sqrt{x+1})(\sqrt{x+2} - \sqrt{x+1})} dx$$

$$= \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(x+2) - (x-1)} dx$$

$$\int \frac{\sqrt{x+2} - \sqrt{x+1}}{1} dx = \frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{3/2} + c$$

31. Question

Evaluate:

$$\int 2^x dx$$

Answer

We know that,

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

32. Question

Evaluate:

$$\int \frac{(1 + \tan x)}{(x + \log \sec x)} dx$$

Answer

Let $(x + \log(\sec x)) = t$

$(1 + \tan x) dx = dt$

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{(1 + \tan x)}{(x + \log \sec x)} = \ln|x + \log(\sec x)| + c$$

33. Question

Evaluate:

$$\int \frac{\sec^2(\log x)}{x} dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int \sec^2 t dt = \tan t + c$$

$$\int \frac{\sec^2(\log x)}{x} dx = \tan(\log x) + c$$

34. Question

Evaluate:

$$\int (2x+1)\left(\sqrt{x^2+x+1}\right) dx$$

Answer

Let $x^2+x+1=t$

$$(2x+1)dx=dt$$

$$\int \sqrt{t}dt = \frac{2}{3}t^{3/2} + c = \frac{2}{3}(x^2+x+1)^{3/2} + c$$

35. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9x^2+16}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2+b^2}} = \frac{1}{a} \log \left| ax + \sqrt{(ax)^2+b^2} \right| + c$$

$$\int \frac{dx}{\sqrt{(3x)^2+4^2}} = \frac{1}{3} \log \left| 3x + \sqrt{9x^2+16} \right| + c$$

36. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4-9x^2}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{b^2-(ax)^2}} = \frac{1}{a} \sin^{-1} \frac{ax}{b} + c$$

$$\int \frac{dx}{\sqrt{2^2-(3x)^2}} = \frac{1}{3} \sin^{-1} \frac{3x}{2} + c$$

37. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4x^2-25}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2-b^2}} = \frac{1}{a} \log \left| ax + \sqrt{(ax)^2-b^2} \right| + c$$

$$\int \frac{dx}{\sqrt{(2x)^2-5^2}} = \frac{1}{2} \log \left| 2x + \sqrt{4x^2-25} \right| + c$$

38. Question

Evaluate:

$$\int \sqrt{4-x^2} dx$$

Answer

We know that,

$$\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$\int \sqrt{2^2-x^2} dx = \frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} + c$$

39. Question

Evaluate:

$$\int \sqrt{9+x^2} dx$$

Answer

We know that,

$$\int \sqrt{a^2+x^2} dx = \frac{x}{2} \sqrt{a^2+x^2} + \frac{a^2}{2} \log|x + \sqrt{a^2+x^2}| + c$$

$$\int \sqrt{3^2+x^2} dx = \frac{x}{2} \sqrt{9+x^2} + \frac{9}{2} \log|x + \sqrt{9+x^2}| + c$$

40. Question

Evaluate:

$$\int \sqrt{x^2-16} dx$$

Answer

We know that,

$$\int \sqrt{x^2-a^2} dx = \frac{x}{2} \sqrt{x^2-a^2} - \frac{a^2}{2} \log|x + \sqrt{x^2-a^2}| + c$$

$$\int \sqrt{x^2-4^2} dx = \frac{x}{2} \sqrt{x^2-16} - 8 \log|x + \sqrt{x^2-16}| + c$$

Objective Questions I

1. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(9+x^2)} = ?$$

A. $\tan^{-1} \frac{x}{3} + C$

B. $\frac{1}{3} \tan^{-1} \frac{x}{3} + C$

C. $3 \tan^{-1} \frac{x}{3} + C$

D. none of these

Answer

$$= \int \frac{dx}{x^2 + 3^2}$$

$$\text{We know, } \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{3} \tan^{-1} \frac{x}{3} + c$$

2. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(4+16x^2)} = ?$$

A. $\frac{1}{32} \tan^{-1} 4x + C$

B. $\frac{1}{16} \tan^{-1} \frac{x}{2} + C$

C. $\frac{1}{8} \tan^{-1} 2x + C$

D. $\frac{1}{4} \tan^{-1} \frac{x}{2} + C$

Answer

$$= \int \frac{dx}{(4x)^2 + 2^2}$$

$$4x=t$$

$$4dx=dt$$

$$dx = \frac{dt}{4}$$

$$= \frac{1}{4} \int \frac{dt}{t^2 + 2^2}$$

$$\text{We know, } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{8} \tan^{-1} \frac{t}{2} + c$$

$$\text{put } t=4x$$

$$= \frac{1}{8} \tan^{-1} \frac{4x}{2} + c$$

$$= \frac{1}{8} \tan^{-1} 2x + c$$

3. Question

Mark (✓) against the correct answer in each of the following:

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$$\int \frac{dx}{(9+4x^2)} dx = ?$$

A. $\frac{1}{2} \tan^{-1} \frac{2x}{3} + C$

B. $\frac{1}{6} \tan^{-1} \frac{2x}{3} + C$

C. $\frac{1}{6} \tan^{-1} \frac{3x}{2} + C$

D. none of these

Answer

$$\int \frac{dx}{(2x)^2 + 3^2}$$

$$2x=t$$

$$2dx=dt$$

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 3^2}$$

$$\text{We know, } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{6} \tan^{-1} \frac{t}{3} + c$$

$$\text{put } t=2x$$

$$= \frac{1}{6} \tan^{-1} \frac{2x}{3} + c$$

4. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{(1+\cos^2 x)} dx = ?$$

A. $-\tan^{-1}(\cos x) + C$

B. $\cot^{-1}(\cos x) + C$

C. $-\cot^{-1}(\cos x) + C$

D. $\tan^{-1}(\cos x) + C$

Answer

$$\int \frac{\sin x}{(\cos x)^2 + 1^2} dx$$

$$\cos x=t$$

$$-\sin x \, dx = dt$$

$$= - \int \frac{dt}{t^2 + 1^2}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= - \tan^{-1} \frac{t}{1} + c$$

$$\text{put } t = \cos x$$

$$= -\tan^{-1}(\cos x) + c$$

5. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos x}{(1 + \sin^2 x)} dx = ?$$

A. $-\tan^{-1}(\sin x) + C$

B. $\tan^{-1}(\cos x) + C$

C. $\tan^{-1}(\sin x) + C$

D. $-\tan^{-1}(\cos x) + C$

Answer

$$\int \frac{\cos x}{(\sin x)^2 + 1^2} dx$$

$$\sin x = t$$

$$\cos x \, dx = dt$$

$$= \int \frac{dt}{t^2 + 1^2}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \tan^{-1} \frac{t}{1} + c$$

$$\text{put } t = \sin x$$

$$= \tan^{-1}(\sin x) + c$$

6. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{e^x}{(e^{2x} + 1)} dx = ?$$

A. $\cot^{-1}(e^x) + C$

B. $\tan^{-1}(e^x) + C$

C. $2 \tan^{-1}(e^x) + C$

D. none of these

Answer

$$= \int \frac{e^x}{(e^x)^2 + 1^2} dx$$

$$e^x = t$$

$$e^x dx = dt$$

$$= \int \frac{dt}{t^2 + 1^2}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \tan^{-1} \frac{t}{1} + c$$

$$\text{put } t = e^x$$

$$\tan^{-1} e^x + c$$

7. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{3x^5}{(1+x^{12})} dx = ?$$

A. $\tan^{-1} x^6 + C$

B. $\frac{1}{4} \tan^{-1} x^6 + C$

C. $\frac{1}{2} \tan^{-1} x^6 + C$

D. none of these

Answer

$$= \int \frac{3x^5}{(x^6)^2 + 1^2} dx$$

$$\text{Let } x^6 = t$$

$$6x^5 dx = dt$$

$$3x^5 dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 1^2}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{t}{1} + c$$

$$\text{put } t = x^6$$

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$$= \frac{1}{2} \tan^{-1} \frac{x^6}{1} + c$$

$$= \frac{1}{2} \tan^{-1} x^6 + c$$

8. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{2x^3}{(4+x^8)} dx = ?$$

A. $\frac{1}{2} \tan^{-1} \frac{x^4}{2} + C$

B. $\frac{1}{4} \tan^{-1} \frac{x^4}{2} + C$

C. $\frac{1}{2} \tan^{-1} x^4 + C$

D. none of these

Answer

$$= \int \frac{2x^3}{(x^4)^2 + 2^2} dx$$

Let $x^4 = t$

$$4x^3 dx = dt$$

$$2x^3 dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \frac{1}{4} \tan^{-1} \frac{t}{2} + c$$

put $t = x^4$

$$= \frac{1}{4} \tan^{-1} \frac{x^4}{2} + c$$

9. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(x^2 + 4x + 8)} = ?$$

A. $\frac{1}{2} \tan^{-1} \left(\frac{x+1}{2} \right) + C$

$$B. \frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + C$$

$$C. \frac{1}{2} \tan^{-1} (x+2) + C$$

$$D. \tan^{-1} \left(\frac{x+2}{2} \right) + C$$

Answer

$$= \int \frac{dx}{x^2 + 4x + 8}$$

Completing the square

$$x^2 + 4x + 8 = x^2 + 4x + 8 + 4 - 4$$

$$= x^2 + 4x + 4 + 4$$

$$= (x+2)^2 + 2^2$$

$$= \int \frac{dx}{(x+2)^2 + 2^2}$$

Let $x+2=t$

$$dx=dt$$

$$= \int \frac{dt}{t^2 + 2^2}$$

$$\text{We know, } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

put $t=x+2$

$$= \frac{1}{2} \tan^{-1} \frac{x+2}{2} + c$$

10. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(2x^2 + x + 3)} = ?$$

$$A. \frac{1}{\sqrt{23}} \tan^{-1} \left(\frac{4x+1}{\sqrt{23}} \right) + C$$

$$B. \frac{1}{\sqrt{23}} \tan^{-1} \left(\frac{x+1}{\sqrt{23}} \right) + C$$

$$C. \frac{2}{\sqrt{23}} \tan^{-1} \left(\frac{4x+1}{\sqrt{23}} \right) + C$$

D. none of these

Answer

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$$= \int \frac{dx}{2x^2 + x + 3}$$

Completing the square

$$\Rightarrow 2x^2 + x + 3 = 2x^2 + \frac{1}{2}x + \frac{3}{2}$$

$$= 2\left(x^2 + \frac{1}{2}x + \frac{3}{2} + \frac{1}{16} - \frac{1}{16}\right)$$

$$= 2\left(\left(x + \frac{1}{4}\right)^2 + \frac{23}{16}\right)$$

$$= \frac{1}{2} \int \frac{dx}{\left(x + \frac{1}{4}\right)^2 + \frac{23}{16}}$$

$$\text{Let } x + \frac{1}{4} = t$$

$$dx = dt$$

$$= \int \frac{dt}{t^2 + \frac{\sqrt{23}^2}{4}}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{4}{2\sqrt{23}} \tan^{-1} \frac{t}{\frac{\sqrt{23}}{4}} + c$$

$$\text{put } t = x + \frac{1}{4}$$

$$= \frac{2}{\sqrt{23}} \tan^{-1} \frac{x + \frac{1}{4}}{\frac{\sqrt{23}}{4}} + c$$

$$= \frac{2}{\sqrt{23}} \tan^{-1} \frac{4x + 1}{\sqrt{23}} + c$$

11. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(e^x + e^{-x})} = ?$$

A. $\tan^{-1}(e^x) + C$

B. $\tan^{-1}(e^{-x}) + C$

C. $-\tan^{-1}(e^{-x}) + C$

D. none of these

Answer

$$= \int \frac{1}{e^x + e^{-x}} dx$$

$$= \int \frac{e^x}{(e^x)^2 + 1^2} dx$$

$$e^x = t \quad e^x$$

$$e^x dx = dt$$

$$= \int \frac{dt}{t^2 + 1^2}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \tan^{-1} \frac{t}{1} + c$$

$$\text{put } t = e^x$$

$$= \tan^{-1} e^x + c$$

12. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2}{(9 + 4x^2)} = ?$$

A. $\frac{x}{4} - \frac{1}{8} \tan^{-1} \frac{x}{3} + C$

B. $\frac{x}{4} - \frac{3}{8} \tan^{-1} \frac{x}{3} + C$

C. $\frac{x}{4} - \frac{3}{8} \tan^{-1} \frac{2x}{3} + C$

D. none of these

Answer

$$\int \frac{x^2}{4x^2 + 9} = \frac{1}{4} \int \frac{4x^2 + 9 - 9}{4x^2 + 9} dx$$

$$= \frac{1}{4} \int 1 + \frac{1}{4} \int \frac{-9}{4x^2 + 9} dx$$

$$= \frac{x}{4} - \frac{9}{4} \int \frac{1}{(2x)^2 + 3^2} dx$$

$$\text{Let } 2x = t$$

$$2 dx = dt$$

$$= \frac{x}{4} - \frac{9}{8} \int \frac{1}{(t)^2 + 3^2} dt$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{x}{4} - \frac{9}{8 \cdot 3} \tan^{-1} \frac{t}{3} + c$$

$$\text{put } t = 2x$$

$$= \frac{x}{4} - \frac{3}{8} \tan^{-1} \frac{2x}{3} + c$$

13. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(x^2 - 1)}{(x^2 + 4)} dx = ?$$

A. $x - 5 \tan^{-1} \frac{x}{2} + C$

B. $x - \frac{5}{2} \tan^{-1} \frac{x}{2} + C$

C. $x - \frac{5}{2} \tan^{-1} \frac{5x}{2} + C$

D. none of these

Answer

$$\begin{aligned} \int \frac{x^2 - 1}{x^2 + 4} &= \int \frac{x^2}{x^2 + 4} - \int \frac{1}{x^2 + 4} \\ &= \int \frac{x^2}{x^2 + 4} - \frac{1}{2} \tan^{-1} \frac{x}{2} \\ &= \int \frac{x^2 + 4 - 4}{x^2 + 4} - \frac{1}{2} \tan^{-1} \frac{x}{2} \\ &= \int \left(1 - \frac{4}{x^2 + 4}\right) - \frac{1}{2} \tan^{-1} \frac{x}{2} \\ &= x - 2 \tan^{-1} \frac{x}{2} - \frac{1}{2} \tan^{-1} \frac{x}{2} + c \\ &= x - \frac{5}{2} \tan^{-1} \frac{x}{2} + c \end{aligned}$$

14. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(4 + 9x^2)} = ?$$

A. $\frac{2}{3} \tan^{-1} \frac{3x}{2} + C$

B. $\frac{1}{6} \tan^{-1} 3x + C$

C. $\frac{1}{6} \tan^{-1} \frac{3x}{2} + C$

D. none of these

Answer

Consider $\int \frac{dx}{(3x)^2 + 2^2}$,

$3x = t$

$$3dx=dt$$

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{dt}{t^2 + 2^2}$$

$$\text{We know, } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{6} \tan^{-1} \frac{t}{2} + c$$

$$\text{put } t=3x$$

$$= \frac{1}{6} \tan^{-1} \frac{3x}{2} + c$$

15. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(4x^2 - 4x + 3)} = ?$$

A. $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{2x-1}{\sqrt{2}} \right) + C$

B. $\frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{2x-1}{\sqrt{2}} \right) + C$

C. $-\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{2x-1}{\sqrt{2}} \right) + C$

D. none of these

Answer

$$\text{Consider } \int \frac{dx}{4x^2 - 4x + 3},$$

Completing the square

$$4x^2 - 4x + 3 = 4\left(x^2 - x + \frac{3}{4}\right)$$

$$= 4\left(x^2 - x + \frac{3}{4} + \frac{1}{4} - \frac{1}{4}\right)$$

$$= 4\left(\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}\right)$$

$$= \frac{1}{4} \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}}$$

$$\text{Let } x - \frac{1}{2} = t$$

$$dx=dt$$

$$= \frac{1}{4} \int \frac{dt}{t^2 + \frac{1}{\sqrt{2}}}$$

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We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \frac{\sqrt{2}}{4} \tan^{-1} \frac{t}{\frac{1}{\sqrt{2}}} + c$$

$$= \frac{1}{2\sqrt{2}} \tan^{-1} \sqrt{2}t + c$$

put $t=x$ - ♦

$$= \frac{1}{2\sqrt{2}} \tan^{-1} \frac{2x-1}{\sqrt{2}} + c$$

16. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(\sin^4 x + \cos^4 x)} = ?$$

A. $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan^2 x - 1}{\sqrt{2} \tan x} \right) + C$

B. $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{\tan^2 x - 1}{\tan x} \right) + C$

C. $\frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{1}{\sqrt{2} \tan x} \right) + C$

D. None of these

Answer

$$\int \frac{dx}{\sin^4 x + \cos^4 x} = \int \frac{1}{\cos^4 x (\tan^4 x + 1)} dx$$

$$= \int \frac{\sec^4 x}{\tan^4 x + 1} dx$$

$$= \int \frac{\sec^2 x \sec^2 x}{\tan^4 x + 1} dx$$

$$= \int \frac{\sec^2 x (1 + \tan^2 x)}{\tan^4 x + 1} dx$$

$\tan x = t$

$\sec^2 x dx = dt$

$$= \int \frac{1 + t^2}{t^4 + 1} dt$$

$$= \int \frac{t^2 + 1}{t^4 + 1} dt$$

$$= \int \frac{1 + t^{-2}}{t^2 + t^{-2}} dt$$

$$= \int \frac{1 + t^{-2}}{t^2 + t^{-2} + 2 - 2} dt$$

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$$= \int \frac{1+t^{-2}}{(t-t^{-1})^2+2} dt$$

Let $t-t^{-1} = u$

$$1+x^{-2} dt=du$$

$$= \int \frac{du}{(u)^2 + \sqrt{2}^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{u}{\sqrt{2}} + c$$

put $u=t-t^{-1}$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t-t^{-1}}{\sqrt{2}} + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t^2-1}{\sqrt{2}t} + c$$

put $t=\tan x$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{\tan^2 x - 1}{\sqrt{2} \tan x} + c$$

17. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = ?$$

A. $\tan^{-1} \frac{(x^2-1)}{\sqrt{3}} + C$

B. $\frac{1}{\sqrt{3}} \tan^{-1} \frac{(x^2-1)}{\sqrt{3}} + C$

C. $\frac{1}{\sqrt{3}} \tan^{-1} \frac{(x^2-1)}{\sqrt{3}x} + C$

D. none of these

Answer

$$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = \int \frac{1+x^{-2}}{x^2+1+x^{-2}} dx$$

$$= \int \frac{1+x^{-2}}{x^2+1+x^{-2}+2-2} dx$$

$$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+3} dx$$

Let $x-x^{-1} = t$

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$$1+x^{-2} dx=dt$$

$$= \int \frac{dt}{(t)^2 + \sqrt{3}^2}$$

$$\text{We know, } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{t}{\sqrt{3}} + c$$

$$\text{put } t=x-x^{-1}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x-x^{-1}}{\sqrt{3}} + c$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{x^2-1}{\sqrt{3}x} + c$$

18. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin 2x}{(\sin^4 x + \cos^4 x)} dx = ?$$

A. $\tan^{-1} (\tan^2 x) + C$

B. $x^2 + C$

C. $-\tan^{-1} (\tan^2 x) + C$

D. none of these

Answer

$$\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx = \int \frac{2 \sin x \cos x}{\cos^4 x (\tan^4 x + 1)} dx$$

$$= \int \frac{2 \tan x \sec^2 x}{(\tan^2 x)^2 + 1} dx$$

$$= \int \frac{2 \tan x \sec^2 x}{(\sec^2 x - 1)^2 + 1} dx$$

$$\text{Let } \sec^2 x - 1 = t$$

$$2 \sec x \sec x \tan x dx = dt$$

$$= \int \frac{dt}{(t)^2 + 1}$$

$$\text{We know, } \int \frac{1}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \tan^{-1} t + c$$

$$\text{put } t = \sec^2 x - 1$$

$$= \tan^{-1} \sec^2 x - 1 + c$$

$$= \tan^{-1} \tan^2 x + c$$

19. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1-9x^2)} = ?$$

A. $\frac{1}{3} \log \left| \frac{1+3x}{1-3x} \right| + C$

B. $\frac{1}{3} \log \left| \frac{1-3x}{1+3x} \right| + C$

C. $\frac{1}{6} \log \left| \frac{1+3x}{1-3x} \right| + C$

D. $\frac{1}{6} \log \left| \frac{1-3x}{1+3x} \right| + C$

Answer

Consider $\int \frac{dx}{(1)^2 - (3x)^2}$

$$3x=t$$

$$3dx=dt$$

$$dx = \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{dt}{1^2 - (t)^2}$$

We know, $\int \frac{1}{a^2-x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{6} \log \frac{1+t}{1-t} + c$$

put $t=3x$

$$\frac{1}{6} \tan^{-1} \frac{1+3x}{1-3x} + c$$

20. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(16-4x^2)} = ?$$

A. $\frac{1}{8} \log \left| \frac{2-x}{2+x} \right| + C$

B. $\frac{1}{16} \log \left| \frac{2-x}{2+x} \right| + C$

C. $\frac{1}{8} \log \left| \frac{2+x}{2-x} \right| + C$

D. $\frac{1}{16} \log \left| \frac{2+x}{2-x} \right| + C$

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Answer

Consider $\int \frac{dx}{(4)^2 - (2x)^2}$

$$2x = t$$

$$2dx = dt$$

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{4^2 - (t)^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{16} \log \frac{4+t}{4-t} + c$$

put $t=2x$

$$= \frac{1}{16} \tan^{-1} \frac{4+2x}{4-2x} + c$$

$$= \frac{1}{16} \tan^{-1} \frac{2+x}{2-x} + c$$

21. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2}{(1-x^6)} dx = ?$$

A. $\frac{1}{6} \log \left| \frac{1+x^3}{1-x^3} \right| + C$

B. $\frac{1}{6} \log \left| \frac{1-x^3}{1+x^3} \right| + C$

C. $\frac{1}{3} \log \left| \frac{1-x^3}{1+x^3} \right| + C$

D. none of these

Answer

$$= \int \frac{x^2}{(1)^2 - (x^3)^2} dx$$

Let $x^3 = t$

$$3x^2 dx = dt$$

$$x^2 dx = \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

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$$= \frac{1}{6} \log \frac{1+t}{1-t} + c$$

put $t=x^3$

$$= \frac{1}{6} \log \frac{1+x^3}{1-x^3} + c$$

22. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x}{(1-x^4)} dx = ?$$

A. $\frac{1}{4} \log \left| \frac{1+x^2}{1-x^2} \right| + C$

B. $\frac{1}{4} \log \left| \frac{1-x^2}{1+x^2} \right| + C$

C. $\frac{1}{2} \log \left| \frac{1+x^2}{1-x^2} \right| + C$

D. none of these

Answer

$$= \int \frac{x}{(1)^2 - (x^2)^2} dx$$

Let $x^2 = t$

$$2x dx = dt$$

$$x dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{1^2 - t^2}$$

$$\text{We know, } \int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$$

$$= \frac{1}{4} \log \frac{1+t}{1-t} + c$$

put $t=x^2$

$$= \frac{1}{4} \log \frac{1+x^2}{1-x^2} + c$$

23. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2}{(a^6 - x^6)} dx = ?$$

$$A. \frac{1}{3a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$$

$$B. \frac{1}{6a^3} \log \left| \frac{a^3 + x^3}{a^3 - x^3} \right| + C$$

$$C. \frac{1}{6a^3} \log \left| \frac{a^3 - x^3}{a^3 + x^3} \right| + C$$

D. none of these

Answer

$$= \int \frac{x^2}{(a^3)^2 - (x^3)^2} dx$$

Let $x^3 = t$

$$3x^2 dx = dt$$

$$x^2 dx = \frac{dt}{3}$$

$$= \frac{1}{3} \int \frac{dt}{(a^3)^2 - t^2}$$

$$\text{We know, } \int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$$

$$= \frac{1}{6a^3} \log \frac{a^3 + t}{a^3 - t} + c$$

put $t = x^3$

$$= \frac{1}{6a^3} \log \frac{a^3 + x^3}{a^3 - x^3} + c$$

24. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(3 - 2x - x^2)} = ?$$

$$A. \frac{1}{4} \log \left| \frac{3+x}{3-x} \right| + C$$

$$B. \frac{1}{4} \log \left| \frac{1+x}{1-x} \right| + C$$

$$C. \frac{1}{4} \log \left| \frac{3+x}{1-x} \right| + C$$

D. none of these

Answer

$$= - \int \frac{dx}{x^2 + 2x - 3}$$

Completing the square

$$x^2 + 2x - 3 = x^2 + 2x - 3 + 1 - 1$$

$$(x+1)^2 - 4$$

$$= - \int \frac{dx}{(x+1)^2 - 4}$$

Let $x+1=t$

$$dx=dt$$

$$= - \int \frac{dt}{t^2 - 2^2}$$

$$= - \int \frac{dt}{2^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{4} \log \frac{2+t}{2-t} + c$$

put $t=x+1$

$$= \frac{1}{4} \log \frac{2+x+1}{2-x-1} + c$$

$$= \frac{1}{4} \log \frac{x+3}{1-x} + c$$

25. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(\cos^2 x - 3 \sin^2 x)} = ?$$

A. $\frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$

B. $\frac{1}{\sqrt{3}} \log \left| \frac{1 - \sqrt{3} \tan x}{1 + \sqrt{3} \tan x} \right| + C$

C. $\frac{1}{2\sqrt{3}} \log \left| \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} \right| + C$

D. none of these

Answer

$$\int \frac{1}{\cos^2 x - 3 \sin^2 x} dx = \int \frac{1}{\cos^2 x (1 - 3 \tan^2 x)} dx$$

$$= \int \frac{\sec^2 x}{(1 - (\sqrt{3} \tan x)^2)} dx$$

Let $\sqrt{3} \tan x = t$

$$\sqrt{3} \sec^2 x dx = dt$$

$$= \frac{1}{\sqrt{3}} \int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{1}{2\sqrt{3}} \log \frac{1+t}{1-t} + c$$

put $t = \sqrt{3} \tan x$

$$= \frac{1}{2\sqrt{3}} \log \frac{1 + \sqrt{3} \tan x}{1 - \sqrt{3} \tan x} + c$$

26. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\operatorname{cosec}^2 x}{(1 - \cot^2 x)} dx = ?$$

A. $\frac{1}{2} \log \left| \frac{1 + \cot x}{1 - \cot x} \right| + C$

B. $-\frac{1}{2} \log \left| \frac{1 + \cot x}{1 - \cot x} \right| + C$

C. $\frac{1}{2} \log \left| \frac{1 - \cot x}{1 + \cot x} \right| + C$

D. none of these

Answer

$$\int \frac{\operatorname{cosec}^2 x}{1 - \cot^2 x} dx$$

Let $\cot x = t$

$$-\operatorname{cosec}^2 x \, dx = dt$$

$$= - \int \frac{dt}{1^2 - t^2}$$

We know, $\int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$

$$= \frac{-1}{2} \log \frac{1+t}{1-t} + c$$

put $t = \cot x$

$$= \frac{-1}{2} \log \frac{1 + \cot x}{1 - \cot x} + c$$

27. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(4x^2 - 1)} = ?$$

A. $\frac{1}{2} \log \left| \frac{2x-1}{2x+1} \right| + C$

B. $\frac{1}{2} \log \left| \frac{2x+1}{2x-1} \right| + C$

C. $\frac{1}{4} \log \left| \frac{2x-1}{2x+1} \right| + C$

D. none of these

Answer

Consider

$$\int \frac{dx}{(2x)^2 - 1^2}$$

$$2x=t$$

$$2dx=dt$$

$$dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 - 1^2}$$

$$\text{We know, } \int \frac{1}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$$

$$= \frac{1}{4} \log \frac{t-1}{t+1} + c$$

put $t=2x$

$$= \frac{1}{4} \log \frac{2x-1}{2x+1} + c$$

28. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x}{(x^4 - 16)} dx = ?$$

A. $\frac{1}{4} \log \left| \frac{x^2 + 4}{x^2 - 4} \right| + C$

B. $\frac{1}{16} \log \left| \frac{x^2 + 4}{x^2 - 4} \right| + C$

C. $\frac{1}{16} \log \left| \frac{x^2 - 4}{x^2 + 4} \right| + C$

D. none of these

Answer

$$= \int \frac{x}{(x^2)^2 - (4)^2} dx$$

Let $x^2 = t$

$2x dx = dt$

$$x dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{1}{(t)^2 - (4)^2} dt$$

We know, $\int \frac{1}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

$$= \frac{1}{16} \log \frac{t-4}{t+4} + c$$

put $t = x^2$

$$= \frac{1}{16} \log \frac{x^2 - 4}{x^2 + 4} + c$$

29. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(\sin^2 x - 4\cos^2 x)} = ?$$

A. $\frac{1}{4} \log \left| \frac{\tan x - 2}{\tan x + 2} \right| + C$

B. $\frac{1}{4} \log \left| \frac{\tan x + 2}{\tan x - 2} \right| + C$

C. $\frac{1}{4} \log \left| \frac{1 - \tan x}{1 + \tan x} \right| + C$

D. none of these

Answer

$$\int \frac{1}{\sin^2 x - 4\cos^2 x} dx = \int \frac{1}{\cos^2 x (\tan^2 x - 4)} dx$$

$$= \int \frac{\sec^2 x}{((\tan x)^2 - 2^2)} dx$$

Let $\tan x = t$

$\sec^2 x dx = dt$

$$= \int \frac{dt}{t^2 - 2^2}$$

We know, $\int \frac{1}{x^2 - a^2} = \frac{1}{2a} \log \frac{x-a}{x+a} + c$

$$= \frac{1}{4} \log \frac{t-2}{t+2} + c$$

put $t = \tan x$

$$= \frac{1}{4} \log \frac{\tan x - 2}{\tan x + 2} + c$$

30. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(4\sin^2 x + 5\cos^2 x)} = ?$$

A. $\frac{1}{2} \tan^{-1} \left(\frac{\tan x}{\sqrt{5}} \right) + C$

B. $\frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{\tan x}{\sqrt{5}} \right) + C$

C. $\frac{1}{2\sqrt{5}} \tan^{-1} \left(\frac{2 \tan x}{\sqrt{5}} \right) + C$

D. none of these

Answer

$$\int \frac{1}{4\sin^2 x + 5\cos^2 x} dx = \int \frac{1}{\cos^2 x(4\tan^2 x + 5)} dx$$

$$\int \frac{\sec^2 x}{((2 \tan x)^2 + \sqrt{5}^2)} dx$$

Let $2 \tan x = t$

$$2 \sec^2 x dx = dt$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + \sqrt{5}^2}$$

$$\text{We know, } \int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \frac{t}{\sqrt{5}} + c$$

put $t = 2 \tan x$

$$= \frac{1}{2\sqrt{5}} \tan^{-1} \frac{2 \tan x}{\sqrt{5}} + c$$

31. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin 3x} dx = ?$$

A. $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \sin x}{\sqrt{3} - \sin x} \right| + C$

B. $\frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \cos x}{\sqrt{3} - \cos x} \right| + C$

$$C. \frac{1}{2\sqrt{3}} \log \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$$

D. none of these

Answer

$$\begin{aligned} \int \frac{\sin x}{\sin 3x} dx &= \int \frac{\sin x}{3 \sin x - 4 \sin^3 x} dx \\ &= \int \frac{1}{3 - 4 \sin^2 x} dx \\ &= \int \frac{1}{\cos^2 x (3 \sec^2 x - 4 \tan^2 x)} dx \\ &= \int \frac{\sec^2 x}{3(1 + \tan^2 x) - 4 \tan^2 x} dx \\ &= \int \frac{\sec^2 x}{3 - \tan^2 x} dx \end{aligned}$$

Let $\tan x = t$

$$\sec^2 x dx = dt$$

$$= \int \frac{dt}{\sqrt{3}^2 - t^2}$$

$$\text{We know, } \int \frac{1}{a^2 - x^2} = \frac{1}{2a} \log \frac{a+x}{a-x} + c$$

$$= \frac{1}{2\sqrt{3}} \log \frac{\sqrt{3} + t}{\sqrt{3} - t} + c$$

put $t = \tan x$

$$= \frac{1}{2\sqrt{3}} \log \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} + c$$

32. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{(x^2 + 1)}{(x^4 + 1)} dx = ?$$

A. $\frac{1}{2} \tan^{-1} \left(\frac{x^2 + 1}{\sqrt{2}x} \right) + C$

B. $\frac{1}{2} \tan^{-1} \left(\frac{x^2 - 1}{\sqrt{2}x} \right) + C$

C. $\frac{1}{\sqrt{2}} \log \left(\frac{x^2 + 1}{x^2 - 1} \right) + C$

D. none of these

Answer

$$\int \frac{(x^2 + 1)}{(x^4 + 1)} dx = \int \frac{1 + x^{-2}}{x^2 + x^{-2}} dx$$

$$= \int \frac{1 + x^{-2}}{x^2 + x^{-2} + 2 - 2} dx$$

$$= \int \frac{1 + x^{-2}}{(x - x^{-1})^2 + 2} dx$$

Let $x - x^{-1} = t$

$$1 + x^{-2} dx = dt$$

$$= \int \frac{dt}{(t)^2 + \sqrt{2}^2}$$

We know, $\int \frac{1}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} + c$$

put $t = x - x^{-1}$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{x - x^{-1}}{\sqrt{2}} + c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \frac{x^2 - 1}{\sqrt{2}x} + c$$

Objective Questions II

1. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4 - 9x^2}} = ?$$

A. $\frac{1}{3} \sin^{-1} \frac{x}{3} + C$

B. $\frac{2}{3} \sin^{-1} \left(\frac{2x}{3} \right) + C$

C. $\frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{4 - 9x^2}} = \int \frac{1}{3} \frac{dx}{\sqrt{\frac{4}{9} - x^2}}$$

$$= \int \frac{1}{3} \frac{dx}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}$$

$$= \frac{1}{3} \sin^{-1} \frac{x}{\frac{2}{3}} + c$$

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$$= \frac{1}{3} \sin^{-1} \frac{3x}{2} + c.$$

2. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16-4x^2}} = ?$$

A. $\frac{1}{2} \sin^{-1} \frac{x}{2} + C$

B. $\frac{1}{4} \sin^{-1} \frac{x}{2} + C$

C. $\frac{1}{2} \sin^{-1} \frac{x}{4} + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{16-4x^2}} = \int \frac{1}{2} \frac{dx}{\sqrt{\frac{16}{4}-x^2}}$$

$$= \int \frac{1}{2} \frac{dx}{\sqrt{(2)^2-x^2}}$$

$$= \frac{1}{2} \sin^{-1} \frac{x}{2} + c$$

3. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos x}{\sqrt{4-\sin^2 x}} = ?$$

A. $\sin^{-1} \frac{x}{2} + C$

B. $\sin^{-1} \left(\frac{1}{2} \cos x \right) + C$

C. $\sin^{-1} (2 \sin x) + C$

D. $\sin^{-1} \left(\frac{1}{2} \sin x \right) + C$

Answer

Put $\sin x = t$

$\Rightarrow \cos x \, dx = dt$

\therefore The given equation becomes

$$\int \frac{dt}{\sqrt{4-t^2}}$$

$$= \sin^{-1} \frac{t}{2} + c$$

But $t = \sin x$

$$= \sin^{-1} \left(\frac{\sin x}{2} \right) + c$$

4. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{2^x}{\sqrt{1-4^x}} dx = ?$$

A. $\sin^{-1} (2^x) \log 2 + C$

B. $\frac{\sin^{-1} (2^x)}{\log 2} + C$

C. $\sin^{-1} (2^x) + C$

D. none of these

Answer

⇒ Let $t=2^x$

$$dt = \log 2 \cdot 2^x \cdot dx$$

$$\Rightarrow \frac{dt}{\log 2} = 2^x \cdot dx$$

$$= \int \frac{dt}{\log 2 \sqrt{1-t^2}}$$

$$= \frac{1}{\log 2} \int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{\log 2} \sin^{-1} t$$

But $t = 2^x$

$$= \frac{1}{\log 2} \sin^{-1} (2^x)$$

5. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x-x^2}} = ?$$

A. $\sin^{-1} (x + 1) + C$

B. $\sin^{-1} (x - 2) + C$

C. $\sin^{-1} (x - 1) + C$

D. none of these

Answer

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$$\begin{aligned} \int \frac{dx}{\sqrt{2x-x^2}} &= \int \frac{dx}{\sqrt{2x-x^2+1-1}} \\ &= \int \frac{dx}{\sqrt{-x^2+2x-1+1}} \\ &= \int \frac{dx}{\sqrt{1-(x-1)^2}} \\ &= \sin^{-1}(x-1)+c \end{aligned}$$

6. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{x(1-2x)} = ?$$

- A. $\frac{1}{\sqrt{2}} \sin^{-1}(2x-1) + C$
 B. $\frac{1}{\sqrt{2}} \sin^{-1}(2x+1) + C$
 C. $\frac{1}{\sqrt{2}} \sin^{-1}(4x+1) + C$
 D. $\frac{1}{\sqrt{2}} \sin^{-1}(4x-1) + C$

Answer

$$\begin{aligned} \int \frac{dx}{\sqrt{x-2x^2}} &= \int \frac{dx}{\sqrt{2}\sqrt{-x^2+\frac{1}{2}x}} \\ &= \int \frac{dx}{\sqrt{2}\sqrt{-(x^2-\frac{1}{2}x)}} \\ &= \int \frac{dx}{\sqrt{2}\sqrt{-(x^2-\frac{1}{2}x)+\frac{1}{16}-\frac{1}{16}}} \\ &= \int \frac{dx}{\sqrt{2}\sqrt{-(x^2-\frac{1}{2}x+\frac{1}{16})+\frac{1}{16}}} \\ &= \int \frac{dx}{\sqrt{2}\sqrt{\frac{1}{16}-(x-\frac{1}{4})^2}} \\ &= \int \frac{dx}{\sqrt{2}\sqrt{(\frac{1}{4})^2-(\frac{4x-1}{4})^2}} \\ &= \frac{1}{\sqrt{2}} \left(\sin^{-1} \left(\frac{\frac{4x-1}{4}}{\frac{1}{4}} \right) \right) \end{aligned}$$

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$$= \frac{1}{\sqrt{2}} \sin^{-1}(4x - 1)$$

7. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{3x^2}{\sqrt{9-16x^6}} dx = ?$$

A. $\frac{1}{4} \sin^{-1}\left(\frac{x^3}{3}\right) + C$

B. $\frac{1}{4} \sin^{-1}\left(\frac{4x^3}{3}\right) + C$

C. $4 \sin^{-1}\left(\frac{x^3}{4}\right) + C$

D. none of these

Answer

$$\Rightarrow \int \frac{3x^2 dx}{\sqrt{9-16x^6}}$$

Let $x^3 = t$

$$\therefore 3x^2 dx = dt$$

$$\therefore x^6 = t^2$$

$$\Rightarrow \int \frac{1}{4} \frac{dt}{\sqrt{\frac{9}{16} - t^2}}$$

$$\Rightarrow \frac{1}{4} \sin^{-1}\left(\frac{4t}{3}\right) + c$$

But $t = x^3$

$$\Rightarrow \frac{1}{4} \sin^{-1}\left(\frac{4x^3}{3}\right) + c$$

8. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2+2x-x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + C$

B. $\sin^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$

C. $\sin^{-1}\sqrt{3}(x-1) + C$

D. none of these

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Answer

$$\Rightarrow \int \frac{dx}{\sqrt{2+2x-x^2}} = \int \frac{dx}{\sqrt{2x-x^2+2+3-3}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{-((x^2-2x+1)-3)}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{3-(x-1)^2}}$$

$$\Rightarrow \sin^{-1}\left(\frac{x-1}{\sqrt{3}}\right) + c.$$

9. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{16-6x-x^2}} = ?$$

A. $\sin^{-1}\left(\frac{x-3}{5}\right) + C$

B. $\sin^{-1}\left(\frac{x+3}{5}\right) + C$

C. $\frac{1}{5} \sin^{-1}(x+3) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{16-6x-x^2}} = \int \frac{dx}{\sqrt{-x^2-6x-9+16+9}}$$

$$= \int \frac{dx}{\sqrt{25-(x+3)^2}}$$

$$= \sin^{-1}\left(\frac{x+3}{5}\right) + c.$$

10. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x-x^2}} = ?$$

A. $\sin^{-1}(x-1) + C$

B. $\sin^{-1}(x+1) + C$

C. $\sin^{-1}(2x-1) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{-x^2+x-}}$$

$$\begin{aligned}
&= \int \frac{dx}{\sqrt{-(x^2 - x) + \frac{1}{4} - \frac{1}{4}}} \\
&= \int \frac{dx}{\sqrt{-(x^2 - x + \frac{1}{4}) + \frac{1}{4}}} \\
&= \int \frac{dx}{\sqrt{(\frac{1}{2})^2 - (x - \frac{1}{2})^2}} \\
&= \sin^{-1} \left(\frac{2x - 1}{\frac{1}{2}} \right) + c
\end{aligned}$$

$$= \sin^{-1}(2x-1) + c$$

11. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}} = ?$$

A. $\frac{1}{\sqrt{3}} \sin^{-1} \left(\frac{3x - 1}{2} \right) + C$

B. $\frac{1}{\sqrt{2}} \sin^{-1} \left(\frac{2x - 1}{3} \right) + C$

C. $\frac{1}{\sqrt{3}} \sin^{-1} \left(\frac{2x - 1}{3} \right) + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{1 + 2x - 3x^2}} = \int \frac{dx}{\sqrt{3} \sqrt{-x^2 + \frac{2}{3}x + \frac{1}{3}}}$$

$$= \int \frac{dx}{\sqrt{3} \sqrt{-(x^2 - \frac{2}{3}x - \frac{1}{3})}}$$

$$= \int \frac{dx}{\sqrt{3} \sqrt{-(x^2 - \frac{2}{3}x - \frac{1}{3}) + \frac{1}{9} - \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3} \sqrt{-(x^2 - \frac{2}{3}x + \frac{1}{9}) + \frac{1}{3} + \frac{1}{9}}}$$

$$= \int \frac{dx}{\sqrt{3} \sqrt{\frac{4}{9} - (x - \frac{1}{3})^2}}$$

$$= \int \frac{dx}{\sqrt{3}\sqrt{\left(\frac{2}{3}\right)^2 - \left(\frac{3x-1}{3}\right)^2}}$$

$$= \frac{1}{\sqrt{3}} \left(\sin^{-1} \left(\frac{\frac{3x-1}{3}}{\frac{2}{3}} \right) \right)$$

$$= \frac{1}{\sqrt{3}} \sin^{-1} \left(\frac{3x-1}{2} \right)$$

12. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 16}} = ?$$

A. $\sin^{-1} \left(\frac{x}{4} \right) + C$

B. $\log \left| x + \sqrt{x^2 - 16} \right| + C$

C. $\log \left| x - \sqrt{x^2 - 16} \right| + C$

D. none of these

Answer

We know

$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right|$$

$$\int \frac{dx}{\sqrt{x^2 - 4^2}} = \log \left| x + \sqrt{x^2 - 16} \right|$$

13. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{4x^2 - 9}} = ?$$

A. $\frac{1}{2} \log \left| 2x + \sqrt{4x^2 - 9} \right| + C$

B. $\frac{1}{4} \log \left| x + \sqrt{4x^2 - 9} \right| + C$

C. $\log \left| 2x + \sqrt{4x^2 - 9} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{(2x)^2 - (3)^2}}$$

Put $t = 2x$

$dt = 2 dx$

$$\Rightarrow dx = \frac{dt}{2}$$

$$= \frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 9}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}|$$

$$= \frac{1}{2} \log |t + \sqrt{t^2 - 9}|$$

But $t = 2x$

$$= \frac{1}{2} \log |2x + \sqrt{4x^2 - 9}|$$

14. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2}{x^6 - 1} dx = ?$$

A. $\frac{1}{2} \log |x^3 + \sqrt{x^6 - 1}| + C$

B. $\frac{1}{3} \log |x^3 + \sqrt{x^6 - 1}| + C$

C. $\frac{1}{3} \log |x^3 - \sqrt{x^6 - 1}| + C$

D. none of these

Answer

$$\Rightarrow \int \frac{x^2 dx}{\sqrt{(x^3)^2 - (1)^2}}$$

Put $t = x^3$

$dt = 3x^2 dx$

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$$\Rightarrow \frac{1}{3} \int \frac{1}{x^2 \sqrt{t^2 - 1}} x^2 dt$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}|$$

$$= \frac{1}{3} \log |t + \sqrt{t^2 - 1}|$$

But $t = x^3$

$$= \frac{1}{3} \log |x^3 + \sqrt{x^6 - 1}|$$

15. Question

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Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sqrt{4\cos^2 x - 1}} = ?$$

A. $-\frac{1}{2} \log \left| 2\cos x + \sqrt{4\cos^2 x - 1} \right| + C$

B. $-\frac{1}{3} \log \left| 2\cos x + \sqrt{4\cos^2 x - 1} \right| + C$

C. $-\frac{1}{6} \log \left| 2\cos x + \sqrt{2\cos^2 x - 1} \right| + C$

D. none of these

Answer

$$\Rightarrow \int \frac{\sin x dx}{\sqrt{(2\cos x)^2 - (1)^2}}$$

Put $t = 2\cos x$

$$dt = -2\sin x dx$$

$$\Rightarrow dx = -\frac{dt}{2\sin x}$$

$$= -\frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right|$$

$$= -\frac{1}{2} \log \left| t + \sqrt{t^2 - 1} \right|$$

But $t = 2\cos x$

$$\Rightarrow -\frac{1}{2} \log \left| 2\cos x + \sqrt{4\cos^2 x - 1} \right|$$

16. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 4}} dx = ?$$

A. $\log \left| \tan x - \sqrt{\tan^2 x - 4} \right| + C$

B. $\log \left| \tan x + \sqrt{\tan^2 x - 4} \right| + C$

C. $\frac{1}{2} \log \left| \tan x + \sqrt{\tan^2 x - 4} \right| + C$

D. none of these

Answer

$$\int \frac{\sec^2 x \, dx}{\sqrt{(\tan x)^2 - (1)^2}}$$

Put $t = \tan x$

$$dt = \sec^2 x$$

$$\Rightarrow dx = -\frac{dt}{\sec^2 x}$$

$$= \int \frac{1}{\sec^2 x} \frac{\sec^2 x \, dt}{\sqrt{t^2 - 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log|x + \sqrt{x^2 - a^2}|$$

$$= \log|t + \sqrt{t^2 - 1}|$$

But $t = \tan x$

$$= \log|\tan x + \sqrt{4 \tan^2 x - 1}|$$

17. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{(1 - e^{2x})} = ?$$

A. $\log|e^x + \sqrt{e^{2x} - 1}| + C$

B. $\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C$

C. $-\log|e^{-x} + \sqrt{e^{-2x} - 1}| + C$

D. none of these

Answer

Differentiating both side with respect to t

$$-2e^{2x} \frac{dx}{dt} = 1 \Rightarrow dx = -\frac{1}{2} \frac{dt}{1-t}$$

$$y = -\frac{1}{2} \int \frac{1}{(1-t)t} dt$$

$$y = -\frac{1}{2} \int \frac{t + (1-t)}{(1-t)t} dt$$

$$y = -\frac{1}{2} \int \frac{1}{(1-t)} + \frac{1}{t} dt$$

$$y = -\frac{1}{2} (-\log(1-t) + \log t) + c$$

Again put, $t = 1 - e^{2x}$

$$y = -\frac{1}{2} (-\log e^{2x} + \log(1 - e^{2x})) + c$$

$$y = -\log \sqrt{\frac{1 - e^{2x}}{e^{2x}}} + c$$

$$y = -\log \sqrt{e^{-2x} - 1} + c$$

18. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = ?$$

A. $\log \left| \left(x - \frac{3}{2} \right) + \sqrt{x^2 - 3x + 2} \right| + C$

B. $\log \left| x + \sqrt{x^2 - 3x + 2} \right| + C$

C. $\log \left| x - \sqrt{x^2 - 3x + 2} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2 - 3x + 2}} = \int \frac{dx}{\sqrt{x^2 - 3x + 2 + \frac{9}{4} - \frac{9}{4}}}$$

$$= \int \frac{dx}{\sqrt{x^2 - 3x + \frac{9}{4} - \frac{1}{4}}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right|$$

$$= \log \left| \left(x - \frac{3}{2}\right) + \sqrt{x^2 - 3x + 2} \right|$$

19. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx = ?$$

A. $\log \left| \sin x + \sqrt{\sin^2 x - 2 \sin x - 3} \right| + C$

B. $\log \left| (\sin x - 1) + \sqrt{\sin^2 x - 2 \sin x - 3} \right| + C$

C. $\log \left| (\sin x - 1) - \sqrt{\sin^2 x - 2 \sin x - 3} \right| + C$

D. none of these

Answer

$$\Rightarrow \int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx$$

Let $t = \sin x$

$$dt = \cos x dx$$

$$\Rightarrow dx = \frac{dt}{\cos x}$$

$$= \frac{\cos x dt}{\cos x \sqrt{t^2 - 2t - 3 + 2 - 2}}$$

$$= \frac{dt}{\sqrt{(t^2 - 2t + 2) - 5}}$$

$$= \frac{dt}{\sqrt{(t-1)^2 - 5}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}|$$

$$\Rightarrow \int \frac{dt}{\sqrt{(t-1)^2 - 5}} = \log |t - 1 + \sqrt{t^2 - 2t - 3}|$$

But $t = \sin x$

$$\therefore \log |\sin x - 1 + \sqrt{\sin^2 x - 2 \sin x - 3}|$$

20. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2 - 4x + x^2}} = ?$$

A. $\log \left| (x-2) + \sqrt{x^2 - 4x + 2} \right| + C$

B. $\log \left| x + \sqrt{x^2 - 4x + 2} \right| + C$

C. $\log \left| x - \sqrt{x^2 - 4x + 2} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2 - 4x + 2}} = \int \frac{dx}{\sqrt{x^2 - 4x + 2 + 4 - 4}}$$

$$= \int \frac{dx}{\sqrt{(x-2)^2 - 2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log |x + \sqrt{x^2 - a^2}|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x-2)^2 - 2}} = \log |x - 2 + \sqrt{x^2 - 4x + 2}|$$

21. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 + 6x + 5}} = ?$$

A. $\log \left| x + \sqrt{x^2 + 6x + 5} \right| + C$

B. $\log \left| x - \sqrt{x^2 + 6x + 5} \right| + C$

C. $\log \left| (x + 3) + \sqrt{x^2 + 6x + 5} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2 + 6x + 5}} = \int \frac{dx}{\sqrt{x^2 + 6x + 5 + 9 - 9}}$$

$$= \int \frac{dx}{\sqrt{(x + 3)^2 - 4}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x + 3)^2 - 4}} = \log \left| x + 3 + \sqrt{x^2 + 6x + 5} \right|$$

22. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{(x - 3)^2 - 1}} = ?$$

A. $\log \left| (x - 3) + \sqrt{x^2 - 6x + 8} \right| + C$

B. $\log \left| x + \sqrt{x^2 - 6x + 8} \right| + C$

C. $\log \left| (x - 3) - \sqrt{x^2 - 6x + 8} \right| + C$

D. none of these

Answer

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x - 3)^2 - 1}} = \log \left| x - 3 + \sqrt{x^2 - 6x + 9 - 1} \right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x - 3)^2 - 1}} = \log \left| x - 3 + \sqrt{x^2 - 6x + 8} \right|$$

23. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}} = ?$$

A. $\log \left| x + \sqrt{x^2 - 6x + 10} \right| + C$

B. $\log \left| (x - 3) + \sqrt{x^2 - 6x + 10} \right| + C$

C. $\log \left| x - \sqrt{x^2 - 6x + 10} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{x^2 - 6x + 10}} = \int \frac{dx}{\sqrt{x^2 - 6x + 10 + 9 - 9}}$$

$$= \int \frac{dx}{\sqrt{(x - 3)^2 + 1}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right|$$

$$\Rightarrow \int \frac{dx}{\sqrt{(x - 3)^2 + 1}} = \log \left| x - 3 + \sqrt{x^2 - 6x + 10} \right|$$

24. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2 dx}{\sqrt{x^6 + a^6}} dx = ?$$

A. $\frac{1}{3} \log \left| x^6 + a^6 \right| + C$

B. $\frac{1}{3} \tan^{-1} \left(\frac{x^3}{a^3} \right) + C$

C. $\frac{1}{3} \log \left| x^3 + \sqrt{x^6 + a^6} \right| + C$

D. none of these

Answer

$$\int \frac{x^2 dx}{\sqrt{(x^3)^2 + (a)^6}}$$

Put $t = x^3$

$dt = 3x^2 dx$

$$\Rightarrow dx = \frac{dt}{3x^2}$$

$$= \frac{1}{3} \int \frac{1}{x^2 \sqrt{t^2 + a^6}} x^2 dt$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}|$$

$$= \frac{1}{3} \log |t + \sqrt{t^2 + a^6}|$$

But $t = x^3$

$$= \frac{1}{3} \log |x^3 + \sqrt{x^6 + a^6}| + c.$$

25. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\sqrt{16 + \tan^2 x}} dx = ?$$

A. $\log |\tan x + \sqrt{\tan^2 x + 16}| + C$

B. $\log |x + \sqrt{\tan^2 x + 16}| + C$

C. $\log |\tan x - \sqrt{\tan^2 x + 16}| + C$

D. none of these

Answer

$$\int \frac{\sec^2 x dx}{\sqrt{(\tan x)^2 + (4)^2}}$$

Put $t = \tan x$

$$dt = \sec^2 x$$

$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

$$= \int \frac{1}{\sec^2 x} \frac{\sec^2 x dt}{\sqrt{t^2 + 16}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}|$$

$$= \log |t + \sqrt{t^2 + 16}|$$

But $t = \tan x$

$$= \log |\tan x + \sqrt{\tan^2 x + 16}|$$

26. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{3x^2 + 6x + 12}} = ?$$

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A. $\log \left| (x+1) + \sqrt{x^2 + 2x + 4} \right| + C$

B. $\frac{1}{3} \log \left| (x+1) + \sqrt{x^2 + 2x + 4} \right| + C$

C. $\frac{1}{\sqrt{3}} \log \left| (x+1) + \sqrt{x^2 + 2x + 4} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{3x^2 + 6x + 12}} = \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 2x + 4}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{x^2 + 2x + 3 + 1}}$$

$$= \int \frac{1}{\sqrt{3}} \frac{dx}{\sqrt{(x+1)^2 + 3}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right|$$

$$\Rightarrow \frac{1}{\sqrt{3}} \int \frac{dx}{\sqrt{(x+1)^2 + 3}} = \log \left| x + 1 + \sqrt{x^2 + 2x + 4} \right|$$

27. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{dx}{\sqrt{2x^2 + 4x + 6}} = ?$$

A. $\frac{1}{2} \log \left| (x+1) + \sqrt{x^2 + 2x + 3} \right| + C$

B. $\frac{1}{\sqrt{2}} \log \left| (x+1) + \sqrt{x^2 + 2x + 3} \right| + C$

C. $\frac{1}{\sqrt{2}} \log \left| x + \sqrt{x^2 + 2x + 3} \right| + C$

D. none of these

Answer

$$\int \frac{dx}{\sqrt{2x^2 + 4x + 6}} = \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

$$= \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{x^2 + 2x + 1 + 2}}$$

$$= \int \frac{1}{\sqrt{2}} \frac{dx}{\sqrt{(x+1)^2 + 2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right|$$

$$\Rightarrow \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x+1)^2 + 2}} = \log |x+1 + \sqrt{x^2 + 2x + 3}|$$

28. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{x^2}{\sqrt{x^6 + 2x^3 + 3}} dx = ?$$

A. $\frac{1}{3} \log |(x^3 + 1) + \sqrt{x^6 + 2x^3 + 3}| + C$

B. $\log |x^3 + \sqrt{x^6 + 2x^3 + 3}| + C$

C. $\frac{1}{3} \log |(x^3 + 1) - \sqrt{x^6 + 2x^3 + 3}| + C$

D. none of these

Answer

$$\int \frac{x^2 dx}{\sqrt{x^6 + 2x^3 + 3}}$$

Let $x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$\Rightarrow \frac{dt}{3x^2} = dx$

$$\int \frac{x^2 dt}{3x^2 \sqrt{t^2 + 2t + 3}} = \frac{1}{3} \int \frac{dt}{\sqrt{t^2 + 2t + 3}}$$

$$= \int \frac{1}{3} \frac{dx}{\sqrt{t^2 + 2t + 1 + 2}}$$

$$= \int \frac{1}{3} \frac{dx}{\sqrt{(t+1)^2 + 2}}$$

$\Rightarrow \int \frac{dx}{\sqrt{x^2 + a^2}} = \log |x + \sqrt{x^2 + a^2}|$

$\Rightarrow \frac{1}{3} \int \frac{dx}{\sqrt{(t+1)^2 + 2}} = \log |t+1 + \sqrt{t^2 + 2t + 3}|$

But $t = x^3$

$= \log |x^3 + 1 + \sqrt{x^6 + 2x^3 + 3}|$

29. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{4-x^2} dx = ?$$

A. $\frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} + C$

B. $x\sqrt{4-x^2} + \sin^{-1}\frac{x}{2} + C$

C. $\frac{1}{2}x\sqrt{4-x^2} - 2\sin^{-1}\frac{x}{2} + C$

D. none of these

Answer

We know

$$\Rightarrow \int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\Rightarrow \int \sqrt{2^2 - x^2} = \frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right) + C$$

$$\Rightarrow \int \sqrt{4 - x^2} = \frac{x}{2}\sqrt{4 - x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$$

30. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{1-9x^2} \, dx = ?$$

A. $\frac{x}{2}\sqrt{1-9x^2} + \frac{1}{18}\sin^{-1}3x + C$

B. $\frac{3x}{2}\sqrt{1-9x^2} + \frac{1}{6}\sin^{-1}3x + C$

C. $\frac{x}{2}\sqrt{1-9x^2} + \frac{1}{6}\sin^{-1}3x + C$

D. none of these

Answer

We know

$$\Rightarrow \int \sqrt{a^2 - x^2} = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\Rightarrow \sqrt{1^2 - (3x)^2} = 3\sqrt{\frac{1}{9} - x^2}$$

$$\Rightarrow 3\sqrt{\frac{1}{9} - x^2} = \frac{3x}{2}\sqrt{\frac{1}{9} - x^2} + \frac{1}{2}\sin^{-1}\left(\frac{x}{\frac{1}{3}}\right) + C$$

$$\Rightarrow \sqrt{1^2 - (3x)^2} = \frac{x}{2}\sqrt{1 - 9x^2} + \frac{3}{18}\sin^{-1}(3x) + C$$

$$\Rightarrow \sqrt{1^2 - (3x)^2} = \frac{x}{2}\sqrt{1 - 9x^2} + \frac{1}{6}\sin^{-1}(3x) + C$$

31. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{9-4x^2} dx = ?$$

A. $\frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$

B. $x\sqrt{9-4x^2} + \frac{9}{2}\sin^{-1}\frac{2x}{3} + C$

C. $\frac{x}{2}\sqrt{9-4x^2} - \frac{9}{4}\sin^{-1}\frac{2x}{3} + C$

D. none of these

Answer

We know

$$\Rightarrow \int \sqrt{a^2-x^2} = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\Rightarrow \sqrt{3^2-(2x)^2} = 2\sqrt{\frac{9}{4}-x^2}$$

$$\Rightarrow 2\sqrt{\frac{9}{4}-x^2} = \frac{x}{2}\sqrt{\frac{9}{4}-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{\frac{3}{2}}\right) + C$$

$$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{2.9}{8}\sin^{-1}(2x) + C$$

$$\Rightarrow \sqrt{9-4x^2} = \frac{x}{2}\sqrt{9-4x^2} + \frac{9}{4}\sin^{-1}(2x) + C$$

32. Question

Mark (✓) against the correct answer in each of the following:

$$\int \cos x \sqrt{9-\sin^2 x} dx = ?$$

A. $\frac{1}{2}\sin x \sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

B. $\frac{\sin x}{2}\sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

C. $\frac{1}{2}\cos x \sqrt{9-\sin^2 x} + \frac{9}{2}\sin^{-1}\left(\frac{\sin x}{3}\right) + C$

D. none of these

Answer

Given: $\int \cos x \sqrt{9-\sin^2 x} dx$

Let $\sin x = t$

$$\cos x dx = dt$$

$$\Rightarrow \frac{dt}{\cos x} = dx$$

$$= \frac{dt}{\cos x} \sqrt{9 - \sin^2 x} \cos x$$

$$= \sqrt{9 - t^2} dt$$

$$\Rightarrow \int \sqrt{a^2 - x^2} = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + C$$

$$\Rightarrow \int \sqrt{3^2 - t^2} = \frac{t}{2} \sqrt{9 - t^2} + \frac{9}{2} \sin^{-1} \left(\frac{t}{3} \right) + C$$

But $t = \sin x$

$$\Rightarrow \int \cos x \sqrt{9 - \sin^2 x} = \frac{\sin x}{2} \sqrt{9 - \sin^2 x} + \frac{9}{2} \sin^{-1} \left(\frac{\sin x}{3} \right) + C$$

33. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{x^2 - 16} dx = ?$$

A. $x\sqrt{x^2 - 16} - 4 \log |x + \sqrt{x^2 - 16}| + C$

B. $\frac{x}{2} \sqrt{x^2 - 16} - 8 \log |x + \sqrt{x^2 - 16}| + C$

C. $\frac{x}{2} \sqrt{x^2 - 16} + 8 \log |x + \sqrt{x^2 - 16}| + C$

D. none of these

Answer

We know

$$\Rightarrow \int \sqrt{x^2 - a^2} = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\Rightarrow \int \sqrt{x^2 - 4^2} = \frac{x}{2} \sqrt{x^2 - 4^2} - \frac{4^2}{2} \log |x + \sqrt{x^2 - 4^2}| + C$$

$$\Rightarrow \int \sqrt{x^2 - 16} = \frac{x}{2} \sqrt{x^2 - 16} - 8 \log |x + \sqrt{x^2 - 16}| + C$$

34. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{x^2 - 4x + 2} dx = ?$$

A. $\frac{1}{2}(x - 2)\sqrt{x^2 - 4x + 2} + \log |(x - 2) + \sqrt{x^2 - 4x + 2}| + C$

B. $(x - 2)\sqrt{x^2 - 4x + 2} + \frac{1}{2} \log |(x - 2) + \sqrt{x^2 - 4x + 2}| + C$

C. $\frac{1}{2}(x - 2)\sqrt{x^2 - 4x + 2} - \log |(x - 2) + \sqrt{x^2 - 4x + 2}| + C$

D. none of these

Answer

$$\sqrt{x^2 - 4x + 2} dx$$

It can be written as

$$\Rightarrow \sqrt{x^2 - 4x + 2 + 2 - 2} = \sqrt{x^2 - 4x + 4 - 2}$$

$$= \sqrt{(x-2)^2 - 2}$$

We know

$$\Rightarrow \int \sqrt{x^2 - a^2} = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + C$$

$$\Rightarrow \int \sqrt{(x-2)^2 - 2} = \frac{(x-2)}{2} \sqrt{(x-2)^2 - 2} - \frac{(\sqrt{2})^2}{2} \log |\sqrt{(x-2)^2 - 2}| + C$$

$$\Rightarrow \int \sqrt{x^2 - 4x + 2} = \frac{x-2}{2} \sqrt{x^2 - 4x + 2} - \log |x^2 - 4x + 2| + C$$

35. Question

Mark (✓) against the correct answer in each of the following:

$$\int \sqrt{9x^2 + 16} dx = ?$$

A. $\frac{x}{2} \sqrt{9x^2 + 16} + \frac{8}{3} \log |3x + \sqrt{9x^2 + 16}| + C$

B. $\frac{x}{2} \sqrt{9x^2 + 16} - \frac{8}{3} \log |3x + \sqrt{9x^2 + 16}| + C$

C. $x \sqrt{9x^2 + 16} + 24 \log |3x + \sqrt{9x^2 + 16}| + C$

D. none of these

Answer

$$\Rightarrow \int \sqrt{x^2 + a^2} = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\Rightarrow 3 \int \sqrt{x^2 + \left(\frac{4}{3}\right)^2} = 3 \left(\frac{x}{2} \sqrt{x^2 + \left(\frac{4}{3}\right)^2} + \frac{16}{9} \log \left| x + \sqrt{x^2 + \left(\frac{4}{3}\right)^2} \right| \right)$$

$$\Rightarrow \int \sqrt{9x^2 + 16} dx = \frac{x}{2} \sqrt{9x^2 + 16} + \frac{8}{3} \log |3x + \sqrt{9x^2 + 16}|$$

36. Question

Mark (✓) against the correct answer in each of the following:

$$\int e^x \sqrt{e^{2x} + 4} dx = ?$$

A. $\frac{1}{2} e^x \sqrt{e^{2x} + 4} - 2 \log |e^x + \sqrt{e^{2x} + 4}| + C$

B. $\frac{1}{2} e^x \sqrt{e^{2x} + 4} + 2 \log |e^x + \sqrt{e^{2x} + 4}| + C$

$$C. e^x \sqrt{e^{2x} + 4} + \frac{1}{2} \log |e^x + \sqrt{e^{2x} + 4}| + C$$

D. none of these

Answer

$$\int e^x \sqrt{e^{2x} + 4} dx$$

Let $e^x = t$

$e^x dx = dt$

$$= \int \sqrt{t^2 + 2^2} dt$$

$$\Rightarrow \int \sqrt{x^2 + a^2} = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\Rightarrow \int \sqrt{t^2 + 2^2} = \frac{t}{2} \sqrt{t^2 + 2^2} + \frac{2^2}{2} \log |t + \sqrt{t^2 + 2^2}| + C$$

But $t = e^x$

$$\Rightarrow \int e^x \sqrt{e^{2x} + 4} dx = \frac{e^x}{2} \sqrt{e^{2x} + 4} + 2 \log |e^x + \sqrt{e^{2x} + 4}| + C$$

37. Question

Mark (✓) against the correct answer in each of the following:

$$\int \frac{\sqrt{16 + (\log x)^2}}{x} dx = ?$$

A. $\frac{1}{2} \log x \cdot \sqrt{16 + (\log x)^2} + 8 \log |\log x + \sqrt{16 + (\log x)^2}| + C$

B. $\frac{1}{2} \log x \cdot \sqrt{16 + (\log x)^2} + 4 \log |\log x + \sqrt{16 + (\log x)^2}| + C$

C. $\log x \cdot \sqrt{16 + (\log x)^2} + 16 \log |\log x + \sqrt{16 + (\log x)^2}| + C$

D. none of these

Answer

$$\int \frac{\sqrt{16 + (\log x)^2}}{x} dx$$

Let $\log x = t$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$= \int \sqrt{t^2 + 4^2} dt$$

$$\Rightarrow \int \sqrt{x^2 + a^2} = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log |x + \sqrt{x^2 + a^2}| + C$$

$$\Rightarrow \int \sqrt{t^2 + 4^2} dt = \frac{t}{2} \sqrt{t^2 + 4^2} + \frac{4^2}{2} \log |t + \sqrt{t^2 + 4^2}| + C$$

But $t = \log x$

$$\begin{aligned} \Rightarrow \int \frac{\sqrt{16 + (\log x)^2}}{x} dx \\ = \frac{\log x}{2} \sqrt{\log^2 x + 16} + 8 \log |\log x + \sqrt{\log^2 x + 16}| + C \end{aligned}$$

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