Properties of Parallel Lines

Q1

Answer :

Given : $l \parallel m$

t is a transversal.

$\angle 5 = 70°$

$\angle 5 = \angle 3 = 70°$ (alternate interior angles)

$\angle 5 + \angle 8 = 180°$ (linear pair)

or $70° + \angle 8 = 180°$

$\angle 8 = 110°$

$\angle 1 = \angle 3 = 70°$ (vertically opposite angles)

$\angle 3 + \angle 4 = 180°$ (linear pair)
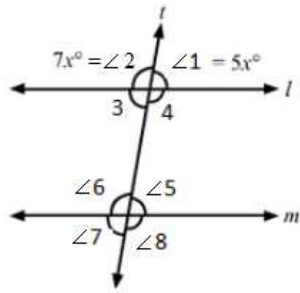
or $\angle 4 = 180 - \angle 3 = 180 - 70 = 110°$

Q2
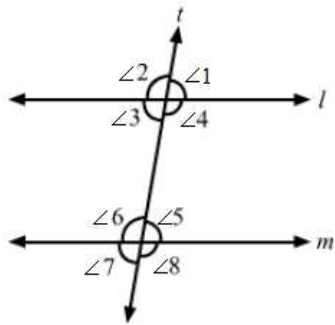
Answer :

Given : $l \parallel m$

t is a transversal.

$\angle 1 : \angle 2 = 5 : 7$

Let the angles measure 5x and 7x.

$\angle 1 + \angle 2 = 180°$ (linear pair)

$\therefore 5x + 7x = 180$

or $12x = 180$

or $x = 15$

$\therefore \angle 1 = 5x = 5(15) = 75°$

and $\angle 2 = 7x = 7(15) = 105°$

$\angle 2 + \angle 3 = 180°$ (linear pair)

$\angle 3 = 180 - 105 = 75°$

$\angle 3 + \angle 6 = 180$ (interior angles on the same side of *the* transversal are supplementary)

$\angle 6 = 180 - \angle 3 = 105°$

and $\angle 6 = \angle 8 = 105°$ (vertically opposite angles)

$\therefore \angle 1 = 75°$

$\angle 2 = 105°$

$\angle 3 = 75°$

$\angle 8 = 105°$

Q3

Answer :

Given : $l \parallel m$

t is a transversal.

Let :

$\angle 1 = (2x - 8)°$

$\angle 2 = (3x - 7)°$

We know that the consecutive interior angles are supplementary.

$\therefore \angle 1 + \angle 2 = 180°$

or $(2x - 8) + (3x - 7) = 180$

or $5x - 15 = 180$
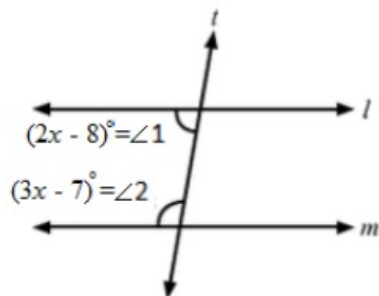
or $5x = 195$

or $x = 39$

$\angle 1 = (2x - 8) = (2 \times 39 - 8) = 70°$
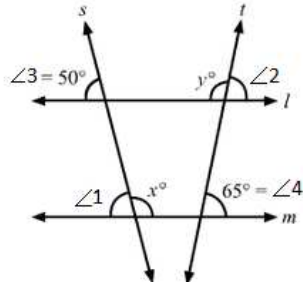
$\angle 2 = (3x - 7) = (3 \times 39 - 7) = 110°$

Q4

Answer:

From the given figure:

$\angle 1 = \angle 3 = 50°$ (corresponding angles)
and $\angle 1 + x° = 180°$ (linear pair)
or $x° = 180° - 50° = 130°$
or $x = 130$
$\angle 2 = \angle 4 = 65°$ (corresponding angles)
and $\angle 2 + y° = 180°$ (linear pair)
or $y° = 180° - 65° = 115°$
or $y = 115$

Q5

Answer:

Given:
$\angle B = 65°$
$\angle C = 45°$
$DAE \parallel BC$
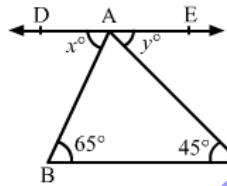The given lines are parallel.
$\therefore x° = \angle B = 65°$ (alternate angles when AB is taken as the transversal)
$y° = \angle C = 45°$ (alternate angles when AC is taken as the transversal)
$\therefore x = 65$
$y = 45$

Q6

Answer:

Given: $CE \parallel BA$
$\angle BAC = 80°$, $\angle ECD = 35°$
(i) $\angle BAC = \angle ACE = 80°$ (alternate angles with AC as a transversal)
(ii) $\angle ACB + \angle ACD = 180°$ (linear pair)
or $\angle ACB + \angle ACE + \angle ECD = 180°$
or $\angle ACB + 80° + 35° = 180°$
or $\angle ACB = 65°$
(iii) In $\triangle ABC$:
$\angle BAC + \angle ACB + \angle ABC = 180°$ (angle sum property)
$80° + 65° + \angle ABC = 180°$
$\angle ABC = 35°$
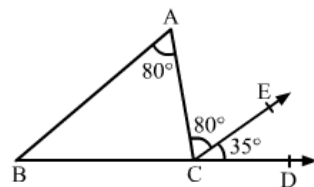
Q7

Answer :

Given : AO ∥ CD

OB ∥ CE

∠AOB = 50˚

∠AOD = ∠CDB = 50˚ (when AO ∥ CD and OB is the transversal)

∠ECD + ∠CDB = 180˚ (consecutive interior angles are supplementary, DB ∥ CE and *CD* is *the* transversal)

∠ECD = 180˚ − 50˚ = 130˚
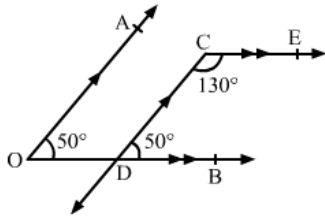
Q8

Answer :

Given : AB ∥ CD

∠ABO = 50˚

∠CDO = 40˚

Construction : Through O, draw EOF ∥ AB.

∠ABO = ∠BOF = 50˚ (alternate angles, when AB ∥ EF and OB is a transversal)

∠FOD = ∠ODC = 40˚ (alternate angles, when CD ∥ EF and OD is a transversal)

∠BOD = ∠BOF + ∠FOD

∠BOD = 50˚ + 40˚ = 90˚
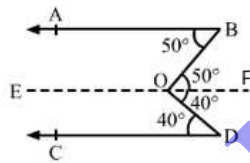
Q9

Answer :

Given : AB ∥ CD

GL and HM are angle bisectors of ∠AGH and ∠GHD, respectively.

∠AGH = ∠GHD (alternate angles)

or $\frac{1}{2}$ ∠AGH = $\frac{1}{2}$ ∠GHD

or ∠LGH = ∠GHM (given)

Therefore, GL ∥ HM as we know that if the angles of any pair of alternate interior angles are equal, then the lines are parallel.
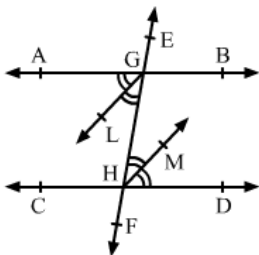
Q10

Answer :

Given : AB ∥ CD

∠ABE = 120°
∠ECD = 100°
∠BEC = x°

Construction : FEG ∥ AB

Now, since *AB* ∥ *FEG* and *AB* ∥ *CD, FEG* ∥ *CD*

∴ *EFG* ∥ *AB* ∥ *CD*

∠ABE = ∠BEG = 120° (alternate angles)

or x° + y° = 120° .... (i)

∠DCE = ∠CEF = 100° (alternate angles)

or $x° + z° = 100°$ ..... (ii)

Also, x° + y° + z° = 180° (FEG is a *s*traight line) ... (iii)

Adding (i) and (ii) :

2x° + y° + z° = 220°

or, x° + 180° = 220° (substituting (iii))

x° = 40°

∴ $x = 40$
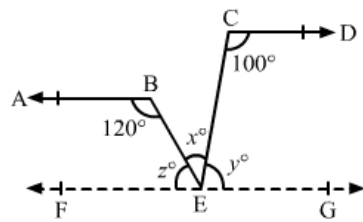
Q11

Answer :

Given : AB ∥ CD

AD ∥ BC

∠1 + ∠2 = 180° (AB ∥ CD and AD is the transversal) ...(i)

∠2 + ∠3 = 180° (AD ∥ BC and AB is the transversal) ...(ii)

From (i) and (ii) :

∠1 + ∠2 = 180° = ∠2 + ∠3

∠1 = ∠3

∠ADC = ∠ABC
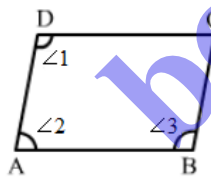
Q12

Answer :

Given :

$l \parallel m$

$p \parallel q$

$\angle 1 = 65°$

$\therefore \angle 1 = \angle a = 65°$ (vertically opposite angles)

$\angle a + \angle d = 180°$ (consecutive interior angles on the same side of a transversal are supplementary)

or $\angle d = 180° - 65° = 115°$

$\angle c + \angle d = 180°$ (consecutive interior angles on the same side of a transversal are supplementary)

or $\angle c = 180° - 115° = 65°$

$\angle c + \angle b = 180°$ (consecutive interior angles on the same side of a transversal are supplementary)

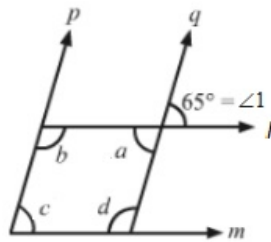or $\angle b = 180° - 65° = 115°$

$\therefore \angle a = 65°$

$\angle b = 115°$

$\angle c = 65°$

$\angle d = 115°$

Q13

Answer :

Given :

$AB \parallel DC$

$AD \parallel BC$

$\angle BAC = 35°$

$\angle CAD = 40°$
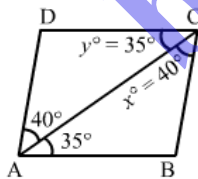
$\therefore \angle BAC = y = 35°$ (alternate angles when $AB \parallel DC$)

$\angle CAD = x = 40°$ (alternate angles when $AD \parallel BC$)

$\therefore x = 40$

$y = 35$

Q14

Answer:

Given :

AB ∥ CD

∠BAE = 125°

∠CAB + ∠BAE = 180°

or 125° + x° = 180°
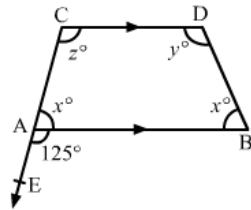
or x = 55

x + z = 180° (consecutive interior angles on the same side of transversal are supplementary)

z = 180 − x = 180 − 55 = 125

y + x = 180° (consecutive interior angles on the same side of transversal are supplementary)

y = 180 − x = 180 − 55 = 125

Q15

Answer:

(i) ∠1 + ∠2 = 180 (linear pair)

or 130° + ∠2 = 180°

or ∠2 = 50° ≠ 40° = ∠3

∴ l ∦ m

(ii) ∠2 + ∠3 = 180° (linear pair)

35° + ∠3 = 180°

∠3 = 145° = 145° = ∠1

∴ l ∥ m

(iii) ∠2 + ∠3 = 180 (linear pair)

∠3 = 180° − 125° = 55°

∠3 = 55° ≠ 60° = ∠1

∴ l ∦ m