12. Indefinite Integral

Exercise 12

1. Question

Evaluate :

Evaluate

i. $\int x^7 dx$

ii. $\int x^{-7} dx$

iii. $\int x^{-1} dx$

iv. $\int x^{5/3} dx$

v. $\int x^{-5/4} dx$

vi. $\int 2^x dx$

vii. $\int \sqrt[3]{x^2}\, dx$

viii. $\int \frac{1}{\sqrt[4]{x^3}} dx$

ix. $\int \frac{2}{x^2} dx$

Answer

i Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^7 dx = \frac{x^{7+1}}{7+1} + c$$

$$= \frac{x^8}{8} + c$$

ii. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-7} dx = \frac{x^{-7+1}}{-7+1} + c$$

$$= \frac{x^{-6}}{-6} + c$$

iii. Given:

$$\int \frac{1}{x} dx = \ln|x| + c$$

iv. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}} dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{3x^{\frac{8}{3}}}{8} + c$$

v. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{5}{4}} dx = \frac{x^{-\frac{5}{4}+1}}{-\frac{5}{4}+1} + c$$

$$= -4x^{-\frac{1}{4}} + c$$

vi. Given:

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

vii. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{2}{3}} dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{3x^{\frac{5}{3}}}{5} + c$$

viii. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{3}{4}} dx = \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c$$

$$= 4x^{\frac{1}{4}} + c$$

ix. Given:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int 2x^{-2} dx = 2\frac{x^{-2+1}}{-2+1} + c$$

$= \frac{-2}{x} + c$

2. Question

Evaluate :

i. $\int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x \right) dx$

ii. $\int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1} \right) dx$

iii. $\int \left(\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax \right) dx$

Answer

i Given:

$$\int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x\right) dx$$
$$= 6\frac{x^{5+1}}{5+1} - 2\frac{x^{-4+1}}{-4+1} - 7\frac{x^2}{2} + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$
$$= 6\frac{x^6}{6} - 2\frac{x^{-3}}{-3} - 7\frac{x^2}{2} + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$
$$= x^6 + \frac{2}{3}x^{-3} - \frac{7}{2}x^2 + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

ii. Given:

$$\int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1}\right) dx$$
$$= 8x - \frac{x^2}{2} + 2\frac{x^{3+1}}{3+1} - 6\frac{x^{-3+1}}{-3+1} + 2\frac{x^{-5+1}}{-5+1} + 5\ln|x| + c$$
$$= 8x - \frac{x^2}{2} + \frac{2}{4}x^4 + \frac{6}{2}x^2 - \frac{2}{4}x^{-4} + 5\ln|x| + c$$
$$= 8x - \frac{x^2}{2} + \frac{1}{2}x^4 + 3x^2 - \frac{1}{2}x^{-4} + 5\ln|x| + c$$

iii. Given:

$$\int \left(\frac{x}{a} + \frac{a}{x} + x^a + a^x + ax\right) dx = \frac{1}{a}\frac{x^2}{2} + a\ln|x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\ln a} + a\frac{x^2}{2} + c$$

3. Question

Evaluate :

i. $\int (2 - 5x)(3 + 2x)(1 - x)\,dx$

ii. $\int \sqrt{x}\left(ax^2 + bx + c\right) dx$

iii. $\int \left(\sqrt{x} - \sqrt[3]{x^4} + \frac{7}{\sqrt[3]{x^2}} - 6x^x + 1 \right) dx$

Answer

i. Given:

$\int(2-5x)(3+2x)(1-x)dx$

$=\int(6-11x-10x^2)(1-x)\,dx$

$=\int(10\,x^3+x^2-17x+6)dx$

$$=\frac{10x^4}{4}+\frac{x^3}{3}-\frac{17x^2}{2}+6x+c$$

$$=\frac{5x^4}{2}+\frac{x^3}{3}-\frac{17x^2}{2}+6x+c$$

ii. Given:

$$=\int\left(ax^{\frac{5}{2}}+bx^{\frac{3}{2}}+cx^{\frac{1}{2}}\right)dx$$

$$=a\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+b\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+c\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$=\frac{2a}{7}x^{\frac{7}{2}}+\frac{2b}{5}x^{\frac{5}{2}}+\frac{2c}{3}x^{\frac{3}{2}}+C$$

iii. Given:

$$\int\left(x^{\frac{1}{2}}-x^{\frac{4}{3}}+7x^{\frac{-2}{3}}-6e^{\ln x^x}+1\right)dx$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}+7\frac{x^{\frac{-2}{3}+1}}{-\frac{2}{3}+1}-6e^{\ln x^x}+x+c$$

$$=\frac{2x^{\frac{3}{2}}}{3}-\frac{3x^{\frac{7}{3}}}{7}-21x^{-\frac{1}{3}}-6x^x+x+c$$

4. Question

Evaluate :

i. $\int\left(x^2-\frac{1}{x^2}\right)^3 dx$

ii. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)dx$

iii. $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$

iv. $\int\frac{(1+2x)^3}{x^4}dx$

v. $\int\frac{(1+x)^3}{\sqrt{x}}dx$

vi. $\int\frac{2x^2+x-2}{(x-2)}dx$

Answer

i. Given:

$=\int x^6 + x^{-6} - 3x^2 - 3x^{-2}\, dx$

$$= \frac{x^7}{7} + \frac{x^{-5}}{5} - x^3 + 3x^{-1} + c$$

ii. Given:

$$= \int x^{\frac{1}{2}} - x^{\frac{-1}{2}} dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$$

iii. Given:

$$= \int \left(x + \frac{1}{x} + 2\right) dx$$

$$= \frac{x^2}{2} + \ln|x| + 2x + c$$

iv. Given:

$$= \int \frac{1 + 8x^3 + 6x + 12x^2}{x^4} dx$$

$$= \int x^{-4} + \frac{8}{x} + 6x^{-3} + 12x^{-2} dx$$

$$= -\frac{x^{-3}}{3} + 8\ln|x| - 3x^{-2} - 12x^{-1} + c$$

v. Given:

$$= \int \frac{1 + x^3 + 3x + 3x^2}{\sqrt{x}} dx$$

$$= \int x^{\frac{-1}{2}} + x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 3x^{\frac{3}{2}} dx$$

$$= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$$

vi. Given:

$$= \int \left(\frac{2x^2}{x-2} + \frac{x-2}{x-2}\right) dx$$

$$= 2\int \left(\frac{x^2 - 4x + 4}{x-2} + \frac{4x}{x-2} - \frac{4}{x-2}\right) dx + \int dx$$

$$= 2\left[\int \frac{(x-2)^2}{x-2} dx + 4\int \frac{x-2+2}{x-2} dx - 4\int \frac{1}{x-2} dx\right] + x + c$$

$$= 2\left[\int (x-2) dx + 4\left(\int dx + 2\int \frac{1}{x-2} dx\right) - 4\ln|x-2|\right] + x + c$$

$$= 2\left[\frac{x^2}{2} - 2x + 4x + 8\ln|x-2| - 4\ln|x-2|\right] + x + c$$

$=x^2 - 4x+8x+8\ln|x-2|+x + c$

$=x^2 +5x+8\ln|x-2|+c$

5. Question

Evaluate :

$$\int\left[1+\frac{1}{\left(1+x^2\right)}-\frac{2}{\sqrt{1-x^2}}+\frac{5}{x\sqrt{x^2-1}}+a^x\right]dx$$

Answer

Given:

Since, $\int \frac{1}{1+x^2}dx = \tan^{-1}x + c$;

$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + c$;

$\int a^x\,dx = \frac{a^x}{\ln a} + c$ &

$$\int \frac{1}{|x|\sqrt{(x^2-1)}}dx = \sec^{-1}x + c$$

So,

$$= x + \tan^{-1}x - 2\sin^{-1}x + 5\sec^{-1}x + \frac{a^x}{\ln a} + c$$

6. Question

Evaluate :

i. $\int\left(\frac{x^2-1}{x^2+1}\right)dx$

ii. $\int\left(\frac{x^6-1}{x^2+1}\right)dx$

iii. $\int\left(\frac{x^4}{1+x^2}\right)dx$

iv. $\int\left(\frac{x^2}{1+x^2}\right)dx$

Answer

i. Given:

$$= \int \frac{x^2+1-2}{x^2+1}dx$$

$$= \int \frac{x^2+1}{x^2+1} - \frac{2}{x^2+1}dx$$

$=x-2\tan^{-1}x + c$

ii. Given:

$$= \int\left[\frac{x^6}{x^2+1} - \frac{1}{x^2+1}\right]dx$$

$$= \int\left[\frac{x^6+3x^2+3x^4+1-3x^2-3x^4-1}{x^2+1} - \frac{1}{x^2+1}\right]dx$$

$$= \int \left[\frac{(x^2+1)^3}{x^2+1} - 3\frac{x^2}{x^2+1} - 3\frac{x^4}{x^2+1} - \frac{1}{x^2+1} - \frac{1}{x^2+1}\right] dx$$

$$= \int (x^2+1)^2\,dx - 3\int \left[\frac{x^2+1-1}{x^2+1}\right] dx - 3\left[\int \frac{x^4+2x^2+1}{x^2+1} + \frac{-2x^2-1}{x^2+1} dx\right]$$
$$- 2\int \frac{1}{x^2+1} dx$$

$$= \int (x^4+2x^2+1)dx - 3\left[\int dx - \int \frac{1}{x^2+1} dx\right]$$
$$- 3\left[\int \frac{(x^2+1)^2}{x^2+1} dx - 2\int \frac{x^2}{x^2+1} dx - \int \frac{1}{x^2+1} dx\right] - 2\int \frac{1}{x^2+1} dx$$

$$= \int (x^4+2x^2+1)dx - 3\left[\int dx - \int \frac{1}{x^2+1} dx\right]$$
$$- 3\left[\int (x^2+1)dx - 2\int \frac{x^2+1-1}{x^2+1} dx - \int \frac{1}{x^2+1} dx\right]$$
$$- 2\int \frac{x^2+1}{x^2+1} dx$$

$$= \frac{x^5}{5} + \frac{2}{3}x^3 + x - 3x + 3\tan^{-1}x - x^3 - 3x + 6x - 3\tan^{-1}x - 2\tan^{-1}x + c$$

$$= \frac{x^5}{5} + \frac{1}{3}x^3 + x - 2\tan^{-1}x + c$$

iii. Given:

$$= \int \frac{x^4+2x^2+1}{x^2+1} + \frac{-2x^2-1}{x^2+1} dx$$

$$= \int \frac{(x^2+1)^2}{x^2+1} dx - 2\int \frac{x^2}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= \int (x^2+1)dx - 2\int \frac{x^2+1-1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= \int (x^2+1)dx - 2\int dx + 2\int \frac{1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$$

$$= \int (x^2+1)dx - 2\int dx + \int \frac{1}{x^2+1} dx$$

$$= \frac{1}{3}x^3 - x + \tan^{-1}x + c$$

iv. Given:

$$= \int \left[\frac{x^2+1-1}{x^2+1}\right] dx$$

$$= \int dx - \int \frac{1}{x^2+1} dx$$

$= x - \tan^{-1} x + c$

7. Question

Evaluate :

$$\int \left(9\sin x - 7\cos x - \frac{6}{\cos^2 x} + \frac{2}{\sin^2 x} + \cot^2 x\right) dx$$

Answer

Given:

$$= \int (9\sin x - 7\cos x - 6(\sec x)^2 + 2(\csc x)^2 + (\csc x)^2 - 1)\, dx$$

=-9 cos x-7 sin x-6 tan x-3 cot x – x + c

8. Question

Evaluate :

$$\int \left(\frac{\cot x}{\sin x} - \tan^2 x - \frac{\tan x}{\cos x} + \frac{2}{\cos^2 x} \right) dx$$

Ans. - cosec x + tan x + x – sec x + C

Answer

Given:

$$= \int (\cot x \csc x - (\sec x)^2 + 1 - \tan x \sec x + 2(\sec x)^2)$$

=-csc x-tan x + x-sec x+2 tan x + c

=-csc x + tan x + x -sec x + c

9. Question

Evaluate :

i. $\int \sec x (\sec x + \tan x) dx$

ii. $\int \operatorname{cosec} x (\operatorname{cosec} x - \cot x) dx$

Answer

i. Given:

$$= \int (\sec x)^2 + \sec x \tan x \, dx$$

=tan x+sec x+c

ii. Given:

$$= \int (\csc x)^2 - \cot x \csc x \, dx$$

=-cot x + csc x + c

10. Question

Evaluate :

i. $\int (\tan x + \cot x)^2 dx$

ii. $\int \left(\frac{1 + 2\sin x}{\cos^2 x} \right) dx$

iii. $\int \left(\frac{3\cos x + 4}{\sin^2 x} \right) dx$

Answer

i. Given:

$$= \int ((\tan x)^2 + (\cot x)^2 + 2)dx$$

$$= \int ((\sec x)^2 - 1 + (\csc x)^2 - 1 + 2)dx$$

$$= \int ((\sec x)^2 + (\csc x)^2)dx$$

=tan x-cot x+c

ii. Given:

$$= \int \left(\frac{1}{(\cos x)^2} + 2\frac{\sin x}{(\cos x)^2}\right)dx$$

$$= \int ((\sec x)^2 + 2\tan x \sec x)dx$$

=tan x+2 sec x + c

iii. Given:

$$= \int (2\cot x \csc x + 4(\csc x)^2)$$

=-2 csc x-4 cot x + c

11. Question

Evaluate :

i. $\int \frac{1}{(1-\cos x)} dx$

ii. $\int \frac{1}{(1-\sin x)} dx$

Answer

i. Given:

Multiply and divide by $(1+\cos x)$

$$= \int \frac{1+\cos x}{1-(\cos x)^2} dx$$

$$= \int \frac{1+\cos x}{(\sin x)^2} dx$$

$$= \int ((\csc x)^2 + \csc x \cot x)dx$$

=-cot x-csc x + c

ii. Given:

Multiply and divide by $(1+\sin x)$

$$= \int \frac{1+\sin x}{1-(\sin x)^2} dx$$

$$= \int \frac{1+\sin x}{(\cos x)^2} dx$$

$$= \int ((\sec x)^2 + \sec x \tan x)dx$$

=tan x + sec x + c

12. Question

Evaluate :

i. $\int \frac{\tan x}{(\sec x + \tan x)} dx$

ii. $\int \frac{\text{cosec } x}{(\text{cosec } x - \cot x)} dx$

Answer

i. Given:

Multiply and divide by $(\sec x - \tan x)$

$$= \int \frac{\tan x \sec x - (\tan x)^2}{(\sec x)^2 - (\tan x)^2} dx$$

$$= \int \tan x \sec x - (\tan x)^2 dx$$

$$= \int (\tan x \sec x - (\sec x)^2 + 1) dx$$

=sec x-tan x + x + c

ii. Given:

Multiply and divide by $(\csc x + \cot x)$

$$= \int \frac{(\csc x)^2 + \csc x \cot x}{(\csc x)^2 - (\cot x)^2} dx$$

$$= \int (\csc x)^2 + \csc x \cot x \, dx$$

=-cot x-csc x + c

13. Question

Evaluate :

i. $\int \frac{\cos x}{1 + \cos x} dx$

ii. $\int \frac{\sin x}{(1 - \sin x)} dx$

Answer

i. Given:

Multiply and divide by $(1 - \cos x)$

$$= \int \frac{\cos x - (\cos x)^2}{1 - (\cos x)^2} dx$$

$$= \int \frac{\cos x - (\cos x)^2}{(\sin x)^2} dx$$

$$= \int (\cot x \csc x - (\cot x)^2) dx$$

$= \int (\cot x \csc x - (\csc x)^2 + 1) dx$

=-csc x + cot x + x + c

ii. Given:

Multiply and divide by $(1 + \sin x)$

$= \int \frac{\sin x + (\sin x)^2}{1 - (\sin x)^2} dx$

$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$

$= \int (\tan x \sec x + (\tan x)^2) dx$

$= \int (\tan x \sec x + (\sec x)^2 - 1) dx$

=sec x + tan x-x + c

14. Question

Evaluate :

i. $\int \sqrt{1 + \cos 2x}\ dx$

ii. $\int \sqrt{1 - \cos 2x}\ dx$

Answer

i. Given:

$= \int \sqrt{2(\cos x)^2} dx$

$= \sqrt{2} \int \cos x\, dx$

$= \sqrt{2} \sin x + c$

ii. Given:

$= \int \sqrt{2(\sin x)^2} dx$

$= \sqrt{2} \int \sin x\, dx$

=-√2 cos x + c

15. Question

Evaluate :

i. $\int \frac{1}{(1 + \cos 2x)} dx$

ii. $\int \frac{1}{(1 - \cos 2x)} dx$

Answer

i. Given:

$$= \int \frac{1}{2(\cos x)^2} dx$$

$$= \frac{1}{2} \int (\sec x)^2 \, dx$$

$$= \frac{1}{2} \tan x + c$$

ii. Given:

$$= \int \frac{1}{2(\sin x)^2} dx$$

$$= \frac{1}{2} \int (\csc x)^2 dx$$

$$= -\frac{1}{2} \cot x + c$$

16. Question

Evaluate :

$$\int \sqrt{1 + \sin 2x} \, dx$$

Answer

Given:

$$= \int \sqrt{\left(1 + \frac{2 \tan x}{1 + (\tan x)^2}\right)} dx$$

$$= \int \sqrt{\left(\frac{(1 + \tan x)^2}{(\sec x)^2}\right)} dx$$

$$= \int \left(\frac{1 + \tan x}{\sec x}\right) dx$$

=∫(cos x + sin x) d

=sin x-cos x+c

17. Question

Evaluate :

$$\int \left(\frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} \right) dx$$

Ans. sec x – cosec x + C

Answer

Given:

$$= \int \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} + \frac{(\sin x)^3}{(\sin x)^2 (\cos x)^2} dx$$

=∫(tan x sec x + csc x cot x)dx

=sec x- csc x + c

18. Question

Evaluate :

$$\int \tan^{-1}\left(\frac{\sin\ 2x}{1+\cos\ 2x}\right)dx$$

Ans. $\frac{x^2}{2}+C$

Answer

Given:

$$=\int \tan^{-1}\left(\frac{2\sin x\cos x}{2(\cos x)^2}\right)dx$$

$=\int\tan^{-1}$ (tan x)dx

$=\int x\ dx$

$=\frac{x^2}{2}+c$

19. Question

Evaluate :

$$\int \cos^{1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)dx$$

Ans. x^2+C

Answer

Given:

$=\int\cos^{-1}$(cos 2x)dx

$=\int 2x\ dx$

$=x^2$ +c

20. Question

Evaluate :

$$\int \cos^{-1}(\sin x)dx$$

Ans. $\left(\frac{\pi x}{2}-\frac{x^2}{2}+C\right)$

Answer

Given:

$$\sin^{-1}(\sin x)+\cos^{-1}(\sin x)=\frac{\pi}{2}$$

$$=\int\left[\frac{\pi}{2}-\sin^{-1}(\sin x)\right]dx$$

$$=\int\left[\frac{\pi}{2}-x\right]dx$$

$$=\frac{\pi}{2}x-\frac{x^2}{2}+c$$

21. Question

Evaluate :

$$\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}\,dx$$

Ans. $\frac{\pi x}{4}-\frac{x^2}{4}+C$

Answer

Given:

$$=\int \tan^{-1}\sqrt{\left(\frac{(1-\sin x)^2}{1-(\sin x)^2}\right)}dx$$

$$=\int \tan^{-1}\left(\frac{1-\sin x}{\cos x}\right)dx$$

$$=\int \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}\right)dx$$

$$=\int \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)dx$$

$$=\int \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$$

$$=\int \left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$=\frac{\pi}{4}x-\frac{x^2}{4}+c$$

22. Question

Evaluate :

$$\int(3\cot x-2\tan x)^2\,dx$$

Ans. 4 tan x – 9 cos x – 25x + C

Answer

Given:

$$=\int(9(\cot x)^2+4(\tan x)^2-12)dx$$

$$=\int(9(\csc x)^2+4(\sec x)^2-25)\,dx$$

=-9 cot x+4 tan x-25x+c

23. Question

Evaluate :

$\int (3 \sin x + 4 \operatorname{cosec} x)^2 \, dx$

Ans. $\frac{57}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + C$

Answer

Given:

$= \int (9(\sin x)^2 + 16(\csc x)^2 + 24) dx$

$= \int \left(\frac{9}{2}(1 - \cos 2x) + 16(\csc x)^2 + 24\right) dx$

$= \frac{9}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + 24x + c$

$= \frac{57}{2}x - \frac{9}{4}\sin 2x - 16 \cot x + c$

24. Question

Evaluate :

$\int \frac{dx}{\left(\sqrt{x+1} + \sqrt{x+2}\right)}$

Ans. $\frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{3/2} + C$

Answer

Given:

Multiply and divide by $\sqrt{(x+1)} - \sqrt{(x+2)}$

$= \int \frac{\left(\sqrt{(x+1)} - \sqrt{(x+2)}\right)}{x+1-x-2} dx$

$= -\int \sqrt{(x+3)} + \sqrt{(x+2)} \, dx$

$= \frac{-2}{3}(x+1)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$

25. Question

Evaluate :

$\int \frac{dx}{\left(\sqrt{x+3} - \sqrt{x+2}\right)}$

Ans. $\frac{2}{3}(x+3)^{3/2} + \frac{2}{3}(x+2)^{3/2} + C$

Answer

Given:

Multiply and divide by $\sqrt{(x+3)} + \sqrt{(x+2)}$

$$= \int \frac{\left(\sqrt{(x+3)} + \sqrt{(x+2)}\right)}{x+3-x-2} dx$$

$$= \int \sqrt{(x+3)} + \sqrt{(x+2)}\, dx$$

$$= \frac{2}{3}(x+3)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

26. Question

Evaluate :

$$\int \left(\frac{1+\cos x}{1-\cos x}\right) dx$$

Ans. $-2\cot\frac{x}{2} - x + C$

Answer

Given:

Multiply and divide by $(1+\cos x)$

$$= \int \left(\frac{(1+\cos x)^2}{1-(\cos x)^2}\right) dx$$

$$= \int \frac{1+(\cos x)^2+2\cos x}{(\sin x)^2} dx$$

$$= \int (\csc x)^2 + (\cot x)^2 + 2\cot x \csc x \; dx$$

$$= \int (\csc x)^2 + (\csc x)^2 - 1 + 2\cot x \csc x \; dx$$

$$= -2\cot x - 2\csc x - x + c$$

$$= -2(\csc x + \cot x) - x + c$$

$$= -2\left(\frac{1+\cos x}{\sin x}\right) - x + c$$

$$= -2\left(\frac{2\cos\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) - x + c$$

$$= -2\cot\frac{x}{2} - x + c$$

27. Question

Evaluate :

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx$$

Ans. -log |cos x – sin x| + C

Answer

Given:

$$= \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} dx$$

$$= \int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let cos x-sin x=t

(-sin x-cos x) dx=dt

-(sin x + cos x)dx=dt

dx=-dt

So, I$= -\int \frac{dt}{t}$

=-ln|t|+c

=-ln|cos x-sin x |+c

28. Question

Evaluate :

$$\int \frac{\cos(x+a)}{\sin(x+b)} dx$$

Ans. cos(a – b)log |sin (x + b)| -x sin(a –b) + C

Answer

Given:

$$= \int \frac{\cos(x+b+a-b)}{\sin(x+b)} dx$$

$$= \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)} dx$$

=cos(a-b)∫cot(x + b) dx-∫sin(a-b)dx

=cos (a-b)ln|sin(x + b)|-x sin(a-b)+c

29. Question

Evaluate :

$$\int \frac{\sin(x-\alpha)}{\sin(x+\alpha)} dx$$

Ans. x cos 2α - sin 2α . log |sin(x + α)| + C

Answer

Given:

$$= \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)} dx$$

$$= \int \frac{\sin(x+\alpha)\cos(2\alpha) - \sin(2\alpha)\cos(x+\alpha)}{\sin(x+\alpha)} dx$$

=∫cos 2α dx-sin2α ∫cot(x+α)dx

=x cos 2α-sin 2α×ln|sin(x+α) |+c

30. Question

Evaluate :

$$\int (1-x)\sqrt{x}\ dx$$

Ans. $\frac{2}{15}x\sqrt{x}(5-3x)+C$

Answer

Given:

$$=\int \left(x^{\frac{1}{2}}-x^{\frac{3}{2}}\right)dx$$

$$=\frac{2}{3}x^{\frac{3}{2}}-\frac{2}{5}x^{\frac{5}{2}}+c$$

$$=\frac{2}{15}x^{\frac{3}{2}}(5-3x)+c$$

31. Question

Evaluate :

$$\int \frac{\sec^2 x}{\text{cosec}^2 x}dx$$

Ans. tan x – x + C

Answer

Given:

$=\int(\tan x)^2\ dx$

$=\int((\sec x)^2-1)dx$

=tan x – x + c

32. Question

Evaluate :

$$\int \left\{\frac{2-3\sin x}{\cos^2 x}\right\}dx$$

Ans. 2 tan x – 3 sec x + C

Answer

Given:

$$=\int \frac{2}{(\cos x)^2}dx-\int \frac{3\sin x}{(\cos x)^2}dx$$

$$=2\int (\sec x)^2\,dx-3\int \tan x\sec x\,dx$$

=2 tan x-3 sec x+c

Objective Questions

1. Question

Mark (√) against the correct answer in each of the following:

$\int x^6 dx = ?$

A. $7x^7 + C$

B. $\frac{x^7}{7} + C$

C. $6x^5 + C$

D. $6x^7 + C$

Answer

Given:

$\int x^6$ dx,

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^6 dx = \frac{x^{6+1}}{6+1} + c$$

$$= \frac{x^7}{7} + c$$

2. Question

Mark (√) against the correct answer in each of the following:

$\int x^{5/3} dx = ?$

A. $\frac{3}{5} x^{2/3} + C$

B. $\frac{8}{3} x^{8/3} + C$

C. $\frac{3}{8} x^{8/3} + C$

D. $\frac{5}{3} x^{8/3} + C$

Answer

Given:

$$\int x^{\frac{5}{3}} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}} dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + c$$

$$= \frac{3}{8}x^{\frac{8}{3}} + c$$

3. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{x^3} dx = ?$$

A. $\frac{-3}{x^2} + C$

B. $\frac{-1}{2x^2} + C$

C. $\frac{-1}{3x^2} + C$

D. $\frac{x^{-2}}{2} + C$

Answer

Given:

$$\int \frac{1}{x^3} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x^3} dx = \frac{x^{-3+1}}{-3+1} + c$$

$$= -\frac{x^{-2}}{2} + c$$

$$= -\frac{1}{2x^2} + c$$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt[3]{x}\, dx = ?$$

A. $\frac{3}{4}x^{3/4} + C$

B. $\frac{4}{3}x^{3/4} + C$

C. $\frac{3}{4}x^{4/3} + C$

D. $\frac{4}{3}x^{4/3} + C$

Answer

Given:

$$\int \sqrt[3]{x}\, dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x}\, dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$$

$$= \frac{x^{\frac{4}{3}}}{4} + c$$

$$= \frac{3}{4} x^{\frac{4}{3}} + c$$

5. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\sqrt[3]{x}} dx = ?$$

A. $\frac{3}{2} x^{2/3} + C$

B. $\frac{3}{2x^{2/3}} + C$

C. $\frac{2}{3x^{2/3}} + C$

D. $\frac{2}{3} x^{3/2} + C$

Answer

Given:

$$\int \frac{1}{\sqrt[3]{x}} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{\sqrt[3]{x}} dx = \frac{x^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} + c$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$$

$$= \frac{3}{2} x^{\frac{2}{3}} + c$$

6. Question

Mark (√) against the correct answer in each of the following:

$\int \sqrt[3]{x^2}\, dx = ?$

A. $\frac{5}{3}x^{5/3} + C$

B. $\frac{3}{5}x^{5/3} + C$

C. $\frac{5}{3}x^{3/5} + C$

D. $\frac{3}{5}x^{3/5} + C$

Answer

Given:

$\int \sqrt[3]{x^2}dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$\int \sqrt[3]{x^2}\, dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$

$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$

$= \frac{3}{5}x^{\frac{5}{3}} + c$

7. Question

Mark (√) against the correct answer in each of the following:

$\int 3^x\, dx = ?$

A. $3^x (\log 3) + C$

B. $3^x + C$

C. $\frac{3^x}{\log 3} + C$

D. $\frac{\log 3}{3^x} + C$

Answer

Given:

$\int 3^x dx$

$\int a^x\, dx = \frac{a^x}{\ln a} + c$

$$\int 3^x dx = \frac{3^x}{\ln 3} + c$$

8. Question

Mark (√) against the correct answer in each of the following:

$$\int 2^{\log x} dx = ?$$

A. $\frac{2^{\log x+1}}{(\log x+1)} + C$

B. $\frac{x^{(\log 2+1)}}{(\log 2+1)} + C$

C. $\frac{2^{\log x}}{\log 2} + C$ 2adc

D. $\frac{2^{\log x}}{2} + C$

Answer

Given:

$$\int 2^{log x} dx$$

As $2^{\log x} = x^{\log 2}$

$$I = \int x^{log 2} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\log 2} dx = \frac{x^{\log 2+1}}{\log 2+1} + c$$

9. Question

Mark (√) against the correct answer in each of the following:

$$\int \text{cosec } x(\text{cosec } x + \cot x) dx = ?$$

A. cot x – cosec x + C

B. - cot x + cosec x + C

C. cot x + cosec x + C

D. - cot x – cosec x + C

Answer

Given:

$$\int \text{cosec} x(\text{cosec} x + cot\, x) dx = \int (\csc x)^2 + \cot x \csc x\, dx$$

=-cot x-csc x+c

10. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec x}{(\sec x + \tan x)} dx = ?$$

A. tan x + sec x + C

B. tan x – sec x + C

C. - tan x + sec x + C

D. - tan x – sec x + C

Answer

Given:

$$\int \frac{sec\, x}{(sec\, x + tan\, x)} dx$$

Multiply and divide by (sec x-tan x)

$$= \int \frac{(\sec x)^2 - \tan x \sec x}{(\sec x)^2 - (\tan x)^2} dx$$

$=\int(\sec x)^2$ -tan x sec x dx

=tan x-sec x + c

11. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = ?$$

A. tan x + x + C

B. tan x – x + C

C. - tan x + x + C

D. - tan x – x + C

Answer

Given:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = \int \frac{(2(\sin x)^2)}{(2(\cos x)^2)} dx$$

$=\int(\tan x)^2$

$=\int((\sec x)^2$ -1) dx

=tan x-x+c

12. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{\sin^2 x \cos^2 x} dx = ?$$

A. tan x + cot x + C

B. - tan x + cot x + C

C. tan x – cot x + C

D. none of these

Answer

Given:

$$\int \frac{1}{sin^2 x\, cos^2 x} dx$$

As we know $\sin^2 x + \cos^2 x = 1$

$$\int \frac{1}{sin^2 x\, cos^2 x} dx = \int \frac{(\sin x)^2 + (\cos x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\cos x)^2} + \frac{1}{(\sec x)^2} dx$$

$=\int(\sec x)^2+(\csc x)^2\, dx$

$=\tan x-\cot x+c$

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = ?$$

A. - cot x – tan x + C

B. - cot x + tan x + C

C. cot x – tan x + C

D. cot x + tan x + C

Answer

Given:

$$\int \frac{cos\, 2x}{cos^2 x\, sin^2 x} dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2} dx$$

$=\int(\csc x)^2-(\sec x)^2\, dx$

$=-\cot x-\tan x+c$

14. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = ?$$

A. 2 sin x + 2x cos α + C

B. 2 sin x – 2x cos α + C

C. - 2 sin x + 2x cos α + C

D. - 2 sin x – 2x cos α + C

Answer

Given:

$$\int \frac{(\cos 2x - \cos 2\alpha)}{(\cos x - \cos \alpha)} dx = \int \frac{2\sin(x+\alpha)\sin(x-\alpha)}{2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)} dx$$

$$= \int 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right) dx$$

=2∫cos (x)+cos(α)dx

=2 sin x+2x cos α+c

15. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{1+\cos 2x}\ dx = ?$$

A. $\sqrt{2}\cos x + C$

B. $\sqrt{2}\sin x + C$

C. $-\sqrt{2}\cos x + C$

D. $-\sqrt{2}\sin x + C$

Answer

Given:

$$\int \sqrt{1+\cos 2x}\,dx = \int \sqrt{2(\cos x)^2}\,dx$$

=√2 ∫cos x dx

=√2 sin x+c

16. Question

Mark (√) against the correct answer in each of the following:

$$\int \sqrt{1+\sin 2x}\ dx = ?$$

A. sin x + cos x + C

B. - sin x + cos x + C

C. sin x – cos x + C

D. - sin x – cos x + C

Answer

Given:

$$\int \sqrt{1+\sin 2x}\,dx = \int \sqrt{1+\frac{2\tan x}{1+(\tan x)^2}}\,dx$$

$$= \int \sqrt{\frac{(1+\tan x)^2}{(\sec x)^2}}\,dx$$

$$= \int \frac{1+\tan x}{\sec x}\,dx$$

=∫cos x + sin x dx

=sin x-cos x+c

17. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx = ?$$

A. cot x + tan x + C

B. - cot x + tan x + C

C. cot x – tan x + C

D. - cot x – tan x + C

Answer

Given:

$$\int \frac{cos\,2x}{sin^2\,x\,cos^2\,x} dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2}\, dx$$

=∫(csc x)^2-(sec x)^2 dx

=-cot x-tan x+c

18. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{(1-\cos 2x)} = ?$$

A. $\frac{1}{2}\cot x + C$

B. 2 cot x + C

C. $-\frac{1}{2}\cot x + C$

D. -2 cot x + C

Answer

Given:

$$\int \frac{dx}{(1-cos\,2x)} = \int \frac{1}{2(\sin x)^2} dx$$

$$= \frac{1}{2}\int (\csc x)^2 dx$$

$$= -\frac{1}{2}\cot x + c$$

19. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin 2x}{\sin x} dx = ?$$

A. 2 sin x + C

B. $\frac{1}{2}\sin x + C$

C. 2 cos x + C

D. $\frac{1}{2}\cos x + C$

Answer

Given:

$$\int \frac{sin\,2x}{sin\,x} dx = \int \frac{2\sin x \cos x}{\sin x} dx$$

=2∫cos x dx

=2 sin x+c

20. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1-\sin x)}{\cos^2 x} dx = ?$$

A. tan x + sec x + C

B. tan x – sec x + C

C. - tan x + sec x + C

D. - tan x – sec x + C

Answer

Given:

$$\int \frac{(1-sin\,x)}{cos^2\,x} dx = \int \frac{1}{(\cos x)^2} - \frac{\sin x}{(\cos x)^2} dx$$

=∫(sec x)^2-tan x sec x dx

=tan x-sec x+c

21. Question

Mark (√) against the correct answer in each of the following:

$$\int \cot^2 x \, dx = ?$$

A. - cot x – x + C

B. cot x – x + C

C. - cot x + x + C

D. cot x + x + C

Answer

Given:

$\int \cot^2 x\, dx = \int ((\csc x)^2 - 1)\, dx$

$= -\cot x - x + c$

22. Question

Mark (√) against the correct answer in each of the following:

$$\int \sec x(\sec x + \tan x)\, dx = ?$$

A. tan x - sec x + C

B. - tan x + sec x + C

C. tan x + sec x + C

D. - tan x – sec x + C

Answer

Given:

$\int \sec x (\sec x + \tan x)\, dx = \int (\sec x)^2 + \sec x \tan x\, dx$

$= \tan x + \sec x + c$

23. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx = ?$$

A. tan x + x + C

B. tan x – x + C

C. - tan x + x + C

D. - tan x - x + C

Answer

Given:

$$\int \frac{sec^2 x}{\operatorname{cosec}^2 x}\, dx = \int \frac{(\sin x)^2}{(\cos x)^2}\, dx$$

$= \int (\tan x)^2\, dx$

$= \int (\sec x)^2 - 1\, dx$

$= \tan x - x + c$

24. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin^2 x}{(1 + \cos x)}\, dx = ?$$

A. x + sin x + C

B. x – sin x + C

C. sin x – x + C

D. - sin x – x + C

Answer

Given: $\int \frac{sin^2 x}{(1+cos\, x)} dx = \int \frac{1-(\cos x)^2}{(1+\cos x)} dx$

$= \int \frac{(1+\cos x)(1-\cos x)}{(1+\cos x)} dx$

=∫(1-cos x)dx

=x-sin x+c

25. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\cot x}{(\text{cosec } x - \cot x)} dx = ?$

A. - cosec x – cot x – x + C

B. cosec x – cot x – x + C

C. - cosec x + cot x – x + C

D. cosec x + cot x – x + C

Answer

Given:

$\int \frac{cot\, x}{(\text{cosec} x - cot\, x)} dx = \int \frac{\cot x\,(\csc x + \cot x)}{((\csc x)^2 - (\cot x)^2)} dx$

=∫cot x csc x+(csc x)^2 dx

=-csc x-cot x+c

26. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\sin x}{(1+\sin x)} dx = ?$

A. sec x + tan x + x + C

B. sec x – tan x + x + C

C. - sec x + tan x + x + C

D. None of these

Answer

Given:

$\int \frac{sin\, x}{(1+sin\, x)} dx$

Multiply and divide by (1-sin x)

$= \int \frac{\sin x - (\sin x)^2}{1-(\sin x)^2} dx$

$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$

$=\int(\tan x \sec x-(\tan x)^2)dx$

$=\int(\tan x \sec x-(\sec x)^2+1)dx$

$=\sec x-\tan x+x+c$

27. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(1+\sin x)}{(1-\sin x)} dx = ?$$

A. 2 tan x + 2 sec x + x + C

B. 2 tan x + 2 sec x – x + C

C. tan x + sec x – x + C

D. None of these

Answer

Given:

$$\int \frac{(1+sin\,x)}{(1-sin\,x)} dx$$

Multiply and divide with (1 + sin x) to get,

$$\int \frac{(1+sin\,x)}{(1-sin\,x)} dx$$

$$= \int \frac{1+(\sin x)^2+2\sin x}{1-(\sin x)^2} dx$$

$$= \int \frac{1+(\sin x)^2+2\sin x}{(\cos x)^2} dx$$

$=\int(\sec x)^2+(\tan x)^2+2 \tan x \sec x\, dx$

$=\int 2(\sec x)^2-1+2 \tan x \sec x\, dx$

$=2 \tan x-x+2 \sec x+c$

28. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{1}{(1+\cos x)} dx = ?$$

A. - cot x + cosec x + C

B. cot x – cosec x + C

C. cot x + cosec x + C

D. None of these

Answer

Given:

$$\int \frac{1}{(1+cos\,x)} dx$$

Multiply and divide by (1-cos x)

$$\int \frac{1}{(1+\cos x)}dx = \int \frac{1-\cos x}{1-(\cos x)^2}dx$$

$$= \int \frac{1-\cos x}{(\sin x)^2}dx$$

$=\int(\csc x)^2$-cot x csc x dx

=-cot x+csc x+c

29. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin^{-1}(\cos x)dx = ?$$

A. cosec x + C

B. $\frac{\pi x}{2} + \frac{x^2}{2} + C$

C. $\frac{\pi x}{2} - \frac{x^2}{2} + C$

D. $\frac{x^2}{2} - \frac{\pi x}{2} + C$

Answer

Given:

$$\int sin^{-1}(\cos x)\,dx$$

$$\sin^{-1}(\cos x) + \cos^{-1}(\cos x) = \frac{\pi}{2}$$

$$= \int \frac{\pi}{2} - \cos^{-1}(\cos x)\,dx$$

$$= \int \frac{\pi}{2} - x\,dx$$

$$= \frac{\pi}{2}x - \frac{x^2}{2} + c$$

30. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}\left\{\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}\right\}dx = ?$$

A. $\frac{-1}{\left(1+x^2\right)} + C$

B. $\frac{1}{\sqrt{1+x^2}} + C$

C. $\frac{1}{\sqrt{1-x^2}}+C$

D. $\frac{x^2}{2}+C$

Answer

Given:

$$\int tan^{-1}\left\{\sqrt{\frac{1-cos\,2x}{1+cos\,2x}}\right\}dx = \int \tan^{-1}\sqrt{\frac{2(\sin x)^2}{2(\cos x)^2}}dx$$

$=\int \tan^{-1}(\tan x)dx$

$=\int x\,dx$

$=\frac{x^2}{2}+c$

31. Question

Mark (√) against the correct answer in each of the following:

$$\int \cot^{-1}\left(\frac{\sin 2x}{1-\cos 2x}\right)dx = ?$$

A. $\frac{-1}{\left(1+x^2\right)}+C$

B. $\frac{-1}{\left(1-x^2\right)}+C$

C. $\frac{x^2}{2}+C$

D. $2x^2+C$

Answer

Given:

$$\int cot^{-1}\left(\frac{sin\,2x}{1-cos\,2x}\right)dx = \int \cot^{-1}\left(\frac{2\sin x\cos x}{1-1+2(\sin x)^2}\right)dx$$

$=\int \cot^{-1}(\cot x)\,dx$

$=\int x\,dx$

$=\frac{x^2}{2}+c$

32. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right)dx = ?$$

A. $-x^2+C$

B. x^2+C

C. $\frac{x^2}{2}+C$

D. $2x^2+C$

Answer

Given:

$$\int sin^{-1}\left(\frac{2\,tan\,x}{1+tan^2\,x}\right)dx=\int \sin^{-1}(\sin 2x)\,dx$$

$=\int 2x\,dx$

$=x^2+c$

33. Question

Mark (√) against the correct answer in each of the following:

$$\int \cos^{-1}\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)dx=?$$

A. x^2+C

B. $-x^2+C$

C. $\frac{1}{\sqrt{1+x^2}}+C$

D. $\frac{1}{\sqrt{1-x^2}}+C$

Answer

Given:

$$\int cos^{-1}\left(\frac{1-tan^2\,x}{1+tan^2\,x}\right)dx=\int \cos^{-1}(\cos 2x)\,dx$$

$=\int 2x\,dx$

$=x^2+c$

34. Question

Mark (√) against the correct answer in each of the following:

$$\int \tan^{-1}(\operatorname{cosec} x-\cot x)\,dx=?$$

A. $\frac{x^2}{4}+C$

B. $\frac{-x^2}{4}+C$

C. $\frac{x^2}{2}+C$

D. $\frac{-x^2}{2}+C$

Answer

Given:

$$\int tan^{-1}(\text{cosec}x - cot\, x)dx = \int \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)dx$$

$$= \int \tan^{-1}\left(\frac{2\sin\frac{x}{2}\sin\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)dx$$

$$= \int \tan^{-1}\left(\tan\frac{x}{2}\right)dx$$

$$= \int \frac{x}{2}dx$$

$$= \frac{x^2}{4}+c$$

35. Question

Mark (√) against the correct answer in each of the following:

$$\int\left(\frac{\left(x^4+1\right)}{\left(x^2+1\right)}\right)dx = ?$$

A. $\frac{x^3}{3}+x-\tan^{-1}x+C$

B. $\frac{x^3}{3}-x-2\tan^{-1}x+C$

C. $\frac{x^3}{3}+x-2\tan^{-1}x+C$

D. None of these

Answer

Given:

$$\int\left(\frac{(x^4+1)}{(x^2+1)}\right)dx = \int \frac{(x^4+2x^2+1)}{(x^2+1)} - \frac{2x^2}{(x^2+1)}dx$$

$$= \int \frac{(x^2+1)^2}{(x^2+1)}dx - 2\left\{\int \frac{(x^2+1)}{(x^2+1)} - \frac{1}{(x^2+1)}dx\right\}$$

$$= \int (x^2+1)dx - 2\{x-\tan^{-1}x\}+c$$

$$= \frac{x^3}{3} - x - 2\tan^{-1}x + c$$

36. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(ax+b)}{(cx+d)}dx = ?$$

A. $\frac{ax}{c} + \log|cx+d| + C$

B. $\frac{a}{c} + \log|cx+d| + C$

C. $\frac{ax}{c} + \frac{(bc-ad)}{c^2}\log|cx+d| + C$

D. None of these

Answer

Given:

$$\int \frac{(ax+b)}{(cx+d)}dx = \int \frac{ax}{cx+d} + \frac{b}{cx+d}dx$$

$$= a\int \frac{x}{cx+d} \times \frac{c}{c}dx + b\int \frac{1}{cx+d}dx$$

$$= \frac{a}{c}\left(\int \frac{cx+d}{cx+d}dx - \frac{d}{cx+d}\right) + b\ln|cx+d| + c$$

$$= \frac{a}{c}\left(x - \frac{d}{c}\ln|cx+d|\right) + \frac{b}{c}\ln|cx+d| + c$$

$$= \frac{a}{c}x + \frac{(bc-ad)}{c^2}\ln|cx+d| + c$$

37. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{(\sin^3 x + \cos^3 x)}{\sin^2 x \cos^2 x}dx = ?$$

A. sin x – cos x + C

B. tan x – cos x + C

C. sec x – cosec x + C

D. None of these

Answer

Given:

$$\int \frac{(sin^3 x + cos^3 x)}{sin^2 x\, cos^2 x}dx = \int \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2} + \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2}dx$$

=∫(tan x sec x + csc x cot x)dx

=sec x-csc x+c

38. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\sin(x-\alpha)} dx = ?$$

A. $x\cos\alpha + (\sin\alpha)\log|\sin(x-\alpha)| + C$

B. $x\sin\alpha + (\sin\alpha)\log|\sin(x-\alpha)| + C$

C. $x\cos\alpha - (\sin\alpha)\log|\sin(x-\alpha)| + C$

D. $x\sin\alpha - (\sin\alpha)\log|\sin(x-\alpha)| + C$

Answer

Given:

$$\int \frac{\sin x}{\sin(x-\alpha)} dx$$

Let x-α=t

dx=dt

$$I = \int \frac{\sin(t+\alpha)}{\sin t} dx$$

$$= \int \frac{\sin t\cos\alpha + \cos t\sin\alpha}{\sin t} dt$$

=∫cos α+sin α cot t dt

=t cos α+sin α ln|sin t |+c

=(x-α) cos α+(sinα)ln|sin(x-α) |+c

=x cos α+ (sin α)ln|sin(x-α) |+c

39. Question

Mark (√) against the correct answer in each of the following:

$$\int \sin 3x \sin 2x \, dx = ?$$

A. $-\frac{1}{5}\cos 5x + C$

B. $\frac{1}{2}\sin x + \frac{1}{10}\sin 5x - C$

C. $\frac{1}{2}\sin x - \frac{1}{10}\sin 5x - C$

D. $-\frac{1}{3}\cos 3x - \frac{1}{2}\sin 2x + C$

Answer

Given:

$$\int \sin 3x \sin 2x \, dx = \frac{1}{2}\int 2\sin 3x \sin 2x \, dx$$

$$= \frac{1}{2}\int \cos x - \cos 5x \, dx$$

$$= \frac{1}{2}\left\{\frac{\sin x}{1} - \frac{\sin 5x}{5}\right\} + c$$

$$= \frac{\sin x}{2} - \frac{\sin 5x}{10} + c$$

40. Question

Mark (√) against the correct answer in each of the following:

$$\int \cos\ 3x\ \sin\ 2x\ dx = ?$$

A. $\frac{1}{2}\cos\ x - \frac{1}{10}\cos\ 5x + C$

B. $-\frac{1}{2}\sin\ x + \frac{1}{10}\sin\ 5x + C$

C. $-\frac{1}{2}\cos\ x + \frac{1}{10}\cos\ 5x + C$

D. None of these

Answer

Given:

$$\int \cos 3x \sin 2x\, dx = \frac{1}{2}\int 2\cos 3x \sin 2x\, dx$$

$$= \frac{1}{2}\int \sin 5x + \cos x\, dx$$

$$= \frac{1}{2}\left\{\frac{-\cos 5x}{5} + \frac{\sin x}{1}\right\} + c$$

$$= -\frac{\cos 5x}{10} + \frac{\sin x}{2} + c$$

41. Question

Mark (√) against the correct answer in each of the following:

$$\int \cos\ 4x\ \cos\ x\ dx = ?$$

A. $\frac{1}{5}\sin\ 5x + \frac{1}{3}\sin\ 3x + C$

B. $\frac{1}{5}\cos\ 5x - \frac{1}{3}\cos\ 3x + C$

C. $\frac{1}{10}\sin\ 5x + \frac{1}{6}\sin\ 3x + C$

D. None of these

Answer

Given:

$$\int \cos 4x \cos x\, dx = \frac{1}{2}\int 2\cos 4x \cos x\, dx$$

$$= \frac{1}{2}\int \cos 5x + \cos 3x \, dx$$

$$= \frac{1}{2}\left\{\frac{\sin 5x}{5} + \frac{\sin 3x}{3}\right\} + c$$

$$= \frac{\sin 5x}{10} + \frac{\sin 3x}{6} + c$$