11. Circles

Exercise 11A

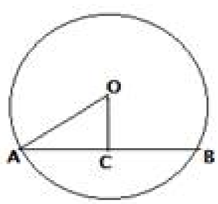
1. Question

A chord of length 16 cm is drawn in a circle of radius 10 cm. Find the distance of the chord from the center of the circle.

Answer

Let AB be a chord of a circle with center O. OC $\perp$ AB, then

AB = 16 cm, and OA = 10 cm.



OC $\perp$ AB

Therefore,

OC bisects AB at C

AC = (1/2) AB

⇒ AC = (1/2) 16

⇒ AC = 8 cm

In triangle OAC,

$OA^2 = OC^2 + AC^2$

⇒ $10^2 = OC^2 + 8^2$

⇒ $100 = OC^2 + 64$

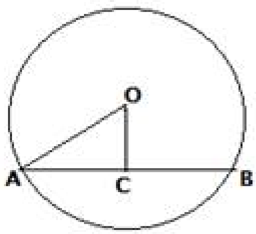
⇒ $OC^2 = 36$

⇒ OC= 6

2. Question

Find the length of a chord which is at a distance of 3 cm from the center of a circle of radius 5 cm.

Answer



Let distance OC = 3 cm

Radius = OA = 5 cm

Draw OC ⊥ AB

In triangle OCA,

$OA^2 = OC^2 + AC^2$

$\Rightarrow 5^2 = 3^2 + AC^2$

$\Rightarrow AC^2 = 16$

⇒ AC = 4 cm __________________ (i)

Now,

AB = 2 AC

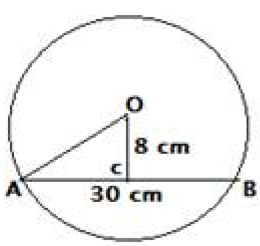
⇒ AB = 8 cm [From equation (i)]

Hence, length of a chord = 8 cm.

3. Question

A chord of a length 30 cm is drawn at a distance of 8 cm from the center of a circle. Find out the radius of the circle.

Answer



Let distance OC = 8 cm

Chord AB = 30 cm

Draw OC ⊥ AB

Therefore,

OC bisects AB at C

AC = (1/2) AB

⇒ AC = (1/2) 30

⇒ AC = 15 cm

In triangle OCA,

$OA^2 = OC^2 + AC^2$

⇒ $OA^2 = 8^2 + 15^2$

⇒ $OA^2 = 64 + 225$

⇒ $OA^2 = 289$

⇒ OA = 17 cm

Hence, radius of the circle = 17 cm.

4. Question

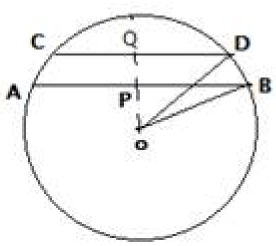
In a circle of radius 5 cm, AB and CD are two parallel chords of lengths 8 cm and 6 cm respectively. calculate the distance between the chords if they are

(i) on the same side of the center

(ii) on the opposite sides of the center.

Answer

(i)



Let radius OB = OD = 5 cm

Chord AB = 8 cm

Chord CD = 6 cm

BP = (1/2) AB

⇒ BP = (1/2) 8 = 4 cm

DQ = (1/2) CD

⇒ DQ = (1/2) 6 = 3 cm

In triangle OPB,

$OP^2 = OB^2 - BP^2$

⇒ $OP^2 = 5^2 - 4^2$

⇒ $OP^2 = 25 - 16$

⇒ $OP^2 = 9$

⇒ OP = 3 cm

In triangle OQD,

$OQ^2 = OD^2 - DQ^2$

⇒ $OQ^2 = 5^2 - 3^2$

⇒ $OQ^2 = 25 - 9$

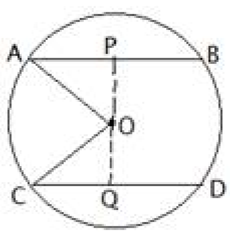
⇒ $OQ^2 = 16$

⇒ OQ = 4 cm

Now,

PQ = OQ – OP = 4 – 3 = 1

Hence, distance between chords = 1 cm.

(ii) 

Let radius OA = OC = 5 cm

Chord AB = 8 cm

Chord CD = 6 cm

AP = (1/2) AB

⇒ AP = (1/2) 8 = 4 cm

CQ = (1/2) CD

⇒ CQ = (1/2) 6 = 3 cm

In triangle OAP,

$OP^2 = OA^2 - AP^2$

⇒ $OP^2 = 5^2 - 4^2$

⇒ $OP^2 = 25 - 16$

⇒ $OP^2 = 9$

⇒ OP = 3 cm

In triangle OQD,

$OQ^2 = OC^2 - CQ^2$

⇒ $OQ^2 = 5^2 - 3^2$

⇒ $OQ^2 = 25 - 9$

⇒ $OQ^2 = 16$

⇒ OQ = 4 cm

Now,

PQ = OP + OQ = 3 + 4 = 7

Hence, distance between chords = 7 cm.

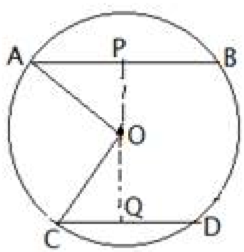
5. Question

Two parallel chords of lengths 30 cm and 16cm are drawn on the opposite sides of the center of a circle of radius 17 cm. Find the distance between the chords.

Answer

Let radius OA = OC = 17 cm

Chord AB = 30 cm and CD = 16 cm



Draw OL and OM

Therefore,

AP = (1/2) AB

⇒ AP = (1/2) 30 = 15 cm

CQ = (1/2) CD

⇒ CQ = (1/2) 16 = 8 cm

In triangle OAP,

$OP^2 = OA^2 - AP^2$

⇒ $OP^2 = 17^2 - 15^2$

⇒ $OP^2 = 289 - 225$

⇒ $OP^2 = 64$

⇒ OP = 8 cm

In triangle OQD,

$OQ^2 = OC^2 - CQ^2$

$\Rightarrow OQ^2 = 17^2 - 8^2$

$\Rightarrow OQ^2 = 289 - 64$

$\Rightarrow OQ^2 = 225$

$\Rightarrow OQ = 15$ cm

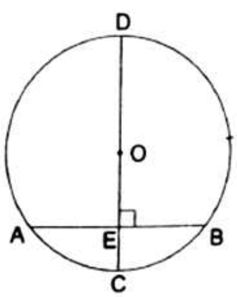
Now,

PQ = OP + OQ = 8 + 15 = 23

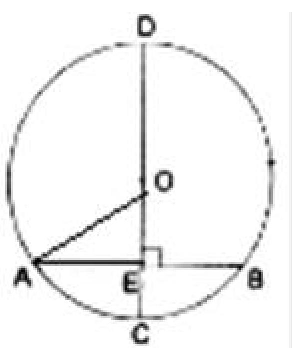
Hence, distance between chords = 23 cm.

6. Question

In the given figure, the diameter CD of a circle with center O is perpendicular to chord AB. If $AB = 12$ cm and $CE = 3$ cm, calculate the radius of the circle.



Answer



Let radius OA = OC = OD = r

Chord AB = 12 cm

OE = OC − CE

⇒ OE = r − 3

AE = (1/2) AB

⇒ AE = (1/2) 12 = 6 cm

In triangle AOE,

$OA^2 = AE^2 + OE^2$

⇒ $r^2 = 6^2 + (r - 3)^2$

⇒ $r^2 = 36 + r^2 + 9 - 6r$

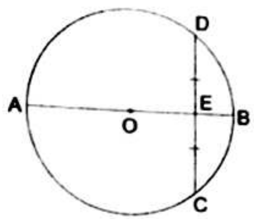
⇒ $6r = 45$

⇒ $r = 7.5$ cm

Hence, radius of circle = 7.5 cm.

7. Question

In the given figure, a circle with center O is given in which a diameter AB bisects the chord CD at a point E such that $CE = ED = 8$ cm and $EB = 4$ cm. Find the radius of the circle.



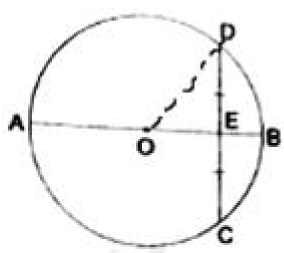
Answer

Let radius OA = OB = OD = r

DE = 8 cm

OE = OB − BE

⇒ OE = r − 4



In triangle ODE,

$OD^2 = DE^2 + OE^2$

⇒ $r^2 = 8^2 + (r - 4)^2$

⇒ $r^2 = 64 + r^2 + 16 - 8r$

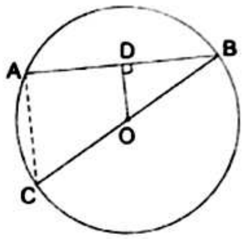
⇒ $8r = 80$

⇒ $r = 10$ cm

Hence, radius of circle = 10 cm.

8. Question

In the adjoining figure, OD is perpendicular to the chord AB of a circle with center O. If BC is a diameter, show that $AC \parallel CD$ and $AC = 2 \times OD$.



Answer

Given OD ⊥ AB

In triangle ABC,

D is the mid-point of AB

∴ AD = DB

O is the mid-point of BC

∴ OC = OB

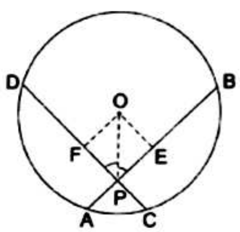
We say, AC||OD

(1/2) AC = OD [Mid-point theorem in triangle ABC]

⇒ AC = 2 × OD Proved.

9. Question

In the given figure, O is the center of a circle in which chords AB and CD intersect at P such that PO bisects $\angle BPD$. Prove that $AB = CD$.



Answer

Proof

In ΔOEP and ΔOFP,

∠OEP = ∠OFP [equal to 90°]

OP = OP [common]

∠OPE = ∠OPF [OP bisects ∠BPD]

Therefore,

ΔOEP = ΔOFP [By angle-side-angle]

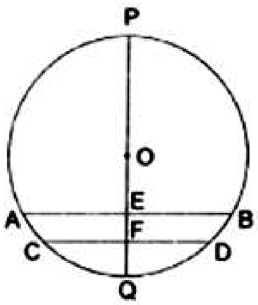
∴ OE = OF

AB = CD [Chords are equidistant from the center]

Hence, AB = CD Proved.

10. Question

Prove that the diameter of a circle perpendicular to one of the two parallel chords of a circle is perpendicular to the other and bisects it.



Answer

∠PFD = ∠PEB [equal to 90°]

∴PF ⊥ CD and OF ⊥ CD

We know that the perpendicular from the center of a circle to chord, bisect the chord.

Therefore,

CF = FD Proved.

11. Question

Prove that two different circles cannot intersect each other at more than two points.

Answer

Let two different circles intersect at three distinct points A, B and C.

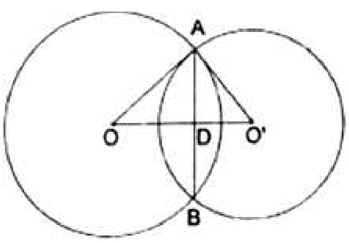
Then, these points are already non-collinear.

A unique circle can be drawn to pass through these points. This is a contradiction.

Hence, two different circles cannot intersect each other at more than two points.

12. Question

Two circles of radii 10 cm and 8 cm intersect each other, and the length of the common chord is 12 cm. Find the distance between their centers.



Answer

Let,

Radius OA = 10 cm and O'A = 8 cm

Chord AB = 12 cm

Now,

AD = (1/2) AB

⇒ AD = (1/2) 12 = 6 cm

In triangle OAD,

$OD^2 = OA^2 - AD^2$

⇒ $OD^2 = 10^2 - 6^2$

⇒ $OD^2 = 100 - 36$

⇒ $OD^2 = 64$

⇒ OD = 8 cm

In triangle O'AD,

$O'D^2 = O'A^2 - AD^2$

⇒ $O'D^2 = 8^2 - 6^2$

⇒ $O'D^2 = 64 - 36$

⇒ $O'D^2 = 28$

⇒ O'D = 2 √7 cm

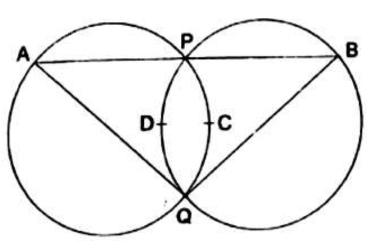
Now,

OO' = OD + O'D = $\left(8+2\sqrt{7}\right)$ cm

Hence, distance between their centers = $\left(8+2\sqrt{7}\right)$ cm

13. Question

Two equal circles intersect in P and Q. A straight line through P meets the circles in A and B. Prove that $QA = QB$.



Answer

Join PQ,

PQ is the common chord of both the circles.

Thus,

arc PCQ = arc PDQ

∴ ∠QAP = ∠QBP

∴ QA = QB Proved.

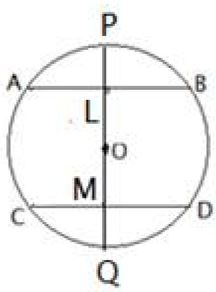
14. Question

If a diameter of a circle bisects each of the two chords of a circle then prove that the chords are parallel.

Answer

Let AB and CD are two chords of a circle with center O.

Diameter POQ bisect s them at L and M.



Then,

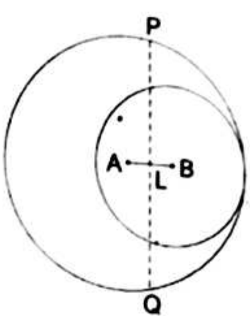
OL ⊥ AB and OM ⊥ CD

∴ ∠ALM = ∠LMD

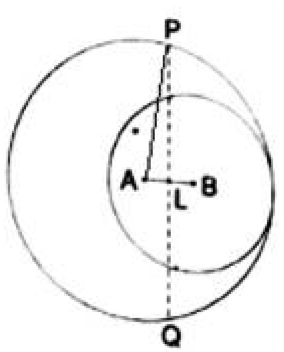
∴ AB||CD [Alternate angles]

15. Question

In the adjoining figure, two circles with centers at A and B, and of radii 5 cm and 3 cm touch each other internally. If the perpendicular bisector of meets the bigger circle in P and Q, find the length of PQ.



Answer



Join AP.

Let PQ intersect AB at L,

Then, AB = 5 – 3 = 2 cm

PQ is the perpendicular bisector of AB,

Then,

AL = (1/2) AB

⇒ AL = (1/2) 2 = 1 cm

In triangle APL,

$PL^2 = PA^2 - AL^2$

⇒ $PL^2 = 5^2 - 1^2$

⇒ $PL^2 = 25 - 1$

⇒ $PL^2 = 24$

⇒ PL = 2 √6 cm

Now,

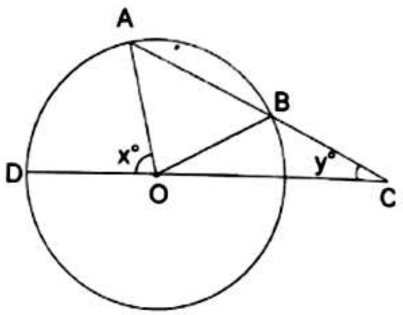
PQ = 2 PL

⇒ PQ = 2 × 2 √6

⇒ PQ = 4 √6 cm

16. Question

In the given figure, AB is a chord of a circle with center O and AB is produced to C such that $BC = OB$. Also, is joined and produced to meet the circle in D. If $\angle ACD = y^\circ$ and $\angle AOD = x^\circ$, prove that $x = 3y$.



Answer

Given, OB = OC

Then, ∠BOC = ∠BCO = y°

External ∠OBA = ∠BOC + ∠BCO = (2y)°

Now,

OA = OB

Then, ∠OAB = ∠OBA = (2y)°

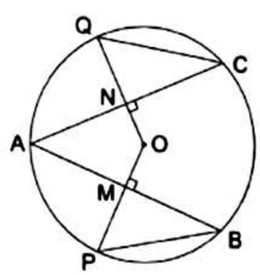
External ∠AOD = ∠OAC + ∠ACO

= ∠OAB + ∠BCO = (3y)°

∴ x° = (3y)°[Given $\angle AOD = x^\circ$,]

17. Question

In the adjoining figure, O is the center of a circle. If AB and AC are chords of the circle such that $AB = AC$, $OP \perp AB$ and $OQ \perp AC$, prove that $PB = QC$.



Answer

Given AB = AC

∴ (1/2)AB = (1/2)AC

OP ⊥ AB and OQ ⊥ AC

∴ MB = NC

⇒ ∠PMB = ∠QNC [90°]

Equal chords are equidistant from the center.

⇒ OM = ON

OP = OQ

⇒ OP – OM = OQ – ON

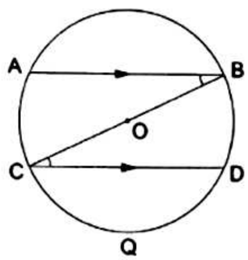
⇒ PM = QN

∴ ΔABC ≅ ΔABC [By side-angle-side criterion of congruence]

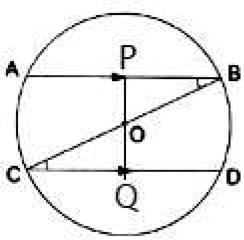
∴ PB = QC Proved.

18. Question

In the adjoining figure, BC is a diameter of a circle with center O. If AB and AC are two chord such that $AB \parallel CD$, prove that $AB = CD$.



Answer



Draw, OP ⊥ AB and OQ ⊥ CD

In triangle OBP and triangle OQC,

∠OPB = ∠OQC [Angle = 90°]

∠OBP = ∠OCD [Alternate angle]

OB = OC [Radius]

By side-angle-side criterion of congruence

ΔOBP ≅ ΔOQC

∴ OP = OQ

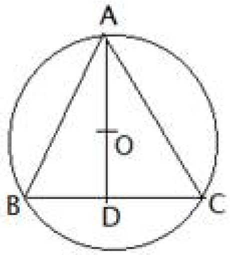
The chords equidistant from the center are equal.

∴ AB = CD Proved.

19. Question

An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Answer



Let ABC be an equilateral triangle of side 9 cm.

And AD be one of its medians.

Then,

AD ⊥ BC

BD = (1/2) BC

⇒ BD = (1/2) 9 = 4.5 cm

In triangle ADB,

$AD^2 = AB^2 - BD^2$

⇒ $AD^2 = 9^2 - (9/2)^2$

⇒ $AD^2 = 81 - (81/4)$

⇒ AD = (9√3)/2

In an equilateral triangle the centroid and circumcenter coincide and AO: OD = 2: 1

∴ radius AO = (2/3) AD

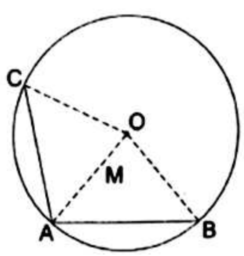
= (2/3) (9√3)/2

= 3√3 cm

Hence, radius of circle = 3√3 cm.

20. Question

In the adjoining figure, AB and AC are two equal chords of a circle with center O. Show that O lies on the bisector of $\angle BAC$.



Answer

In triangle OAB and triangle OAC,

AB = AC [Given]

OB = CO [Radius]

OA = OA [Common]

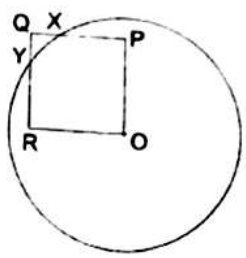
By side-side-side criterion of congruence

ΔOAB ≅ ΔOAC

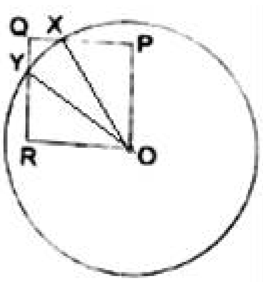
∴ ∠OAB = ∠OAC Proved.

21. Question

In the adjoining figure, $OPQR$ is a square. A circle drawn with center O cuts the suare in X and Y. Prove that $QX = QY$.



Answer



In triangle OPX and triangle ORY,

OX = OY [Radius]

∠OPX = ∠ORY [Common]

OP = OR [Sides of square]

By side-angle-side criterion of congruence,

ΔOPX ≅ ΔORY

∴ PX = RY

⇒ PQ – PX = QR – RY [PQ = QR]

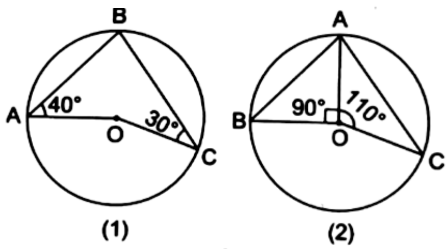
⇒ QX = QY Proved.

Exercise 11B

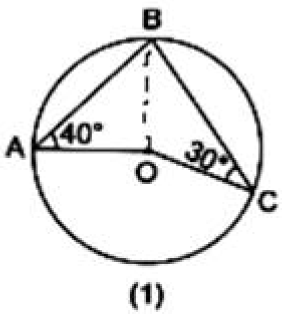
1. Question

(i) In Figure (1), O is the center of the circle. If $\angle OAB = 40°$ and $\angle OCB = 30°$, find $\angle AOC$. (ii) In figure (2), A, B and C are three points on the circle with center O such that $\angle AOB = 90°$ and $\angle AOC = 110°$.

Find $\angle BAC$.



Answer



(i) Join OB.

∠OAB = ∠OBA = 40°[Because OB = OA]

∠OCB = ∠OBC = 30°[Because OB = OC]

∠ABC = ∠OBA + ∠OBC

⇒ ∠ABC = 40°+ 30°

⇒ ∠ABC = 70°

∠AOC = 2 × ∠ABC

⇒ ∠AOC = 2 × ∠ABC

⇒ ∠AOC = 2 × 70°

⇒ ∠AOC = 140°

(ii) $\angle BAC = 80°$

∠BOC = 360° - (∠AOB + ∠AOC) [Sum of all angles at a point = 360°]

⇒ ∠BOC = 360° - (90° + 110°)

⇒ ∠BOC = 360° - 200°

⇒ ∠BOC = 160°

We know that ∠BOC = 2 × ∠BAC

⇒ ∠BAC = (1/2) × ∠BOC

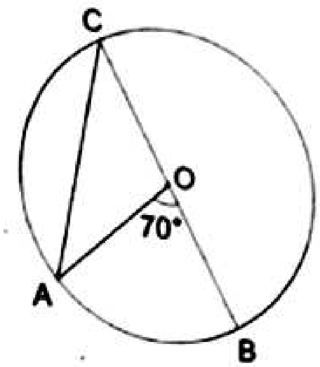
⇒ ∠BAC = (1/2) × 160°

⇒ ∠BAC = 80°

2. Question

In the given figure, O is the center of the circle and $\angle AOB = 70°$.

Calculate the values of (i) $\angle OCA$, (ii) $\angle OAC$.



Answer

(i) ∠AOC + ∠AOB = 180°[Because BC is a straight line]

⇒ ∠AOC + 70°= 180°

⇒ ∠AOC + 70°= 180°

⇒ ∠AOC = 110°

OA = OC [Radius]

∴ ∠OAC = ∠OCA ________________ (i)

In triangle AOC,

∠OAC + ∠OCA + ∠AOC = 180°[Sum of angles of triangle]

⇒ 2 ∠OCA + 110° = 180°[From equation (i)]

⇒ 2 ∠OCA = 70°

⇒ 2 ∠OCA = 70°

⇒ ∠OCA = 35°

(ii) $\angle OAC = 35°$

∠AOC + ∠AOB = 180°[Because BC is a straight line]

⇒ ∠AOC + 70°= 180°

⇒ ∠AOC + 70°= 180°

⇒ ∠AOC = 110°

OA = OC [Radius]

∴ ∠OAC = ∠OCA ________________ (i)

In triangle AOC,

$\angle OAC + \angle OCA + \angle AOC = 180°$[Sum of angles of triangle]

$\Rightarrow 2\angle OAC + 110° = 180°$[From equation (i)]

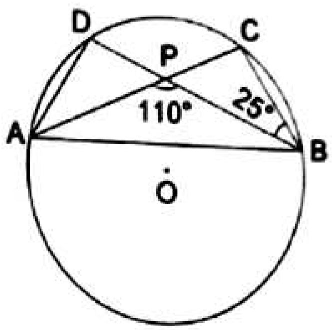
$\Rightarrow 2\angle OAC = 70°$

$\Rightarrow 2\angle OAC = 70°$

$\Rightarrow \angle OAC = 35°$

3. Question

In the given figure, O is the center of the circle. If $\angle PBC = 25°$ and $\angle APB = 110°$, find the value of $\angle ADB$.



Answer

$\angle BPC + \angle APB = 180°$[Because APC is a straight line]

$\Rightarrow \angle BPC + 110° = 180°$

$\Rightarrow \angle BPC = 70°$

In triangle BPC,

$\angle BPC + \angle PBC + \angle PCB = 180°$[Sum of angles of triangle]

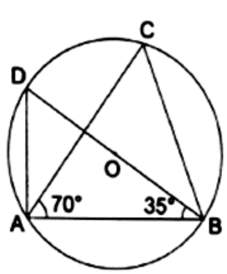
$\Rightarrow 70° + 25° + \angle PCB = 180°$

$\Rightarrow \angle PCB = 85°$

$\therefore \angle ADB = \angle PCB = 85°$[Angles in the same segment of a circle]

4. Question

In the given figure, O is the center of the circle. If $\angle ABD = 35°$ and $\angle BAC = 70°$, find $\angle ACB$.



Answer

In triangle ABD,

∠ABD + ∠BAD + ∠ADB = 180°[Sum of angles of triangle]

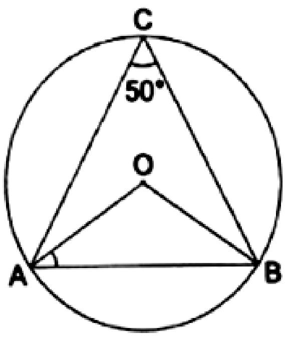
⇒ 35°+ 90° + ∠ADB = 180°

⇒ ∠ADB = 55°

∴ ∠ACB = ∠ADB = 55°[Angles in the same segment of a circle]

5. Question

In the given figure, O is the center of the circle. If $\angle ACB = 50^\circ$, find $\angle OAB$.



Answer

∠AOB = 2 × ∠ACB

⇒ ∠AOB = 2 × 50°

⇒ ∠AOB = 100°

OA =OB [Radius of the circle]

∴ ∠OAB = ∠OBA __________________ (i)

In triangle AOB,

∠OAB + ∠OBA + ∠AOB = 180°[Sum of angles of triangle]

⇒ 2 ∠OAB + 100° = 180°[From equation (i)]

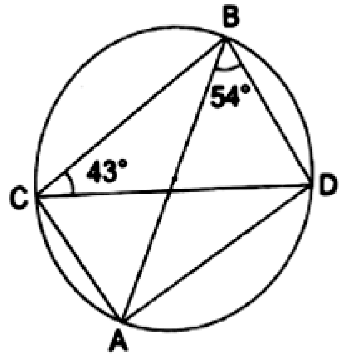
⇒ 2 ∠OAB = 80°

⇒ ∠OAB = 40°

6. Question

In the given figure, $\angle ABD = 54^\circ$ and $\angle BCD = 43^\circ$, calculate

(i) $\angle ACD$ (ii) $\angle BAD$ (iii) $\angle BDA$.



Answer

(i) $\angle ACD = 54^{\circ}$

∠ABD and ∠ACD are in the segment AD.

∴ ∠ACD = ∠ABD [Angles in the same segment of a circle]

∠ACD = 54°

(ii) ∠BAD = 43°

∠BAD and ∠BCD are in the segment BD.

∴ ∠BAD = ∠BCD [Angles in the same segment of a circle]

∠BAD = 43°

(iii) $\angle BDA = 83^{\circ}$

In triangle ABD,

∠ABD + ∠BAD + ∠BAD = 180°[Sum of angles of triangle]

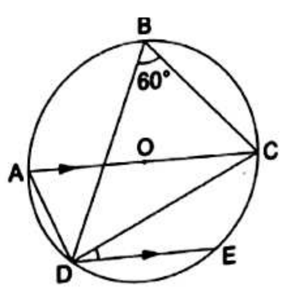
⇒ 54° + 43° + ∠BAD = 180°

⇒ 97° + ∠BAD = 180°

⇒ ∠BAD = 83°

7. Question

In the adjoining figure, DE is a chord parallel to diameter AC of the circle with center O. If $\angle CBD = 60^{\circ}$, calculate $\angle CDE$.



Answer

∠CAD and ∠CBD are in the segment BD.

∴ ∠CAD = ∠CBD [Angles in the same segment of a circle]

∠CAD = 60°

In triangle ACD,

∠CAD + ∠ADC + ∠ACD = 180°[Sum of angles of triangle]

⇒ 60° + 90° + ∠ACD = 180°

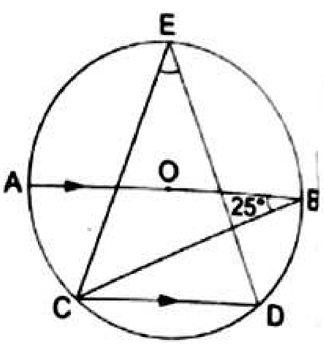
⇒ 150° + ∠ACD = 180°

⇒ ∠ACD = 30°

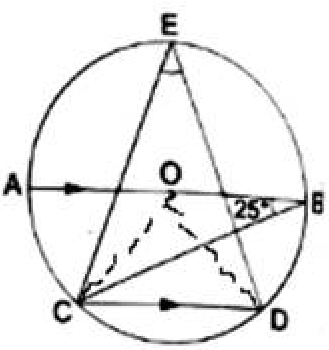
∴ ∠CDE = ∠ACD = 30°[Alternate angles]

8. Question

In the adjoining figure, O is the center of a circle. Chord CD is parallel to diameter AB. If $\angle ABC = 25^{\circ}$, calculate $\angle CED$.



Answer



Join OC and OD.

∠ABC = ∠BCD = 25°[Alternate angles]

The angle subtended by an arc of a circle at the center is double the angle subtended by the arc at any point on the circumference.

∴ ∠BOD = 2 × ∠BCD

⇒ ∠BOD = 2 × 25°

⇒ ∠BOD = 50°

Similarly,

$\angle AOC = 2 \times \angle ABC$

$\Rightarrow \angle AOC = 2 \times 25°$

$\Rightarrow \angle AOC = 50°$

Now,

$\angle AOB = 180°$ [AOB is a straight line]

$\Rightarrow \angle AOC + \angle COD + \angle BOD = 180°$

$\Rightarrow 50° + \angle COD + 50° = 180°$

$\Rightarrow 100° + \angle COD = 180°$

$\Rightarrow \angle COD = 80°$

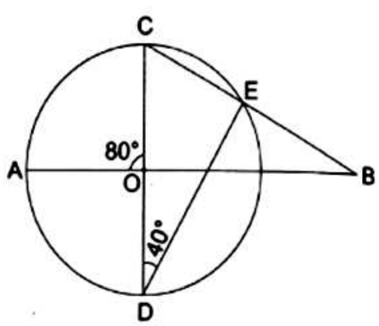
$\therefore \angle CED = (1/2) \angle COD$

$\Rightarrow \angle CED = (1/2)\ 80°$

$\Rightarrow \angle CED = 40°$

9. Question

In the given figure, AB and CD are straight lines through the center O of a circle. If $\angle AOC = 80°$ and $\angle CDE = 40°$, find (i) $\angle DCE$, (ii) $\angle ABC$.



Answer

(i) $\angle DCE = 50°$

In triangle CDE,

$\angle CDE + \angle CED + \angle DCE = 180°$[Sum of angles of triangle]

$\Rightarrow 40° + 90° + \angle DCE = 180°$

$\Rightarrow 130° + \angle DCE = 180°$

$\Rightarrow \angle DCE = 50°$

(ii) $\angle ABC = 30°$

$\angle AOC + \angle BOC = 180°$[Because AOB is a straight line]

$\Rightarrow 80° + \angle BOC = 180°$

⇒ ∠BOC = 100°

In triangle BOC,

∠OCB + ∠BOC + ∠OBC = 180°[Sum of angles of triangle]

⇒ 50° + 100° + ∠OBC = 180°[∠DCE = 50°]

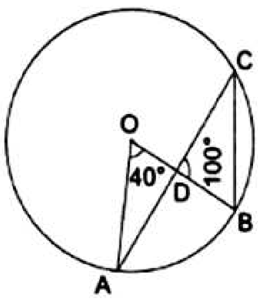
⇒ 150° + ∠OBC = 180°

⇒ ∠OBC = 30°

∴ ∠ABC = ∠OBC = 30°

10. Question

In the adjoining figure, O is the center of a circle, $\angle AOB = 40^\circ$ and $\angle BDC = 100^\circ$, find $\angle OBC$.



Answer

∠DCB = (1/2) ∠AOB [∠DCB = ∠ACB]

⇒ ∠DCB = (1/2) 40°

⇒ ∠DCB = 20°

In triangle BCD,

∠BDC + ∠DCB + ∠DBC = 180°[Sum of angles of triangle]

⇒ 100° + 20° + ∠OBC = 180°

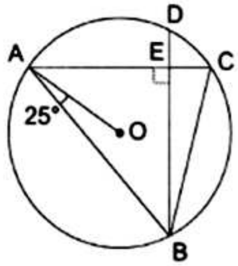
⇒ 120° + ∠DBC = 180°

⇒ ∠DBC = 60°

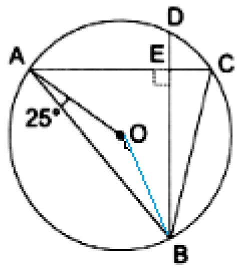
∴ ∠OBC = ∠DBC = 60°

11. Question

In the adjoining figure, chords AC and BD of a circle with center O, intersect at right angles at E. If $\angle OAB = 25^\circ$, calculate $\angle EBC$.



Answer



Join OB,

∴ OA = OB [Radius]

∴ ∠OAB = ∠OBA = 25°

In triangle AOB,

∠AOB + ∠OAB + ∠OBA = 180°[Sum of angles of triangle]

⇒ ∠AOB + 25° + 25° = 180°

⇒ ∠AOB + 50° = 180°

⇒ ∠AOB = 130°

Now,

∠ACB = (1/2) ∠AOB

⇒ ∠ACB = (1/2) 130°

⇒ ∠ACB = 65°

In triangle BEC,

∠EBC + ∠ECB + ∠BEC = 180°[Sum of angles of triangle]

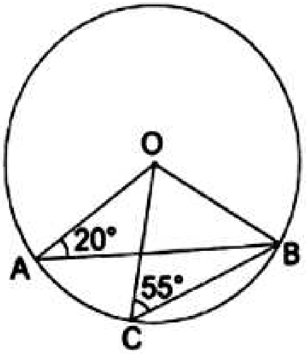
⇒ ∠EBC + 65° + 90° = 180°

⇒ ∠EBC + 155° = 180°

⇒ ∠EBC = 25°

12. Question

In the given figure, O is the center of a circle in which $\angle OAB = 20^\circ$ and $\angle OCB = 55^\circ$. Find (i) $\angle BOC$, (ii) $\angle AOC$.



Answer

(i) $\angle BOC = 70°$

OB = OC [Radius]

∴ ∠OBC = ∠OCB = 55°

In triangle OCB,

∠OBC + ∠OCB + ∠BOC = 180°[Sum of angles of triangle]

⇒ 55° + 55° + ∠BOC = 180°

⇒ 110° + ∠BOC = 180°

⇒ ∠BOC = 70°

(ii) $\angle AOC = 70°$

OA = OB [Radius]

∴ ∠OBA = ∠OAB = 20°

In triangle AOB,

∠OBA + ∠OAB + ∠AOB = 180°[Sum of angles of triangle]

⇒ 20° + 20° + ∠AOB = 180°

⇒ 40° + ∠AOB = 180°

⇒ ∠AOB = 140°

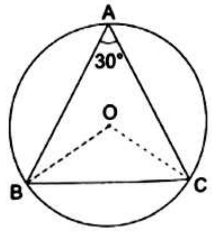
∴ ∠AOC = ∠AOB - ∠BOC

⇒ ∠AOC = 140° - 70°

⇒ ∠AOC = 70°

13. Question

In the given figure, $\angle BAC = 30°$. Show that BC is equal to the radius of the circumcircle of ΔABC whose center is O.



Answer

∠BOC = 2 × ∠BAC

⇒ ∠BOC = 2 × 30°

⇒ ∠BOC = 60°_______________ (i)

OB = OC

∴ ∠OBC = ∠OCB _________________ (ii)

In triangle AOB,

∠OBC + ∠OCB + ∠BOC = 180°[Sum of angles of triangle]

⇒ 2 ∠OCB + 60° = 180°

⇒ 2 ∠OCB = 120°

⇒ ∠OCB = 60°

∴ ∠OBC = 60°[From equation (ii)]

From equation (i) and (ii),

∠OBC = ∠OCB = ∠BOC = 60°

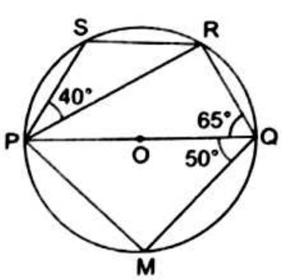
∴ BOC is an equilateral triangle.

∴ OB = OC = BC

Hence, BC is the radius of the circumcircle.

14. Question

In the given figure, PQ is a diameter of a circle with center O. If $\angle PQR = 65^\circ$, $\angle SPR = 40^\circ$ and $\angle PQM = 50^\circ$, find $\angle OPR, \angle OPM$ and $\angle PRS$.



Answer

In triangle PQR,

∠QPR + ∠PQR + ∠PRQ = 180°[Sum of angles of triangle]

⇒ ∠QPR + 65° + 90° = 180°

⇒ ∠QPR + 155° = 180°

⇒ ∠QPR = 25°_______________ (i)

In triangle PMQ,

∠QPM + ∠PMQ + ∠PQM = 180°[Sum of angles of triangle]

⇒ ∠QPM + 90° + 50° = 180°

⇒ ∠QPM + 140° = 180°

⇒ ∠QPM = 40°

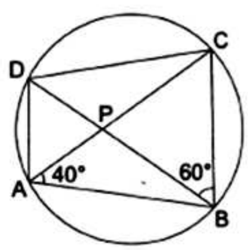
Now,

∠PRS = ∠QPR = 25°[Alternate angles]

Exercise 11C

1. Question

In the given figure, $ABCD$ is a cyclic quadrilateral whose diagonals intersect at P such that $\angle DBC = 60^\circ$ and $\angle BAC = 40^\circ$. Find (i) $\angle BCD$, (ii) $\angle CAD$.



Answer

(i) $\angle BCD = 80^\circ$

∠BAC = ∠BDC = 40°[Angles in the same segment]

In triangle BCD,

∠BCD + ∠DBC + ∠BDC = 180°[Sum of angles of triangle]

⇒ ∠BCD + 60° + 40° = 180°

⇒ ∠BCD + 100° = 180°

⇒ ∠BCD = 80°

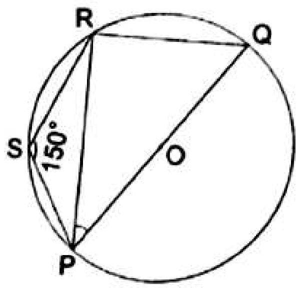
(ii) $\angle BCD = 80^\circ$

∠CAD = ∠CBD [Angles in the same segment]

⇒ ∠CAD = 40°

2. Question

In the given figure, POQ is a diameter and $PORS$ is a cyclic quadrilateral. If $\angle PSR = 150°$, find $\angle RPO$.



Answer

In cyclic quadrilateral PQRS,

∠PSR + ∠PQR = 180°[Opposite angles]

⇒ 150° + ∠PQR = 180°

⇒ ∠PQR = 30°

In triangle PQR,

∠RPQ + ∠PQR + ∠PRQ = 180°[Sum of angles of triangle]

⇒ ∠RPQ + 30° + 90° = 180°

⇒ ∠RPQ + 120° = 180°

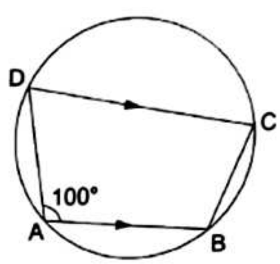
⇒ ∠RPQ = 60°

3. Question

In the given figure, $ABCD$ is a cyclic quadrilateral in which $AB \parallel DC$.

If $\angle BAD = 100°$, find

(i) $\angle BCD$ (ii) $\angle ADC$ (iii) $\angle ABC$.



Answer

(i) $\angle BCD = 80°$

∠BAD + ∠BCD = 180°[Opposite angles of a cyclic quadrilateral arc supplementary]

⇒ 100° + ∠BCD = 180°

⇒ ∠BCD = 80°

(ii) $\angle ADC = 80^\circ$

∠BAD + ∠ADC = 180°[Interior angles of same side]

⇒ 100° + ∠ADC = 180°

⇒ ∠ADC = 80°

(iii) $\angle ABC = 100^\circ$

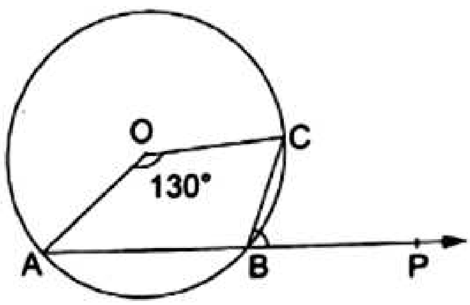
∠BCD + ∠ABC = 180°[Interior angles of same side]

⇒ 80° + ∠ABC = 180°

⇒ ∠ABC = 100°

4. Question

In the given figure, O is the center of the circle and arc ABC subtends an angle of 130° at the center. If AB is extended to P, find $\angle PBC$.



Answer

Reflex ∠AOC = 360° - ∠AOC

= 360° - 130°

= 230°

∴∠ABC = (1/2) ∠AOC

⇒ ∠ABC = (1/2) 230°

⇒ ∠ABC = 115°

Now,

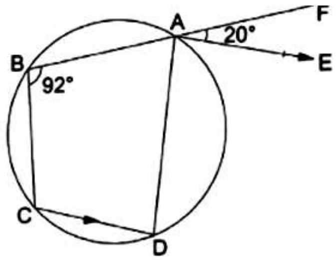
∠ABC + ∠PBC = 180°[Because ABP is a straight line]

⇒ 115° + ∠PBC = 180°

⇒ ∠PBC = 65°

5. Question

In the given figure, $ABCD$ is a cyclic quadrilateral in which AE is drawn parallel to CD, and BA is produced. If $\angle ABC = 92^\circ$ and $\angle FAE = 20^\circ$, find $\angle BCD$.



Answer

ABCD is cyclic quadrilateral.

∴ ∠ABC + ∠ADC = 180°

⇒ 92° + ∠ADC = 180°

⇒ ∠ADC = 88°

AE || CD

∴∠EAD = ∠ADC = 88°

Now,

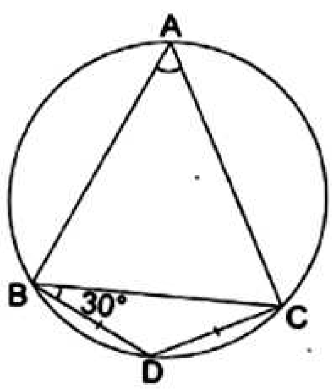
∠BCD = 180° - ∠DAB⇒ ∠BCD = ∠DAF = ∠EAD + ∠EAF

⇒ ∠BCD = 88° + 20°

⇒ ∠BCD = 108°

6. Question

In the given figure, $BD = DC$ and $\angle CBD = 30^\circ$, find $m(\angle BAC)$.



Answer

BD = CD

∴∠CBD = ∠BCD = 30°

In triangle BCD,

∠BDC + ∠BCD + ∠CBD = 180°[Sum of angles of triangle]

⇒ ∠BDC + 30° + 30° = 180°

⇒ ∠BDC + 60° = 180°

⇒ ∠BDC = 120°

Now,

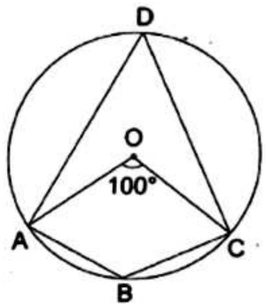
∠BDC + ∠BAC = 180°[ABCD is a cyclic quadrilateral]

⇒ 120° + ∠BAC = 180°

⇒ ∠BAC = 60°

7. Question

In the given figure, O is the center of the given circle and measure of arc ABC is 100°. Determine $\angle ADC$ and $\angle ABC$.



Answer

∠ADC = (1/2) ∠AOC

⇒ ∠ADC = (1/2) 100°

⇒ ∠ADC = 50°

Now,

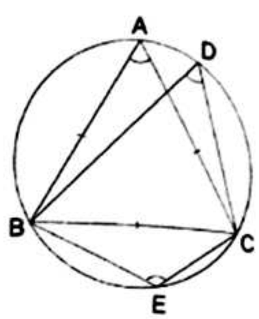
∠ADC + ∠ABC = 180°[ABCD is a cyclic quadrilateral]

⇒ 50° + ∠ABC = 180°

⇒ ∠ABC = 130°

8. Question

In the given figure, ΔABC is equilateral. Find (i) $\angle BDC$. (ii) $\angle BEC$.



Answer

(i) $\angle BDC = 60^{\circ}$

ABC is equilateral triangle.

∴ ∠ABC = ∠ACB = ∠BAC = 60°_____________________ (i)

∠BDC = ∠BAC = 60°[Angles in the same segment of a circle are equal]

(ii) $\angle BEC = 120^{\circ}$

ABCD is a cyclic quadrilateral

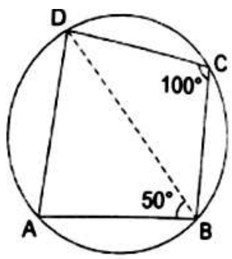
∴ ∠BAC + ∠BEC = 180°

⇒ 60° + ∠BEC = 180°

⇒ ∠BEC = 120°

9. Question

In the adjoining figure, $ABCD$ is a cyclic quadrilateral in which $\angle BCD = 100^{\circ}$ and $\angle ABD = 50^{\circ}$. Find $\angle ADB$.



Answer

ABCD is a cyclic quadrilateral

∴ ∠BCD + ∠BAD = 180°[Opposite angle of a cyclic quadrilateral are supplementary]

⇒ 100° + ∠BAD = 180°

⇒ ∠BAD = 80°

In triangle ABD,

∠ADB + ∠ABD + ∠BAD = 180°[Sum of angles of triangle]

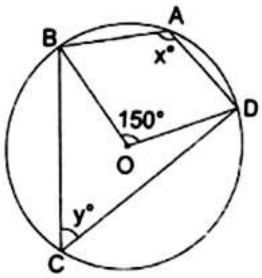
⇒ ∠ADB + 50° + 80° = 180°

⇒ ∠ADB + 130° = 180°

⇒ ∠ADB = 50°

10. Question

In the given figure, O is the center of a circle and $\angle BOD = 150^{\circ}$. Find the values of x and y.



Answer

Reflex ∠BOD = (360° – ∠BOD)

⇒ Reflex ∠BOD = (360° – 150°)

⇒ Reflex ∠BOD = 210°

Now,

X = (1/2) (Reflex ∠BOD)

⇒ X = (1/2) 210°

⇒ X = 105°

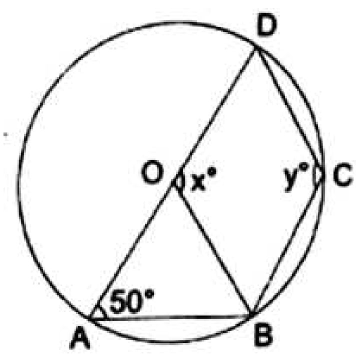
X + Y = 180°

⇒ 105° + Y = 180°

⇒ Y = 75°

11. Question

In the given figure, O is the center of a circle and $\angle DAB = 50°$. Find the values of x and y.



Answer

OA = OB [Radius]

∴ ∠OAB = ∠OBC = 50°

In triangle AOB,

∠AOB + ∠OAB + ∠OBC = 180°[Sum of angles of triangle]

⇒ ∠AOB + 50° + 50° = 180°

⇒ ∠AOB + 100° = 180°

⇒ ∠AOB = 80°

∴ x = 180°– ∠AOB [AOD is a straight line]

⇒ x = 180°- 80°

⇒ x = 100°

∴ X + Y = 180°[Opposite angle of a cyclic quadrilateral are supplementary]

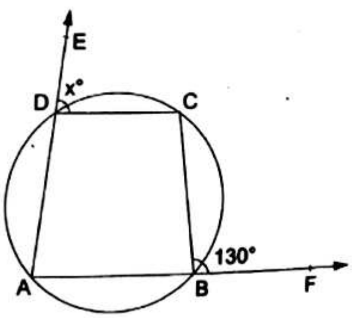
⇒ 100° + Y = 180°

⇒ Y = 80°

12. Question

In the given figure, sides AD and AB of cyclic quadrilateral $ABCD$ are produced to E and F respectively.

If $\angle CBF = 130°$ and $\angle CDE = x°$, find the value of x.



Answer

∠ABC + ∠CBF = 180°[Because ABF is a straight line]

⇒ ∠ABC + 130° = 180°

⇒ ∠ABC = 50°

∴ x = ∠ABC = 50°[Exterior angle = interior opposite angle]

13. Question

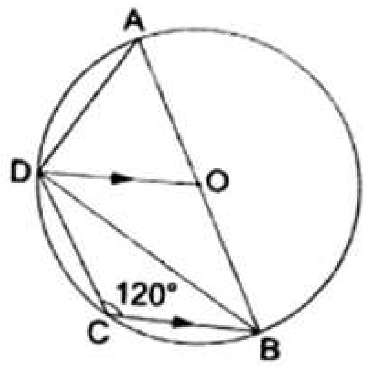
In the given figure, AB is a diameter of a circle with center O and $DO \parallel CB$.

If $\angle BCD = 120°$. calculate

(i) $\angle BAD$ (ii) $\angle ABD$

(iii) $\angle CBD$ (iv) $\angle ADC$.

Also, show that ΔAOD is an equilateral triangle.



Answer

(i) $\angle BAD = 60°$

ABCD is a cyclic quadrilateral.

∴ ∠BAD + ∠BCD = 180°

⇒ ∠BAD + 120° = 180°

⇒ ∠BAD = 60°

(ii) $\angle ABD = 30°$

∠BDA = 90°[Angle in a semi-circle]

In triangle ABD,

∠ABD + ∠BDA + ∠BAD = 180°[Sum of angles of triangle]

⇒ ∠ABD + 90° + 60° = 180°

⇒ ∠ABD + 150° = 180°

⇒ ∠ABD = 30°

(iii) $\angle CBD = 30°$

OD = OA [Radius]

∴ ∠OAD = ∠ODA = ∠BAD = 180°

∴∠ODB = 90° - ∠ODA

⇒ ∠ODB = 90° - 60°

⇒ ∠ODB = 30°

(iv) $\angle ADC = 120°$

∠ADC = ∠ADB + ∠CDB

⇒ ∠ADC = 90° + 30°

⇒ ∠ADC = 120°

In triangle AOD,

∠AOD + ∠OAD + ∠ODA = 180°[Sum of angles of triangle]

⇒ ∠AOD + 60° + 60° = 180°

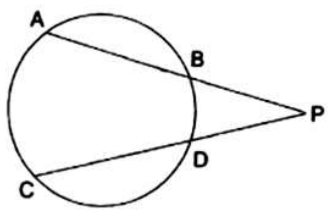
⇒ ∠AOD + 120° = 180°

⇒ ∠AOD = 60°

∴ Triangle AOD is an equilateral triangle.

14. Question

Two chords AB and CD of a circle intersect each other at P outside the circle. If $AB = 6$ cm, $BP = 2$ cm and $PD = 2.5$ cm, find CD.



Answer

Two chords AB and CD of a circle intersect each other at P outside the circle.

∴ AP × BP = CP × PD

⇒ (AB + BP) × BP = (CD + PD) × PD

⇒ (6 + 2) × 2 = (CD + 2.5) × 2.5

⇒ 16 = 2.5 CD + 6.25

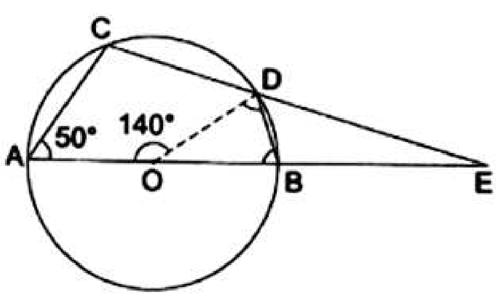
⇒ 2.5 CD = 9.75

⇒ CD = 3.9 cm

15. Question

In the given figure, O is the center of a circle. If $\angle AOD = 140^\circ$ and $\angle CAB = 50^\circ$, calculate

(i) $\angle EDB$, (ii) $\angle EBD$.



Answer

(i) $\angle EDB = 50^\circ$

∠BOD + ∠AOD = 180°[AOB is a straight line]

$\Rightarrow \angle BOD + 140^\circ = 180^\circ$

$\Rightarrow \angle BOD = 40^\circ$

$OB = OD$

$\therefore \angle OBD = \angle ODB$

In triangle AOD,

$\angle BOD + \angle OBD + \angle ODB = 180^\circ$[Sum of angles of triangle]

$\Rightarrow 40^\circ + 2\angle OBD = 180^\circ$

$\Rightarrow 2\angle OBD = 140^\circ$

$\Rightarrow \angle OBD = 70^\circ$

$\therefore \angle OBD = \angle ODB = 70^\circ$

ABDC is a cyclic quadrilateral.

$\therefore \angle CAB + \angle BDC = 180^\circ$

$\Rightarrow \angle CAB + \angle ODB + \angle ODC = 180^\circ$

$\Rightarrow 50^\circ + 70^\circ + \angle ODC = 180^\circ$

$\Rightarrow \angle ODC = 60^\circ$

Now,

$\angle EDB = 180^\circ - \angle BDC$ [Because CDE is a straight line]

$\Rightarrow \angle EDB = 180^\circ - (\angle ODB + \angle ODC)$

$\Rightarrow \angle EDB = 180^\circ - (70^\circ + 60^\circ)$

$\Rightarrow \angle EDB = 180^\circ - 130^\circ$

$\Rightarrow \angle EDB = 50^\circ$

(ii) $\angle EBD = 110^\circ$

$\angle BOD + \angle AOD = 180^\circ$[AOB is a straight line]

$\Rightarrow \angle BOD + 140^\circ = 180^\circ$

$\Rightarrow \angle BOD = 40^\circ$

$OB = OD$

$\therefore \angle OBD = \angle ODB$

In triangle AOD,

$\angle BOD + \angle OBD + \angle ODB = 180^\circ$[Sum of angles of triangle]

$\Rightarrow 40^\circ + 2\angle OBD = 180^\circ$

$\Rightarrow 2\angle OBD = 140^\circ$

⇒ ∠OBD = 70°

∴ ∠OBD = ∠ODB = 70°

Now,

∠EBD + ∠OBD = 180°[Because OBE is a straight line]

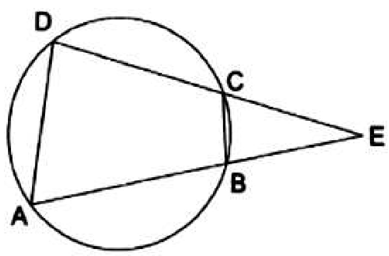
⇒ ∠EBD + 70° = 180°

⇒ ∠EBD = 110°

16. Question

In the given figure, $ABCD$ is a cyclic quadrilateral whose sides AB and DC are produced to meet in E.

Prove that $\Delta EBC - \Delta EDA$.



Answer

In ΔEBC and ΔEDA,

∠EBC = ∠CDA

⇒ ∠EBC = ∠CDA __________________ (i)

∠ECB = ∠BAD

⇒ ∠ECB = ∠EAD __________________ (ii)

∠BEC = ∠DEA __________________ (iii)

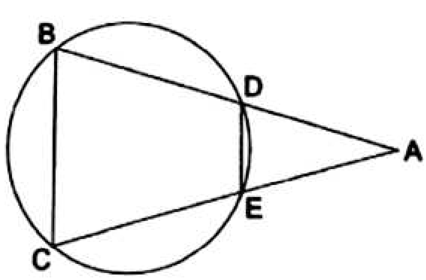
From equation (i), (ii) and (iii),

ΔEBC ≅ ΔEDA Proved.

17. Question

In the given figure, ΔABC is an isosceles triangle in which $AB = AC$ and a circle passing through B and C intersects AB and AC at D and E respectively.

Prove that $DE \parallel BC$.



Answer

Given AB = AC

∴ ∠ACB = ∠ABC

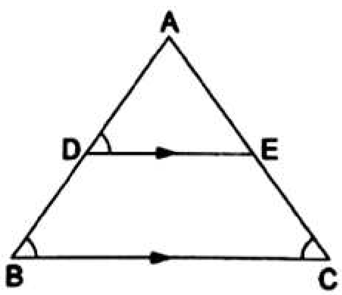
Ext. ∠ADE = ∠ACB = ∠ABC

∴ ∠ADE = ∠ABC

∴ DE || BC Proved.

18. Question

ABC is an isosceles triangle in which $AB = AC$. If D and E are midpoints of AB and AC respectively, prove that the points D, B, C, E are concyclic.



Answer

Given, ABC is an isosceles triangle in which AB = AC. D and E are midpoints of AB and AC respectively.

∴ DE || BC

⇒ ∠ADE = ∠ABC _______________ (i)

AB = AC

⇒ ∠ABC = ∠ACB _______________ (ii)

Now,

∠ADE + ∠EDB = 180°[Because ADB is a straight line]

⇒ ∠ACB + ∠EDB = 180°

The opposite angles are supplementary.

∴ D, B, C, E are concyclic.

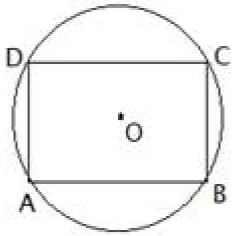
19. Question

Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

Answer

Let, ABCD be a cyclic quadrilateral and O be the center of

the circle passing through A, B, C, and D.



Then,

Each of AB, BC, CD and DA being a chord of the

circle, its right bisector must pass through O.

Therefore,

The right bisectors of AB, BC, CD and DA pass through and are concurrent.

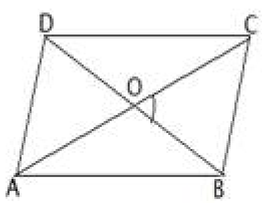
20. Question

Prove that the circles described with the four sides of a rhombus, as diameters, pass through the point of intersection of its diagonals.

Answer

Let diagonals BD and AC of the rhombus ABCD intersect at O.

We know that the diagonals of a rhombus bisect each other at right angles.



∴ ∠BOC = 90°

∴∠BOC lies in a circle.

The circle drawn with BC as diameter will pass through O.

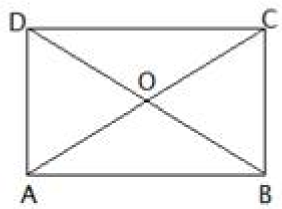
21. Question

$ABCD$ is a rectangle. Prove that the center of the circle through A, B, C, D is the point of intersection of its diagonals.

Answer

Let O be the point of intersection of the diagonals BD and AC of rectangle ABCD.

Since, the diagonals of a rectangle are equal and bisect each other.



∴ OA = OB = OC = OD

Hence, O is the center of the circle through A, B, C, D.

22. Question

Give a geometrical construction for finding the fourth point lying on a circle passing through three given points, without finding the center of the circle. Justify the construction.

Answer

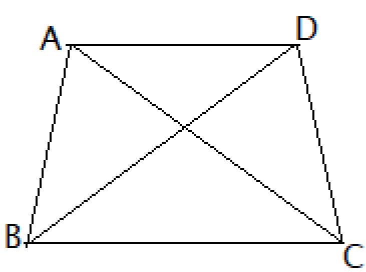
Let A, B, C, D be the given points.

With B as center and radius equal to AC draw an arc.

With C as center and AB as radius draw another arc.

Which cuts the previous arc at D,

Then, D is the required point BD and CD.



In ΔABC and ΔDCB,

AB = DC

AC = DB

BC = CB

∴ ΔEBC ≅ ΔEDA

⇒ ∠BAC = ∠CDB

Thus, BC subtends equal angles, ∠BAC and ∠CDB on the same side of it.

Therefore, points A, B, C, D are concyclic.

23. Question

In a cyclic quadrilateral $ABCD,$ if $(\angle B - \angle D) = 60^\circ,$ show that the smaller of the two is 60°.

Answer

Given, ∠B - ∠D = 60°______________________ (i)

ABCD is a cyclic quadrilateral,

∴ ∠B + ∠D = 180°______________________ (ii)

From equation (i) and (ii),

2 ∠B = 240°

⇒ ∠B = 120°__________________ (iii)

From equation (ii),

∠B + ∠D = 180°

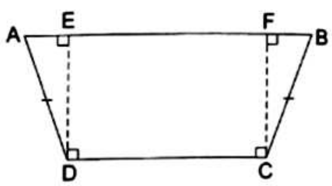
⇒ 120° + ∠D = 180°[From equation (iii)]

⇒ ∠D = 60°

Hence, the smaller of the two angle ∠D = 60°.

24. Question

In the given figure, $ABCD$ is a quadrilateral in which $AD = BC$ and $\angle ADC = \angle BCD$. Show that the points A, B, C, D lie on a circle.



Answer

In ΔADE and ΔBCF,

AD = BC

∠AED = ∠BFC

∠ADE = ∠BCF [∠ADC - 90° = ∠BCD - 90°]

∴ ΔADE ≅ ΔBCF

The Cross ponding parts of the congruent triangles are equal.

∴ ∠A = ∠B

Now,

∠A + ∠B + ∠C + ∠D = 360°

⇒ 2 ∠B + 2 ∠D = 360°

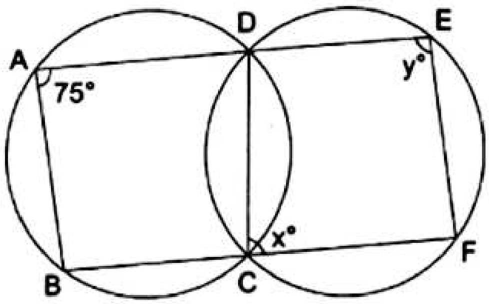
⇒ ∠B + ∠D = 180°

∴ ABCD is a cyclic quadrilateral.

25. Question

In the given figure, $\angle BAD = 75^\circ, \angle DCF = x^\circ$ and $\angle DEF = y^\circ$.

Find the values of x and y.



Answer

∠DCF = ∠DAB

⇒ x = 75° [Exterior angle is equal to the interior opposite angle.]

Now,

∠DCF + ∠DEF = 180° [Opposite angles of a cyclic quadrilateral]

⇒ x + y = 180°

⇒ 75° + y = 180°

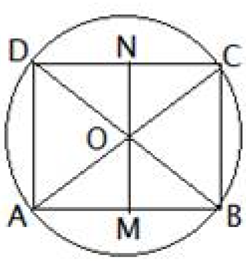
⇒ y = 105°

26. Question

The diagonals of a cyclic quadrilateral are at right angles. Prove that the perpendicular from the point of their intersection on any side when produced backwards, bisects the opposite side.

Answer

Given: Let ABCD be a cyclic quadrilateral, diagonals AC and BD intersect at O at right angles.



∠OCN = ∠OBM [Angles in the same segment] ____________ (i)

∠OBM + ∠BOM = 90° [Because ∠OLB = 90°] _______________ (ii)

∠BOM + ∠CON = 90°[LOM is a straight line and ∠BOC = 90°] _______________ (iii)

From equation (ii) and (iii),

∠OBM + ∠BOM = ∠BOM + ∠CON

⇒ ∠OBM = ∠CON

Thus, ∠OCN = ∠OBM and ∠OBM = ∠CON

⇒ ∠OCN = ∠CON

∴ON = CN ______________________ (iv)

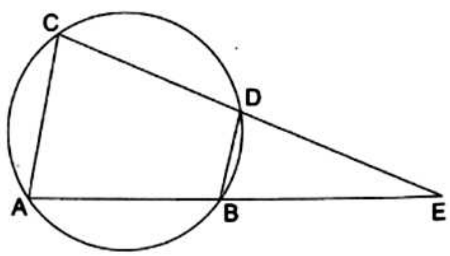
Similarly, ON = ND _____________________ (v)

From equation (iv) and (v),

CN = ND Proved.

27. Question

In the given figure, chords AB and CD of a circle are produced to meet at E. Prove that ΔEDB and ΔEAC are similar.



Answer

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

Chord AB of a circle is produced to E.

∴ ext. ∠BDE = ∠BAC = ∠EAC _____________________ (i)

Chord CD of a circle is produced to E.

∴ ext. ∠DBE = ∠ACD = ∠ACE ______________________ (ii)

In ΔEDB and ΔEAC,

∠BDE = ∠CAE [From equation (i)]

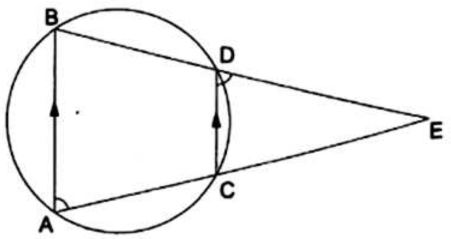
∠DBE = ∠ACE [From equation (ii)]

∠E = ∠E [Common angle]

∴ ΔEDB ~ ΔEAC Proved.

28. Question

In the given figure, AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E, prove that ΔAEB is isosceles.



Answer

Given: AB and CD are two parallel chords of a circle.

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

∴ ext. ∠DCE = ∠B and ext. ∠EDC = ∠A

A || B

∴ ∠EDC = ∠B and ∠DCE = ∠A

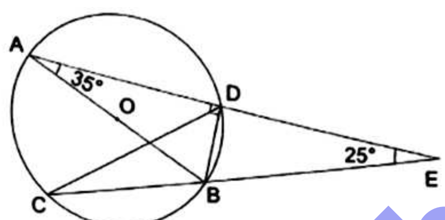
∴∠A = ∠B

Hence, ΔAEB is isosceles.

29. Question

In the given figure, AB is a diameter of a circle with center O. If ADE and CBE are straight lines, meeting at E such that $\angle BAD = 35^\circ$ and $\angle BED = 25^\circ$, find

(i) $\angle DBC$ (ii) $\angle DCB$ (iii) $\angle BDC$.



Answer

(i) $\angle DBC = 115^\circ$

∠BDA = 90°= ∠EDB [Semi circle angle]

In triangle EBD,

∠DBE + ∠EDB + ∠BED = 180°

⇒ ∠DBE + 90° + 25° = 180°

⇒ ∠DBE + 115° = 180°

⇒ ∠DBE = 65°

Now,

∠DBC + ∠DBE = 180°[CBE is a straight line]

⇒ ∠DBC + 65° = 180°

⇒ ∠DBC = 115°

(ii) $\angle DCB = 35°$

∠DCB = ∠BAD [Angle in the same segment]

∴∠DCB = 35°

(iii) ∠BDC = 30°

In triangle BCD,

∠BDC + ∠DCB + ∠DBC = 180°

⇒ ∠BDC + 35° + 115° = 180°

⇒ ∠BDC + 150° = 180°

⇒ ∠BDC = 30°

CCE Questions

1. Question

The radius of a circle is 13 cm and the length of one of its chords is 10 cm. The distance of the chord from the centre is

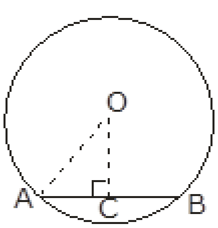
A. 11.5 cm

B. 12 cm

C. $\sqrt{69}$ cm

D. 23 cm

Answer



Given radius(AO) = 13cm

Length of the chord (AB) = 10cm

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC = 5cm

Now in Δ AOC,

Using Pythagoras theorem

$AO^2 = AC^2 + OC^2$

$13^2 = 5^2 + OC^2$

$OC^2 = 13^2 - 5^2$

$OC^2 = 169 - 25$

$OC^2 = 144$

OC = 12cm

∴ The distance of the chord from the centre is 12cm.

2. Question

A chord is at a distance of 8 cm from the centre of a circle of radius 17 cm. The length of the chord is

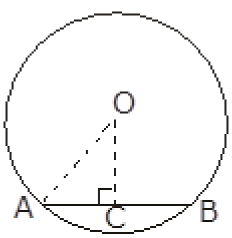
A. 25 cm

B. 12.5 cm

C. 30 cm

D. 9 cm

Answer



Given radius(AO) = 17cm

Length of the chord (AB) = x

distance of the chord from the centre is 8cm.

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC

Now in Δ AOC

Using Pythagoras theorem

$AO^2 = AC^2 + OC^2$

$17^2 = AC^2 + 8^2$

$AC^2 = 17^2 - 8^2$

$AC^2 = 289 - 64$

$AC^2 = 225$

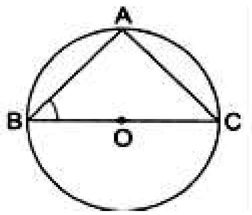
AC = 15cm

∴ BC = 15cm

∴ The length of the chord is AC + BC = 15 + 15 = 30 cm.

3. Question

In the given figure, BOC is a diameter of a circle and AB = AC. Then, ∠ABC = ?



A. 30^o

B. 45^o

C. 60^o

D. 90^o

Answer

Given: BOC is the diameter of the circle

AB = AC

Here, BAC forms a semicircle.

We know that angle in a semicircle is always 90°

∴ ∠BAC = 90°

Here ∠ABC = ∠ACB (since angles opposite equal sides are equal in a triangle)

We know that sum of all the angles in the triangle is 180°

That is

∠ABC + ∠ACB + ∠BAC = 180°

⇒ 2 × ∠ABC + ∠BAC = 180°

⇒ 2 × ∠ABC + 90 = 180°

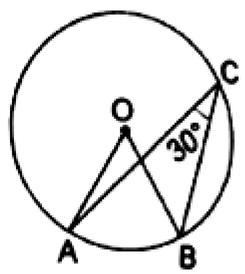
⇒ 2 × ∠ABC = 180° – 90°

⇒ 2 ×∠ABC = 90°

⇒ ∠ABC = 45°

4. Question

In the given figure, O is the centre of a circle and ∠ACB = 30°. Then, ∠AOB = ?



A. 30^{o}

B. 15^{o}

C. 60^{o}

D. 90^{o}

Answer

Given: $\angle ACB = 30^{0}$.

We know that

2 × ∠ACB = ∠AOB (∵The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

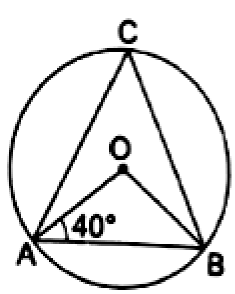
∴2 × 30^{o} = ∠AOB

∠AOB = 60^{o}.

∴ ∠AOB = 60^{o}

5. Question

In the given figure, O is a centre of a circle. If ∠OAB = 40° and C is a point on the circle, then ∠ACB = ?



A. 40^{o}

B. 50^{o}

C. 80^{o}

D. 100^{o}

Answer

In Δ AOB OA = OB(radius)

∠OAB = ∠OBA (Angles opposite to equal sides are equal)

∴ ∠OBA = 40°

By angle sum property

∠OAB + ∠OBA + ∠AOB = 180°

∠AOB = 180° – ∠OAB – ∠OBA

∠AOB = 180° – 40° – 40° = 100°

We know that

2 ×∠ACB = ∠AOB (∵The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

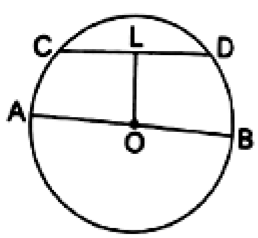
∴2 ×∠ACB = 100°

∠ACB = $\frac{100}{2}$

∴ ∠ACB = 50°

6. Question

In the given figure, AOB is a diameter of a circle with centre O such that AB = 34 cm and CD is a chord of length 30 cm. Then, the distance of CD from AB is



A. 8 cm

B. 15 cm

C. 18 cm

D. 6 cm

Answer

Given: AB 34 cm and CD = 30 cm

Here OL is the perpendicular bisector to CD

∴ CL = LD = 15 cm

Construction: Join OD(radius)

OD = 17cm

Now in Δ ODL

By Pythagoras theorem

$OD^2 = OL^2 + LD^2$

$17^2 = OL^2 + 15^2$

$OL^2 = 17^2 + 15^2$

$OL^2 = 289—225$

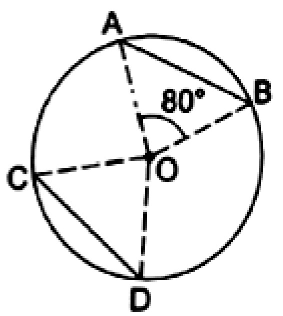
$OL^2 = 64$

OL = 8

∴The distance of CD from AB is = OL = 8cm

7. Question

AB and CD are two equal chords of a circle with centre O such that ∠AOB = 80°, then ∠COD = ?



A. 100^o

B. 80^o

C. 120^o

D. 40^o

Answer

Given: $\angle AOB = 80^o$,

AB = CD

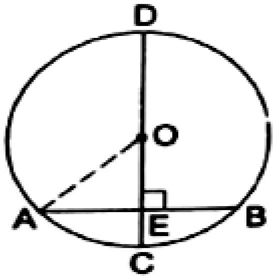
We know that angles subtended from equal chords at center are equal.

∴ ∠ AOB = ∠ COD

∴ ∠ COD = 80°

8. Question

In the given figure, CD is the diameter of a circle with centre O and CD is perpendicular to chord AB. If AB = 12 cm and CE = 3 cm, then radius of the circle is



A. 6 cm

B. 9 cm

C. 7.5 cm

D. 8 cm

Answer

Given: AB = 12cm, CE = 3cm

AB = AE + EB

AE = EB (OC is perpendicular bisector to AB)

∴ AE = 6 cm

Let CD = 2x (diameter)

AO = OC = x (radius)

In Δ AOE

$AO^2 = AE^2 + OE^2$

$x^2 = 6^2 + (OC - EC)^2$

$x^2 = 6^2 + (x - 3)^2$

$x^2 = 6^2 + x^2 + 3^2 - 2(x)(3)$

$x^2 = 36 + x^2 + 9 - 6x$

$6x = 36 + 9 + x^2 - x^2$

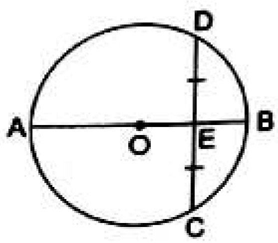
6x = 45

$x = \frac{45}{6} = 7.5$

∴ Radius = x = 7.5 cm

9. Question

In the given figure, O is the centre of a circle and diameter AB bisects the chord CD at a point such that CE = ED = 8cm and EB = 4 cm. The radius of the circle is



A. 10 cm

B. 12 cm

C. 6 cm

D. 8 cm

Answer

Given: CE = ED = 8 cm and EB = 4 cm

Construction: Join OC (OC is radius)

Let AB = 2x (diameter)

OB = OC = x (radius)

In Δ COE

$CO^2 = CE^2 + OE^2$

$x^2 = 8^2 + (OB - EB)^2$

$x^2 = 8^2 + (x - 4)^2$

$x^2 = 8^2 + x^2 + 4^2 - 2(x)(4)$

$x^2 = 64 + x^2 + 16 - 8x$

$8x = 64 + 16 + x^2 - x^2$

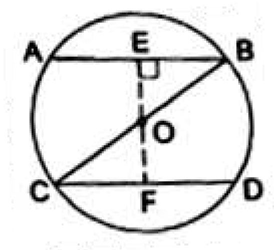
$8x = 80$

$x = \frac{80}{8} = 10$

∴ Radius = x = 10 cm

10. Question

In the given figure, BOC is a diameter of a circle with centre O. If AB and CD are two chords such that AB || CD. If AB = 10 cm, then CD = ?



A. 5cm

B. 12.5cm

C.15cm

D. 10cm

Answer

Given: AB||CD and AB = 10cm

Construction: Drop perpendiculars OE and OF on to AB and CD respectively.

Now,

Consider ΔBOE and ΔCOF

Here,

OB = OC (radius)

∠OEB = ∠OFC (right angle)

∠COF = ∠BOE (vertically opposite angles)

∴ By AAS congruency ΔBOE ≅ ΔCOF

∴ OE = OF (by congruent parts of congruent triangles)

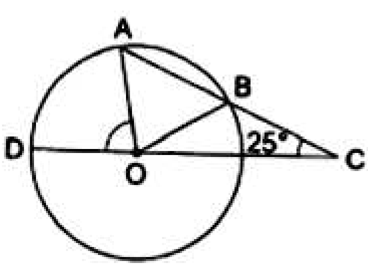
Chords equidistant from center are equal in length

That is CD = AB = 10cm

∴ CD = 10cm

11. Question

In the given figure, AB is a chord of a circle with centre O and AB is produced to C such that BC = OB. Also, CO is joined and produced to meet the circle in D. If ∠AOC = 25° ∠ACD = 25°, then ∠AOD = ?



A. 50^o

B. 75°

C. 90°

D. 100°

Answer

Given: BC = OB and ∠ACD = 25°

Here in Δ OBC

∠BOC = ∠BCO (angles opposite to equal sides are equal)

∴ ∠BOC = 25°

By angle sum property

∠BOC + ∠BCO + ∠OBC = 180°

25° + 25° + ∠OBC = 180°

50° + ∠OBC = 180°

∠OBC = 180° – 50°

∴ ∠OBC = 130°

Here

∠ABC = ∠ABO + ∠OBC = 180°

∠ABO + 130° = 180°

∠ABO = 180° – 130°

∴ ∠ABO = 50°

Now, in ΔAOB

OB = OA (radius)

∠ABO = ∠BAO = 50° (angles opposite to equal sides are equal)

By angle sum property

∠ABO + ∠BAO + ∠AOB = 180°

50° + 50° + ∠AOB = 180°

∠AOB = 180° – (50° + 50°) = 180° – 100° = 80°

∴ ∠AOB = 80°

Here

∠DOC = ∠AOD + ∠AOB + ∠BOC = 180°

∠AOD + 80° + 25° = 180°

∠AOD + 105° = 180°

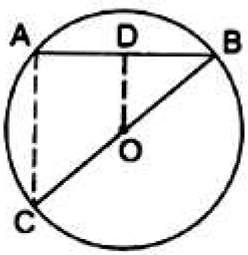
∠AOD = 180° – 105°

∠AOD = 75°

∴ ∠AOD = 75°

12. Question

In the given figure, AB is a chord of a circle with centre O and BOC is a diameter. If OD ⊥ AB such that OD = 6cm then AC = ?



A. 9 cm

B. 12 cm

C. 15 cm

D. 7.5 cm

Answer

Given: $OD \perp AB$ and OD = 6cm

Here OB is radius

Let OB = x cm

In ΔBOD, By Pythagoras theorem

$OB^2 = BD^2 + OD^2$

$x^2 = BD^2 + 6^2$

$x^2 = BD^2 + 36$

$BD^2 = x^2 – 36$

Now consider Δ ABC

Here BC = 2x

By Pythagoras theorem

$BC^2 = AB^2 + AC^2$

$(2x)^2 = 4(x^2 – 36)+ AC^2$

$4x^2 = 4x^2 –144 + AC^2$

$AC^2 = 144$

AC = 12 cm

∴ AC = 12 cm

13. Question

An equilateral triangle of side 9 cm is inscribed in a circle. The radius of the circle is

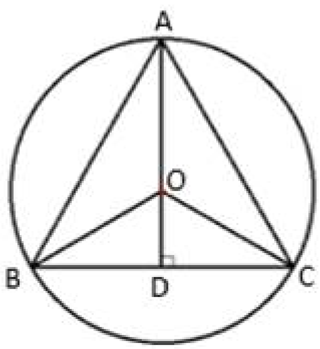
A. 3 cm

B. $3\sqrt{2}$ cm

C. $3\sqrt{3}$ cm

D. 6 cm

Answer



Given: Equilateral triangle of side 9 cm is inscribed in a circle.

Construction: Join OA, OB, OC and drop a perpendicular bisector from center O to BC.

Here,

Area (ΔABC) = 3× area (ΔOBC)

Area (ΔABC) = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 9^2 = \frac{81\sqrt{3}}{4}$

Now,

Area (ΔOBC) = $\frac{1}{2} \times AC \times OD = \frac{1}{2} \times 9 \times OD$

We know that,

Area (ΔABC) = 3× area (ΔOBC)

$\frac{81\sqrt{3}}{4} = \frac{1}{2} \times 9 \times OD$

$OD = \frac{3\sqrt{3}}{2}$

Now, in ΔODC

By Pythagoras theorem

$OC^2 = OD^2 + DC^2$

$OC^2 = \left(\frac{3\sqrt{3}}{2}\right)^2 + \left(\frac{9}{2}\right)^2$

$OC^2 = \frac{27}{4} + \frac{81}{4} = \frac{108}{4} = 27$

$OC = 3\sqrt{3}$

∴ Radius = OC = $3\sqrt{3}$

14. Question

The angle in a semicircle measures

A. 45°

B. 60°

C. 90°

D. 36°

Answer

Angle in a semicircle measures 90°

15. Question

Angles in the same segment of a circle area are

A. equal

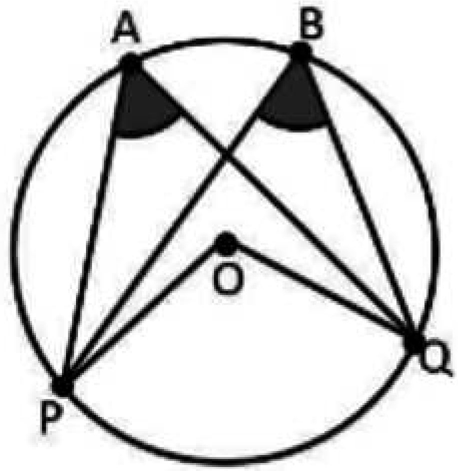
B. complementary

C. supplementary

D. none of these

Answer

Angles in the same segment of a circle are always equal.



Proof: As we know angle subtended by an arc is double the angle subtended at any other point.

So,∠POQ = 2∠PAQ .... (1)

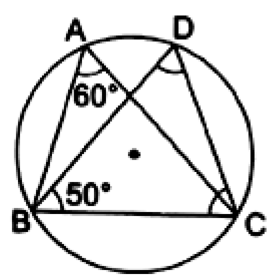
∠POQ= 2∠PBQ .... (2)

From (1) and (2),∠PAQ = ∠PBQ

Hence proved

16. Question

In the given figure, ΔABC and ΔDBC are inscribed in a circle such that ∠BAC = 60° and ∠DBC = 50°. Then, ∠BCD = ?



A. 50^{o}

B. 60^{o}

C. 70^{o}

D. 80^{o}

Answer

Given: Two triangles ΔABC and ΔBCD, ∠BAC = 60° and ∠DBC = 50°

We know that ∠BAC = ∠BDC = 60° (∵ angles in the same segment drawn from same chord are equal).

Now consider ΔBCD

By angle sum property

∠DBC + ∠BDC + ∠BCD = 180°

50° + 60° + ∠BCD = 180°

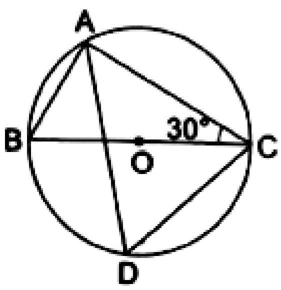
∠BCD = 180° – 50° – 60°

∠BCD = 70°

∴ ∠BCD = 70°

17. Question

In the given figure, BOC is a diameter of a circle with centre O. If ∠BCA = 30°, then ∠CDA = ?



A. 30^{o}

B. 45^{o}

C. 60^{o}

D. 50^{o}

Answer

Given: $\angle BCA = 30^{\circ},$

Here,

∠BAC = 90° (angle in the semicircle)

Now, in ΔABC

By angle sum property

∠BCA + ∠BAC + ∠ABC = 180°

30° + 90° + ∠ABC = 180°

∠ABC = 180° – 30° – 90°

∠ABC = 60°

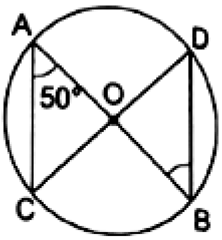
Here,

∠ABC = ∠ADC (angles in the same segment)

∴ ∠CDA = 60°

18. Question

In the given figure, O is the centre of a circle. If ∠OAC = 50°, then ∠ODB = ?



A. 40°

B. 50°

C. 60°

D. 75°

Answer

Given: ∠OAC = 50°

Consider ΔAOC

∠OAC = ∠OCA = 50° (∵ OA = OC = radius, angles opposite to equal sides are equal)

Now, by angle sum property

∠OAC + ∠OCA + ∠AOC = 180°

50° + 50° + ∠AOC = 180°

∠AOC = 180° – 50° – 50°

∠AOC = 80°

Now angle ∠BOD = ∠AOC = 80° (vertically opposite angles)

Now, consider ΔBOD

Here,

OB = OD (radius)

∠OBD = ∠ODB (angles opposite to equal angles are equal)

Let ∠ODB = x

By angle sum property

∠ODB + ∠OBD + ∠BOD = 180°

x + x + 80° = 180°

2x = 180° – 80°

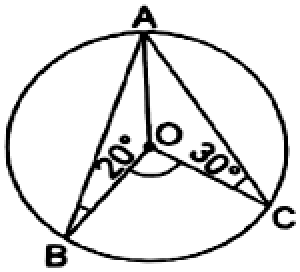
2x = 100°

x = 50°

∴ ∠ODB = 50°

19. Question

In the given figure, O is the centre of a circle in which ∠OBA = 20° and ∠OCA = 30°. Then, ∠BOC = ?



A. 50^o

B. 90^o

C. 100^o

D. 130^o

Answer

Given: $\angle OBA = 20^o$ and $\angle OCA = 30^o$.

Consider ΔOAB

Here,

OA = OB (radius)

∠OBA = ∠OAB = 20° (angles opposite to equal sides are equal)

By angle sum property

∠AOB + ∠OBA + ∠OAB = 180°

∠AOB + 20° + 20° = 180°

∠AOB = 180° – 20° – 20°

∠AOB = 140°

Similarly, in ΔAOC

OA = OC (radius)

∠OCA = ∠OAC = 30° (angles opposite to equal sides are equal)

By angle sum property

∠AOC + ∠OCA + ∠OAC = 180°

∠AOC + 30° + 30° = 180°

∠AOC = 180° – 30° – 30°

∠AOC = 120°

Here,

∠CAB = ∠OAB + ∠OAC = 50°

Here,

2∠CAB = ∠BOC (∵The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

∴2∠CAB = ∠BOC

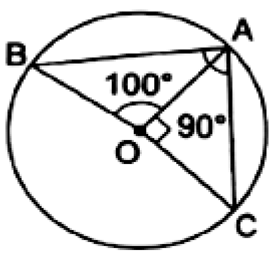
∴2 × 50° = ∠BOC

∠BOC = 100°.

∴ ∠BOC = 100°

20. Question

In the given figure, O is the centre of a circle. If ∠AOB = 100° and ∠AOC = 90°, then ∠BAC = ?



A. 85°

B. 80°

C. 95°

D. 75°

Answer

Given: ∠AOB = 100° and ∠AOC = 90°,

Consider ΔOAB

Here,

OA = OB (radius)

Let ∠OBA = ∠OAB = x (angles opposite to equal sides are equal)

By angle sum property

∠AOB + ∠OBA + ∠OAB = 180°

100° + x + x = 180°

2x = 180° − 100°

2x = 80°

x = 40°

Similarly, in ΔAOC

OA = OC (radius)

Let ∠OCA = ∠OAC = y (angles opposite to equal sides are equal)

By angle sum property

∠AOC + ∠OCA + ∠OAC = 180°

90° + y + y = 180°

2y = 180° – 90°

2y = 90°

y = 45°

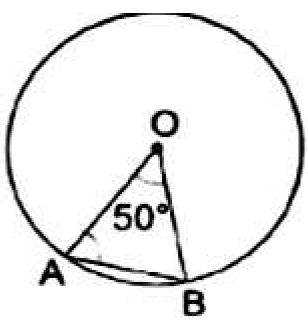
Here,

∠BAC = ∠OAB + ∠OAC = x + y = 40° + 45° = 85°

∴ ∠BAC = 85°

21. Question

In the given figure, O is the centre of a circle. Then, ∠OAB = ?



A. 50^{o}

B. 60^{o}

C. 55^{o}

D. 65^{o}

Answer

Given: $\angle AOB = 100^{o}$ and $\angle AOC = 90^{o}$,

In ΔOAB

Here,

OA = OB (radius)

Let ∠OBA = ∠OAB = x (angles opposite to equal sides are equal)

By angle sum property

∠AOB + ∠OBA + ∠OAB = 180°

50° + x + x = 180°

2x = 180° – 50°

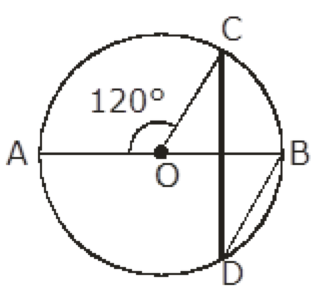
2x = 130°

x = 60°

∴ ∠OAB = 60°

22. Question

In the given figure, O is the centre of a circle and ∠AOC = 120°. Then, ∠BDC = ?



A. 60°

B. 45°

C. 30°

D. 15°

Answer

Given: ∠AOC = 120°

Construction: Join OD

We know that,

∠AOC = 2 × ∠ADC

120° = 2 ∠ADC

∠ADC = 60°

Here,

∠ADB = 90° (angle in a semicircle)

∠ADB = ∠ADC + ∠CDB = 90°

∠ADC + ∠CDB = 90°

60° + ∠CDB = 90°

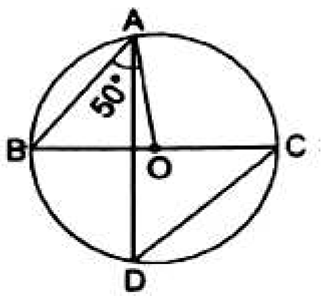
∠CDB = 90° – 60°

∠CDB = 30°

∴ ∠BDC = 30°

23. Question

In the given figure, O is the centre of a circle and ∠OAB = 50°. Then, ∠CDA = ?



A. 40^{o}

B. 50^{o}

C. 75^{o}

D. 25^{o}

Answer

Given: ∠OAB = 50°

Construction: Join AC

Here,

In ΔAOB

OA = OB (radius)

∠OAB = ∠OBA (angles opposite to equal sides are equal)

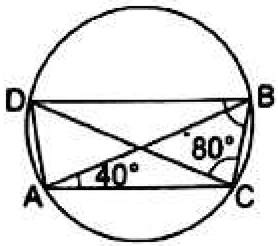
∴ ∠OBA = 50°

∠OBA = ∠CDA (angles in the same segment)

∴ ∠CDA = 50°

24. Question

In the give figure, and are two intersecting chords of a circle. If ∠CAB = 40° and ∠BCD = 80°, then ∠CBD = ?



A. 80°

B. 60°

C. 50°

D. 70°

Answer

Given: $\angle CAB = 40^0$ and ∠BCD = 80°

Here,

∠CAB = ∠CDB = 40° (∵ angles in the same segment drawn from same chord are equal).

Now, in ΔBCD

By angle sum property

∠BCD + ∠CDB + ∠CBD = 180°

80° + 40° + ∠CBD = 180°

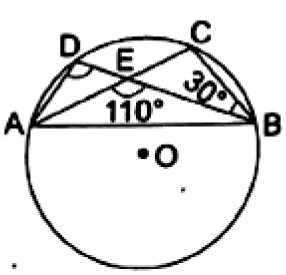
∠CBD = 180° – 40° – 80°

∠CBD = 60°

∴ ∠CBD = 60°

25. Question

In the given figure, O is the centre of a circle and chords AC and BD intersect at E. If ∠AEB = 110° and ∠CBE = 30° then ∠ADB = ?



A. 70°

B. 60°

C. 80°

D. 90°

Answer

Given: ∠AEB = 110° and ∠CBE = 30°

∠AEC = ∠AEB + ∠BEC = 180°

∠AEB + ∠BEC = 180°

110° + ∠BEC = 180°

∠BEC = 180° – 110°

∠BEC = 70°

In ΔBEC

By angle sum property

∠CBE + ∠BEC + ∠ECB = 180°

30° + 70° + ∠ECB = 180°

∠ECB = 180° – 30° – 70°

∠ECB = 80°

Here,

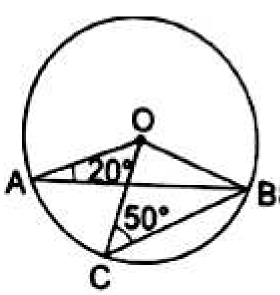
∠ECB = ∠ADB (angles in the same segment)

∴ ∠ECB = ∠ADB = 80°

∴ ∠ADB = 80°

26. Question

In the given figure, O is the centre of a circle in which ∠OAB = 20° and ∠OCB = 50°. Then, ∠AOC = ?



A. 50°

B. 70°

C. 20°

D. 60°

Answer

Given: ∠OAB = 20° and ∠OCB = 50°

Here,

In ΔAOB

OA = OB (radius)

∠OAB = ∠OBA (angles opposite to equal sides are equal)

∴ ∠ OBA = 20°

Now, by angle sum property

∠AOB + ∠OBA + ∠OAB = 180°

∠AOB + 20° + 20° = 180°

∠AOB = 180° – 20° – 20°

∠AOB = 140°

Now, Consider Δ BOC

OC = OB (radius)

∠OCB = ∠OBC (angles opposite to equal sides are equal)

∴ ∠ OBA = 50°

Now, by angle sum property

∠COB + ∠OBC + ∠OCB = 180°

∠COB + 50° + 50° = 180°

∠COB = 180° – 50° – 50°

∠COB = 80°

Here,

∠AOB = ∠AOC + ∠COB

140° = ∠AOC + 80°

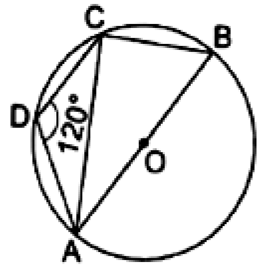
∠AOC = 140° – 80°

∠AOC = 60°

∴ ∠AOC = 60°

27. Question

In the given figure, AOB is a diameter and ABCD is a cyclic quadrilateral. If ∠ADC = 120°, then ∠BAC = ?



A. 60°

B. 30°

C. 20°

D. 45°

Answer

Given: ABCD is cyclic quadrilateral and ∠ADC = 120°

Here,

∠ADC + ∠ABC = 180° (opposite angles in cyclic quadrilateral are supplementary)

120° + ∠ABC = 180°

∠ABC = 180° – 120°

∠ABC = 60°

Here,

∠ACB = 90° (angle in semicircle)

Now, consider ΔABC

By angle sum property

∠BAC + ∠ABC + ∠ACB = 180°

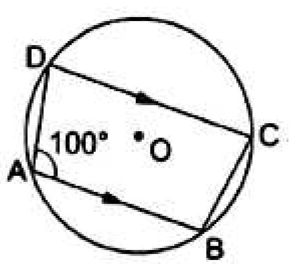
∠BAC + 60° + 90° = 180°

∠BAC = 180° – 60° – 90°

∠BAC = 30°

28. Question

In the given figure, ABCD is a cyclic quadrilateral in which AB || DC and ∠BAD = 100°. Then ∠ABC = ?



A. 80^o

B. 100^o

C. 50^o

D. 40^o

Answer

Given: ABCD is a cyclic quadrilateral, AB||DC and ∠BAD = 100°

Here,

∠BAD + ∠BCD = 180° (opposite angles in cyclic quadrilateral are supplementary)

100° + ∠BCD = 180°

∠BCD = 180° – 100°

∠BCD = 80°

Here, AB||DC and BC is the transversal

∠ABC + ∠BCD = 180° (interior angles along the transversal are supplementary)

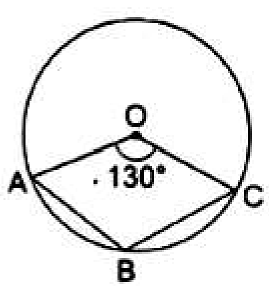
∠ABC + 80° = 180°

∠ABC = 180° – 80° = 100°

∴ ∠ABC = 100°

29. Question

In the given figure, O is the centre of a circle and ∠AOC = 130°. Then, ∠ABC = ?



A. 50^o

B. 65^o

C. 115^o

D. 130^o

Answer

Given: ∠AOC = 130°

Here,

(Exterior ∠AOC) = 360° – (interior ∠AOC)

(Exterior ∠AOC) = 360° – 130°

(Exterior ∠AOC) = 230°

We know that,

(Exterior ∠AOC) = 2 × ∠ABC

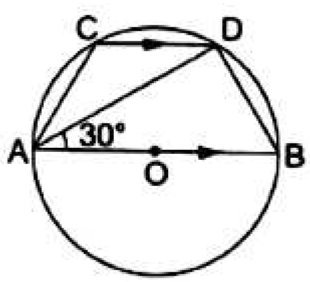
230° = 2 × ∠ABC

∠ABC = $\frac{230}{2}$ = 115°

∴ ∠ABC = 115°

30. Question

In the given figure, AOB is a diameter of a circle and CD || AB. If ∠BAD = 30°, then ∠CAD = ?



A. 30^{o}

B. 60^{o}

C. 45^{o}

D. 50^{o}

Answer

Given: CD||AB and ∠BAD = 30°

Consider ΔABD

∠ADB = 90° (angle in semicircle)

Now, by angle sum property

∠ABD + ∠BAD + ∠ADB = 180°

∠ABD + 30° + 90° = 180°

∠ABD = 180° – 30° – 90°

∠ABD = 60°

Here,

∠ABD + ∠ACD = 180° (opposite angles in cyclic quadrilateral are supplementary)

60° + ∠ACD = 180°

∠BCD = 180° – 60°

∠BCD = 120°

Here, CD||AB and AC is the transversal

∠CAB + ∠ACD = 180° (interior angles along the transversal are supplementary)

∠CAB + 120° = 180°

∠ABC = 180° – 120° = 60°

∠ABC = 60°

∠ABC = ∠CAD + ∠DAB

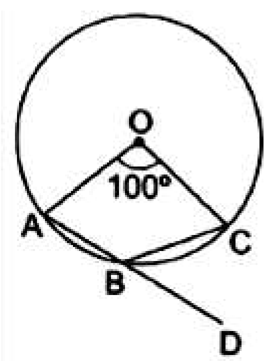
60° = ∠CAD + 30°

∠CAD = 60° – 30° = 30°

∴ ∠CAD = 30°

31. Question

In the given figure, O is the centre of a circle in which ∠AOC = 100°. Side AB of quad. OABC has been produced to D. Then, ∠CBD = ?



A. 50°

B. 40°

C. 25°

D. 80°

Answer

Given: ∠AOC = 100°

Here,

(Exterior ∠AOC) = 360° – (interior ∠AOC)

(Exterior ∠AOC) = 360° – 100°

(Exterior ∠AOC) = 260°

We know that,

(Exterior ∠AOC) = 2 × ∠ADC

260° = 2 × ∠ABC

∠ABC = $\frac{260}{2}$ = 130°

∴ ∠ABC = 130°

Here,

∠ABD = ∠ABC + ∠CBD

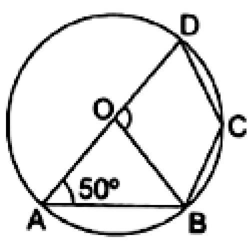
180° = 130° + ∠CBD

∠CBD = 180° – 130° = 50°

∴ ∠CBD = 50°

32. Question

In the given figure, O is the centre of a circle and ∠ = 50°. Then, ∠BOD = ?



A. 130°

B. 50°

C. 100°

D. 80°

Answer

Given: ∠OAB = 50°

Consider ΔAOB

Here,

OA = OB (radius)

∠OAB = ∠OBA = 50° (In a triangle, angles opposite to equal sides are equal)

By angle sum property

∠AOB + ∠OAB + ∠OBA = 180°

∠AOB + 50° + 50° = 180°

∠AOB = 180° – 50° – 50°

∠AOB = 80°

Here,

∠AOD = ∠AOB + ∠BOD

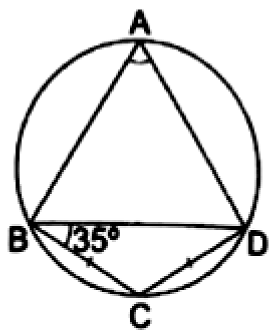
180° = 80° + ∠BOD

∠BOD = 180° – 80° = 100°

∴ ∠BOD = 100°

33. Question

In the given figure, ABCD is a cyclic quadrilateral in which BC = CD and ∠CBD = 35°. Then, ∠BAD = ?



A. 65^{o}

B. 70^{o}

C. 110^{o}

D. 90^{o}

Answer

Given: CB = CD and ∠CBD = 35°

Consider ΔBCD

Here,

CB = CD (given)

∠CBD = ∠CDB = 35° (In a triangle, angles opposite to equal sides are equal)

By angle sum property

∠BCD + ∠CBD + ∠CDB = 180°

∠BCD + 35° + 35° = 180°

∠BCD = 180° – 35° – 35° = 110°

We know that,

In a cyclic quadrilateral opposite angles are supplementary

∴ ∠BCD + ∠BAD = 180°

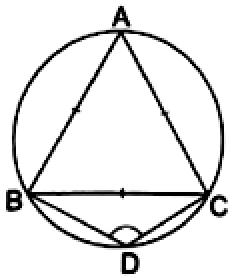
110° + ∠BAD = 180°

∠BAD = 180° – 110° = 70°

∴ ∠BAD = 70°

34. Question

In the given figure, equilateral ΔABC is inscribed in a circle and ABDC is a quadrilateral, as shown. Then, ∠BDC = ?



A. 90°

B. 60°

C. 120°

D. 150°

Answer

Given: ΔABS is equilateral

In ΔABC

∠BAC = 60° (All angles in equilateral triangle are equal to 60°)

We know that,

In a cyclic quadrilateral opposite angles are supplementary

∴ ∠BAC + ∠BDC = 180°

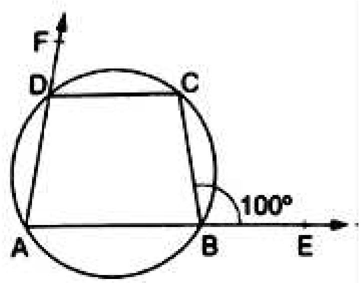
60° + ∠BDC = 180°

∠BDC = 180° – 60° = 120°

∴ ∠BDC = 120°

35. Question

In the given figure, sides AB and AD of quad. ABCD are produced to E and F respectively. If ∠CBE = 100°, then ∠CDF = ?



A. 100°

B. 80°

C. 130°

D. 90°

Answer

Given: ∠CBE = 100°

Here,

∠ABE = ∠ABC + ∠CBE

180° = ∠ABC + 100°

∠ABC = 180° – 100° = 80°

We know that,

In a cyclic quadrilateral opposite angles are supplementary

∴ ∠ABC + ∠ADC = 180°

80° + ∠ADC = 180°

∠ADC = 180° – 80° = 100°

Here,

∠ADF = ∠ADC + ∠CDF

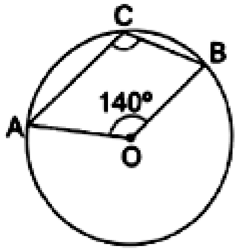
180° = 100° + ∠CDF

∠CDF = 180° – 100° = 80°

∴ ∠CDF = 80°

36. Question

In the given figure, O is the centre of a circle and ∠AOB = 140°. Then, ∠ACB = ?



A. 70°

B. 80°

C. 110°

D. 40°

Answer

Given: ∠AOB = 140°

Here,

(Exterior ∠AOB) = 360° – (interior ∠AOB)

(Exterior ∠AOB) = 360° – 140°

(Exterior ∠AOB) = 220°

We know that,

(Exterior ∠AOB) = 2 × ∠ACB

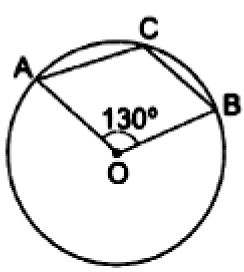
220° = 2 × ∠ACB

∠ACB = $\frac{220}{2}$ = 110°

∴ ∠ACB = 110°

37. Question

In the given figure, O is the centre of a circle and ∠AOB = 130°. Then, ∠ACB = ?



A. 50°

B. 65°

C. 115°

D. 155°

Answer

Given: ∠AOB = 130°

Here,

(Exterior ∠AOB) = 360° – (interior ∠AOB)

(Exterior ∠AOB) = 360° – 130°

(Exterior ∠AOB) = 230°

We know that,

(Exterior ∠AOB) = 2 × ∠ACB

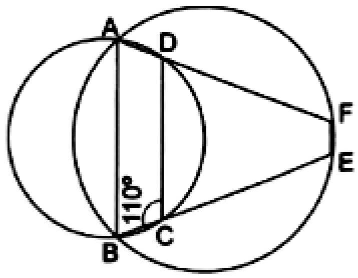
230° = 2 × ∠ACB

∠ACB = $\frac{230}{2}$ = 115°

∴ ∠ACB = 115°

38. Question

In the given figure, ABCD and ABEF are two cyclic quadrilaterals. If ∠BCD = 110°, then ∠BEF = ?



A. 55°

B. 70°

C. 90°

D. 110°

Answer

Given: ABCD, ABEF are two cyclic quadrilaterals and ∠BCD = 110°

In Quadrilateral ABCD

We know that,

In a cyclic quadrilateral opposite angles are supplementary

∴ ∠BCD + ∠BAD = 180°

110° + ∠BAD = 180°

∠BAD = 180° – 110° = 70°

Similarly in Quadrilateral ABEF

∴ ∠BAD + ∠BEF = 180°

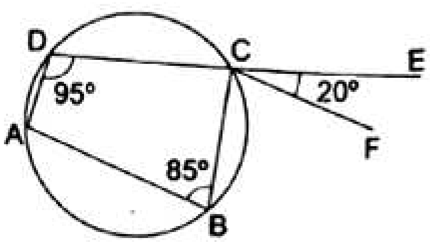
70° + ∠BEF = 180°

∠BEF = 180° – 70° = 110°

∴ ∠BEF = 110°

39. Question

In the given figure, ABCD is a cyclic quadrilateral in which DC is produced to E and CF is drawn parallel to AB such that ∠ADC = 90° and ∠ECF = 20°. Then, ∠BAD = ?



A. 95^{o}

B. 85^{o}

C. 105^{o}

D. 75^{o}

Answer

Given: $ABCD$ is a cyclic quadrilateral, CF||AB, $\angle ADC = 95^{0}$ and $\angle ECF = 20^{0}$.

Here, CF|| AB

Hence BC is transversal

∴ ∠ABC = ∠BCF = 85° (Alternate interior angles)

Here,

∠DCB + ∠BCF + ∠ECF = ∠DCE

∠DCB + 85° + 20° = 180°

∠DCB = 180° – 85° – 20° = 75°

We know that,

In a cyclic quadrilateral opposite angles are supplementary

∴ ∠DCB + ∠BAD = 180°

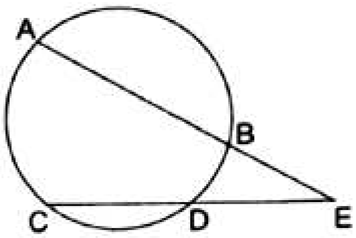
75° + ∠BAD = 180°

∠BAD = 180° – 75° = 105°

∴ ∠BAD = 105°

40. Question

Two chords AB and CD of a circle intersect each other at a point E outside the circle. If AB = 11 cm, BE = 3 cm and DE = 3.5 cm, then CD = ?



A. 10.5 cm

B. 9.5 cm

C. 8.5 cm

D. 7.5 cm

Answer

Given: AB = 11cm, BE = 3cm and DE = 3.5cm

Construction: Join AC

Here,

AE: CE = DE: BE

AE×BE = DE×CE

(AB + BE)×BE = DE×(CD + DE)

(11 + 3)×3 = 3.5×(CD + 3.5)

14×3 = 3.5×(CD + 3.5)

3.5×(CD + 3.5) = 42

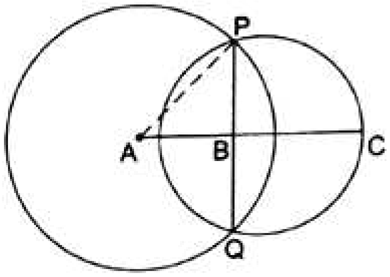
(CD + 3.5) = $\frac{42}{3.5}$ = 12

CD = 12—3.5 = 8.5

∴ CD = 8.5

41. Question

In the given figure, A and B are the centers of two circles having radii 5 cm and 3 cm respectively and intersecting at points P and Q respectively. If AB = 4 cm, then the length of common chord PQ is



A. 3 cm

B. 6 cm

C. 7.5 cm

D. 9 cm

Answer

Given: $AB = 4$ cm, two circles having radii 6 cm and 3 cm

Construction: join AP

Consider ΔABP

Here,

$AP^2 = AB^2 + BP^2$

$5^2 = 4^2 + 3^2$

25 = 16 + 9

25 = 25

∴ ΔABP is right angled triangle

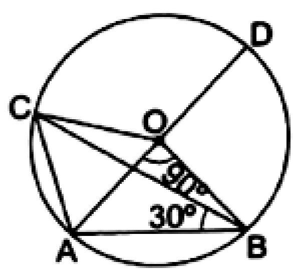
PQ = 2×BP

PQ = 2×3 = 6cm

∴PQ = 6cm

42. Question

In the given figure, ∠AOB = 90° and ∠ABC = 30°. Then, ∠CAO = ?



A. 30°

B. 45^o

C. 60^o

D. 90^o

Answer

Given: $\angle AOB = 90^0$ and $\angle ABC = 30^0$.

Construction: join CD

We know that,

∠AOB = 2 × ∠ACB

90° = 2 × ∠ACB

∠ACB = $\frac{90}{2}$ = 45°

Similarly,

∠COA = 2 × ∠CBA

∠COA = 2 × 30

∠COA = 60°

Here,

∠COD + ∠COA = ∠AOD

∠COD + 60° = 180°

∠COD = 180° – 60° = 120°

Again

∠COD = 2 × ∠CAO

∠CAO = $\frac{120}{2}$ = 60°

∴∠CAO = 60°

43. Question

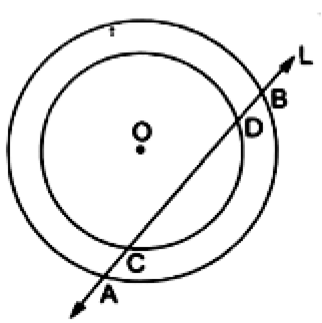
Three statements are given below:

I. If a diameter of a circle bisects each of the two chords of a circle, then the chords are parallel.

II. Two circles of radii 10 cm and 17 cm intersect each other and the length of the common chord is 16 cm. Then, the distance between their centres is 23 cm.

III. ∠ is the line intersecting two concentric circles with centre O at points A, B, C and D as shown. Then, AC = DB.

Which is true?



A. I and II

B. I and III

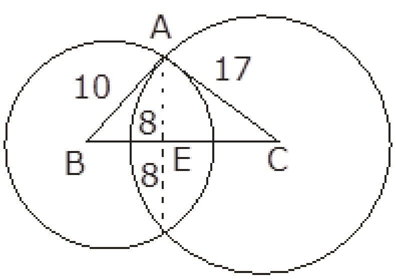
C. II and III

D. II only

Answer

Here, Clearly I and III are correct.

Let us check for II statement



Construction: Let B and C be the centers of two circles having radii 10cm and 17 cm respectively and let AD be the common chord cutting BC at E.

Here,

AE = ED = 8cm

Now, in ΔABE

$BE^2 = AB^2 - AE^2$

$BE^2 = (10)^2 - (8)^2$

$BE^2 = 100 - 64 = 36$

BE = 6cm

Now, in ΔAEC

$EC^2 = AC^2 - AE^2$

$EC^2 = (17)^2 - (8)^2$

$EC^2 = 289 - 64 = 225$

EC = 25cm

Here,

BC = BE + EC = 6 + 15 = 21cm

But, it is given BC = 23cm

∴ Statement II is false

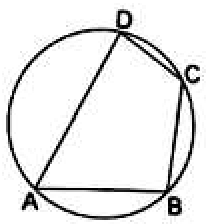
44. Question

Two statements I and II are given and a question is given. The correct answer is

Is ABCD a cyclic quadrilateral?

I. Points A, B, C and D lie on a circle.

II. ∠B + ∠D = 180°.



A. if the given question can be answered by any one of the statements but not the other;

B. if the given question can be answered by using either statement alone;

C. if the given question can be answered by using both the statements together but cannot be answered by using either statement;

D. if the given question cannot be answered by using both the statements together.

Answer

Here,

ABCD is said to be cyclic quadrilateral

If either of any point is satisfied

i)Points A, B, C and D lie on a circle.

ii)∠ B + ∠C = 180°

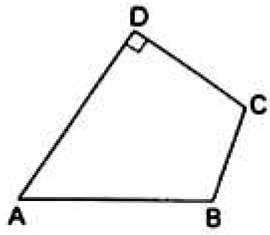
45. Question

Two statements I and II are given and a question is given. The correct answer is

Is ΔABC right – angled at B?

I. ABCD is a cyclic quadrilateral.

II. ∠D = 90°.



A. if the given question can be answered by any one of the statements but not the other;

B. if the given question can be answered by using either statement alone;

C. if the given question can be answered by using both the statements together but cannot be answered by using either statement;

D. if the given question cannot be answered by using both the statements together.

Answer

Here,

ΔABC right – angled at B

If both the conditions satisfy

i) $ABCD$ is a cyclic quadrilateral

ii) $\angle D = 90^\circ$.

46. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	**Reason (R)**
The circle drawn taking any one of the equal sides of an isosceles right triangle as diameter bisects the base.	The angle in a semicircle is 1 right angle.

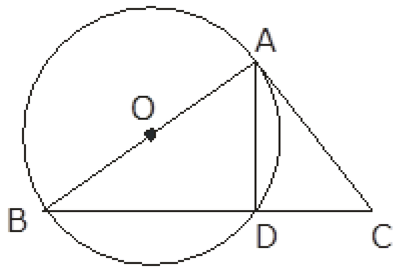
A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer



Assertion (A):

Construction: Draw a Δ ABC in which AB = AC, Let O be the midpoint of AB and with O as centre and OA as radius draw a circle, meeting BC at D

Now, In Δ ABD

∠ ADB = 90° (angle in semicircle)

Also, ∠ ADB + ∠ ADC = 180°

90° + ∠ ADC = 180°

∠ ADC = 180° – 90°

∠ ADC = 90°

Consider Δ ADB and Δ ADC

Here,

AB = AC (given)

AD = AD (common)

∠ ADB = ∠ ADC (90°)

∴ By SAS congruency, Δ ADB ≅ Δ ADC

So, BD = DC(C.P.C.T)

Thus, the given circle bisects the base. So, Assertion (A) is true

Reason (R) :

Let ∠ BAC be an angle in a semicircle with centre O and diameter BOC

Now, the angle subtended by arc BOC at the centre is ∠ BOC = 2× 90°

∠ BOC = 2× ∠ BAC = 2× 90°

So, ∠ BAC = 90° (right angle)

So, reason (R) is true

Clearly, reason (R) gives assertion (A)

Hence, correct choice is A

47. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	**Reason (R)**
The radius of a circle is 10 cm and the length of one of its chords is 16 cm. Then, the distance of the chord from the centre is 6 cm.	The perpendicular from the centre of a circle to a chord (other than the diameter) bisects the chord.

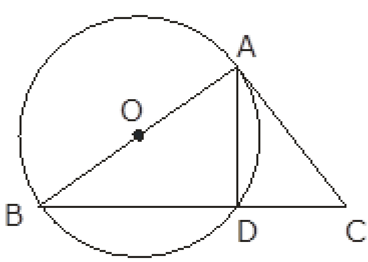
A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer



Assertion (A) :

Let O be the centre of the circle and AB be the chord

Construction: Draw, L is the midpoint of AB

Here,

OA = 10cm

$AL = \frac{1}{2} AB = 8cm$

In ΔOAL,

$OL^2 = OA^2 - AL^2$

$OL^2 = (10)^2 - (8)^2$

$OL^2 = 100 - 64$

$OL = \sqrt{36} = 6cm$

Thus, Assertion (A) is true.

Clearly, reason (R) given Assertion (A).

Hence, the correct choice is A.

48. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	**Reason (R)**
In a circle of radius 13 cm, there is a chord of length 10 cm at a distance of 12 cm from the centre of the circle.	A unique circle can be drawn to pass through three give non – collinear points.

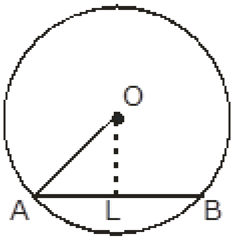
A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer



Clearly, reason (R) is true.

Assertion (A) :

OA = 13cm

OL = 12cm

In ΔOAL,

$AL^2 = OA^2 - OL^2$

$AL^2 = (13)^2 - (12)^2$

$AL^2 = 169 - 144$

OL = $\sqrt{25}$ = 5cm

Now, AB = 2 × AL = 2 × 5 = 10cm

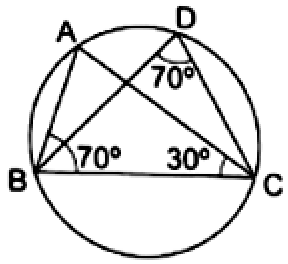
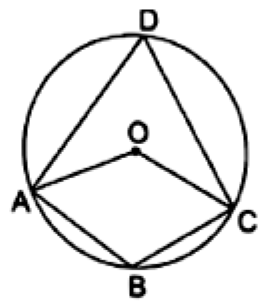
Thus, Assertion (A) is true

∴ Reason (R) and Assertion (A) are both true but reason (R) does not gives Assertion (A).

Hence, correct choice is B

49. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	**Reason (R)**
In the given figure, ∠ABC = 70° and ∠ACB = 30°. Then, ∠BDC = 70°. 	In the given figure, ∠AOC = 130°, then ∠ABC = 115°. 

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Assertion (A) :

Here, in ΔABC

By angle sum property

∠ABC + ∠BCA + ∠CAB = 180°

70° + 30° + ∠CAB = 180°

∠CAB = 180° – 70° – 30° = 80°

∠CAB = ∠BDC = 80° (angles in same segment)

But given that ∠BDC = 70°

∴ Assertion(A) is wrong.

Reason (R) :

$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 130° = 65°$

$\angle ABC + \angle ADC = 180°$

$\angle ABC + 65° = 180°$

$\angle ABC = 180° - 65° = 115°$

Reason (R) is true

Assertion (A) :

$\angle ABC + \angle BCA + \angle BAC = 180°$

$70° + 30° + \angle BAC = 180°$

$\angle BAC = 180° - 70° - 30°$

$\angle BAC = 80°$

$\therefore \angle BDC = \angle BAC = 80°$ (angles in the same segment)

This is false.

Thus, Assertion (A) is false and Reason (R) is true.

Hence, correct choice is D

50. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	**Reason (R)**
A cyclic parallelogram is a square.	Diameter is the largest chord in a circle.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Clearly, Assertion (A) is false and Reason (R) is true.

Hence, correct choice is D

51. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). For selecting the correct answer:

Assertion (A)	**Reason (R)**
If two circles intersect at two points, then the line joining their centres is perpendicular to the common chord.	The perpendicular bisectors of two chords of a circle intersect at its centre.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) and Reason (R) are true but Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Clearly, Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

52. Question

Write T for true and F for false

(i) The degree measures of a semicircle is 180°.

(ii) The perimeter of a circle is called its circumference.

(iii) A circle divides the plane into three parts.

(iv) Let O be the centre of a circle with radius r. Then a point P such that OP < r is called an interior point of the circle.

(v) A circle can have only a finite number of equal chords.

Answer

(i) T

(ii) T

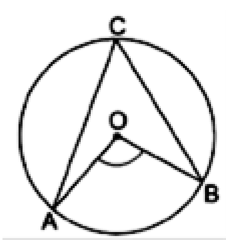
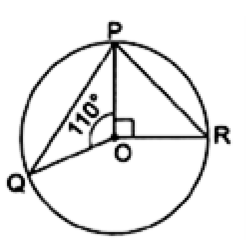
(iii) T (The region inside the circle, region outside the circle and region on the circle).

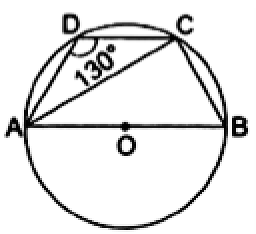
(iv) T (because point P lies inside the circle)

(v) F (A circle can have infinite number of chords)

53. Question

Match the following columns:

Column I	**Column II**
(a) Angle in a semicircle measures	(p) 40^o
(b) In the given figure, O is the centre of a circle. If ∠AOB = 120°, then ∠ACB = ? 	(q) 80^o
(c) In the given figure, O is the centre of a circle. If ∠POR = 90° and ∠POQ = 110°, then ∠QPR = ? 	(r) 90^o

(d) In cyclic quadrilateral ABCD, it is given that ∠ADC = 130° and AOB is a diameter of the circle through A, B, C and D. Then, ∠BAC = ? 	(s) 60°

The correct answer is:

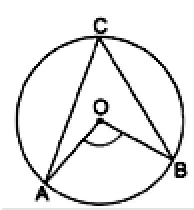
(a) – ……, (b) – ………,

(c) – …….., (d) – ………,

Answer

(a) Angle in a semicircle measures – 90° (r)

(b) In the given figure, O is the centre of a circle. If ∠AOB = 120°, then ∠ACB = ?

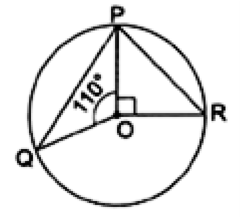


$\frac{1}{2}$∠AOB = ∠ACB

∠ACB = $\frac{1}{2}$ × 120° = 60°

∠ACB = 60° (s)

(c) In the given figure, O is the centre of a circle. If $\angle POR = 90^0$ and $\angle POQ = 110^0$, then $\angle QPR = ?$



Here, OP = OR = OQ (radius)

In ΔPOR

∠OPR = ∠ORP (angles opposite to equal sides are equal)

By angle sum property

∠POR + ∠OPR + ∠ORP = 180°

90° + 2×∠OPR = 180°

2×∠OPR = 180°—90°

2×∠OPR = 90°

∠OPR = 45°

Similarly in Δ POQ

∠OPQ = ∠OQP (angles opposite to equal sides are equal)

By angle sum property

∠POQ + ∠OPQ + ∠OQP = 180°

110° + 2×∠OQP = 180°

2×∠OQP = 180°–110°

2×∠OQP = 70°

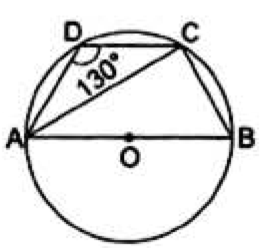
∠OQP = 35°

∠QPR = ∠QPO + ∠OPR = 45° + 35° = 80°

∴ ∠QPR = 80° (q)

(d) In cyclic quadrilateral $ABCD$, it is given that $\angle ADC = 130^0$ and AOB is a diameter of the circle

through A, B, C and D. Then, $\angle BAC = ?$



Here,

∠ADC + ∠ABC = 180° (opposite angles in cyclic quadrilateral are supplymentary)

130° + ∠ABC = 180°

∠ABC = 180°–130° = 50°

In ΔABC

By angle sum property

∠BAC + ∠ABC + ∠ACB = 180°

∠BAC + 50° + 90° = 180°

∠BAC = 180°–50°–90° = 40°

∴∠BAC = 40° (p)

∴ Answers are: (a) – (r), (b) – (s), (c) – (q), (d) – (p)

54. Question

Fill in the blanks

(i) Two circles having the same centre and different radii are called _________ circles.

(ii) Diameter is the __________ chord of a circle.

(iii) A continuous piece of a circle is called the _________ of the circle.

(iv) An arc of a circle is called a ___________ if the ends of the arc are the ends of a diameter.

(v) A segment of a circle is the region between an arc and a _________ of the circle.

(vi) A line segment joining the centre to any point on the circle is called its _________.

Answer

(i) Two circles having the same centre and different radii are called *concentric* cricles.

(ii) Diameter is the *longest* chord of a circle.

(iii) A continuous piece of a circle is called the *arc* of the circle.

(iv) An arc of a circle is called a *semicircle* if the ends of the arc are the ends of a diameter.

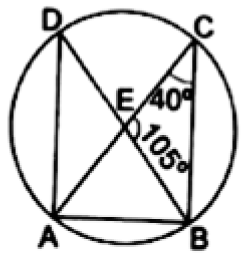
(v) A segment of a circle is the region between an arc and a *chord* of the circle.

(vi) A line segment joining the centre to any point on the circle is called its *radius.*

Formative Assessment (Unit Test)

1. Question

In the given figure, ∠ECB = 40° and ∠CEB = 105°. Then, ∠EAD = ?



A. 50°

B. 35°

C. 20°

D. 40°

Answer

Given: $\angle ECB = 40^{\circ}$ and $\angle CEB = 105^{\circ}$.

Here,

∠ACB = ∠ADB = 40° (angles in same segment)

∠BEC = ∠AED = 105° (vertically opposite angles)

In ΔAED

By angle sum property

∠ADE + ∠AED + ∠EAD = 180°

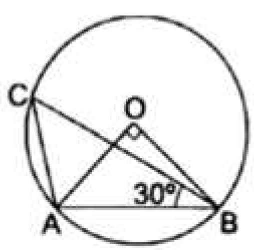
40° + 105° + ∠EAD = 180°

∠EAD = 180° – 40° – 105° = 35°

∴ ∠EAD = 35°

2. Question

In the given figure, O is the centre of a circle, ∠AOB = 90° and ∠ABC = 30°. Then, ∠CAO = ?



A. 30°

B. 45°

C. 60°

D. 90°

Answer

Given: $\angle AOB = 90^{o}$ and $\angle ABC = 30^{o}$.

We know that,

∠AOB = 2× ∠ACB

$\frac{1}{2}$∠AOB = ∠ACB

$\frac{1}{2}$ ×90° = ∠ACB

∠ACB = 45°

Now, consider ΔABC

By angle sum property

∠ACB + ∠ABC + ∠CAB = 180°

45° + 30° + ∠CAB = 180°

∠CAB = 180° — 45° — 30° = 105°

Consider ΔAOB

Here,

OA = OB (radius)

Let OA = OB = x

By angle sum property

∠AOB + ∠OAB + ∠OBA = 180°

90° + x + x = 180°

2x = 180° – 90° = 90°

x = 45°

Now,

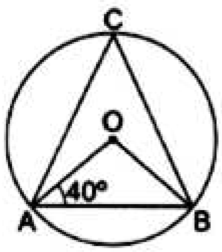
∠CAB = ∠BAO + ∠CAO = 105°

∠CAO = 105° – 45° = 60°

∴ ∠CAO = 60°

3. Question

In the given figure, O is the centre of a circle. If ∠OAB = 40°, then ∠ACB = ?



A. 40^o

B. 50^o

C. 60^o

D. 70^o

Answer

Given: ∠OAB = 40°

Consider ΔAOB

Here,

OA = OB (radius)

∠OBA = ∠OAB = 40° (angles opposite to equal sides are equal)

By angle sum property

∠OBA + ∠OAB + ∠AOB = 180°

40° + 40° + ∠AOB = 180°

∠AOB = 180° — 40° — 40° = 100°

We know that,

∠AOB = 2× ∠ACB

$\frac{1}{2}$∠AOB = ∠ACB

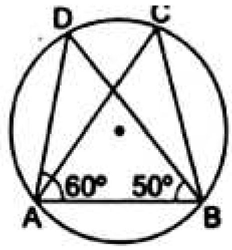
$\frac{1}{2}$ ×100° = ∠ACB

∠ACB = 50°

∴∠ACB = 50°

4. Question

In the given figure, ∠DAB = 60° and ∠ABD = 50°, then ∠ACB = ?



A. 50^o

B. 60^o

C. 70^o

D. 80^o

Answer

Given: ∠DAB = 60° and ∠ABD = 50°

In ΔABD

By angle sum property

∠DAB + ∠ABD + ∠ADB = 180°

60° + 50° + ∠ADB = 180°

110° + ∠ADB = 180°

∠ADB = 180° – 110° = 70°

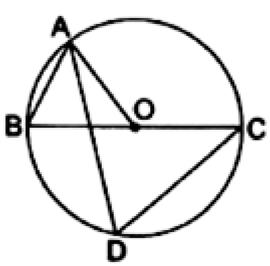
Here,

∠ADB = ∠ACB = 70° (angles in same segment)

∴∠ACB = 70°

5. Question

In the given figure, O is the centre of a circle, BC is a diameter and ∠BAO = 60°. Then, ∠ADC = ?



A. 30^o

B. 45^o

C. 60^o

D. 120^o

Answer

Given: $\angle BAO = 60^{o}$.

Consider ΔAOB

Here,

OA = OB (radius)

∠OBA = ∠OAB = 60° (angles opposite to equal sides are equal)

By angle sum property

∠OBA + ∠OAB + ∠AOB = 180°

60° + 60° + ∠AOB = 180°

∠AOB = 180° — 60° — 60° = 60°

Here,

∠BOC = ∠BOA + ∠AOC = 180°

60° + ∠AOC = 180°

∠AOC = 180° – 60° = 120°

We know that,

∠AOC = 2× ∠ADC

$\frac{1}{2}$∠AOC = ∠ADC

$\frac{1}{2}$ ×120° = ∠ADC

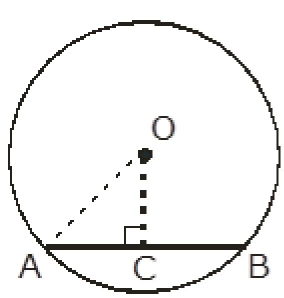
∠ADC = 60°

∴∠ADC = 60°

6. Question

Find the length of a chord which is at a distance of 9 cm from the centre of a circle of radius 15 cm.

Answer



Given radius(AO) = 15cm

Length of the chord (AB) = x

distance of the chord from the centre is 9cm.

Draw a perpendicular bisector from center to the chord and name it OC.

∴ AC = BC

Now in Δ AOC

Using Pythagoras theorem

$AO^2 = AC^2 + OC^2$

$15^2 = AC^2 + 9^2$

$AC^2 = 15^2 - 9^2$

$AC^2 = 225 - 81$

$AC^2 = 144$

AC = 12cm

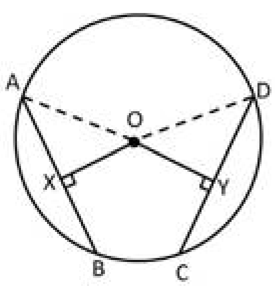
∴ BC = 12cm

∴ The length of the chord is AC + BC = 12 + 12 = 24 cm.

7. Question

Prove that equal chords of a circle are equidistant from the centre.

Answer



Given: AB = CD

Construction: Drop perpendiculars OX and OY on to AB and CD respectively and join OA and OD.

Here, OX ⊥AB (perpendicular from center to chord divides it into two equal halves)

$AX = BX = \frac{AB}{2}$ – – (1)

OY ⊥CD (perpendicular from center to chords divides it into equal halves

$CY = DY = \frac{CD}{2}$ – – (2)

Now, given that

AB = CD

$\therefore \frac{AB}{2} = \frac{CD}{2}$

AX = DY (from −1 and −2) − − (3)

In ΔAOX and ΔDOY

∠OXA = ∠OYD (right angle)

OA = OD (radius)

AX = DY (from −3)

∴ BY RHS congruency

ΔAOX≅ΔDOY

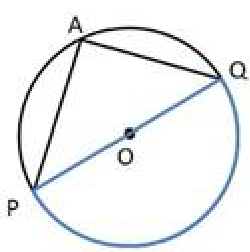
OX = OY (by C.P.C.T)

Hence proved.

8. Question

Prove that an angle in a semicircle is a right angle.

Answer



We know that,

∠POQ = 2∠PAQ

$\frac{\angle POQ}{2}$ = ∠PAQ

$\frac{180^\circ}{2}$ = ∠PAQ

90° = ∠PAQ

∠PAQ = 90°

Hence proved

9. Question

Prove that a diameter is the largest chord in a circle.

Answer

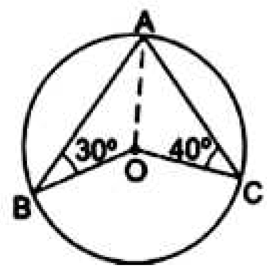
We know that,

A chord nearer to the center is longer than the chord which is far from the center

∴ Diameter is the longest chord in the circle (because it passes through the center and other chords are far from the center)

10. Question

A circle with centre O is given in which ∠OBA = 30° and ∠OCA = 40°. Find ∠BOC.



Answer

Given: $\angle OBA = 30^0$ and $\angle OCA = 40^0$.

Consider ΔOAB

Here,

OA = OB (radius)

∠OBA = ∠OAB = 30° (angles opposite to equal sides are equal)

Similarly, in ΔAOC

OA = OC (radius)

∠OCA = ∠OAC = 40° (angles opposite to equal sides are equal)

Here,

∠CAB = ∠OAB + ∠OAC = 30° + 40° = 70°

Here,

2×∠CAB = ∠BOC (∵The angle subtended by an arc at the center is twice the angle subtended by the same arc on any point on the remaining part of the circle).

∴2×∠CAB = ∠BOC

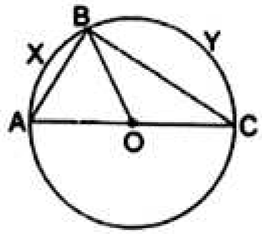
∴2×70° = ∠BOC

∠BOC = 140°.

∴ ∠BOC = 140°

11. Question

In the given figure, AOC is a diameter of a circle with centre O and arc $AXB = \frac{1}{2}$ arc BYC. Find ∠BOC.



Answer

Given: $AXB = \frac{1}{2}$ arc $BYC.$

Here,

2×AXB = BYC

∴ 2×∠AOB = ∠BOC

∠AOB = $\frac{1}{2}$∠BOC –1

Here,

∠AOC = ∠AOB + ∠BOC = 180°

$\frac{1}{2}$∠BOC + ∠BOC = 180° (from –1)

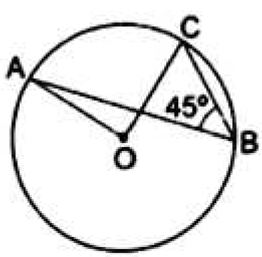
$\frac{3}{2}$∠BOC = 180°

∠BOC = $\frac{2}{3}$×180° = 120°

∴ $\angle BOC = 120^0$

12. Question

In the given figure, O is the centre of a circle and ∠ABC = 45°. Prove that OA ⊥ OC.



Answer

Given: ∠ABC = 45°

We know that ,

∠ AOC = 2×∠ABC

∠ AOC = 2×45 = 90°

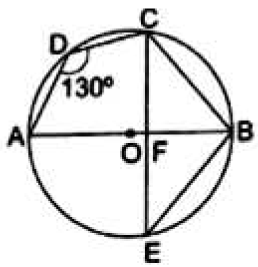
∴ ∠AOC = 90°

Therefore $OA \perp OC$.

Hence proved.

13. Question

In the given figure, O is the centre of a circle, ∠ADC = 130° and chord BC = chord BE. Find ∠CBE.



Answer

Given: ∠ADC = 130° , BC = BE

We know that,

(exterior ∠AFC) = (2×∠ADC)

(exterior ∠AFC) = (2×130)

(exterior ∠AFC) = 260

∠AFC = 360° – (exterior ∠AFC) = 360°–260° = 100°

∠AFB = ∠AFC + ∠CFB = 180°

∠AFC + ∠CFB = 180°

100° + ∠CFB = 180°

∠CFB = 180° – 100° = 80°

In quadrilateral ABCD

∠ADC + ∠ABC = 180° (opposite angles in cyclic quadrilateral are supplementary)

130° + ∠ABC = 180°

∠ABC = 180° – 130° = 50°

In ΔBCF

By angle sum property

∠CBF + ∠CFB + ∠BCF = 180°

50° + 80° + ∠BCF = 180°

∠BCF = 180° – 50° – 80° = 50°

Now,

∠CFE = ∠CFB + ∠BFE = 180°

∠CFB + ∠BFE = 180°

80° + ∠BFE = 180°

∠BFE = 180° – 80° = 100°

Here,

In ΔBCE

BC = BE (given)

∠BCE = ∠BEC = 50° (angles opposite to equal sides are equal)

By angle sum property

∠BCE + ∠BEC + ∠CBE = 180°

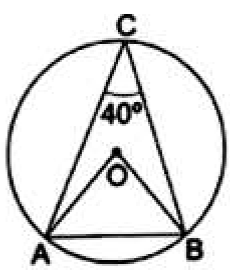
50° + 50° + ∠CBE = 180°

∠CBE = 180° – 50° – 50° = 100°

$\therefore \angle CBE = 100^\circ$

14. Question

In the given figure, O is the centre of a circle, ∠ACB = 40°. Find ∠OAB.



Answer

Given: ∠ACB = 40°

We know that ,

∠ AOB = 2×∠ACB

∠ AOB = 2×40 = 80°

∴ ∠AOB = 80°

In ΔAOB

OA = OB (radius)

∠OAB = ∠OBA (angles opposite to equal sides are equal)

Let ∠OAB = ∠OBA = x

By angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180°$

$80 + x + x = 180°$

$80 + 2x = 180°$

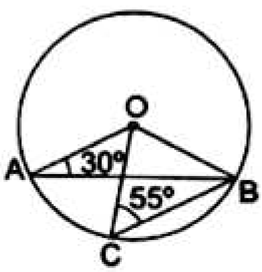
$2x = 180° - 80° = 100°$

$x = \frac{100}{2} = 50°$

$\therefore \angle OAB = 50^0$

15. Question

In the given figure, O is the centre of a circle, ∠OAB = 30° and ∠OCB = 55°. Find ∠BOC and ∠AOC.



Answer

Given: $\angle OAB = 30^0$ and $\angle OCB = 55^0$.

Here,

In ΔAOB

OA = OB (radius)

∠OAB = ∠OBA (angles opposite to equal sides are equal)

∴ ∠ OBA = 30°

Now, by angle sum property

$\angle AOB + \angle OBA + \angle OAB = 180°$

$\angle AOB + 30° + 30° = 180°$

$\angle AOB = 180° - 30° - 30°$

$\angle AOB = 120°$

Now, Consider Δ BOC

OC = OB (radius)

∠OCB = ∠OBC (angles opposite to equal sides are equal)

∴ ∠ OBA = 55°

Now, by angle sum property

∠BOC + ∠OBC + ∠OCB = 180°

∠BOC + 55° + 55° = 180°

∠BOC = 180° – 55° – 55° = 70°

∴ ∠BOC = 70°

Here,

∠AOB = ∠AOC + ∠BOC

120° = ∠AOC + 70°

∠AOC = 120° – 70°

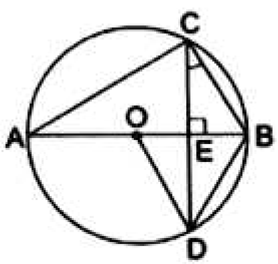
∠AOC = 50°

∴ ∠AOC = 50°

∴ $\angle BOC = 70^0, \angle AOC = 50^0$

16. Question

In the given figure, O is the centre of the circle, BD = OD and CD ⊥ AB. Find ∠CAB.



Answer

Given: $BD = OD$ and $CD \perp AB.$

In ΔOBD

OB = OD = DB

∴ Δ OBD is equilateral

∴ ∠ODB = ∠DBO = ∠BOD = 60°

Consider ΔDEB and ∠BEC

Here,

BE = BE (common)

∠CEB = ∠DEB (right angle)

CE = DE (OE is perpendicular bisector)

∴ By SAS congruency

$\angle CAB = 30^{\circ}$

ΔDEB ≅ ∠BEC

∴∠DEB = ∠EBC (C.P.C.T)

∴∠EBC = 60°

Now, in ΔABC

∠EBC = 60°

∠ACB = 90° (angle in semicircle)

By angle sum property

∠EBC + ∠ACB + ∠CAB = 180°

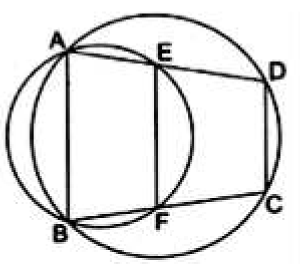
60° + 90° + ∠CAB = 180°

∠CAB = 180° − 60° − 90° = 30°

∴∠CAB = 30°

17. Question

In the given figure, ABCD is a cyclic quadrilateral. A circle passing through A and B meets AD and BC in the points E and F respectively. Prove that EF || DC.



Answer

Here,

In cyclic Quadrilateral ABFE

∠ABF + ∠AEF = 180° (opposite angles in cyclic quadrilateral are supplementary) −1

In cyclic Quadrilateral ABCD

∠ABC + ∠ADC = 180° (opposite angles in cyclic quadrilateral are supplementary) −2

From −1 and −2

∠ABF + ∠AEF = ∠ABC + ∠ADC

∠AEF = ∠ADC (∠ABF = ∠ABC)

Since these are corresponding angles

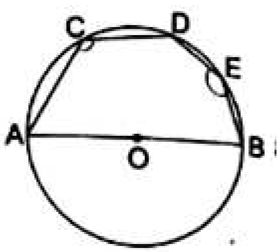
We can say that EF || DC

∴ EF||DC

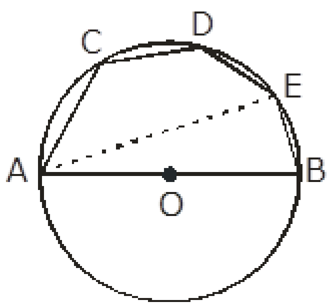
Hence proved.

18. Question

In the given figure, AOB is a diameter of the circle and C, D, E are any three points on the semicircle. Find the value of ∠ACD + ∠BED.



Answer



Construction: Join AE

Consider cyclic quadrilateral ACDEA

Here,

∠ACD + ∠DEA = 180° (opposite angles in cyclic quadrilateral are supplementary)

Also,

∠AEB = 90° (angle in semicircle)

∴∠ACD + ∠DEA + ∠AEB = 180° + 90°

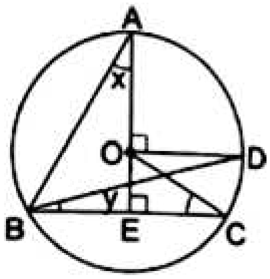
∠ACD + ∠BED = 270° (∠DEA + ∠AEB = ∠BED)

∴∠ACD + ∠BED = 270°

Hence proved.

19. Question

In the given figure, O is the centre of a circle and ∠BCO = 30°. Find x and y.



Answer

Given: $\angle BCO = 30^0$.

In ΔEOC

By angle sum property

∠EOC + ∠OEC + ∠OCE = 180°

∠EOC + 90° + 30° = 180°

∠EOC = 180° – 90° – 30° = 60°

∠EOC = 60°

Here,

∠EOD = ∠EOC + ∠COD = 90°

∠EOC + ∠COD = 90°

60° + ∠COD = 90°

∠COD = 90° – 60° = 30°

Now,

∠ AOC = ∠AOD + ∠COD = 90° + 30° = 120°

We know that ,

∠ COD = 2×∠CBD

$\frac{1}{2}$∠ COD = ∠CBD

∠CBD = $\frac{1}{2}$× 120° = 60°

Consider ΔABE

By angle sum property

∠AEB + ∠ABE + ∠BAE = 180°

90° + 60° + ∠BAE = 180°

∠BAE = 180° – 90° – 60° = 30°

∴ x = 30°

We know that ,

$\angle AOC = 2\times\angle ABC$

$\frac{1}{2}\angle AOC = \angle ABC$

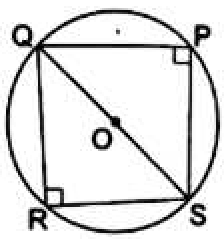
$\angle ABC = \frac{1}{2}\times 30° = 15°$

$\therefore y = 15°$

$\therefore x = 30, \; y = 15$

20. Question

PQ and RQ are the chords of a circle equidistant from the centre. Prove that the diameter passing through Q bisects ∠PQR and ∠PSR.



Answer

Given: chords PQ and RQ are equidistant from center.

Here consider ΔPQS and ΔRQS

Here,

QS = QS (common)

∠QPS = ∠QRS (right angle)

PQ = QS (chords equidistant from center are equal in length)

∴ By RHS congruency ΔPQS ≅ ΔRQS

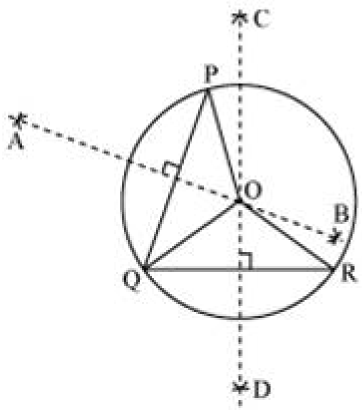
∴ ∠RQS = ∠SQP and ∠RSQ = ∠QSP (by C.P.C.T)

Therefore we can say that diameter passing through Q bisects $\angle PQR$ and $\angle PSR.$

21. Question

Prove that there is one and only one circle passing through three non – collinear points.

Answer



Given: Three non collinear points P, Q and R

Construction: Join PQ and QR.

Draw perpendicular bisectors AB of PQ and CD of QR. Let the perpendicular bisectors intersect at the point O.

Now join OP, OQ and OR.

A circle is obtained passing through the points P, Q and R.

Proof:

We know that,

Every point on the perpendicular bisector of a line segment is equidistant from its ends

points.

Thus, OP = OQ (Since, O lies on the perpendicular bisector of PQ)

and OQ = OR. (Since, O lies on the perpendicular bisector of QR)

So, OP = OQ = OR.

Let OP = OQ = OR = r.

Now, draw a circle C(O, r) with O as centre and r as radius.

Then, circle C(O, r) passes through the points P, Q and R.

Next, we prove this circle is the only circle passing through the points P, Q and R.

If possible, suppose there is a another circle C(O′, t) which passes through the points P, Q, R.

Then, O′ will lie on the perpendicular bisectors AB and CD.

But O was the intersection point of the perpendicular bisectors AB and CD.

So, O ′ must coincide with the point O. **(Since, two lines cannot intersect at more than one point**)

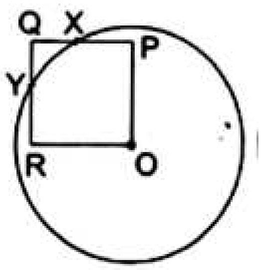
As, O′P = t and OP = r; and O ′ coincides with O, we get $t = r$.

Therefore, C(O, r) and C(O, t) are congruent.

Thus, there is one and only one circle passing through three the given non – collinear points.

22. Question

In the give figure, OPQR is a square. A circle drawn with centre O cuts the square in x and y. Prove that QX = XY.



Answer

Construction: Join OX and OY

In Δ OPX and ΔORY,

OX = OY (radii of the same circle)

OP = OR (sides of the square)

∴ΔOPX≅ Δ ORY (RHS rule)

∴ PX = RY (CPCT)—1

OPQR is a square

∴ PQ = RQ

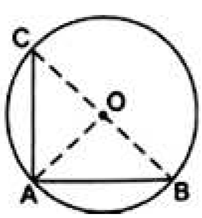
∴ PX + QX = RY + QY

QX = QY (from –1)

Hence proved

23. Question

In the given figure, AB and AC we two equal chords of a circle with centre O. Show that O lies on the bisectors of ∠BAC.



Answer

Given: AB = AC

Construction: join OA, OB and OC

Proof:

Consider ΔAOB and ΔAOC

Here,

OC = OB (radius)

OA = OA (common)

AB = AC (given)

∴ By SSS congruency

ΔAOB ≅ ΔAOC

∴ ∠OAC = ∠OAB (by C.P.C.T)

Hence, we can say that OA is the bisector of ∠BAC, that is O lies on the bisector of ∠BAC.