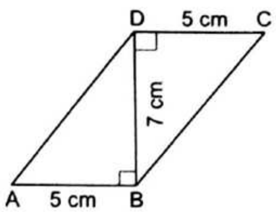
10. Area

Exercise 10A

1. Question

In the adjoining figure, show that ABCD is a parallelogram.

Calculate the area of ‖gm ABCD.



Answer

In the given figure consider ΔABD and ΔBCD

Area of ΔABD = $\frac{1}{2}$ x base x height = $\frac{1}{2}$ x AB x BD

= $\frac{1}{2}$ x 5 x 7 = $\frac{35}{2}$ -------- 1

Area of ΔBCD = $\frac{1}{2}$ x base x height = $\frac{1}{2}$ x DC x DB

= $\frac{1}{2}$ x 5 x 7 = $\frac{35}{2}$ ----------2

From 1 and 2 we can tell that area of two triangle that is ΔABD and ΔBCD are equal

Since the diagonal BD divides ABCD into two triangles of equal area and opp sides AB = DC

∴ABCD is a parallelogram

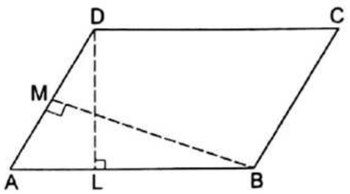
∴ Area of parallelogram ABCD = Area of ΔABD + Area of ΔBCD

= $\left(\frac{35}{2} + \frac{35}{2}\right)$ = $\frac{70}{2}$ cm^2 = 35 cm^2

∴ Area of parallelogram ABCD = 35cm^2

2. Question

In a parallelogram ABCD, it is being given that AB = 10 cm and the altitudes corresponding to the sides AB and AD are DL = 6 cm and BM = 8 cm, respectively. Find AD.



Answer

Given

AB = 10 cm

DL = 6 cm

BM = 8 cm

AD = ? (To find)

Here, Area of parallelogram = base x height

In the given figure if we consider AB as base Area = AB x DL

If we consider DM as base Area = AD x BM

∴ Area = AB x DL = AD x BM

⇒ 10 x 6 = AD x 8

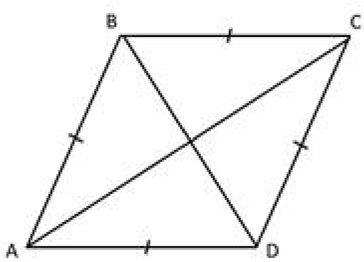
⇒ 60 = 8 x AD

$\Rightarrow AD = \frac{60}{8} = 7.5\ cm$

3. Question

Find the area of a rhombus, the lengths of whose diagonals are 16 cm and 24 cm respectively.

Answer



Here, Let ABCD be Rhombus with diagonals AC and BD

Here let AC = 24 and BD = 16

We know that, in a Rhombus, diagonals are perpendicular bisectors to each other

∴ if we consider ΔABC AC is base and OB is height

Similarly, in ΔADC AC is base and OD is height

Now, Area of Rhombus = Area of ΔABC + Area of ΔADC

$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$

$= \frac{1}{2} \times 24 \times \frac{BD}{2} + \frac{1}{2} \times 24 \times \frac{BD}{2}$ (Since AC and BC are perpendicular bisectors $\therefore$ OB = OD = $\frac{BD}{2}$)

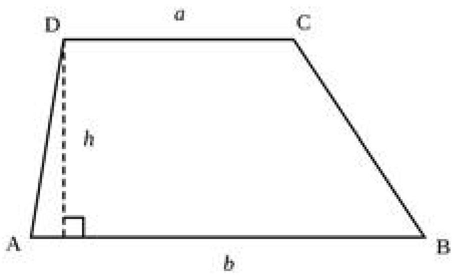
$= \frac{1}{2} \times 24 \times \frac{16}{2} + \frac{1}{2} \times 24 \times \frac{16}{2} = 96 + 96 = 192\ cm^2$

$\therefore$ Area of Rhombus ABCD is $192 cm^2$

4. Question

Find the area of a trapezium whose parallel sides are 9 cm and 6 cm respectively and the distance between these sides is 8 cm.

Answer



Given

AB = a = 9 cm

DC = b = 6 cm

Height (h) = 8 cm

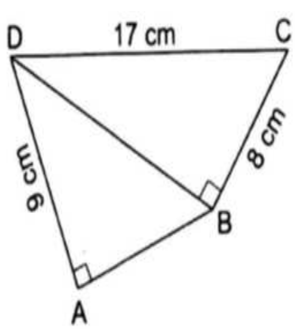
We know that area of trapezium is $\frac{1}{2}$ x (sum of parallel sides) x height

Therefore, Area of trapezium ABCD $= \frac{1}{2} \times (AB + DC) \times h = \frac{1}{2} \times (9 + 6) \times 8 = 60\ cm^2$

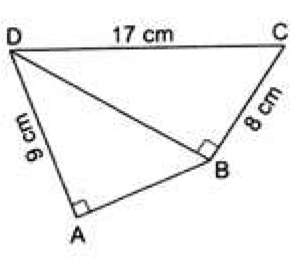
$\therefore$ Area of Trapezium ABCD = $60 cm^2$

5A. Question

Calculate the area of quad. ABCD, given in Fig. (i).



Answer



(i)

Given

AD = 9 cm

BC = 8 cm

DC = 17 cm

Here Area of Quad ABCD = Area of ΔABD + Area of ΔBCD

$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times BC \times BD$

By Pythagoras theorem in ΔBCD

$DC^2 = BD^2 + BC^2$

$17^2 = BD^2 + 8^2$

$BD^2 = 17^2 - 8^2 = 289 - 64 = 225$

$\therefore$BD = 15 cm

Similarly in ΔABD using Pythagoras theorem

$BD^2 = AD^2 + AB^2$

$15^2 = 9^2 + AB^2$

$AB^2 = 15^2 - 9^2 = 225 - 81 = 144$

$\therefore$AB = 12 cm

Now, Area of Quad ABCD = Area of ΔABD + Area of ΔBCD

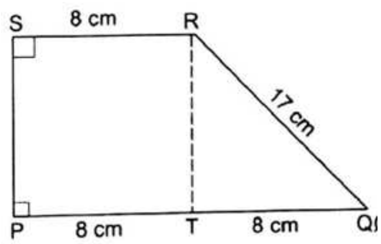
$= \frac{1}{2} \times AB \times AD + \frac{1}{2} \times BC \times BD$

$= \frac{1}{2} \times 12 \times 9 + \frac{1}{2} \times 8 \times 15 = 54 + 60 = 114\ cm^2$

$\therefore$ Area of Quadrilateral ABCD = 114 cm^2

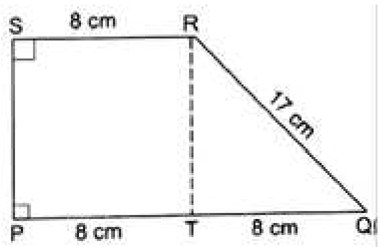
5B. Question

Calculate the area of trap. PQRS, given in Fig. (ii).



(ii)

Answer



(ii)

Given :- Right trapezium

RS = 8 cm

PT = 8cm

TQ = 8 cm

RQ = 17 cm

Here PQ = PT + TQ = 8 + 8 = 16

We know that area of trapezium is $\frac{1}{2}$ x (sum of parallel sides) x height

That is $\frac{1}{2}$ x (AB + DC) x RT

Consider ΔTQR

By Pythagoras theorem

$RQ^2 = TQ^2 + RT^2$

$17^2 = 8^2 + RT^2$

$RT^2 = 17^2 - 8^2 = 289 - 64 = 225$

$\therefore$ RT = 15 cm

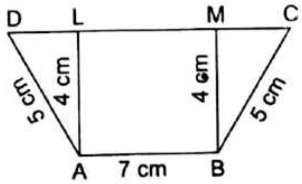
$\therefore$ Area of trapezium = $\frac{1}{2}$ x (RS + PQ) x RT

= $\frac{1}{2}$ x (8 + 16) x 15 = 180 cm^2

∴ Area of trapezium PQRS = 180cm^2

6. Question

In the adjoining figure, ABCD is a trapezium in which AB ∥ DC; AB = 7 cm; AD = BC = 5 cm and the distance between AB and DC is 4 cm. Find the length of DC and hence, find the area of trap. ABCD.



Answer

Given

AB = 7 cm

AD = BC 5 cm

AL = BM = 4cm (height)

DC = ?

Here in the given figure AB = LM

∴ LM = 7 cm ------------1

Now Consider ∆ALD

By Pythagoras theorem

$AD^2 = AL^2 + DL^2$

$5^2 = 4^2 + DL^2$

$DL^2 = 5^2 - 4^2 = 25 - 16 = 9$

∴ DL = 3 cm --------------2

Similarly in ∆BMC

By Pythagoras theorem

$BC^2 = BM^2 + MC^2$

$5^2 = 4^2 + MC^2$

$MC^2 = 5^2 - 4^2 = 25 - 16 = 9$

∴ MC = 3 cm --3

∴ from 1 2 and 3

DC = DL + LM + MC = 3 + 7 + 3 = 13 cm

We know that area of trapezium is $\frac{1}{2}$ x (sum of parallel sides) x height

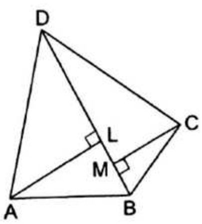
∴ Area of trapezium = $\frac{1}{2}$ x (AB + DC) x AL

= $\frac{1}{2}$ x (7 + 13) x 4 = 40 cm^2

∴ Area of trapezium ABCD = 180cm^2

7. Question

BD is one of the diagonals of a quad. ABCD. If AL ⊥ BD and CM ⊥ BD, show that ar(quad. ABCD) = $\frac{1}{2}$ x BD x (AL + CM).



Answer

Given :

AL ⊥ BD and CM ⊥ BD

To prove : ar (quad. ABCD) = $\frac{1}{2}$ x BD x (AL + CM)

Proof:

Area of ΔABD = $\frac{1}{2}$ x BD x AM

Area of ΔABD = $\frac{1}{2}$ x BD x CM

Now area of Quad ABCD = Area of ΔABD + Area of ΔBCD

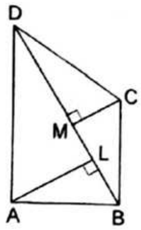
= $\frac{1}{2}$ x BD x AL + $\frac{1}{2}$ x BD x CM

= $\frac{1}{2}$ x BD x (AL + CM)

Hence proved

8. Question

In the adjoining figure, ABCD is a quadrilateral in which diag. BD = 14 cm. If AL ⊥ BD and CM ⊥ BD such that AL = 8 cm and CM = 6 cm, find the area of quad. ABCD.



Answer

Given

AL ⊥ BD and CM ⊥ BD

BD = 14 cm

AL = 8 cm

CM = 6 cm

Here,

Area of ΔABD = $\frac{1}{2}$ x BD x AM

Area of ΔABD = $\frac{1}{2}$ x BD x CM

Now area of Quad ABCD = Area of ΔABD + Area of ΔBCD

= $\frac{1}{2}$ x BD x AL + $\frac{1}{2}$ x BD x CM

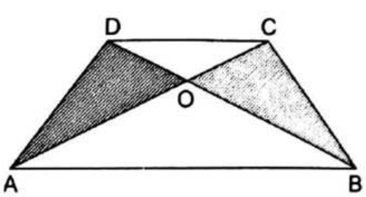
= $\frac{1}{2}$ x BD x (AL + CM)

∴ Area of quad ABCD = $\frac{1}{2}$ x BD x (AL + CM) = $\frac{1}{2}$ x 14 x (8 + 6) = 98cm^2

∴ Area of quad ABCD = 98cm^2

9. Question

In the adjoining figure, ABCD is a trapezium in which AB ∥ DC and its diagonals AC and BD intersect at O. Prove that ar(ΔAOD) = ar(ΔBOC).



Answer

Given

AB ∥ DC

To prove that: area(ΔAOD) = area(ΔBOC)

Here in the given figure Consider ΔABD and ΔABC,

we find that they have same base AB and lie between two parallel lines AB and CD

According to the theorem: triangles on the same base and between same parallel lines have equal areas.

∴ Area of ΔABD = Area of ΔBCA

Now,

Area of ΔAOD = Area of ΔABD - Area of ΔAOB ---1

Area of ΔCOB = Area of ΔBCA - Area of ΔAOB ---2

∴ From 1 and 2

We can conclude that area(ΔAOD) = area(ΔBOC) (Since Area of ΔAOB is common)

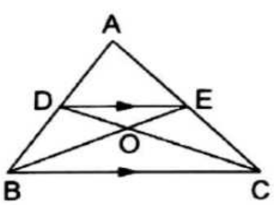
Hence proved

10. Question

In the adjoining figure, DE ∥ BC. Prove that

(i) ar(ΔACD) = ar(ΔABE),

(ii) ar(ΔOCE) = ar(ΔOBD).



Answer

Given

AB ∥ DC

To prove that : (i) area(ΔACD) = area(ΔABE)

(ii) area(ΔOCE) = area(ΔOBD)

(i)

Here in the given figure Consider ΔBDE and ΔECD,

we find that they have same base DE and lie between two parallel lines BC and DE

According to the theorem: triangles on the same base and between same parallel lines have equal areas.

∴ Area of ∆BDE = Area of ∆ECD

Now,

Area of ∆ACD = Area of ∆ECD + Area of ∆ADE ---1

Area of ∆ABE = Area of ∆BDE + Area of ∆ADE ---2

∴ From 1 and 2

We can conclude that area(ΔAOD) = area(ΔBOC) (Since Area of ∆ADE is common)

Hence proved

(ii)

Here in the given figure Consider ∆BCD and ∆BCE,

we find that they have same base BC and lie between two parallel lines BC and DE

According to the theorem : triangles on the same base and between same parallel lines have equal areas.

∴ Area of ∆BCD = Area of ∆BCE

Now,

Area of ∆OBD = Area of ∆BCD - Area of ∆BOC ---1

Area of ∆OCE = Area of ∆BCE - Area of ∆BOC ---2

∴ From 1 and 2

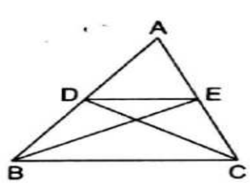
We can conclude that area(ΔOCE) = area(ΔOBD) (Since Area of ∆BOC is common)

Hence proved

11. Question

In the adjoining figure, D and E are points on the sides AB and AC of ΔABC such that ar(ΔBCE) = ar(ΔBCD).

Show that DE ∥ BC.



Answer

Given

A triangle ABC in which points D and E lie on AB and AC of ΔABC such that ar(ΔBCE) = ar(ΔBCD).

To prove: DE ∥ BC

Proof:

Here, from the figure we know that ΔBCE and ΔBCD lie on same base BC and

It is given that area(ΔBCE) = area(ΔBCD)

Since two triangle have same base and same area they should equal altitude(height)

That means they lie between two parallel lines

That is DE ∥ BC

∴ DE ∥ BC

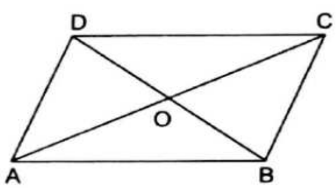
Hence proved

12. Question

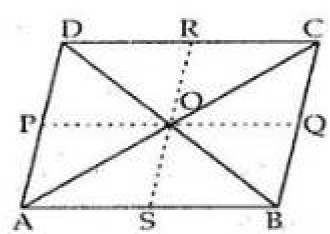
In the adjoining figure, O is any point inside a parallelogram ABCD. Prove that

(i) ar(ΔOAB) + ar(ΔOCD) = $\frac{1}{2}$ ar(∥gm ABCD),

(ii)ar(ΔOAD) + ar(ΔOBC) = $\frac{1}{2}$ ar(∥gm ABCD).



Answer



Given : A parallelogram ABCD with a point 'O' inside it.

To prove : (i) area(ΔOAB) + area(ΔOCD) = $\frac{1}{2}$ area(∥gm ABCD),

(ii)area(ΔOAD) + area(ΔOBC) = $\frac{1}{2}$ area(∥gm ABCD).

Construction : Draw PQ ∥ AB and RS ∥ AD

Proof:

(i)

ΔAOB and parallelogram ABQP have same base AB and lie between parallel lines AB and PQ.

According to theorem: If a triangle and parallelogram are on the same base and between the same parallel lines, then the area of the triangle is equal to half of the area of the parallelogram.

$\therefore$ area(ΔAOB) = $\frac{1}{2}$ area(‖gm ABQP) ---1

Similarly, we can prove that area(ΔCOD) = $\frac{1}{2}$ area(‖gm PQCD) ---2

$\therefore$ Adding –1 and –2 we get,

area(ΔAOB) + area(ΔCOD) = $\frac{1}{2}$ area(‖gm ABQP) + $\frac{1}{2}$ area(‖gm PQCD)

area(ΔAOB) + area(ΔCOD) = $\frac{1}{2}$ [area(‖gm ABQP) + area(‖gm PQCD)] = $\frac{1}{2}$ area(‖gm ABCD)

$\therefore$ area(ΔAOB) + area(ΔCOD) = $\frac{1}{2}$ area(‖gm ABCD)

Hence proved

(ii)

ΔOAD and parallelogram ASRD have same base AD and lie between parallel lines AD and RS.

According to theorem: If a triangle and parallelogram are on the same base and between the same parallel lines, then the area of the triangle is equal to half of the area of the parallelogram.

$\therefore$ area(ΔOAD) = $\frac{1}{2}$ area(‖gm ASRD) ---1

Similarly, we can prove that area(ΔOBC) = $\frac{1}{2}$ area(‖gm BCRS) ---2

$\therefore$ Adding –1 and –2 we get,

area(ΔOAD) + area(ΔOBC) = $\frac{1}{2}$ area(‖gm ASRD) + $\frac{1}{2}$ area(‖gm BCRS)

area(ΔOAD) + area(ΔOBC) = $\frac{1}{2}$ [area(‖gm ASRD) + area(‖gm BCRS)] = $\frac{1}{2}$ area(‖gm ABCD)

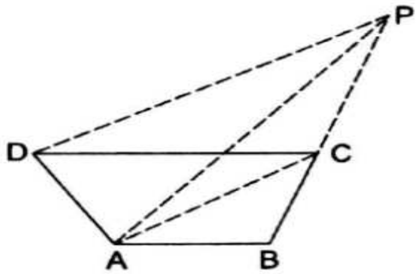
$\therefore$ area(ΔOAD) + area(ΔOBC) = $\frac{1}{2}$ area(‖gm ABCD)

Hence proved

13. Question

In the adjoining figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P.

Prove that ar(ΔABP) = (quad.ABCD).



Answer

Given : ABCD is a quadrilateral in which a line through D drawn parallel to AC which meets BC produced in P.

To prove: area of (ΔABP) = area of (quad ABCD)

Proof:

Here, in the given figure

ΔACD and ΔACP have same base and lie between same parallel line AC and DP.

According to the theorem : triangles on the same base and between same parallel lines have equal areas.

∴ area of (ΔACD) = area of (ΔACP) -------------1

Now, add area of (ΔABC) on both side of (1)

∴ area of (ΔACD) + (ΔABC) = area of (ΔACP) + (ΔABC)

Area of (quad ABCD) = area of (ΔABP)

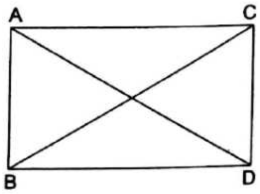
∴ area of (ΔABP) = Area of (quad ABCD)

Hence proved

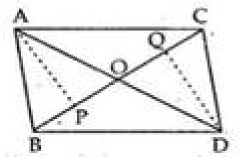
14. Question

In the adjoining figure, ΔABC and ΔDBC are on the same base BC with A and D on opposite sides of BC such that ar(ΔABC) = ar(ΔDBC).

Show that BC bisects AD.



Answer



Given : ΔABC and ΔDBC having same base BC and area(ΔABC) = area(ΔDBC).

To prove: OA = OD

Construction : Draw AP ⊥ BC and DQ ⊥ BC

Proof :

Here area of ΔABC = $\frac{1}{2}$ x BC x AP and area of ΔABC = $\frac{1}{2}$ x BC x DQ

since, area(ΔABC) = area(ΔDBC)

∴ $\frac{1}{2}$ x BC x AP = $\frac{1}{2}$ x BC x DQ

∴ AP = DQ -------------- 1

Now in ΔAOP and ΔQOD, we have

∠APO = ∠DQO = 90° and

∠AOP = ∠DOQ [Vertically opposite angles]

AP = DQ [from 1]

Thus by AAS congruency

ΔAOP ≅ ΔQOD [AAS]

Thus By corresponding parts of congruent triangles law [C.P.C.T]

∴ OA = OD [C.P.C.T]

Hence BC bisects AD

Hence proved

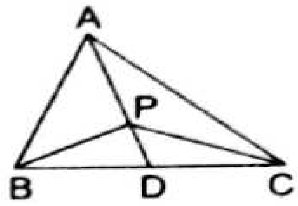
15. Question

In the adjoining figure, AD is one of the medians of a ΔABC and P is a point on AD.

Prove that

(i) ar(ΔBDP) = ar(ΔCDP)

(ii) ar(ΔABP) = ar(ΔACP)



Answer

Given : A ΔABC in which AD is the median and P is a point on AD

To prove: (i) ar(ΔBDP) = ar(ΔCDP),

(ii) ar(ΔABP) = ar(ΔACP).

(i)

In ΔBPC, PD is the median. Since median of a triangle divides the triangles into two equal areas

So, area(ΔBDP) = area(ΔCDP)----1

Hence proved

(ii)

In ΔABC AD is the median

So, area(ΔABD) = area(ΔADC) ----2 and

area(ΔBDP) = area(ΔCDP) [from 1]

Now subtracting area(ΔBDP) from ---2 , we have

area(ΔABD) - area(ΔBDP) = area(ΔADC) - area(ΔBDP)

area(ΔABD) - area(ΔBDP) = area(ΔADC) - area(ΔCDP) [since area(ΔBDP) = area(ΔCDP) from –1]

∴ area(ΔABP) = area(ΔACP)

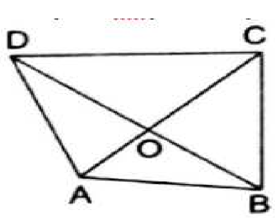
Hence proved.

16. Question

In the adjoining figure, the diagonals AC and BD of a quadrilateral ABCD intersect at O.

If BO = OD, prove that

Ar(ΔABC) = ar(ΔADC).



Answer

Given : A quadrilateral ABCD with diagonals AC and BD and BO = OD

To prove: Area of (ΔABC) = area of (ΔADC)

Proof : BO = OD [given]

Here AO is the median of ΔABD

∴ Area of (ΔAOD) = Area of (ΔAOB) ---------------- 1

And OC is the median of ΔBCD

∴ Area of (ΔCOD) = Area of (ΔBOC) ---------------- 2

Now by adding –1 and –2 we get

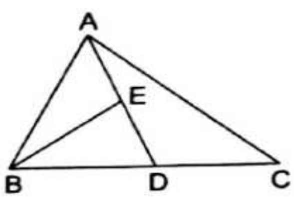
Area of (ΔAOD) + Area of (ΔCOD) = Area of (ΔAOB) + Area of (ΔBOC)

∴ Area of (ΔABC) = Area of (□#x2206;ADC)

Hence proved

17. Question

ABC is a triangle in which D is the midpoint of BC and E is the midpoint of AD. Prove that ar(ΔBED) = $\frac{1}{4}$ ar(ΔABC).



Answer

Given : A ΔABC in which AD is the median and E is the midpoint on line AD

To prove: area(ΔBED) = $\frac{1}{4}$ area(ΔABC)

Proof : here in ΔABC AD is the midpoint

∴ Area of (ΔABD) = Area of (ΔADE)

Hence Area of (ΔABD) = $\frac{1}{2}$ [Area of (ΔABC)] ------------------ 1

No in ΔABD E is the midpoint of AD and BE is the median

∴ Area of (ΔBDE) = Area of (ΔABE)

Hence Area of (ΔBED) = $\frac{1}{2}$ [Area of (ΔABD)] -------------- 2

Substituting (1) in (2), we get

Hence Area of (ΔBED) = $\frac{1}{2}$ [$\frac{1}{2}$ Area of (ΔABC)]

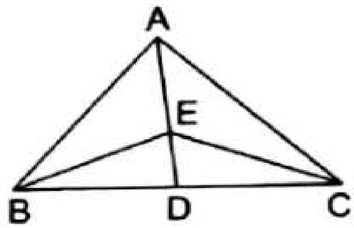
∴ area(ΔBED) = $\frac{1}{4}$area(ΔABC)

Hence proved

18. Question

The vertex A of ΔABC is joined to a point D on the side BC. The midpoint of AD is E. Prove that

ar(ΔBEC) = $\frac{1}{2}$ ar(ΔABC).



Answer

Given : A ΔABC in which AD is a line where D is a point on BC and E is the midpoint of AD

To prove: ar(ΔBEC) = $\frac{1}{2}$ ar(ΔABC)

Proof: In ΔABD E is the midpoint of side AD

∴ Area of (ΔBDE) = Area of (ΔABE)

Hence Area of (ΔBDE) = $\frac{1}{2}$ [Area of (ΔABD)] −1

Now, consider ΔACD in which E is the midpoint of side AD

∴ Area of (ΔECD) = Area of (ΔAEC)

Hence Area of (ΔECD) = $\frac{1}{2}$ [Area of (ΔACD)] −2

Now, adding −1 and −2, we get

Area of (ΔBDE) + Area of (ΔECD) = $\frac{1}{2}$ [Area of (ΔABD)] + $\frac{1}{2}$ [Area of (ΔACD)]

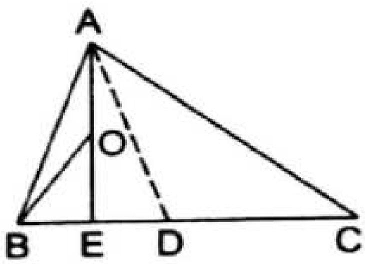
∴ area(ΔBEC) = $\frac{1}{2}$ [area(ΔABD) + area(ΔACD)]

∴ Area(ΔBEC) = $\frac{1}{2}$ Area(ΔABC)

Hence proved

19. Question

D is the midpoint of side BC of ΔABC and E is the midpoint of BD. If O is the midpoint of AE, prove that ar(ΔBOE) = $\frac{1}{8}$ ar(ΔABC).



Answer

Given : D is the midpoint of side BC of ΔABC and E is the midpoint of BD and O is the midpoint of AE

To prove : ar(ΔBOE) = $\frac{1}{8}$ ar(ΔABC)

Proof : Consider ΔABC here D is the midpoint of BC

∴ Area of (ΔABD) = Area of (ΔACD)

∴ Area(ΔABD) = $\frac{1}{2}$ Area(ΔABC)—1

Now, consider ΔABD here E is the midpoint of BD

∴ Area of (ΔABE) = Area of (ΔAED)

∴ Area(ΔABE) = $\frac{1}{2}$ Area(ΔABD)—2

Substituting −1 in −2 , we get

∴ Area(ΔABE) = $\frac{1}{2}$ ($\frac{1}{2}$ Area(ΔABC))

Area(ΔABE) = $\frac{1}{4}$ Area(ΔABC)—3

Now consider ΔABE here O is the midpoint of AE

∴ Area of (ΔBOE) = Area of (ΔAOB)

∴ Area(ΔBOE) = $\frac{1}{2}$ Area(ΔABE)—4

Now, substitute −3 in −4 , we get

Area(ΔBOE) = $\frac{1}{2}$ ($\frac{1}{4}$ Area(ΔABC))

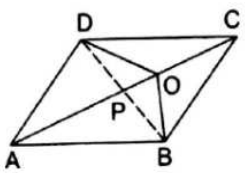
∴ area(ΔBOE) = $\frac{1}{8}$ area(ΔABC)

Hence proved

20. Question

In the adjoining figure, ABCD is a parallelogram and O is any point on the diagonal AC.

Show that ar(ΔAOB) = ar(ΔAOD).



Answer

Given : A parallelogram ABCD in which AC is the diagonal and O is some point on the diagonal AC

To prove: area(ΔAOB) = area(ΔAOD)

Construction : Draw a diagonal BD and mark the intersection as P

Proof:

We know that in a parallelogram diagonals bisect each other, hence P is the midpoint of ΔABD

∴ Area of (ΔAPB) = Area of (ΔAPD)—1

Now consider ΔBOD here OP is the median, since P is the midpoint of BD

∴ Area of (ΔOPB) = Area of (ΔOPD)—2

Adding –1 and –2 we get

Area of (ΔAPB) + Area of (ΔOPB) = Area of (ΔAPD) + Area of (ΔOPD)

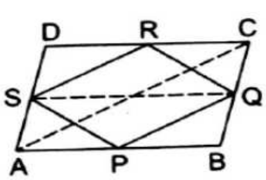
∴ Area of (ΔAOB) = Area of (ΔAOD)

Hence proved

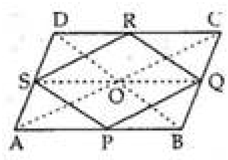
21. Question

P.Q.R.S are respectively the midpoints of the sides AB, BC, CD and DA of ∥ gm ABCD. Show that PQRS is a parallelogram and also show that

Ar(∥gm PQRS) = $\frac{1}{2}$ x ar(∥gm ABCD).



Answer



Given : ABCD is a parallelogram and P,Q,R,S are the midpoints of AB,BC,CD,AD respectively

To prove: (i) PQRS is a parallelogram

(ii) Area(∥gm PQRS) = $\frac{1}{2}$ x area(∥gm ABCD)

Construction : Join AC ,BD,SQ

Proof:

(i)

As S and R are midpoints of AD and CD respectively, in ΔACD

SR || AC [By midpoint theorem] ------------------- (1)

Similarly in ΔABC , P and Q are midpoints of AB and BC respectively

PQ || AC [By midpoint theorem] ------------------ (2)

From (1) and (2)

SR || AC || PQ

∴ SR || PQ ------------------- (3)

Again in ΔACD as S and P are midpoints of AD and CB respectively

SP || BD [By midpoint theorem] ------------------ (4)

Similarly in ΔABC , R and Q are midpoints of CD and BC respectively

RQ || BD [By midpoint theorem] -------------------- (5)

From (4) and (5)

SP || BD || RQ

∴ SP || RQ ----------- (6)

From (3) and (6)

We can say that opposite sides are Parallel in PQRS

Hence we can conclude that PQRS is a parallelogram.

(ii)

Here ABCD is a parallelogram

S and Q are midpoints of AD and BC respectively

∴ SQ || AB

∴SQAB is a parallelogram

Now, area(ΔSQP) = $\frac{1}{2}$ x area of (SQAB) -------------- 1

[Since ΔSQP and ||gm SQAB have same base and lie between same parallel lines]

Similarly

S and Q are midpoints of AD and BC respectively

∴ SQ || CD

∴SQCD is a parallelogram

Now, area(ΔSQR) = $\frac{1}{2}$ x area of (SQCD) ------------------- 2

[Since ΔSQR and ||^{gm} SQCD have same base and lie between same parallel lines]

Adding (1) and (2) we get

area(ΔSQP) + area(ΔSQR) = $\frac{1}{2}$ x area of (SQAB) + $\frac{1}{2}$ x area of (SQCD)

∴area(PQRS) = $\frac{1}{2}$(area of (SQAB) + area of (SQCD))

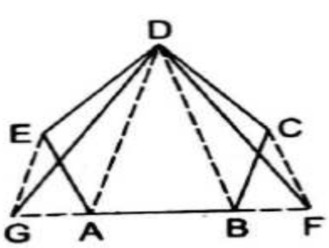
∴ Area(∥gm PQRS) = $\frac{1}{2}$ x area(∥gm ABCD)

Hence proved

22. Question

The given figure shows a pentagon ABCDE, EG, drawn parallel to DA, meets BA produced at G, and CF, drawn parallel to DB, meets AB produced at F.

Show that ar(pentagon ABCDE) = ar(ΔDGF).



Answer

Given : ABCDE is a pentagon EG is drawn parallel to DA which meets BA produced at G and CF is drawn parallel to DB which meets AB produced at F

To prove: area(pentagon ABCDE) = area(ΔDGF)

Proof:

Consider quadrilateral ADEG. Here,

area(ΔAED) = area(ΔADG) ------------- (1)

[since two triangles are on same base AD and lie between parallel line i.e, AD||EG]

Similarly now, Consider quadrilateral BDCF. Here,

area(ΔBCD) = area(ΔBDF) ---------------- (2)

[since two triangles are on same base AD and lie between parallel line i.e, AD||EG]

Adding Eq (1) and (2) we get

area(ΔAED) + area(ΔBCD) = area(ΔADG) + area(ΔBDF) ------------------ (3)

Now add area(ΔABD) on both sides of Eq (3), we get

∴ area(ΔAED) + area(ΔBCD) + area(ΔABD) = area(ΔADG) + area(ΔBDF) + area(ΔABD)

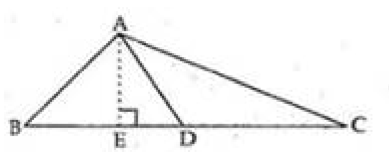
∴ area(pentagon ABCDE) = area(ΔDGF)

Hence proved

23. Question

Prove that a median divides a triangle into two triangles of equal area.

Answer



Given : A ΔABC with D as median

To prove : Median D divides a triangle into two triangles of equal areas.

Constructions: Drop a perpendicular AE onto BC

Proof: Consider ΔABD

area(ΔABD) = $\frac{1}{2}$ x BD x AE

Now , Consider ΔACD

area(ΔACD) = $\frac{1}{2}$ x CD x AE

since D is the median

BD = CD

∴ $\frac{1}{2}$ x BD x AE = $\frac{1}{2}$ x CD x AE

Hence , area(ΔABD) = area(ΔACD)

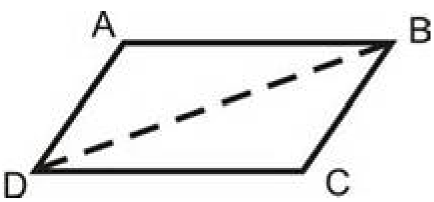
∴ we can say that Median D divides a triangle into two triangles of equal areas.

Hence proved

24. Question

Show that a diagonal divides a parallelogram into two triangles of equal area.

Answer



Given: A parallelogram ABCD with a diagonal BD

To prove: area(ΔABD) = area(ΔBCD)

Proof:

We know that in a parallelogram opposite sides are equal, that is

AD = BC and AB = CD

Now, consider ΔABD and ΔBCD

Here AD = BC

AB = CD

BD = BD (common)

Hence by SSS congruency

ΔABD ≅ ΔBCD

By this we can conclude that both the triangles are equal

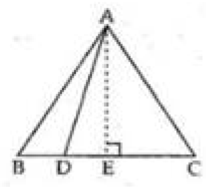
∴ area(ΔABD) = area(ΔBCD)

Hence proved

25. Question

The base BC of ΔABC is divided at D such BD = $\frac{1}{2}$ DC. Prove that ar(ΔABD) = $\frac{1}{3}$ x ar(ΔABC).

Answer



Given: A ΔABC with a point D on BC such that BD = $\frac{1}{2}$DC

To prove: area(ΔABD) = $\frac{1}{3}$ x area(ΔABC)

Construction: Drop a perpendicular onto BC

Proof: area(ΔABC) = $\frac{1}{2}$ x BC x AE ---------------(1)

and, area(ΔABD) = $\frac{1}{2}$ x BD x AE ----------------- (2)

given that BD = $\frac{1}{2}$DC ------------------ (3)

so, BC = BD + DC = BD + 2BD = 3BD [from 2]

∴ BD = $\frac{1}{3}$(BC)

Sub BD in (1), we get

area(ΔABD) = $\frac{1}{2}$ x ($\frac{1}{3}$(BC) X AE)

area(ΔABD) = $\frac{1}{3}$ x ($\frac{1}{2}$ BC X AE)

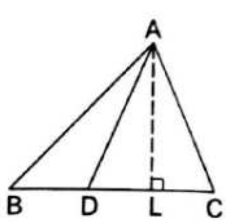
∴ area(ΔABD) = $\frac{1}{3}$ x area(ΔABC) [from 1]

Hence proved

26. Question

In the adjoining figure, the points D divides the

Side BC of ΔABC in the ratio m:n. prove that area(ΔABD): area(ΔABC) = m:n



Answer

Given : A ΔABC in which a point D divides the Side BC in the ratio m:n.

To prove: area(ΔABD): area(ΔABC) = m:n

Construction : Drop a perpendicular AL on BC

Proof:

area(ΔABD) = $\frac{1}{2}$ x BD x AL ---------------- (1)

and, area(ΔADC) = $\frac{1}{2}$ x DC x AL ------------------ (2)

BD:DC = m:n

$\frac{BD}{DC} = \frac{m}{n}$

$\therefore\ BD = \frac{m}{n} \times DC$ --------------(3)

sub Eq (3) in eq (1)

area(ΔABD) = $\frac{1}{2}$ x ($\frac{m}{n}$ x DC) x AL

area(ΔABD) = $\frac{m}{n}$ x ($\frac{1}{2}$ x DC x AL)

area(ΔABD) = $\frac{m}{n}$ x area(ΔADC)

$\therefore \frac{\text{area}(\Delta ABD)}{\text{area}(\Delta ADC)} = \frac{m}{n}$

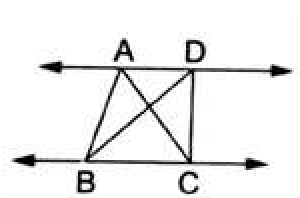
∴ Area(ΔABD): Area(ΔABC) = m:n

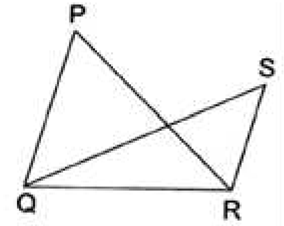
Hence proved

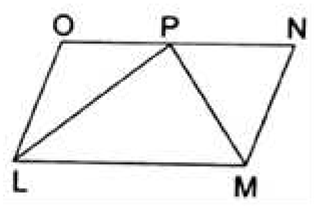
CCE Questions

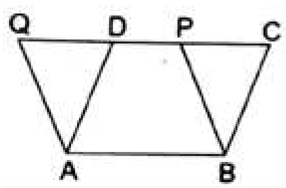
1. Question

Out of the following given figures which are on the same base but not between the same parallels?

A. 

B. 

C. 

D. 

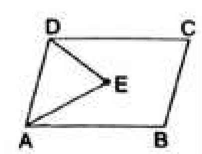
Answer

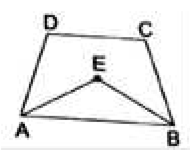
Here, ΔPQR and ΔSQR are on the same base QR but there is no parallel line to QR.

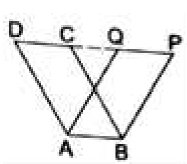
∴ Here, Figure in option B is on the same base but not between the same parallels.

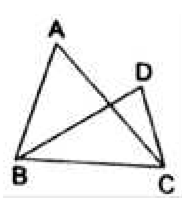
2. Question

In which of the following figures, you find polynomials on the same base and between the same parallels?

A. 

B. 

C. 

D. 

Answer

Here parallelogram ABCD and parallelogram ABQP lie on the same base AB and lie between the parallel line AB and DP.

∴ Here, Figure in option C is on the same base and between the same parallels.

3. Question

The median of a triangle divides it into two

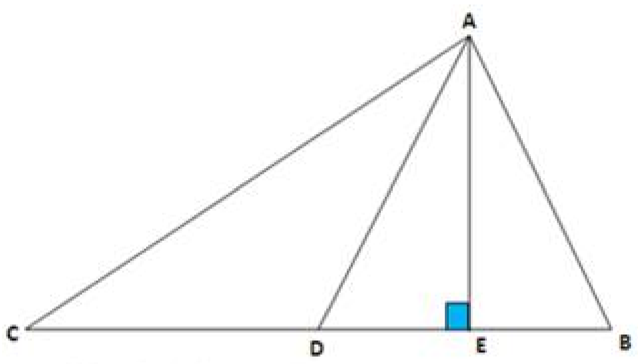
A. Triangles of equal area

B. Congruent triangles

C. Isosceles triangles

D. Right triangles

Answer



In ΔABC, AD is the median

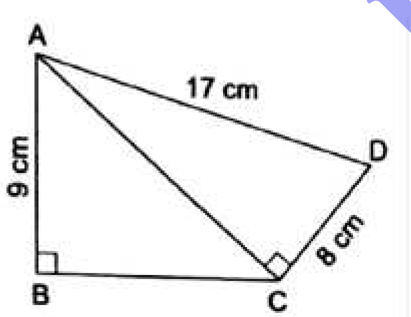
Hence BD = DCDraw AE ⊥ BC

Area of ΔABD = Area of ΔADC

Thus median of a triangle divides it into two triangles of equal area.

4. Question

The area of quadrilateral ABCD in the given figure is



A. $57cm^2$

B. $108cm^2$

C. $114cm^2$

D. $195cm^2$

Answer

Given:

$\angle ABC = 90°$

$\angle ACD = 90°$

CD = 8cm

AB = 9cm

AD = 17cm

Consider ΔACD

Here, By Pythagoras theorem : $AD^2 = CD^2 + AC^2$

$17^2 = 8^2 + AC^2$

$\Rightarrow AC^2 = 17^2 - 8^2$

$\Rightarrow AC^2 = 289 - 64 = 225$

$\Rightarrow AC = 15$

Now, Consider ΔABC

Here, By Pythagoras theorem : $AC^2 = AB^2 + BC^2$

$15^2 = 9^2 + AC^2$

$\Rightarrow BC^2 = 15^2 - 9^2$

$\Rightarrow BC^2 = 225 - 81 = 144$

$\Rightarrow BC = 12$

Here,

Area (quad.ABCD) = Area (ΔABC) + Area (ΔACD)

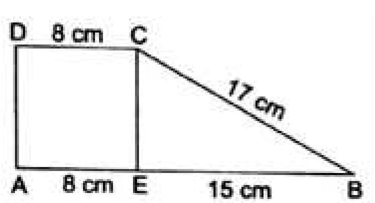
Area (quad.ABCD) = $1/2 \times AB \times BC + 1/2 \times AC \times CD$

Area (quad.ABCD) = $1/2 \times 9 \times 12 + 1/2 \times 15 \times 8 = 54 + 60 = 104cm^2$

$\therefore$ Area (quad.ABCD) = $114cm^2$

5. Question

The area of trapezium ABCD in the given figure is



A. $62cm^2$

B. $93cm^2$

C. $124cm^2$

D. $155cm^2$

Answer

Given:

$\angle BEC = 90°$

$\angle DAE = 90°$

CD = AE = 8cm

BE = 15cm

BC = 17cm

Consider ΔCEB

Here, By Pythagoras theorem

$BC^2 = CE^2 + EB^2$

$17^2 = CE^2 + 15^2$

$CE^2 = 17^2 - 15^2$

$CE^2 = 289 - 225 = 64$

CE = 8

Here,

$\angle AEC = 90°$

CD = CE = 8cm

∴ AECD is a Square.

∴ Area (Trap. ABCD) = Area (Sq. AECD) + Area (ΔCEB)

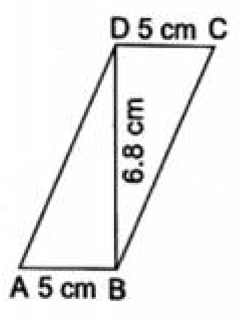
Area (Trap. ABCD) = AE×EC + 1/2×CE×EB

Area (Trap. ABCD) = 8×8 + 1/2×8×15 = 64 + 60 = $104cm^2$

∴ Area (Trap. ABCD) = $124cm^2$

6. Question

In the given figure, ABCD is a ∥gm in which AB = CD = 5cm and BD⊥DC such that BD = 6.8cm. Then, the area of ∥gm ABCD = ?



A. $17cm^2$

B. $25cm^2$

C. $34cm^2$

D. $68cm^2$

Answer

Given:

AB = CD = 5cm

BD⊥DC

BD = 6.8cm

Now, consider the parallelogram ABCD

Here, let DC be the base of the parallelogram then BD becomes its altitude (height).

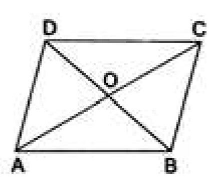
Area of the parallelogram is given by: Base × Height

∴ area of ∥gm ABCD = CD×BD = 5×6.8 = $34cm^2$

∴area of ∥gm ABCD = $34cm^2$.

7. Question

In the given figure, ABCD is a ∥gm in which diagonals AC and BD intersect at O. If ar(∥gm ABCD) is $52cm^2$, then the ar(ΔOAB) = ?



A. $26cm^2$

B. $18.5cm^2$

C. $39cm^2$

D. $13cm^2$

Answer

Given: ABCD is a ∥gm in which diagonals AC and BD intersect at O and ar(∥gm ABCD) is $52cm^2$.

Here,

Ar (ΔABD) = ar(ΔABC)

(∵ ΔABD and ΔABC on same base AB and between same parallel lines AB and CD)

Here,

ar(ΔABD) = ar(ΔABC) = 1/2 × ar(||gm ABCD)

(∵ ΔABD and ΔABC on same base AB and between same parallel lines AB and CD are half the area of the parallelogram)

∴ ar(ΔABD) = ar(ΔABC) = 1/2 × 52 = $26cm^2$

Now, consider ΔABC

Here OB is the median of AC

(∵ diagonals bisect each other in parallelogram)

∴ ar(ΔAOB) = ar(ΔBOC)

(∵median of a triangle divides it into two triangles of equal area)

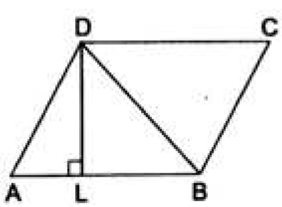
ar(ΔAOB) = 1/2 × ar(ΔABC)

ar(ΔAOB) = 1/2 × 26 = $13cm^2$

∴ ar(ΔAOB) = $13cm^2$

8. Question

In the given figure, ABCD is a ∥gm in which DL ⊥ AB. If AB = 10cm and DL = 4cm, then the ar(∥gm ABCD) = ?



A. $40cm^2$

B. $80cm^2$

C. $20cm^2$

D. $196cm^2$

Answer

Area of parallelogram is: base × height

Here,

Base = AB = 10cm

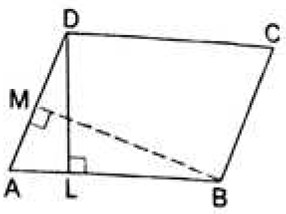
Height = DL = 4cm

∴ ar(∥gm ABCD) = AB ×DL = 10×4 = 40cm^2

∴ ar(∥gm ABCD) = 40cm^2

9. Question

In ∥gm ABCD, it is given that AB = 10cm, DL ⊥ AB and BM ⊥ AD such that DL = 6cm and BM = 8cm. Then, AD = ?



A. 7.5cm

B. 8cm

C. 12cm

D. 14cm

Answer

Given:

AB = 10cm

DL ⊥ AB

BM ⊥ AD

DL = 6cm

BM = 8cm

Now, consider the parallelogram ABCD

Here, let AB be the base of the parallelogram then DL becomes its altitude (height).

Area of the parallelogram is given by: Base × Height

∴ area of ∥gm ABCD = AB×DL = 10×6 = 60cm^2

Now,

Consider AD as base of the parallelogram then BM becomes its altitude (height)

∴ area of ∥gm ABCD = AD × BM = 60cm^2

AD × 8 = 60cm^2

AD = 60/8 = 7.5cm

∴length of AD = 7.5cm.

10. Question

The lengths of the diagonals of a rhombus are 12cm and 16cm. The area of the rhombus is

A. 192cm^2

B. 96cm^2

C. 64cm^2

D. 80cm^2

Answer

Given:

Length of diagonals of rhombus: 12cm and 16cm.

Area of the rhombus is given by: $\frac{\text{product of diagonals}}{2}$

∴ Area of the rhombus = $\frac{12\times16}{2}$ = 96cm^2

11. Question

Two parallel sides of a trapezium are 12cm and 8cm long and the distance between them 6.5cm. The area of the trapezium is

A. 74cm^2

B. 32.5cm^2

C. 65cm^2

D. 130cm^2

Answer

Given:

Lengths of parallel sides of trapezium: 12cm and 8cm

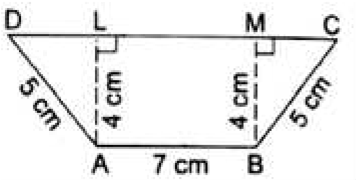
Distance between two parallel lines (height): 6.5cm

Area of the trapezium is given by: $\frac{(\text{sum of parallel sides})\times\text{height}}{2}$

∴ Area of the trapezium = $\frac{(12+8)\times6.5}{2}$ = 65cm^2

12. Question

In the given figure ABCD is a trapezium such that AL ⊥ DC and BM ⊥ DC. If AB = 7cm, BC = AD = 5cm and AL = BM = 4cm, then ar(trap. ABCD) = ?



A. $24cm^2$

B. $40cm^2$

C. $55cm^2$

D. $27.5cm^2$

Answer

Given:

AL ⊥ DC

BM ⊥ DC

AB = 7cm

BC = AD = 5cm

AL = BM = 4cm

Here,

MC = DL and AB = LM = 7 cm

Consider the ΔBMC

Here, by Pythagoras theorem

$BC^2 = BM^2 + MC^2$

$5^2 = 4^2 + MC^2$

$MC^2 = 25—16$

$MC^2 = 9$

MC = 3cm

∴ MC = DL = 3cm

CD = DL + LM + MC = 3 + 7 + 3 = 13cm

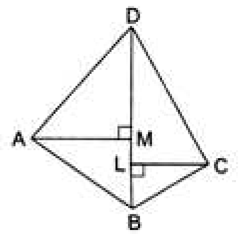
Now,

Area of the trapezium is given by: $\frac{\text{(sum of parallel sides)}\times\text{height}}{2}$

∴ Area of the rhombus = $\frac{(13+7)\times 4}{2}$ = 40cm^2

13. Question

In a quadrilateral ABCD, it is given that BD = 16cm. If AL ⊥ BD and CM ⊥ BD such that AL = 9cm and CM = 7cm, then ar(quad.ABCD) = ?



A. 256cm^2

B. 128cm^2

C. 64cm^2

D. 96cm^2

Answer

Given:

BD = 16cm

AL ⊥ BD

CM ⊥ BD

AL = 9cm

CM = 7cm

Here,

Area of quadrilateral ABCD = area(ΔABD) + area(ΔBCD)

Area of triangle = 1/2 × base × height

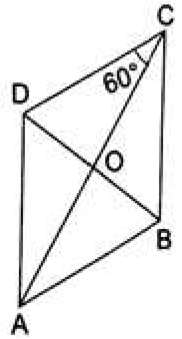
area(ΔABD) = 1/2 × base × height = 1/2 × BD × CM = 1/2 × 16 × 7 = 56cm^2

area(ΔBCD) = 1/2 × base × height = 1/2 × BD × AL = 1/2 × 16 × 9 = 64cm^2

∴ Area of quadrilateral ABCD = area(ΔABD) + area(ΔBCD) = 56 + 64 = 120cm^2

14. Question

ABCD is a rhombus in which ∠C = 60°. Then, AC : BD = ?



A. 3:1

B. 3:2

C. 3:1

D. 3:2

Answer

Given:∠DCB = 60°

Let the length of the side be x

Here, consider ΔBCD

BC = DC (all sides of rhombus are equal)

∴ ∠CDB = ∠CBD (angles opposite to equal sides are equal)

Now, by angle sum property

∠CDB + ∠CBD + ∠BCD = 180°

2× ∠CBD = 180° −60°

2 × ∠CBD = 180° – 60°

∴ 2× ∠CBD = 120°

∠ CBD = $\frac{120}{2}$ = 60°

∴ ∠CDB = ∠CBD = 60°

∴ Δ ADC is equilateral triangle

∴ BC = CD = BD = x cm

In Rhombus diagonals bisect each other.

Consider Δ COD

By Pythagoras theorem

$CD^2 = OD^2 + OC^2$

$x^2 = \left[\frac{x}{2}\right]^2 + OC^2$

$OC^2 = x^2 - \left[\frac{x}{2}\right]^2$

$OC = \left[\frac{\sqrt{4x^2 - x^2}}{2}\right]$

$OC = \frac{\sqrt{3} \times x}{2}$ cm

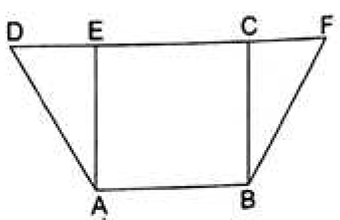
$\therefore AC = 2\times OC = 2 \times \frac{\sqrt{3}\times x}{2} = \sqrt{3}x$

$AC: BD = \sqrt{3}x : x = \sqrt{3} : 1$

$\therefore AC: BD = \sqrt{3} : 1$

15. Question

In the given figure ABCD and ABFE are parallelograms such that ar(quad. EABC) = 17cm^2 and ar(∥gm ABCD) = 25cm^2. Than, ar(ΔBCF) = ?



A. 4cm^2

B.4.8cm^2

C. 6cm^2

D. 8cm^2

Answer

Given: ar(quad. EABC) = 17cm^2 and ar(∥gm ABCD) = 25cm^2

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

∴ Area (∥gm ABCD) = Area (||gm ABFE) = 25cm^2

Here,

Area (||gm ABFE) = Area (quad. EABC) + Area (ΔBCF)

25cm^2 = 17cm^2 + Area (ΔBCF)

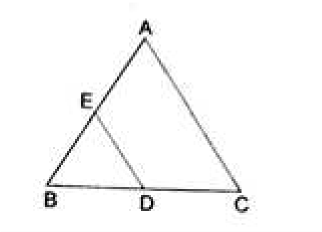
Area (ΔBCF) = 25 – 17 = 8cm^2

∴ Area (ΔBCF) = 8cm^2

16. Question

ΔABC and ΔBDE are two equilateral triangles such that D is the midpoint of BC. Then,

ar(ΔBDE): ar(ΔABC) = ?



A. 1:2

B. 1:4

C. 3:2

D. 3:4

Answer

Given: ΔABC and ΔBDE are two equilateral triangles, D is the midpoint of BC.

Consider ΔABC

Here, let AB = BC = AC = x cm (equilateral triangle)

Now, consider ΔBED

Here,

BD = 1/2 BC

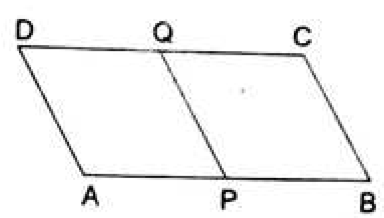
∴ BD = ED = EB = 1/2 BC = x/2 (equilateral triangle)

Area of the equilateral triangle is given by: $\frac{\sqrt{3}}{4}a^2$ (a is side length)

∴ ar(ΔBDE): ar(ΔABC) = $\frac{\sqrt{3}}{4} \times \left(\frac{x}{2}\right)^2 : \frac{\sqrt{3}}{4}x^2 = \frac{1}{4}:1$ = 1:4

17. Question

In a ∥gm ABCD, if Point P and Q are midpoints of AB and CD respectively and ar(∥gm ABCD) = 16cm^2, then ar(∥gmAPQD) = ?



A. 8cm^2

B. 12cm^2

C. 6cm^2

D. 9cm^2

Answer

Given:

P and Q are midpoints of AB and CD respectively

ar(∥gm ABCD) = 16cm^2

Now, consider the (∥gm ABCD)

Here,

Q is the midpoint of DC and P is the midpoint of AB.

∴ By joining P and Q (∥gm ABCD) is divided into two equal parallelograms.

That is, ar(∥gm ABCD) = ar(∥gmAPQD) + ar(∥gmPQCB)

ar(∥gm ABCD) = 2×ar(∥gmAPQD) (∵ar(∥gmAPQD) = ar(∥gmPQCB))

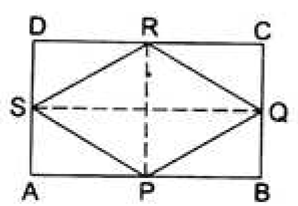
2×ar(∥gmAPQD) = 16cm^2 (∵ar(∥gm ABCD) = 16cm^2)

ar(∥gmAPQD) = 16/2 = 8cm^2

∴ ar(∥gmAPQD) = 8cm^2

18. Question

The figure formed by joining the midpoints of the adjacent sides of a rectangle of sides 8cm and 6cm is a



A. Rectangle of area 24cm^2

B. Square of area 24cm^2

C. Trapezium of area 24cm^2

D. Rhombus of area 24cm^2

Answer

Given: A rectangle with sides 8cm and 6cm.

Consider the Rectangle ABCD

Here DR = RD = AP = PB = 8/2 = 4cm (∵ P and R are the midpoints of DC and AB respectively) and AS = SD = BQ = QC = 6/2 = 3cm($\because$ S and Q are the midpoints of AD and BC respectively)

Now, consider the ΔRSD

By Pythagoras theorem

$SR^2 = SD^2 + DR^2$

$SR^2 = 3^2 + 4^2$

$SR^2 = 9 + 16$

$SR^2 = 25$

SR = 5 cm

Similarly using Pythagoras theorem in ΔQRC , ΔPBQ and ΔAPS

We get RQ = QP = PS = 5cm

$\therefore$ SR = RQ = QP = PS = 5cm

$\therefore$ PQSR is Rhombus of side length 5cm

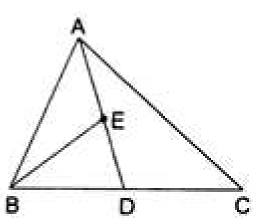
Area of the rhombus is given by: $\frac{\text{product of diagonals}}{2}$

$\therefore$ Area of the rhombus = $\frac{PR \times SQ}{2} = \frac{8 \times 6}{2} = 24cm^2$

$\therefore$ Area(PQRS) = $24cm^2$

19. Question

In ΔABC, if D is the midpoint of BC and E is the midpoint of AD, then ar(ΔBED) = ?



A. $\frac{1}{2}$ ar(ΔABC)

B. $\frac{1}{3}$ ar(ΔABC)

C. $\frac{1}{4}$ ar(ΔABC)

D. $\frac{2}{3}$ ar(ΔABC)

Answer

Given: D is the midpoint of BC and E is the midpoint of AD

Here,

D is the midpoint of BC and AD is the median of ΔABC

Area (Δ ABD) = Area (Δ ADC) (∵ median divides the triangle into two triangles of equal areas)

∴ Area (Δ ABD) = Area (Δ ADC) = $\frac{1}{2}$Area (ΔABC)

Now, consider Δ ABD

Here, BE is the median

Area (Δ ABE) = Area (Δ BED)

∴ Area (Δ ABE) = Area (Δ BED) = $\frac{1}{2}$Area (ΔABD)

Area (Δ BED) = $\frac{1}{2}$Area (ΔABD)

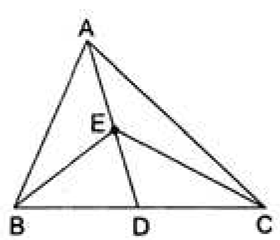
Area (Δ BED) = $\frac{1}{2} \times \left[\frac{1}{2} \text{Area } (\Delta ABC)\right]$ (∵Area (Δ ABD) = $\frac{1}{2}$Area (ΔABC))

Area (Δ BED) = $\frac{1}{4}$Area (ΔABC)

∴ Area (Δ BED) = $\frac{1}{4}$Area (ΔABC)

20. Question

The vertex A of ΔABC is joined to a point D on BC. If E is the midpoint of AD, then ar(ΔBEC) = ?



A. $\frac{1}{2}$ ar(ΔABC)

B. $\frac{1}{3}\text{ar}(\Delta ABC)$

C. $\frac{1}{4}\text{ar}(\Delta ABC)$

D. $\frac{1}{6}\text{ar}(\Delta ABC)$

Answer

Given:

Here,

D is the midpoint of BC and AD is the median of ΔABC

Area (Δ ABD) = Area (Δ ADC) (∵ median divides the triangle into two triangles of equal areas)

∴ Area (Δ ABD) = Area (Δ ADC) = $\frac{1}{2}$Area (ΔABC)

Now, consider Δ ABD

Here, BE is the median

Area (Δ ABE) = Area (Δ BED)

∴ Area (Δ ABE) = Area (Δ BED) = $\frac{1}{2}$Area (ΔABD)

Area (Δ BED) = $\frac{1}{2}$Area (ΔABD)

Area (Δ BED) = $\frac{1}{2} \times \left[\frac{1}{2}\text{Area }(\Delta ABC)\right]$ (∵Area (Δ ABD) = $\frac{1}{2}$Area (ΔABC)) −1

Area (Δ BED) = $\frac{1}{4}$Area (ΔABC)

Similarly,

Area (Δ EDC) = $\frac{1}{4}$Area (ΔABC) −2

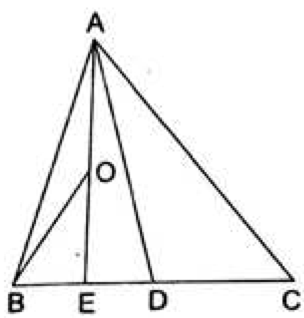
Add −1 and −2

Area (Δ BED) + Area (Δ EDC) = $\frac{1}{4}$Area (ΔABC) + $\frac{1}{4}$Area (ΔABC) = $\frac{1}{2}$Area (ΔABC)

∴ Area (Δ BEC) = $\frac{1}{2}$Area (ΔABC)

21. Question

In ΔABC, it given that D is the midpoint of BC; E is the midpoint of BD and O is the midpoint of AE. Then ar(ΔBOE) = ?



A. $\frac{1}{3}$ ar(ΔABC)

B. $\frac{1}{4}$ ar(ΔABC)

C. $\frac{1}{6}$ ar(ΔABC)

D. $\frac{1}{8}$ ar(ΔABC)

Answer

Given: D is the midpoint of BC; E is the midpoint of BD and O is the midpoint of AE.

Here,

D is the midpoint of BC and AD is the median of ΔABC

Area (Δ ABD) = Area (Δ ADC) (∵ median divides the triangle into two triangles of equal areas)

∴ Area (Δ ABD) = Area (Δ ADC) = $\frac{1}{2}$Area (ΔABC)

Now, consider Δ ABD

Here, AE is the median

Area (Δ ABE) = Area (Δ BED)

∴ Area (Δ ABE) = Area (Δ BED) = $\frac{1}{2}$Area (ΔABD)

Area (Δ ABE) = $\frac{1}{2}$Area (ΔABD)

Area (Δ ABE) = $\frac{1}{2} \times \left[\frac{1}{2}\text{Area }(\Delta ABC)\right]$ (∵Area (Δ ABD) = $\frac{1}{2}$Area (ΔABC)) −1

Area (Δ ABE) = $\frac{1}{4}$Area (ΔABC)

Consider Δ ABE

Here, BO is the median

Area (Δ BOE) = Area (Δ BOA)

∴ Area (Δ BOE) = Area (Δ BOA) = $\frac{1}{2}$Area (ΔABE)

Area (Δ BOE) = $\frac{1}{2} \times \left[\frac{1}{4}\text{Area }(\Delta ABC)\right]$ (∵Area (Δ ABE) = $\frac{1}{4}$Area (ΔABC))

Area (Δ BOE) = $\frac{1}{8}$Area (ΔABC)

∴ Area (Δ BOE) = $\frac{1}{8}$Area (ΔABC)

22. Question

If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the parallelogram is

A. 1:2

B. 1:3

C. 1:4

D. 3:4

Answer

Given:

We know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

Area (ΔABF) = 1/2 Area(||gm ABCD) −1

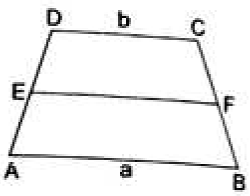
Area (ΔABF) : Area (||gm ABCD) = 1/2 Area(||gm ABCD) : Area(||gm ABCD) (from −1)

Area (ΔABF) : Area (||gm ABCD) = 1/2 : 1 = 1:2

∴ Area (ΔABF) : Area (||gm ABCD) = 1:2

23. Question

In the given figure ABCD is a trapezium in which AB∥DC such that AB = a cm and DC = b cm. If E and F are the midpoints of AD and BC respectively. Then, ar (ABFE) : ar(EFCD) = ?



A. A:b

B. (a + 3b):(3a + b)

C. (3a + b):(a + 3b)

D. (2a + b):(3a + b)

Answer

Given: ABCD is a trapezium, AB∥DC, AB = a cm and DC = b cm, E and F are the midpoints of AD and BC.

Since E and F are midpoints of AD and BC, EF will be parallel to both AB and CD.

EF = $\frac{a+b}{2}$

Height between EF and DC and height between EF and AB are equal, because E and F are midpoints OF AD and BC and EF||AB||DC.

Let height between EF and DC and height between EF and AB be h cm.

Area of trapezium = 1/2 × (sum of parallel lines) × height

Now,

Area (Trap.ABFE) = 1/2 × (a + $\frac{a+b}{2}$) × h.

and

Area (Trap.ABFE) = 1/2 × (b + $\frac{a+b}{2}$) × h.

Area (Trap.ABFE) : Area (Trap.ABFE) = 1/2 × (a + $\frac{a+b}{2}$) × h : 1/2 × (b + $\frac{a+b}{2}$) × h

Area (Trap.ABFE) : Area (Trap.ABFE) = $\frac{2a+a+b}{2}$: $\frac{2b+a+b}{2}$ = 3a + b : a + 3b

∴ Area (Trap.ABFE) : Area (Trap.ABFE) = 3a + b : a + 3b

24. Question

ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD is

A. a rectangle

B. a∥gm

C. a rhombus

D. all of these

Answer

Given: a quadrilateral whose diagonal AC divides it into two parts, equal in area.

Here,

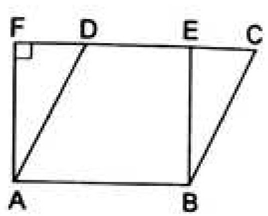
A quadrilateral is any shape having four sides, it is given that diagonal AC of the quadrilateral divides it into two equal parts.

We know that the rectangle, parallelogram and rhombus are all quadrilaterals, in these quadrilaterals if a diagonal is drawn say AC it divides it into equal areas.

∵ This diagonal divide the quadrilateral into two equal or congruent triangles.

25. Question

In the given figure, a ∥gm ABCD and a rectangle ABEF are of equal area. Then,



A. Perimeter of ABCD = perimeter of ABEF

B. Perimeter of ABCD < perimeter of ABEF

C. Perimeter of ABCD > perimeter of ABEF

D. Perimeter of ABCD = $\frac{1}{2}$(perimeter of ABEF)

Answer

Given: Area (∥gm ABCD) = Area (rectangle ABEF)

Consider ΔAFD

Clearly AD is the hypotenuse

∴ AD > AF

Perimeter of Rectangle ABEF = 2× (AB + AF) −1

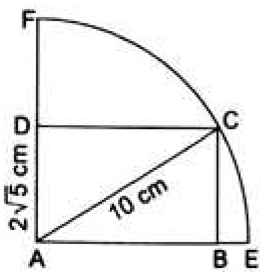
Perimeter of Parallelogram ABCD = 2× (AB + AD) −2

On comparing −1 and −2, we can see that

Perimeter of ABCD > perimeter of ABEF (∵AD > AF)

26. Question

In the given figure, ABCD is a rectangle inscribed in a quadrant of a circle of radius 10cm. If AD = 25cm, then area of the rectangle is



A. $32cm^2$

B. $40cm^2$

C. $44cm^2$

D. $48cm^2$

Answer

Given: ABCD is a rectangle inscribed in a quadrant of a circle of radius 10cm and AD = 25cm

Consider Δ ADC

By Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$10^2 = (25)^2 + AC^2$

$AC^2 = 10^2 — (25)^2$

$AC^2 = 100 - 20 = 80$

AC = 45

Area of rectangle = length × breadth = DC × AD

Area of rectangle = 45 × 25 = $40cm^2$

∴ Area of rectangle = $40cm^2$

27. Question

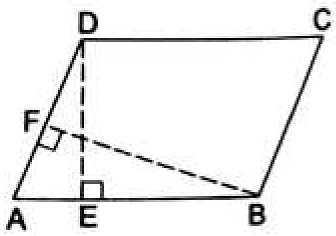
Look at the statements given below:

(I) A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

(II) In a ‖gm ABCD, it is given that AB = 10cm. The altitudes DE on AB and BF on AD being 6cm and 8cm respectively, then AD = 7.5 cm.

(III) Area of a ‖gm = $\frac{1}{2}$x base x altitude.

Which is true?



A. I only

B. II only

C. I and II

D. II and III

Answer

Consider Statement (I) :

Two or more parallelograms on the same base and between the same parallels are equal in area. Rectangle is also a parallelogram.

∴ It is true.

Consider Statement (II) :

Here, let AB be the base of the parallelogram then DE becomes its altitude (height).

Area of the parallelogram is given by: Base × Height

∴ Area of ‖gm ABCD = AB×DE = 10×6 = 60cm^2

Now,

Consider AD as base of the parallelogram then BF becomes its altitude (height)

∴ area of ‖gm ABCD = AD × BF = 60cm^2

AD × 8 = 60cm^2

AD = $\frac{60}{8}$ = 7.5cm

∴length of AD = 7.5cm.

∴ Statement (II) is correct.

Consider Statement (III)

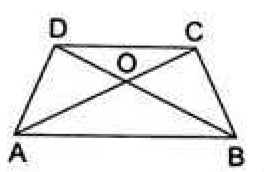
Area of parallelogram is base× height

∴ Statement (III) is false

∴ Statement (I) and (II) are true and statement (III) is false

28. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	**Reason (R)**
In a trapezium ABCD we have AB ∥ DC and the diagonals AC and BD intersect at O. Then, ar(ΔAOD) = ar(ΔBOC)  	Triangles on the same base and between the same parallels are equal in areas.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Assertion:

Here, Area (ΔABD) = Area(ΔABC) (∵ Triangles on same base and between same parallel lines) –1

Subtract Area (Δ AOB) on both sides of –1

Area (ΔABD) – Area (Δ AOB) = Area (ΔABC) – Area (Δ AOB)

Area (ΔAOD) = Area (ΔBOC)

∴ Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

29. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	**Reason (R)**
If ABCD is a rhombus whose one angle is 60°, then the ratio of the lengths of its diagonals is 3:1.	Median of a triangle divides it into two triangles of equal area.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Given: ∠DCB = 60°

Let the length of the side be x

Here, consider ΔBCD

BC = DC (all sides of rhombus are equal)

∴ ∠CDB = ∠CBD (angles opposite to equal sides are equal)

Now, by angle sum property

∠CDB + ∠CBD + ∠BCD = 180°

2× ∠CBD = 180° –60°

2 × ∠CBD = 180° – 60°

∴ 2× ∠CBD = 120°

$\angle$ CBD = $\frac{120}{2}$ = 60°

∴ ∠CDB = ∠CBD = 60°

∴ Δ ADC is equilateral triangle

∴ BC = CD = BD = x cm

In Rhombus diagonals bisect each other.

Consider Δ COD

By Pythagoras theorem

$CD^2 = OD^2 + OC^2$

$x^2 = \left[\frac{x}{2}\right]^2 + OC^2$

$OC^2 = x^2 - \left[\frac{x}{2}\right]^2$

$OC = \left[\frac{\sqrt{4x^2 - x^2}}{2}\right]$

$OC = \frac{\sqrt{3} \times x}{2}$ cm

∴ $AC = 2\times OC = 2 \times \frac{\sqrt{3}\times x}{2} = \sqrt{3}x$

$AC : BD = \sqrt{3}x : x = \sqrt{3} : 1$

∴ $AC : BD = \sqrt{3} : 1$

∴ Both Assertion but Reason are true and Reason is not a correct explanation of Assertion.

30. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	**Reason (R)**
The diagonals of a ‖ gm divide it into four triangles of equal area.	A diagonal of a ‖ gm divides it into two triangles of equal area.

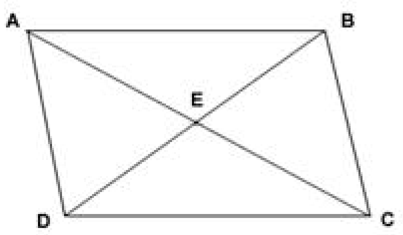
A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer



Consider Δ ABD

We know that diagonals in a parallelogram bisect each other

∴ E is the midpoint of BD, AE is median of Δ ABD

∴ Area (Δ ADE) = Area (Δ AEB) (∵ Median divides the triangle into two triangles of equal areas)

Similarly we can prove

Area (Δ ADE) = Area (Δ DEC)

Area (Δ DEC) = Area (Δ CEB)

Area (Δ CEB) = Area (Δ AEB)

∴ Diagonals of a ‖ gm divide into four triangles of equal area.

∴ Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

31. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	**Reason (R)**
The area of a trapezium whose parallel sides measure 25cm and 15cm respectively and the distance between them is 6cm, is $120cm^2$.	The area of an equilateral triangle of side 8cm is $163cm^2$.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Area of trapezium = 1/2 × (sum of parallel sides) × height = 1/2 × (25 + 15)×6 = $120cm^2$

∴ Area of trapezium = $120cm^2$

∴ Assertion is correct.

Area of an equilateral triangle is given by: $\frac{\sqrt{3}}{4} \times a^2$ (here ‘a’ is length of the side)

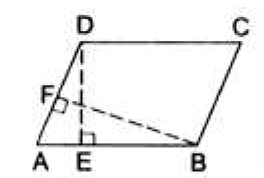
∴ Area of an equilateral triangle with side length 8 cm = $\frac{\sqrt{3}}{4} \times 8^2 = 16\sqrt{3}$

∴ Reason is correct

∴ Both Assertion but Reason are true and Reason is not a correct explanation of Assertion.

32. Question

The question consists of two statements, namely, Assertion (A) and Reason (R). Choose the correct answer.

Assertion (A)	**Reason (R)**
In the given figure, ABCD is a ∥ gm in which DE ⊥ AB and BF ⊥ AD. If AB = 16cm, DE = 8cm and BF = 10cm, then AD is 12cm. 	Area of a ∥gm = base x height.

A. Both Assertion (A) and Reason (R) are true and Reason (R) is a correct explanation of Assertion (A).

B. Both Assertion (A) but Reason (R) are true and Reason (R) is not a correct explanation of Assertion (A).

C. Assertion (A) is true and Reason (R) is false.

D. Assertion (A) is false and Reason (R) is true.

Answer

Here, let AB be the base of the parallelogram then DE becomes its altitude (height).

Area of the parallelogram is given by: Base × Height

∴ Area of ∥gm ABCD = AB×DE = 16×8 = 128cm^2

Now,

Consider AD as base of the parallelogram then BF becomes its altitude (height)

∴ area of ∥gm ABCD = AD × BF = 128cm^2

AD × 10 = 128cm^2

AD = $\frac{128}{10}$ = 12.8cm

∴length of AD = 12.8cm

∴Assertion is false and Reason is true

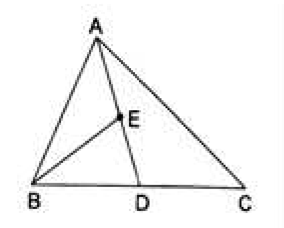
33. Question

Which of the following is a false statement?

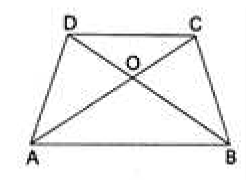
(A) A median of a triangle divides it into two triangles of equal areas.

(B) The diagonals of a ∥gm divide it into four triangles of equal areas.

(C) In a ΔABC, if E is the midpoint of median AD, then ar(ΔBED) = $\frac{1}{4}$ar(ΔABC).



(D) In a trap. ABCD, it is given that AB∥DC and the diagonals AC and BD intersect at O. Then, ar(ΔAOB) = ar(ΔCOD).



Answer

The correct answer is Option (D)

Δ ABC and Δ BCD does not lie between parallel lines and also Δ AOB and Δ COD are not congruent.

34. Question

Which of the following is a false statement?

A) If the diagonals of a rhombus are 18cm and 14cm, then its area is 126cm^2.

B) Area of a ∥gm = $\frac{1}{2}$x base x corresponding height.

C) A parallelogram and a rectangle on the same base and between the same parallels are equal in area.

D) If the area of a ∥ gm with one side 24cm and corresponding height h cm is 192cm^2, then h = 8cm.

Answer

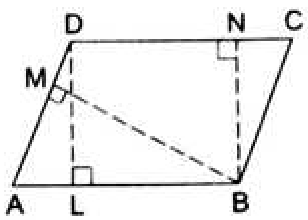
The correct answer is Option (B)

Area of parallelogram = base × corresponding height.

Formative Assessment (Unit Test)

1. Question

The area of ‖ gm ABCD is



A. AB X BM

B. BC X BN

C. DC X DL

D. AD X DL

Answer

Area of the ||gm is Base×Height

Here, height is distance between the Base and its corresponding parallel side.

∴ Area (||gm ABCD) = Base × Height = DC × DL

(∵ Here DC is taken as length and DL is the distance between DC and its corresponding parallel side AB).

2. Question

Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

A. 1:2

B. 1:1

C. 2:1

D. 3:1

Answer

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

Consider two ||gms ABCD and PQRS which are on same base and lie between same parallel lines.

∴ ar(||gm ABCD) = ar(||gm PQRS) −1

∴ ar(||gm ABCD) : ar(||gm PQRS) = 1:1 (∵ eq −1)

3. Question

ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area. Then, ABCD

A. Is a rectangle

B. is a rhombus

C. is a parallelogram

D. need not be any of (A), (B), (C)

Answer

Quadrilateral is any closed figure which has four sides.

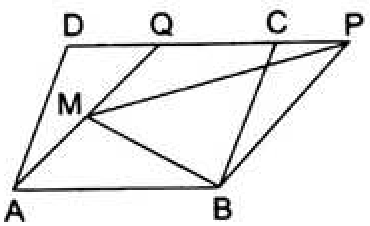
Rhombus, Rectangle, Parallelograms are few Quadrilaterals.

When a Diagonal AC of a quadrilateral divides it into two parts of equal areas, it is not necessary for the figure to be a Rhombus or a Rectangle or a Parallelogram, it can be any Quadrilateral.

4. Question

In the given figure, ABCD and ABPQ are two parallelograms and M is a point on AQ and BMP is a triangle.

Then, ar(ΔBMP) = $\frac{1}{2}$ar(‖gm ABCD).



A. True

B. False

Answer

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

∴ ar(||gm ABCD) = ar(||gm ABPQ) −1

We also know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

∴ ar(ΔBMP) = $\frac{1}{2}$ar(‖gm ABPQ)

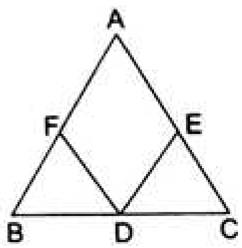
But, from −1

ar(||gm ABCD) = ar(||gm ABPQ)

∴ ar(ΔBMP) = $\frac{1}{2}$ar(‖gm ABCD)

5. Question

The midpoints of the sides of a triangle along with any of the vertices as the fourth point makes a parallelogram of area equal to



A. 1/2 (ar ΔABC)

B. 1/3 (arΔABC)

C. 1/4 (arΔABC)

D. ar (ΔABC)

Answer

Join EF

Here Area (ΔAEF) = Area (ΔBDF) = Area (ΔDEF) = Area (ΔDEC) = $\frac{1}{4}$ Area (ΔABC) – 1

Consider any vertex of the triangle.

Let us consider Vertex B

Here, BDEF form a parallelogram.

Area (||gm BDEF) = Area (ΔBDF) + Area (ΔDEF)

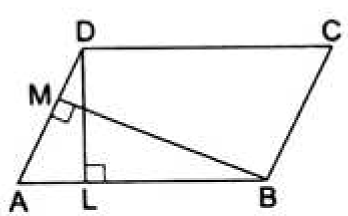
Area (||gm BDEF) = $\frac{1}{4}$ Area (ΔABC) + $\frac{1}{4}$ Area (ΔABC) = $\frac{1}{2}$ Area (ΔABC) (from –1)

∴ Area (||gm BDEF) = $\frac{1}{2}$ Area (ΔABC)

Similarly, we can prove for other vertices.

6. Question

Let ABCD be a ∥ gm in which DL ⊥ AB and BM ⊥ AD such that AD = 6 cm, BM = 10 and DL = 8 cm. Find AB.



Answer

Given:

AD = 6cm

DL ⊥AB

BM ⊥ AD

DL = 8cm

BM = 10cm

Now, consider the parallelogram ABCD

Here, let AD be the base of the parallelogram then BM becomes its altitude (height).

Area of the parallelogram is given by: Base × Height

∴ area of ∥gm ABCD = AD×BM = 6×10 = 60cm^2

Now,

Consider AB as base of the parallelogram then DL becomes its altitude (height)

∴ area of ∥gm ABCD = AB × DL = 60cm^2

AB × 8 = 60cm^2

AB = $\frac{60}{8}$ = 7.5cm

∴length of AB = 7.5cm.

7. Question

Find the area of the trapezium whose parallel sides are 14 cm and 10 cm and whose height is 6 cm.

Answer

Given: Length of parallel sides 14 cm and 10 cm, height is 6cm

We know that area of trapezium is given by: 1/2 (sum of parallel sides)×height

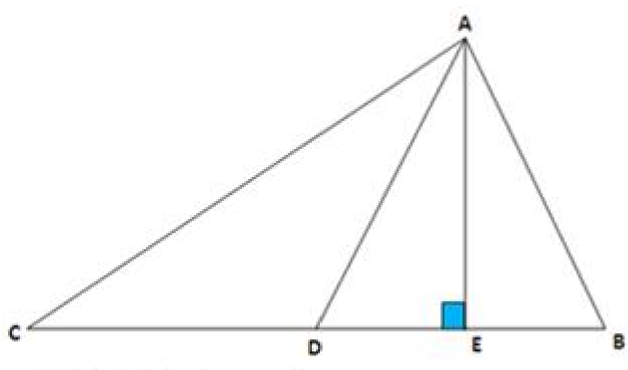
∴ Area of trapezium = 1/2 (14 + 10)×6 = 72cm^2

∴ Area of trapezium = 72cm^2

8. Question

Show that the median of a triangle divides it into two triangles of equal area.

Answer



Consider the Figure

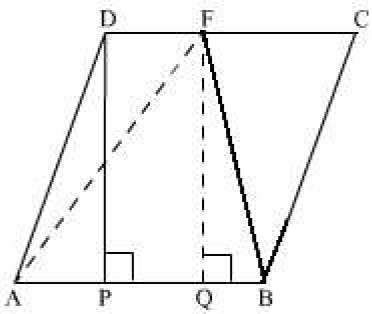
Here,

In ΔABC, AD is the medianHence BD = DCDraw AE ⊥ BCArea of ΔABD = Area of ΔADCThus median of a triangle divides it into two triangles of equal area.

9. Question

Prove that area of a triangle = $\frac{1}{2}$X base X altitude.

Answer



We know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

Consider the figure,

Here,

Area(ΔABF) = 1/2 Area(||gm ABCD) (From above statement) −1

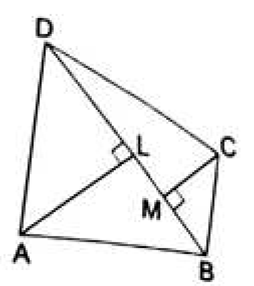
Area(||gm ABCD) = Base× Height −2

Sub −2 in −1

Area(ΔABF) = 1/2 × Base× Height

10. Question

In the adjoining figure, ABCD is a quadrilateral in which diagonal BD = 14cm. If AL ⊥ BD and CM ⊥ BD such that AL = 8 cm and CM = 6 cm, find the area of quad. ABCD.



Answer

Given: BD = 14cm, AL = 8 cm, CM = 6 cm and also, AL ⊥ BD and CM ⊥ BD.

Here,

Area (Quad.ABCD) = Area (ΔABD) + Area (ΔABC)

Area (ΔABD) = 1/2 base × height = 1/2 × BD×AL = 1/2 × 14 × 8 = $56cm^2$

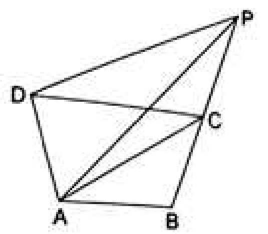
Area (ΔABC) = 1/2 base × height = 1/2 × BD×CM = 1/2 × 14 × 6 = $42cm^2$

∴ Area (Quad.ABCD) = Area (ΔABD) + Area (ΔABC) = 56 + 42 = 98 cm^2

∴ Area (Quad.ABCD) = 98 cm^2

11. Question

In the adjoining figure, ABCD is a quadrilateral. A line through D, parallel to AC, meets BC produced in P. Prove that ar(ΔABP) = ar(quad. ABCD).



Answer

Given: AC ||DP

We know that any two or Triangles having the same base and lying between the same parallel lines are equal in area.

∴ Area (Δ ACD) = Area (Δ ACP) −1

Add Area (Δ ABC) on both sides of eq −1

We get,

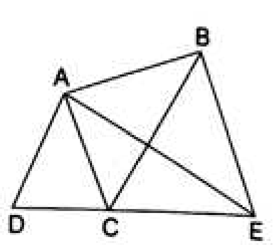
Area (Δ ACD) + Area (Δ ABC) = Area (Δ ACP) + Area (Δ ABC)

That is,

Area (quad.ABCD) = Area (Δ ABP)

12. Question

In the given figure, ABCD is a quadrilateral and BE ∥ AC and also BE meets DC produced at E. Show that the area of ΔADE is equal to the area of quad. ABCD.



Answer

Given: BE ||AC

We know that any two or more Triangles having the same base and lying between the same parallel lines are equal in area.

∴ Area (Δ ACE) = Area (Δ ACB) –1

Add Area (Δ ADC) on both sides of eq –1

We get,

Area (Δ ACE) + Area (Δ ADC) = Area (Δ ACB) + Area (Δ ADC)

That is,

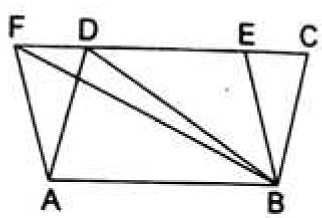
Area (Δ ADE) = Area (quad. ABCD)

13. Question

In the given figure, area of ∥ gm ABCD is 80 cm^2.

Find (i) ar(∥gm ABEF)

(ii) ar(ΔABD) and (iii) ar(ΔBEF).



Answer

Given: area of ∥ gm ABCD is 80 cm^2

We know that any two or parallelogram having the same base and lying between the same parallel lines are equal in area.

∴ ar(||gm ABCD) = ar(||gm ABEF) –1

We also know that when a parallelogram and a triangle lie on same base and between same parallel lines then, area of the triangle is half the area of the parallelogram.

∴ ar(ΔABD) = 1/2 × ar(||gm ABCD) and,

ar(ΔBEF) = 1/2 × ar(||gm ABEF)

(i)

ar(||gm ABCD) = ar(||gm ABEF)

∴ ar(||gm ABEF) = 80cm^2 (∵ar(||gm ABCD) = 80cm^2)

(ii)

ar(ΔABD) = 1/2 × ar(||gm ABCD)

ar(ΔABD) = 1/2 × 80 = 40cm^2 (∵ar(||gm ABCD) = 80cm^2)

∴ ar(ΔABD) = $40cm^2$

(iii)

ar(ΔBEF) = 1/2 × ar(||gm ABEF)

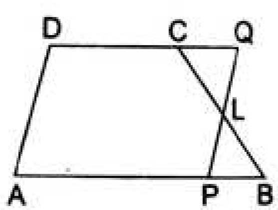
ar(ΔBEF) = 1/2 × 80 = $40cm^2$ (∵ar(||gm ABEF) = $80cm^2$)

∴ ar(ΔBEF) = $40cm^2$

14. Question

In trapezium ABCD, AB∥DC and L is the midpoint of BC. Through L, a line PQ ∥ AD has been drawn which meets AB in Point P and DC produced in Q.

Prove that ar(trap. ABCD) = ar(∥gm APQD).



Answer

Given: AB∥DC and L is the midpoint of BC, PQ ∥ AD

Construction: Drop a perpendicular DM from D onto AP

Consider ΔPBL and ΔCQL

Here,

∠LPB = ∠LQC (Alternate interior angles, AB|| DQ)

BL = LC (L is midpoint of BC)

∠PLB = ∠QLC (vertically opposite angles)

∴ By AAS congruency

ΔPBL ≅ ΔCQL

∴ PB = CQ (C.P.C.T)

Area (||gm APQD) = base× height = AP × DM –1

Area (Trap.ABCD) = 1/2 × (sum of parallel sides) × height = 1/2 × (AB + DC) × DM

Area (Trap.ABCD) = 1/2 × (AB + DC) × DM = 1/2 × (AP + PB + DC) × DM (∵ AB = AP + PB)

Area (Trap.ABCD) = 1/2 × (AP + CQ + DC) × DM (∵ PB = CQ)

Area (Trap.ABCD) = 1/2 × (AP + DQ) × DM (∵ DC + CQ = DQ)

Area (Trap.ABCD) = 1/2 × (2× AP) × DM (∵ AP = DQ)

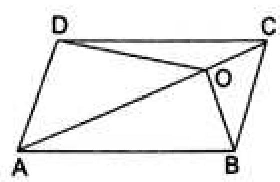
Area (Trap.ABCD) = AP × DM –2

From −1 and −2

Area (Trap.ABCD) = Area (||gm APQD)

15. Question

In the adjoining figure, ABCD is a ∥ gm and O is a point on the diagonal AC. Prove that ar(ΔAOB) = ar(ΔAOD).



Answer

Given: ABCD is a ∥ gm and O is a point on the diagonal AC.

Construction: Drop perpendiculars DM and BN onto diagonal AC.

Here,

DM = BN (perpendiculars drawn from opposite vertices of a ||gm to the diagonal are equal)

Now,

Area (ΔAOB) = 1/2 × base × height = 1/2 × AO × BN −1

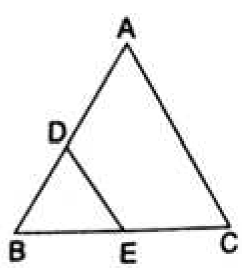
Area (ΔAOD) = 1/2 × base × height = 1/2 × AO × DM −2

From −1 and −2

Area (ΔAOB) = Area (ΔAOD) (∵ BN = DM)

16. Question

Δ ABC and ΔBDE are two equilateral triangles such that D(E) is the midpoint of BC. Then, prove that ar(ΔBDE) = $\frac{1}{4}$ar(ΔABC).



Answer

Given:ΔABC and ΔBDE are two equilateral triangles, D is the midpoint of BC.

Consider ΔABC

Here, let AB = BC = AC = x cm (equilateral triangle)

Now, consider ΔBED

Here,

BD = 1/2 BC

∴ BD = ED = EB = 1/2 BC = x/2 (equilateral triangle)

Area of the equilateral triangle is given by: $\frac{\sqrt{3}}{4}a^2$ (a is side length)

∴ ar(ΔBDE): ar(ΔABC) = $\frac{\sqrt{3}}{4} \times (\frac{x}{2})^2 : \frac{\sqrt{3}}{4} x^2 = \frac{1}{4} : 1$ = 1:4

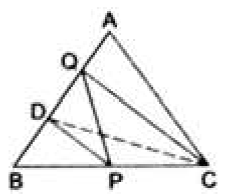
That is $\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{1}{4}$

∴ ar(ΔBDE) = $\frac{1}{4}$ ar(ΔABC)

Hence Proved

17. Question

In ΔABC, D is the midpoint of AB and P Point is any point on BC. If CQ ∥ PD meets AB in Q, then prove that ar(ΔBPQ) = $\frac{1}{2}$ar(ΔABC).



Answer

Given: D is the midpoint of AB and P Point is any point on BC, CQ∥ PD

In Quadrilateral DPQC

Area (Δ DPQ) = Area (Δ DPC)

Add Area (Δ BDP) on both sides

We get,

Area (Δ DPQ) + Area (Δ BDP) = Area (Δ DPC) + Area (Δ BDP)

Area (Δ BPQ) = Area (Δ BCD) −1

D is the midpoint BC, and CD is the median

∴ Area (Δ BCD) = Area (Δ ACD) = 1/2 × Area (Δ ABC) −2

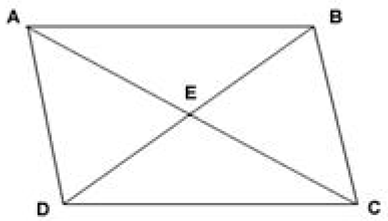
Sub −2 in −1

Area (Δ BPQ) = 1/2 × Area (Δ ABC) (∵Area (Δ BCD) = 1/2 × Area (Δ ABC))

18. Question

Show that the diagonals of a ∥ gm divide into four triangles of equal area.

Answer



Consider Δ ABD

We know that diagonals in a parallelogram bisect each other

∴ E is the midpoint of BD, AE is median of Δ ABD

∴ Area (Δ ADE) = Area (Δ AEB) (∵ Median divides the triangle into two triangles of equal areas)

Similarly we can prove

Area (Δ ADE) = Area (Δ DEC)

Area (Δ DEC) = Area (Δ CEB)

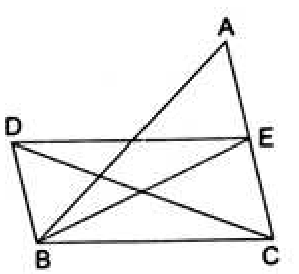
Area (Δ CEB) = Area (Δ AEB)

∴ Diagonals of a ∥ gm divide into four triangles of equal area.

19. Question

In the given figure, BD ∥ CA, E is the midpoint of CA and BD = $\frac{1}{2}$CA.

Prove that ar(ΔABC) = 2×ar(ΔDBC).



Answer

Given: BD ∥ CA, E is the midpoint of CA and BD = $\frac{1}{2}$CA

Consider Δ BCD and Δ DEC

Here,

BD = EC (∵ E is the midpoint of AC that is CE = $\frac{1}{2}$CA, BD = $\frac{1}{2}$CA)

CD = CD (common)

∠BDC = ∠ECD (alternate interior angles, DB||AC)

∴ By SAS congruency

Δ BCD ≅ Δ DEC

∴ Area (Δ BCD) = Area (Δ DEC) −1

Here,

Area (Δ BCE) = Area (Δ DEC) (triangles on same base CE and between same parallel lines) −2

E is the midpoint of AC, BE is the median of ΔABC

∴ Area (Δ BCE) = Area (Δ ABE) = 1/2 × Area (Δ ABC)

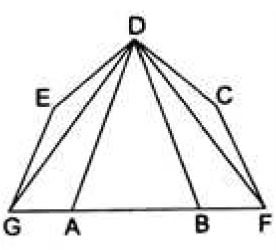
∴ Area (Δ DEC) = 1/2 × Area (Δ ABC) (∵Area (Δ BCE) = Area (Δ DEC))

∴ Area (Δ BCD) = 1/2 × Area (Δ ABC) (∵Area (Δ DEC) = Area (Δ BCD))

20. Question

The given figure shows a pentagon ABCDE in which EG, drawn parallel to DA, meets BA produced at G and CF drawn parallel to DB meets AB produced at F.

Show that ar(pentagon ABCDE) = ar(ΔDGF).



Answer

Given: EG||DA, CF||DB

Here, in Quadrilateral ADEG

Area (Δ AED) = Area (Δ ADG) −1

In Quadrilateral CFBD

Area (Δ CBD) = Area (Δ BCF) −2

Add −1 and −2

Area (Δ AED) + Area (Δ CBD) = Area (Δ ADG) + Area (Δ BCF) −3

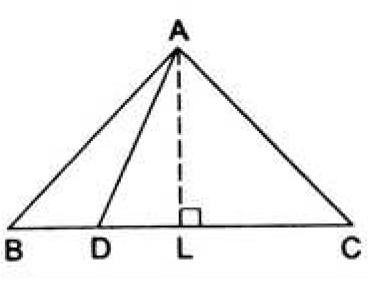
Add Area (Δ ABD) to −3

Area (Δ AED) + Area (Δ CBD) + Area (Δ ABD) = Area (Δ ADG) + Area (Δ BCF) + Area (Δ ABD)

Area (pentagon ABCDE) = Area (ΔDGF)

21. Question

In the adjoining figure, the point D divides the side BC of ΔABC in the ratio m:n. Prove that ar(ΔABD): ar(ΔADC) = m:n.



Answer

Given: D divides the side BC of ΔABC in the ratio m:n

Area (Δ ABD) = 1/2 × BD × AL

Area (Δ ADC) = 1/2 × CD × AL

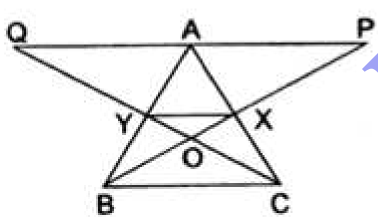
Area (ΔABD): Area (ΔADC) = 1/2 × BD × AL: 1/2 × CD × AL

Area (ΔABD): Area (ΔADC) = BD: CD

Area (ΔABD): Area (ΔADC) = m: n (∵ BD:CD = m:n)

22. Question

In the give figure, X and Y are the midpoints of AC and AB respectively, QP ∥ BC and CYQ and BXP are straight lines. Prove that ar(ΔABP) = ar(ΔACQ).



Answer

Given: X and Y are the midpoints of AC and AB respectively, QP ∥ BC and CYQ and BXP are straight lines.

Construction: Join QB and PC

In Quadrilateral BCQP

Area (Δ QBC) = Area (Δ BCP) (Triangles on same base BC and between same parallel lines are equal in area) −1 and,

Area (||gm ACBQ) = Area (||gm ABCP) (parallelograms on same base BC and between same parallel lines are equal in area) −2

Subtract −1 from −2

Area (||gm ACBQ) – Area (Δ QBC) = Area (||gm ABCP) – Area (Δ BCP)

Area (ΔACQ) = Area (ΔABP)

∴ Area(ΔABP) = Area(ΔACQ)