5 Index Numbers and Moving Averages

5.1 INDEX NUMBERS

The value of money is going down, we hear everyday. This means that since prices of things are going up, we get lesser and lesser quantities of the same item for a rupee. The workers say the increases in wages are not keeping up with inflation, and so actual wages are going down — or that standard of living is going down. People in Delhi say that property prices have skyrocketed, compared to cities like Kolkata and Chennai, or even Hong Kong. Similarly, crime rate in Delhi is increasing, even outstripping increase in population. In all these cases, we are making comparisons, either in terms of time, or in terms of geographic locations. This leads us to define a very useful and widely used statistic—the index number. An index number is simply a ratio of two quantities, such as prices, values or other economic variables taken at two different periods of time. Thus, it helps to compare the change with similar data collected in the *base period* or *fixed period.*

Index number *is a specialised average designed to measure the change in the level of an activity or item, either with respect to time or geographic location or some other characteristic.* It is described either as a ratio or a percentage. For example, when we say that consumer price index for 2008 is 175 compared to 2001, it means that consumer prices have risen by 75% over these seven years.

Study of index numbers reveals long term trends also. By using suitable time frame to calculate index numbers, we can find seasonal variations, cyclical variation, irregular (or abnormal) changes and long term trends of any activity - whether it is sale of ice-cream, or absence from school, or literacy level in a district, or unemployment problem, or sale of Ambassador cars by Birlas, and so on.

Wholesale Price Index (WPI) and ***Consumer Price Index (CPI)*** are widely used terms. They indicate the inflation rates, and also changes in standard of living. Consumer price index is based on prices of five sets of items — Food, Housing (Rent), Household goods, Fuel and light, and Miscellaneous. Each item is based on study of a number of items — *e.g.* Food includes Rice, Wheat, Dal, Milk, and so on.

Thus, the **characteristics of index** numbers are :

— they are expressed as ratio or percentage.

— they are specialised averages.

— they measure the change in the level of a phenomenon.

— they measure the effect of change over a period of time.

— they measure changes not capable of direct measurement *i.e.* they measure relative changes in an economic activity by measuring those factors which affect that activity.

5.1.1 Uses of Index Numbers

Index numbers are important tools of business and economic activity. Their main uses are :

1. They are used to feel the pulse of the economy. Thus, the index numbers work as *barometers of economic activity.*
2. They help in framing suitable policies and take decisions relating to wages, prices, consumption etc.
3. They reveal trends and tendencies. They are used as indicators of inflationary or deflationary tendencies.
4. They are used to measure the purchasing power of money.
5. They help in forecasting future economic activity.

5.1.2 Classification of Index Numbers

According to the activity they measure, the index numbers are classified as

(*i*) Price indexes (*ii*) Quantity indexes

(*iii*) Value indexes (*iv*) Special purpose indexes.

Price indexes measure changes in some price characteristic. Wholesale price index and consumer price index are two examples of Price indexes.

Quantity indexes measure changes in some quantity (volume) characteristic, for example, index of Industrial production, or index of scooters sold.

Value indexes measure change in some criterion of value, while **Special Purpose indexes** are constructed from time to time to measure certain special characteristic.

5.1.3 Problems in the construction of Index Numbers

The following points should be kept in mind while constructing index numbers.

(*i*) *Defining the purpose of the Index clearly.* There is no all-purpose index. If you are constructing a consumer price index, then don't include wholesale prices, and so on.

(*ii*) *Selecting base year (or base period) carefully.* The period against which relative change is to be measured should be chosen carefully. It should not be too distant in the past. It should be normal period - free of abnormalities like wars, floods, epidemics etc. Sometimes, instead of a fixed base, the *chain base method* may be used, for example, where the prices of a year are linked to the previous year and not with the fixed year.

(*iii*) *Selecting the numbers of items to be included.* As every item cannot be included, only the relevant and representative items should be chosen. Also items should be standardised so that after a time lapse they can be easily identified.

(*iv*) *Selection of price quotations and choice of places.* Once the items and their number has been decided, the locations (markets, shops) should be selected carefully so that a representative sample of price quotations can be obtained.

(*v*) *Choice of an average.* Since index numbers are specialised averages, we have to decide which average (arithmetic mean, median, mode, geometric mean or harmonic mean) is to be used while constructing the index. Though geometric mean gives best results, usually arithmetic mean is used to save calculation work.

(*vi*) *Selection of appropriate weights.* Since different items are consumed in different quantities, suitable weights may be used to reflect the relative importance of different items.

5.1.4 Methods of construction of Index Numbers

If only one item is involved and its two different values are given at two different times (or places etc.), then index number is simply the ratio of two numbers, expressed as a percentage. For example, if in 1990, only 2 lac cars were registered, and in the year 2000, ten lac cars were registered, then the (quantity) index is $\frac{10 \text{ lac}}{2 \text{ lac}} \times 100 = 500$. Similarly, if in

Mumbai the commercial space rent is \$1 per sq. foot per month, while in New York it is \$2·50 per sq. foot per month, then index of rental of New York compared to Mumbai is $\frac{2 \cdot 50}{1 \cdot 00} \times 100 = 250$.

Generally instead of one item, rates of a number of items are given, for current year as well as for base year. Sometimes different weights, or quantities are also given for those items. There are a number of ways to calculate index numbers in such cases.

Index number

- **Unweighted**
  - **Simple Aggregative**
  - **Simple Average of price relatives**
- **Weighted (or Arithmetic Mean method)**
  - **Weighted Aggregate**
  - **Weighted average of price relatives**

(*i*) **Simple aggregative method**

If Σp_1 is the sum total of current prices of commodities under consideration, and Σp_0 is the sum total of prices of these commodities in the base year, then the price index number for the current year is

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100$$

(*ii*) **Simple average of price relatives method**

Price Relative means the ratio of price of a certain item in current year to the price of that item in base year, expressed as a percentage *i.e.* Price Relative $= \frac{p_1}{p_0} \times 100$.

For example, if a colour TV cost ₹ 12000 in 1995 and ₹ 18000 in 2008, the price relative is $\frac{18000}{12000} \times 100 = 150$.

When a number of items are involved, we first calculate the price relative of each item and then simply take their average to calculate the index number. Thus, the formula for computing price index using this method is

$$P_{01} = \frac{\Sigma\left(\frac{p_1}{p_0} \times 100\right)}{N},$$ where N is the number of items.

Sometimes, to simplify calculations, the following form is used :

$$P_{01} = \left(\Sigma \frac{p_1}{p_0}\right) \times \frac{100}{N} \text{ or } \frac{1}{N}\ \Sigma\left(\frac{p_1}{p_0} \times 100\right)$$

(*iii*) **Weighted aggregate method**

If along with base prices, and current prices of a number of items, the weights or quantities of each are given, then index number based on weighted aggregates is given by

$$P_{01} = \frac{\Sigma\, p_1 w}{\Sigma\, p_0 w} \times 100$$

(*iv*) **Weighted average of price relatives method**

This is the commonly used method to construct consumer or wholesale price index when base and current prices of a number of items, along with weights or quantities are given. Weighted average of price relatives is given by

$$P_{01} = \frac{\Sigma\left(\frac{p_1}{p_0} \times 100\right) \times w}{\Sigma w}, \text{ or}$$

$$P_{01} = \frac{\Sigma I w}{\Sigma w}, \text{ where } I = \frac{p_1}{p_0} \times 100, \text{ the price relative.}$$

ILLUSTRATIVE EXAMPLES

Example 1. *Find by simple aggregate method, the index number from the following data :*

Commodity	*Base Price (₹)*	*Current Price (₹)*
Rice	*30*	*35*
Wheat	*22*	*25*
Fish	*54*	*64*
Potato	*20*	*25*
Coal	*15*	*18*

Solution. We construct the following table :

Commodity	*Base Price (₹)* p_0	*Current Price (₹)* p_1
Rice	30	35
Wheat	22	25
Fish	54	64
Potato	20	25
Coal	15	18
Total	$\Sigma p_0 = 141$	$\Sigma p_1 = 167$

Hence, required index number by simple aggregate method,

$$P_{01} = \frac{\Sigma p_1}{\Sigma p_0} \times 100 = \frac{167}{141} \times 100 = 118{\cdot}44$$

Thus, we see that there is an average increase of about 18·44% in the price of commodities.

Example 2. *In above example, calculate price relative of Fish and Coal.*

Solution. Price relative of fish $= \dfrac{\text{current price of fish}}{\text{base price of fish}} \times 100$

$$= \frac{64}{54} \times 100 = 118{\cdot}5$$

Price relative of coal $= \dfrac{\text{current price of coal}}{\text{base price of coal}} \times 100$

$$= \frac{18}{15} \times 100 = 120.$$

Example 3. *For data in example 1, calculate price index using the price relative method.*

Solution. We construct the table as below :

Commodity	*Base Price (₹)* p_0	*Current Price (₹)* p_1	*Price relative* $\frac{p_1}{p_0} \times 100$
Rice	30	35	116·67
Wheat	22	25	113·64
Fish	54	64	118·52
Potato	20	25	125
Coal	15	18	120
Total			$\Sigma \frac{p_1}{p_0} \times 100 = 593{\cdot}83$

Hence, the required index number is simple average of price relatives,

$$P_{01} = \frac{1}{N}\left(\Sigma \frac{p_1}{p_0} \times 100\right) = \frac{593 \cdot 83}{5} = 118 \cdot 77$$

Note that by using simple aggregate method in example 1, we had calculated the price index as 118·44.

Example 4. *Let us assume that with prices given in example 1, a Bengali family buys quantities of rice, wheat, fish, potato and coal in the ratio 3 : 1 : 3 : 2 : 2. Find weighted aggregate price index.*

Solution. We construct the table as below :

Commodity	*Base Price* (₹) p_0	*Current Price* (₹) p_1	*Weight* w	$p_0 w$	$p_1 w$
Rice	30	35	3	90	105
Wheat	22	25	1	22	25
Fish	54	64	3	162	192
Potato	20	25	2	40	50
Coal	15	18	2	30	36
Total				344	408

The required index number using weighted aggregates is

$$P_{01} = \frac{\Sigma p_1 w}{\Sigma p_0 w} \times 100 = \frac{408}{344} \times 100 = 118 \cdot 60$$

Example 5. *With above data (as in example 4), calculate price index using weighted average of price relatives.*

Solution. Construct the table as below :

Commodity	*Base Price* (₹) p_0	*Current Price* (₹) p_1	*Weight* w	*Price relative* $I = \frac{p_1}{p_0} \times 100$	$I\,w$
Rice	30	35	3	116·67	350
Wheat	22	25	1	113·64	113·64
Fish	54	64	3	118·52	355·56
Potato	20	25	2	125	250
Coal	15	18	2	120	240
Total			$\Sigma w = 11$		$\Sigma Iw = 1309 \cdot 2$

Hence, the index using weighted average of price relatives,

$$P_{01} = \frac{\Sigma Iw}{\Sigma w} = \frac{1309 \cdot 2}{11} = 119 \cdot 02$$

Example 6. *With data from Example 1, consider the case of a Punjabi family which uses more wheat than rice or fish. Calculate price index using weighted aggregate as well as using weighted average of price relatives, assuming that weights are 10, 50, 10, 20, 20.*

Solution. Using weighted aggregate, the required price index $= \frac{\Sigma p_1 w}{\Sigma p_0 w} \times 100$

Commodity	*Base Price* (₹) p_0	*Current Price* (₹) p_1	*Weight* w	$p_0 w$	$p_1 w$
Rice	30	35	10	300	350
Wheat	22	25	50	1100	1250
Fish	54	64	10	540	640
Potato	20	25	20	400	500
Coal	15	18	20	300	360
Total				2640	3100

Hence, the price index using weighted aggregates is

$$P_{01} = \frac{\Sigma p_1 w}{\Sigma p_0 w} \times 100 = \frac{3100}{2640} \times 100 = 117{\cdot}42$$

Now, to calculate the price index using weighted average of price relatives, we construct the following table :

Commodity	*Base Price (₹)* p_0	*Current Price (₹)* p_1	*Weight* w	*Price relative* $I = \frac{p_1}{p_0} \times 100$	$I\,w$
Rice	30	35	10	116·67	1166·7
Wheat	22	25	50	113·64	5682
Fish	54	64	10	118·52	1185·2
Potato	20	25	20	125	2500
Coal	15	18	20	120	2400
Total			110		12933·9

Hence, the price index using weighted average of price relatives is

$$P_{01} = \frac{\Sigma I w}{\Sigma w} = \frac{12933{\cdot}9}{110} = 117{\cdot}58$$

Comparing these results with those of examples 4 and 5, we see that Bengali family has suffered more than Punjabi family. Note that price rise is less for wheat than for rice and fish, and Bangali family consumes more fish and rice compared to wheat (see the weights), while Punjabi family consumes more wheat than rice and fish (see the weights). This demonstrates how the weights affect the price index.

Example 7. *Construct the index number for 1991 taking 1990 as the base year by simple average of price relatives method :*

Commodity	*A*	*B*	*C*	*D*	*E*
Price in 1990 (₹)	*100*	*80*	*160*	*220*	*40*
Price in 1991 (₹)	*140*	*120*	*180*	*240*	*40*

Solution. Construct the table as below :

Commodity	*Price in 1990 (₹)* p_0	*Price in 1991 (₹)* p_1	*Price relative* $\frac{p_1}{p_0} \times 100$
A	100	140	140
B	80	120	150
C	160	180	112·5
D	220	240	109·1
E	40	40	100
Total			= 611·6

Hence, the required price index using simple average of price relatives,

$$P_{01} = \frac{1}{N}\left(\Sigma \frac{p_1}{p_0} \times 100\right) \times 100 = \frac{611{\cdot}6}{5} = 122{\cdot}32$$

Example 8. *The price index for the following data for the year 2011 taking 2001 as the base year was 127. The simple average of price relatives method was used. Find the value of x :*

Items	*A*	*B*	*C*	*D*	*E*	*F*
Price (₹ per unit) in year 2001	*80*	*70*	*50*	*20*	*18*	*25*
Price (₹ per unit) in year 2011	*100*	*87·50*	*61*	*22*	*x*	*32·50*

Solution. Construct the table as below :

Items	Price (₹ per unit) in year 2001 p_0	Price (₹ per unit) in year 2011 p_1	Price relative $\frac{p_1}{p_0} \times 100$
A	80	100	$\frac{100}{80} \times 100 = 125$
B	70	87·50	$\frac{87 \cdot 50}{70} \times 100 = 125$
C	50	61	$\frac{61}{50} \times 100 = 122$
D	20	22	$\frac{22}{20} \times 100 = 110$
E	18	x	$\frac{x}{18} \times 100 = \frac{50x}{9}$
F	25	32·50	$\frac{32 \cdot 50}{25} \times 100 = 130$
Total			$612 + \frac{50}{9}x$

Here, N = total number of items = 6.
Using simple average of price relative method,

$$\text{price index} = \frac{1}{N} \Sigma \left(\frac{p_1}{p_0} \times 100 \right) = \frac{1}{6}\left(612 + \frac{50}{9}x\right) = 127 \text{ (given)}$$

$$\Rightarrow \quad 612 + \frac{50}{9}x = 6 \times 127 \Rightarrow \frac{50}{9}x = 762 - 612$$

$$\Rightarrow \quad \frac{50}{9}x = 150 \Rightarrow x = 27.$$

Hence, the value of $x = 27$.

Example 9. *Calculate the index number for 2005 with 2000 as the base year by weighted aggregate method :*

Commodity	Price (in ₹) in the year 2000	Price (in ₹) in the year 2005	Weights
A	140	180	10
B	400	550	7
C	100	250	6
D	125	150	8
E	200	300	4

(I.S.C. 2007)

Solution. Construct the table as below :

Commodity	Base Price (₹) in 2000, p_0	Current Price (₹) in 2005, p_1	Weight w	$p_0 w$	$p_1 w$
A	140	180	10	1400	1800
B	400	550	7	2800	3850
C	100	250	6	600	1500
D	125	150	8	1000	1200
E	200	300	4	800	1200
Total				6600	9550

Using weighted aggregate method,

$$\text{index number} = \frac{\Sigma p_1 w}{\Sigma p_0 w} \times 100 = \frac{9550}{6600} \times 100 = 144{\cdot}696$$
$$= 144{\cdot}7 \text{ (approximately).}$$

Example 10. *Calculate the index number for the year 2006 with 1996 as the base year by the weighted average of price relative method from the following data :*

Commodity	*A*	*B*	*C*	*D*	*E*
Weight	*40*	*25*	*5*	*20*	*10*
Price (₹ per unit) year 1996	*32·00*	*80·00*	*1·00*	*10·24*	*4·00*
Price (₹ per unit) year 2006	*40·00*	*120·00*	*1·00*	*15·36*	*3·00*

(I.S.C. 2009)

Solution. We construct the table as below :

Commodity	*Weight*	*Base year 1996* p_0	*year 2006* p_1	*Price relative* $I = \frac{p_1}{p_0} \times 100$	Iw
A	40	32·00	40·00	125	5000
B	25	80·00	120·00	150	3750
C	5	1·00	1·00	100	500
D	20	10·24	15·36	150	3000
E	10	4·00	3·00	75	750
Total	100				13000

Using weighted average of price relative method,

$$\text{index number} = \frac{\Sigma Iw}{\Sigma w} = \frac{13000}{100} = 130.$$

Example 11. *The price relatives and weights of a set of commodities are given below :*

Commodity	*A*	*B*	*C*	*D*
Price Relative	*125*	*120*	*127*	*119*
Weight	x	$2x$	y	$y + 3$

If the sum of weights is 40 and the index for the set is 122, find the numerical values of x and y.

Solution. The above data can be written in a table as :

Commodity	*Weight* w	*Price relative* I	Iw
A	x	125	$125x$
B	$2x$	120	$240x$
C	y	127	$127y$
D	$y + 3$	119	$119y + 357$
Total	$\Sigma w = 3x + 2y + 3$		$\Sigma Iw = 365x + 246y + 357$

As it is given that sum of weights is 40, we get

$$3x + 2y + 3 = 40$$
$$\Rightarrow \quad 3x + 2y = 37 \qquad \ldots(i)$$

As index number is given to be 122, we get

$$\frac{\Sigma Iw}{\Sigma w} = 122 \Rightarrow \frac{365x + 246y + 357}{40} = 122$$

$$\Rightarrow \quad 365x + 246y = 122 \times 40 - 357 = 4523 \quad \ldots(ii)$$

To solve (*i*) and (*ii*), multiply (*i*) by 123,

$$369x + 246y = 4551 \quad \ldots(iii)$$

Subtracting (*ii*) from (*iii*), we get

$$4x = 28 \Rightarrow x = 7$$

Putting this value of x in (*i*), we get $y = 8$.

Example 12. *The wholesale price index (or price relative) of rice in 2002 compared to 2000 is 130. If the cost of rice was ₹ 12 per kg in 2000, calculate the cost in 2002.*

Solution. Let the cost of rice be ₹ p per kg in 2002.

Then, by given,

$$130 = \frac{p}{12} \times 100$$

$$\Rightarrow \quad p = \frac{130 \times 12}{100} = 15{\cdot}60$$

Hence, the price of rice in 2002 is ₹ 15·60 per kg.

Example 13. *During a certain period, the cost of living index number goes from 110 to 200 and the salary of a worker is also raised from ₹ 325 to ₹ 500. Does the worker really gains or loses, and by how much amount in real terms ?*

Solution. $\textbf{Real wage} = \dfrac{\textbf{Actual wage}}{\textbf{Cost of living index}} \times \textbf{100}$

So real wage of ₹ 325 $= ₹\ \dfrac{325}{110} \times 100 = ₹\ 295{\cdot}45$

and real wage of ₹ 500 $= ₹\ \dfrac{500}{200} \times 100 = ₹\ 250$

So the worker actually loses *i.e.* ₹ (295·45 – 250)

= ₹ 45·45 in real terms.

EXERCISE 5.1

1. Fill in the blanks :

   (*i*) Index numbers are __________ types of ratio.

   (*ii*) Index numbers are barometers of __________ .

   (*iii*) Quantity indexes measure changes in __________ characteristic, compared to __________ period.

   (*iv*) Weighted indexes are __________ to unweighted indexes.

   (*v*) Base period is the period of __________ activities.

   (*vi*) If the price index is 132, it means that price has increased by __________ compared to base period.

   (*vii*) If the price index is 88, it means that price has decreased by __________ compared to base period.

2. (*i*) Price relative of coal is 125 in 2001 compared to 2000. If the coal cost ₹ 8 per kg in 2000, find its cost in 2001.

   (*ii*) Price relative of TV set is 90 in 2001 compared to 2000. If a TV set cost ₹ 9000 in 2000, find its cost in 2001.

(*iii*) Price relative of sugar is 110 in 2002 compared to 2001. If sugar costs ₹ 16·50 per kg in 2002, what did it cost in 2001 ?

(*iv*) Price relative of maize is 80 in 2002 compared to 2001. If maize costs ₹ 12 per kg in 2002, what did it cost in 2001 ?

3. A small industrial concern used three raw materials A, B and C in its manufacturing process. The prices of the materials was as shown below :

Commodities	*Price in ₹ in the year 1995*	*Price in ₹ in the year 2005*
A	4	5
B	60	57
C	36	42

Using 1995 as the base year, calculate a simple aggregate price index for 2005. (*I.S.C. 2008*)

4. Construct the consumer price index for 1990 taking 1989 as the base year, and using simple average of price relative method for the following data :

Commodities	*Price in 1989*	*Price in 1990*
Butter	20	21
Cheese	16	12
Milk	3	3
Eggs	2·80	2·80

(*I.S.C 2003*)

5. From the following data, compute price index by using simple average of price relatives :

Commodities and unit	*Price in 1989 (₹)*	*Price in 1990 (₹)*
Butter (kg)	20·00	21·00
Cheese (kg)	15·00	14·00
Milk (kg)	3·00	3·00
Bread (1)	2·80	2·80
Eggs (Doz.)	6·00	8·00
Ghee (1 tin)	250·00	260·00

(*I.S.C. 2004*)

6. Consider the following data :

Items	*Units*	*Price (in ₹)*	
		In 1994 (p_0)	*In 1998* (p_1)
Wheat	1 kg	5·60	7·20
Rice	1 kg	17·20	24·80
Pulses	1 kg	36·00	44·00
Milk	1 *l*	24·00	30·00
Clothing	1 m	199·00	130·00

Using 1994 as the base year, calculate the index for 1998 correct up to one decimal using

(*a*) simple aggregate method

(*b*) simple average of relatives method.

7. Taking 2003 as the base year, with an index number 100, calculate an index number for 2007, based on

(*i*) simple aggregate (*ii*) price relatives derived from the table given below :

Commodity	*A*	*B*	*C*	*D*
Price per unit in 2003	20	10	25	40
Price per unit in 2007	24	20	30	40

8. Calculate a cost of living index from the following table of prices and weights.

	Weight	*Price Index*
Food	35	108·5
Rent	9	102·6
Clothes	10	97·0
Fuel	7	100·9
Miscellaneous	39	103·7

9. Construct Index number for following data :

Commodity	*Butter*	*Bread*	*Tea*	*Bacon*
Relative Index	181	116	110	152
Weight	4	12	3	7

10. Find the consumer index number for the year 2010 using year 2000 as the base year by using method of weighted aggregates :

Commodity	A	B	C	D	E
2000 Price per unit (₹)	16	40	0·50	5·12	2
2010 Price per unit (₹)	20	60	0·50	6·25	1·50
Weights	40	25	5	20	10

(*I.S.C. 2012*)

11. Based on year 1988 as base, the index numbers for 1988, 1989, 1990, 1991 and 1992 are 100, 110, 120, 200 and 400. Now taking 1992 as base year, calculate index numbers for years 1988, 1989, 1990, 1991 and 1992.

12. The price quotations of four different commodities for 2001 and 2009 are as given below. Calculate the index number for 2009 with 2001 as the base year by using weighted average of price relative method.

Commodity	*Weight*	*Price (in ₹)*	
		2009	*2001*
A	10	9·00	4·00
B	49	4·40	5·00
C	36	9·00	6·00
D	4	3·60	2·00

(*I.S.C. 2011*)

13. Calculate the index number for the year 1979 with 1970 as base from the following data using weighted average of price relatives :

Commodity	*weights*	*Price (in ₹)*	
		1970	*1979*
A	22	2·50	6·20
B	48	3·30	4·40
C	17	6·25	12·75
D	13	0·65	0·90

(*I.S.C. 2005*)

19. Find the consumer price index for 1994 on the base of 1988, from the following data, using the method of weighted relatives :

Item	*Food*	*Rent*	*Clothing*	*Fuel*	*Miscellaneous*
Price in 1988 (in ₹)	200	100	150	50	100
Price in 1994 (in ₹)	280	200	120	100	200
Weight	30	20	20	10	20

(*I.S.C. 1997*)

20. The following table shows the prices per unit in 1980 and 1984 with weights of commodities A, B, C, D :

Commodity	*Weights*	*Price per unit in 1980*	*Price per unit in 1984*
A	20	25	30
B	25	20	30
C	15	50	70
D	40	5	10

Taking 1980 as base year with index number 100, calculate the index number of 1984 based on weighted average of price relatives. (*I.S.C. 2000*)

21. Taking 1975 as the base year, with an index number 100, calculate an index number for 1979, based on weighted average of price relatives from the table given below :

Commodity	*A*	*B*	*C*	*D*
Weight	30	15	25	30
Price per unit in 1975	20	10	5	40
Price per unit in 1979	24	20	30	40

(*I.S.C. 2002*)

5.2 MOVING AVERAGES

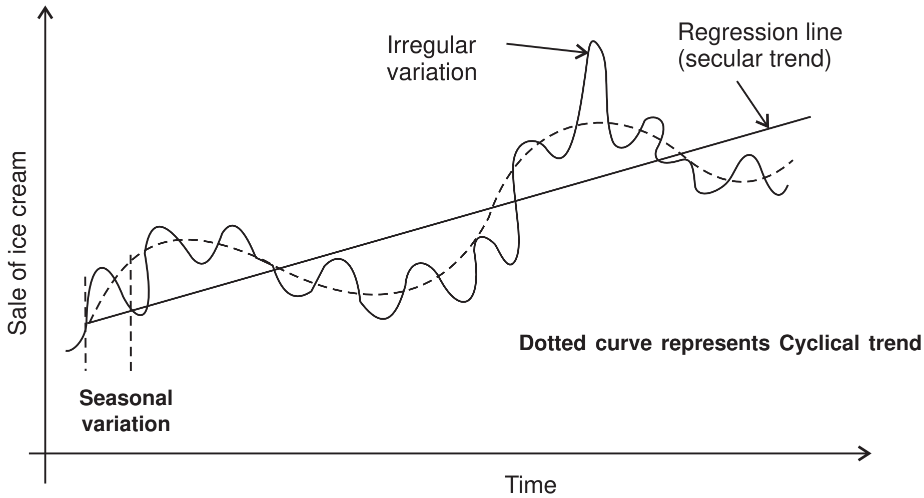
Consider the following data : monthly sale of ice cream in last one year ; annual rainfall in last 20 years ; weekly price index for last 52 weeks. This type of data, where observations are taken at specified times is called **time series.** Usually, equal intervals are used. Many times, long term or short term analysis of time series is required. Long term trend, called *secular trend,* is usually calculated by finding regression line,

$$y - \bar{y} = b_{yx}(x - \bar{x}).$$

There are three other kinds of variations which are important:

1. **Seasonal variation.** For example, sale of soft drinks and ice creams is higher in summer than in winter ; crockery sales are higher in festival season (diwali, christmas etc.) than at other times, and so on.
2. **Cyclical variation.** You must have heard about rise and fall of Roman empire. In fashion magazines, you read about rise and fall of hemlines. Share markets rise, fall, rise, fall like a yoyo. Only thing is we are not sure about the duration of the cycle (otherwise we would be millionaires!), but such cyclical trends are found in many time series.
3. **Irregular variations.** With sudden ban on mustard oil, Soya oil shows a marked, irregular upward sales. With announcement of elections, there is unusual rise in income of printing presses. With floods, there is irregular fall in crop yield. Such *spikes* in data can be attributed to some unusual phenomenon.

Above analysis shows that for analysis of data or for prediction, regression lines may not always be useful.



Basically analysis/prediction requires "smoothening of curve".

5.2.1 Purpose of moving averages

Moving averages are used in cyclical variations to eliminate fluctuations due to cyclical changes in time series. The cyclical variations are smoothened by averaging the values for the variate for a specified number of successive years (months or weeks etc.). The number of years (months or weeks etc.) over which the values are averaged depends upon the length of the cycles found in the time series. The time-interval over which the averages are taken is called the **period** of the cycle.

5.2.2 Method for finding moving averages

The average value for a number of years (months or weeks etc.) is taken and placed against the middle of the period. If the period taken is equal to the length of one cycle (or two cycles, or more cycles), then this results in elimination of cycles.

If $x_1, x_2, x_3, \ldots, x_n$ is the given annual time series, then

(*i*) 3-yearly moving averages are

$$\frac{x_1 + x_2 + x_3}{3}, \frac{x_2 + x_3 + x_4}{3}, \frac{x_3 + x_4 + x_5}{3}, \ldots \text{ which are placed}$$

against years 2, 3, 4, … respectively.

(*ii*) 5-yearly moving averages are

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}, \frac{x_2 + x_3 + x_4 + x_5 + x_6}{5}, \ldots \text{ which are placed}$$

against years 3, 4, … respectively.

(*iii*) 4-yearly moving averages are

$$\frac{x_1 + x_2 + x_3 + x_4}{4}, \frac{x_2 + x_3 + x_4 + x_5}{4}, \ldots \text{ which are placed}$$

against years 2·5, 3·5, … respectively. Further, to synchronise time frame for moving averages and original data, we have to average every two moving averages; average of first and second moving average in this case would be placed against $\frac{2\cdot5 + 3\cdot5}{2}$ = 3rd year; average of second and third moving average would be placed against $\frac{3\cdot5 + 4\cdot5}{2}$ = 4th year, and so on.

This is called **4-yearly centred moving average.**

Note. If the period is even, then the centred moving average is to be found out.

Following examples will make the above concept very clear.

ILLUSTRATIVE EXAMPLES

Example 1. *(i) Obtain the three year moving averages for the following series of observations.*

Year	*1995*	*1996*	*1997*	*1998*	*1999*	*2000*	*2001*	*2002*
Annual Sales (In 0000 ₹)	*3·6*	*4·3*	*4·3*	*3·4*	*4·4*	*5·4*	*3·4*	*2·4*

(ii) Obtain the five year moving average.

(iii) Construct also the 4-year centred moving average.

Solution. (*i*) First 3-year moving average is $\frac{3·6 + 4·3 + 4·3}{3} = \frac{12·2}{3}$ = 4·067, and is placed against 2nd year *i.e.* 1996; second 3-year moving average is $\frac{4·3 + 4·3 + 3·4}{3} = \frac{12·0}{3}$ = 4·0, and is placed against 3rd year *i.e.* 1997, and so on. Thus, we have :

Calculation of 3-year moving averages :

Year	*Annual sale*	*3-year moving total*		*3-year moving average*
1995	3·6	–	1/3	–
1996	4·3	12·2		4·067
1997	4·3	12·0		4·00
1998	3·4	12·1		4·03
1999	4·4	13·2		4· 40
2000	5·4	13·2		4· 40
2001	3·4	11·2		3·73
2002	2·4	–		–

(*ii*) First 5-yearly moving average is $\frac{3·6 + 4·3 + 4·3 + 3·4 + 4·4}{5} = \frac{20·0}{5}$ = 4·00, and is placed against 3rd year *i.e.* 1997. Second 5-yearly moving average is $\frac{4·3 + 4·3 + 3·4 + 4·4 + 5·4}{5} = \frac{21·8}{5}$ = 4·36, and is placed against 4th year *i.e.* 1998, and so on. Thus, we have :

Calculation of 5-year moving averages :

Year	*Annual sale*	*5-year moving total*		*5-year moving average*
1995	3·6	–		–
1996	4·3	–	1/5	–
1997	4·3	20·0		4·00
1998	3·4	21·8		4·36
1999	4·4	20·9		4·18
2000	5·4	19·0		3·80
2001	3·4	–		–
2002	2·4	–		–

(*iii*) In the 4-year moving averages, the first step of averaging of 4 values each results in placing these in between years — so we take averages of each two successive moving averages to synchronise them with given time frame. Thus, we have the following table :

5. The following data relate to the pay of workers employed at a factory.

Type of worker	*Rate of pay (₹/hour)*	*Average number of hours worked per week*		*Number of workers employed*
		June	*November*	
Skilled	6·50	37	38·5	26
Semi-skilled	5·10	38·5	39·5	14
Unskilled	3·50	40·5	40	42

Calculate a weighted aggregate index of average *weekly pay* for November (June = 100), using the number of workers employed as a weighting factor.

6. Calculate the 5 yearly moving averages of the number of students in a college from the following data and plot them on a graph paper :

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
No. of students	332	317	357	392	402	405	510	427	405	438

(*I.S.C. 2012*)

7. The number of traffic offences committed in a certain city over a period of 3 years is given in the following table :

	Jan.-March	*April-June*	*July-Sept.*	*Oct.-Dec.*
2000	74	56	48	69
2001	83	52	49	81
2002	94	60	48	79

Draw a graph illustrating these figures. Calculate suitable moving averages and plot them on the same graph. Comment on the result.

ANSWERS

EXERCISE 5.1

1. (*i*) specialised (*ii*) economic activity
(*iii*) quantity (or volume), base (or fixed) (*iv*) preferred
(*v*) normal (*vi*) 32% (*vii*) 12%.

2. (*i*) ₹ 10 per kg (*ii*) ₹ 8100 (*iii*) ₹ 15 per kg (*iv*) ₹ 15 per kg.

3. 104. **4.** 95. **5.** 105·9. **6.** (*i*) 129·1 (*ii*) 130·0.

7. (*i*) 114 (*ii*) 135. **8.** 104·4. **9.** 135.

10. 138·39.

11. 25·0, 27·5, 30·0, 50·0, 100. **12.** 128·10. **13.** 171·24. **14.** 164·05.

15. 137·27. **16.** 171·23. **17.** $x = 80$, $y = 70$.

18. $x = 40$, $y = 53$. **19.** 158. **20.** 162·5. **21.** 246.

EXERCISE 5.2

1. 3·667, 5·333, 6·667, 8·333, 10·333. **2.** 47, 55, 52·33, 60·33, 64·33.

3. 4·67, 5·33, 6, 7, 8, 8·33, 9.

4. 2·33, 5·67, 8·33, 16·67, 27·67, 27·67, 29·67, 20·67, 20, 10·33, 5·33, 2

5. The 3-day moving averages are 2·3, 5·6, 12·3, 19·6, 31·0, 34·3, 26·3, 20·0, 11·3, 5·3, 2·0. This shows a steady increase (arrival of fresh cases every day) and then decrease (control over epidemic).

6. 470, 484·6, 503·2, 515·2, 517·8, 523·6.

7. 368, 381·6, 390·4, 402·2, 419·8, 439·8.