7. Statistics

Exercise 7.1

1. Question

Calculate the mean for the following distribution:

x :	5	6	7	8	9
y :	4	8	14	11	3

Answer

X	y	yx
5	4	20
6	8	48
7	14	98
8	11	88
9	3	27
	N = 40	∑yx = 281

Mean = $\frac{\sum yx}{N}$

= $\frac{281}{40}$ = 7.025

2. Question

Find the mean of the following data:

x:	19	21	23	25	27	29	31
y:	13	15	16	18	16	15	13

Answer

X	y	yx
19	13	247
21	15	315
23	16	368
25	18	450
27	16	432
29	15	435
31	13	403
	N = 106	$\sum yx = 2650$

Mean = $\frac{\sum yx}{N}$

= $\frac{2650}{106}$ = 25

3. Question

If the mean of the following data is 20.6. Find the value of p.

x:	10	15	p	25	35
y:	3	10	25	7	5

Answer

X	y	yx
10	3	30
15	10	150
P	25	25p
25	7	175
35	5	175
	N = 50	∑yx = 530 + 25p

Given,

Mean = 20.6

$\frac{\sum yx}{N}$ = 20.6

$\frac{530+25p}{50}$ = 20.6

530 + 25p = 20.6 (50)

25p = 20.6 (50) – 530

$p = \frac{500}{25} = 20$

4. Question

If the mean of the following data is 15, find p.

x :	5	10	15	20	25
y :	6	p	6	10	5

Answer

X	y	yx
5	6	30
10	p	10p
15	6	90
20	10	200
25	5	125
	N = p + 27	∑yx = 10p + 445

Given,

Mean = 15

$\frac{\sum yx}{N}$ = 15

$\frac{10p+445}{p+27}$ = 15

10p + 445 = 15 (p + 27)

10p + 445 = 15p + 405

15p – 10p = 445 – 405

5p = 40

p = $\frac{40}{5}$ = 8

5. Question

Find the value of p for the following distribution whose mean is 16.6

x :	8	12	15	p	20	25	30
y :	12	16	20	24	16	8	4

Answer

X	y	yx
8	12	96
12	16	192
15	20	300
P	24	24p
20	16	320
25	8	200
30	4	120
	N = 100	∑yx = 24p + 1228

Given,

Mean = 16.6

$\frac{\sum yx}{N}$ = 16.6

$\frac{24p+1228}{100}$ = 16.6

24p + 1228 = 1660

24p = 1660 – 1228

$24p = 432$

$p = \frac{432}{24} = 18$

6. Question

Find the missing value of p for the following distribution whose mean is 12.58

x:	5	8	10	12	p	20	25
y:	2	5	8	22	7	4	2

Answer

X	y	yx
5	2	10
8	5	40
10	8	80
12	22	264
P	7	7p
20	4	80
25	2	50
	N = 50	∑yx = 524 + 7p

Given,

Mean = 12.58

$\frac{\sum yx}{N}$ = 12.58

$\frac{524+7p}{50}$ = 12.58

524 + 7p = 12.58 (50)

7p = 629 – 524

$p = \frac{105}{7} = 15$

7. Question

Find the missing frequency (p) for the following distribution whose mean is 7.68.

x:	3	5	7	9	11	13
y:	6	8	15	p	8	4

Answer

X	y	yx
3	6	18
5	8	40
7	15	105
9	P	9p
11	8	88
13	4	52
	N = p + 41	∑yx = 9p + 303

Given,

Mean = 7.68

$\frac{\sum yx}{N}$ = 7.68

$\frac{9p+303}{p+41}$ = 7.68

9p + 303 = 7.68 (p + 41)

9p – 7.68p = 314.88 – 303

1.32p = 11.88

p = $\frac{11.88}{1.32}$ = 9

8. Question

Find the value of p, if the mean of the following distribution is 20.

x:	15	17	19	20+p	23
y:	2	3	4	5p	6

Answer

X	y	yx
15	2	30
17	3	51
19	4	76
20 + p	5p	$100p + 5p^2$
23	6	138
	N = 5p + 15	$\sum yx = 295 + 100p + 5p^2$

Given,

Mean = 20

$\frac{\sum yx}{N} = 20$

$\frac{295+100p+5\,(p*p)}{5p+15}$ = 20

295 + 100p + $5p^2$ = 100p + 300

295 + $5p^2$ = 300

$5p^2$ = 300 – 295

$5p^2$ – 5 = 0

5 (p^2 – 1) = 0

p^2 – 1 = 0

(p + 1) (p – 1) = 0

p = � 1

If p + 1 = 0, p = - 1 (Reject)

Or p – 1 = 0, p = 1

9. Question

The following table gives the number of boys of a particular age in a class of 40 students. Calculate the mean age of the students.

Age (in year):	15	16	17	18	19	20
No. of students:	3	8	10	10	5	4

Answer

Age (x)	No. of students (y)	yx
15	3	45
16	8	128
17	10	170
18	10	180
19	5	95
20	4	80
	N = 40	$\sum yx = 698$

Mean = $\frac{\sum yx}{N}$

= $\frac{698}{40}$ = 17.45

Therefore, mean age is 17.45 years.

10. Question

Candidates of four schools appear in a mathematics test. The data were as follows:

Schools	No. of Candidates	Average Score
I	60	75
II	48	80
III	Not available	55
IV	40	50

If the average score of the candidates of all the four schools is 66, find the number of candidates that appeared from school III.

Answer

Let the number of candidates from school III be P.

Schools	No. of candidates (N ;)	Average score (x ;)
I	60	75
II	48	80
III	P	55
IV	40	50

Given,

Average score of all schools = 66

$\frac{N1x1+N2x2+N3x3+N4x4}{N1+N2+N3+N4}$ = 66

$\frac{60*75+48*80+P*55+40*50}{60+48+P+40}$ = 66

$\frac{4500+3840+55P+2000}{148+P}$ = 66

10340 + 55P = 66P + 9768

10340 – 9768 = 66P – 55P

P = $\frac{572}{11}$ = 52

11. Question

Five coins were simultaneously tossed 1000 times and at each toss the numbers of heads were observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss.

No. of heads per toss	No. of tosses
0	38
1	144
2	342
3	287
4	164
5	25
Total	1000

Answer

No. of heads per toss (x)	No. of tosses (y)	yx
0	38	0
1	144	144
2	342	684
3	287	861
4	164	656
5	25	125

Mean number of heads per toss = $\frac{\Sigma yx}{N}$

= $\frac{2470}{1000}$ = 2.47

Therefore, mean = 2.47

12. Question

Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution 50.

x: 10	30	50	70	90
y: 17	f1	32	f2	19

Answer

X	y	yx
10	17	170
30	f1	30f1
50	32	1600
70	f2	70f2
90	19	1710
	N = 120	∑yx = 30f1 + 70f2 + 3480

Given, mean = 50

$\frac{\sum yx}{N}$ = 50

$\frac{30f1+70f2+3480}{120}$ = 50

30f1 + 70f2 + 3480 = 50 * 120

30f1 + 70f2 + 3480 = 6000 (i)

Also, ∑y = 120

17 + f1 + 32 + f2 + 19 = 120

f1 + f2 = 52

f1 = 52 – f2

Substituting value of f1 in (i), we get

30 (52 – f2) + 70f2 + 3480 = 6000

40f2 = 960

f2 = 24

Hence, f1 = 52 – 24 = 28

Therefore, f1 = 28 and f2 = 24

13. Question

The arithmetic mean of the following data is 14. Find the value of k.

x_i:	5	10	15	20	25
y_i:	7	k	8	4	5

Answer

X	y	yx
5	7	35
10	K	10k
15	8	120
20	4	80
25	5	125
	N = 24 + k	∑yx = 360 + 10k

Given,

Mean = 14

$\frac{\sum yx}{N}$ = 14

$\frac{360+10k}{24+k}$ = 14

360 + 10k = 336 + 14k

24 = 4k

k = 6

Hence, the value of k is 6

14. Question

The arithmetic mean of the following data is 25, find the value of k.

x_i:	5	15	25	35	45
y_i:	3	k	3	6	2

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Answer

X	y	yx
5	3	15
15	K	15k
25	3	75
35	6	210
45	2	90
	N = 14 + k	Σyx = 15k + 390

Given,

Mean = 25

$\frac{\Sigma yx}{N}$ = 25

$$\frac{15k+390}{14+k} = 25$$

15k + 390 = 25 (14 + k)

15k + 390 = 350 + 25k

40 = 10k

k = 4

15. Question

If the mean of the following data is 18.75. Find the value of p.

x_i:	10	15	p	25	30
y_i:	5	10	7	8	2

Answer

X	y	yx
10	5	50
15	10	150
P	7	7p
25	8	200
30	2	60
	N = 32	$\sum yx = 7p + 460$

Given,

Mean = 18.75

$\frac{\sum yx}{N} = 18.75$

$\frac{7p+460}{32} = 18.75$

7p + 460 = 18.75 (32)

7p + 460 = 600

7p = 140

p = 20

Hence, the value of p is 20

Exercise 7.2

1. Question

The number of telephone calls received at an exchange per interval for 250 successive one-minute intervals are given in the following frequency table.

No. of calls (x):	0	1	2	3	4	5	6
No. of intervals (f):	15	24	29	46	54	43	39

Compute the mean number of calls per interval.

Answer

Let the assumed mean (A) = 3

Number of calls (x_i)	Number of intervals (f_i �)	$u_i = x_i - A = x_i - 3$	f_iu_i
0	15	-3	-45
1	24	-2	-48
2	29	-1	-29
3	46	0	0
4	54	1	54
5	43	2	86
6	39	3	117
	N = 250		$\sum f_iu_i = 135$

Mean number of calls = A + $\frac{\sum fiui}{N}$

$= 3 + \frac{135}{250} = \frac{885}{250}$

= 3.54

2. Question

Five coins were simultaneously tossed 1000 times, and at each toss the number of heads was observed. The number of tosses during which 0, 1, 2, 3, 4 and 5 heads were obtained are shown in the table below. Find the mean number of heads per toss

No. of heads pre toss(x):	0	1	2	3	4	5
No. of tosses (f):	38	144	342	287	164	25

Answer

Let the assumed mean (A) = 2

No. of heads per toss (x_i)	No. of intervals (f_i)	$U_i = x_i - A = x_i - 2$	f_iu_i
0	38	-2	-76
1	144	-1	-144
2	342	0	0
3	287	1	287
4	164	2	328
5	25	3	75
	N = 1000		$\sum f_iu_i = 470$

Mean number of heads per toss = A + $\frac{\sum fiui}{N}$

$= 2 + \frac{470}{1000} = 2 + 0.47$

$= 2.47$

3. Question

The following table gives the number of branches and number of plants in the garden of a school.

No. of branches (x):	2	3	4	5	6
No. of plants (f):	49	43	57	38	13

Answer

Let the assumed mean (A) = 4

No. of branches (x_i)	No. of plants (f_i)	$u_i = x_i - A = x_i - 4$	f_iu_i
2	49	-2	-98
3	43	-1	-43
4	57	0	0
5	38	1	38
6	13	2	26
	N = 200		$\sum f_iu_i = -77$

Average number of branches per plant = A + $\frac{\sum fiui}{N}$

= 4 + $\frac{-77}{200}$ = $\frac{800-77}{200}$

= 3.62 (approx.)

4. Question

The following table gives the number of children of 150 families in a village

No. of children (x):	0	1	2	3	4	5
No. of families (f):	10	21	55	42	15	7

Find the average number of children per family.

Answer

Let the assumed mean (A) = 2

No. of children (x_i)	No. of families (f_i)	$u_i = x_i - A = x_i - 2$	f_iu_i
0	10	-2	-20
1	21	-1	-21
2	55	0	0
3	42	1	42
4	15	2	30
5	7	3	21
	N = 150		$\sum f_iu_i = 52$

Average number of children per family = A + $\frac{\sum fiui}{N}$

= 2 + $\frac{52}{150}$ = $\frac{300+52}{150}$

= 2.35 (approx)

5. Question

The marks obtained out of 50, by 102 students in a Physics test are given in the frequency table below:

Marks (x):	15	20	22	24	25	30	33	38	45
Frequency (f):	5	8	11	20	23	18	13	3	1

Find the average number of marks.

Answer

Let the assumed mean (A) = 25

Marks (x_i)	Frequency (f_i)	$u_i = x_i - A = x_i - 25$	f_iu_i
15	5	-10	-50
20	8	-5	-40
22	11	-3	-33
24	20	-1	-20
25	23	0	0
30	18	5	90
33	13	8	104
38	3	13	39
45	1	20	20
	N = 102		$\sum f_iu_i = 110$

Average number of marks = A + $\frac{\sum fiui}{N}$

$= 25 + \frac{110}{102} = \frac{2550+110}{102}$

$= \frac{2660}{102} = 26.08$ (approx)

6. Question

The number of students absent in a class were recorded every day for 120 days and the information is given in the following frequency table:

No. of students absent (x):	0	1	2	3	4	5	6	7
No. of days (f):	1	4	10	50	34	15	4	2

Find the mean number of students absent per day.

Answer

Let the assumed mean (A) = 3

No. of students absent (x_i)	No. of days (f_i)	$u_i = x_i - A = x_i - 3$	$f_i u_i$
0	1	-3	-3
1	4	-2	-8
2	10	-1	-10
3	50	0	0
4	34	1	34
5	15	2	30
6	4	3	12
7	2	4	8
	N = 120		$\sum f_i u_i = 63$

Mean number of students absent per day = A + $\frac{\sum fiui}{N}$

$= 3 + \frac{63}{120} = \frac{360+63}{120}$

= 3.53 (approx)

7. Question

In the first proof reading of a book containing 300 pages the following distribution of misprints was obtained:

No. of misprints per pages (x):	0	1	2	3	4	5
No. of pages (f):	154	95	36	9	5	1

Find the average number of misprints per page.

Answer

To find : the average number of misprints per page.

Solution : Use the shortcut method to find the mean of given data.For that,Let the assumed mean be (A) = 2,The deviation of values x_i from assumed mean be $d_i = x_i - A$.

Now to find the mean:First multiply the frequencies in column (ii) with the value of deviations in column (iii) as $f_i d_i$.

Construct the table using above information.The table is as follows:

No. of misprints per pages (x_i)	No. of pages (f_i)	$d_i = x_i - A = x_i - 2$	$f_i d_i$
0	154	-2	-308
1	95	-1	-95
2	36	0	0
3	9	1	9
4	5	2	10
5	1	3	3
	N = 300		$\sum f_i d_i = -381$

Now add the sum of all entries in column (iii) to obtain $\sum_{i=1}^{n} f_i d_i$

and the sum of all frequencies in the column (ii) to obtain $\sum_{i=1}^{n} f_i = N$

So,Average number of misprints per day = A + $\frac{\sum f_i d_i}{N}$

where, N = total number of observations

⇒ Mean = 2 + $\frac{-381}{300}$

Mean = $\frac{600-381}{300}$

Mean = $\frac{219}{300}$

Mean = 0.73

8. Question

The following distribution gives the number of accidents met by 160 workers in a factory during a month.

No. of accidents (x):	0	1	2	3	4
No. of workers (f):	70	52	34	3	1

Find the average number of accidents per worker.

Answer

Let the assumed mean (A) = 2

No. of accidents (x_i)	No. of workers (f_i)	$u_i = x_i - A = x_i - 2$	f_iu_i
0	70	-2	-140
1	52	-1	-52
2	34	0	0
3	3	1	3
4	1	2	2
	N = 160		$\sum f_iu_i = -187$

Average number of accidents per day workers = A + $\frac{\sum fiui}{N}$

$= 2 + (-\frac{187}{160}) = \frac{320-187}{160}$

= 0.83

9. Question

Find the mean from the following frequency distribution of marks at a test in statistics:

Marks (x):	5	10	15	20	25	30	35	40	45	50
No. of students (f):	15	50	80	76	72	45	39	9	8	6

Answer

Let the assumed mean (A) = 25

Marks (x_i)	No. of students (f_i)	$u_i = x_i - A = x_i - 25$	f_iu_i
5	15	-20	-300
10	50	-15	-750
15	80	-10	-800
20	76	-5	-380
25	72	0	0
30	45	5	225
35	39	10	390
40	9	15	135
45	8	20	160
50	6	25	150

	N = 400		$\sum f_i u_i$ = -1170

Mean = A + $\frac{\sum fiui}{N}$

= 25 + $\frac{-1170}{400}$ = $\frac{10000-1170}{400}$

= 22.075

Exercise 7.3

1. Question

The following table gives the distribution of total household expenditure (in rupees) of manual workers in a city.

Expenditure (in rupees) (in rupees)	Frequency (f)	Expenditure (in rupees) (x)	Frequency (f)
100-150	24	300-350	30
150-200	40	350-400	22
200-250	33	400-450	16
250-300	28	450-500	7

Find the average expenditure (in rupees) per household.

Answer

Let the assumed mean (A) = 275

Class interval	Mid value (x_i)	$d_i = x_i - 275$	$u_i = \frac{xi-275}{50}$	Frequency (f_i)	f_iu_i
100 - 150	125	-150	-3	24	-72
150 - 200	175	-100	-2	40	-80
200 - 250	225	-50	-1	33	-33
250 - 300	275	0	0	28	0
300 - 350	325	50	1	30	30
350 - 400	375	100	2	22	44
400 - 450	425	150	3	16	48
450 - 500	475	200	4	7	28
				N = 200	$\sum f_iu_i = -35$

We have, A = 275

h = 50

Mean = A + h * $\frac{\sum fiui}{N}$

$= 275 + 50 * \frac{-35}{200}$

$= 275 - 8.75$

$= 266.25$

2. Question

A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Numbers of plants:	0-2	2-4	4-6	6-8	8-10	10-12	12-14
Numbers of houses:	1	2	1	5	6	2	3

Which method did you use for finding the mean, and why?

Answer

Let us find class marks (x_i) for each interval by using the relation.

Class mark (x_i) = $\frac{Upperclass\ limit + Lowerclass\ limit}{2}$

Now we may compute x_i and f_ix_i as follows:

Number of plants	Number of house (f_i)	x_i	$f_i x_i$
0 - 2	1	1	1
2 - 4	2	3	6
4 - 6	1	5	5
6 - 8	5	7	35
8 - 10	6	9	54
10 - 12	2	11	22
12 - 14	3	13	39
	N = 20		$\sum f_i x_i = 162$

Fro, the table we may observe that,

$\sum f_i = 20$

$\sum f_i x_i = 162$

Mean, $\overline{X} = \frac{\sum fixi}{\sum fi}$

$= \frac{162}{20} = 8.1$

So mean number of plants per house is 8.1

We have used for the direct method values x_i and f_i are very small.

3. Question

Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages (in Rs):	100-120	120-140	140-160	160-180	180-200
Numbers of workers:	12	14	8	6	10

Find the mean daily wages of the workers of the factory by using an appropriate method.

Answer

Let the assumed mean (A) = 150

Class interval	Mid value (x_i)	$d_i = x_i - 150$	$u_i = \frac{xi-150}{20}$	Frequency (f_i)	$f_i u_i$
100 – 120	110	-40	-2	12	-24
120 – 140	130	-20	-1	14	-14
140 – 160	150	0	0	8	0
160 – 180	170	20	1	6	6
180 - 200	190	40	2	10	20
				N = 50	$\sum f_i u_i = -12$

We have, A = 150

h = 20

Mean = A + h * $\frac{\sum fiui}{N}$

= 150 + 20 * $\frac{-12}{50}$

= 150 - $\frac{24}{5}$ = 150 – 4.8

= 145.2

4. Question

Thirty women were examined in a hospital by a doctor and the number of heart beats per minute recorded and summarized as follows. Find the mean heart beats per minute for these women, choosing a suitable method.

Number of heart beats per minute:	65-68	68-71	71-74	74-77	77-80	80-83	83-86
Number of women:	2	4	3	8	7	4	2

Answer

We may find class mark of each interval (x_i) by using the relation.

x_i = $\frac{Upper\ class\ limit+Lower\ class\ limit}{2}$

Class size of this data = 3

Now taking 75.5 as assumed mean (A), we may calculate d_i, u_i, f_iu_i as following:

No. of heart beat per minute	Number of women (f_i)	x_i	$d_i = x_i - 75.5$	$u_i = \frac{xi-75.5}{h}$	f_iu_i
65 – 68	2	66.5	-9	-3	-6
68 – 71	4	69.5	-6	-2	-8
71 – 74	3	72.5	-3	-1	-3
74 – 77	8	75.5	0	0	0
77 – 80	7	78.5	3	1	7
80 – 83	4	81.5	6	2	8
83 – 86	2	84.5	9	3	6
	N = 30				$\sum f_iu_i = 4$

Now we may observe from table that $\Sigma f_i = 30$, $\Sigma f_iu_i = 4$

Mean $(\overline{X}) = d_i + \frac{\sum iui}{\sum fi} * h$

$= 75.5 + \frac{4}{30} * 3$

= 75.5 + 0.4 = 75.9

So mean of heart beat per minute of women are 75.9 beats per minute.

5. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-6	6-12	12-18	18-24	24-30
Frequency:	6	8	10	9	7

Answer

Let A assumed mean be 15.

Class interval	Mid value (x_i)	$d_i = x_i - 15$	$u_i = \frac{xi-15}{6}$	f_i	f_iu_i
0 – 6	3	-12	-2	6	-12
6 – 12	9	-6	-1	8	-8
12 – 18	15	0	0	10	0
18 – 24	21	6	1	9	9
24 – 30	27	12	2	7	14
				N = 40	$\sum f_iu_i = 3$

A = 15, h = 6

Mean = A + h * $\frac{\Sigma f_i x_i}{N}$

= 15 + 6 * $\frac{3}{40}$

= 15 + 0.45

= 15.45

6. Question

Find the mean of each of the following frequency distributions:

Class interval:	50-70	70-90	90-110	110-130	130-150	150-170
Frequency:	18	12	13	27	8	22

Answer

Let the assumed mean (A) be 100.

Class interval	Mid value (x_i)	$d_i = x_i - 100$	$u_i = \frac{xi-100}{20}$	f_i	f_iu_i
50 – 70	60	-40	-2	18	-36
70 – 90	80	-20	-1	12	-12
90 – 110	100	0	0	13	0
110 – 130	120	20	1	27	27
130 – 150	140	40	2	8	16
150 – 170	160	60	3	22	66
				N = 100	$\sum f_iu_i = 61$

A = 100, h = 20

Mean = A + h * $\frac{\sum fiui}{N}$

= 100 + 20 * $\frac{61}{100}$

= 100 + 12.2

= 112.2

7. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-8	8-16	16-24	24-32	32-40
Frequency:	6	7	10	8	9

Answer

Let the assumed mean (A) = 20

Class interval	Mid – value (x_i)	$d_i = x_i - 20$	$u_i = \frac{xi-20}{8}$	Frequency (f_i)	f_iu_i
0 – 8	4	-16	-2	6	-12
8 – 16	12	-8	-1	7	-17
16 – 24	20	0	0	10	0
24 – 32	28	8	1	8	8
32 - 40	36	16	2	9	18
				N = 40	$\sum f_iu_i = 7$

We have, A = 20

h = 8

Mean = A + h * $\frac{\sum fiui}{N}$

= 20 + 8 * $\frac{7}{40}$

= 20 + 1.4 = 21.4

8. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-6	6-12	12-18	18-24	24-30
Frequency:	7	5	10	12	6

Answer

Let the assumed mean (A) = 15

Class interval	Mid value (x_i)	$d_i = x_i - 15$	$u_i = \frac{xi-15}{6}$	Frequency (f_i)	f_iu_i
3 – 6	3	-12	-2	7	-14
6 – 12	9	-6	-1	5	-5
12 – 18	15	0	0	10	0
18 – 24	21	6	1	12	12
24 - 30	27	12	2	6	12
				N = 40	$\sum f_iu_i = 5$

We have, A = 15

h = 6

Mean = A + h * $\frac{\sum fiui}{N}$

$= 15 + 6 * \frac{5}{40}$

$= 15 + 0.75 = 15.75$

9. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-10	10-12	20-30	30-40	40-50
Frequency:	9	12	15	10	14

Answer

Let the assumed mean (A) = 25

Class interval	Mid value (x_i)	$d_i = x_i - 25$	$u_i = \frac{xi-25}{10}$	Frequency (f_i)	f_iu_i
0 – 10	5	-20	-2	9	-18
10 – 20	15	-10	-1	12	-12
20 – 30	25	0	0	15	0
30 – 40	35	10	1	10	10
40 – 50	45	20	2	14	28
				N = 60	$\sum f_iu_i = 8$

We have, A = 25

h = 10

Mean = A + h * $\frac{\sum fiui}{N}$

= 25 + 10 * $\frac{8}{60}$

= 25 + $\frac{8}{6}$

= 25 + $\frac{4}{3}$ = 26.333

10. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-8	8-16	16-24	24-32	32-40
Frequency:	5	9	10	8	8

Answer

Let the assumed mean (A) = 20

Class interval	Mid value (x_i)	$d_i = x_i - 20$	$u_i = \frac{xi-20}{8}$	Frequency (f_i)	f_iu_i
0 – 8	4	-16	-2	5	-10
8 – 16	12	-8	-1	9	-9
16 – 24	20	0	0	10	0
24 – 32	28	8	1	8	8
32 – 40	36	16	2	8	16
				N = 40	$\sum f_iu_i = 5$

We have, A = 20

h = 8

Mean = A + h * $\frac{\sum fiui}{N}$

= 20 + 8 * $\frac{5}{40}$

= 20 + 1 = 21

11. Question

Find the mean of each of the following frequency distributions:

Class interval:	0-8	8-16	16-24	24-32	32-40
Frequency:	5	6	4	3	2

Answer

Let the assumed (A) = 20

Class interval	Mid value (x_i)	$d_i = x_i - 20$	Frequency (f_i)	$u_i = \frac{xi-20}{8}$	f_1u_1
0 – 8	4	-16	5	-2	-10
8 -16	12	-8	6	-1	-6
16-24	20	0	4	0	0
24-32	28	8	3	1	3
32-40	36	16	2	2	4
			N = 20		$\sum f_iu_i = -9$

We have, A = 20

h = 8

Mean = A + h * $\frac{\sum fiui}{N}$

= 20 + 8 * $\frac{-9}{20}$

= 20 - $\frac{72}{20}$ = 20 − 3.6

= 16.4

12. Question

Find the mean of each of the following frequency distributions:

Class interval:	10-30	30-50	50-70	70-90	90-110	110-130
Frequency:	5	8	12	20	3	2

Answer

Let the assumed mean (A) = 60

Class interval	Mid value (x_i)	$d_i = x_i - 60$	$u_i = \frac{xi-60}{20}$	Frequency (f_i)	f_iu_i
10-30	20	-40	-2	5	-10
30-50	40	-20	-1	8	-8
50-70	60	0	0	12	0
70-90	80	20	1	20	20
90-110	100	40	2	3	6
110-130	120	60	3	2	6
				N = 50	$\sum f_iu_i = 14$

We have, A = 60

h = 20

Mean = A + h * $\frac{\sum fiui}{N}$

= 60 + 20 * $\frac{14}{50}$

= 60 + 5.6

= 65.6

13. Question

Find the mean of each of the following frequency distributions:

Class interval:	25-35	35-45	45-55	55-65	65-75
Frequency:	6	10	8	12	4

Answer

Let the assumed mean (A) = 50

Class interval	Mid value	$d_i = x_i - 50$	$u_i = \frac{xi-50}{10}$	Frequency (f_i)	f_iu_i
25-35	30	-20	-2	6	-12
35-45	40	-10	-1	10	-10
45-55	50	0	0	8	0
55-65	60	10	1	12	12
65-75	70	20	2	4	8
				N = 40	$\sum f_iu_i = -2$

We have, A = 50

h = 10

Mean = A + h * $\frac{\sum fiui}{N}$

= 50 + 10 * $\frac{-2}{40}$

= 50 – 0.5

= 49.5

14. Question

Find the mean of each of the following frequency distributions:

Classes:	25-29	30-34	35-39	40-44	45-49	50-54	55-59
Frequency:	14	22	16	6	5	3	4

Answer

Let the assumed mean (A) = 42

Class interval	Mid value (x_i)	$d_i = x_i - 42$	$U_i = \frac{xi-42}{5}$	Frequency (f_i)	f_iu_i
25-29	27	-15	-3	14	-42
30-34	32	-10	-2	22	-44
35-39	37	-5	-1	16	-16
40-44	42	0	0	6	0
45-49	47	5	1	5	5
50-54	52	10	2	3	6
55-59	57	15	3	4	12
				N = 10	$\sum f_iu_i = -79$

We have, A = 42

h = 5

Mean = A + h * $\frac{\sum fiui}{N}$

= 42 + 5 * $\frac{-79}{10}$

= 42 - $\frac{79}{14}$

$= \frac{588-79}{14}$

= 36.357

15. Question

For the following distribution, calculate mean using all suitable methods:

Size of items:	1-4	4-9	9-16	16-27
Frequency:	6	12	26	20

Answer

By direct method

Class interval	Mid value (x_i)	Frequency (f_i)	$f_i x_i$
1-4	2.5	6	15
4-9	6.5	12	78
9-16	12.5	26	325
16-27	21.5	20	430
		N = 64	$\sum f_i u_i = 848$

Mean = $\frac{\sum fixi}{N} + A$

$= \frac{848}{64} = 13.25$

By assumed mean method

Let, the assumed mean (A) = 6.5

Class interval	Mid value (x_i)	$u_i = x_i - A = x_i - 6.5$	Frequency (f_i)	f_iu_i
1-4	2.5	-4	6	-24
4-9	6.5	0	12	0
9-16	12.5	6	26	156
16-27	21.5	15	20	300
			N = 64	$\sum f_iu_i = 432$

Mean = A + $\frac{\sum fiui}{N}$

$= 6.5 + \frac{432}{64}$

= 6.5 + 6.75

= 13.25

16. Question

The weekly observations on cost of living index in a certain city for the year 2004-2005 are given below. Compute the weekly cost of living index.

Cost of living Cost of living	Number of Students	Cost of living Index	Number of students
1400-1500	3	1700-1800	9
1500-1600	10	1800-1900	6
1600-1700	20	1900-2000	2

Answer

Let the assumed mean (A) = 1650

Class interval	Mid value (x_i)	$d_i = x_i - A = x_i - 1650$	$u_i = \frac{xi-1650}{100}$	Frequency (f_i)	f_iu_i
1400-1500	1450	-200	-2	3	-10
1500-1600	1550	-100	-1	10	-10
1600-1700	1650	0	0	20	0
1700-1800	1750	100	1	9	9
1800-1900	1850	200	2	6	12
1900-2000	1950	300	3	2	6
				N = 52	$\sum f_iu_i = 7$

We have, A = 1650

h = 100

Mean = A + h * $\frac{\sum fiui}{N}$

$= 1650 + 100 * \frac{7}{52}$

$= 1650 + \frac{1650}{13}$

$= \frac{21450+175}{13} = \frac{21625}{13}$

= 1663.46

17. Question

The following table shows the marks scored by 140 students in an examination of a certain paper:

Marks:	0-10	10-20	20-30	30-40	40-50
Number of students:	20	24	40	36	20

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

Answer

From Direct method:

Class interval	Mid value (x_i)	f_i	f_ix_i
0-10	5	20	100
10-20	15	24	360
20-30	25	40	1000
30-40	35	36	1260
40-50	45	20	900
		N = 140	$\sum f_ix_i$ = 3620

Mean = $\frac{\sum fixi}{N}$

= $\frac{3620}{140}$ = 25.857

Assumed mean method: let assumed mean (A) = 25

Mean = A + $\frac{\sum fiui}{N}$

Class interval	Mid value (x_i)	$u_i = x_i - A$	f_i	f_iu_i
0-10	5	-20	20	-400
10-20	15	-10	24	-240
20-30	25	0	40	0
30-40	35	10	36	360
40-50	45	20	20	400
			N = 140	$\sum f_iu_i = 120$

Mean = A + $\frac{\sum fixi}{N}$

= 25 + $\frac{120}{140}$ = 25 + 0.857

= 25.857

Step deviation method: Let the assumed mean (A) = 25

Class interval	Mid value (x_i)	$d_i = x_i - A = x_i - 25$	$u_i = \frac{xi-25}{10}$	Frequency (f_i)	f_iu_i
0-10	5	-20	-2	20	-40
10-20	15	-10	-1	24	-24
20-30	25	0	0	40	0
30-40	35	10	1	36	36
40-50	45	20	2	20	40
				N = 140	$\sum f_iu_i = 12$

Mean = A + $\frac{\sum fiui}{N}$ * h

= 25 + $\frac{12}{140}$ * 10

= 25 + 0.857 = 25.857

18. Question

The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequency is 50. Compute the missing frequency.

Class:	0-20	20-40	40-60	60-80	80-100	100-120
Frequency:	5	f1	10	f2	7	8

Answer

Class interval	Mid value (x_i)	Frequency (f_i)	$f_i u_i$
0-20	10	5	50
20-40	30	f1	30f1
40-60	50	10	500
60-80	70	f2	70f2
80-100	90	7	630
100-120	110	8	880
		N = 50	$\sum f_i u_i = 30f1 + 70f2 + 2060$

Given, Sum of frequency = 50

5+ f1 + 10 + f2 + 7 + 8 = 50

$f1 + f2 = 50 – 5 – 10 – 7 – 8$

$f1 + f2 = 20$

$3f1 + 3f2 = 60$ (i) [Multiply by 3]

And mean = 62.8

$\frac{\sum fiui}{N} = 62.8$

$\frac{30f1+70f2+2060}{50} = 62.8$

$30f1 + 70f2 = 3140 – 2060$

$30f1 + 70f2 = 1080$

$3f1 + 7f2 = 108$ (ii) [Divide by 10]

Subtract (i) from (ii), we get

$3f1 + 7f2 – 3f1 – 3f2 = 108 – 60$

$4f2 = 48$

$f2 = 12$

Put value of f2 in (i), we get

$3f1 + 3 * 12 = 60$

$3f1 + 36 = 60$

$3f1 = 24$

$f1 = 8$

So, f1 = 8 and f2 = 12

19. Question

The following distribution shows the daily pocket allowance given to the children of a multistory building. The average pocket allowance is Rs. 18.00. Find out the missing frequency.

Class interval:	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency:	7	6	9	13	-	5	4

Answer

Given, Mean = 18

Let missing frequency be V

Class interval	Mid value (x_i)	f_i	$f_i x_i$
11-13	12	7	84
13-15	14	6	84
15-17	16	9	144
17-19	18	13	234
19-21	20	V	20V
21-23	22	5	110
23-25	24	4	56
		N = 44 + V	$\sum f_i x_i = 752 + 20V$

Mean = $\frac{\sum fixi}{N}$

18 = $\frac{752+20V}{44+V}$

792 + 18V = 752 + 20V

792 – 752 = 20V – 18V

40 = 2V

V = 20

20. Question

If the mean of the following distribution is 27, find the value of p.

Class:	0-10	10-20	20-30	30-40	40-50
Frequency:	8	p	12	13	10

Answer

Class interval	Mid value (x_i)	Frequency (f_i)	$f_i x_i$
0-10	5	8	40
10-20	15	p	15p
20-30	25	12	300
30-40	35	13	455
40-50	45	10	450
		N = 43 + p	$\sum f_i x_i$ = 1245 + 15p

Given, mean = $\frac{\sum fixi}{N}$

27 = $\frac{1245+15p}{43+p}$

1161 + 27p = 1245 + 15p

27p – 15p = 1245 – 1161

12p = 84

p = 7

21. Question

In a retail market, fruit vendor were-selling mangoes kept in packing boxes. These boxes contained varying number of mangoes. The following was the distribution of mangoes according to the number of boxes.

Number of mangoes:	50-52	53-55	56-58	59-61	62-64
Number of boxes:	15	110	135	115	25

Find the mean number of mangoes kept in a packing box. Which method of finding the mean did you choose?

Answer

Number of mangoes	Number of boxes (f_i)
50-52	15
53-55	110
56-58	135
59-61	115
62-64	25

We may observe that the class intervals are not continuous. There is a gap between two class intervals so we have to add $\frac{1}{2}$ from lower class limit of each interval.

Class size (h) of this data = 3

Now taking 57 as assumed mean, we can calculate as follows:

Class interval	f_i	x_i	$d_i = x_i - 57$	$u_i = \frac{xi-57}{h}$	f_iu_i
49.5-52.5	15	51	-6	-2	-30
52.5-55.5	110	54	-3	-1	-110
55.5-58.5	135	57	0	0	0
58.5-61.5	115	60	6	1	115
61.5-64.5	25	63	3	2	50
	N = 400				$\sum f_iu_i = 25$

Mean = A + $\frac{\sum fiui}{N}$ * h

= 57 + $\frac{25}{400}$ * 3

= 57 + $\frac{3}{16}$

= 57 + 0.1875 = 57.1875

= 57.19

Number of mangoes kept in packing box is 57.19

22. Question

The table below shows the daily expenditure on food of 25 households in a locality.

Daily expenditure (in Rs):	100-150	150-200	200-250	250-300	300-350
Number of households:	4	5	12	2	2

Find the mean daily expenditure on food by a suitable method.

Answer

We may calculate class marks (x_i) for each interval by using the relation

$x_i = \frac{Upper\ class\ limit + lower\ class\ limit}{2}$

Class size = 50

Now taking 225 as assumed mean we can calculate as follows:

Daily expenditure (In Rs)	f_i	$d_i = x_i - A = x_i - 225$	$u_i = \frac{xi-225}{h}$	x_i	f_iu_i
100-150	4	-100	-2	125	-8
150-200	5	-50	-1	175	-5
200-250	12	0	0	225	0
250-300	2	50	1	275	2
300-350	2	100	2	325	4
	N = 25				$\sum f_iu_i = -7$

Mean $(\overline{x}) = A + \frac{\sum fiui}{N} * h$

$= 225 + \frac{-7}{25} * 50$

$= 225 - 14 = 211$

So mean expenditure on food is 211

23. Question

To find out the concentration of SO_2 in the air (in parts per million, i.e., ppm), the data was collected for 30 localities in a certain city and is presented below:

Concentration of SO_2 (In ppm)	Frequency
0.00-0.04	4
0.04-0.08	9
0.08-0.12	9
0.12-0.16	2
0.16-0.20	4
0.20-0.24	2

Find the mean concentration of SO_2 in the air.

Answer

We may calculate class marks (x_i) for each interval by using the relation

$x_i = \frac{Upper\ class\ limit + lower\ class\ limit}{2}$

Class size = 0.04

Now taking 0.14 as assumed mean (A), we can calculate as follows:

Concentration So_2 (ppm)	Frequency (f_i)	x_i	$d_i = x_i - 0.14$	u_i	f_iu_i
0.00-0.04	4	0.02	-0.12	-3	-12
0.04-0.08	9	0.06	-0.08	-2	-18
0.08-0.12	9	0.10	-0.04	-1	-9
0.12-0.16	2	0.14	0	0	0
0.16-0.20	4	0.18	0.04	1	4
0.20-0.24	2	0.22	0.08	2	4
	N = 30				$\sum f_iu_i = -31$

Mean $(\bar{x}) = A + \frac{\sum fiui}{N} * h$

$= 0.14 + \frac{-31}{30} * (0.04)$

= 0.14 – 0.04133

= 0.099ppm

SO, mean concentration of SO_2 in the air is 0.099ppm

24. Question

A class teacher has the following absentee record of 40 students of a class for the whole term. Find the mean number of days a student was absent.

Numbers of days:	0-6	6-10	10-14	14-20	20-28	28-38	38-40
Number of students:	11	10	7	4	4	3	1

Answer

We may calculate class marks (x_i) for each interval by using the relation

$x_i = \frac{Upper\ class\ limit + lower\ class\ limit}{2}$

Now taking assumed mean (A) = 16

Number of days	Number of students (f_i)	x_i	$d_i = x_i - 16$	$f_i d_i$
0-6	11	3	-13	-143
6-10	10	5	-8	-80
10-14	7	12	-4	-28
14-20	4	16	0	0
20-28	4	24	8	32
28-38	3	33	17	51
38-40	1	39	23	23
	N = 40			$\sum f_i d_i = -145$

Mean $(\overline{x}) = A + \frac{\sum fidi}{N}$

$= 16 + \frac{-145}{40}$

= 16 – 3.62 = 12.38

So, mean number of days is 12.38 days for which students were absent.

25. Question

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %):	45-55	55-65	65-75	75-85	85-95
Number of cities:	3	10	11	8	3

Answer

We may calculate class marks (x_i) for each interval by using the relation

$x_i = \frac{\text{Upper class limit} + \text{lower class limit}}{2}$

Class size (h) for this data = 10

Now taking 70 as assumed mean (A) we can calculate as follows:

Literacy rate (in %)	Number of cities (f_i)	x_i	$d_i = x_i - 70$	$u_i = \frac{di}{10}$	f_iu_i
45-55	3	50	-20	-2	-6
55-65	10	60	-10	-1	-10
65-75	11	70	0	0	0
75-85	8	80	10	1	8
85-95	3	90	20	2	6
	N = 35				$\sum f_iu_i = -2$

Mean $(\overline{x}) = A + \frac{\Sigma fiui}{N} * h$

$= 70 + \frac{-2}{35} * 10$

= 70 – 0.57

= 69.43

So, mean literacy rate is 69.437

Exercise 7.4

1. Question

Following are the lives in hours of 15 pieces of the components of aircraft engine. Find the median:

715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719

Answer

Lives in hours of 15 pieces are = 715, 724, 725, 710, 729, 745, 694, 699, 696, 712, 734, 728, 716, 705, 719

Arrange the above in ascending order:

694, 696, 699, 705, 710, 712, 716, 719, 724, 725, 728, 729, 734, 745

N = 15 (Odd)

Median = $(\frac{N+1)}{2})$ Term

$= (\frac{15+1}{2})$ Term

= 16^{th} Term = 716

2. Question

The following is the distribution of height of students of a certain class in a certain city.

Height (in cms):	160-162	163-165	166-168	169-171	172-174
No. of students:	15	118	142	127	18

Find the median height.

Answer

Class interval (exclusive)	Class interval (inclusive)	Class interval (Frequency)	Cumulative frequency
160-162	159.5-162.5	15	15
163-164	162.5-165.5	118	133 (F)
166-168	165.5-168.5	142 (f)	275
169-171	168.5-171.5	127	402
172-174	171.5-174.5	18	420
		N = 420	

We have, N = 420

$\frac{N}{2} = \frac{420}{2}$ = 210

The cumulative frequency just greater than $\frac{N}{2}$ is 275 then 165.5-168.5 is the median class such that,

l = 165.5, f = 142, F = 133 and h = 168.5-165.5 = 3

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 165.5 + $\frac{210-133}{142}$ * 3

= 165.5 + $\frac{77}{142}$ * 3

= 165.5 + 1.63

= 167.13

3. Question

Following is the distribution of I.Q of 100 students. Find the median I.Q.

I.Q:	55-64	65-74	75-84	85-94	95-104	105-114	115-124	125-134	135-144
No. of students:	1	2	9	22	33	22	8	2	1

Answer

Class interval (exclusive)	Class interval (inclusive)	Frequency	Cumulative frequency
55-64	54.5-64.5	1	1
65-74	64.5-74.5	2	3
75-84	74.5-84.5	9	12
85-94	84.5-94.5	22	34 (F)
95-104	94.5-104.5	33 (f)	37
105-114	104.5-114.5	22	89

115-124	114.5-124.5	8	97
125-134	124.5-134.5	2	99
135-144	134.5-144.5	1	100
		N = 100	

We have, N = 100

$\frac{N}{2} = \frac{100}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 67 then the median class 94.5-104.5 such that,

l = 94.5, f = 33, F = 34, h = 104.5 – 94.5 = 10

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 94.5 + $\frac{50-34}{33}$ * 10

= 94.5 + $\frac{16}{33}$ * 10

= 94.5 + 4.85 = 99.35

4. Question

Calculate the median from the following data:

Rent (in Rs):	15-25	25-35	35-45	45-55	55-65	65-75	75-85	85-95
No. of Houses:	8	10	15	25	40	20	15	7

Answer

Class interval	Frequency	Cumulative frequency
15-25	8	8
25-35	10	18
35-45	15	33
45-55	25	58 (F)
55-65	40 (f)	98
65-75	20	48
75-85	15	133
85-95	7	140
	N = 140	

We have, N = 140

$\frac{N}{2} = \frac{140}{2} = 70$

The cumulative frequency is just greater than 98 then median class is 55-65 such that

l = 55, f = 40, F = 58, h = 65 – 55 = 10

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 55 + $\frac{70-58}{40}$ * 10

= 55 + $\frac{12}{40}$ * 10

= 55 + 3 = 58

Therefore, Median = 58

5. Question

Calculate the median from the following data:

Marks below:	10	20	30	40	50	60	70	80
No. of students:	15	35	60	84	96	127	198	250

Answer

140 kg below	No. of students	Class interval	Frequency	Cumulative frequency
10	15	0-10	15	15
20	35	10-20	20	35
30	60	20-30	25	60
40	84	30-40	24	84
50	96	40-50	12	96 (F)
60	127	50-60	31 (f)	127
70	198	60-70	71	198
80	250	70-80	52	250
			N = 250	

We have, N = 250

$\frac{N}{2} = \frac{250}{2} = 125$

The cumulative frequency is just greater than $\frac{N}{2}$ is 127 then median class is 50-60 such that:

l = 50, f = 31, F = 96, h = 60-50 = 10

Median = l + $\frac{\frac{N}{2} - F}{f}$

= 50 + $\frac{125-96}{31}$ * 10

= 50 + $\frac{29*10}{31}$

= $\frac{445}{31}$ = 59.35

6. Question

An incomplete distribution is given as follows:

Variable:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency:	10	20	?	40	?	25	15

You are given that the median value is 35 and the sum of all the frequencies is 170. Using the median formula fill up the missing frequency.

Answer

Class interval	Frequency	Cumulative frequency
0-10	10	10
10-20	20	30
20-30	f_1	$30 + f_1$ (F)
30-40	40 (f)	$70 + f_1$
40-50	f_2	$70 + f_1 + f_2$
50-60	25	$95 + f_1 + f_2$
60-70	15	$110 + f_1 + f_2$
	N = 170	

Given, Median = 35

The median class = 30-40

l = 30, h = 10, f = 40 and F = 30 + f_1

Median = l + $\frac{\frac{N}{2} - F}{f}$

35 = 30 + $\frac{85-(30+f1)}{40}$ * 10

$5 = \frac{55 - f1}{4}$

$f_1 = 55 - 20 = 35$

Given, Sum of frequencies = 170

$= 10 + 20 + f_1 + 40 + f_2 + 25 + 15 = 170$

$= 10 + 20 + 35 + 40 + f_2 + 25 + 15 = 170$

$= f_2 = 170 - 145 = 25$

Therefore, $f_1 = 35$ and $f_2 = 25$

7. Question

Calculate the missing frequency form the following distribution, it being given that the median of the distribution is 24.

Age in years:	0-10	10-20	20-30	30-40	40-50
No. of persons:	5	25	?	18	7

Answer

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	25	30 (F)
20-30	x (f)	30 + x
30-40	18	48 + x
40-50	7	55 + x
	N = 55 + x	

Given, Median = 24

Then median class = 20-30

l = 20, h = 10, f = x and F = 30

Median = l + $\frac{\frac{N}{2} - F}{f}$

24 = 20 + $\frac{55 + \frac{x}{2} - 30}{x}$ * 10

4x = 275 + 5x – 300

4x – 5x = -25

-x = -25

x = 25

Therefore, missing frequency = 25

8. Question

Find the missing frequencies and the median for the following distribution if the mean is 1.46.

No. of accidents:	0	1	2	3	4	5	Total
Frequency (No. of days):	46	?	?	25	10	5	200

Answer

No. of accidents (x)	No. of days (f)	Fx
0	46	0
1	X	X
2	Y	2y
3	25	75
4	10	40
5	5	25
	N = 200	$\sum fx = x + 2y + 140$

Given, N = 200

= 46 + x + y + 25 + 10 + 5 = 200

= x + y = 200 – 46 – 25 – 10 – 5

= x + y = 114 (i)

And Mean = 1.46

$\frac{\sum fx}{N}$ = 1.46

= $\frac{x+2y+140}{200}$ = 1.46

= x + 2y + 140 = 292

= x + 2y = 292 – 140

= x + 2y = 152 (ii)

Subtract (i) from (ii), we get

X + 2y – x – y = 152 – 114

y = 38

Put the value of y in (i), we get

x = 114 – 38 = 76

No. of accidents	No. of days	Cumulative frequency
0	46	46
1	76	122
2	38	160
3	25	185
4	10	195
5	5	200
	N = 200	

We have, N = 200

$\frac{N}{2} = \frac{200}{2} = 100$

The cumulative frequency just more than $\frac{N}{2}$ is 122 so the median is 1

9. Question

An incomplete distribution is given below:

Variable:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	12	30	-	65	-	25	18

You are given that the median value is 46 and the total number of items is 230.

(i) Using the median formula fill up missing frequencies.

(ii) Calculate the AM of the completed distribution.

Answer

(i)

Class interval	Frequency	Cumulative frequency
10-20	12	12
20-30	30	42
30-40	X	42 + x (F)
40-50	65 (f)	107 + x
50-60	Y	107 + x + y
60-70	25	132 + x + y
70-80	18	150 + x + y
	N = 230	

Given, Median = 46

Then, median class = 40-50

Therefore, l = 40, h = 10, f = 65, F = 42 + x

Median = l + $\frac{\frac{N}{2} - F}{f}$

46 = 40 + $\frac{115-(42+x)}{65}$ * 10

$\frac{6*65}{10} = 73 - x$

39 = 73 – x

x = 73 – 39

x = 34

Given, N = 230

= 12 + 30 + 34 + 65 + y + 25 + 18 = 230

= 184 + y = 230

= y = 230 – 184 = 46

(ii)

Class interval	Mid – value	Frequency	fx
10-20	15	12	180
20-30	25	30	750
30-40	35	34	1190
40-50	45	65	2925
50-60	55	46	2530
60-70	65	25	1625
70-80	75	18	1350
		N = 230	$\sum$fx = 10550

Mean = $\frac{\sum fx}{N}$

= $\frac{10550}{230}$ = 45.87

10. Question

The following table gives the frequency distribution of married women by age at marriage

Age (in years)	Frequency	Age (in years)	Frequency
15-19	53	40-44	9
20-24	140	45-49	5
25-29	98	50-54	3
30-34	32	55-59	3
35-39	12	60 and above	2

Calculate the median and interpret the results.

Answer

Class interval (exclusive)	Class interval (inclusive)	Frequency	Cumulative frequency
15-19	14.5-19.5	53	53 (F)
20-24	19.5-24.5	140 (f)	193
25-29	24.5-29.5	98	291

30-34	29.5-34.5	32	593
35-39	34.5-39.5	12	335
40-44	39.5-44.5	9	344
45-49	44.5-49.5	5	349
50-54	49.5-54.5	3	352
54-59	54.5-59.5	3	355
60 and above	59.5 and above	2	357
		N = 357	

N = 357

$\frac{N}{2} = \frac{357}{2}$ = 178.5

The cumulative frequency just greater than $\frac{N}{2}$ is 193 then the median class is 19.5-24.5 such that:

l = 19.5, f = 140, F = 53, h = 5

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 19.5 + $\frac{178.5-53}{140}$ * 5 = 23.98

Nearly half the women were married between the age 15 and 25

11. Question

If the median of the following frequency distribution is 28.5 find the missing frequencies:

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	Total
Total Frequency:	5	f_1	20	15	f_2	5	60

Answer

Given: The frequency table.

To find: The missing frequencies f_1 and f_2 .

Solution: Construct a table to calculate cumulative frequencies,The table is shown below:

Class interval	Frequency	Cumulative frequency
0-10	5	5
10-20	f_1	$5 + f_1$ (F)
20-30	20 (f)	$25 + f_1$
30-40	15	$40 + f_1$
40-50	f_2	$40 + f_1 + f_2$
50-60	5	$45 + f_1 + f_2$
	N = 60	

Given, Median = 28.5

As it lies in the interval 20-30,

So, median class is 20-30

Now,

l, lower class = 20, frequency of median class, f = 20, Cumulative frequency of class preceding the median class, F = $5 + f_1$, height of class, h = 10N is 60,

we know,

$$Median = l + \left(\frac{\frac{N}{2} - F}{f} \right) \times h$$

$28.5 = 20 + \frac{30-(5+f1)}{20} * 10$

$28.5 - 20 = \frac{30-5-f1}{20} * 10$

$8.5 = \frac{25-f1}{2}$ $17 = 25 - f_1$

$f_1 = 25 - 17$

$f_1 = 8$

Given, sum of frequencies = 60

i.e. $5 + f_1 + 20 + 15 + f_2 + 5 = 60$ Put the value of f_1

therefore, $5 + 8 + 20 + 15 + f_2 + 5 = 60$

hence, $f_2 = 7$

Therefore, $f_1 = 8$ and $f_2 = 7$

12. Question

The median of the following data is 525. Find he missing frequency, if it is given that there are 100 observations in the data:

\

Class Interval	Frequency	Class interval	Frequency
0-100	2	500-600	20
100-200	5	600-700	f_2
200-300	f_1	700-800	9
300-400	12	800-900	7
400-500	17	900-1000	4

Answer

Class interval	Frequency	Cumulative frequency
0-100	2	2
100-200	5	7
200-300	f_1	$7 + f_1$
300-400	12	$19 + f_1$

400-500	17	$36 + f_1$ (F)
500-600	20 (f)	$56 + f_1$
600-700	f_2	$56 + f_1 + f_2$
700-800	9	$65 + f_1 + f_2$
800-900	7	$72 + f_1 + f_2$
900-1000	4	$76 + f_1 + f_2$
	N = 100	

Given, Median = 525

Then median class = 500-600

$l = 500$, $f = 20$, $F = 36 + f_1$, $h = 100$

Median = $l + \frac{\frac{N}{2} - F}{f}$

$525 = 500 + \frac{50-(36+f1)}{20} * 100$

$525 - 500 = \frac{50-36-f1}{20} * 100$

$25 = (14 - f_1)\ 5$

$5f_1 = 45$

$f_1 = 9$

Given, sum of frequencies = 100

$= 2 + 5 + f_1 + 12 + 17 + 20 + f_2 + 9 + 7 + 4 = 100$

$= 2 + 5 + 9 + 12 + 17 + 20 + f_2 + 9 + 7 + 4 = 100$

$=85 + f_2 = 100$

$f_2 = 15$

Therefore, $f_1 = 9$ and $f_2 = 15$

13. Question

If the median of the following data is 32.5, find the missing frequencies.

Class interval:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	Total
Frequency:	f_1	5	9	12	f_2	3	2	40

Answer

Class interval	Frequency	Cumulative frequency
0-10	f_1	f_1
10-20	5	$5 + f_1$
20-30	9	$14 + f_1$ (F)
30-40	12 (f)	$26 + f_1$
40-50	f_2	$26 + f_1 + f_2$
50-60	3	$29 + f_1 + f_2$
60-70	2	$31 + f_1 + f_2$
	N = 40	

Given, Median = 32.5

Then median class = 30-40

$l = 30$, $h = 10$, $f = 12$, $F = 14 + f_1$

Median = $l + \frac{\frac{N}{2} - F}{f}$

$32.5 = 30 + \frac{20-(14+f1)}{12} * 10$

$2.5 = \frac{6-f1}{6} * 5$

$15 = (6 - f_1)\ 5$

$3 = 6 - f_1$

$f_1 = 3$

Given, sum of frequencies = 40

$= 3 + 5 + 9 + 12 + f_2 + 3 + 2 = 40$

$= 34 + f_2 = 40$

$= f_2 = 6$

Therefore, $f_1 = 3$ and $f_2 = 6$

14. Question

Compute the median for each of the following data

(i) Marks	No. of students	(ii) Marks	No. of students
Less than 10	0	More than150	0
Less than 30	10	More than140	12
Less than 50	25	More than130	27
Less than 70	43	More than120	60
Less than 90	65	More than110	105
Less than 110	87	More than100	124
Less than 130	96	More than90	141
Less than 150	100	More than80	150

Answer

(i)

Marks	No. of students	Class interval	Frequency	Cumulative frequency
Less than 10	0	0-10	0	0
Less than 30	10	10-30	10	10
Less than 50	25	30-50	15	25
Less than 70	43	50-70	18	43 (F)
Less than 90	65	70-90	22 (f)	65
Less than 110	87	90-110	22	87
Less than 130	96	110-130	9	96
Less than 150	100	130-150	4	100
			N = 100	

We have, N = 100

$\frac{N}{2} = \frac{100}{2} = 50$

The cumulative frequency just greater than $\frac{N}{2}$ is 65 then median Class 70-90, such that:

l = 70, f = 22, F = 43, h = 20

Median = l + $\frac{\frac{N}{2} - F}{f} \times h$

= 70 + $\frac{50-43}{22} \times 20$

= 70 + $\frac{7*20}{22}$

= 70 + 6.36 = 76.36

(ii)

Marks	No. of students	Class interval	Frequency	Cumulative frequency
More than 80	150	80-90	9	9
More than 90	141	90-100	17	16
More than 100	124	100-110	19	45 (F)
More than 110	105	110-120	45 (f)	90
More than 120	60	120-130	33	123
More than 130	27	130-140	15	138
More than 140	12	140-150	12	150
More than 150	0	150-160	0	150
			N = 150	

We have, N = 150

$\frac{N}{2} = \frac{150}{2} = 7$

The cumulative frequency is just more than $\frac{N}{2}$ is 90 then, the Median Class is 110-120 such that:

l = 110, f = 45, F = 45, h = 10

Median = l + $\frac{\frac{N}{2} - F}{f} \times$ h

= 110 + $\frac{75-45}{45} \times 10$

= 110 + $\frac{30*10}{45}$

= 110 + 6.67

= 116.67

15. Question

A survey regarding the height (in cm) of 51 girls of class X of a school was conducted and the following data was obtained:

Height in cm	Number of Girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Find the median height.

Answer

To calculate the median height we need to find the class interval and their corresponding frequencies.

The given distribution being of the less than type 140, 145, 150,….165 give the upper limit of the corresponding class intervals. So, the classes should be below 140, 140-145, 145-150,….160-165. Observe that from the given distribution, we find that there are 4 girls with height less than 145 and 4 girls with height less than 140. Therefore, the number of girls with height in the interval 140-145 is 11 – 4 = 7

Similarly, the frequency of 145-150 is 29 – 11 = 19, for 150-155 it is 40 – 29 = 11 and so on so our frequency distribution table with the given cumulative frequency becomes:

Class interval	Frequencies	Cumulative frequency
Below 140	4	4
140-145	7	11
145-150	18	29
150-155	11	40
155-160	6	46
160-165	5	51

Now N = 51

So, $\frac{N}{2} = \frac{51}{2}$ = 25.5

This observation lies in the class 145 – 150

Then, l (lower limit) = 145

f = 11 and h = 5

Median = 145 + $\frac{25.5-11}{18}$ * 5

= 145 + 4.03

= 149.03

So, the median height of the girls is 149.03 cm. This means that the height of the about 50% of the girls is less than this height and 50% are taller than this height.

16. Question

A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are only given to persons having age 18 years onwards but less than 60 years.

Age in years	Number of policy holders
Blow 20	2
Blow 25	6
Blow 30	24
Blow 35	45
Blow 40	78
Blow 45	89
Blow 50	92
Blow 55	98
Blow 60	100

Answer

: Here class width is not same. There is no need to adjust the frequencies according to class intervals. Now given frequency table is of less than type represented with upper class limits. As policies were given only to persons having age 18 years onwards but less than 60 years we can define class intervals with their respective cumulative frequencies as below:

Age (in years)	No. of policy holders	Cumulative frequency
18-20	2	2
20-25	6 - 2 = 4	6
25-30	24 - 6 = 18	24
30-35	45 - 24 = 21	45
35-40	78 – 45 = 33	78
40-45	89 – 78 = 11	89
45-50	92 – 89 = 3	92
50-55	98 – 92 = 6	98
55-60	100 – 98 = 2	100

Now from the table we may observe that N = 100

Cumulative frequency just greater than $\frac{N}{2}$ (N = 50) is 78belonging to interval 35-40.

So, Median Class = 35 – 40

Lower limit (l) = 35

Class size (h) = 5

Frequency (f) = 33 and F = 45

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 35 + ($\frac{50-45}{33}$) * 5

= 35 + $\frac{5}{33}$ * 5

= 35 + 0.76

= 35.76

So, Median age is 35.76 years.

17. Question

The lengths of 40 leaves of a plant are measured correct to the nearest millimeter, and the data obtained is represented in the following table:

Length (in mm):	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No. of leaves:	3	5	9	12	5	4	2

Find the median life.

Answer

: The given data is not having continuous class intervals. So, we have to add and subtract 0.5 to upper class limit and lower class limit.

Length (in mm)	Number of leaves	Cumulative frequency
117.5-126.5	3	3
126.5-135.5	5	8 = 3 + 5
135.5-144.5	9	17 = 8 + 9
144.5-153.5	12	29 = 17 + 12
153.5-162.5	5	34 = 29 + 5
162.5-171.5	4	38 = 34 + 4
171.5-180.5	2	40 = 38 + 2

Median class = 144.5-153.5

l = 144.5, h = 9, f = 12 and F = 17

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 144.5 + ($\frac{20-17}{12}$ * 9

= 144.5 + $\frac{9}{4}$

= 146.75

So, median length is 146.75 mm.

18. Question

The following table gives the distribution of the life time of 400 neon lamps:

Light time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the median life.

Answer

We can find cumulative frequencies with their respective class intervals as below:

Life time	Number of lams	Cumulative frequency
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400
	N = 400	

Median class = 3000-3500

l = 3000, f = 86, F = 130, h = 500

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 3000 + $\frac{70*500}{86}$

= 3406.98 hours

So, median life time is 3406.98 hours.

19. Question

The distribution below gives the weight of 30 students in a class. Find the median weight of students:

Weight (in kg):	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No. of students:	2	3	8	6	6	3	2

Answer

We may find cumulative frequencies with their respective class intervals as below:

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
No. of students	2	3	8	6	6	3	2
Cumulative frequency	2	5	13	19	25	28	30

Median class = 55-60

l = 55, f = 6, F = 13 and h = 5

Median = l + $\frac{\frac{N}{2} - F}{f}$ * h

= 55 + $\left(\frac{15-13}{6}\right)$ * 5

= 55 + $\frac{10}{6}$

= 56.666

So, median weight is 56.67 kg.

Exercise 7.5

1. Question

Find the mode of the following data:

(i) 3, 5, 7, 4, 5, 3, 5, 6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(ii) 3, 3, 7, 4, 5, 3, 5,6, 8, 9, 5, 3, 5, 3, 6, 9, 7, 4

(iii) 15, 8, 26, 25, 24, 15, 18, 20, 24, 15, 19, 15

Answer

Mode is the value which occurs maximum number of times in a data.(i)

Value(x)	3	4	5	6	7	8	9
Frequency(f)	4	2	5	2	2	1	2

Mode = 5 (Since, its frequency is 5 which is maximum)

(ii)

Value(x)	3	4	5	6	7	8	9
Frequency(f)	5	2	4	2	2	1	2

Mode = 3 (Since, its frequency is 5 which is maximum)

(iii)

Value(x)	8	15	18	19	20	24	25	26
Frequency(f)	1	4	1	1	1	2	1	1

Mode = 15 (Since, its frequency is 4 which is maximum)

2. Question

The shirt sizes worn by a group of 200 persons, who bought the shirt from a store, are as follows:

Shirt size:	37	38	39	40	41	42	43	44
Number of persons:	15	25	39	41	36	17	15	12

Find the model shirt size worn by the group.

Answer

Shirt size	37	38	39	40	41	42	43	44
No. of persons	15	25	39	41	36	17	15	12

Model shirt size= 40 (Since, it occurs maximum number of times)

3. Question

Find the mode of the following distribution.

(i)

Class-interval:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	5	8	7	12	28	20	10	10

(ii)

Class-interval:	10-15	15-20	20-25	25-30	30-35	35-40
Frequency:	30	45	75	35	25	15

(iii)

Class-interval:	25-30	30-35	35-40	40-45	45-50	50-60
Frequency:	25	34	50	42	38	14

Answer

Mode = l + $\frac{f-f1}{2f-f1-f2}\times h$Where l = lower limit of the modal classh=width of the modal classf$_1$ = frequency of the class preceding the modal classf$_2$ = frequency of the class following the modal class

(i)

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

Here, the maximum frequency is 28 then the corresponding class 40-50 is the model class

l= 40, h= 50-40= 10, f= 28, f$_1$=12, f$_2$ =20

Mode = l + $\frac{f-f1}{2f-f1-f2}\times h$

Mode = 40 + $\frac{28-12}{2\times 28-12-20}\times 10$

Mode =40 + 6.67

Mode =46.67

(ii)

Class interval	10 -15	15 -20	20 -25	25 -30	30 -35	35 -40
Frequency	30	45	75	35	25	15

Here, the maximum frequency is 75, then the corresponding interval 20- 25 is modal class

l= 20, h=5, f=75, f$_1$=45, f$_2$ =35

Mode = l + $\frac{f-f1}{2f-f1-f2}\times h$

Mode = 20 + $\frac{75-45}{2\times 75-45-35}\times 5$

Mode =20 + 2.14

Mode =22.14

(iii)

Class interval	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

Here, the maximum frequency is 50, then the corresponding interval 35-40 is modal class l= 35, h=5, f=50, f_1=34, f_2 =42

Mode = l + $\frac{f-f1}{2f-f1-f2} \times h$

Mode = 35 + $\frac{50-34}{2\times 50-34-42} \times 5$

Mode =35 + 3.33

Mode =38.33

4. Question

Compare the modal ages of two groups of students appearing for an entrance test:

Age (in years):	16-18	1-20	20-22	22-24	24-26
Group A:	50	78	46	28	23
Group B:	54	89	40	25	17

Answer

For Group A:

Here, the maximum frequency is 78, the corresponding class interval 18 -20 is modal class

l=18, h=2, f=78, f_1=50, f_2 =46

Mode = l + $\frac{f-f1}{2f-f1-f2} \times h$

= 18 + $\frac{78-50}{2\times 78-50-46} \times 2$

=18+ $\frac{56}{60}$

=18 + 0.93 =18.93 years

For Group B:

Here, the maximum frequency is 89, the corresponding class interval 18 -20 is modal class

l=18, h=2, f=89, f_1=54, f_2 =40

Mode = l + $\frac{f-f1}{2f-f1-f2} \times h$

= 18 + $\frac{89-54}{2\times 89-54-40} \times 2$

=18+ $\frac{70}{84}$ =18 + 0.83 =18.33

Hence, the modal age of group A is higher than that of group B.

5. Question

The marks in science of 80 students of class X are given below: Find the mode of the marks obtained by the students in science.

Marks:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency:	3	5	16	12	13	20	5	4	1	1

Answer

To find: The mode of the marks obtained in science

Solution: Mode = l + $\frac{f-f1}{2f-f1-f2}\times h$

Where l = lower limit of the modal class

h = width of the modal class

f_1 = frequency of the class preceding modal class

f_2 = frequency of the class following modal class

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
frequency	3	5	16	12	13	20	5	4	1	1

Here, the maximum frequency is 20, the corresponding class interval 50-60 is modal class

l=50, h=10, f=20, f_1=13, f_2 =5

Mode = l + $\frac{f-f1}{2f-f1-f2}\times h$

Mode = 50 + $\frac{20-13}{2\times20-13-5}\times 10$

Mode =50+ $\frac{70}{22}$ =50 + 3.18

Mode =53.18

6. Question

The following is the distribution of height of students of certain class in a certain city:

Height (in cms):	160-162	163-165	166-168	169-171	172-174
No. of students:	15	118	142	127	18

Find the average height of maximum number of students.

Answer

Height(exclusive)	10-162	163-165	166-168	169-171	172-174
Height(inclusive)	159.5-162.5	162.5-165.5	165.5-168.5	168.5-171.5	171.5-174.5
No. of students	15	118	142	127	18

Here, the maximum frequency is 142, the corresponding class interval 165.5-168.5 is modal class

$l=165.5$, $h=3$, $f=142$, $f_1=118$, $f_2=127$

Mode = $l + \frac{f-f1}{2f-f1-f2} \times h$

$= 165.5 + \frac{142-118}{2\times142-118-127} \times 3$

$=18+ \frac{72}{39} =165.5 +1.85$

$=167.35$ cm

7. Question

The following table shows the ages of the patients admitted in a hospital during a year:

Age (in years):	5-15	15-25	25-35	35-45	45-55	55-65
No. of students:	6	11	21	23	14	5

Find the mode and the mean of the data given above. Compare and interpret the two measures of central tendency.

Answer

We may compute class marks (x_i) as per the relation

$X_i = \frac{upperclass\ limit + lowerclass\ limit}{2}$

Now, let assumed mean (A) = 30

Age(in years)	No. of patients(f_i)	Class marks (x_i)	$d_i = x_i - 30$	$f_i d_i$
5-15	6	10	-20	-120
15-25	11	20	-10	-110
25-35	21	30	0	0
35-45	23	40	10	230
45-55	14	50	20	180
55-65	5	60	30	150
Total	80			430

$\Sigma f_i = 80$, $\Sigma f_i d_i = 430$

Mean=$A + \frac{\Sigma f i d i}{\Sigma f i}$

$= 30 + \frac{430}{80} = 30 + 5.375$

$=35.38$

It represents that on an average the age of patients admitted was 35.38 years. As we can observe that the maximum class frequency 23 belonging to class interval 35-45.

So, modal class= 35-45

Lower limit (l) of modal class =35

Frequency (f_1) of the modal class=23

h=10,

Frequency (f_0)of class preceding the modal class=21

Frequency (f_2) of class succeeding the modal class =14

Now, $Mode = l + \left(\frac{f-f0}{2f-f0-f2}\right)h$

$= 35 + \left(\frac{23-21}{2(23)-21-14}\right)10$

=35 + 1.81 =36.8years

8. Question

The following data gives the information on the observed lifetimes (in hours) of 225 electrical components:

Lifetimes (in hours):	0-20	20-40	40-60	60-80	80-100	100-120
No. of components:	10	35	52	61	38	29

Determine the modal lifetimes of the components.

Answer

From the data as given above we may observe that maximum class frequency 61 belonging to the class interval 60 -80

So, modal class = 60-80

l=60, f_1=61, f_0=52, f_2=38, h=20

$$Mode = l + \left(\frac{f-f0}{2f-f0-f2}\right)h$$

$= 35 + \left(\frac{61-52}{2(61)-52-38}\right)20$

$=60 + \frac{90}{16} = 60 + 5.625$

=65.625hours

9. Question

The following data gives the distribution of total monthly household expenditure of 200 families of a village. Find the modal monthly expenditure of the families. Also, find the men monthly expenditure:

Expenditure (in Rs.)	Frequency
1000-1500	24
1500-2000	40
2000-2500	33
2500-3000	28
3000-3500	30
3500-4000	22
4000-4500	16
4500-5000	7

Answer

We may observe that the given data be maximum class frequency is 40 belonging to 1500- 2000 intervals

So, modal class = 1500- 2000

$l=1500$, $f=40$, $f_0=24$, $f_2=33$, $h=50$

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right) h$$

$$= 1500 + \left(\frac{40-24}{2(40)-24-33}\right) 50$$

=1500 + 347.826

=1847.826

So, modal class monthly expenditure was Rs. 1847.83

We may compute class marks (x_i) as per the relation:

$X_i = \frac{upperclass\ limit + lowerclass\ limit}{2}$

h= 500, A=2750

Expenditure(in Rs)	N0.of families(f_i)	x_i	d_i=x_i -2750	u_i	f_iu_i
1000-1500	24	1250	-1500	-3	-72
1500-2000	40	1750	-1000	-2	-80
2000-2500	33	2250	-500	-1	-33
2500-3000	28	2750	0	0	0
3000-3500	30	3250	500	1	30
3500-4000	22	3750	1000	2	44
4000-4500	16	4250	1500	3	48
4500-5000	7	4750	2000	4	28
Total	200				-35

Σx_i=200, $\sum fiui$

($\bar{x}$) mean = $A + \frac{\sum fiui}{\sum fi} \times h$

($\bar{x}$) = $2750 + \frac{-35}{200} \times 500$

=2750-87.5

=2662.5

So, mean monthly expenditure was Rs. 2662.50

10. Question

The following distribution gives the state-wise teacher-student ratio in higher secondary schools of India. Find the mode and mean of this data. Interpret, the two measures:

Number of students Per Teacher	Number of States/U.T.
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

Answer

To find: The mean and mode of the given table.

Solution:

Number of students Per Teacher	Number of States/U.T.
15-20	3
20-25	8
25-30	9
30-35	10
35-40	3
40-45	0
45-50	0
50-55	2

Mode is calculated as:

$$Mode = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

Where

l=lower limit of modal class

h=width of the modal class

f=frequency of the modal class

f_1=frequency of the class preceding the modal class

f_2= frequency of the class following the modal class

Since, the maximum class frequency is 10

Hence, modal class interval= 30-35

h= 5, l=30, f= 10,f_1=9 and f_2=3

$$\Rightarrow Mode = 30 + \frac{10-9}{2(10)-9-3} \times 5$$

$$\Rightarrow Mode = 30 + \frac{1}{20-9-3} \times 5$$

$$\Rightarrow Mode = 30 + \frac{5}{8}$$

⇒ Mode = 30 + 0.625⇒ Mode = 30.625

Now to find mean,

Use the formula, $Mean = A + h\left\{\frac{1}{N}\sum f_i u_i\right\}$

Where

A = assumed mean

$d_i = x_i - A$

h=length of class intervals

$u_i = (x_i - A)/h$

N=sum of all frequencies

Number of students Per Teacher	Mid values (x_i)	Number of States/U.T. (f_i)	$d_i = x_i - 32.5$	$u_i = (x_i - 32.5)/5$	$f_i u_i$
15-20	17.5	3	-15	-3	-9
20-25	22.5	8	-10	-2	-16
25-30	27.5	9	-5	-1	-9
30-35	32.5	10	0	0	0
35-40	37.5	3	5	1	3
40-45	42.5	0	10	2	0
45-50	47.5	0	15	3	0
50-55	52.5	2	20	4	8
		$\sum f_i=35$			$\sum f_i u_i=-23$

Here A = 32.5h = 5N = 35So, $Mean = 32.5 + 5\left\{\frac{-23}{35}\right\}$

$$Mean = 32.5 + \left\{\frac{-23}{7}\right\}$$

⇒ Mean = 32.5-3.28⇒ Mean = 29.21

11. Question

The given distribution shows the number of runs scored by some top batsmen of the world in one-day international cricket matches.

Runs Scored	Number of batsman
3000-4000	4
4000-5000	18
5000-6000	9
6000-7000	7
7000-8000	6
8000-9000	3
9000-10000	1
10000-10000	1

Find the mode of the data.

Answer

From the given data we may observe that maximum class frequency is 18 belonging to the class interval 4000-5000

So, modal class= 4000-5000

Lower limit, l= 4000

f_0=4, f_2=9, f=18,h = 1000

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right)h$$

$= 4000 + \left(\frac{18-4}{2(18)-4-9}\right)1000$

$=4000 + \frac{14000}{23} = 4608.7$ runs

12. Question

A student noted the number of cars passing through a spot on a road for 100 periods each of 3 minutes and summarized it in the table given below. Find the mode of the data:

Number of cars:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	7	14	13	12	20	11	15	8

Answer

From the given data we may observe that maximum class frequency is 20 belonging to the class interval 40-50

So, modal class= 40-50

Lower limit, l= 40

f_0=12, f_2=11, f=20,h = 10

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right)h$$

$= 40 + \left(\frac{20-12}{40-12-11}\right)10$

$=40 + \frac{80}{17}$

=40 + 4.7 =44.7

13. Question

The following frequency distribution gives the monthly consumption of electricity of the consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthly consumption (in units):	65-85	85-105	105-125	125-145	145-165	165-185	185-205
No. of consumers:	4	5	13	20	14	8	4

Answer

Class interval	Mid value (x)	Frequency(f)	fx	Cumulative frequency
65-85	75	4	300	4
85-105	95	5	475	9
105-125	115	13	1495	22
125-145	135	20	2700	42
145-165	155	14	2170	56
165-185	175	8	1400	64
185-205	195	4	780	68
total		N=68	∑fx=9320	

Mean= $\frac{\sum fx}{N} = \frac{9320}{68} = 137.05$

We have, N= 68,

N/2 = 34

Hence, medium class =125- 145, such that

l=125, f′=20, f=22, h=20

Median = l + $\frac{\frac{n}{2}-f}{f'} \times h = 125 + \frac{34-22}{20} \times 20 = 137$

Here, we may observe that maximum class frequency is 20 belonging to the class interval 125-145

So, modal class= 125-145

Lower limit, l= 125

f_0=13, f_2=14, f=20,h = 20

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right)h$$

$$= 125 + \left(\frac{20-13}{40-13-14}\right)20 = 125 + \frac{140}{13} = 135.77$$

14. Question

100 surnames were randomly picked up from a local telephone directly and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows:

Numbers of letters:	1-4	4-7	7-10	10-13	13-16	16-19
Number surnames:	6	30	40	16	4	4

Determine the median number of letters in the surnames. Find the mean number of letters in the surnames. Also, find the modal size of the surnames.

Answer

Class interval	Mid value (x)	Frequency (f)	fx	Cumulative frequency
1-4	2.5	6	15	6
4-7	5.5	30	165	36
7-10	8.5	40	340	76
10-13	11.5	16	185	92
13-16	14.5	4	58	96
16-19	17.5	4	70	100
Total		N=100		

Mean= $\frac{\sum fx}{N} = \frac{832}{100} = 8.32$

We have, N= 100,

N/2 = 50

Hence, median class =7-10, such that

l=7, f'=40, f=36, h=3

Median = l + $\frac{\frac{n}{2}-f}{f'} \times h = 7 + \frac{50-36}{40} \times 3 = 8.05$

Here, we may observe that maximum class frequency is 40 belonging to the class interval 7-10

So, modal class= 7-10

Lower limit, l= 7

f_0=30, f_2=16, f=40,h = 3

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right)h$$

$$= 7 + \left(\frac{40-30}{2(40)-30-16}\right)3 = 7 + \frac{30}{34} = 7.88$$

15. Question

Find the mean, median and mode of the following data:

Classes:	0-20	20-40	40-60	60-80	80-100	100-120	120-140
Frequency:	6	8	10	12	6	5	3

Answer

Class interval	Mid value(x)	Frequency(f)	fx	Cummulative frequency
0-20	10	6	60	6
20-40	30	8	240	14
40-60	50	10	500	24
60-80	70	12	840	36
80-100	90	6	540	42
100-120	110	5	550	47
120-140	130	3	390	50
Total		N=50		

Mean= $\frac{\sum fx}{N} = \frac{320}{50} = 62.4$

We have, N= 50,

N/2 = 25

Hence, median class =60-80, such that

l=60, f'=12, f=24, h=20

Median = l + $\frac{\frac{n}{2}-f}{f'} \times h = 60 + \frac{25-24}{12} \times 20 = 60 + 1.67 = 61.67$

Here, we may observe that maximum class frequency is 12 belonging to the class interval 60-80

So, modal class= 60-80

Lower limit, l= 60

f_0=10, f_2=6, f=12,h = 20

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right) h$$

$$= 60 + \left(\frac{12-10}{24-10-6}\right) 20 = 60 + \frac{40}{8} = 65$$

16. Question

Find the mean, median and mode of the following data:

Classes:	0-50	50-100	100-150	150-200	200-250	250-300	300-350
Frequency:	2	3	5	6	5	3	1

Answer

Class interval	Mid value(x)	Frequency(f)	fx	Cumulative frequency
0-50	25	2	50	2
50-100	75	3	225	5
100-150	125	5	625	10
150-200	175	6	1050	16
200-250	225	5	1125	21
250-300	275	3	825	24
300-350	325	1	325	25
Total		N=25		

Mean= $\frac{\sum fx}{N} = \frac{4225}{25} = 169$

We have, N= 25,

N/2 = 12.5

Hence, median class =150-200, such that

l=150, f'=6, f=10, h=50

Median = l + $\frac{\frac{n}{2}-f}{f'} \times h = 150 + \frac{12.5-10}{6} \times 50 = 170.83$

Here, we may observe that maximum class frequency is 6 belonging to the class interval 150-200

So, modal class= 150-200

Lower limit, l= 150

f_0=5, f_2=5, f=6, h = 50

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right) h$$

$$= 150 + \left(\frac{6-5}{12-5-5}\right) 50 = 150 + \frac{1}{2} 50 = 175$$

17. Question

The following table gives the daily income of 50 workers of a factory:

Daily income (in Rs):	100-120	120-140	140-160	160-180	180-200
Number of workers:	12	14	8	6	10

Answer

Class interval	Mid value(x)	Frequency(f)	fx	Cumulative frequency
100-120	110	12	1320	12
120-140	130	14	1820	26
140-160	150	8	1200	34
160-180	170	6	1000	40
180-200	190	10	1900	50
Total		N=50		

Mean= $\frac{\sum \text{fx}}{N} = \frac{7260}{50} = 145.2$

We have, N= 50,

N/2 = 25

Hence, medium class =120-140, such that

l=120, f'=14, f=12, h=20

Median = l + $\frac{\frac{n}{2}-f}{f'} \times h = 120 + \frac{25-12}{14} \times 20 = 138.57$

Here, we may observe that maximum class frequency is 14 belonging to the class interval 120-140

So, modal class= 120-140

Lower limit, l= 120

f_0=12, f_2=8, f=14,h = 20

$$Mode = l + \left(\frac{f - f0}{2f - f0 - f2}\right) h$$

$= 120 + \left(\frac{14-12}{28-12-8}\right) 20 = 120 + \frac{25}{5} = 125$

Exercise 7.6

1. Question

Draw an ogive by less than method for the following data:

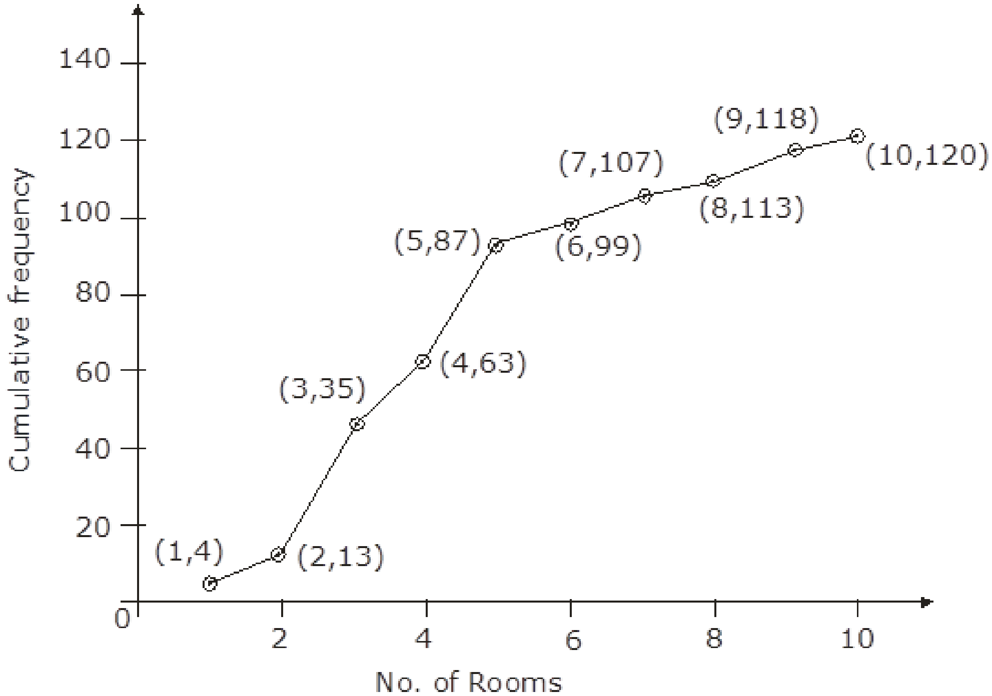
No. of rooms:	1	2	3	4	5	6	7	8	9	10
No. of houses:	4	9	22	28	24	12	8	6	5	2

Answer

We first prepare the cumulative frequency distribution by less than method as given below:

No. of rooms	No. of houses	Cumulative frequency
Less than or equal to 1	4	4
Less than or equal to 2	9	13
Less than or equal to 3	22	35
Less than or equal to 4	28	63
Less than or equal to 5	24	87
Less than or equal to 6	12	99
Less than or equal to 7	8	107
Less than or equal to 8	6	113
Less than or equal to 9	5	118
Less than or equal to 10	2	120

Now we mark the upper class limits along x-axis and cumulative frequency along y-axis. Thus, we plot the points (1,4); (2,13); (3,35); (4,63); (5,87); (6,99); (7,107); (8,113); (9,118); (10,120)



2. Question

The marks scored by 750 students in an examination are given in the form of a frequency distribution table:

Marks	No. of Students
600-400	16
640-680	45
680-720	156
720-760	284
760-800	172
800-840	59
840-880	18

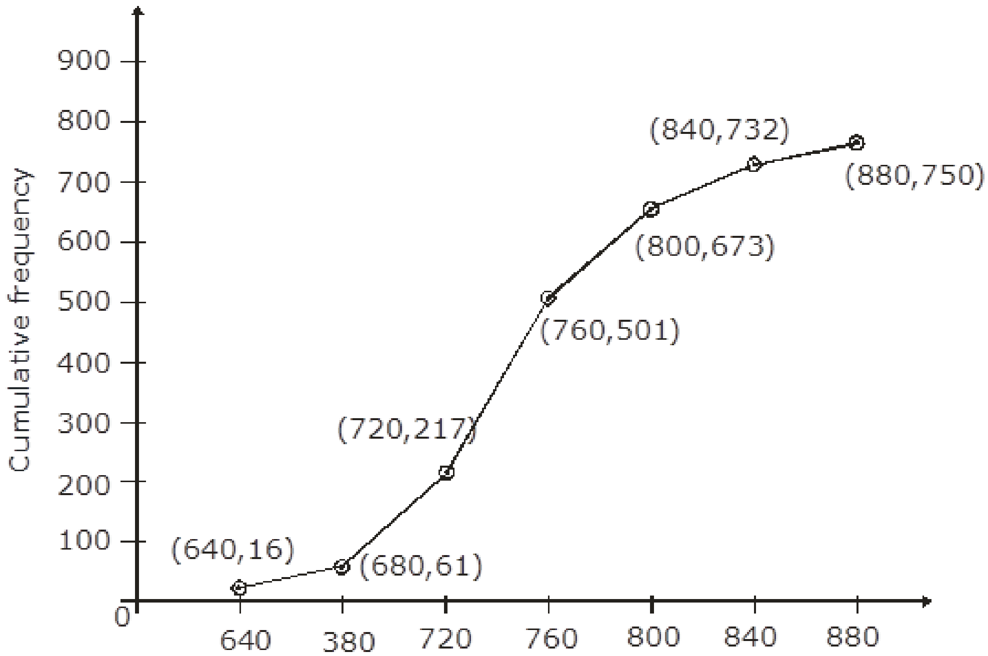
Prepare a cumulative frequency table by less than method and draw an ogive.

Answer

We first prepare the cumulative frequency distribution by less than method as given below:

Marks	No. of students	Marks less than	Cumulative frequency
600-640	16	640	16
640-680	45	680	61
680-720	156	720	217
720-760	284	760	501
760-800	172	800	673
800-840	59	840	732
840-880	18	880	750

Now we mark the upper class limits along x-axis and cumulative frequency along y-axis. Thus, we plot the points: (640,16); (680,61); (720,217); (760,501); (800,673); (840,732); (880,750)



3. Question

Draw an ogive to represent the following frequency distribution:

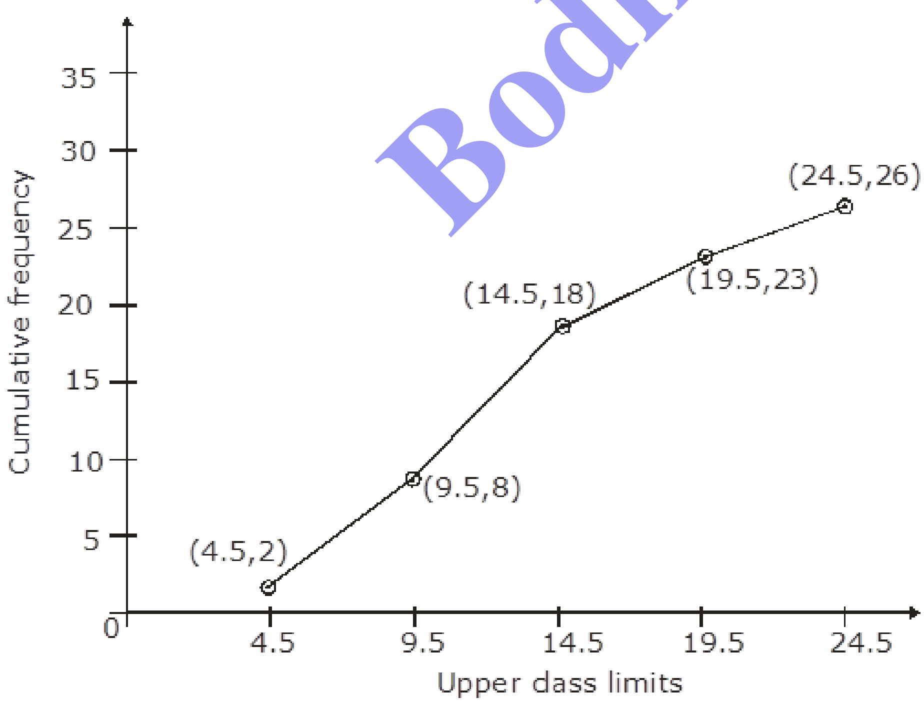
Class-interval:	0-4	5-9	10-14	15-19	20-24
No. of students:	2	6	10	5	3

Answer

The given frequency of distribution is not continuous. So, we first make it continuous and prepare cumulative frequency distribution as under:

Class interval	No. of students	Less than	Cumulative frequency
0.5-4.5	2	4.5	2
4.5-9.5	6	9.5	8
9.5-14.5	10	14.5	18
14.5-19.5	5	19.5	23
19.5-24.5	3	24.5	26

Now we mark the upper class limits along x-axis and cumulative frequency along y-axis. Thus, we plot the points: (4,5,2); (9,5,8); (14,5,18); (19,5,23); (24,5,26)



4. Question

The monthly profits (in Rs.) of 100 shops are distributed as follows:

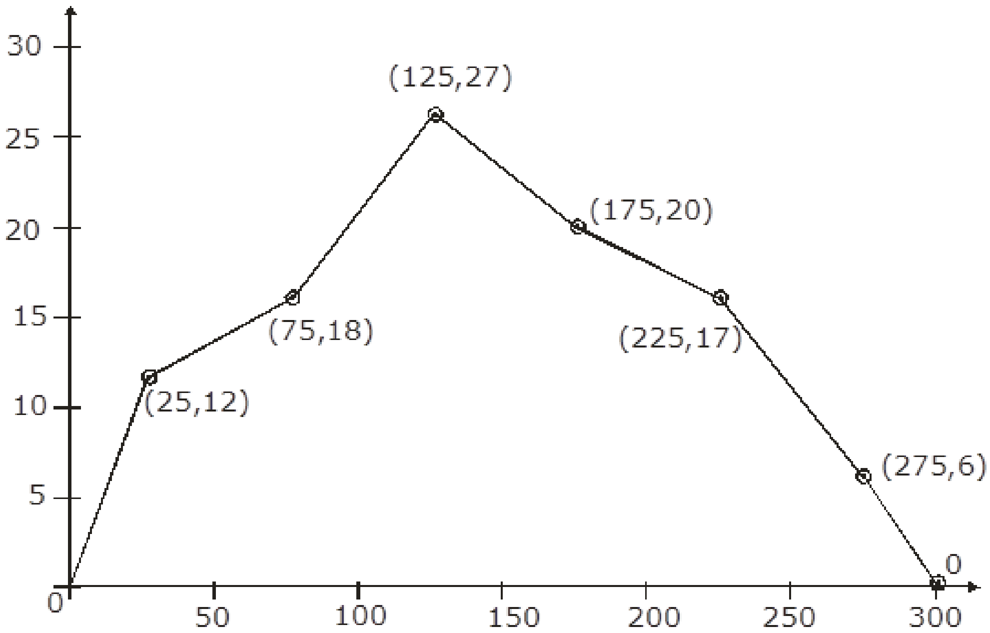
Profits per shop:	0-50	50-100	100-150	150-200	200-250	250-300
No. of shops:	12	18	27	20	17	6

Draw the frequency polygon for it.

Answer

We have,

Profit per shop	Mid value	No. of shops
Less than 0	0	0
0-50	25	12
50-100	75	18
100-150	125	27
150-200	175	20
200-250	225	17
250-300	275	6
Above 300	300	0



5. Question

The following table gives the height of trees:

Height	No. of trees
Less than 7	26
Less than 14	57
Less than 21	92
Less than 28	134
Less than 35	216
Less than 42	287
Less than 49	341
Less than 56	360

Draw ‘less than’ ogive and ‘more than’ ogive.

Answer

Less than method, it is given that:

Height	No. of trees
Less than 7	26
Less than 14	57
Less than 21	92
Less than 28	134
Less than 35	216
Less than 42	287
Less than 49	341
Less than 56	360

Now we mark the upper class limits along x-axis and cumulative frequency along y-axis. Thus, we plot the points: (7,26); (14,57); (21,92); (28,134); (35,216); (42,287); (49,341); (56,360)

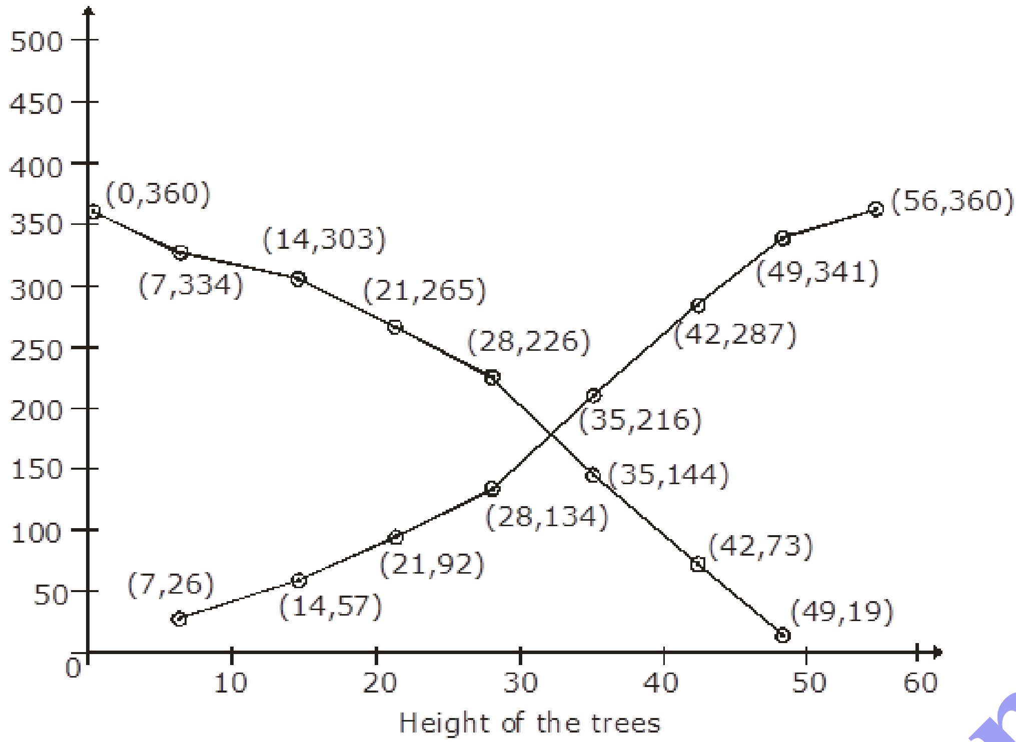
More than method: We will prepare the cumulative frequency table by more than method as given below:

Height	Frequency	Height more than	Cumulative frequency
0-7	26	0	360
7-14	31	7	334
14-21	35	14	303
21-28	42	21	263
28-35	82	28	226
35-42	71	35	144
42-49	54	42	73
49-56	19	49	19

Now we mark,

On x-axis lower class limit and on y-axis Cumulative frequency

Thus, we plot graph as (0,360); (7,334); (14,303); (21,263); (28,226); (35,144); (42,73); (49,19)



6. Question

The annual profits earned by 30 shops of a shopping complex in a locality give rise to the following distribution:

Profit (in lakhs in Rs.)	Number of shops (frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Draw both ogives for the above data and hence obtain the median.

Answer

More than method:

Profit (in lakhs in Rs.)	No. of shops (Frequency)
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3

Now we mark,

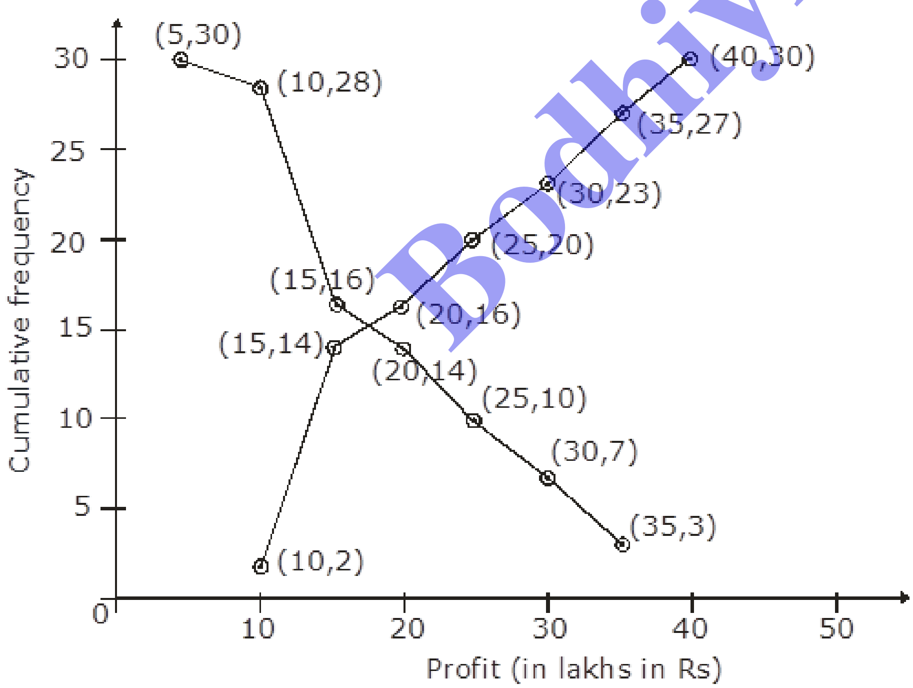
On x-axis lower class limit and on y-axis Cumulative frequency

Thus, we plot graph as: (5,30); (10,28); (15,16); (20,14); (25,10); (30,7); (35,3)

Less than method:

Profit (in lakhs in Rs.)	No. of shops (Frequency)	Profit in less than	Cumulative frequency
0-10	2	10	2
10-15	12	15	14
15-20	2	20	16
20-25	4	25	20
25-30	3	30	23
30-35	4	35	27
35-40	3	40	30

Now we mark the upper class limit on x-axis and the cumulative frequency on y-axis. Thus, we plot the points: (10,2); (15,14); (20,16); (25,20); (30,23); (35,27); (40,30)



7. Question

The following distribution gives the daily income of 50 workers of a factory:

Daily income (in Rs.):	100-120	120-140	140-160	160-180	180-200
Number of workers:	12	14	8	6	10

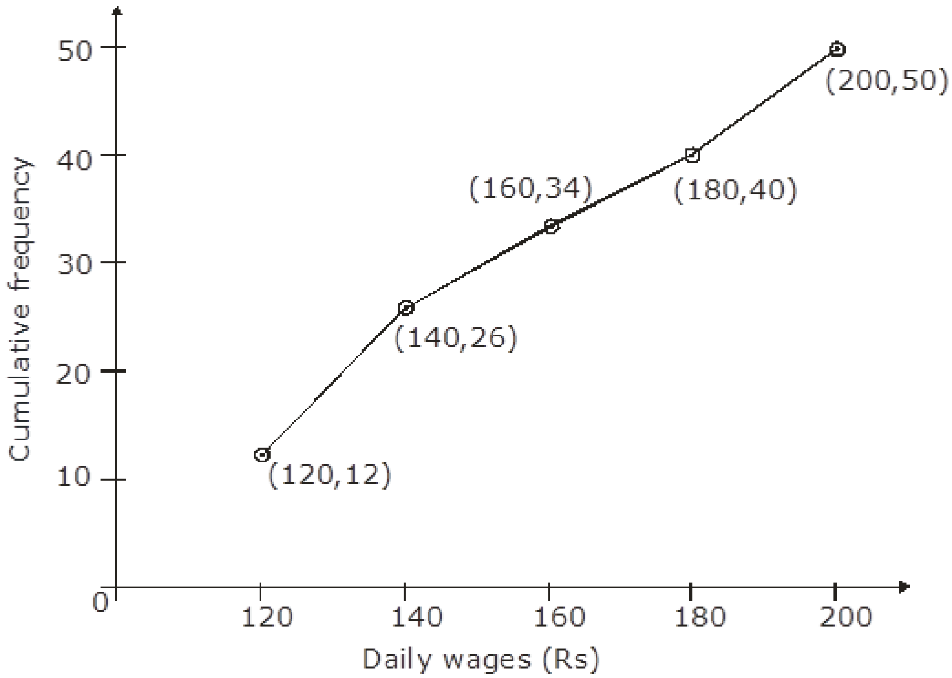
Convert the above distribution to a less than type cumulative frequency distribution and draw its ogive.

Answer

We first prepare cumulative frequency table by less than method as given below:

Daily income (in Rs.)	Cumulative frequency
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

Now we mark on x-axis upper class limit and on y-axis cumulative frequency. Thus, we plot the points: (120, 12); (140,26); (160,34); (180,40); (200,50)



8. Question

The following table gives production yield per hectare of wheat of 100 farms of a village:

Number of farms:	50-55	55-60	60-65	65-70	70-75	75-80 in kg per hectare
Number of farms:	2	8	12	24	38	16

Draw 'less than' ogive and 'more than' ogive.

Answer

Production yield kg per hectare	Cumulative frequency
More than or equal to 50	100
More than or equal to 55	98
More than or equal to 60	90
More than or equal to 65	78
More than or equal to 70	54
More than or equal to 75	16

Less than method:

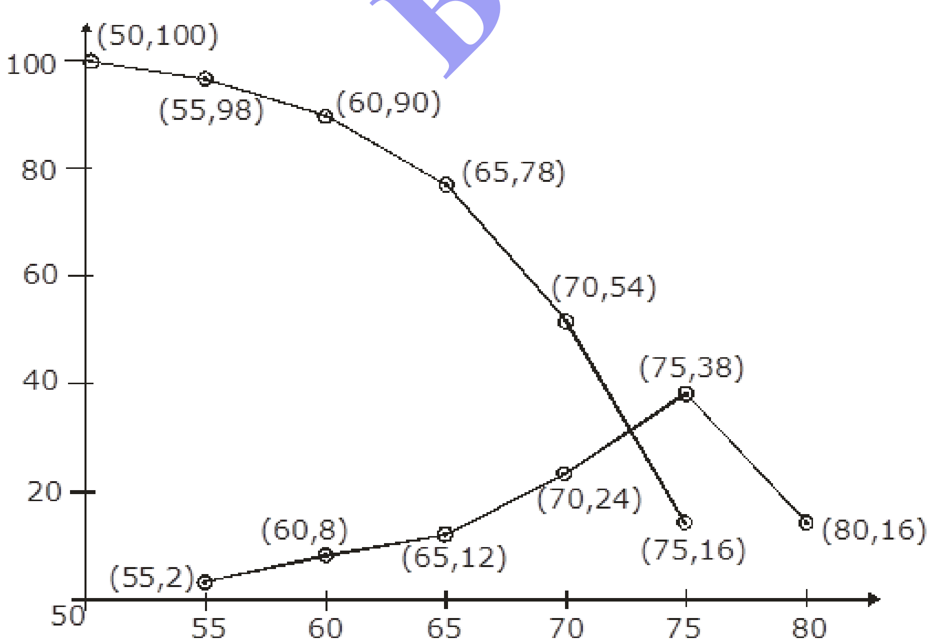
Production yield (in kg/hectare)	No. of farms	Less than	Cumulative frequency
50-55	2	55	2
55-60	8	60	10
60-65	12	65	22
65-70	24	70	46
70-75	38	75	84
75-80	16	80	100

Now on x-axis upper class limits and on y-axis cumulative frequency, we plot the points: (55,2); (60,10) ; (65,22); (70,46); (75,84); (80,100)

More than method:

Production yield (in kg/hectare)	No. of farms	More than	Cumulative frequency
50-55	2	50	100
55-60	8	55	98
60-65	12	60	90
65-70	24	65	78
70-75	38	70	54
75-80	16	75	16

Now, Mark on x-axis lower class limit and on y-axis cumulative frequency. We plot the points: (50,100); (55,98); (60,90); (65,78); (70,54); (75,16)



9. Question

During the medical check-up of 35 students of a class, their weights were recorded as follows:

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence, obtain the median weight from the graph and verify the result by using the formula.

Answer

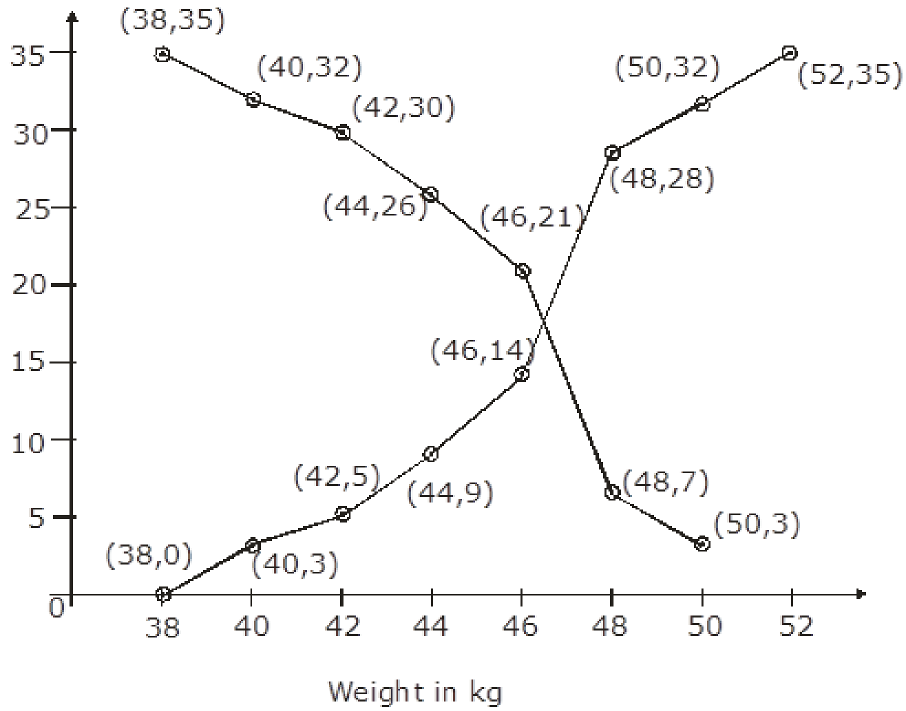
Less than method:

It is given that on x-axis upper class limit and on y-axis cumulative frequency. We plot the points: (38,0); (40,3); (42,5); (49,9); (46,14); (48,28); (50,32); (52,35)

More than method:

Weight (in kg)	No. of students	More than	Cumulative frequency
38-40	3	38	35
40-42	2	40	32
42-44	4	42	30
44-46	5	44	26
46-48	14	46	21
48-50	4	48	7
50-52	3	50	3

X -axis lower class limit and y-axis cumulative frequency, we plot the points: (38,35); (40,32); (42,30); (44,26); (46,21); (48,7); (50,3)



We find the two types of cumulative frequency curves intersect at point P.

The value of M is 46.5 kg

Verification,

We have

Weight (in kg)	No. of students	Cumulative frequency
36-38	0	0
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Now, N = 35

Therefore, $\frac{N}{2} = \frac{35}{2} = 17.5$

The cumulative frequency is just greater than $\frac{N}{2}$ is 28 and the corresponding classes 46-48

Thus, 46-48 is the median class such that,

$l = 46$, $f = 14$, $C_1 = 14$ and $h = 2$

Median = l + $\frac{\frac{N}{2}-C1}{f}$ * h

= 46 + $\frac{17.5-14}{14}$ * 2

= 46 + $\frac{7}{14}$ = 46 + 0.5

= 46.5 kg

Hence, verified.

CCE - Formative Assessment

1. Question

Define mean.

Answer

The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

If $x_1, x_2, \ldots, x_n$ are observations with frequencies $f_1, f_2, \ldots, f_n$ i.e. x_1 occurs f_1 times and x_2 occurs f_2 times and so on, then we have

Sum of the values of the observations = $f_1x_1 + f_2x_2 + \ldots + f_nx_n$

and Number of observations = $f_1 + f_2 + \ldots + f_n$

So, mean($\bar{x}$) of observations is given by

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n}$$

Or,

In summation form, it can be shorted to

$$\bar{x} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

Which also can also be written as,

$$\bar{x} = \frac{\sum f_ix_i}{\sum f_i}$$

And it is understood i varies from 1 to n.

2. Question

What is the algebraic sum o5f deviations of a frequency distribution about its mean?

Answer

Suppose $x_1, x_2, \ldots, x_n$ are n observations with mean as x.

By definition of mean, [i.e. The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations]

We have,

$x = \frac{x_1 + x_2 + \cdots + x_n}{n}$ and

$nx = x_1 + x_2 + \ldots + x_n$ ...[1]

So, in this case we have assumed mean(a) is equal to mean of the observations(x)

And we know that

$d_i = x_i - a$

where, d_i is deviation of a (i.e. assumed mean) from each of x_i i.e. observations.

So, In the above case we have

$d_1 = x_1 - x$

$d_2 = x_2 - x$

.

.

.

$d_n = x_n - x$

and sum of deviations

$d_1 + d_2 + \ldots + d_n = x_1 - x + x_2 - x + \ldots + x_n - x$

$= x_1 + x_2 + \ldots + x_n - (x + x + \ldots$ {upto n times})

$= nx - nx$ [Using 1]

$= 0$

Hence, sum of deviations is zero.

3. Question

Which measure of central tendency is given by the x-coordinate of the point of intersection of the 'more than' ogive and 'less than' ogive?

Answer

Median

As we know that, the x-coordinate of the point of intersection of the more than ogive and less than ogive give us median of the data.

4. Question

What is the value of the median of the data using the graph in the following figure of less than ogive and more than ogive?

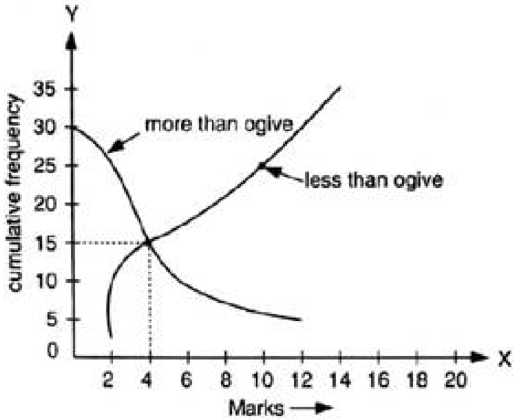


Fig. 7.11

Answer

4

As we know that, the x-coordinate of the point of intersection of the more than ogive and less than ogive give us median of the data.

5. Question

Write the empirical relation between mean, mode and median.

Answer

We know that,

Mode = 3 Median – 2 Mean

6. Question

Which measure of central tendency can be determined graphically?

Answer

As we know that, the x-coordinate of the point of intersection of the more than ogive and less than ogive give us median of the data.

So, median can be determined graphically.

7. Question

Write the modal class for the following frequency distribution:

Class-interval:	10-15	15-20	20-25	25-30	30-35	35-40
Frequency:	30	35	75	40	30	15

Answer

As class of maximum frequency is called modal class.

Modal class in above case is 20-25 as 75 is maximum frequency.

8. Question

A student draws a cumulative frequency curve for the marks obtained by 40 students of a class as shown below. Find the median marks obtained by the students of the class.

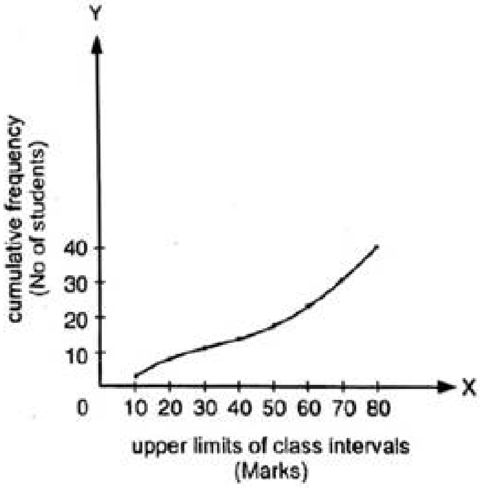


Fig. 7.12

Answer

We know that, For finding median from a less than ogive or more than ogive curve, we follow below steps.

1. we find the sum of all frequencies or the last cumulative frequency in our given data, let that be N

2. Then we calculate $\frac{N}{2}$ and locate the point corresponding to $\frac{N}{2}th$ on the curve.

3. The X coordinate of the point located i.e. the class corresponding to $\frac{N}{2}th$ cumulative frequency is the median of data.

From the graph, we locate last cumulative frequency as 40 i.e. sum of all the frequencies is 40.

i.e. N = 40 and $\frac{N}{2} = 20$

Median is the marks corresponding to $\frac{N}{2}th$ student.

In order to find the median, we first locate the point corresponding to 20^{th} student on Y axis.

And from graph, that point is (50, 20)

So, marks corresponding to 20^{th} student is 50.

So, the median of above data is 50

9. Question

Write the median class for the following frequency distribution:

Class-interval:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	5	8	7	12	28	20	10	10

Answer

First, we prepare the cumulative frequency table for above data

Class	Frequency	Cumulative Frequency
0-10 10-20 20-30 30-40 40-50 50-60 60-70 70-80	5 8 7 12 28 20 10 10	5 13 20 32 60 80 90 100
	N = 100	

We know that median class of a data is the class-interval corresponding to cumulative frequency just greater than $\frac{N}{2}$

Where,

N = sum of all frequencies

As N = 100 therefore $\frac{N}{2} = 50$

And

Cumulative frequency just greater than 50 is 60 which lies corresponding to class 40-50

Hence, 40-50 is median class.

10. Question

In the graphical representation of a frequency distribution, if the distance between mode and mean is k times the distance between median and mean, then write the value of k.

Answer

Distance between mode and mean = mode - mean

Distance between median and mean = median - mean

Given that,

(mode - mean) = k(median - mean)

⇒ mode - mean = k median - k mean

⇒ mode = k median - k mean + mean

⇒ mode = k median - (k-1) mean

Comparing it with empirical relation, i.e.

mode = 3 Median - 2 mode

We get,

k = 3

11. Question

Find the class marks of classes 10-25 and 35-55

Answer

We know, class marks of a class interval is

$$= \frac{1}{2}(\text{lower limit} + \text{upper limit})$$

For 10-25

Lower limit = 10

Upper limit = 25

Class mark $= \frac{1}{2}(10 + 25) = 17.5$

For 35-55

Lower limit = 35

Upper limit = 55

Class mark $= \frac{1}{2}(35+55) = 45$

12. Question

Write the median class of the following distribution:

Classes:	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency:	4	4	8	10	12	8	4

Answer

First, we prepare the cumulative frequency table for above data

Class	Frequency	Cumulative Frequency
0-10	4	4
10-20	4	8
20-30	8	16
30-40	10	26
40-50	12	38
50-60	8	46
60-70	4	50
	N = 50-	

We know that median class of a data is the class-interval corresponding to cumulative frequency just greater than $\frac{N}{2}$

Where,

N = sum of all frequencies

As N = 25 therefore $\frac{N}{2} = 25$

And

Cumulative frequency just greater than 25 is 26 which lies corresponding to class 30-40

Hence, 30-40 is median class.

1. Question

Which of the following is not a measure of central tendency?

A. Mean

B. Median

C. Mode

D. Standard deviation

Answer

There are three measures of central tendency

1) Mean 2) Median 3) Mode

2. Question

The algebraic sum of the deviations of a frequency distribution from its mean is

A. always positive

B. always negative

C. 0

D. a non-zero number

Answer

Suppose x_1, x_2, … , x_n are n observations with mean as x.

By definition of mean, [i.e. The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations]

We have,

$x = \frac{x_1 + x_2 + \cdots + x_n}{n}$ and

$nx = x_1 + x_2 + \ldots + x_n$ …[1]

So, in this case we have assumed mean(a) is equal to mean of the observations(x)

And we know that

$d_i = x_i - a$

where, d_i is deviation of a (i.e. assumed mean) from each of x_i i.e. observations.

So, In the above case we have

$d_1 = x_1 - x$

$d_2 = x_2 - x$

.

.

.

$d_n = x_n - x$

and sum of deviations

$d_1 + d_2 + \ldots + d_n = x_1 - x + x_2 - x + \ldots + x_n - x$

$= x_1 + x_2 + \ldots + x_n - (x + x + \ldots$ {upto n times})

= nx - nx [Using 1]

= 0

Hence, sum of deviations is zero.

3. Question

The arithmetic mean of 1, 2, 3, ..., n is

A. $\frac{n+1}{2}$

B. $\frac{n-1}{2}$

C. $\frac{n}{2}$

D. $\frac{n}{2}+1$

Answer

We know that mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

and, we have given series

1, 2, 3, …, n

Clearly the above series is an AP(Arithmetic progression) with

first term, a = 1 and

common difference, d = 1

And no of terms is clearly n.

And last term is also n.

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2} \ldots[1]$$

As,

$$\text{Mean} = \frac{\text{Sum of all terms}}{\text{no of terms}} = \frac{1+2+3+\cdots+n}{n}$$

$$\Rightarrow \text{Mean} = \frac{\left(\frac{n(n+1)}{2}\right)}{n} = \frac{n+1}{2}$$

4. Question

For a frequency distribution, mean, median and mode are connected by the relation

A. Mode = 3 Mean − 2 Median

B. Mode = 2 Median − 3 Mean

C. Mode = 3 Median − 2 Mean

D. Mode = 3 Median + 2 Mean

Answer

We know that empirical relation between mean, median and mode is

Mode = 3 Median - 2 Mean

5. Question

Which of the following cannot be determined graphically?

A. Mean

B. Median

C. Mode

D. None of these

Answer

Median can be find graphically by drawing any of the ogive or both ogives.

And Mode can be find graphically by drawing histogram of the given data.

But mean can't be determined graphically.

6. Question

The median of a given frequency distribution is found graphically with the help of

A. Histogram

B. Frequency curve

C. Frequency polygon

D. Ogive

Answer

There are two ways in which median can be determined graphically.

(1) By drawing any of the ogive

In this case, we first compute $\frac{N}{2}$, where N is the sum of frequencies and then we locate the point M corresponding to Nth cumulative frequency on curve, and the x-coordinate of M gives the median.

(2) By drawing both of the ogives

We draw both ogive curves [i.e. less than ogive and greater than ogive] and intersection of both ogives gives the value of median.

7. Question

The mode of a frequency distribution can be determined graphically from

A. Histogram

B. Frequency polygon

C. Ogive

D. Frequency curve

Answer

The following steps must be followed to find the mode graphically.

1. Represent the given data in the form of a Histogram. The frequency determines the height of each bar. Identify the highest rectangle. This corresponds to the modal class of the series.

2. Join the top corners of the modal bar with the immediately next corners of the adjacent bars. The two lines must be cutting each other.

3. Let the point where the joining lines cut each other be 'A'. Draw a perpendicular line from point A onto the x-axis. The point 'P' where the perpendicular will meet the x-axis will give the mode.

8. Question

Mode is

A. least frequent value

B. middle most value

C. most frequent value

D. None of these

Answer

By Definition of mode, mode is most frequent value.

9. Question

The mean of n observations is $\overline{X}$. If the first item is increased by 1, second by 2 and so on, then the new mean is

A. $\overline{X}+n$

B. $\overline{X}+\frac{n}{2}$

C. $\overline{X}+\frac{n+1}{2}$

D. None of these

Answer

Given, mean is $\overline{x}$,

Let x_1, x_2, …, x_n are n observations.

And we know

The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

i.e.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$\Rightarrow n\bar{x} = x_1 + x_2 + \cdots + x_n$ ...[1]

Given as the first term is increased by 1 and 2nd term is increased by 2 and so on. Then the terms will be

$x_1 + 1, x_2 + 2, \ldots, x_n + n$

Let the new mean be x

$$x = \frac{x_1 + 1 + x_2 + 2 + \cdots + x_n + n}{n}$$

$\Rightarrow x = \frac{(x_1 + x_2 + \cdots + x_n) + (1 + 2 + \cdots + n)}{n}$ ...[2]

Now, we have series

1, 2, 3, ..., n

Clearly the above series is an AP(Arithmetic progression) with

first term, a = 1 and

common difference, d = 1

And no of terms is clearly n.

And last term is also n.

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$$

Using this in equation [2] and using equation [1] we have

$$x = \frac{n\bar{x} + \frac{n(n-1)}{2}}{n} = \bar{x} + \frac{n-1}{2}$$

10. Question

One of the methods of determining mode is

A. Mode = 2 Median − 3 Mean

B. Mode = 2 Median + 3 Mean

C. Mode = 3 Median − 2 Mean

D. Mode = 3 Median + 2 Mean

Answer

We know that empirical relation between mean, median and mode is

Mode = 3 Median - 2 Mean

11. Question

If the mean of the following distribution is 2.6, then the value of y is

Variable (x):	1	2	3	4	5
Frequency:	4	5	y	1	2

A. 3

B. 8

C. 13

D. 24

Answer

Let the draw the frequency distribution table for the above data

Variable(x_i)	Frequency(f_i)	$f_i x_i$
1	4	4
2	5	10
3	y	3y
4	1	4
5	2	10
	$\sum f_i = 12 + y$	$\sum f_i x_i = 28 + 3y$

As we know the mean($\overline{x}$)

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

In this case, $\overline{x} = 2.6$

So we have

$$2.6 = \frac{28 + 3y}{12 + y}$$

⇒ 31.2 + 2.6y = 28 + 3y

⇒ 3.2 = 0.4y

⇒ y = 8

12. Question

The relationship between mean, median and mode for a moderately skewed distribution is

A. Mode = 2 Median – 3 Mean

B. Mode = Median – 2 Mean

C. Mode = 2 Median − Mean

D. Mode = 3 Median − 2 mean

Answer

We know that empirical relation between mean, median and mode is

Mode = 3 Median - 2 Mean

13. Question

The mean of a discrete frequency distribution x_i/f_i; i = 1, 2, ..., n is given by

A. $\frac{\sum f_i x_i}{\sum f_i}$

B. $\frac{1}{n}\sum_{i=1}^{n} f_i x_i$

C. $\frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} x_i}$

D. $\frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} i}$

Answer

If $x_1, x_2, \ldots, x_n$ are observations with frequencies $f_1, f_2, \ldots, f_n$ i.e. x_1 occurs f_1 times and x_2 occurs f_2 times and so on, then we have

Sum of the values of the observations = $f_1x_1 + f_2x_2 + \ldots + f_nx_n$

and Number of observations = $f_1 + f_2 + \ldots + f_n$

So, mean($\overline{x}$) of observations is given by

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + \cdots + f_nx_n}{f_1 + f_2 + \cdots + f_n}$$

Or,

In summation form, it can be shorted to

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Which also can also be written as,

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

And it is understood i varies from 1 to n.

14. Question

If the arithmetic mean of x, x + 3, x + 6, x + 9, and x + 12 is 10, the x =

A. 1

B. 2

C. 6

D. 4

Answer

Terms are x, x + 3, x + 6, x + 9, x + 12

No of terms = 5

We know that

Mean $= \frac{\text{Sum of all observation}}{\text{No of observations}}$

$$\Rightarrow 10 = \frac{x + x + 3 + x + 6 + x + 9 + x + 12}{5}$$

⇒ 50 = 5x + 30

⇒ 5x = 20

⇒ x = 4

15. Question

If the median of the data: 24, 25, 26, x + 2, x + 3, 30, 31, 34 is 27.5, then x =

A. 27

B. 25

C. 28

D. 30

Answer

Terms are 24, 25, 26, x + 2, x + 3, 30, 31, 34

No of terms = 8

We know that, if even no of terms or observations are given, then the median of data is mean of the values of $\left(\frac{n}{2}\right)^{th}$ term and $\left(\frac{n}{2} + 1\right)^{th}$ term. Where n is no of terms.

In this case, n = 8

$$\frac{n}{2} = 4 \text{ and } \frac{n}{2} + 1 = 5$$

i.e. median of above data is mean of 4^{th} and 5^{th} term

⇒ median $= \frac{(x+2)+(x+3)}{2} = \frac{2x+5}{2}$

$$\Rightarrow 27.5 = \frac{2x+5}{2}$$

⇒ 55 = 2x + 5

⇒ 2x = 50

⇒ x = 25

16. Question

If the median of the data: 6, 7, x − 2, x, 17, 20, written in ascending order, is 16. Then x =

A. 15

B. 16

C. 17

D. 18

Answer

Terms are 6, 7, x - 2, x, 17, 20

No of terms = 6

We know that, if even no of terms or observations are given, then the median of data is mean of the values of $\left(\frac{n}{2}\right)^{th}$ term and $\left(\frac{n}{2}+1\right)^{th}$ term. Where n is no of terms.

In this case, n = 6

$$\frac{n}{2} = 3 \text{ and } \frac{n}{2} + 1 = 4$$

i.e. median of above data is mean of 3^{rd} and 4^{th} term

⇒ median $= \frac{(x-2)+x}{2} = \frac{2x-2}{2}$

⇒ 16 = x - 1 [As median is 16]

⇒ x = 17

17. Question

The median of first 10 prime numbers is

A. 11

B. 12

C. 13

D. 14

Answer

The first ten prime no's are :

2, 3, 5, 7, 11, 13, 17, 23, 29, 31

Clearly, the data is in ascending order. and

No of terms, n = 10

We know that, if even no of terms or observations are given, then the median of data is mean of the values of $\left(\frac{n}{2}\right)^{th}$ term and $\left(\frac{n}{2}+1\right)^{th}$ term. Where n is no of terms.

In this case, n = 10

$$\frac{n}{2} = 5 \text{ and } \frac{n}{2} + 1 = 6$$

i.e. median of above data is mean of 5^{th} and 6^{th} term

⇒ median $= \frac{11+13}{2} = \frac{24}{2}$

⇒ median = 12

18. Question

If the mode of the data: 64, 60, 48, x, 43, 48, 43, 34 is 43, then x + 3 =

A. 44

B. 45

C. 46

D. 48

Answer

As we know, mode of any data is the observation which occurs most.

In this case, 48 occurs two times and 43 is the mode of data

Therefore, 43 should occur more than two times.

And this is possible if and only if

x = 43

⇒ x + 3 = 43 + 3 = 46

Hence, correct option is (C).

19. Question

If the mode of the data: 16, 15, 17, 16, 15, x, 19, 17, 14 is 15, then x =?

A. 15

B. 16

C. 17

D. 19

Answer

Given: The mode of the data: 16, 15, 17, 16, 15, x, 19, 17, 14 is 15.

To find: The value of x.

Solution: As we know, mode of any data is the observation which occurs most.

In this case, 17 occurs two times which implies 17 is the mode.But it is given that 15 is the mode of data.In order for 15 to be mode it has to occur more than 2 times.As 15 is already occurring 2 times, the possibility of it occurring more than 2 times is that x should be 15.

⇒ x = 15Hence, correct option is (B).

20. Question

The mean of 1, 3, 4, 5, 7, 4 is m. The numbers 3, 2, 2, 4, 3, 3, p have mean m − 1 and median q. Then, p + q =

A. 4

B. 5

C. 6

D. 7

Answer

First data is:

1, 3, 4, 5, 7, 4

Given, mean = m

And we know,

Mean $= \frac{\text{Sum of all observations}}{\text{No of observations}}$ ...[1]

No of observations = 6

Sum of all observations = 1 + 3 + 4 + 5 + 7 + 4 = 24

Hence,

Mean $= \frac{24}{6} = 4$

⇒ m = 4 ...[2]

Second data is :

3, 2, 2, 4, 3, 3, p

No of observations = 7

Sum of observations = 3 + 2 + 2 + 4 + 3 + 3 + p = 17 + p

Given,

Mean = m - 1

Using [1]

$$m - 1 = \frac{(17 + p)}{7}$$

$$4 - 1 = \frac{17 + p}{7} \Rightarrow 3 = \frac{17 + p}{7}$$

$\Rightarrow 21 = 17 + p$

$\Rightarrow p = 4$ …[3]

Hence, series is 3, 2, 2, 4, 3, 3, 4

For median, let us write our data in increasing order

2, 2, 3, 3, 3, 4, 4,

Also, as the no of terms in this data is odd

We know that if there are odd number of terms in a data, then the median of data is $\left(\frac{n+1}{2}\right)^{th}$ term. Where n is no of terms

$n = 7$

$$\Rightarrow \frac{n + 1}{2} = 4$$

median = 4^{th} term = 3

$\Rightarrow q = 3$ …[4]

From [3] and [4]

$p + q = 4 + 3 = 7$

21. Question

If the mean of a frequency distribution is 8.1 and $\Sigma f_i x_i = 132 + 5k$, $\Sigma f_i = 20$, then k =

A. 3

B. 4

C. 5

D. 6

Answer

If $x_1, x_2, \ldots, x_n$ are observations with frequencies $f_1, f_2, \ldots, f_n$ i.e. x_1 occurs f_1 times and x_2 occurs f_2 times and so on, then we have

mean($\overline{x}$) of observations is given by

$$\overline{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Given, mean = 8.1

$$\sum f_i x_i = 132 + 5k$$

$$\sum f_i = 20$$

Putting this values in formula:

$$8.1 = \frac{132 + 5k}{20}$$

⇒ 162 = 132 + 5k

⇒ 5k = 30

⇒ k = 6

22. Question

If the mean of 6, 7, x, 8, y, 14 is 9, then

A. x + y = 21

B. x + y = 19

C. x − y = 19

D. x − y =21

Answer

Terms are 6, 7, x, 8, y, 14

No of terms = 6

We know that

Mean $= \frac{\text{Sum of all observation}}{\text{No of observations}}$

$$\Rightarrow 9 = \frac{6 + 7 + x + 8 + y + 14}{6}$$

⇒ 54 = x + y + 35

⇒ x + y = 19

Hence, correct option is (B)

23. Question

The mean of n observation is $\overline{x}$. If the first observation is increased by 1, the second by 2, the third by 3, and so on, then the new mean is

A. $\overline{x} + (2n + 1)$

B. $\overline{x} + \frac{n+1}{2}$

C. $\overline{x} + (n + 1)$

D. $\overline{x} - \frac{n+1}{2}$

Answer

Given, mean is $\overline{x}$,

Let $x_1, x_2, \ldots, x_n$ are n observations.

And we know

The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

i.e.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\Rightarrow n\bar{x} = x_1 + x_2 + \cdots + x_n \ldots[1]$$

Given as the first term is increased by 1 and 2^{nd} term is increased by 2 and so on. Then the terms will be

$x_1 + 1, x_2 + 2, \ldots, x_n + n$

Let the new mean be x

$$x = \frac{x_1 + 1 + x_2 + 2 + \cdots + x_n + n}{n}$$

$$\Rightarrow x = \frac{(x_1 + x_2 + \cdots + x_n) + (1 + 2 + \cdots + n)}{n} \ldots[2]$$

Now, we have series

1, 2, 3, …, n

Clearly the above series is an AP(Arithmetic progression) with

first term, a = 1 and

common difference, d = 1

And no of terms is clearly n.

And last term is also n.

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}$$

Using this in equation [2] and using equation [1] we have

$$x = \frac{n\bar{x} + \frac{n(n-1)}{2}}{n} = \bar{x} + \frac{n-1}{2}$$

24. Question

If the mean of first n natural numbers is $\frac{5n}{9}$, then n =

A. 5

B. 4

C. 9

D. 10

Answer

First n natural numbers are

1, 2, 3, 4, …, n

We know that mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

and, we have given series

1, 2, 3, …, n

Clearly the above series is an AP(Arithmetic progression) with

first term, a = 1 and

common difference, d = 1

And no of terms is clearly n.

And last term is also n.

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2} \text{ ....[1]}$$

As,

Mean$= \frac{\text{Sum of all terms}}{\text{no of terms}} = \frac{1+2+3+\cdots+n}{n}$

⇒ Mean$= \frac{\left(\frac{n(n+1)}{2}\right)}{n} = \frac{n+1}{2}$

Given, mean $= \frac{5n}{9}$

$$\Rightarrow \frac{n+1}{2} = \frac{5n}{9}$$

⇒ 9n + 9 = 10n

⇒ n = 9

25. Question

The arithmetic mean and mode of a data are 24 and 12 respectively, then its median is

A. 25

B. 18

C. 20

D. 22

Answer

We know that empirical relation between mean, median and mode is

Mode = 3 Median - 2 Mean

Given,

Mean = 24

Mode = 12

Putting values in the formula,

12 = 3 Median - 2(24)

⇒ 12 = 3 Median - 48

⇒ 3 Median = 60

⇒ Median = 20

26. Question

The mean of first n odd natural number is

A. $\frac{n+1}{2}$

B. $\frac{n}{2}$

C. n

D. n^2

Answer

We know that mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

and, we have first n odd natural numbers as

1, 3, …, 2n - 1

Clearly the above series is an AP(Arithmetic progression) with

first term, a = 1 and

common difference, d = 2

And no of terms is clearly n.

And last term is (2n - 1)

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + 2n - 1) = \frac{n(2n)}{2} = n^2 \ldots[1]$$

As,

Mean$= \frac{\text{Sum of all terms}}{\text{no of terms}}$

$\Rightarrow$ Mean$= \frac{n^2}{n} = n$

27. Question

The mean of first n odd natural numbers is $\frac{n^2}{81}$, then n =

A. 9

B. 81

C. 27

D. 18

Answer

We know that mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

and, we have first n odd natural numbers as

1, 3, ..., 2n - 1

Clearly the above series is an AP(Arithmetic progression) with first term, a = 1 and common difference, d = 2

And no of terms is clearly n.

And last term is (2n - 1)

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + 2n - 1) = \frac{n(2n)}{2} = n^2 \ldots[1]$$

As,

Mean$= \frac{\text{Sum of all terms}}{\text{no of terms}}$

$\Rightarrow$ Mean$= \frac{n^2}{n} = n$

Now, given mean $= \frac{n^2}{81}$

$$\Rightarrow \; n = \frac{n^2}{81}$$

⇒ n = 81

28. Question

If the difference of mode and median of a data is 24, then the difference of medina and mean is

A. 12

B. 24

C. 8

D. 36

Answer

Difference of mode and median, mode - median = 24 …[1]

We know that empirical relation between mean, median and mode is

Mode = 3 Median - 2 Mean

⇒ 3 Mode - 2 Mode = 3 Median - 2 Mean

⇒ 3 Mode - 3 Median = 2 Mode - 2 Mean

⇒ 3(Mode - Median) = 2(Mode - Mean)

From [1] we have

3(24) = 2(Mode - Mean)

⇒ Mode - Mean = 36 …[2]

on substracting [1] from [2]

Mode - Mean - (Mode - Median) = 36 - 24

Mode - Mean - Mode + Median = 8

Median - Mode = 8

Hence, difference between median and mode is 8.

29. Question

If the arithmetic mean of 7, 8, x, 11, 14 is x, then x =

A. 9

B. 9.5

C. 10

D. 10.5

Answer

Terms are 7, 8, x, 11, 14

No of terms = 5

We know that

Mean $= \frac{\text{Sum of all observation}}{\text{No of observations}}$

$$\Rightarrow x = \frac{7 + 8 + x + 11 + 14}{5}$$

⇒ 5x = x + 40

⇒ 4x = 40

⇒ x = 10

Hence, correct option is (C)

30. Question

If mode of a series exceeds its mean by 12, then mode exceeds the median by

A. 4

B. 8

C. 6

D. 10

Answer

Given, mode exceeds mean by 12 i.e. mode - mean = 12 …[1]

We know that empirical relation between mean, median and mode is

Mode = 3 Median - 2 Mean

⇒ 3 Mode - 2 Mode = 3 Median - 2 Mean

⇒ 3 Mode - 3 Median = 2 Mode - 2 Mean

⇒ 3(Mode - Median) = 2(Mode - Mean)

From [1] we have

3(Mode - Median) = 2(12)

⇒ Mode - Median = 8

i.e. Mode exceeds Median by 8.

31. Question

If the mean of first n natural number is 15, then n =

A. 15

B. 30

C. 14

D. 29

Answer

First n natural numbers are

1, 2, 3, 4, …, n

We know that mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

and, we have given series

1, 2, 3, …, n

Clearly the above series is an AP(Arithmetic progression) with

first term, a = 1 and

common difference, d = 1

And no of terms is clearly n.

And last term is also n.

We know, sum of terms of an AP if first and last terms are known is:

$$S_n = \frac{n}{2}(a + a_n)$$

Putting the values in above equation we have sum of series i.e.

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2} \ldots[1]$$

As,

Mean$= \frac{\text{Sum of all terms}}{\text{no of terms}} = \frac{1+2+3+\cdots+n}{n}$

⇒ Mean$= \frac{\left(\frac{n(n+1)}{2}\right)}{n} = \frac{n+1}{2}$

Given, mean = 15

$$\Rightarrow \frac{n+1}{2} = 15$$

⇒ n + 1 = 30

⇒ n = 29

32. Question

If the mean observations $x_1, x_2, \ldots, x_n$ is $\overline{X}$, then the means of $x_1 + a, x_2 + a, \ldots, x_n + a$ is

A. $a\overline{X}$

B. $\overline{X} - a$

C. $\overline{X} + a$

D. $\frac{\overline{X}}{a}$

Answer

Given, mean is $\overline{X}$,

and $x_1, x_2, \ldots, x_n$ are n observations.

And we know

The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

i.e.

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$\Rightarrow n\bar{x} = x_1 + x_2 + \cdots + x_n$ ...[1]

And we have another series

$x_1 + a, x_2 + a, \ldots, x_n + a$

Let the new mean be x

$$x = \frac{x_1 + 1 + x_2 + 2 + \cdots + x_n + n}{n}$$

$$\Rightarrow x = \frac{(x_1 + x_2 + \cdots + x_n) + (a + a + \cdots \text{ upto n times})}{n}$$

Now, From [1] we have

$$x = \frac{n\bar{x} + na}{n} = \bar{x} + a$$

33. Question

Mean of a certain number of observations is $\bar{x}$. If each observation is divided by m(m ≠ 0) and increased by n, then the mean of new observation is

A. $\frac{\bar{x}}{m} + n$

B. $\frac{\bar{x}}{n} + m$

C. $\bar{x} + \frac{n}{m}$

D. $\bar{x} + \frac{m}{n}$

Answer

Given, mean is $\bar{x}$,

Let $x_1, x_2, \ldots, x_k$ are k observations.

And we know

The mean or average of observations, is the sum of the values of all the observations divided by the total number of observations.

i.e.

$\bar{x} = \frac{x_1 + x_2 + \cdots + x_k}{k}$ ....[1]

Given, the terms are divided by m and increased by n. Then the terms will be

$$\frac{x_1}{m}+n, \frac{x_2}{m}+n, \ldots, \frac{x_k}{m}+n$$

Let the new mean be x

$$x=\frac{\left(\frac{x_1}{m}+n+\frac{x_2}{m}+n+\cdots+\frac{x_k}{m}+n\right)}{k}$$

$$\Rightarrow x=\frac{\frac{(x_1+x_2+\cdots+x_k)}{m}+kn}{k}$$

$$\Rightarrow \; x=\frac{x_1+x_2+\cdots+x_k}{mk}+n$$

Now, From [1] we have

$$x=\frac{\bar{x}}{m}+n$$

Hence correct option is (A)

34. Question

If $u_i=\frac{x_i-25}{10}, \Sigma f_i u_i=20, \Sigma f_i=100$, then $\overline{x}$ =

A. 23

B. 24

C. 27

D. 25

Answer

We have given,

$$\sum f_i u_i=20$$

Also,

$$u_i=\frac{x_i-25}{10}$$

Putting this in above equation

$$\sum f_i\left(\frac{x_i-25}{10}\right)=20$$

$$\Rightarrow \frac{1}{10}\sum(f_i x_i-25 f_i)=20$$

$$\Rightarrow \sum f_i x_i-\sum 25 f_i=200$$

$$\Rightarrow \sum f_i x_i - 25 \sum f_i = 200$$

Now, given $\sum f_i = 100$ ...[1]

using this

We have,

$$\sum f_i x_i - 25(100) = 200$$

$$\Rightarrow \sum f_i x_i - 2500 = 200$$

$$\Rightarrow \sum f_i x_i = 2700 \text{ ...[2]}$$

We know,

If $x_1, x_2, \ldots, x_n$ are observations with frequencies $f_1, f_2, \ldots, f_n$ i.e. x_1 occurs f_1 times and x_2 occurs f_2 times and so on, then mean($\bar{x}$) of observations is given by

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

From [1] and [2]

$$\bar{x} = \frac{2700}{100} = 27$$

35. Question

If 35 is removed from the data: 30, 34, 35, 36, 37, 38, 39, 40, then the median increases by

A. 2

B. 1.5

C. 1

D. 0.5

Answer

Given series is,

30, 34, 35, 36, 37, 38, 39, 40

We know that, if even no of terms or observations are given, then the median of data is mean of the values of $\left(\frac{n}{2}\right)^{th}$ term and $\left(\frac{n}{2}+1\right)^{th}$ term. Where n is no of terms.

In this case, no of terms,n = 8

$$\frac{n}{2} = 4 \text{ and } \frac{n}{2} + 1 = 5$$

i.e. median of above data is mean of 4^{th} and 5^{th} term

In this case,

4^{th} term = 36

5^{th} term = 37

$\Rightarrow$ median $= \frac{36+37}{2} = \frac{73}{2}$

$\Rightarrow$ median = 37.5

If 35 is removed, the series will be

30, 34, 36, 37, 38, 39, 40

No of terms = 7

We know that if there are odd number of terms in a data, then the median of data is $\left(\frac{n+1}{2}\right)^{th}$ term. Where n is no of terms

n = 7

$$\Rightarrow \frac{n+1}{2} = 4$$

median = 4^{th} term = 37

Difference in both medians = 37.5 - 37 = 0.5

Hence, median increases by 0.5.