7. Factorization

Exercise 7.1

1. Question

Find the greatest common factor (GCF/HCF) of the following polynomials

$2x^2$ and $12x^2$

Answer

The numerical coefficients of given numerical are 2, 12

Greatest common factor of 2, 12 is 2

Common literals appearing in given numerical is x

Smallest power of x in two monomials = 2

Monomials of common literals with smallest power= x^2

Hence, the greatest common factor = $2x^2$

2. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$6x^3y$ and $18x^2y^3$

Answer

The numerical coefficients of given numerical are 6,18

Greatest common factor of 6, 18 is 6

Common literals appearing in given numerical are x and y

Smallest power of x in both monomials= 2

Smallest power of y in both monomials = 1

Binomials of common literals with smallest power= x^2y

Hence, the greatest common factor = $6x^2y$

3. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$7x, 21x^2$ and $14xy^2$

Answer

The numerical coefficients of given numerical are 7, 21, 14

Greatest common factor of 7, 21, 14 is 7

Common literals appearing in given numerical are x and y

Smallest power of x in three monomials = 1

Smallest power of y in three monomials = 0

Monomials of common literals with smallest power= x

Hence, the greatest common factor = 7x

4. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$42x^2yz$ and $63x^3y^2z^3$

Answer

The numerical coefficients of given numerical are 42 and 63.

Greatest common factor of 42, 63 is 21.

Common literals appearing in given numerical are x, y and z

Smallest power of x in two monomials = 2

Smallest power of y in two monomials = 1

Smallest power of z in two monomials = 1

Monomials of common literals with smallest power= x^2yz

Hence, the greatest common factor = $21x^2yz$

5. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$12ax^2$, $6a^2x^3$ and $2a^3x^5$

Answer

The numerical coefficients of given numerical are 12, 6, 2

Greatest common factor of 12, 6, 2 is 2.

Common literals appearing in given numerical are a and x

Smallest power of x in three monomials = 2

Smallest power of a in three monomials = 1

Monomials of common literals with smallest power= ax^2

Hence, the greatest common factor = $2ax^2$

6. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$9x^2, 15x^2y^3, 6xy^2$ and $21x^2y^5$

Answer

The numerical coefficients of given numerical are 9, 15, 16, 21

Greatest common factor of 9, 15, 16, 21 is 3.

Common literals appearing in given numerical are x and y

Smallest power of x in four monomials = 1

Smallest power of y in four monomials = 0

Monomials of common literals with smallest power= x

Hence, the greatest common factor = 3x

7. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$4a^2b^3, -21a^3b, 18a^4b^3$

Answer

The numerical coefficients of given numerical are 4, -12, 18.

Greatest common factor of 4, -12, 18 is 2.

Common literals appearing in given numerical are a and b

Smallest power of a in three monomials = 2

Smallest power of b in three monomials = 1

Monomials of common literals with smallest power= a^2b

Hence, the greatest common factor = $2a^2b$

8. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$6x^2y^2, -9xy^3, 3x^3y^2$

Answer

The numerical coefficients of given numerical are 6, 9, 3

Greatest common factor of 6, 9, 3 is 3.

Common literals appearing in given numerical are x and y

Smallest power of x in three monomials = 1

Smallest power of y in three monomials = 2

Monomials of common literals with smallest power= xy^2

Hence, the greatest common factor = $3xy^2$

9. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

a^2b^3, a^3b^2

Answer

The numerical coefficients of given numerical are 0

Common literals appearing in given numerical are a and b

Smallest power of a in two monomials = 2

Smallest power of b in two monomials = 2

Monomials of common literals with smallest power= the greatest common factor = a^2b^2

10. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$36a^2b^2c^4, 54a^4c^2, 90a^4b^2c^2$

Answer

The numerical coefficients of given numerical are 36, 54, 90

Greatest common factor of 36, 54, 90 is 18.

Common literals appearing in given numerical are a, b and c

Smallest power of a in three monomials = 2

Smallest power of b in three monomials = 0

Smallest power of c in three monomials = 2

Monomials of common literals with smallest power= a^2c^2

Hence, the greatest common factor = $18a^2c^2$

11. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

x^3, yx^2

Answer

The numerical coefficients of given numerical are 0

Common literals appearing in given numerical are x and y

Smallest power of x in two monomials = 2

Smallest power of y in two monomials = 0

Monomials of common literals with smallest power= x^2

Hence, the greatest common factor = x^2

12. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$15a^3$, - $54a^2$, -150a

Answer

The numerical coefficients of given numerical are 15, -45, -150

Greatest common factor of 15, -45, -150 is 15.

Common literals appearing in given numerical is smallest power of a in three monomials = 1

Monomials of common literals with smallest power= a

Hence, the greatest common factor = 15a

13. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$2x^3y^2, -10x^2y^3, 14xy$

Answer

The numerical coefficients of given numerical are 2, 10, 14.

Greatest common factor of 2, 10, 14 is 2.

Common literals appearing in given numerical are x and y

Smallest power of x in three monomials = 1

Smallest power of y in three monomials = 1

Monomials of common literals with smallest power= xy

Hence, the greatest common factor = 2xy

14. Question

Find the greatest common factor (GCF/HCF) of the following polynomials:

$14x^3y^5, -10x^5y^3, 12x^2y^2$

Answer

The numerical coefficients of given numerical are 14, 10, 2.

Greatest common factor of 14, 10, 2 is 2.

Common literals appearing in given numerical are x and y

Smallest power of x in three monomials = 2

Smallest power of y in three monomials = 2

Monomials of common literals with smallest power= x^2y^2

Hence, the greatest common factor = $2x^2y^2$

15. Question

Find the greatest common factor of the terms in each of the following expressions:

$5a^5 + 10a^5 - 15a^2$

Answer

The highest common factor of three terms = $5a^2$

$=5a^2(a^2 + 2a - 3)$

16. Question

Find the greatest common factor of the terms in each of the following expressions:

$2xyz + 3x^2y + 4y^2$

Answer

The highest common factor of three terms = y

Therefore,

$= y(2xz + 3x^2 + 4y)$

17. Question

Find the greatest common factor of the terms in each of the following expressions:

$3a^2b^2 + 4b^2c^2 + 12a^2b^2c^2$

Answer

The highest common factor of three terms = b^2

Therefore,

$5a^2b^2 + 4b^2c^2 + 12a^2b^2c^2 = b^2(3a^2 + 4c^2 + 12a^2c^2)$

Exercise 7.2

1. Question

Factorize the following:

$3x - 9$

Answer

Greatest common factor of the two terms namely 3x and -9 of expression 3x - 9 is 3

$3x = 3 \times x$ and $-9 = 3 \times (-3)$

3x - 9 = 3(x - 3)

2. Question

Factorize the following:

$5x - 15x^2$

Answer

Greatest common factor of the two terms namely 5x and $-15x^2$ of expression $5x - 15x^2$ is $5x -15x^2$

$5x = 5x(1)$ and $-15x^2 = 5x(-3x)$

$5x -15x^2 = 5x(1 - 3x)$

3. Question

Factorize the following:

$20a^{12}b^2 - 15a^8b^4$

Answer

Greatest common factor of the two terms namely 20a12b2 and -15a8b4 of expression 20a12b2 - 15a8b4 is 5a8b2

$20a12b2 = 5a8b2\ (4a4)$ and $- 15a8b^4 = 5a^8b^2\ (-3b^2)$

$20a^{12}b^2 - 15a^8b^4 = 5a^8b^2\ (4a^4 - 3b^2) = 5a^8b^2((2a)^2 - (b\sqrt{3})^2) = 5a^8b^2(2a + b\sqrt{3})(2a - b\sqrt{3})$

4. Question

Factorize the following:

$72x^6y^7 - 96x^7y^6$

Answer

Greatest common factor of the two terms namely $72x^6y^7$ and $- 96x^7y^6$ of expression $72x^6y^7 - 96x^7y^6$ is $24x^6y^6$

$72x^6y^7 = 24x^6y^6\ (3y)$ and $- 96x^7y^6 = 24x^6y^6(-4x)$

$72x^6y^7 - 96x^7y^6 = 24x^6y^6\ (3y - 4y)$

5. Question

Factorize the following:

$20x^3 - 40x^2 + 80x$

Answer

Greatest common factor of the two terms namely $20x^3$, $-40x^2$ and 80x of expression $20x^3 - 40x^2 + 80x$ is 20x

$20x^3 - 40x^2 + 80x = 20x(x^2 - 2x + 4)$

6. Question

Factorize the following:

$2x^3y^2 - 4x^2y^3 + 8xy^4$

Answer

Greatest common factor of the two terms namely $2x^3y^2$, $- 4x^2y^3$, $- 8xy^4$ of expression $2x^3y^2 - 4x^2y^3 - 8xy^4$ is $2xy^2$

$2x^3y^2 - 4x^2y^3 - 8xy^4 = 2xy^2\ (x^2 - 2xy + 4y)$

7. Question

Factorize the following:

$10m^3n^2 + 15m^4n - 20m^2n^3$

Answer

Greatest common factor of the two terms namely $10m^3n^2$, $15m^4n$, $-20m^2n^3$ of expression $10m^3n^2 + 15m^4n - 20m^2n^3$ is $5mn^2$

$10m^3n^2 + 15m^4n - 20m^2n^3 = 5mn^2(2mn + 3m^2 - 4n)$

8. Question

Factorize the following:

$2a^4b^4 - 3a^3b^5 + 4a^2b^5$

Answer

Greatest common factor of the two terms namely $2a^4b^4$, $-3a^3b^5$, $4a^2b^5$ of expression $2a^4b^4 - 3a^3b^5 + 4a^2b^5$ is a^2b^4

$2a^4b^4 - 3a^3b^5 + 4a^2b^5 = a^2b^4 (2a^2 - 3ab + 4b)$

9. Question

Factorize the following:

$28a^2 + 14a^2b^2 - 21a^4$

Answer

Greatest common factor of the two terms namely $28a^2$, $14a^2b^2$, $-21a^4$ of expression $28a^2 + 14a^2b^2 - 21a^4$ is $7a^2$

$28a^2 + 14a^2b^2-21a^4 = 7a^2(4 + 2b^2 - 3a^2)$

10. Question

Factorize the following:

$a^4b - 3a^2b^2 - 6ab^3$

Answer

Greatest common factor of the two terms namely a^4b, $-3a^2b^2$, $-6ab^3$ of expression $a^4b - 3a^2b^2 - 6ab^3$ is ab

$a^4b - 3a^2b^2 - 6ab^3 = ab (a^3 - 3ab -6ab^2)$

11. Question

Factorize the following:

$2l^2mn - 3lm^2n + 4lmn^2$

Answer

Greatest common factor of the two terms namely $21lmn$, $-3lm^2n$, $4lmn^2$ of expression $21lmn - 3lm^2n + 4lmn^2$ is lm

$21lmn - 3lm^2n + 4lmn^2 = lm(21 - 3m + 4n)$

12. Question

Factorize the following:

$x^4y^2 - x^2y^4 - x^4y^4$

Answer

Greatest common factor of the two terms namely x^4y^2, $-x^2y^4$, $-x^4y^4$ of expression $x^4y^2 - x^2y^4 - x^4 y^4$ is x^2y^2

$x^4y^2 - x^2y^4 - x^4y^4 = x^2y^2 (x^2 - y^2 -x^2y^2)$

13. Question

Factorize the following:

$9x^2y + 3axy$

Answer

Greatest common factor of the two terms namely $9x^2y$ and $3axy$ of expression $9x^2y + 3axy$ is $3xy$

$9x^2y + 3axy = 3xy(3x^2 + a)$

14. Question

Factorize the following:

$16m - 4m^2$

Answer

Greatest common factor of the two terms namely $16m - 4m^2$ of expression $16m - 4m^2$ is $4m$

$16m - 4m^2 = 4m(4 - m)$

15. Question

Factorize the following:

$-4a^2 + 4ab - 4ca$

Answer

Greatest common factor of the two terms namely -4a, 4ab, -4ca of expression -4a + 4ab -4ca is -4a

-4a + 4ab -4ca = -4a(a - b + c)

16. Question

Factorize the following:

$16m - 4m^2$

Answer

Greatest common factor of the two terms namely x^2yz, xy^2z, xyz^2 of expression $x^2yz + xy^2z + xyz^2$ is xyz

$x^2yz + xy^2z + xyz^2 = xyz(x + y + z)$

17. Question

Factorize the following:

$ax^2y + bxy^2 + cxyz$

Answer

Greatest common factor of the two terms namely -4a, 4ab, -4ca of expression -4a + 4ab -4ca is -4a

$ax^2y + bxy^2 + cxyz = xy\ (ax + by + cz)$

Exercise 7.3

1. Question

Factorize each of the following algebraic expressions:

$6x(2x - y) + 7y(2x - y)$

Answer

(6x + 7y) (2x – y) [Therefore, taking (2x – y) common)]

2. Question

Factorize each of the following algebraic expressions:

$2r(y-z)+s(x-y)$

Answer

-2r (x – y) + s (x – y) [Therefore, taking – 1 common]

= (x – y) (-2r + s) [Therefore, taking (x – y) common]

= (x – y) (s – 2r)

3. Question

Factorize each of the following algebraic expressions:

$7a(2x-3)+3b(2x-3)$

Answer

(7a + 3b) (2x – 3) [Therefore, taking (2x – 3) common]

4. Question

Factorize each of the following algebraic expressions:

$9a(6a-5b)-12a^2(6a-5b)$

Answer

(9a – $12a^2$) (6a – 5b) [Therefore, taking (6a – 5b) common]

5. Question

Factorize each of the following algebraic expressions:

$5(x-2y)^2+3(x-2y)$

Answer

(x – 2y) [5 (x – 2y) + 3] [Therefore, taking (x – 2y) common]

= (x – 2y) (5x – 10y + 3)

6. Question

Factorize each of the following algebraic expressions:

$16(2l-3m)^2-12(3m-2l)$

Answer

16 (2l – $3m^2$) + 12 (2l – 3m) [Therefore, 3m – 2l = - (2l – 3m)]

= 4 (2l – 3m) [4 (2l – 3m) + 3] [Therefore, taking 4 (2l – 3m) common]

= 4 (3l – 2m) (8l – 12m + 3)

7. Question

Factorize each of the following algebraic expressions:

$3a(x-2y)-b(x-2y)$

Answer

(3a – b) (x – 2y) [Therefore, taking (x – 2y) as common]

8. Question

Factorize each of the following algebraic expressions:

$a^2(x+y)+b^2(x+y)+c^2(x+y)$

Answer

$(a^2 + b^2 + c^2)$ (x + y) [Therefore, taking (x + y) common in each term]

9. Question

Factorize each of the following algebraic expressions:

$(x-y)^2+(x-y)$

Answer

(x – y) (x – y + 1) [Therefore, taking (x – y) common)

10. Question

Factorize each of the following algebraic expressions:

$6(a+2b)-4(aa+2b)^2$

Answer

[6 – 4 (a + 2b)] (a + 2b) [Therefore, taking (a + 2b) common]

= (6 – 4a – 8b) (a + 2b)

11. Question

Factorize each of the following algebraic expressions:

$a(x-y)+2b(y-x)+c(x-y)^2$

Answer

a (x – y) – 2b (x – y) + c $(x – y)^2$ [Therefore, (y – x) = - (x – y)]

= (x – y) [a – 2b + c (x – y)]

= (x – y) (a – 2b + cx – cy)

12. Question

Factorize each of the following algebraic expressions:

$-4(x-2y)^2+8(x-2y)$

Answer

- (x – 2y) [4 (x – 2y – 8] [Therefore, taking – (x – 2y) as common]

= - (x – 2y) (4x – 8y – 8)

13. Question

Factorize each of the following algebraic expressions:

$x^3(a-2b)+x^2(a-2b)$

Answer

x^2 (a – 2b) (x + 1) [Therefore, taking x^2 (a – 2b) as common]

14. Question

Factorize each of the following algebraic expressions:

$(2x-3y)(a+b)+(3x-2y)(a+b)$

Answer

(a + b) (2x – 3y + 3x – 2y) [Therefore, taking (a + b) common]

= (a + b) (5x – 5y)

15. Question

Factorize each of the following algebraic expressions:

$4(x+y)(3a-b)+6(x+y)(2b-3a)$

Answer

2 (x + y) [2 (3a – b) + 3 (2b – 3a)] [Therefore, by taking 2 (x + y) common]

= 2 (x + y) (4b – 3a)

Exercise 7.4

1. Question

Factorize each of the following expressions:

$qr-pr+qs-ps$

Answer

q (r + s) – p (r + s)

= (q – p) (r + s)

2. Question

Factorize each of the following expressions:

$p^2q-pr^2-pq+r^2$

Answer

p (pq – r^2) – 1 (pq – r^2)

= (p – 1) (pq – r^2)

3. Question

Factorize each of the following expressions:

$1+x+xy+x^2y$

Answer

1 (1 + xy) + x (1 + xy)

= (1 + x) (1 + xy)

4. Question

Factorize each of the following expressions:

$ax+ay-bx-by$

Answer

a (x + y) – b (x + y)

= (a – b) (x + y)

5. Question

Factorize each of the following expressions:

$xa^2+xb^2-ya^2-yb^2$

Answer

$x(a^2 + b^2) - y(a^2 + b^2)$

$= (x - y)(a^2 + b^2)$

6. Question

Factorize each of the following expressions:

$x^2 + xy + xzyz$

Answer

$x(x + 3) + y(x + 3)$

$= (x + y)(x + 3)$

7. Question

Factorize each of the following expressions:

$2ax + bx + 2ay + by$

Answer

$2a(x + y) + b(x + y)$

$= (2a + b)(x + y)$

8. Question

Factorize each of the following expressions:

$ax - by - ay + y^2$

Answer

$a(b - y) - y(b - y)$

$= (a - y)(b - y)$

9. Question

Factorize each of the following expressions:

$axy + bcxy - az - bcz$

Answer

$a(xy - z) + bc(xy - z)$

$= (a + bc)(xy - z)$

10. Question

Factorize each of the following expressions:

$lm^2 - mn^2 - lm + n^2$

Answer

$2m(m - 1) - n^2(m - 1)$

$= (2m - n^2)(m - 1)$

11. Question

Factorize each of the following expressions:

$x^3 - y^2 + x - x^2y^2$

Answer

$y^2 (1 + x^2) + x (1 + x^2)$

$= (x – y^2) (1 + x^2)$

12. Question

Factorize each of the following expressions:

$6xy + 6 – 9y – 4x$

Answer

$2x (3y – 2) – 3 (3y – 2)$

$= (2x – 3) (3y – 2)$

13. Question

Factorize each of the following expressions:

$x^2 – 2ax – 2ab + bx$

Answer

$x (x + b) – 2a (x + b)$

$= (x – 2a) (x + b)$

14. Question

Factorize each of the following expressions:

$x^3 – 2x^2y + 3xy^2 – 6y^3$

Answer

$x (x^2 + 3y^2) – 2y (x^2 + 3y^2)$

$=(x – 2y) (x^2 + 3y^2)$

15. Question

Factorize each of the following expressions:

$abx^2 + (ay – b) x – y$

Answer

$abx^2 – ayx – bx – y$

$= bx (ax – 1) + y (ax – 1)$

$= (bx + y) (ax – 1)$

16. Question

Factorize each of the following expressions:

$(ax + by)^2 + (bx – ay)^2$

Answer

$a^2x^2 + b^2y^2 + 2axby + b^2x^2 + a^2y^2 – 2axby$

$= a^2 (x^2 + y^2) + b^2 (x^2 + y^2)$

$= (a^2 + b^2) (x^2 + y^2)$

17. Question

Factorize each of the following expressions:

$16(a-b)^3 - 24(a-b)^2$

Answer

$8(a-b)^2 [2(a-b) - 3]$

$= 8(a-b)^2 [2a - 2b - 3]$

18. Question

Factorize each of the following expressions:

$ab(x^2+1) + x(a^2+b^2)$

Answer

$abx^2 + ab + xa^2 + xb^2$

$= ax(bx + a) + b(bx + a)$

$= (ax + b)(bx + a)$

19. Question

Factorize each of the following expressions:

$a^2x^2 + (ax^2+1)x + a$

Answer

$a^2x^2 + ax^3 + x + a$

$= x(ax^2 + 1) + a(ax^2 + 1)$

$= (x + a)(ax^2 + 1)$

20. Question

Factorize each of the following expressions:

$a(a-2b-c) + 2bc$

Answer

$a^2 - 2ab - ac + 2bc$

$= a(a - c) - 2b(a - c)$

$= (a - 2b)(a - c)$

21. Question

Factorize each of the following expressions:

$a(a+b-c) - bc$

Answer

$a^2 + ab + ac - bc$

$= a(a - c) + b(a - c)$

$= (a + b)(a - c)$

22. Question

Factorize each of the following expressions:

$x^2 - 11xy - x + 11y$

Answer

x (x – 1) – 11y (x – 1)

= (x – 11y) (x – 1)

23. Question

Factorize each of the following expressions:

ab – a – b + 1

Answer

a (b – 1) – 1 (b – 1)

= (a – 1) (b – 1)

24. Question

Factorize each of the following expressions:

$x^2 + y - xy - x$

Answer

x (x – 1) – y (x – 1)

= (x – y) (x – 1)

Exercise 7.5

1. Question

Factorize each of the following expressions:

$16x^2 - 25y^2$

Answer

$(4x)^2 - (5y)^2$

= (4x + 5y) (4x – 5y)

2. Question

Factorize each of the following expressions:

$27x^2 - 12y^2$

Answer

Consider $27x^2 - 12y^2$,Taking 3 common we get,3 $[(3x)^2 - (2y)^2]$As we know $a^2 - b^2 = (a-b)(a+b)$

= 3 (3x + 2y) (3x – 2y)

3. Question

Factorize each of the following expressions:

$144a^2 - 289b^2$

Answer

$(12a)^2 - (17b)^2$

= (12a + 17b) (12a – 17b)

4. Question

Factorize each of the following expressions:

$12m^2 - 27$

Answer

$3 (4m^2 - 9)$

$= 3 [(2m)^2 - 3^2]$

$= 3 (2m + 3) (2m - 3)$

5. Question

Factorize each of the following expressions:

$125x^2 - 45y^2$

Answer

$5 (25x^2 - 9y^2)$

$= 5 [(5x)^2 - (3y)^2]$

$= 5 (5x + 3y) (5x - 3y)$

6. Question

Factorize each of the following expressions:

$144a^2 - 169b^2$

Answer

$(12a)^2 - (13b)^2$

$= (12a + 13b) (12a - 13b)$

7. Question

Factorize each of the following expressions:

$(2a - b)^2 - 16c^2$

Answer

$(2a - b)^2 - (4c)^2$

$= (2a - b + 4c) (2a - b - 4c)$

8. Question

Factorize each of the following expressions:

$(x + 2y)^2 - 4(2x - y)^2$

Answer

$(x + 2y)^2 - [2 (2x - y)]^2$

$= [(x + 2y) + 2 (2x - y)] [x + 2y - 2 (2x - y)]$

$= (x + 4x + 2y - 2y) (x - 4x + 2y + 2y)$

$= (5x) (4y - 3x)$

9. Question

Factorize each of the following expressions:

$3a^5 - 48a^3$

Answer

$3a^3 (a^2 - 16)$

$= 3a^3 (a^2 - 4^2)$

$= 3a^3 (a + 4) (a - 5)$

10. Question

Factorize each of the following expressions:

$a^4 - 16b^4$

Answer

$(a^2)^2 - (4b^2)^2$

$= (a^2 + 4b^2) (a^2 - 4b^2)$

11. Question

Factorize each of the following expressions:

$x^8 - 1$

Answer

$(x^4)^2 - (1)^2$

$= (x^4 + 1) (x^4 - 1)$

12. Question

Factorize each of the following expressions:

$64 - (a + 1)^2$

Answer

$8^2 - (a + 1)^2$

$= [8 + (a + 1)] [8 - (a + 1)]$

$= (a + 9) (7 - a)$

13. Question

Factorize each of the following expressions:

$36l^2 - (m + n)^2$

Answer

$(6l)^2 - (m + n)^2$

$= (6l + m + n) (6l - m - n)$

14. Question

Factorize each of the following expressions:

$25x^4y^4 - 1$

Answer

$(5x^2y^2)^2 - (1)^2$

$= (5x^2y^2 - 1) (5x^2y^2 + 1)$

15. Question

Factorize each of the following expressions:

$a^4 - \frac{1}{b^4}$

Answer

$(a^2)^2 - (\frac{1}{b*b})^2$

$= (a^2 + \frac{1}{b*b})\ (a^2 - \frac{1}{b*b})$

16. Question

Factorize each of the following expressions:

$x^3 - 144x$

Answer

$x\ [x^2 - (12)^2]$

$= x\ (x + 12)\ (x - 12)$

17. Question

Factorize each of the following expressions:

$(x - 4y)^2 - 625$

Answer

$(x - 4y)^2 - (25)^2$

$= (x - 4y + 25)\ (x - 4y - 25)$

18. Question

Factorize each of the following expressions:

$9(a - b)^2 - 100(x - y)^2$

Answer

$[3\ (a - b)]^2 - [10\ (x - y)]^2$

$= [3\ (a - b) + 10\ (x + y)]\ [3\ (a - b) - 10\ (x - y)]$

$= [3a - 3b + 10x - 10y]\ [3a - 3b - 10x + 10y]$

19. Question

Factorize each of the following expressions:

$(3 + 2a)^2 - 25a^2$

Answer

$(3 + 2a)^2 - (5a)^2$

$= (3 + 2a + 5a)\ (3 + 2a - 5a)$

$= (7a + 3)\ (3 - 3a)$

20. Question

Factorize each of the following expressions:

$(x + y)^2 - (a - b)^2$

Answer

$[(x + y) + (a - b)]\ [(x + y) - (a - b)]$

= (x + y + a – b) (x + y – a + b)

21. Question

Factorize each of the following expressions:

$\frac{1}{16}x^2y^2 - \frac{4}{49}y^2z^2$

Answer

$(\frac{1}{4}xy)^2 - (\frac{2}{7}yz)^2$

$= (\frac{xy}{4} + \frac{2yz}{7})(\frac{xy}{4} - \frac{2yz}{7})$

$= y^2(\frac{x}{4} + \frac{2}{7}z)(\frac{x}{4} - \frac{2}{7}z)$

22. Question

Factorize each of the following expressions:

$75a^3b^2 - 108ab^4$

Answer

$3ab^2(25a^2 - 36b^2)$

$= 3ab^2[(5a)^2 - (6b)^2]$

$= 3ab^2(5a + 6b)(5a - 6b)$

23. Question

Factorize each of the following expressions:

$x^5 - 16x^3$

Answer

$x^3(x^2 - 16)$

$= x^3(x^2 - 4^2)$

$= x^3(x + 4)(x - 4)$

24. Question

Factorize each of the following expressions:

$\frac{50}{x^2} - \frac{2x^2}{81}$

Answer

$2(\frac{25}{x*x} - \frac{x*x}{81})$

$= 2[(\frac{5}{x})^2 - (\frac{x}{9})^2]$

$= 2(\frac{5}{x} + \frac{x}{9})(\frac{5}{x} - \frac{x}{9})$

25. Question

Factorize each of the following expressions:

$256x^5 - 81x$

Answer

$x(256x^4 - 81)$

$= x[(16x^2)^2 - 9^2]$

$= x(16x + 9)(16x - 9)$

26. Question

Factorize each of the following expressions:

$a^4 - (2b + c)^4$

Answer

$(a^2)^2 - [(2b + c)^2]^2$

$= [a^2 + (2b + c)^2][a^2 - (2b + c)^2]$

$= [a^2 + (2b + c)^2][a + 2b + c][a - 2b - c]$

27. Question

Factorize each of the following expressions:

$(3x + 4y)^4 - x^4$

Answer

$[(3x + 4y)^2]^2 - (x^2)^2$

$= [(3x + 4y)^2 + x^2][(3x + 4y)^2 - x^2]$

$= [(3x + 4y)^2 + x^2][3x + 4y + x][3x + 4y - x]$

28. Question

Factorize each of the following expressions:

$p^2q^2 - p^4q^4$

Answer

$(pq)^2 - (p^2q^2)^2$

$= (pq + p^2q^2)(pq - p^2q^2)$

$= (pq)^2(1 + pq)(1 - pq)$

29. Question

Factorize each of the following expressions:

$3x^3y - 24xy^3$

Answer

$3xy(x^2 - 81y^2)$

$= 3xy[x^2 - (9y)^2]$

$= (3xy)(x + 9y)(x - 9y)$

30. Question

Factorize each of the following expressions:

$a^4b^4 - 16c^4$

Answer

$(a^2b^2)^2 - (4c^2)^2$

$= (a^2b^2 + 4c^2)(a^2b^2 - 4c^2)$

$= (a^2b^2 + 4c^2)(ab + 2c)(ab - 2c)$

31. Question

Factorize each of the following expressions:

$x^4 - 625$

Answer

$(x^2)^2 - (25)^2$

$= (x^2 + 25)(x^2 - 25)$

$= (x^2 + 25)(x + 5)(x - 5)$

32. Question

Factorize each of the following expressions:

$x^4 - 1$

Answer

$(x^2)^2 - (1)^2$

$= (x^2 + 1)(x^2 - 1)$

$= (x^2 + 1)(x + 1)(x - 1)$

33. Question

Factorize each of the following expressions:

$49(a-b)^2 - 25(a+b)^2$

Answer

$[7(a - b)]^2 - [5(a + b)]^2$

$= [7(a - b) + 5(a + b)][7(a - b) - 5(a + b)]$

$= (7a - 7b + 5a + 5b)(7a - 7b - 5a - 5b)$

$= (12a - 2b)(2a - 12b)$

$= 2(6a - b)\,2(a - 6b)$

$= 4(6a - b)(a - 6b)$

34. Question

Factorize each of the following expressions:

$x - y - x^2 + y^2$

Answer

$x - y - (x^2 - y^2)$

$= x - y - (x + y)(x - y)$

$= (x - y)(1 - x - y)$

35. Question

Factorize each of the following expressions:

$16(2x-1)^2 - 25y^2$

Answer

$[4(2x-1)]^2 - (5y)^2$

$= (8x - 4 + 5y)(8x - 4 - 5y)$

36. Question

Factorize each of the following expressions:

$4(xy+1)^2 - 9(x-1)^2$

Answer

$[2x(xy + 1)]^2 - [3(x - 1)]^2$

$= (2xy + 2 + 3x - 3)(2xy + 2 - 3x + 3)$

$= (2xy + 3x - 1)(2xy - 3x + 5)$

37. Question

Factorize each of the following expressions:

$(2x+1)^2 - 9x^4$

Answer

$(2x + 1)^2 - (3x^2)^2$

$= (2x + 1 + 3x^2)(2x + 1 - 3x^2)$

$= (3x^2 + 2x + 1)(-3x^2 + 2x + 1)$

38. Question

Factorize each of the following expressions:

$x^4 - (2y-3z)^2$

Answer

$(x^2)^2 - (2y - 3z)^2$

$= (x^2 + 2y - 3z)(x^2 - 2y + 3z)$

39. Question

Factorize each of the following expressions:

$a^2 - b^2 + a - b$

Answer

$(a + b)(a - b) + (a - b)$

$= (a - b)(a + b + 1)$

40. Question

Factorize each of the following expressions:

$16a^4 - b^4$

Answer

$(4a^2)^2 - (b^2)^2$

$= (4a^2 + b^2)(4a^2 - b^2)$

$= (4a^2 + b^2)(2a + b)(2a - b)$

41. Question

Factorize each of the following expressions:

$a^4 - 16(b - c)^4$

Answer

$(a^2)^2 - [4(b - c)^2]$

$= [a^2 + 4(b - c)^2][a^2 - 4(b - c)^2]$

$= [a^2 + 4(b - c)^2][(a + 2b - 2c)(a - 2b + 2c)]$

42. Question

Factorize each of the following expressions:

$2a^4 - 32a$

Answer

$2a(a^4 - 16)$

$= 2a[(a)^2 - (4)^2]$

$= 2a(a^2 + 4)(a^2 - 4)$

$= 2a(a^2 + 4)(a + 2)(a - 2)$

43. Question

Factorize each of the following expressions:

$a^4b^4 - 81c^4$

Answer

$(a^2b^2)^2 - (9c^2)^2$

$= (a^2b^2 + 9c^2)(a^2b^2 - 9c^2)$

$= (a^2b^2 + 9c^2)(ab + 3c)(ab - 3c)$

44. Question

Factorize each of the following expressions:

$xy^9 - yx^9$

Answer

$xy(y^8 - x^8)$

$= xy[(y^4)^2 - (x^4)^2]$

$= xy(y^4 + x^4)(y^4 - x^4)$

$= xy(y^4 + x^4)(y^2 + x^2)(y^2 - x^2)$

$= xy(y^4 + x^4)(y^2 + x^2)(y + x)(y - x)$

45. Question

Factorize each of the following expressions:

$x^3 - x$

Answer

$x (x^2 – 1)$

$= x (x + 1) (x – 1)$

46. Question

Factorize each of the following expressions:

$18^2x^2 - 32$

Answer

$2 [(3ax)^2 – (4)^2]$

$= 2 (3ax + 4) (3ax – 4)$

Exercise 7.6

1. Question

Factorize each of the following algebraic expressions:

$4x^2 + 12xy + 9y^2$

Answer

$4x^2 + 12xy + 9y^2$

$= (2x)^2 + (3y)^2 + 2 (2x) (3y)$

$= (2x + 3y)^2$

2. Question

Factorize each of the following algebraic expressions:

$9a^2 - 24ab + 16b^2$

Answer

Consider $9a^2 – 24ab + 16b^2$,As we know $(x - y)^2 = x^2 + y^2 - 2xy$Here $x = 3a$, $y = 4b$So,

$(3a)^2 + (4b)^2 – 2 (3a) (4a)$

$= (3a – 4b)^2$

3. Question

Factorize each of the following algebraic expressions:

$p^2q^2 - 6pqr + 9r^2$

Answer

$(pq)^2 + (3r)^2 – 2 (pq) (3r)$

$= (pq – 3r)^2$

4. Question

Factorize each of the following algebraic expressions:

$36a^2 + 36a + 9$

Answer

$9 (4a^2 + 4a + 1)$

$= 9 [(2a)^2 + 2 (2a) + 1^1]$

$= 9\ (2a + 1)^2$

5. Question

Factorize each of the following algebraic expressions:

$a^2 + 2ab + b^2 - 16$

Answer

$(a + b)^2 - 4^2$

$= (a + b + 4)\ (a + b - 4)$

6. Question

Factorize each of the following algebraic expressions:

$9z^2 - x^2 + 4xy - 4y^2$

Answer

$(3z)^2 - [x^2 - 2\ (x)\ (2y) + (2y)^2]$

$= (3z)^2 - (x - 2y)^2$

$= [3z + (x - 2y)]\ [3z - (x - 2y)]$

7. Question

Factorize each of the following algebraic expressions:

$9a^4 - 24a^2b^2 + 16b^4 - 256$

Answer

$(3a^2)^2 - 2\ (4a^2)\ (3b^2) + (4b^2)^2 - (16)^2$

$= (3a^2 - 4b^2)^2 - (16)^2$

$= (3a^2 - 4b^2 + 16)\ (3a^2 - 4b^2 - 16)$

8. Question

Factorize each of the following algebraic expressions:

$16 - a^6 + 4a^3b^3 - 4b^6$

Answer

$4^2 - [(a^3)^2 - 2\ (a^3)\ (2b^3) + (2b^3)^2]$

$= 4^2 - (a^3 - 2b^3)^2$

$= [4 + (a^3 - 2b^3)]\ [4 - (a^3 - 2b^3)]$

9. Question

Factorize each of the following algebraic expressions:

$a^2 - 2ab + b^2 - c^2$

Answer

$(a + b)^2 - c^2$

$= (a + b + c)\ (a + b - c)$

10. Question

Factorize each of the following algebraic expressions:

$x^2 + 2x + 1 - 9y^2$

Answer

$(x + 1)^2 - (3y)^2$

$= (x + 3y + 1)(x - 3y + 1)$

11. Question

Factorize each of the following algebraic expressions:

$a^2 + 4ab + 3b^2$

Answer

$a^2 + ab + 3ab + 3b^2$

$= a(a + b) + 3b(a + b)$

$= (a + 3b)(a + b)$

12. Question

Factorize each of the following algebraic expressions:

$96 - 4x - x^2$

Answer

$-x^2 - 4x + 96$

$= -x^2 - 12x + 8x + 96$

$= -x(x + 12) + 8(x + 12)$

$= (x + 12)(-x + 8)$

13. Question

Factorize each of the following algebraic expressions:

$a^4 + 3a^2 + 4$

Answer

$(a^2)^2 + (a^2)^2 + 2(2a^2) + 4 - a^2$

$= (a^2 + 2)^2 + (-a^2)$

$= (a^2 + 2 + a)(a^2 + 2 - a)$

14. Question

Factorize each of the following algebraic expressions:

$4x^4 + 1$

Answer

$(2x^2)^2 + 1 + 4x^2 - 4x^2$

$= (2x^2 + 1)^2 - 4x^2$

$= (2x^2 + 2x + 1)(2x^2 - 2x + 1)$

15. Question

Factorize each of the following algebraic expressions:

$4x^4 + y^4$

Answer

$(2x^2)^2 + (y^2)^2 + 4x^2y^2 - 4x^2y^2$

$= (2x^2 + y^2)^2 - 4x^2y^2$

$= (2x^2 + y^2 + 2xy)\,(2x^2 + y^2 - 2xy)$

16. Question

Factorize each of the following algebraic expressions:

$(x+2)^2 - 6(x+2) + 9$

Answer

$x^2 + 4 + 4x - 6x - 12 + 9$

$= x^2 + 1 - 2x$

$= (x - 1)^2$

17. Question

Factorize each of the following algebraic expressions:

$25 - p^2 - q^2 - 2pq$

Answer

$25 - (p^2 + q^2 + 2pq)$

$= (5)^2 - (p + q)^2$

$= (5 + p + q)\,(5 - p - q)$

$= -(p + q - 5)\,(p + q + 5)$

18. Question

Factorize each of the following algebraic expressions:

$x^2 + 9y^2 - 6xy - 25a^2$

Answer

$(x - 3y)^2 - (5a)^2$

$= (x - 3y + 5a)\,(x - 3y - 5a)$

19. Question

Factorize each of the following algebraic expressions:

$49 - a^2 + 8ab - 16b^2$

Answer

$49 - (a^2 - 8ab + 16b^2)$

$= 49 - (a - 4b)^2$

We know: $a^2 - b^2 = (a + b)(a-b)$

$= (7 + a - 4b)\,(7 - a + 4b)$

$= -(a - 4b + 7)\,(a - 4b - 7)$

20. Question

Factorize each of the following algebraic expressions:

$a^2 - 8ab + 16b^2 - 25c^2$

Answer

$(a - 4b)^2 - (5c)^2$

$= (a - 4b + 5c)(a - 4b - 5c)$

21. Question

Factorize each of the following algebraic expressions:

$x^2 - y^2 + 6y - 9$

Answer

$x^2 + 6y - (y^2 - 6y + 9)$

$= x^2 - (y - 3)^2$

$= (x + y - 3)(x - y + 3)$

22. Question

Factorize each of the following algebraic expressions:

$25x^2 - 10x + 1 - 36y^2$

Answer

$(5x)^2 - 2(5x) + 1 - (6y)^2$

$= (5x - 1)^2 - (6y)^2$

$= (5x - 1 + 6y)(5x - 1 - 6y)$

23. Question

Factorize each of the following algebraic expressions:

$a^2 - b^2 + 2bc - c^2$

Answer

$a^2 - (b^2 - 2bc + c^2)$

$= a^2 - (b - c)^2$

$= (a + b - c)(a - b + c)$

24. Question

Factorize each of the following algebraic expressions:

$a^4 + 2b + b^2 - c^2$

Answer

$(a + b)^2 - c^2$

$= (a + b + c)(a + b - c)$

25. Question

Factorize each of the following algebraic expressions:

$49 - x^2 - y^2 + 2xy$

Answer

$49 - (x^2 + y^2 - 2xy)$

$= 7^2 - (x - y)^2$

$= [7 + (x - y)] [7 - x + y]$

26. Question

Factorize each of the following algebraic expressions:

$a^2 + 4b^2 - 4ab - 4c^2$

Answer

$a^2 - 2\ (a)\ (2b) + (2b)^2 - (2c)^2$

$= (a - 2b)^2 - (2c)^2$

$= (a - 2b + 2c)\ (a - 2b - 2c)$

27. Question

Factorize each of the following algebraic expressions:

$x^2 - y^2 - 4xz + 4z^2$

Answer

$x^2 - 2\ (x)\ (2z) + (2z)^2 - y^2$As $(a-b)^2 = a^2 + b^2 - 2ab$

$= (x - 2z)^2 - y^2$

As $a^2 - b^2 = (a+b)(a-b)$

$= (x - 2z + y)\ (x - 2z - y)$

Exercise 7.7

1. Question

Factorize each of the following algebraic expressions:

$x^2 + 12x - 45$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

$p + q = 12,\ pq = -45$

Clearly,

$15 - 3 = 12,\ 15\ (-3) = -45$

Therefore, split 12x as 15x – 3x

Therefore,

$x^2 + 12x - 45 = x^2 + 15x - 3x - 45$

$= x\ (x + 15) - 3\ (x + 15)$

$= (x - 3)\ (x + 15)$

2. Question

Factorize each of the following algebraic expressions:

$40 + 3x - x^2$

Answer

$-\ (x^2 - 3x - 40)$

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = - 3, pq = - 40

Clearly,

5 – 8 = -3, 5 (-8) = -40

Therefore, split -3x as 5x – 8x

Therefore,

x^2 - 3x – 40 = x^2 + 5x – 8x – 40

= x (x + 5) – 8 (x + 5)

= (x – 8) (x + 5)

3. Question

Factorize each of the following algebraic expressions:

$a^2 + 3a - 88$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = 3, pq = -88

Therefore, split 3a as 11a – 8a

Therefore,

a^2 + 3a – 88 = a^2 + 11a – 8a – 88

= a (a + 11) – 8 (a + 11)

= (x – 8) (a + 11)

4. Question

Factorize each of the following algebraic expressions:

$a^2 - 14a - 51$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = -14, pq = -51

Clearly,

3 – 17 = -14, 3 (-17) = -51

Therefore, split 14a as 3a – 17a

Therefore,

a^2 – 14a – 51 = a^2 + 3a – 17a – 51

= a (a + 3) – 17 (a + 3)

= (a – 17) (a + 3)

5. Question

Factorize each of the following algebraic expressions:

$x^2 + 14x + 45$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = 14, pq = 45

Clearly,

5 + 9 = 14, 5 (9) = 45

Therefore, split 14x as 5x + 9x

Therefore,

$x^2 + 14x + 45 = x^2 + 5x + 9x + 45$

= x (x + 5) – 9 (x + 5)

= (x + 9) (x + 5)

6. Question

Factorize each of the following algebraic expressions:

$x^2 - 22x + 120$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = -22, pq = 120

Clearly,

-12 – 10 = -22, (-12) (-10) = -120

Therefore, split -22x as -12x – 10x

Therefore,

$x^2 - 22x + 120 = x^2 - 12x - 10x + 120$

= x (x - 12) – 10 (x - 12)

= (x – 10) (x - 12)

7. Question

Factorize each of the following algebraic expressions:

$x^2 - 11x - 42$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = -11, pq = -42

Clearly,

3 – 14 = -11, 3 (-14) = -42

Therefore, split (-11x) as 3x – 14x

Therefore,

$x^2 - 11x - 42 = x^2 + 3x - 14x - 42$

= x (x + 3) – 14 (x + 3)

= (x – 14) (x + 3)

8. Question

Factorize each of the following algebraic expressions:

$a^2 + 2a - 3$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = 2, pq = -3

Clearly,

p = 3, q = -1

Therefore, split (2a) as (3a – a)

Therefore,

$a^2 + 2a - 3 = a^2 + 3a - a - 3$

= a (a + 3) – 1 (a + 3)

= (a – 1) (a + 3)

9. Question

Factorize each of the following algebraic expressions:

$a^2 + 14a + 48$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = 14, pq = 48

Clearly,

8 + 6 = 14, 8 (6) = 48

Therefore, split (14a) as 8a + 6a

Therefore,

$a^2 + 14a + 48 = a^2 + 8a + 6a + 48$

= a (a + 8) + 6 (a + 8)

= (a + 6) (a + 8)

10. Question

Factorize each of the following algebraic expressions:

$x^2 - 4x - 21$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = -4, pq = -21

Clearly,

3 – 7 = -4, 3 (-7) = -21

Therefore, split (-4x) as 3x – 7x

Therefore,

$x^2 + 4x - 21 = x^2 + 3x - 7x - 21$

= x (x + 3) – 7 (x + 3)

= (x – 7) (x + 3)

11. Question

Factorize each of the following algebraic expressions:

$y^2 + 5y - 36$

Answer

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = 5, pq = -36

Clearly,

9 – 4 = 5, 9 (-4) = -36

Therefore, split 5y as 9y – 4y

Therefore,

$y^2 + 5y - 36 = y^2 + 9y - 4y - 36$

= y (y + 9) – 4 (y + 9)

= (y – 4) (y + 9)

12. Question

Factorize each of the following algebraic expressions:

$(a^2 - 5a)^2 - 36$

Answer

It can be written as $(a^2 - 5a)^2 - 6^2$

Using $a^2 - b^2 = (a + b)(a - b)$

$(a^2 - 5a)^2 - 6^2 = (a^2 - 5a + 6)(a^2 - 5a - 6)$

To factorize $(a^2 - 5a + 6)$, we need to find p and q where,

p + q = -5, pq = 6

Clearly,

-2 – 3 = -5, (-2) (-3) = 6

Therefore, split -5a as a – 6a

Therefore,

$a^2 - 5a - 6 = a^2 - a - 6a + 6$

= (a – 6) (a – 1)

Therefore,

$(a^2 - 5a)^2 - 3b = (a^2 - 5a + b)(a^2 - 5a - 6)$

= (a – 1) (a – 2) (a – 3) (a – 6)

13. Question

Factorize each of the following algebraic expressions:

$(a + 7)(a - 10) + 16$

Answer

$a^2 - 3a - 54$

In order to factorize the given expression, we find to find two numbers p and q such that:

p + q = -3, pq = -54

Clearly,

6 – 9 = - 3, 6 (-9) = -54

Therefore, split – 3a as 6a – 9a

Therefore,

$a^2 – 3a – 54 = a^2 + 6a – 9a – 54$

= (a - 9) (a + 6)

Therefore,

(a + 7) (a – 10) + 16 = (a – 9) (a + 6)

Exercise 7.8

1. Question

Resolve each of the following quadratic trinomials into factors:

$2x^2 + 5x + 3$

Answer

Here, coefficient of x^2 = 2, coefficient of x = 5and constant term = 3

We shall now split up the coefficient of x i.e., 5 into two parts whose sum is 5 and product is 2 * 3 = 6

So, we write middle term 5x as 2x + 3x

Thus, we have

$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$

= 2x (x + 1) + 3 (x + 1)

= (2x + 3) (x + 1)

2. Question

Resolve each of the following quadratic trinomials into factors:

$2x^2 – 3x – 2$

Answer

Here, coefficient of x^2 = 2, coefficient of x = - 3 and constant term = -2

We shall now split up the coefficient of x i.e., -3 into two parts whose sum is -3 and product is 2 * -2 = - 4

So, we write middle term -3x as -4x + x

Thus, we have

$2x^2 - 3x – 2 = 2x^2 - 4x + x – 2$

= 2x (x – 2) + 1 (x – 2)

= (x – 2) (2x + 1)

3. Question

Resolve each of the following quadratic trinomials into factors:

$3x^2 + 10x + 3$

Answer

Here, coefficient of x^2 = 3, coefficient of x = 10 and constant term = 3

We shall now split up the coefficient of x i.e., 10 into two parts whose sum is 10 and product is 3 * 3 = 9

So, we write middle term 10x as 9x + x

Thus, we have

$3x^2 + 10x + 3 = 3x^2 + 9x + x + 3$

= 3x (x + 3) + 1 (x + 3)

= (3x + 1) (x + 3)

4. Question

Resolve each of the following quadratic trinomials into factors:

$7x - 6 - 2x^2$

Answer

$7x - 6 - 2x^2$ = $- 2x^2 + 7x - 6$

Here, coefficient of x^2 = -2, coefficient of x = 7and constant term = -6

We shall now split up the coefficient of x i.e., 7 into two parts whose sum is 7 and product is -2 * -6 = 12

Clearly,

4 + 3 = 7 and,

4 * 3 = 12

So, we write middle term 7x as 4x + 3x

Thus, we have

$-2x^2 + 7x - 6 = -2x^2 + 4x + 3x - 6$

= -2x (x – 2) + 3 (x – 2)

= (x – 2) (3 – 2x)

5. Question

Resolve each of the following quadratic trinomials into factors:

$7x^2 - 19x - 6$

Answer

Here, coefficient of x^2 = 7, coefficient of x = -19 and constant term = -6

We shall now split up the coefficient of x i.e., -19 into two parts whose sum is -19 and product is 7 * -6 = -42

Clearly,

2 - 21 = -19 and,

2 * (-21) = - 42

So, we write middle term - 19x as 2x - 21x

Thus, we have

$7x^2 - 19x - 6 = 7x^2 + 2x - 21x - 6$

= x (7x + 2) - 3 (7x + 2)

= (7x + 2) (x – 3)

6. Question

Resolve each of the following quadratic trinomials into factors:

$28 - 31x - 5x^2$

Answer

$28 - 31x - 5x^2 = - 5x^2 - 31x + 28$

Here, coefficient of $x^2 = -5$, coefficient of $x = - 31$ and constant term $= 28$

We shall now split up the coefficient of x i.e., - 31 into two parts whose sum is - 31 and product is -5 (28) = - 140

Clearly,

4 - 35 = - 31 and,

4 (-35) = - 140

So, we write middle term - 31x as 4x - 35x

Thus, we have

$- 5x^2 - 31x + 28 = -5x^2 + 4x - 35x + 28$

= -x (5x – 4) - 7 (5x – 4)

= - (x + 7) (5x - 4)

7. Question

Resolve each of the following quadratic trinomials into factors:

$3 + 23y - 8y^2$

Answer

$3 + 23y - 8y^2 = - 8y^2 + 23y + 3$

Here, coefficient of $y^2 = -8$, coefficient of $y = 23$ and constant term $= 3$

We shall now split up the coefficient of x i.e., 23 into two parts whose sum is 23 and product is -8 (3) = - 24

Clearly,

24 - 1 = 23 and,

24 (-1) = - 24

So, we write middle term 23y as 24y - y

Thus, we have

$-8y^2 + 23y + 3 = - 8^2 + 24y - y + 3$

= -8y (y – 3) - 1 (y – 3)

= - (8y + 1) (y – 3)

8. Question

Resolve each of the following quadratic trinomials into factors:

$11x^2 - 54x + 63$

Answer

$11x^2 - 54x + 63$

Here, coefficient of $x^2 = 11$, coefficient of $x = - 54$ and constant term $= 63$

We shall now split up the coefficient of x i.e., -54 into two parts whose sum is - 54 and product is 11 * 63 =

693

Clearly,

-33x - 21x = - 54x and,

(-33) * (-21) = 693

So, we write middle term - 54x as - 33x - 21x

Thus, we have

$11x^2 – 54x + 63 = 11x^2 - 33x - 21x – 6$

= 11x (x – 3) - 21 (x – 3)

= (11x – 21) (x – 3)

9. Question

Resolve each of the following quadratic trinomials into factors:

$7x - 6x^2 + 20$

Answer

$7x – 6x^2 + 20 = - 6x^2 + 7x + 20$

Here, coefficient of x^2 = -6, coefficient of x = 7and constant term = 20

We shall now split up the coefficient of x i.e., 7 into two parts whose sum is 7 and product is -6 * 20 = - 120

Clearly,

15 - 8 = 7 and,

15 (-8) = - 120

So, we write middle term 7x as 15x - 8x

Thus, we have

$-6x^2 + 7x + 20 = -6x^2 + 15x - 8x + 20$

= -3x (2x – 5) - 4 (2x – 5)

= - (3x + 4) (2x - 5)

10. Question

Resolve each of the following quadratic trinomials into factors:

$3x^2 + 22x + 35$

Answer

$3x^2 + 22x + 35$

Here, coefficient of x^2 = 3, coefficient of x = 22 and constant term = 35

We shall now split up the coefficient of x i.e., 22 into two parts whose sum is 22 and product is 3 * 35 = 105

So, we write middle term 22x as 15x + 7x

Thus, we have

$3x^2 + 22x + 35= 3x^2 + 15x + 7x + 35$

= 3x (x + 5) + 7 (x + 5)

= (3x + 7) (x+ 5)

11. Question

Resolve each of the following quadratic trinomials into factors:

$12x^2 - 17xy + 6y^2$

Answer

$12x^2 - 17xy + 6y^2$

Here, coefficient of $x^2 = 12$, coefficient of x = -17and constant term = $6y^2$

We shall now split up the coefficient of middle term i.e., -17y into two parts whose sum is -17y and product is $12 * 6y^2 = 72y^2$

Clearly,

-9y – 8y = -17y and,

$(-9y)(-8y) = 72y^2$

So, we replace middle term -17xy = - 9xy – 8xy

Thus, we have

$12x^2 - 17xy + 6y^2 = 12x^2 - 9xy - 8xy + 6y^2$

= 3x (4x – 3y) – 2y (4x – 3y)

= (3x – 2y) (4x – 3y)

12. Question

Resolve each of the following quadratic trinomials into factors:

$6x^2 - 5xy - 6y^2$

Answer

Here, coefficient of $x^2 = 6$, coefficient of x = -5y and constant term = $- 6y^2$

We shall now split up the coefficient of middle term i.e., -5y into two parts whose sum is -5y and product is $6 (-6y^2) = - 36y^2$

Clearly,

4y – 9y = -5y and,

$(4y)(-9y) = - 36y^2$

So, we replace middle term -5xy = 4xy – 9xy

Thus, we have

$6x^2 - 5xy - 6y^2 = 6x^2 + 4xy - 9xy - 6y^2$

= (2x – 3y) (3x + 2y)

13. Question

Resolve each of the following quadratic trinomials into factors:

$6x^2 - 13xy + 2y^2$

Answer

Here, coefficient of $x^2 = 6$, coefficient of x = -13y and constant term = $2y^2$

We shall now split up the coefficient of middle term i.e., -13y into two parts whose sum is -13y and product is $6 (2y^2) = 12y^2$

Clearly,

-12y – y = -13y and,

(-12y) (-y) = $12y^2$

So, we replace middle term -13xy = -12xy – xy

Thus, we have

$6x^2$ -13xy+ $2y^2$ = $6x^2$ - 12xy - xy - $2y^2$

= (6x – y) (x - 2y)

14. Question

Resolve each of the following quadratic trinomials into factors:

$14x^2 + 11xy - 15y^2$

Answer

Here, coefficient of x^2 = 14, coefficient of x = 11y and constant term = - $15y^2$

We shall now split up the coefficient of middle term i.e., 11y into two parts whose sum is 11y and product is 14 (-$15y^2$) = - $210y^2$

Clearly,

21y – 10y = 11y and,

(21y) (-10y) = - $210y^2$

So, we replace middle term 11xy = 21xy – 10xy

Thus, we have

$14x^2$ + 11xy- $15y^2$ = $14x^2$ + 21xy - 10xy - $15y^2$

= 2x (7x – 5y) + 3y (7x – 5y)

= (2x + 3y) (7x - 5y)

15. Question

Resolve each of the following quadratic trinomials into factors:

$6a^2 + 17ab - 3b^2$

Answer

Here, coefficient of a^2 = 6, coefficient of a = 17b and constant term = - $3b^2$

We shall now split up the coefficient of middle term i.e., 17b into two parts whose sum is 17b and product is 6 (-$3b^2$) = - $18b^2$

Clearly,

18b – b = 17b and,

6 (-$3b^2$) = - $36y^2$

So, we replace middle term 17ab = 18ab – ab

Thus, we have

$6a^2$ +17ab– $3b^2$ = $6a^2$ + 18ab - ab – $3b^2$

= 6a (a + 3b) – b (a + 3b)

= (6a – b) (a + 3b)

16. Question

Resolve each of the following quadratic trinomials into factors:

$36a^2 + 12abc - 15b^2c^2$

Answer

Here, coefficient of $a^2 = 36$, coefficient of $a = 12bc$ and constant term $= -15b^2c^2$

We shall now split up the coefficient of middle term i.e., 12bc into two parts whose sum is 12bc and product is $36 (-15b^2c^2) = -500b^2c^2$

So, we replace middle term $12abc = 30abc - 18abc$

Thus, we have

$36a^2 - 12abc - 15b^2c^2 = 36a^2 + 30abc - 18abc - 15b^2c^2$

$= (6a + 5bc)(6a - 3bc)$

17. Question

Resolve each of the following quadratic trinomials into factors:

$15x^2 - 16xyz - 15y^2z^2$

Answer

Here, coefficient of $x^2 = 15$, coefficient of $x = -16yz$ and constant term $= -15y^2z^2$

We shall now split up the coefficient of middle term i.e., -16yz into two parts whose sum is -16yz and product is $15 (-15y^2z^2) = -225y^2z^2$

Clearly,

$-25yz + 9yz = -16yz$ and,

$(-25yz)(9yz) = -225y^2z^2$

So, we replace middle term $-16xyz = -25yz - 9yz$

Thus, we have

$15x^2 - 16xyz - 15y^2z^2 = 15x^2 - 25yz + 9yz - 15y^2z^2$

$= 5x(3x - 5yz) + 3yz(3x - 5yz)$

$= (5x + 3yz)(3x - 5yz)$

18. Question

Resolve each of the following quadratic trinomials into factors:

$(x - 2y)^2 - 5(x - 2y) + 6$

Answer

$x^2 + 4y^2 - 4xy - 5x + 10y + 6$

Here, coefficient of $(x - 2y)^2 = 1$, coefficient of $(x - 2y) = -5$ and constant $= 6$

We shall now split up the coefficient of middle term i.e., -5 into two parts whose sum is -5 and product is $6 (1) = 6$

Clearly,

$-2 - 3 = -5$ and,

$-2(-3) = 6$

So, we replace $-5(x - 3y) = -2(x - 2y) - 3(x - 2y)$

Thus, we have

$(x - 2y)^2 - 5(x - 2y) + 6 = (x - 2y)^2 - 2(x - 2y) - 3(x - 2y) + 6$

$= (x - 2y - 2)(x - 2y - 3)$

19. Question

Resolve each of the following quadratic trinomials into factors:

$(2a - b)^2 + 2(2a - b) - 8$

Answer

Here, coefficient of $(2a - b)^2 = 1$, coefficient of $(2a - b) = 2$ and constant term $= -8$

We shall now split up the coefficient of middle term i.e., 2 into two parts whose sum is 2 and product is $-8\ (1) = -8$

Clearly,

$4 - 2 = 2$ and,

$4(-2) = -8$

So, we replace $2(2a - b) = 4(2a - b) - 2(2a - b)$

Thus, we have

$(2a - b)^2 + 2(2a - b) - 8 = (2a - b)^2 + 4(2a - b) - 2(2a - b) - 8$

$= (2a - b)(2a - b + 4) - 2(2a - b + 4)$

$= (2a - b - 2)(2a - b + 4)$

Exercise 7.9

1. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$p^2 + 6p + 8$

Answer

$p^2 + 6p + 8$

Here, coefficient of p^2 is unity so we add and subtract square of half of coefficient of p

Therefore,

$p^2 + 6p + 8 = p^2 + 6p + 3^2 - 3^2 + 8$ (Adding and subtracting 3^2)

$= (p + 3)^2 - 1^2$ (By completing the square)

$= (p + 3 - 1)(p + 3 + 1)$

$= (p + 2)(p + 4)$

2. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$q^2 - 10q + 21$

Answer

$q^2 - 10q + 21$ Coefficient of q^2 is 1 so we add and subtract square of half of coefficient of q

Therefore,

$q^2 - 10q + 21 = q^2 - 10q + 5^2 - 5^2 + 21$ (Adding and subtracting 5^2)

$= (q - 5)^2 - 2^2$ (By completing the square)

$= (q - 5 - 2)\ (q - 5 + 2)$

$= (q - 7)\ (q - 3)$

3. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$4y^2 + 12y + 5$

Answer

$4y^2 + 12y + 5$

We have $4y^2 + 12y + 5 = 4\ (y^2 + 3y + \frac{5}{4})$ [Therefore, coefficient of $y^2 = 1$]

$= 4\ [y^2 + 3y + (\frac{3}{2})^2 - (\frac{3}{2})^2 + \frac{5}{4}]$

$= 4\ [(y + \frac{3}{2})^2 - 1^2]$ (Completing the square)

$= 4\ (y + \frac{3}{2} + 1)\ (y + \frac{3}{2} - 1)$

$= (2y + 5)\ (2y + 1)$

4. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$p^2 + 6p - 16$

Answer

$p^2 + 6p - 16$

Coefficient of $p^2 = 1$

Therefore, we have

$p^2 + 6p + 3^2 - 3^2 - 16$ (Adding and subtracting 3^2)

$= (p + 3)^2 - 5^2$ (Completing the square)

$= (p + 3 + 5)\ (p + 3 - 5)$

$= (p + 8)\ (p - 2)$

5. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$x^2 + 12x + 20$

Answer

$x^2 + 12x + 20$

Coefficient of $x^2 = 1$

Therefore, we have

$x^2 + 12x + 6^2 - 6^2 + 20$ (Adding and subtracting 6^2)

$= (x + 6)^2 - 4^2$ (Completing the square)

$= (x + 6 + 4)\ (x + 6 - 4)$

$= (x + 10)\ (x + 2)$

$= 4 [x - \frac{3}{2} + 1] [x - \frac{3}{2} - 1]$

$= (2x - 1) (2x - 5)$

6. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$a^2 - 14a - 51$

Answer

$a^2 - 14a - 51$

Coefficient of $a^2 = 1$

Therefore, we have

$a^2 - 14a - 51 = a^2 - 14a + 7^2 - 7^2 - 51$ (Therefore, adding and subtracting 7^2)

$= (a - 7)^2 - 10^2$ (Completing the square)

$= (a - 7 + 10) (9 - 7 - 10)$

$= (a + 3) (a - 17)$

7. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$a^2 + 2a - 3$

Answer

$a^2 + 2a - 3$

Coefficient of $a^2 = 1$

Therefore, we have

$a^2 + 2a - 3 = a^2 + 2a + 1^2 - 1^2 - 3$ (Adding and subtracting 1^2)

$= (a + 1)^2 - 2^2$ (Completing the square)

$= (a + 1 + 2) (a + 1 - 2)$

$= (a + 3) (a - 1)$

8. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$4x^2 - 12x + 5$

Answer

$4x^2 - 12x + 5$

We have,

$4x^2 - 12x + 5 = 4 (x^2 - 3x + \frac{5}{4})$

$= 4 [x^2 - 3x + (\frac{3}{2})^2 - (\frac{3}{2})^2 + \frac{5}{4})]$ [Therefore, adding and subtracting $(\frac{3}{2})^2$]

$= 4 [(x - \frac{3}{2})^2 - 1^2]$ (Therefore, completing the square)

9. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$y^2 - 7y + 12$

Answer

$y^2 - 7y + 12$

Coefficient of $y^2 = 1$

Therefore, we have

$y^2 - 7y + 12 = y^2 - 7y + (\frac{7}{2})^2 - (\frac{7}{2})^2 + 12$ [By adding and subtracting $(\frac{7}{2})^2$]

$= (y - \frac{7}{2})^2 - (\frac{1}{2})^2$ (Completing the square)

$= (y - \frac{7}{2} \frac{1}{2}) (y - \frac{7}{2} + \frac{1}{2})$

$= (y - 4) (y - 3)$

10. Question

Factorize each of the following quadratic polynomials by using the method of completing;

$z^2 - 4z - 12$

Answer

$z^2 - 4z - 12$

Coefficient of $z^2 = 1$

Therefore, we have

$z^2 - 4z - 12 = z^2 - 4z + 2^2 - 2^2 - 12$ [By adding and subtracting 2^2]

$= (z - 2)^2 - 4^2$ (Completing the square)

$= (z - 2 + 4) (z - 2 - 4)$

$= (z + 2) (z - 6)$