6. Graphs of Trigonometric Functions

Exercise 6.1

1 A. Question

Sketch the graphs of the following functions :

f(x) = 2 sin x, 0 ≤ x ≤ π

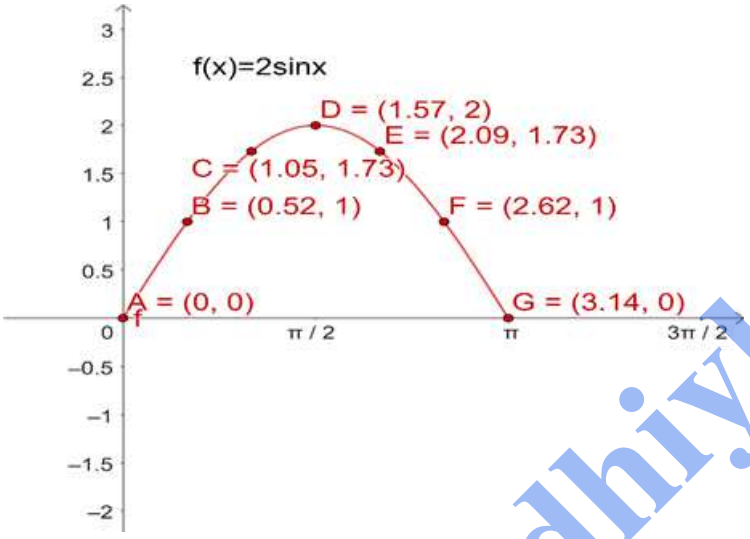
Answer

We know that g (x) = sin x is a periodic function with period π.

∴ f (x) = 2 sin x is a periodic function with period π. So, we will draw the graph of f (x) = 2 sin x in the interval [0, π]. The values of f (x) = 2 sin x at various points in [0, π] are listed in the following table:

X	0 (A)	π/6 (B)	π/3 (C)	π/2 (D)	2π/3 (E)	5π/6 (F)	π (G)
F(x) = 2 sin x	0	1	√3 = 1.73	2	√3 = 1.73	1	0

By plotting the above points, we obtain the required curve.



1 B. Question

Sketch the graphs of the following functions :

g(x) = 3 sin $\left(x-\frac{\pi}{4}\right), 0 \le x \le \frac{5\pi}{4}$

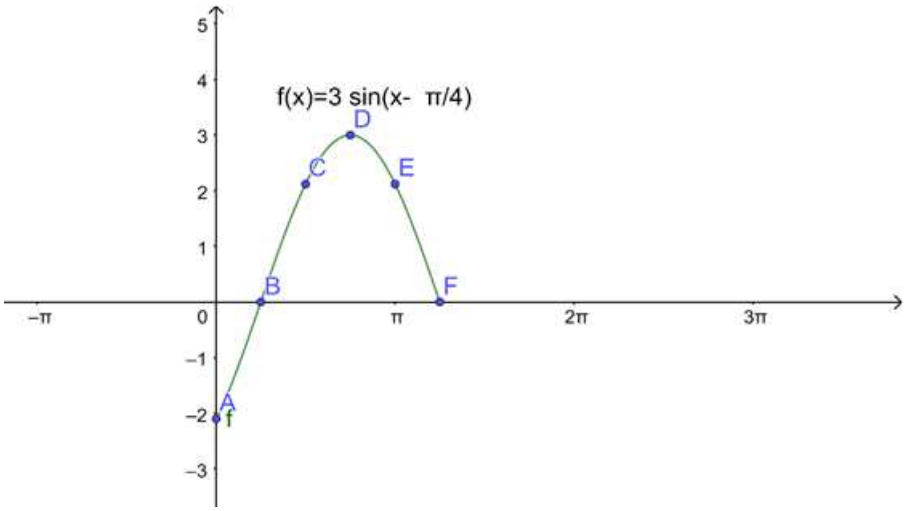
Answer

We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.

∴ $g(x) = 3\sin(x-\frac{\pi}{4})$ is a periodic function with period π. So, we will draw the graph of $g(x) = 3\sin(x-\frac{\pi}{4})$ in the interval [0, 5π/4]. The values of $g(x) = 3\sin(x-\frac{\pi}{4})$ at various points in [0, 5π/4] are listed in the following table:

X	0 (A)	π/4 (B)	π/2 (C)	3π/4 (D)	π (E)	5π/4 (F)
g(x) = 3 sin $\left(x-\frac{\pi}{4}\right)$	-3/√2 = -2.1	0	$\frac{3}{\sqrt{2}}$ = 2.12	3	$\frac{3}{\sqrt{2}}$ = 2.12	0

By plotting the above points, we obtain the required curve.



1 C. Question

Sketch the graphs of the following functions :

h(x) = 2 sin 3x, 0 ≤ x ≤ 2 π/3

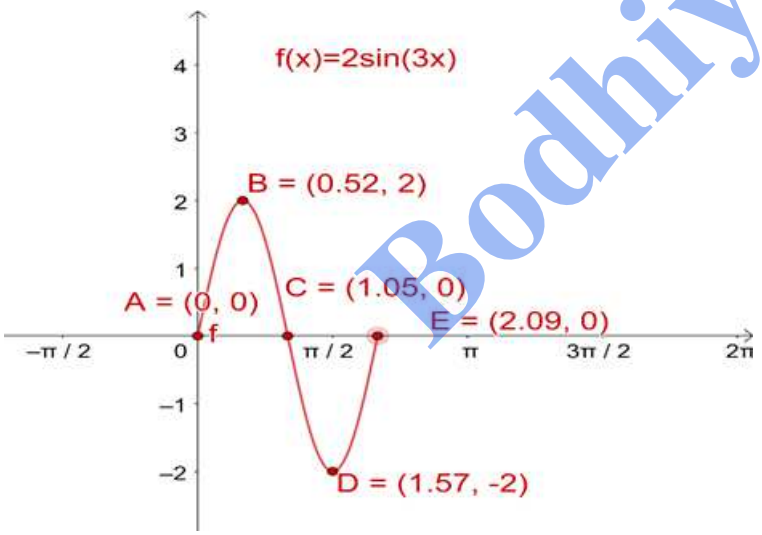
Answer

We know that g (x) = sin x is a periodic function with period 2π.

∴ h (x) = 2 sin 3x is a periodic function with period 2π/3. So, we will draw the graph of h (x) = 2 sin 3x in the interval [0, 2π/3]. The values of h (x) = 2 sin 3x at various points in [0, 2π/3] are listed in the following table:

X	0(A)	π/6(B)	π/3 (C)	π/2 (D)	2π/3 (E)
H(x) = 2 sin 3x	0	2	0	-2	0

By plotting the above points, we obtain the required curve.



1 D. Question

Sketch the graphs of the following functions :

$$\phi(x) = 2\sin\left(2x - \frac{\pi}{3}\right), 0 \le x \le \frac{7\pi}{5}$$

Answer

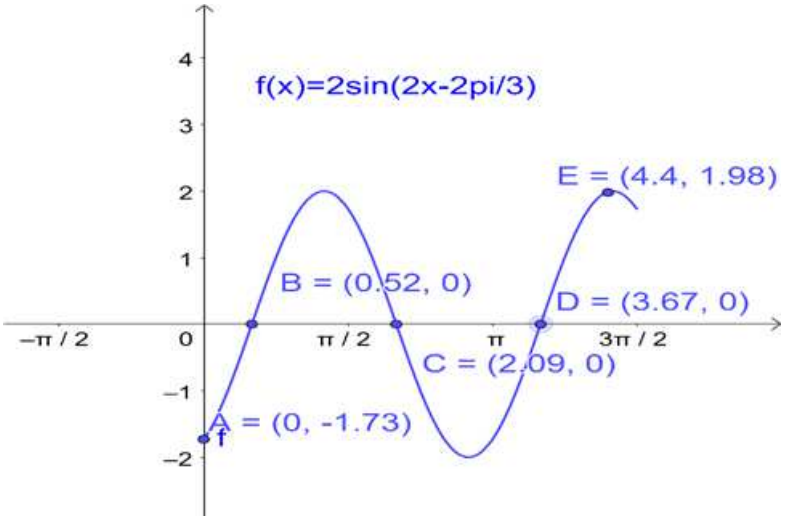
We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.

∴ $\phi\ (x)\ =\ 2\sin(2x - \frac{\pi}{3})$ is a periodic function with period π. So, we will draw the graph of $\phi\ (x)\ =\ 2\sin(2x - \frac{\pi}{3})$ in the interval [0, 7π/5]. The values of $\phi\ (x)\ =\ 2\sin(2x - \frac{\pi}{3})$ at various points in [0,

7π/5] are listed in the following table:

X	0	π/6	2π/3	7π/6	7π/5
$\phi(x) = 2\sin\left(2x-\frac{\pi}{3}\right)$	-√3 = -1.73	0	0	0	1.98

By plotting the above points, we obtain the required curve.



1 E. Question

Sketch the graphs of the following functions :

$$\psi(x) = 4\sin 3\left(x-\frac{\pi}{4}\right), 0 \le x \le 2\pi$$

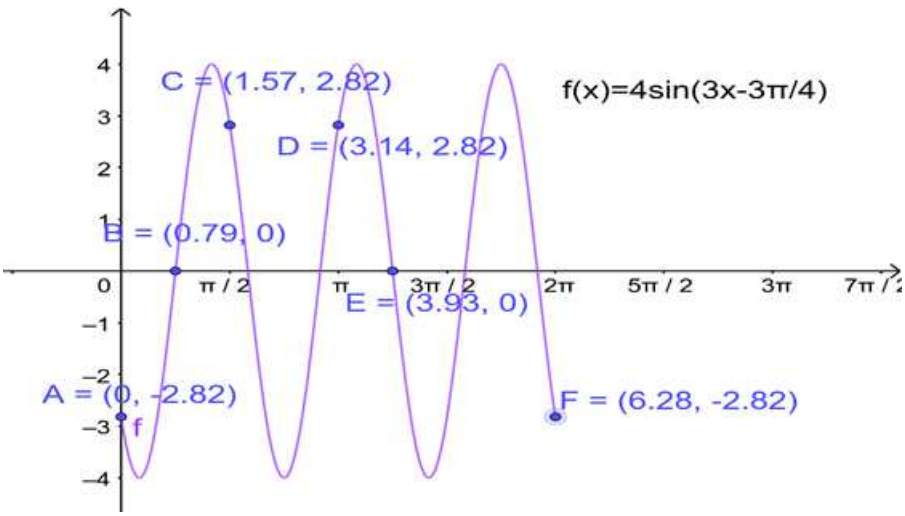
Answer

We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.

$\therefore \psi(x) = 4\sin 3(x-\frac{\pi}{4})$ is a periodic function with period 2π. So, we will draw the graph of $\psi(x) = 4\sin 3(x-\frac{\pi}{4})$ in the interval [0, 2π]. The values of $\psi(x) = 4\sin 3(x-\frac{\pi}{4})$ at various points in [0, 2π] are listed in the following table:

X	0	π/4	π/2	π	5π/4	2π
$\psi(x) = 4\sin 3\left(x-\frac{\pi}{4}\right)$	-2√2 = -2.82	0	2√2 = 2.82	2√2 =2.82	0	-2√2 = -2.82

By plotting the above points, we obtain the required curve.



1 F. Question

Sketch the graphs of the following functions :

$$\theta(x) = \sin\left(\frac{x}{2} - \frac{\pi}{4}\right), 0 \le x \le 4\pi$$

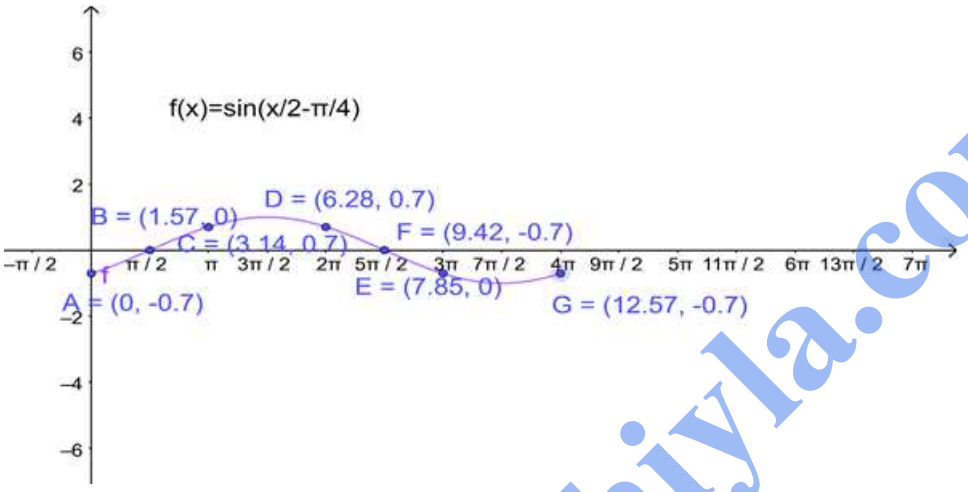
Answer

We know that if f(x) is a periodic function with period T, then f (ax + b) is periodic with period T/|a|.

∴ $\theta\ (x)\ =\ \sin(\frac{x}{2} - \frac{\pi}{4})$ is a periodic function with period 4π. So, we will draw the graph of $\theta\ (x)\ =\ \sin(\frac{x}{2} - \frac{\pi}{4})$ in the interval [0, 4π]. The values of $\theta\ (x)\ =\ \sin(\frac{x}{2} - \frac{\pi}{4})$ at various points in [0, 4π] are listed in the following table:

X	0	π/2	π	2π	5π/2	3π	4π
$\theta(x) = \sin(\frac{x}{2} - \frac{\pi}{4})$	-0.7	0	$1/\sqrt{2}$ = 0.7	$1/\sqrt{2}$ = 0.7	0	$-1/\sqrt{2}$ = -0.7	$-1/\sqrt{2}$ =-0.7

By plotting the above points, we obtain the required curve.



1 G. Question

Sketch the graphs of the following functions :

u(x) = $\sin^2 x$, 0 ≤ x ≤ 2π υ(x) = |sin x|, 0 ≤ x ≤ 2π

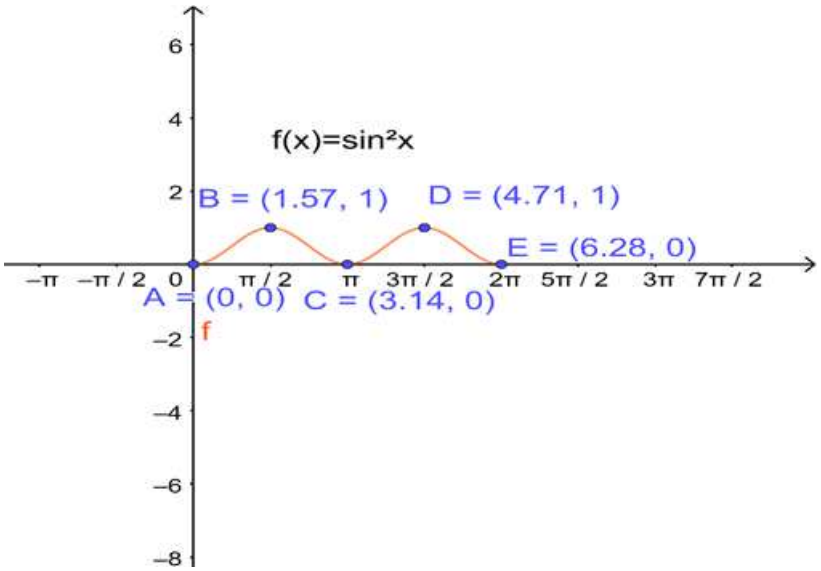
Answer

We know that g (x) = sin x is a periodic function with period π.

∴ u (x) = $\sin^2 x$ is a periodic function with period 2π. So, we will draw the graph of u (x) = $\sin^2 x$ in the interval [0, 2π]. The values of u (x) = $\sin^2 x$ at various points in [0, 2π] are listed in the following table:

X	0	π/2	π	3π/2	2π
U(x) = $\sin^2 x$	0	1	0	1	0

By plotting the above points, we obtain the required curve.

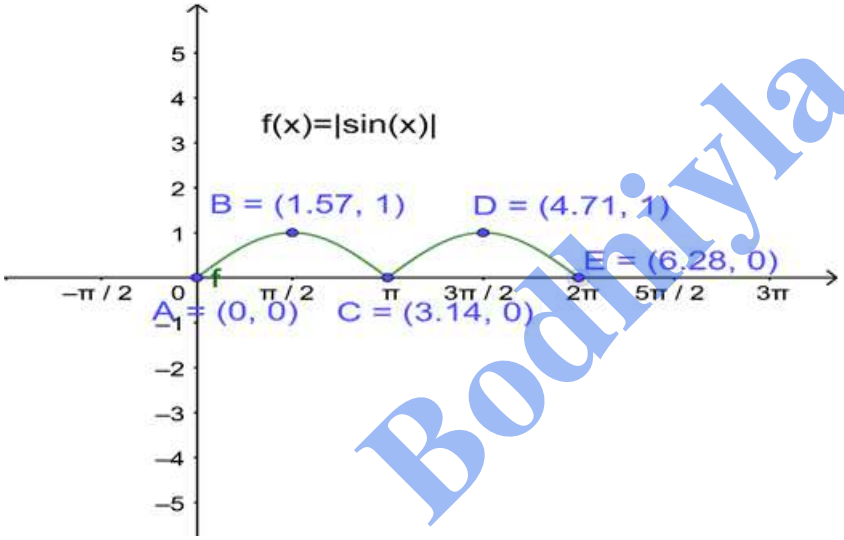


Then,

∴ u (x) = |sin x| is a periodic function with period 2π. So, we will draw the graph of u (x) = |sin x| in the interval [0, 2π]. The values of u (x) = |sin x| at various points in [0, 2π] are listed in the following table:

X	0	π/2	π	3π/2	2π
U(x) = |sin x|	0	1	0	1	0

By plotting the above points, we obtain the required curve.



1 G. Question

Sketch the graphs of the following functions :

f(x) = 2 sin πx, 0 ≤ x ≤ 2.

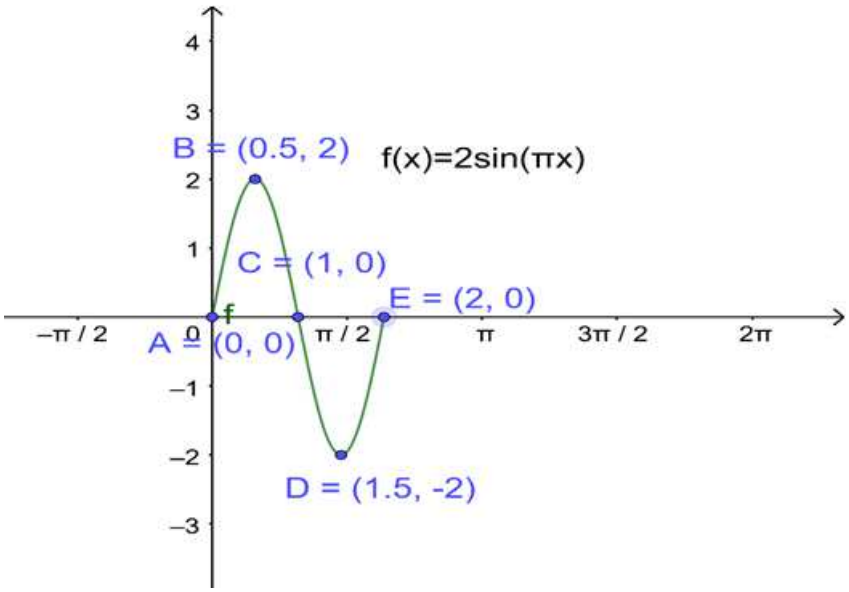
Answer

We know that g (x) = sin x is a periodic function with period 2π.

∴ f (x) = 2 sin πx is a periodic function with period 2. So, we will draw the graph of f (x) = 2 sin πx in the interval [0, 2]. The values of f (x) = 2 sin πx at various points in [0, 2] are listed in the following table:

X	0	1/2	1	3/2	2
f(x) = 2 sin πx	0	2	0	-2	0

By plotting the above points, we obtain the required curve.



2 A. Question

Sketch the graphs of the following pairs of functions on the same axes :

f(x) = sin x, g(x) = sin $\left(x+\frac{\pi}{4}\right)$

Answer

We observe that the functions f(x) = sin x and g (x) = sin (x + π/4) are periodic functions with periods 2π and 7π/4.

The values of these functions are tabulated below:

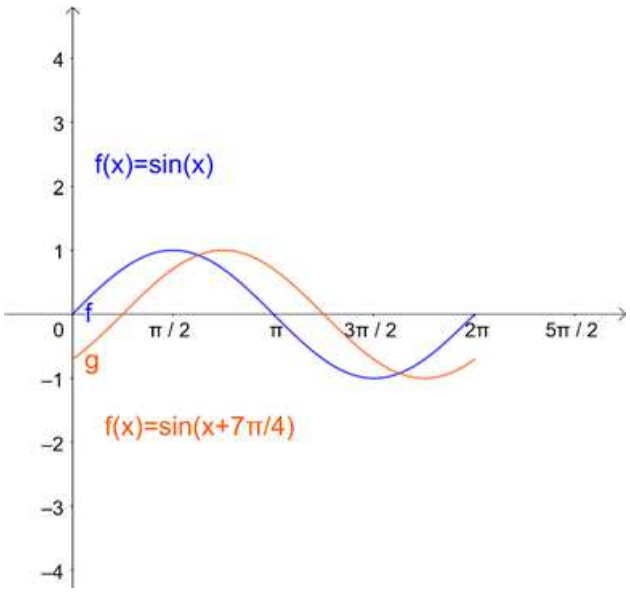
Values of f (x) = sin x in [0, 2π]

X	0	π/2	π	3π/2	2π
f(x) = sin x	0	1	0	-1	0

Values of g (x) = sin (x + π/4) in [0, 7π/4]

X	0	π/4	3π/4	5π/4	7π/4
g(x) = $\sin\left(x+\frac{\pi}{4}\right)$	1/√2 = 0.7	1	0	-1	0

By plotting the above points, we obtain the required curve.



2 B. Question

Sketch the graphs of the following pairs of functions on the same axes :

f(x) = sin x, g(x) = sin 2x

Answer

We observe that the functions f(x) = sin x and g (x) = sin 2x are periodic functions with periods 2π and π.

The values of these functions are tabulated below:

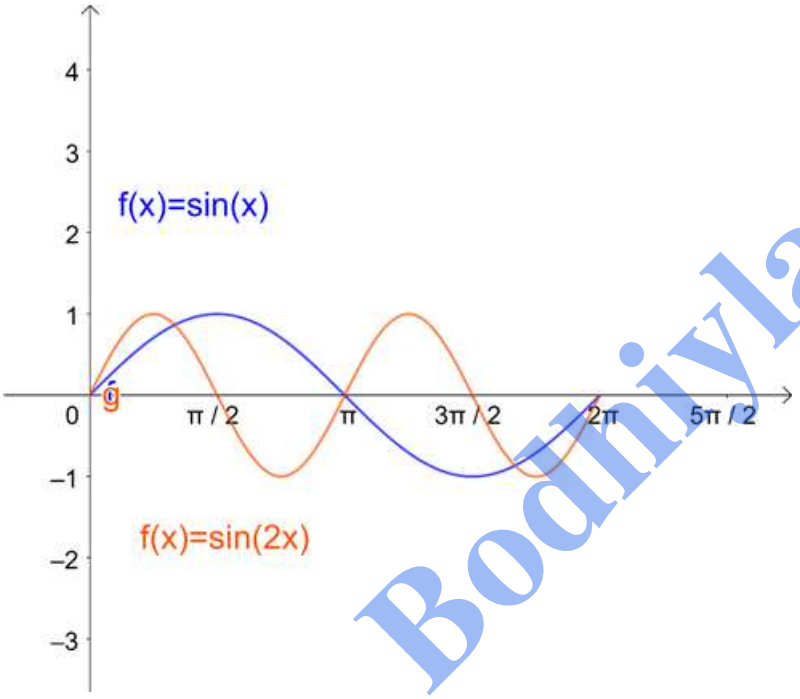
Values of f (x) = sin x in [0, 2π]

X	0	π/2	π	3π/2	2π
f(x) = sin x	0	1	0	-1	0

Values of g (x) = sin (2x) in [0, π]

X	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4	2π
g(x) = sin (2x)	0	1	0	-1	0	1	0	-1	0

By plotting the above points, we obtain the required curve.



2 C. Question

Sketch the graphs of the following pairs of functions on the same axes :

f(x) = sin 2x, g(x) = 2 sin x

Answer

We observe that the functions f(x) = sin 2x and g (x) = 2 sin x are periodic functions with periods π and π.

The values of these functions are tabulated below:

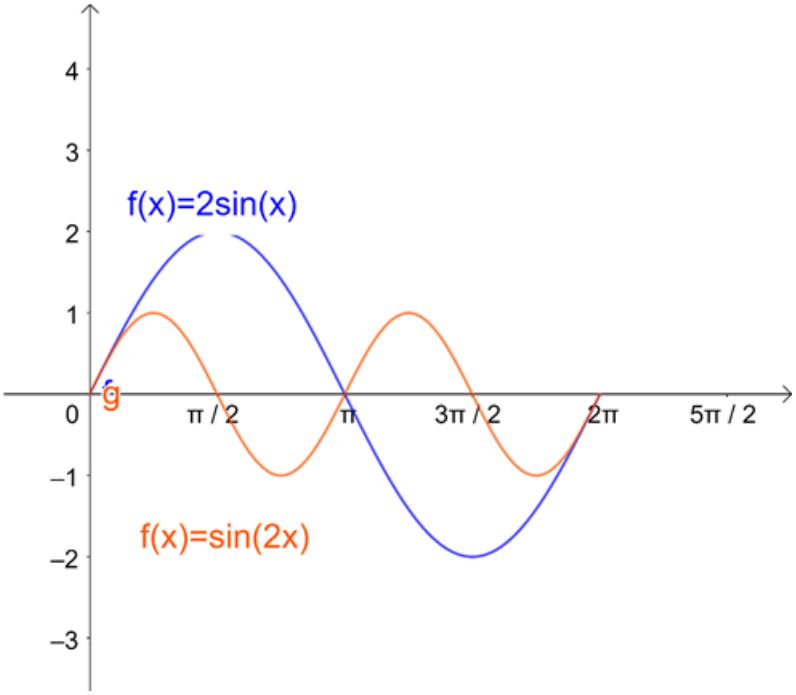
Values of f (x) = sin (2x) in [0, π]

X	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4	2π
f(x) = sin (2x)	0	1	0	-1	0	1	0	-1	0

Values of g (x) = 2 sin x in [0, π]

X	0	π/2	π	3π/2	2π
g(x) = 2 sin x	0	1	0	-1	0

By plotting the above points, we obtain the required curve.



2 D. Question

Sketch the graphs of the following pairs of functions on the same axes :

$f(x) = \sin\frac{x}{2}$, g(x) = sin x

Answer

We observe that the functions f(x) = sin x/2 and g (x) = sin x are periodic functions with periods π and 2π.

The values of these functions are tabulated below:

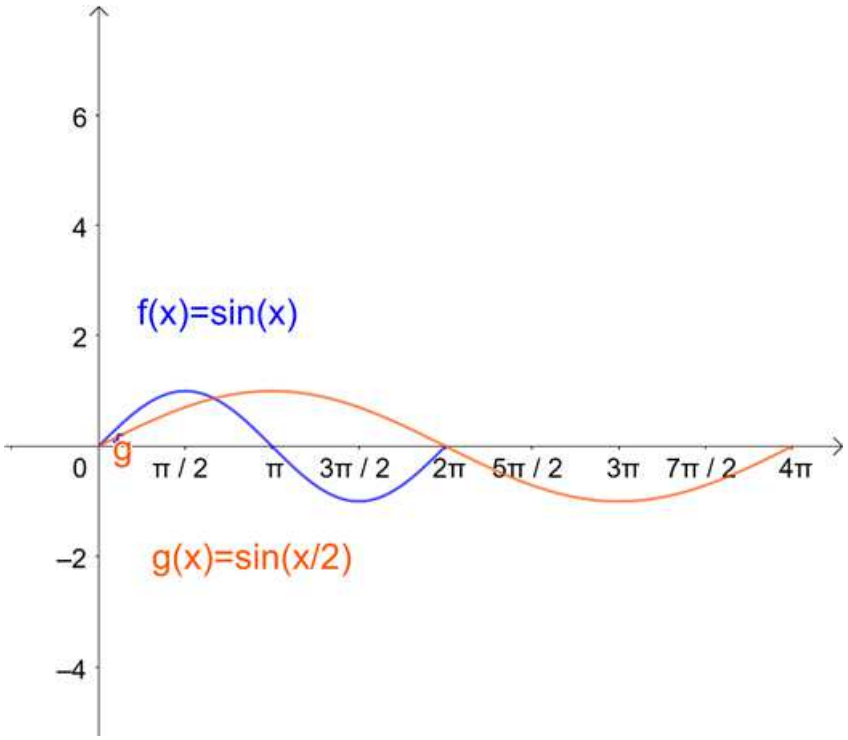
Values of f (x) = sin x/2 in [0, π]

X	0	π	2π	3π	4π
f(x) = sin x/2	0	1	0	-1	0

Values of g (x) = sin (x) in [0, 2π]

X	0	π/2	Π	3π/2	2π	5π/2	3π	7π/2	4π
g(x) = sin (2x)	0	1	0	-1	0	1	0	-1	0

By plotting the above points, we obtain the required curve.



Exercise 6.2

1 A. Question

Sketch the graphs of the following trigonometric functions :

f(x) = $\cos\left(x-\frac{\pi}{4}\right)$

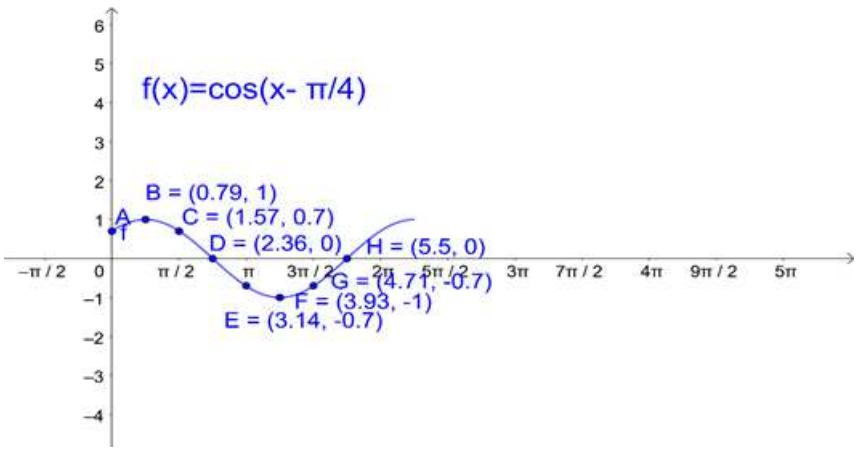
Answer

We know that g (x) = cos x is a periodic function with period 2π.

∴ f (x) = cos (x - π/4) is a periodic function with period π. So, we will draw the graph of f (x) = cos (x - π/4) in the interval [0, π]. The values of f (x) = cos (x - π/4) at various points in [0, π] are listed in the following table:

X	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4
f (x) = cos (x - π/4)	1/√2 = 0.7	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0

By plotting the above points, we obtain the required curve.



1 B. Question

Sketch the graphs of the following trigonometric functions :

g(x) = cos $\left(x+\frac{\pi}{4}\right)$

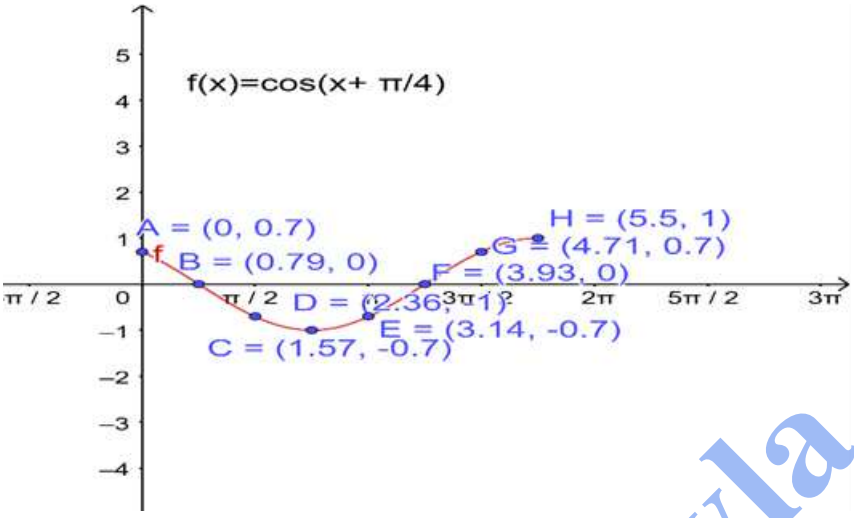
Answer

We know that f (x) = cos x is a periodic function with period 2π.

∴ g (x) = cos (x + π/4) is a periodic function with period π. So, we will draw the graph of g (x) = cos (x + π/4) in the interval [0, π]. The values of g (x) = cos (x + π/4) at various points in [0, π] are listed in the following table:

X	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4
g (x) = cos (x + π/4)	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0	1/√2 = 0.7	1

By plotting the above points, we obtain the required curve.



1 C. Question

Sketch the graphs of the following trigonometric functions :

h(x) = $\cos^2 2x$

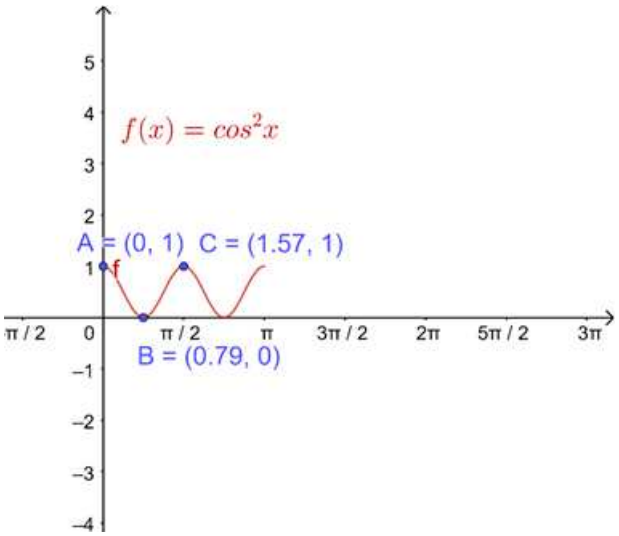
Answer

We know that f (x) = cos x is a periodic function with period 2π.

∴ h (x) = $\cos^2 2x$ is a periodic function with period π. So, we will draw the graph of h (x) = $\cos^2 2x$ in the interval [0, π]. The values of h (x) = $\cos^2 2x$ at various points in [0, π] are listed in the following table:

X	0	π/4	π/2	3π/4	π	5π/4	3π/2
h (x) = $\cos^2 2x$	1	0	1	0	1	0	1

By plotting the above points, we obtain the required curve.



1 D. Question

Sketch the graphs of the following trigonometric functions :

$$\phi(x) = 2\cos\left(x - \frac{\pi}{6}\right)$$

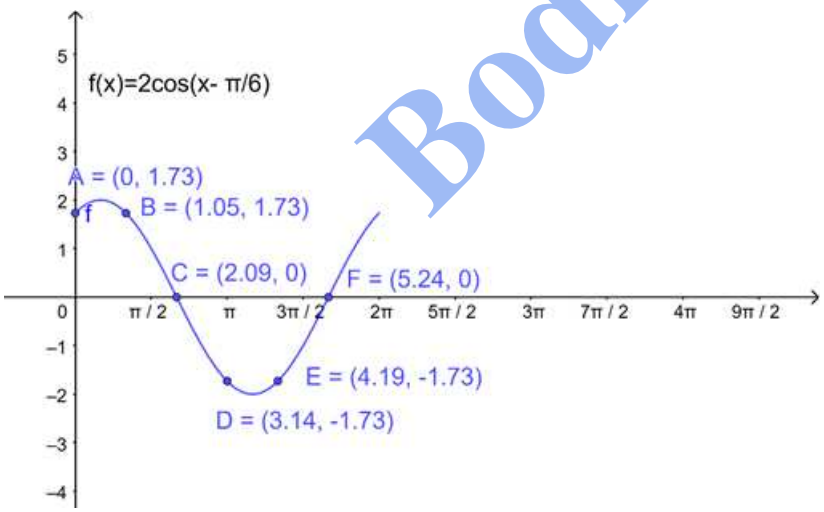
Answer

We know that f (x) = cos x is a periodic function with period 2π.

∴ ϕ (x) = 2cos (x - π/6) is a periodic function with period π. So, we will draw the graph of ϕ (x) = 2cos (x - π/6) in the interval [0, π]. The values of ϕ (x) = 2cos (x - π/6) at various points in [0, π] are listed in the following table:

X	0	π/3	2π/3	π	4π/3	5π/3
ϕ (x) = 2cos (x - π/6)	√3 = 1.73	√3 = 1.73	0	-√3 = -1.73	-√3 = -1.73	0

By plotting the above points, we obtain the required curve.



1 E. Question

Sketch the graphs of the following trigonometric functions :

$$\psi(x) = \cos 3x$$

Answer

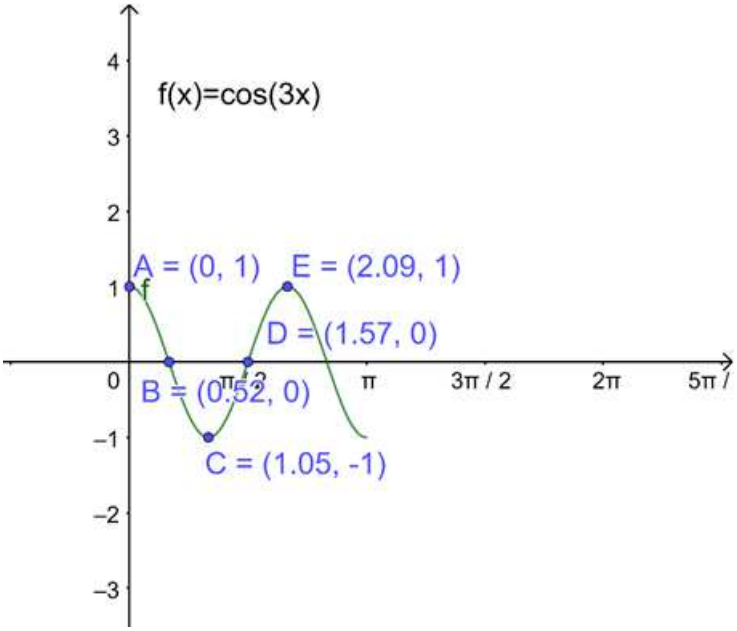
We know that f (x) = cos x is a periodic function with period 2π.

∴ ψ (x) = cos (3x) is a periodic function with period 2π/3. So, we will draw the graph of ψ (x) = cos (3x) in the

interval [0, 2π/3]. The values of ψ (x) = cos (3x) at various points in [0, 2π/3] are listed in the following table:

X	0	π/6	π/3	π/2	2π/3	5π/6
ψ(x) = cos (3x)	1	0	-1	0	1	0

By plotting the above points, we obtain the required curve.



1 F. Question

Sketch the graphs of the following trigonometric functions :

$$u(x) = \cos^2 \frac{x}{2}$$

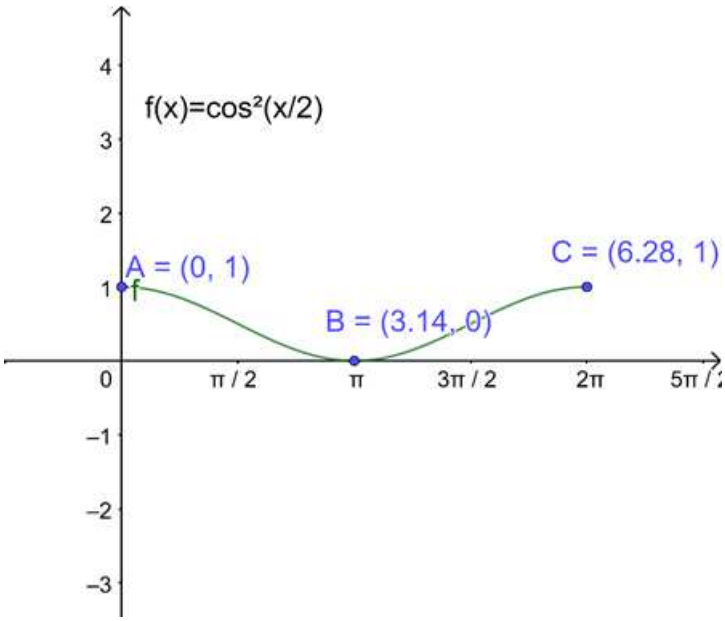
Answer

We know that f (x) = cos x is a periodic function with period 2π.

∴ u (x) = $\cos^2$ (x/2) is a periodic function with period π. So, we will draw the graph of u (x) = $\cos^2$ (x/2) in the interval [0, π]. The values of u (x) = $\cos^2$ (x/2) at various points in [0, π] are listed in the following table:

X	0	π	2π	3π
u (x) = $\cos^2$ (x/2)	1	0	1	0

By plotting the above points, we obtain the required curve.



1 G. Question

Sketch the graphs of the following trigonometric functions :

f(x) = cos π x

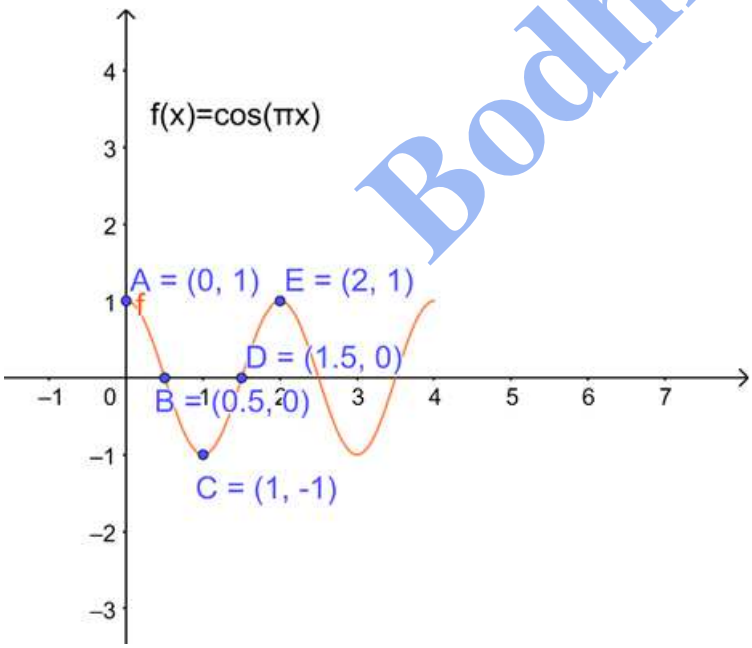
Answer

We know that g (x) = cos x is a periodic function with period 2π.

∴ f (x) = cos (πx) is a periodic function with period 2. So, we will draw the graph of f (x) = cos (πx) in the interval [0, 2]. The values of f (x) = cos (πx) at various points in [0, 2] are listed in the following table:

X	0	1/2	1	3/2	2	5/2
f (x) = cos (πx)	1	0	-1	0	1	0

By plotting the above points, we obtain the required curve.



1 H. Question

Sketch the graphs of the following trigonometric functions :

g(x) = cos 2π x

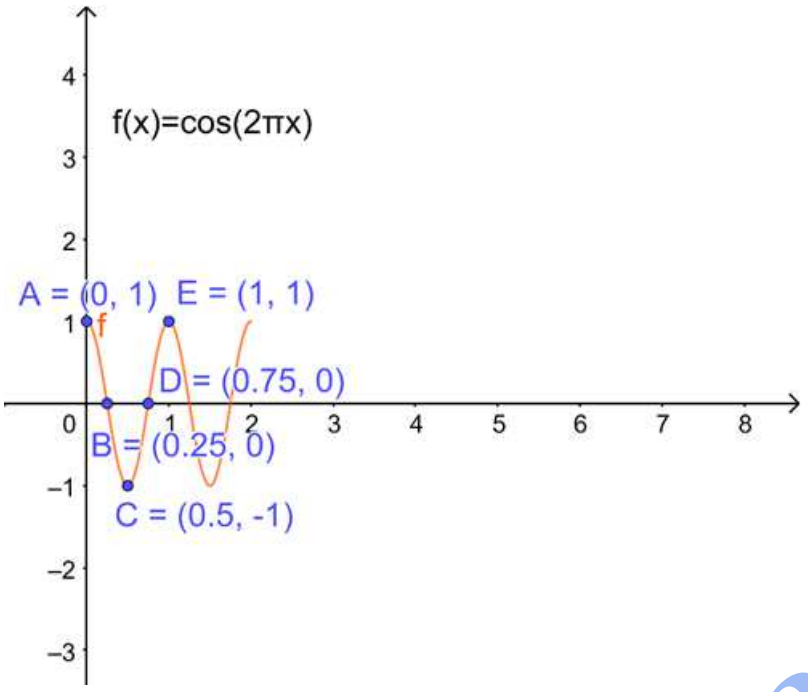
Answer

We know that f (x) = cos x is a periodic function with period 2π.

∴ g (x) = cos (2πx) is a periodic function with period 1. So, we will draw the graph of g (x) = cos (2πx) in the interval [0, 1]. The values of g (x) = cos (2πx) at various points in [0, 1] are listed in the following table:

X	0	1/4	1/2	3/4	1	5/4	3/2	7/4	2
g (x) = cos (2πx)	1	0	-1	0	1	0	-1	0	1

By plotting the above points, we obtain the required curve.



2 A. Question

Sketch the graphs of the following curves on the same scale and the same axes :

y = cos x and y = $\cos\left(x-\frac{\pi}{4}\right)$

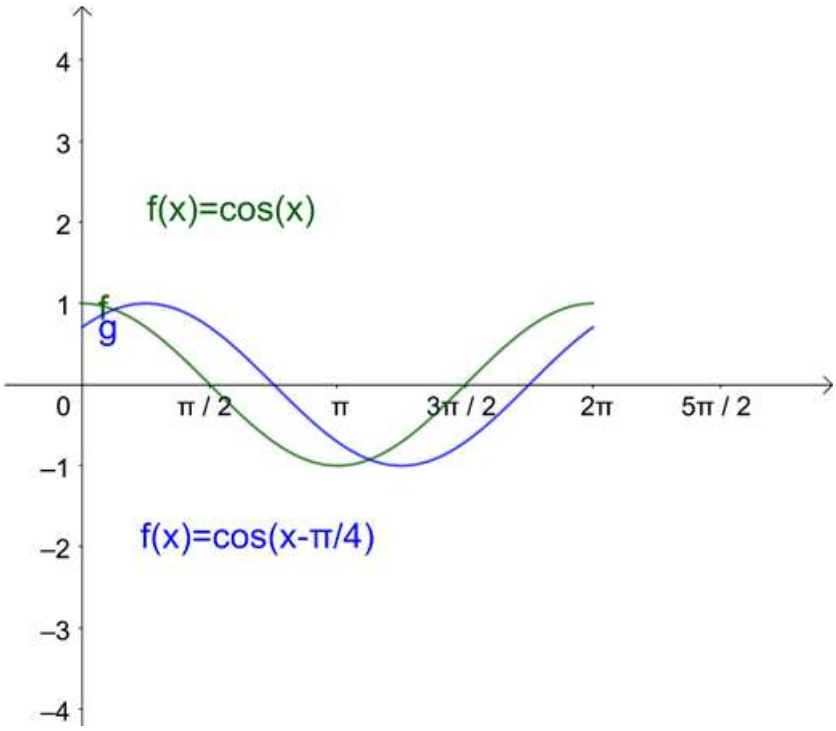
Answer

We observe that the functions y = cos x and y = cos (x - π/4) are periodic functions with periods π and π.

The values of these functions are tabulated below:

X	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4
y = cos x	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0	1
y = cos (x-π/4)	1/√2 = 0.7	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1	-1/√2 = -0.7	0

By plotting the above points, we obtain the required curve.



2 B. Question

Sketch the graphs of the following curves on the same scale and the same axes :

y = cos 2x and y = cos $\left(x-\frac{\pi}{4}\right)$

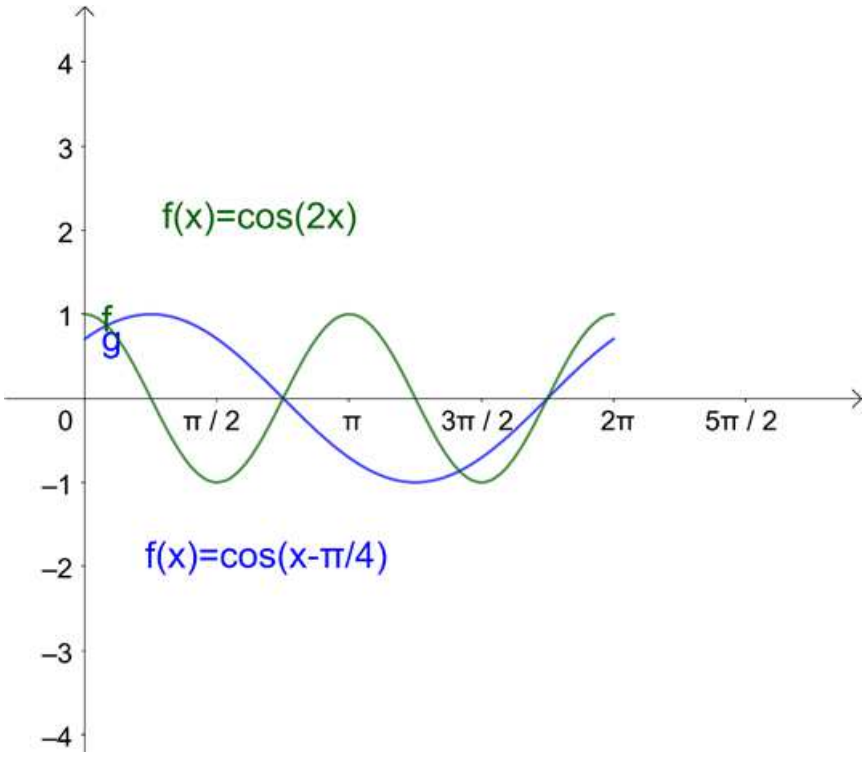
Answer

We observe that the functions y = cos 2x and y = cos 2(x - π/4) are periodic functions with periods π and π.

The values of these functions are tabulated below:

X	0	π/4	π/2	3π/4	π	5π/4	3π/2	7π/4
y = cos 2x	1	0	-1	0	1	0	-1	0
y = cos 2(x-π/4)	0	1	0	-1	0	1	0	-1

By plotting the above points, we obtain the required curve.



2 C. Question

Sketch the graphs of the following curves on the same scale and the same axes :

y = cos x and y = cos $\frac{x}{2}$

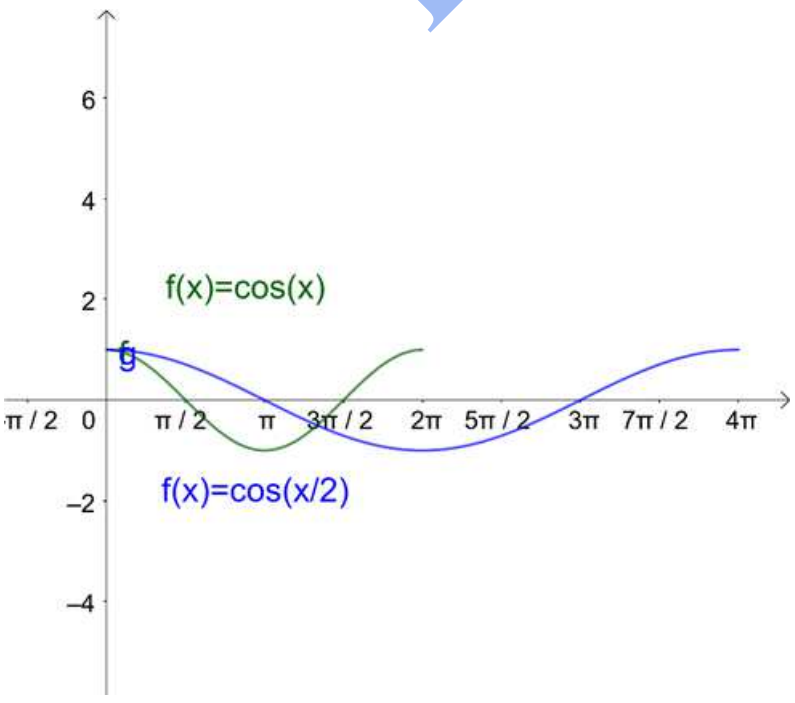
Answer

We observe that the functions y = cos x and y = cos (x/2) are periodic functions with periods π and π.

The values of these functions are tabulated below:

X	0	π/2	π	3π/2	2π
y = cos x	1	0	-1	0	1
y = cos (x/2)	1	1/√2 = 0.7	0	-1/√2 = -0.7	-1

By plotting the above points, we obtain the required curve.



2 D. Question

Sketch the graphs of the following curves on the same scale and the same axes :

$y = \cos^2 x$ and $y = \cos x$

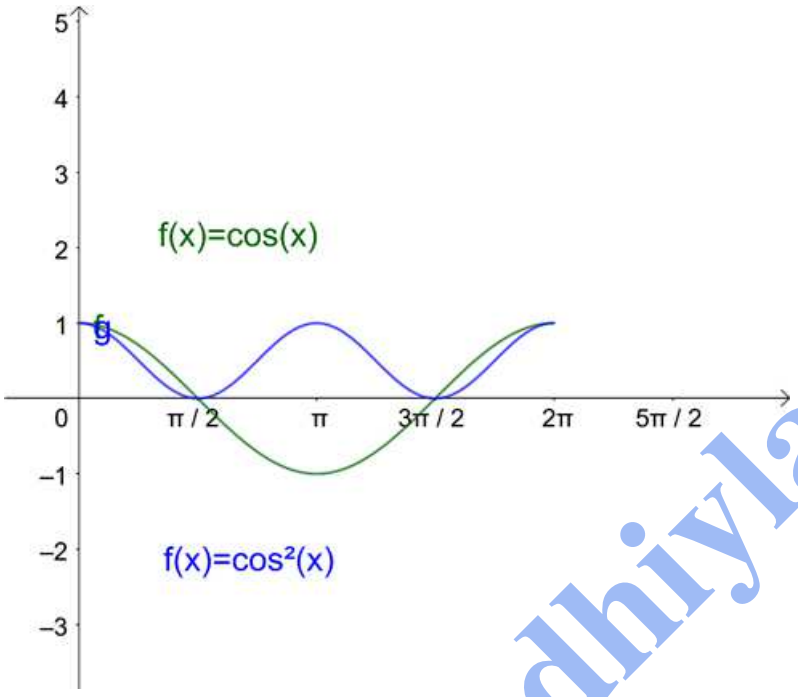
Answer

We observe that the functions $y = \cos^2 x$ and $y = \cos (x)$ are periodic functions with period 2π.

The values of these functions are tabulated below:

X	0	π/2	π	3π/2	2π
$y = \cos^2 x$	1	0	1	0	1
$y = \cos x$	1	0	-1	0	1

By plotting the above points, we obtain the required curve.



Exercise 6.3

1. Question

Sketch the graphs of the following functions :

f(x) = 2 cosec πx

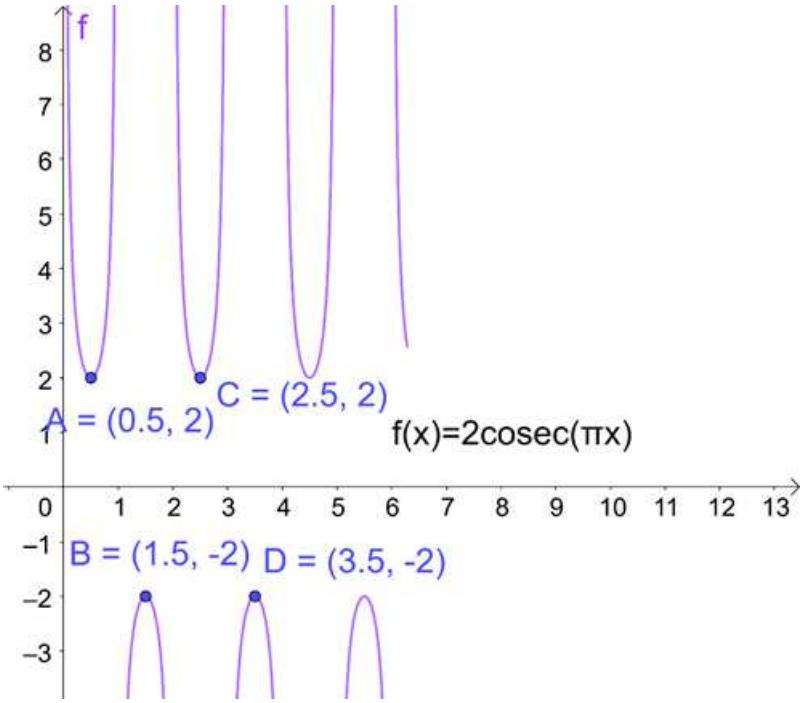
Answer

We know that g (x) = cosec x is a periodic function with period 2π.

∴ f (x) = 2 cosec (πx) is a periodic function with period 2. So, we will draw the graph of f (x) = 2 cosec (πx) in the interval [0, 2]. The values of f (x) = 2 cosec (πx) at various points in [0, 2] are listed in the following table:

X	0	1/2	1	1-	3/2	2-	2	5/2
f (x) = 2 cosec (πx)	∞	2	∞	-∞	-2	-∞	∞	2

By plotting the above points, we obtain the required curve.



2. Question

Sketch the graphs of the following functions :

f(x) = 3 sec x

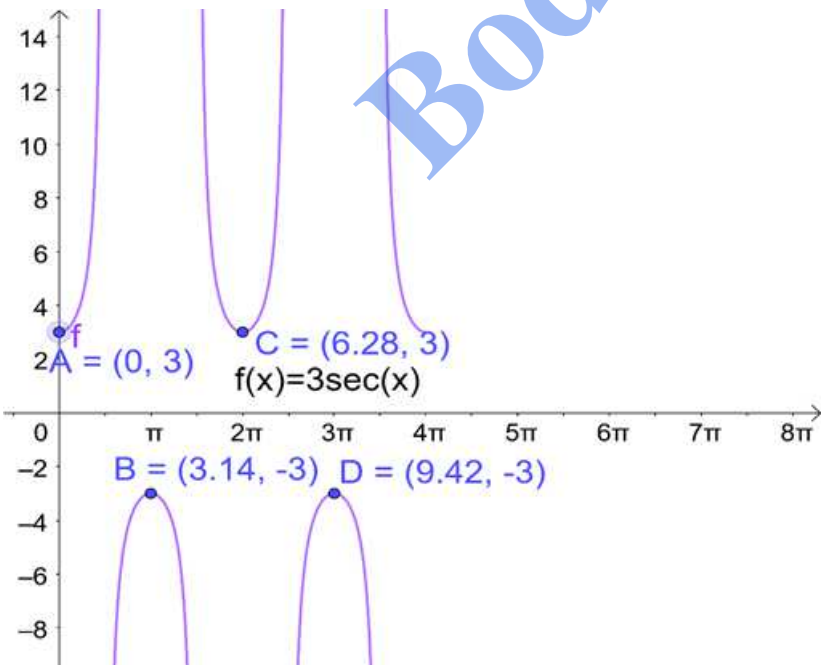
Answer

We know that g (x) = sec x is a periodic function with period π.

∴ f (x) = 3 sec (x) is a periodic function with period π. So, we will draw the graph of f (x) = 3 sec (x) in the interval [0, π]. The values of f (x) = 3 sec (x) at various points in [0, π] are listed in the following table:

X	0	π/2	π/2-	π	3π/2-	3π/2	2π	5π/2
f (x) = 3 sec (x)	3	∞	-∞	-3	-∞	∞	3	∞

By plotting the above points, we obtain the required curve.



3. Question

Sketch the graphs of the following functions :

f(x) = cot 2x

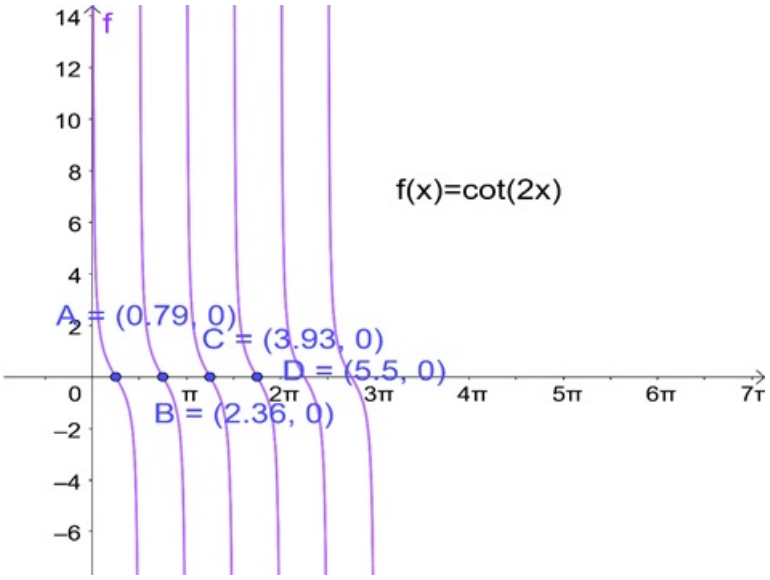
Answer

We know that g (x) = cot x is a periodic function with period π.

∴ f (x) = cot (2x) is a periodic function with period π. So, we will draw the graph of f (x) = cot (2x) in the interval [0, π]. The values of f (x) = cot (2x) at various points in [0, π] are listed in the following table:

X	0	π/4	π/2-	π/2 +	3π/4	Π-
f (x) = cot (2x)	→ ∞	0	-∞	→∞	0	-∞

By plotting the above points, we obtain the required curve.



4. Question

Sketch the graphs of the following functions :

f(x) = 2sec πx

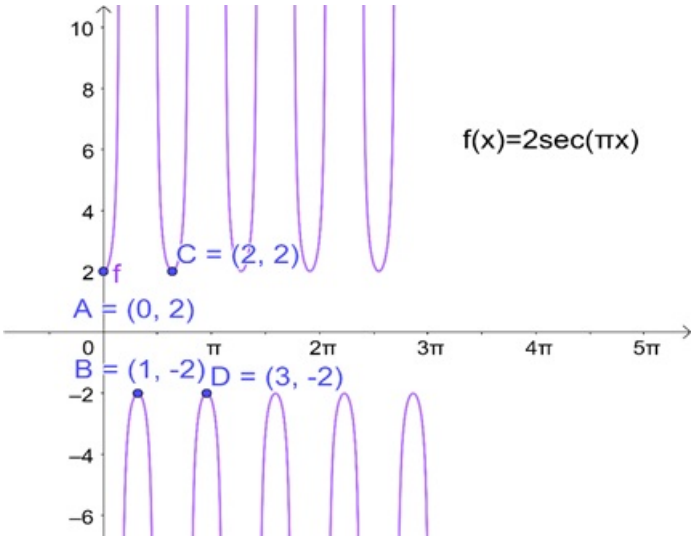
Answer

We know that g (x) = sec x is a periodic function with period π.

∴ f (x) = 2 sec (πx) is a periodic function with period 1. So, we will draw the graph of f (x) = 2 sec (πx) in the interval [0, 1]. The values of f (x) = 2 sec (πx) at various points in [0, 1] are listed in the following table:

X	0	1/2+	1/2-	1	3/2 -	3/2	2
f (x) = 3 sec (x)	2	∞	→-∞	-2	-∞	∞	2

By plotting the above points, we obtain the required curve.



5. Question

Sketch the graphs of the following functions :

$f(x) = \tan^2 x$

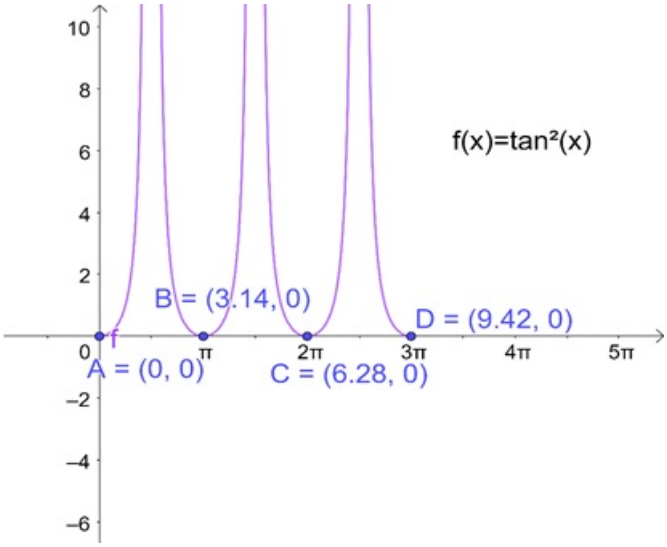
Answer

We know that g (x) = tan x is a periodic function with period π.

∴ f (x) = $\tan^2$ (x) is a periodic function with period π. So, we will draw the graph of f (x) = $\tan^2$ (x) in the interval [0, π]. The values of f (x) = $\tan^2$ (x) at various points in [0, π] are listed in the following table:

X	0	π/2	π/2	Π	3π/2	3π/2	2π
f (x) = $\tan^2$ (x)	0	∞	→ ∞	0	∞	→ ∞	0

By plotting the above points, we obtain the required curve.



6. Question

Sketch the graphs of the following functions :

$f(x) = \cot^2 x$

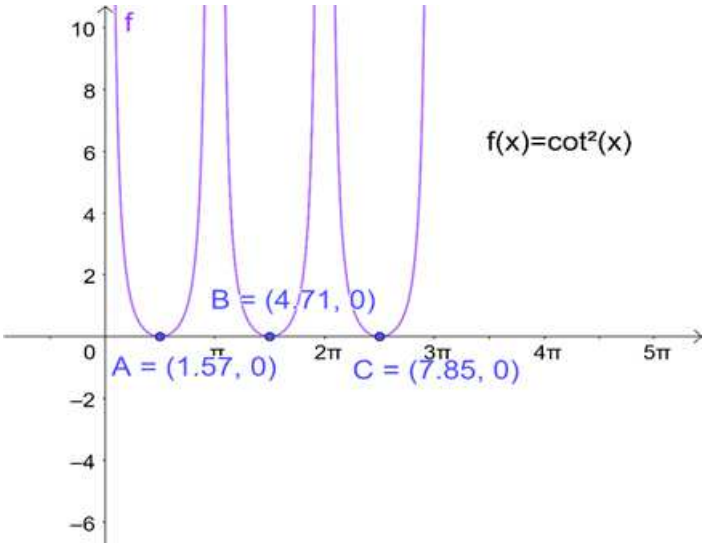
Answer

We know that g (x) = cot x is a periodic function with period π.

∴ f (x) = $\cot^2$ (x) is a periodic function with period π. So, we will draw the graph of f (x) = $\cot^2$ (x) in the interval [0, π]. The values of f (x) = $\cot^2$ (x) at various points in [0, π] are listed in the following table:

X	0	π/2	π	π	3π/2	2π
f (x) = $\cot^2$ (x)	→ ∞	0	∞	→ ∞	0	∞

By plotting the above points, we obtain the required curve.



7. Question

Sketch the graphs of the following functions :

$$f(x)=\cot\frac{\pi x}{2}$$

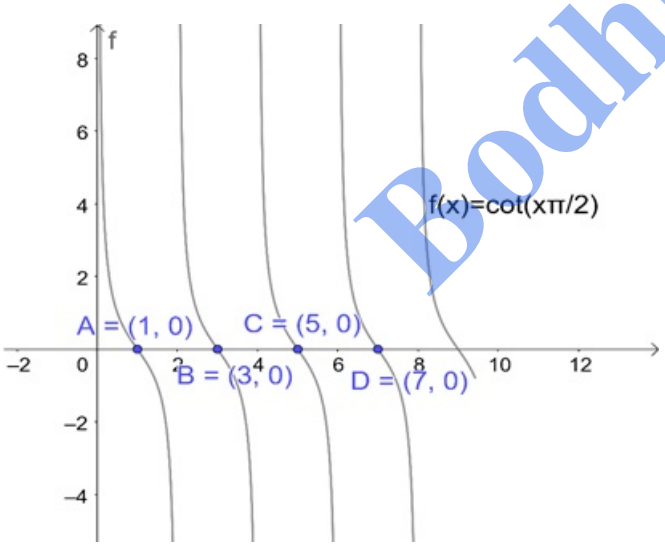
Answer

We know that g (x) = cot x is a periodic function with period π.

∴ f (x) = cot (πx/2) is a periodic function with period 2. So, we will draw the graph of f (x) = cot (πx/2) in the interval [0, 2]. The values of f (x) = cot (πx/2) at various points in [0, 2] is listed in the following table:

X	-2	-1	0-	0 +	1	2
f (x) = cot (πx/2)	→ ∞	0	→ -∞	→ ∞	0	→ -∞

By plotting the above points, we obtain the required curve.



8. Question

Sketch the graphs of the following functions :

$f(x) = \sec^2 x$

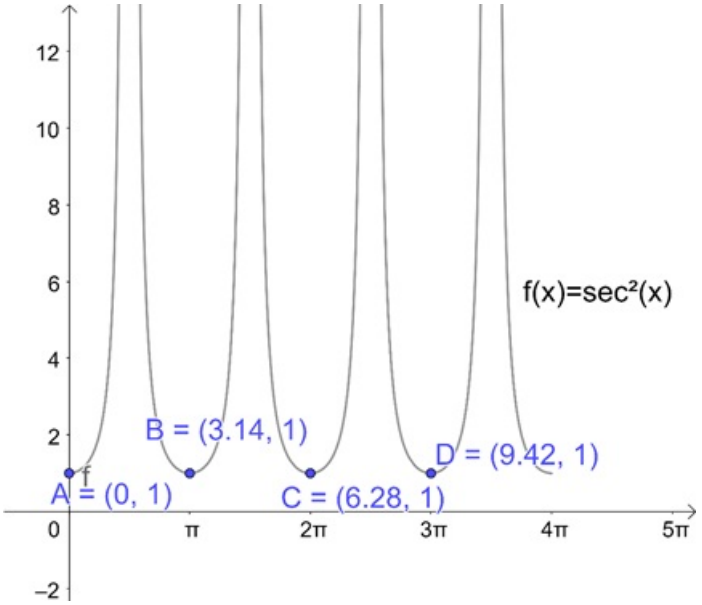
Answer

We know that g (x) = sec x is a periodic function with period π.

∴ f (x) = $\sec^2$ (x) is a periodic function with period π. So, we will draw the graph of f (x) = $\sec^2$ (x) in the interval [0, π]. The values of f (x) = $\sec^2$ (x) at various points in [0, π] are listed in the following table:

X	0	π/2	π/2	π	3π/2	3π/2	2π
f (x) = $\sec^2$ (x)	1	→∞	→-∞	1	→∞	→-∞	1

By plotting the above points, we obtain the required curve.



9. Question

Sketch the graphs of the following functions :

f(x) = cosec^2 x

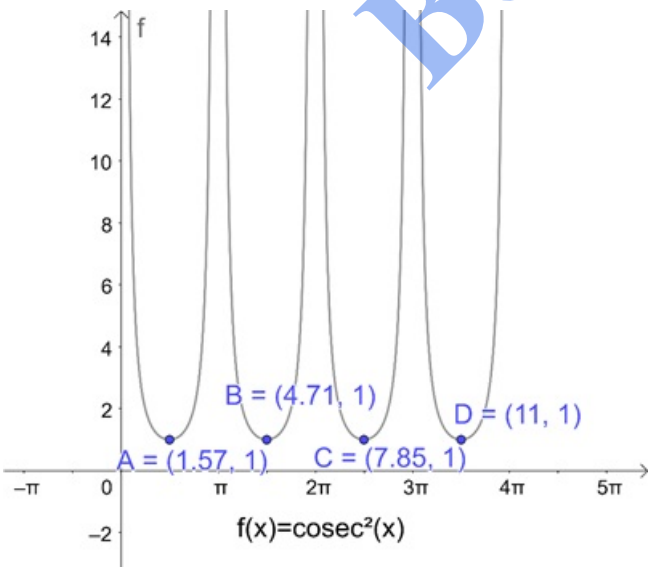
Answer

We know that g (x) = cosec x is a periodic function with period 2π.

∴ f (x) = cosec^2 (x) is a periodic function with period 2π. So, we will draw the graph of f (x) = cosec^2 (x) in the interval [0, 2π]. The values of f (x) = cosec^2 (x) at various points in [0, 2π] are listed in the following table:

X	0	π/2	Π	π	3π/2	2π
f (x) = cosec^2 (x)	→-∞	1	→∞	→-∞	1	→∞

By plotting the above points, we obtain the required curve.



10. Question

Sketch the graphs of the following functions :

f(x) = tan 2x

Answer

We know that g (x) = tan x is a periodic function with period π.

∴ f (x) = tan (2x) is a periodic function with period π/2. So, we will draw the graph of f (x) = tan (2x) in the interval [0, π/2]. The values of f (x) = tan (2x) at various points in [0, π/2] are listed in the following table:

X	-3π/4	-π/2	-π /4	-π/4	0	π/4	π/4	π /2	3π/4
f (x) = tan (2x)	→-∞	0	→ ∞	→ -∞	0	→∞	→ -∞	0	→ ∞

By plotting the above points, we obtain the required curve.