5. Trigonometric Functions

Exercise 5.1

1. Question

Prove the following identities

$\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Answer

LHS = $\sec^4 x - \sec^2 x$

$= (\sec^2 x)^2 - \sec^2 x$

We know $\sec^2 \theta = 1 + \tan^2 \theta$.

$= (1 + \tan^2 x)^2 - (1 + \tan^2 x)$

$= 1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$

$= \tan^4 x + \tan^2 x$ = RHS

Hence proved.

2. Question

Prove the following identities

$\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$

Answer

LHS = $\sin^6 x + \cos^6 x$

$= (\sin^2 x)^3 + (\cos^2 x)^3$

We know that $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$= (\sin^2 x + \cos^2 x)[(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$

We know that $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$= 1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x$

$= 1^2 - 3\sin^2 x \cos^2 x$

$= 1 - 3\sin^2 x \cos^2 x$ = RHS

Hence proved.

3. Question

Prove the following identities

(cosecx – sinx) (secx – cosx) (tanx + cotx) = 1

Answer

LHS = (cosecx – sinx) (secx – cosx) (tanx + cotx)

We know that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}; \tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= \left(\frac{1}{\sin x} - \sin x\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \frac{1 - \sin^2 x}{\sin x} \times \frac{1 - \cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x}$$

= 1 = RHS

Hence proved.

4. Question

Prove the following identities

cosecx (secx – 1) – cotx (1 – cosx) = tanx – sinx

Answer

LHS = cosecx (secx – 1) – cotx (1 – cosx)

We know that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}; \tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= \frac{1}{\sin x}\left(\frac{1}{\cos x} - 1\right) - \frac{\cos x}{\sin x}(1 - \cos x)$$

$$= \frac{1}{\sin x}\left(\frac{1 - \cos x}{\cos x}\right) - \frac{\cos x}{\sin x}(1 - \cos x)$$

$$= \left(\frac{1 - \cos x}{\sin x}\right)\left(\frac{1}{\cos x} - \cos x\right)$$

$$= \left(\frac{1 - \cos x}{\sin x}\right)\left(\frac{1 - \cos^2 x}{\cos x}\right)$$

We know that $1 - \cos^2 x = \sin^2 x$.

$$= \left(\frac{1 - \cos x}{\sin x}\right)\left(\frac{\sin^2 x}{\cos x}\right)$$

$$= (1 - \cos x)\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\sin x}{\cos x} - \sin x$$

$$= \tan x - \sin x$$

= RHS

Hence proved.

5. Question

Prove the following identities

$$\frac{1 - \sin x \cos x}{\cos x(\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} = \sin x$$

Answer

LHS $= \frac{1 - \sin x \cos x}{\cos x(\sec x - \operatorname{cosec} x)} \times \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x}$

We know that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}$

$$= \frac{1 - \sin x \cos x}{\cos x\left(\frac{1}{\cos x} - \frac{1}{\sin x}\right)} \times \frac{(\sin x)^2 - (\cos x)^2}{(\sin x)^3 + (\cos x)^3}$$

We know that $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$= \frac{1 - \sin x \cos x}{\cos x\left(\frac{\sin x - \cos x}{\cos x \sin x}\right)} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$= \frac{\sin x(1 - \sin x \cos x)}{\sin x - \cos x} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$= \frac{\sin x(1 - \sin x \cos x)}{1} \times \frac{1}{[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \sin x(1 - \sin x \cos x) \times \frac{1}{(1 - \sin x \cos x)}$$

= sinx

= RHS

Hence proved.

6. Question

Prove the following identities

$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = (\sec x \operatorname{cosec} x + 1)$$

Answer

LHS $= \frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x}$

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= \frac{\frac{\sin x}{\cos x}}{1 - \frac{\cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{1 - \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{\frac{\cos x - \sin x}{\cos x}}$$

$$= \frac{\sin^2 x}{\cos x(\sin x - \cos x)} - \frac{\cos^2 x}{\sin x(\sin x - \cos x)}$$

$$= \frac{\sin^3 x - \cos^3 x}{\sin x \cos x(\sin x - \cos x)}$$

We know that $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x(\sin x - \cos x)}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$= \frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

We know that $\text{cosec}\,\theta = \frac{1}{\sin\theta}; \sec\theta = \frac{1}{\cos\theta}$

= cosecx × secx + 1

= secx cosecx + 1

= RHS

Hence proved.

7. Question

Prove the following identities

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Answer

LHS $= \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$

We know that $a^3 \pm b^3 = (a \pm b)(a^2 + b^2 \mp ab)$

$$= \frac{(\sin x + \cos x)\,[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x} + \frac{(\sin x - \cos x)\,[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know that $\sin^2 x + \cos^2 x = 1$.

= 1 - sinx cosx + 1 + sinx cosx

= 2

= RHS

Hence proved.

8. Question

Prove the following identities

$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2 = 1$

Answer

LHS $= (\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$

$= [(\sec x \sec y)^2 + (\tan x \tan y)^2 + 2(\sec x \sec y)(\tan x \tan y)] - [(\sec x \tan y)^2 + (\tan x \sec y)^2 + 2(\sec x \tan y)(\tan x \sec y)]$

$= [\sec^2 x \sec^2 y + \tan^2 x \tan^2 y + 2(\sec x \sec y)(\tan x \tan y)] - [\sec^2 x \tan^2 y + \tan^2 x \sec^2 y + 2(\sec^2 x \tan^2 y)(\tan x \sec y)]$

$= \sec^2 x \sec^2 y - \sec^2 x \tan^2 y + \tan^2 x \tan^2 y - \tan^2 x \sec^2 y$

$= \sec^2 x (\sec^2 y - \tan^2 y) + \tan^2 x (\tan^2 y - \sec^2 y)$

$= \sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$

We know that $\sec^2 x - \tan^2 x = 1$.

$= \sec^2 x \times 1 - \tan^2 x \times 1$

$= \sec^2 x - \tan^2 x$

= 1

= RHS

Hence proved.

9. Question

Prove the following identities

$$\frac{\cos x}{1-\sin x}=\frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

Answer

RHS $= \frac{1+\cos x+\sin x}{1+\cos x-\sin x}$

$$=\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}$$

$$=\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}\times\frac{(1+\cos x)+(\sin x)}{(1+\cos x)+(\sin x)}$$

$$=\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$=\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$=\frac{1+\cos^2 x+2\cos x+\sin^2 x+2\sin x+2\sin x\cos x}{1+\cos^2 x+2\cos x-\sin^2 x}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$=\frac{1+1+2\cos x+2\sin x+2\sin x\cos x}{(1-\sin^2 x)+\cos^2 x+2\cos x}$$

We know that $1 - \cos^2 x = \sin^2 x$.

$$=\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$=\frac{2+2\cos x+2\sin x+2\sin x\cos x}{2\cos^2 x+2\cos x}$$

$$=\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$=\frac{1+\cos x+\sin x+\sin x\cos x}{\cos x(\cos x+1)}$$

$$=\frac{1(1+\cos x)+\sin x(\cos x+1)}{\cos x(\cos x+1)}$$

$$=\frac{(1+\sin x)(\cos x+1)}{\cos x(\cos x+1)}$$

$$=\frac{1+\sin x}{\cos x}\times\frac{\cos x}{\cos x}$$

$$=\frac{(1+\sin x)\cos x}{\cos^2 x}$$

We know that $1 - \sin^2 x = \cos^2 x$.

$$=\frac{(1+\sin x)\cos x}{1-\sin^2 x}$$

$$= \frac{(1+\sin x)\cos x}{(1-\sin x)(1+\sin x)}$$

$$= \frac{\cos x}{1-\sin x}$$

= LHS

Hence proved.

10. Question

Prove the following identities

$$\frac{\tan^3 x}{1+\tan^2 x} + \frac{\cot^3 x}{1+\cot^2 x} = \frac{1-2\sin^2 x\cos^2 x}{\sin x\cos x}$$

Answer

LHS $= \frac{\tan^3 x}{1+\tan^2 x} + \frac{\cot^3 x}{1+\cot^2 x}$

We know that $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \mathrm{cosec}^2 x$

$$= \frac{\tan^3 x}{\sec^2 x} + \frac{\cot^3 x}{\mathrm{cosec}^2 x}$$

$$= \frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}} + \frac{\frac{\cos^3 x}{\sin^3 x}}{\frac{1}{\sin^2 x}}$$

$$= \frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x}$$

$$= \frac{\sin^4 x + \cos^4 x}{\cos x\sin x}$$

$$= \frac{(\sin^2 x)^2 + (\cos^2 x)^2}{\cos x\sin x}$$

We know that $a^2 + b^2 = (a + b)^2 - 2ab$

$$= \frac{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x}{\sin x\cos x}$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \frac{1^2 - 2\sin^2 x\cos^2 x}{\sin x\cos x}$$

$$= \frac{1 - 2\sin^2 x\cos^2 x}{\sin x\cos x}$$

= RHS

Hence proved.

11. Question

Prove the following identities

$1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x}$ = sinx cosx

Answer

LHS $= 1 - \frac{\sin^2 x}{1+\cot x} - \frac{\cos^2 x}{1+\tan x}$

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= 1 - \frac{\sin^2 x}{1+\frac{\cos x}{\sin x}} - \frac{\cos^2 x}{1+\frac{\sin x}{\cos x}}$$

$$= 1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\sin x + \cos x}$$

$$= \frac{\sin x + \cos x - (\sin^3 x + \cos^3 x)}{\sin x + \cos x}$$

We know that $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$= \frac{\sin x + \cos x - ((\sin x + \cos x)((\sin x)^2 + (\cos x)^2 - \sin x \cos x))}{\sin x + \cos x}$$

$$= \frac{(\sin x + \cos x)(1 - \sin^2 x - \cos^2 x + \sin x \cos x)}{\sin x + \cos x}$$

$= 1 - (\sin^2 x + \cos^2 x) + \sin x \cos x$

We know that $\sin^2 x + \cos^2 x = 1$.

$= 1 - 1 + \sin x \cos x$

$= \sin x \cos x$

= RHS

Hence proved.

12. Question

Prove the following identities

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x}\right)\sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

Answer

LHS $= \left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x}\right)\sin^2 x \cos^2 x$

We know that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$

$$= \left(\frac{1}{\frac{1}{\cos^2 x} - \cos^2 x} + \frac{1}{\frac{1}{\sin^2 x} - \sin^2 x}\right)\sin^2 x \cos^2 x$$

$$= \left(\frac{\cos^2 x}{1 - \cos^4 x} + \frac{\sin^2 x}{1 - \sin^4 x}\right)\sin^2 x \cos^2 x$$

$$= \left(\frac{\cos^2 x(1 - \sin^4 x) + \sin^2 x(1 - \cos^4 x)}{(1 - \cos^4 x)(1 - \sin^4 x)}\right)\sin^2 x \cos^2 x$$

$$= \left(\frac{\cos^2 x - \cos^2 x \sin^4 x + \sin^2 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)(1 - \sin^2 x)(1 + \cos^2 x)(1 - \cos^2 x)}\right)\sin^2 x \cos^2 x$$

We know that $\sin^2 x + \cos^2 x = 1$.

$$= \left(\frac{1 - \cos^2 x \sin^4 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)\cos^2 x (1 + \cos^2 x)\sin^2 x}\right)\sin^2 x \cos^2 x$$

$$= \left(\frac{1 - \cos^2 x \sin^2 x (\sin^2 x + \cos^2 x)}{(1 + \sin^2 x)(1 + \cos^2 x)}\right)$$

$$= \left(\frac{1 - \cos^2 x \sin^2 x}{2 + \sin^2 x \cos^2 x}\right)$$

= RHS

Hence proved.

13. Question

Prove the following identities

$(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 = \sec^2\alpha \sec^2\beta$

Answer

LHS = $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2$

$= 1 + \tan^2\alpha \tan^2\beta + 2\tan\alpha\tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha\tan\beta$

$= 1 + \tan^2\alpha \tan^2\beta + \tan^2\alpha + \tan^2\beta$

$= \tan^2\alpha(\tan^2\beta + 1) + 1(1 + \tan^2\beta)$

$= (1 + \tan^2\beta)(1 + \tan^2\alpha)$

We know that $1 + \tan^2\theta = \sec^2\theta$

$= \sec^2\alpha \sec^2\beta$

= RHS

Hence proved.

14. Question

Prove the following identities

$$\frac{(1 + \cot x + \tan x)(\sin x - \cos x)}{\sec^3 x - \operatorname{cosec}^3 x} = \sin^2 x \cos^2 x$$

Answer

LHS $= \frac{(1+\cot x+\tan x)(\sin x-\cos x)}{\sec^3 x-\operatorname{cosec}^3 x}$

We know that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$; $\sec\theta = \frac{1}{\cos\theta}$; $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$= \frac{\left(1 + \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)(\sin x - \cos x)}{\frac{1}{\cos^3 x} - \frac{1}{\sin^3 x}}$$

$$= \frac{(\sin x \cos x + \cos^2 x + \sin^2 x)(\sin x - \cos x)(\sin^2 x \cos^2 x)}{\sin^3 x - \cos^3 x}$$

We know that $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$= \frac{(1 + \sin x \cos x)(\sin x - \cos x)(\sin^2 x \cos^2 x)}{(\sin x - \cos x)(\sin^2 x + \cos^2 x + \sin x \cos x)}$$

$$= \frac{(1 + \sin x \cos x)(\sin x - \cos x)(\sin^2 x \cos^2 x)}{(\sin x - \cos x)(1 + \sin x \cos x)}$$

= $\sin^2$x $\cos^{2x}$

= RHS

Hence proved.

15. Question

Prove the following identities

$$\frac{2 \sin x \cos x - \cos x}{1 - \sin x + \sin^2 x - \cos^2 x} = \cot x$$

Answer

LHS $= \frac{2 \sin x \cos x - \cos x}{1 - \sin x + \sin^2 x - \cos^2 x}$

We know that 1 – $\cos^2$x = $\sin^2$x

$$= \frac{\cos x\,(2 \sin x - 1)}{\sin^2 x + \sin^2 x - \sin x}$$

$$= \frac{\cos x\,(2 \sin x - 1)}{2 \sin^2 x - \sin x}$$

$$= \frac{\cos x\,(2 \sin x - 1)}{\sin x\,(2 \sin x - 1)}$$

$$= \frac{\cos x}{\sin x}$$

= cotx

= RHS

Hence proved.

16. Question

Prove the following identities

cosx (tanx + 2) (2 tanx + 1) = 2 secx + 5 sinx

Answer

LHS = cosx (tanx + 2) (2 tanx + 1)

= cosx (2 $\tan^2$x + 5 tanx + 2)

$$= \cos x \left(\frac{2 \sin^2 x}{\cos^2 x} + \frac{5 \sin x}{\cos x} + 2 \right)$$

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$= \frac{2 \sin^2 x + 5 \sin x \cos x + 2 \cos^2 x}{\cos x}$$

$$= \frac{2 + 5 \sin x \cos x}{\cos x}$$

$$= \frac{2}{\cos x} + \frac{5 \sin x \cos x}{\cos x}$$

= 2 secx + 5 sinx

= RHS

Hence proved.

17. Question

If $a = \frac{2\sin x}{1+\cos x+\sin x}$, then prove that $\frac{1-\cos x+\sin x}{1+\sin x}$ is also equal to a.

Answer

Given $a = \frac{2\sin x}{1+\cos x+\sin x}$

Rationalizing the denominator,

$$= \frac{2\sin x}{1+\cos x+\sin x} \times \frac{(1+\sin x)-\cos x}{(1+\sin x)-\cos x}$$

$$= \frac{2\sin x\,[(1+\sin x)-\cos x]}{(1+\sin x)^2-\cos^2 x}$$

$$= \frac{2\sin x\,[(1+\sin x)-\cos x]}{1+\sin^2 x+2\sin x-\cos^2 x}$$

$$= \frac{2\sin x\,[(1+\sin x)-\cos x]}{2\sin^2 x+2\sin x}$$

$$= \frac{2\sin x\,[(1+\sin x)-\cos x]}{2\sin x\,(1+\sin x)}$$

$$= \frac{(1+\sin x)-\cos x}{1+\sin x}$$

$$\therefore a = \frac{1-\cos x+\sin x}{1+\sin x}$$

Hence proved.

18. Question

If $\sin x = \frac{a^2-b^2}{a^2+b^2}$, find the values of tanx, secx and cosecx

Answer

Given $\sin x = \frac{a^2-b^2}{a^2+b^2}$

We know that $\sin^2 x + \cos^2 x = 1 \rightarrow \cos^2 x = 1 - \sin^2 x$

$$\Rightarrow \cos^2 x = 1 - \left(\frac{a^2-b^2}{a^2+b^2}\right)^2$$

$$= \frac{(a^4+b^4+2a^2b^2)-(a^4+b^4-2a^2b^2)}{(a^2+b^2)^2}$$

$$= \frac{4a^2b^2}{(a^2+b^2)^2}$$

$$\Rightarrow \cos x = \frac{2ab}{(a^2+b^2)^2}$$

$$\Rightarrow \tan x = \frac{\sin x}{\cos x} = \frac{\frac{a^2 - b^2}{a^2 + b^2}}{\frac{2ab}{(a^2 + b^2)^2}} = \frac{(a^2 - b^2)}{2ab}$$

$$\Rightarrow \sec x = \frac{1}{\cos x} = \frac{1}{\frac{2ab}{(a^2 + b^2)^2}} = \frac{(a^2 + b^2)^2}{2ab}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{\frac{a^2 - b^2}{a^2 + b^2}} = \frac{a^2 + b^2}{a^2 - b^2}$$

19. Question

If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Answer

Given tanx = b/a

$$\Rightarrow \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{1+\frac{b}{a}}{1-\frac{b}{a}}} + \sqrt{\frac{1-\frac{b}{a}}{1+\frac{b}{a}}}$$

$$= \sqrt{\frac{1+\tan x}{1-\tan x}} + \sqrt{\frac{1-\tan x}{1+\tan x}}$$

$$= \frac{\tan x + 1 + 1 - \tan x}{\sqrt{1 - \tan^2 x}}$$

$$= \frac{2}{\sqrt{1 - \tan^2 x}}$$

$$= \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

$$\therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{2 \cos x}{\sqrt{\cos^2 x - \sin^2 x}}$$

20. Question

If $\tan x = \frac{a}{b}$, show that $\frac{a \sin x - b \cos x}{a \sin x + b \cos x} = \frac{a^2 - b^2}{a^2 + b^2}$.

Answer

Given tanx = a/b

LHS $= \frac{a \sin x - b \cos x}{a \sin x + b \cos x}$

Dividing by b cosx,

$$= \frac{\frac{a \tan x}{b} - 1}{\frac{a \tan x}{b} + 1}$$

Substituting value of tanx,

$$= \frac{a^2 - b^2}{a^2 + b^2}$$

= RHS

Hence proved.

21. Question

If cosecx – sinx = a^3, secx – cosx = b^3, then prove that a^2 b^2 (a^2 + b^2) = 1.

Answer

Given cosecx – sinx = a^3

We know that cosecx = 1/ sinx.

$$\Rightarrow \frac{1}{\sin x} - \sin x = a^3$$

$$\Rightarrow \frac{1 - \sin^2 x}{\sin x} = a^3$$

We know that 1 – $\sin^2$x = $\cos^2$x

$$\therefore a = \left(\frac{\cos^2 x}{\sin x}\right)^{\frac{1}{3}} \ldots (1)$$

Also given secx – cosx = b^3

We know that secx = 1/ cosx

$$\Rightarrow \frac{1}{\cos x} - \cos x = a^3$$

$$\Rightarrow \frac{1 - \cos^2 x}{\cos x} = a^3$$

We know that 1 – $\cos^2$x = $\sin^2$x

$$\therefore a = \left(\frac{\sin^2 x}{\cos x}\right)^{\frac{1}{3}} \ldots (2)$$

Consider LHS = a^2b^2 (a^2 + b^2)

$$= \left(\left(\frac{\cos^2 x}{\sin x}\right)^{\frac{1}{3}}\left(\frac{\sin^2 x}{\cos x}\right)^{\frac{1}{3}}\right)\left(\left(\left(\frac{\cos^2 x}{\sin x}\right)^{\frac{1}{3}}\right)^2 + \left(\left(\frac{\sin^2 x}{\cos x}\right)^{\frac{1}{3}}\right)^2\right)$$

$$= (\sin x \cos x)^{\frac{2}{3}}\left(\frac{(\cos^2 x)^{\frac{2}{3}}}{(\sin x)^{\frac{2}{3}}} + \frac{(\sin^2 x)^{\frac{2}{3}}}{(\cos x)^{\frac{2}{3}}}\right)$$

$$= (\sin x \cos x)^{\frac{2}{3}}\left(\frac{(\cos^3 x)^{\frac{2}{3}} + (\sin^3 x)^{\frac{2}{3}}}{(\sin x)^{\frac{2}{3}}(\cos x)^{\frac{2}{3}}}\right)$$

$$= (\sin x \cos x)^{\frac{2}{3}}\left(\frac{\cos^2 x + \sin^2 x}{(\sin x \cos x)^{\frac{2}{3}}}\right)$$

We know that $\cos^2$ + $\sin^2$x = 1

= 1

= RHS

Hence proved.

22. Question

If cotx(1 + sinx) = 4m and cotx(1 – sinx) = 4n, prove that $(m^2 - n^2)^2 = mn$.

Answer

Given 4m = cotx (1+ sinx) and 4n = cotx (1 – sinx)

Multiplying both equations, we get

⇒ 16mn = $\cot^2x\ (1 - \sin^2x)$

We know that $1 - \sin^2x = \cos^2x$

⇒ 16mn = $\cot^2x \cos^2x$

$\Rightarrow mn = \frac{\cos^4 x}{16\sin^2 x}$ … (1)

Squaring the given equations and then subtracting,

⇒ $16m^2 = \cot^2x\ (1+ \sin x)^2$ and $16n^2 = \cot^2x\ (1 - \sin x)^2$

⇒ $16m^2 - 16n^2 = \cot^2x\ (4 \sin x)$

$\therefore m^2 - n^2 = \frac{\cot^2 x \sin x}{4}$

Squaring both sides,

$\Rightarrow (m^2 - n^2)^2 = \frac{\cot^4 x \sin^2 x}{16}$

$\Rightarrow (m^2 - n^2)^2 = \frac{\cos^4 x \sin^2 x}{16 \sin^2 x}$… (2)

From (1) and (2),

⇒ $(m^2 - n^2) = mn$

Hence proved.

23. Question

If sinx + cosx = m, then prove that $\sin^6x + \cos^6x = \frac{4-3(m^2-1)^2}{4}$, where $m^2 \le 2$

Answer

Given sinx + cosx = m

We have to prove that $\sin^6 x + \cos^6 x = \frac{4-3(m^2-1)^2}{4}$

Proof:

LHS = $\sin^6x + \cos^6x$

= $(\sin^2x)^3 + (\cos^2x)^3$

We know that $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

= $(\sin^2x + \cos^2x)^3 - 3\sin^2x \cos^2x(\sin^2x + \cos^2x)$

= $1 - 3 \sin^2x \cos^2x$

RHS $= \frac{4-3(m^2-1)^2}{4}$

$= \frac{4 - 3((\sin x + \cos x)^2 - 1)^2}{4}$

$$= \frac{4 - 3(\sin^2 x + \cos^2 x + 2\sin x\cos x - 1)^2}{4}$$

$$= \frac{4 - 3(\sin^2 x - (1 - \cos^2 x) + 2\sin x\cos x)^2}{4}$$

$$= \frac{4 - 3 \times 4\sin^2 x\cos^2 x}{4}$$

= 1 – 3 $\sin^2x \cos^2x$

LHS = RHS

Hence proved.

24. Question

If a = secx – tanx and b = cosecx + cotx, then show that ab + a – b + 1 = 0.

Answer

Given a = secx – tanx and b = cosecx + cotx

a $= \frac{1-\sin x}{\cos x}$ and b $= \frac{1+\cos x}{\sin x}$

LHS = ab + a – b + 1

$$= \left(\frac{1 - \sin x}{\cos x}\right)\left(\frac{1 + \cos x}{\sin x}\right) + \frac{1 - \sin x}{\cos x} - \frac{1 + \cos x}{\sin x} + 1$$

$$= \frac{1 - \sin x + \cos x - \sin x\cos x + \sin x - \sin^2 x - \cos x - \cos^2 x + \sin x\cos x}{\sin x\cos x}$$

$$= \frac{1 - \sin^2 x - \cos^2 x}{\sin x\cos x}$$

= 0 = RHS

Hence proved.

25. Question

Prove that :

$$\left|\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}}\right| = -\frac{2}{\cos x}, \text{ where } \frac{\pi}{2} < x < \pi$$

Answer

LHS $= \left|\sqrt{\frac{1-\sin x}{1+\sin x}} + \sqrt{\frac{1+\sin x}{1-\sin x}}\right|$

$$= \left|\sqrt{\frac{1 - \sin x(1 - \sin x)}{1 + \sin x(1 - \sin x)}} + \sqrt{\frac{1 + \sin x(1 + \sin x)}{1 - \sin x(1 + \sin x)}}\right|$$

$$= \left|\sqrt{\frac{(1 - \sin x)^2}{(1 - \sin^2 x)}} + \sqrt{\frac{(1 + \sin x)^2}{1 - \sin^2 x}}\right|$$

$$= \left|\frac{1 - \sin x + 1 + \sin x}{\cos x}\right|$$

$$= \left|\frac{2}{\cos x}\right|$$

$= -\frac{2}{\cos x}$[∵ π/2 <x < π and in second quadrant, cosx is negative]

= RHS

Hence proved.

26 A. Question

If $T_n = \sin^n x + \cos^n x$, prove that

$$\frac{T_3 - T_5}{T_1} = \frac{T_5 - T_7}{T_3}$$

Answer

Given $T_n = \sin^n x + \cos^n x$

LHS $= \frac{T_3 - T_5}{T_1}$

$$= \frac{(\sin^3 x + \cos^3 x) - (\sin^5 x + \cos^5 x)}{\sin x + \cos x}$$

$$= \frac{\sin^3 x - \sin^5 x + \cos^3 x - \cos^5 x}{\sin x + \cos x}$$

$$= \frac{\sin^3 x (1 - \sin^2 x) + \cos^3 x (1 - \cos^2 x)}{\sin x + \cos x}$$

$$= \frac{\sin^3 x \cos^2 x + \cos^3 x \sin^2 x}{\sin x + \cos x}$$

$$= \frac{\sin^2 x \cos^2 x (\sin x + \cos x)}{\sin x + \cos x}$$

$= \sin^2 x \cos^2 x$

RHS $= \frac{T_5 - T_7}{T_3}$

$$= \frac{(\sin^5 x + \cos^5 x) - (\sin^7 x + \cos^7 x)}{\sin^3 x + \cos^3 x}$$

$$= \frac{\sin^5 x - \sin^7 x + \cos^5 x - \cos^7 x}{\sin^3 x + \cos^3 x}$$

$$= \frac{\sin^5 x (1 - \sin^2 x) + \cos^5 x (1 - \cos^2 x)}{\sin x + \cos x}$$

$$= \frac{\sin^5 x \cos^2 x + \cos^5 x \sin^2 x}{\sin x + \cos x}$$

$$= \frac{\sin^2 x \cos^2 x (\sin^3 x + \cos^3 x)}{\sin^3 x + \cos^3 x}$$

$= \sin^2 x \cos^2 x$

LHS = RHS

Hence proved.

26 B. Question

If $T_n = \sin^n x + \cos^n x$, prove that

$2 T_6 - 3 T_4 + 1 = 0$

Answer

Given $T_n = \sin^n x + \cos^n x$

LHS = $2T_6 - 3T_4 + 1$

$= 2 (\sin^6 x + \cos^6 x) - 3 (\sin^4 x + \cos^4 x) + 1$

$= 2 (\sin^2 x + \cos^2 x) (\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x) - 3 (\sin^4 x + \cos^4 x) + 1$

We know that $\sin^2 x + \cos^2 x = 1$.

$= 2 (1) (\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x) - 3 (\sin^4 x + \cos^4 x) + 1$

$= 2\sin^4 x + 2\cos^4 x - 2\sin^2 x \cos^2 x - 3\sin^4 x - 3\cos^4 x + 1$

$= - (\sin^4 x + \cos^4 x) - 2\sin^2 x \cos^2 x + 1$

$= - (\sin^2 x + \cos^2 x)^2 + 1$

$= - 1 + 1$

$= 0$

= RHS

Hence proved.

26 C. Question

If $T_n = \sin^n x + \cos^n x$, prove that

$6 T_{10} - 15 T_8 + 10 T_6 - 1 = 0$

Answer

Given $T_n = \sin^n x + \cos^n x$

LHS = $6T_{10} - 15 T_8 + 10T_6 - 1$

$= 6 (\sin^{10} x + \cos^{10} x) - 15 (\sin^8 x + \cos^8 x) + 10 (\sin^6 x + \cos^6 x) - 1$

$= 6 (\sin^6 x + \cos^6 x) (\sin^4 x + \cos^4 x) - \cos^4 x \sin^4 x (\sin^2 x + \cos^2 x) - 15 (\sin^6 x + \cos^6 x) (\sin^2 x + \cos^2 x) - \cos^2 x \sin^2 x (\sin^4 x + \cos^4 x) + 10 (\sin^2 x + \cos^2 x) (\sin^4 x + \cos^4 x - \cos^2 x \sin^2 x) - 1$

We know that $\sin^2 x + \cos^2 x = 1$.

$= 6 [(1 - 3 \sin^2 x \cos^2 x) (1 - 2 \sin^2 x \cos^2 x) - \sin^4 x \cos^4 x] - 15 [(1 - 3 \sin^2 x \cos^2 x) - \sin^2 x \cos^2 x (1 - 2 \sin^2 x \cos^2 x)] + 10 (1 - 3 \sin^2 x \cos^2 x) - 1$

$= 6 (1 - 5 \sin^2 x \cos^2 x + 5 \sin^4 x \cos^4 x) - 15 (1 - 4 \sin^2 x \cos^2 x + 2 \sin^4 x \cos^4 x) + 10 (1 - 3 \sin^2 x \cos^2 x) - 1$

$= 6 - 30 \sin^2 x \cos^2 x + 30 \sin^4 x \cos^4 x - 15 + 60 \sin^2 x \cos^2 x - 30 \sin^4 x \cos^4 x + 10 - 30 \sin^2 x \cos^2 x - 1$

$= 6 - 15 + 10 - 1$

$= 0$

= RHS

Hence proved.

Exercise 5.2

1 A. Question

Find the values of the other five trigonometric functions in each of the following:

cot $x = \frac{12}{5}, x$ in quadrant III

Answer

Given cotx = 12/5 andx is in quadrant III

In third quadrant, tanx and cotx are positive and sinx, cosx and secx & cosecx are negative.

We know that $\tan x = \frac{1}{\cot x}$; $\operatorname{cosec} x = \sqrt{1+\cot^2 x}$; $\sin x = \frac{1}{\operatorname{cosec} x}$; $\cos x = -\sqrt{1-\sin^2 x}$ and $\sec x = \frac{1}{\cos x}$

$$\Rightarrow \tan x = \frac{1}{\frac{12}{5}} = \frac{5}{12}$$

$$\Rightarrow \operatorname{cosec} x = -\sqrt{1+\left(\frac{12}{5}\right)^2}$$

$$= -\sqrt{\frac{25+144}{25}}$$

$$= -\sqrt{\frac{169}{25}}$$

$$= -\frac{13}{5}$$

$$\Rightarrow \sin x = \frac{1}{-13/5} = -\frac{5}{13}$$

$$\Rightarrow \cos x = -\sqrt{1-\left(\frac{-5}{13}\right)^2}$$

$$= -\sqrt{\frac{169-25}{169}}$$

$$= -\sqrt{\frac{144}{169}}$$

$$= -\frac{12}{13}$$

$$\Rightarrow \sec x = \frac{1}{\frac{-12}{13}} = -\frac{13}{12}$$

1 B. Question

Find the values of the other five trigonometric functions in each of the following:

cos $x = -\frac{1}{2}, x$ in quadrant II

Answer

Given cotx = -1/2 andx is in quadrant II

In second quadrant, sinx and cosecx are positive and tanx, cotx and cosx & secx are negative.

We know that $\sin\ x = \sqrt{1-\cos^2 x}$; $\tan x\ = \frac{\sin x}{\cos x}$; $\cot x = \frac{1}{\tan x}$; $\mathrm{cosec}\ x = \frac{1}{\sin x}$ and $\sec x\ = \frac{1}{\cos x}$

$$\Rightarrow \sin x = \sqrt{1-\left(\frac{-1}{2}\right)^2}$$

$$= \sqrt{\frac{4-1}{4}}$$

$$= \sqrt{\frac{3}{4}}$$

$$= \frac{\sqrt{3}}{2}$$

$$\Rightarrow \tan x = \frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = -\sqrt{3}$$

$$\Rightarrow \cot x = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow \mathrm{cosec} x = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec x = \frac{1}{\frac{-1}{2}} = -2$$

1 C. Question

Find the values of the other five trigonometric functions in each of the following:

tan $x = \frac{3}{4}, x$ in quadrant III

Answer

Given tanx = 3/4 andx is in quadrant III

In third quadrant, tanx and cotx are positive and sinx, cosx, secx and cosecx are negative.

We know that $\sin\ x = \sqrt{1-\cos^2 x}$; $\tan x\ = \frac{\sin x}{\cos x}$; $\cot x = \frac{1}{\tan x}$; $\mathrm{cosec}\ x = \frac{1}{\sin x}$ and $\sec x\ = -\sqrt{1+\tan^2 x}$

$$\Rightarrow \cot x = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

$$\Rightarrow \sec x = -\sqrt{1+\left(\frac{3}{4}\right)^2}$$

$$= -\sqrt{\frac{16+9}{16}}$$

$$= -\sqrt{\frac{25}{16}}$$

$= -\frac{5}{4}$

$\Rightarrow \cos x = \frac{1}{-\frac{5}{4}} = -\frac{4}{5}$

$\Rightarrow \sin x = -\sqrt{1 - \left(\frac{-4}{5}\right)^2}$

$= -\sqrt{\frac{25-16}{25}}$

$= -\sqrt{\frac{9}{25}}$

$= -\frac{3}{5}$

$\Rightarrow \operatorname{cosec} x = \frac{1}{-\frac{3}{5}} = \frac{-5}{3}$

1 D. Question

Find the values of the other five trigonometric functions in each of the following:

sin $x = \frac{3}{5}, x$ in quatrant I

Answer

Given sinx = 3/5 andx is in first quadrant.

In first quadrant, all trigonometric ratios are positive.

We know that $\tan x = \frac{\sin x}{\cos x}$; $\operatorname{cosec} x = \frac{1}{\sin x}$; $\sin x = \frac{1}{\operatorname{cosec} x}$; $\cos x = \sqrt{1-\sin^2 x}$ and $\sec x = \frac{1}{\cos x}$

$\Rightarrow \cos x = \sqrt{1 - \left(\frac{-3}{5}\right)^2}$

$= \sqrt{\frac{25-9}{25}}$

$= \sqrt{\frac{16}{25}}$

$= \frac{4}{5}$

$\Rightarrow \tan x = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

$\Rightarrow \cot x = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

$\Rightarrow \text{cosec}x = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

$\Rightarrow \sec x = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

2. Question

If sin $x = \frac{12}{13}$ and lies in the second quadrant, find the value of secx + tanx.

Answer

Given sinx = 12/13 andx lies in the second quadrant.

In second quadrant, sinx and cosecx are positive and all other ratios are negative.

We know that $\cos x = \sqrt{1 - \sin^2 x}$

$\cos x = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$

$= -\sqrt{1 - \frac{144}{169}}$

$= -\sqrt{\frac{(169 - 144)}{169}}$

$= -\sqrt{\frac{25}{169}}$

$= -\frac{5}{13}$

We know that tanx = sinx /cosx and secx = 1/cosx

$\Rightarrow \tan x = \frac{\frac{12}{13}}{\frac{-5}{13}} = -\frac{12}{5}$

$\Rightarrow \sec x = \frac{1}{\frac{-5}{13}} = -\frac{13}{5}$

$\therefore \sec x + \tan x = -\frac{12}{5} + \left(-\frac{13}{5}\right)$

$= \frac{-12 - 13}{5}$

$= -\frac{25}{5} = -5$

3. Question

If sin $x = \frac{3}{5}$, tan $y = \frac{1}{2}$ and $\frac{\pi}{2} < x < \pi < y < \frac{3\pi}{2}$, find the value of 8 tan $x - \sqrt{5}\sec y$.

Answer

Given sinx = 3/5, tan y = 1/2 and $\frac{\pi}{2} < x < \pi < y < \frac{3\pi}{2}$

Thus, x is in second quadrant and y is in third quadrant.

In second quadrant, cosx and tanx are negative.

In third quadrant, sec y is negative.

We know that $\cos x = -\sqrt{1-\sin^2 x}$ and $\tan x = \frac{\sin x}{\cos x}$

$\Rightarrow \cos x = -\sqrt{1-\left(\frac{3}{5}\right)^2}$

$= -\sqrt{1-\frac{9}{25}}$

$= -\sqrt{\frac{25-9}{25}}$

$= -\sqrt{\frac{16}{25}}$

$= -\frac{4}{5}$

$\Rightarrow \tan x = \frac{\frac{3}{5}}{\frac{-4}{5}} = \frac{-3}{5}$

We know that $\sec y = -\sqrt{1+\tan^2 y}$

$\Rightarrow \sec y = -\sqrt{1+\left(\frac{1}{2}\right)^2}$

$= -\sqrt{1+(\frac{1}{4})}$

$= -\sqrt{\frac{4+1}{4}}$

$= -\sqrt{\frac{5}{4}}$

$= -\frac{\sqrt{5}}{4}$

$\therefore 8\tan x - \sqrt{5}\sec y = 8\left(-\frac{3}{4}\right) - \sqrt{5}\left(-\frac{\sqrt{5}}{2}\right)$

$= -6 + \frac{5}{2}$

$= -\frac{7}{2}$

4. Question

If sinx + cosx = 0 andx lies in the fourth quadrant, find sinx and cosx.

Answer

Given sinx + cosx = 0 andx lies in fourth quadrant.

⇒sinx = -cosx

$\Rightarrow \frac{\sin x}{\cos x} = -1$

∴ tanx = -1

In fourth quadrant, cosx and secx are positive and all other ratios are negative.

We know that $\sec x = \sqrt{1+\tan^2 x}$; $\cos x = \frac{1}{\sec x}$; $\sin x = -\sqrt{1-\cos^2 x}$

$\Rightarrow \sec x = \sqrt{1+(-1)^2 = \sqrt{2}}$

$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$

$\Rightarrow \sin x = -\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}$

$= -\sqrt{1-\frac{1}{2}}$

$= -\sqrt{\frac{2-1}{2}}$

$= -\sqrt{\frac{1}{2}}$

$= -\frac{1}{\sqrt{2}}$

$\therefore \sin x = -\frac{1}{\sqrt{2}}$ and $\cos x = \frac{1}{\sqrt{2}}$

5. Question

If cos $x = -\frac{3}{5}$ and $\pi < x < \frac{3\pi}{2}$, find the values of other five trigonometric functions and hence evaluate $\frac{\operatorname{cosec} x + \cot x}{\sec x - \tan x}$.

Answer

Given cosx= -3/5 and π <x < 3π/2

It is in the third quadrant. Here, tanx and cotx are positive and all other rations are negative.

We know that $\sin x = -\sqrt{1-\cos^2 x}$; $\tan x \frac{\sin x}{\cos x}$; $\cot x = \frac{1}{\tan x}$; $\sec x = \frac{1}{\cos x}$ and $\operatorname{cosec} x = \frac{1}{\sin x}$

$\Rightarrow \sin x = -\sqrt{1-\left(\frac{-3}{5}\right)^2}$

$= -\sqrt{1-\frac{9}{25}}$

$= -\sqrt{\frac{25-9}{25}}$

$$= -\sqrt{\frac{16}{25}}$$

$$= -\frac{4}{5}$$

$$\Rightarrow \tan x = \frac{\frac{-4}{5}}{\frac{-3}{5}} = \frac{4}{3}$$

$$\Rightarrow \cot x = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

$$\Rightarrow \sec x = \frac{1}{\frac{-3}{5}} = -\frac{5}{3}$$

$$\Rightarrow \operatorname{cosec} x = \frac{1}{\frac{-4}{5}} = -\frac{5}{4}$$

$$\therefore \frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta} = \frac{\frac{-5}{4} + \frac{3}{4}}{\frac{-5}{3} - \frac{4}{3}}$$

$$= \frac{\frac{-5+3}{4}}{\frac{-5-4}{3}}$$

$$= \frac{\frac{-2}{4}}{\frac{-9}{3}}$$

$$= \frac{\frac{-1}{2}}{-3}$$

$$= \frac{1}{6}$$

Exercise 5.3

1 A. Question

Find the values of the following trigonometric ratios:

$$\sin\frac{5\pi}{3}$$

Answer

Given $\sin\frac{5\pi}{3}$

$$\Rightarrow \frac{5\pi}{3} = \left(\frac{5\pi}{3} \times 180\right)^\circ = 300^\circ$$

$$= (90^\circ \times 3 + 30^\circ)$$

300° lies in fourth quadrant in which sine function is negative.

$$\therefore \sin\left(\frac{5\pi}{3}\right) = \sin(300^\circ)$$

$$= \sin(90^\circ \times 3 + 30^\circ)$$

$$= -\cos 30^\circ$$

$$= \frac{-\sqrt{3}}{2}$$

1 B. Question

Find the values of the following trigonometric ratios:

sin 17 π

Answer

Given sin 17π

⇒ sin 17π =sin 3060°

⇒ 3060° =90° × 34 + 0°

3060° is in negative direction of x-axis i.e. on boundary line of II and III quadrants.

∴ sin (3060°)= sin(90° × 34 + 0°)

= -sin 0°

= 0

1 C. Question

Find the values of the following trigonometric ratios:

$\tan\frac{11\pi}{6}$

Answer

Given tan(11π/6)

⇒ $\frac{11\pi}{6} = (\frac{11}{6} \times 180°)$

=330°

330° lies in fourth quadrant in which tangent function is negative.

∴ $\left(\frac{11\pi}{6}\right) = \tan(330°)$

=tan (90° × 3+60°)

=-cot 60°

$= -\frac{1}{\sqrt{3}}$

1 D. Question

Find the values of the following trigonometric ratios:

$\cos\left(-\frac{25\pi}{4}\right)$

Answer

Given $\cos\left(\frac{-25\pi}{4}\right)$

⇒ $\cos\left(\frac{-25\pi}{4}\right) = \cos(-1125°)$

⇒ cos (-1125°) = cos (1125°)

= cos (90° × 12 + 45°)

1125° lies in first quadrant in which cosine function is positive.

∴ cos (1125°) = cos (90°× 12 + 45°)

= cos (45°)

= 1/√2

1 E. Question

Find the values of the following trigonometric ratios:

$\tan \frac{7\pi}{4}$

Answer

Given tan 7π/4

$\Rightarrow \tan \frac{7\pi}{4} = \tan 315°$

⇒ 315° = (90° × 3 + 45°)

315° lies in fourth quadrant in which tangent function is negative.

∴ tan (315°) = tan (90° × 3 + 45°)

=-cot 45°

=-1

1 F. Question

Find the values of the following trigonometric ratios:

$\sin \frac{17\pi}{6}$

Answer

Given $\sin \frac{17\pi}{6}$

$\Rightarrow \sin \frac{17\pi}{6} = \sin 510°$

⇒ 510° = (90° × 5 + 60°)

510° lies in second quadrant in which sine function is positive.

∴sin (510°) = sin (90° × 5 + 60°)

= cos (60°)

= 1/2

1 G. Question

Find the values of the following trigonometric ratios:

$\cos \frac{19\pi}{6}$

Answer

Given $\cos \frac{19\pi}{6}$

$\Rightarrow \cos \frac{19\pi}{6} = \cos 570°$

⇒ 570° = (90° × 6 + 30°)

570° lies in third quadrant in which cosine function is negative.

∴ cos (570°) = cos (90° × 6 + 30°)

= -cos (30°)

$= -\frac{\sqrt{3}}{2}$

1 H. Question

Find the values of the following trigonometric ratios:

$\sin\left(-\frac{11\pi}{6}\right)$

Answer

Given $\sin\left(\frac{-11\pi}{6}\right)$

$\Rightarrow \sin\frac{-11\pi}{6} = \sin -330°$

⇒ -sin 330° = -sin (90° × 3 + 60°)

330° lies in the fourth quadrant in which the sine function is negative.

∴ sin (-330)° = -sin (90° × 3 + 60°)

= - (-cos 60°)

= - (-1/2)

= 1/2

1 I. Question

Find the values of the following trigonometric ratios:

$\operatorname{cosec}\left(-\frac{20\pi}{3}\right)$

Answer

Given $\operatorname{cosec}\left(-\frac{20\pi}{3}\right)$

$\Rightarrow \operatorname{cosec}\left(-\frac{20\pi}{3}\right) = \operatorname{cosec}(-1200°)$

⇒ cosec (-1200°) = cosec (1200°)

=cosec (90° × 13 + 30)

1200° lies in second quadrant in which cosec function is positive.

∴ cosec (-1200°)= -cosec (90° × 13 + 30°)

= -sec 30°

$= -\frac{2}{\sqrt{3}}$

1 J. Question

Find the values of the following trigonometric ratios:

$\tan\left(-\frac{13\pi}{4}\right)$

Answer

Given $\tan\left(\frac{-13\pi}{4}\right)$

$\Rightarrow \tan\frac{-13\pi}{4} = \tan -585°$

⇒ -tan 585° = -tan (90° × 6 + 45°)

585° lies in the third quadrant in which the tangent function is positive.

∴ tan (-585)° = -tan (90° × 6 + 45°)

= - (tan 45°)

= -1

1 K. Question

Find the values of the following trigonometric ratios:

$\cos\frac{19\pi}{4}$

Answer

Given $\cos\frac{19\pi}{4}$

$\Rightarrow \cos\frac{19\pi}{4} = \cos 855°$

⇒ 855° = 90° × 9 + 45°

855° lies in the second quadrant in which the cosine function is negative.

∴ cos 855° = cos (90° × 9 + 45°)

= -sin 45°

$= \frac{-1}{\sqrt{2}}$

1 L. Question

Find the values of the following trigonometric ratios:

$\sin\frac{41\pi}{4}$

Answer

Given $\sin\frac{41\pi}{4}$

$\Rightarrow \sin\frac{41\pi}{4} = \sin 1845°$

⇒ sin 1845° = 90° × 20 + 45°

1845° lies in the first quadrant in which the sine function is positive.

∴ sin 1845° = sin (90° × 20 + 45°)

= sin 45°

$= \frac{1}{\sqrt{2}}$

1 M. Question

Find the values of the following trigonometric ratios:

$$\cos \frac{39\pi}{4}$$

Answer

Given $\cos \frac{39\pi}{4}$

$\Rightarrow \cos \frac{39\pi}{4} = \cos 1755°$

⇒ 1755° = 90° × 19 + 45°

1755° lies in the fourth quadrant in which the cosine function is positive.

∴ cos 1755° = cos (90° × 19 + 45°)

= sin 45°

$= \frac{1}{\sqrt{2}}$

1 N. Question

Find the values of the following trigonometric ratios:

$$\sin \frac{151\pi}{6}$$

Answer

Given $\sin \frac{151\pi}{6}$

$\Rightarrow \sin \frac{151\pi}{6} = \sin 4530°$

⇒ sin 4530° = 90° × 50 + 30°

4530° lies in the third quadrant in which the sine function is negative.

∴ sin 4530° = sin (90° × 50 + 30°)

= - sin 30°

= -1/2

2 A. Question

prove that :

tan 225^{o} cot 405^{o} + tan 765^{o} cot 675^{o} = 0

Answer

LHS = tan 225° cot 405° + tan 765° cot 675°

= tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)

<u>We know that when n is odd, cot → tan.</u>

= tan 45° cot 45° + tan 45° [-tan 45°]

= tan 45° cot 45° - tan 45° tan 45°

= 1 × 1 – 1 × 1

= 1 – 1

= 0

= RHS

Hence proved.

2 B. Question

prove that :

$$\sin\frac{8\pi}{3}\cos\frac{23\pi}{6}+\cos\frac{13\pi}{3}\sin\frac{35\pi}{6}=\frac{1}{2}$$

Answer

LHS $=\sin\frac{8\pi}{3}\cos\frac{23\pi}{6}+\cos\frac{13\pi}{3}\sin\frac{35\pi}{6}$

= sin 480° cos 690° + cos 780° sin 1050°

= sin (90° × 5 + 30°) cos (90° × 7 + 60°) + cos (90° × 8 + 60°) sin (90° × 11 + 60°)

We know that when n is odd, sin → cos and cos → sin.

= cos 30° sin 60° + cos 60° [-cos 60°]

$=\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}-\frac{1}{2}\times\frac{1}{2}$

= 3/4 - 1/4

= 2/4

= 1/2

= RHS

Hence proved.

2 C. Question

prove that :

cos 24^{o} + cos 55^{o} + cos 125^{o} + cos 204^{o} + cos 300^{o} = $\frac{1}{2}$

Answer

LHS = cos 24° + cos 55° + cos 125° + cos 204° + cos 300°

= cos 24° + cos (90° × 1 – 35°) + cos (90° × 1 + 35°) + cos (90° × 2 + 24°) + cos (90° × 3 + 30°)

We know that when n is odd, cos → sin.

= cos 24° + sin 35° - sin 35° - cos 24° + sin 30°

= 0 + 0 + 1/2

= 1/2

= RHS

Hence proved.

2 D. Question

prove that :

tan (-225^{o}) cot (-405^{o}) – tan (-765^{o}) cot (675^{o}) = 0

Answer

LHS = tan (-225^{o}) cot (-405^{o}) – tan (-765^{o}) cot (675^{o})

We know that tan (-x) = -tan (x) and cot (-x) = -cot (x).

= [-tan (225°)] [-cot (405°)] – [-tan (765°)] cot (675°)

= tan (225°) cot (405°) + tan (765°) cot (675°)

= tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)

= tan 45° cot 45° + tan 45° [-tan 45°]

= 1 × 1 + 1 × (-1)

= 1 – 1

= 0

= RHS

Hence proved.

2 E. Question

prove that :

cos 570^o sin 510^o + sin (-330^o) cos (-390^o) = 0

Answer

LHS = cos 570^o sin 510^o + sin (-330^o) cos (-390^o)

We know that sin (-x) = -sin (x) and cos (-x) = +cos (x).

= cos 570^o sin 510^o + [-sin (330^o)] cos (390^o)

= cos 570^o sin 510^o - sin (330^o) cos (390^o)

= cos (90° × 6 + 30°) sin (90° × 5 + 60°) – sin (90° × 3 + 60°) cos (90° × 4 + 30°)

We know that cos is negative at 90° + θ i.e. in Q_2 and when n is odd, sin → cos and cos → sin.

= -cos 30° cos 60° - [-cos 60°] cos 30°

= -cos 30° cos 60° + cos 60° cos 30°

= 0

= RHS

Hence proved.

2 F. Question

prove that :

$$\tan\frac{11\pi}{3}-2\sin\frac{4\pi}{6}-\frac{3}{4}\text{cosec}^2\frac{\pi}{4}+4\cos^2\frac{17\pi}{6}=\frac{3-4\sqrt{3}}{2}$$

Answer

LHS $= \tan\frac{11\pi}{3}-2\sin\frac{4\pi}{6}-\frac{3}{4}\text{cosec}^2\frac{\pi}{4}+4\cos^2\frac{17\pi}{6}$

$$=\tan\frac{11\times180°}{3}-2\sin\frac{4\times180°}{6}-\frac{3}{4}\text{cosec}^2\frac{180°}{4}+4\cos^2\frac{17\times180°}{6}$$

$$=\tan 660°-2\sin 120°-\frac{3}{4}[\text{cosec }45]^2+4[\cos 510°]^2$$

= tan (90° × 7 + 30°) – 2 sin (90° × 1 + 30°) – 3/4 [cosec 45°]^2 + 4 [cos (90° × 5 + 60°)]^2

We know that tan and cos is negative at 90° + θ i.e. in Q_2 and when n is odd, tan → cot, sin → cos and cos → sin.

= [-cot 30°] – 2 cos 30° - 3/4 [cosec 45°]^2 + [-sin 60°]^2

= - cot 30° - 2 cos 30° - 3/4 [cosec 45°]^2 + [sin 60°]^2

$= -\sqrt{3} - \frac{2\sqrt{3}}{2} - \frac{3}{4}[\sqrt{2}]^2 + 4\left[\frac{\sqrt{3}}{2}\right]^2$

$= -\sqrt{3} - \sqrt{3} - \frac{6}{4} + \frac{12}{4}$

$= \frac{3 - 4\sqrt{3}}{2}$

= RHS

Hence proved.

2 G. Question

prove that :

$$3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4} = 1$$

Answer

LHS $= 3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$

$= 3\sin\frac{180°}{6}\sec\frac{180°}{3} - 4\sin\frac{5(180°)}{6}\cot\frac{180°}{4}$

= 3 sin 30° sec 60° - 4 sin 150° cot 45°

= 3 sin 30° sec 60° - 4 sin (90° × 1 + 60°)cot 45°

<u>We know that when n is odd, sin → cos.</u>

= 3 sin 30° sec 60° - 4 cos 60° cot 45°

= 3 (1/2) (2) – 4 (1/2) (1)

= 3 – 2

= 1

= RHS

Hence proved.

3 A. Question

Prove that :

$$\frac{\cos(2\pi + x)\operatorname{cosec}(2\pi + x)\tan(\pi/2 + x)}{\sec(\pi/2 + x)\cos x\cot(\pi + x)} = 1$$

Answer

LHS $= \frac{\cos(2\pi+x)\operatorname{cosec}(2\pi+x)\tan\left(\frac{\pi}{2}+x\right)}{\sec\left(\frac{\pi}{2}+x\right)\cos x\cot(\pi+x)}$

$= \frac{\cos x \operatorname{cosec} x[-\cot x]}{[-\operatorname{cosec} x]\cos x \cot x}$

$= \frac{-\cos x \operatorname{cosec} x \cot x}{-\operatorname{cosec} x \cos x \cot x}$

= 1

= RHS

Hence proved.

3 B. Question

Prove that :

$$\frac{\operatorname{cosec}\left(90^{\circ}+x\right)+\cot\left(450^{\circ}+x\right)}{\operatorname{cosec}\left(90^{\circ}+x\right)+\tan\left(180^{\circ}-x\right)}+\frac{\tan\left(180^{\circ}+x\right)+\sec\left(180^{\circ}-x\right)}{\tan\left(360^{\circ}+x\right)-\sec\left(-x\right)}=2$$

Answer

LHS $= \frac{\operatorname{cosec}\,(90^{\circ}+x)+\cot(450^{\circ}+x)}{\operatorname{cosec}(90^{\circ}-x)+\tan(180^{\circ}-x)}+\frac{\tan(180^{\circ}+x)+\sec(180^{\circ}-x)}{\tan(360^{\circ}+x)-\sec(-x)}$

$$=\frac{\operatorname{cosec}\,(90^{\circ}+x)+\cot(90^{\circ}\times 5+x)}{\operatorname{cosec}(90^{\circ}-x)+\tan(90^{\circ}\times 2-x)}+\frac{\tan(90^{\circ}\times 2+x)+\sec(90^{\circ}\times 2-x)}{\tan(90^{\circ}\times 4+x)-\sec(-x)}$$

We know that when n is odd, cosec → sec and also sec (-x) = secx.

$$=\frac{\sec x+\cot(90^{\circ}\times 5+x)}{\operatorname{cosec}(90^{\circ}-x)+\tan(90^{\circ}\times 2-x)}+\frac{\tan(90^{\circ}\times 2+x)+\sec(90^{\circ}\times 2-x)}{\tan(90^{\circ}\times 4+x)-\sec(x)}$$

$$=\frac{\sec x-\tan x}{\sec x-\tan x}+\frac{\tan x-\sec x}{\tan x-\sec x}$$

= 1 + 1

= 2

= RHS

Hence proved.

3 C. Question

Prove that :

$$\frac{\sin\left(\pi-x\right)\cos\left(\frac{\pi}{2}+x\right)\tan\left(\frac{3\pi}{2}-x\right)\cot\left(2\pi-x\right)}{\sin\left(2\pi-x\right)\cos\left(2\pi+x\right)\operatorname{cosec}\left(-x\right)\sin\left(\frac{3\pi}{2}-x\right)}=1$$

Answer

LHS $= \frac{\sin(\pi-x)\cos\left(\frac{\pi}{2}+x\right)\tan\left(\frac{3\pi}{2}-x\right)\cot(2\pi-x)}{\sin(2\pi-x)\cos(2\pi+x)\operatorname{cosec}\,(-x)\sin\left(\frac{3\pi}{2}-x\right)}$

$$=\frac{\sin(180^{\circ}-x)\cos(90^{\circ}+x)\tan(270^{\circ}-x)\cot(360^{\circ}-x)}{\sin(360^{\circ}-x)\cos(360^{\circ}+x)\operatorname{cosec}\,(-x)\sin(270^{\circ}-x)}$$

We know that cosec (-x) = -cosecx.

$$=\frac{\sin(90^{\circ}\times 2-x)\cos(90^{\circ}\times 1+x)\tan(90^{\circ}\times 3-x)\cot(90^{\circ}\times 4-x)}{\sin(90^{\circ}\times 4-x)\cos(90^{\circ}\times 4+x)\left[-\operatorname{cosec}\,(x)\right]\sin(90^{\circ}\times 3-x)}$$

We know that when n is odd, cos → sin, tan → cot and sin → cos.

$$=\frac{[-\sin x][-\sin x]\cot x\,[-\cot x]}{[-\sin x]\cos x\,[-\operatorname{cosec}\,x][-\cos x]}$$

$$= \frac{\sin^2 x \cot^2 x}{\sin x \operatorname{cosec} x \cos x \cos x}$$

$$= \frac{\sin^2 x \times \frac{\cos^2 x}{\sin^2 x}}{\sin x \times \frac{1}{\sin x} \times \cos^2 x}$$

$$= \frac{\cos^2 x}{\cos^2 x}$$

= 1

= RHS

Hence proved.

3 D. Question

Prove that :

$$\left\{1 + \cot x - \sec\left(\frac{\pi}{2} + x\right)\right\}\left\{1 + \cot x + \sec\left(\frac{\pi}{2} + x\right)\right\} = 2 \cot x$$

Answer

LHS $= \left\{1 + \cot x - \sec\left(\frac{\pi}{2} + x\right)\right\}\left\{1 + \cot x + \sec\left(\frac{\pi}{2} + x\right)\right\}$

= {1 + cotx – (-cosecx)} {1 + cotx + (-cosecx)}

= {1 + cotx + cosecx} {1 + cotx – cosecx}

= {(1 + cotx) + (cosecx)} {(1 + cotx) – (cosecx)}

We know that (a + b) (a – b) = $a^2 - b^2$

= $(1 + \cot x)^2 - (\operatorname{cosec} x)^2$

= $1 + \cot^2 x + 2 \cot x - \operatorname{cosec}^2 x$

We know that $1 + \cot^2 x = \operatorname{cosec}^2 x$

= $\operatorname{cosec}^2 x + 2 \cot x - \operatorname{cosec}^2 x$

= 2 cotx

= RHS

Hence proved.

3 E. Question

Prove that :

$$\frac{\tan\left(\frac{\pi}{2} - x\right)\sec(\pi - x)\sin(-x)}{\sin(\pi + x)\cot(2\pi - x)\operatorname{cosec}\left(\frac{\pi}{2} - x\right)} = 1$$

Answer

LHS $= \frac{\tan\left(\frac{\pi}{2} - x\right)\sec(\pi - x)\sin(-x)}{\sin(\pi + x)\cot(2\pi - x)\operatorname{cosec}\left(\frac{\pi}{2} - x\right)}$

$$= \frac{\tan(90° - x)\sec(180° - x)\sin(-x)}{\sin(180° + x)\cot(360° - x)\operatorname{cosec}(90° - x)}$$

We know that sin (-x) = -sinx.

$$= \frac{\tan(90° \times 1 - x)\sec(90° \times 2 - x)\ [-\sin(x)]}{\sin(90° \times 2 + x)\cot(90° \times 4 - x)\,\mathrm{cosec}(90° \times 1 - x)}$$

We know that when n is odd, tan → cot and cosec → sec.

$$= \frac{[\cot x][-\sec x][-\sin x]}{[-\sin x][-\cot x][\sec x]}$$

$$= \frac{\cot x \sec x \sin x}{\cot x \sec x \sin x}$$

= 1

= RHS

Hence proved.

4. Question

Prove that :

$$\sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9} = 2$$

Answer

LHS $= \sin^2\frac{\pi}{18} + \sin^2\frac{\pi}{9} + \sin^2\frac{7\pi}{18} + \sin^2\frac{4\pi}{9}$

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \sin^2\frac{7\pi}{18} + \sin^2\frac{8\pi}{18}$$

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \sin^2\left(\frac{\pi}{2} - \frac{2\pi}{18}\right) + \sin^2\left(\frac{\pi}{2} - \frac{\pi}{18}\right)$$

We know that when n is odd, sin → cos.

$$= \sin^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \cos^2\frac{2\pi}{18} + \cos^2\frac{\pi}{18}$$

$$= \sin^2\frac{\pi}{18} + \cos^2\frac{\pi}{18} + \sin^2\frac{2\pi}{18} + \cos^2\frac{2\pi}{18}$$

We know that $\sin^2 + \cos^2 x = 1$.

= 1 + 1

= 2

= RHS

Hence proved.

5. Question

Prove that :

$$\sec\left(\frac{3\pi}{2} - x\right)\sec\left(x - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + x\right)\tan\left(x - \frac{3\pi}{2}\right) = -1.$$

Answer

LHS $= \sec\left(\frac{3\pi}{2} - x\right)\sec\left(x - \frac{5\pi}{2}\right) + \tan\left(\frac{5\pi}{2} + x\right)\tan\left(x - \frac{3\pi}{2}\right)$

$$= \sec\left(\frac{3\pi}{2} - x\right)\left[-\sec\left(\frac{5\pi}{2} - x\right)\right] + \tan\left(\frac{5\pi}{2} + x\right)\left[-\tan\left(\frac{3\pi}{2} - x\right)\right]$$

We know that sec (-x) = sec (x) and tan (-x) = -tan (x).

$= \sec\left(\frac{3\pi}{2} - x\right)\left[\sec\left(\frac{5\pi}{2} - x\right)\right] - \tan\left(\frac{5\pi}{2} + x\right)\left[\tan\left(\frac{3\pi}{2} - x\right)\right]$

$= \sec\left(\frac{\pi}{2} \times 3 - x\right)\sec\left(\frac{\pi}{2} \times 5 - x\right) - \tan\left(\frac{\pi}{2} \times 5 + x\right)\tan\left(\frac{\pi}{2} \times 3 - x\right)$

We know that when n is odd, sec → cosec and tan → cot.

= [-cosecx] [cosecx] - [-cotx] [cotx]

= $-\text{cosec}^2x + \cot^2x$

= - $[\text{cosec}^2x - \cot^2x]$

We know that $\text{cosec}^2x - \cot^2x = 1$

= -1

= RHS

Hence proved.

6. Question

In a ΔABC, prove that :

i. cos (A + B) + cos C = 0

ii. $\cos\left(\frac{A+B}{2}\right) = \sin\frac{C}{2}$

iii. $\tan\frac{A+B}{2} = \cot\frac{C}{2}$

Answer

We know that in ΔABC, A + B + C = π

(i) Here A + B = π – C

LHS = cos (A + B) + cos C

= cos (π – C) + cos C

We know that cos (π – C) = -cos C

= -cos C + cos C

= 0

= RHS

Hence proved.

(ii) ⇒ A + B = π – C

$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$

$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

LHS = $\cos\left(\frac{A+B}{2}\right)$

$= \cos(\frac{\pi}{2} - \frac{C}{2})$

We know that $\cos(\frac{\pi}{2} - x) = \sin x$

$= \sin\left(\frac{C}{2}\right)$

= RHS

Hence proved.

(iii)

⇒ A + B = π - C

$\Rightarrow \frac{A+B}{2} = \frac{\pi - C}{2}$

$\Rightarrow \frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}$

LHS $= \tan\left(\frac{A+B}{2}\right)$

$= \tan(\frac{\pi}{2} - \frac{C}{2})$

We know that $\tan(\frac{\pi}{2} - x) = \cot x$

$= \cot\left(\frac{C}{2}\right)$

= RHS

Hence proved

7. Question

If A, B, C, D be the angles of a cyclic quadrilateral taken in order prove that :

cs(180° – A) + cos (180° + B) + cos (180° + C) – sin (90° + D) = 0

Answer

Given A, B, C and D are the angles of a cyclic quadrilateral.

∴ A + C = 180° and B + D = 180°

⇒ A = 180° – C and B = 180° - D

Now, LHS = cos (180° – A) + cos (180° + B) + cos (180° + C) – sin (90° + D)

= -cos A + [-cos B] + [-cos C] + [-cos D]

= -cos A – cos B – cos C – cos D

= -cos (180° - C) – cos (180° - D) – cos C – cos D

= -[-cos C] – [-cos D] – cos C – cos D

= cos C + cos D – cos C – cos D

= 0

= RHS

Hence proved.

8 A. Question

Find x from the following equations:

$$\text{cosec}\left(\frac{\pi}{2}+\theta\right)+x\cos\theta\cot\left(\frac{\pi}{2}+\theta\right)=\sin\left(\frac{\pi}{2}+\theta\right)$$

Answer

$\Rightarrow \text{cosec}\left(\frac{\pi}{2}+\theta\right)+x\cos\theta\cot\left(\frac{\pi}{2}+\theta\right)=\sin\left(\frac{\pi}{2}+\theta\right)$

⇒ cosec (90° + θ) +x cos θ cot (90° + θ) = cos θ

We know that when n is odd, cot → tan.

⇒ sec θ +x cos θ [-tan θ] = cos θ

⇒ sec θ –x cos θ tan θ = cos θ

⇒ sec θ –x cos θ (sin θ/ cos θ) = cos θ

⇒ sec θ –x sin θ = cos θ

⇒ sec θ – cos θ =x sin θ

$\Rightarrow \frac{1}{\cos\theta}-\cos\theta = x\sin\theta$

$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta}=x\sin\theta$

We know that 1 – $\cos^2$ θ = $\sin^2$ θ

$\Rightarrow \frac{\sin^2\theta}{\cos\theta}=x\sin\theta$

$\Rightarrow \frac{\sin^2\theta}{\cos\theta\sin\theta}=x$

⇒ tan θ =x

∴x = tan θ

8 B. Question

Find x from the following equations:

$$x\cot\left(\frac{\pi}{2}+\theta\right)+\tan\left(\frac{\pi}{2}+\theta\right)\sin\theta+\text{cosec}\left(\frac{\pi}{2}+\theta\right)=0$$

Answer

Given $x\cot\left(\frac{\pi}{2}+\theta\right)+\tan\left(\frac{\pi}{2}+\theta\right)\sin\theta+\text{cosec}\left(\frac{\pi}{2}+\theta\right)=0$

⇒x cot (90° + θ) + tan (90° + θ) sin θ + cosec (90° + θ) = 0

⇒x [-tan θ] + [-cot θ] sin θ + sec θ = 0

$\Rightarrow -x\left[\frac{\sin\theta}{\cos\theta}\right]-\frac{\cos\theta}{\sin\theta}\sin\theta+\frac{1}{\cos\theta}=0$

$\Rightarrow -x\left[\frac{\sin\theta}{\cos\theta}\right]-\cos\theta+\frac{1}{\cos\theta}=0$

$\Rightarrow \frac{-x\sin\theta-\cos^2\theta+1}{\cos\theta}=0$

⇒ -x sin θ – $\cos^2$ θ + 1 = 0

We know that 1 – $\cos^2$ θ = $\sin^2$ θ

⇒ -x sin θ + $\sin^2$ θ = 0

⇒x sin θ = $\sin^2$ θ

⇒x = $\sin^2$ θ/ sin θ

∴x = sin θ

9 A. Question

Prove that:

$$\tan 4\pi - \cos\frac{3\pi}{2} - \sin\frac{5\pi}{6}\cos\frac{2\pi}{3} = \frac{1}{4}$$

Answer

LHS $= \tan 4\pi - \cos\frac{3\pi}{2} - \sin\frac{5\pi}{6}\cos\frac{2\pi}{3}$

= tan 720° - cos 270° - sin 150° cos 120°

= tan (90° × 8 + 0°) – cos (90° × 3 + 0°) – sin (90° × 1 + 60°) cos (90° × 1 + 30°)

We know that when n is odd, cos → sin and sin → cos.

= tan 0° - sin 0° - cos 60° [-sin 30°]

= tan 0° - sin ° + cos 60° sin 30°

= 0 – 0 + 1/2 (1/2)

= 1/4

= RHS

Hence proved.

9 B. Question

Prove that:

$$\sin\frac{13\pi}{3}\sin\frac{8\pi}{3} + \cos\frac{2\pi}{3}\sin\frac{5\pi}{6} = \frac{1}{2}$$

Answer

LHS $= \sin\frac{13\pi}{3}\sin\frac{8\pi}{3} + \cos\frac{2\pi}{3}\sin\frac{5\pi}{6}$

= sin 780° sin 480° + cos 120° sin 150°

= sin (90° × 8 + 60°) sin (90° × 5 + 30°) + cos (90° × 1 + 30°) sin (90° × 1 + 60°)

We know that when n is odd, cos → sin and sin → cos.

= sin 60° cos 30° + [-sin 30°] cos 60°

= sin 60° cos 30° - sin 30° cos 60°

We know that sin A cos B – cos A sin B = sin (A – B)

= sin (60° - 30°)

= sin 30°

= 1/2

= RHS

Hence proved.

9 C. Question

Prove that:

$$\sin\frac{13\pi}{3}\sin\frac{2\pi}{3}+\cos\frac{4\pi}{3}\sin\frac{13\pi}{6}=\frac{1}{2}$$

Answer

LHS $=\sin\frac{13\pi}{3}\sin\frac{2\pi}{3}+\cos\frac{4\pi}{3}\sin\frac{13\pi}{6}$

= sin 780° sin 120° + cos 240° sin 390°

= sin (90° × 8 + 60°) sin (90° × 1 + 30°) + cos (90° × 2 + 60°) sin (90° × 4 + 30°)

We know that when n is odd, sin → cos.

= sin 60° cos 30° + [-cos 60°] sin 30°

= sin 60° cos 30° - sin 30° cos 60°

We know that sin A cos B – cos A sin B = sin (A – B)

= sin (60° - 30°)

= sin 30°

= 1/2

= RHS

Hence proved.

9 D. Question

Prove that:

$$\sin\frac{10\pi}{3}\cos\frac{13\pi}{6}+\cos\frac{8\pi}{3}\sin\frac{5\pi}{6}=-1$$

Answer

LHS $=\sin\frac{10\pi}{3}\cos\frac{13\pi}{3}+\cos\frac{8\pi}{3}\sin\frac{5\pi}{6}$

= sin 600° cos 390° + cos 480° sin 150°

= sin (90° × 6 + 60°) cos (90° × 4 + 30°) + cos (90° × 5 + 30°) sin (90° × 1 + 60°)

We know that when n is odd, sin → cos and cos → sin.

= [-sin 60°] cos 30° + [-sin 30°] cos 60°

= -sin 60° cos 30° - sin 30° cos 60°

= -[sin 60° cos 30° + cos 60° sin 30°]

We know that sin A cos B + cos A sin B = sin (A + B)

= -sin (60° + 30°)

= -sin 90°

= -1

= RHS

Hence proved.

9 E. Question

Prove that:

$$\tan\frac{5\pi}{4}\cot\frac{9\pi}{4}+\tan\frac{17\pi}{4}\cot\frac{15\pi}{4}=0$$

Answer

LHS $= \tan\frac{5\pi}{4}\cot\frac{9\pi}{4}+\tan\frac{17\pi}{4}\cot\frac{15\pi}{4}$

= tan 225° cot 405° + tan 765° cot 675°

= tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)

We know that when n is odd, cot → tan.

= tan 45° cot 45° + tan 45° [-tan 45°]

= tan 45° cot 45° - tan 45° tan 45°

= 1 × 1 – 1 × 1

= 1 – 1

= 0

= RHS

Hence proved.

Very Short Answer

1. Question

Write the maximum and minimum values of cos (cos x).

Answer

Let cos x = t

Range of t =(-1,1)

∴ Maximum and Minimum value of cos x is 1 and -1 respectively.

Now,

cos(-x) = cos x

∴ Range of cos(cos x) = [cos(1),cos(0)]

⇒ cos(cos x) = [cos1,0]

2. Question

Write the maximum and minimum values of sin (sin x).

Answer

sin(x) has maximum value at x = π/2 and its minimum at

x = -π/2 which are 1 and -1 respectively.

As 1 < π/2;

so, the argument of the outer sin always lies within the interval

[-π/2, π/2]

So the maximum and minimum of the given function are

sin 1 and - sin 1.

3. Question

Write the maximum value of sin (cos x).

Answer

Value of cos(x) varies from -1 to 1 for all R and sin(x) is increasing in [-π/2,π/2]

∴ sin(cos x) has max value of sin1.

4. Question

If $\sin x = \cos^2 x$, then write the value of $\cos^2 x\,(1 + \cos^2 x)$.

Answer

Given $\sin x = \cos^2 x$

To find the value of $\cos^2 x\,(1 + \cos^2 x)$.

$\Rightarrow \cos^2 x\,(1 + \cos^2 x)$.

$\Rightarrow \cos^2 x + \cos^4 x$.

As $\cos^2 x = 1 - \sin^2 x$ the above equation becomes

$\Rightarrow 1 - \sin^2 x + \sin^2 x$

$\Rightarrow 1$.

5. Question

If $\sin x = \text{cosec } x = 2$, then write the value of $\sin^n x + \text{cosec}^n x$.

Answer

(Question might be different)

$\sin x + \text{cosec } x = 2$

$\Rightarrow \sin x + \frac{1}{\sin x} = 2$

$\Rightarrow \sin^2 x + 1 = 2\sin x$

$\Rightarrow \sin^2 x - 2\sin x + 1 = 0$

$\Rightarrow (\sin x - 1)^2 = 0$

$\Rightarrow \sin x = 1$

As $\sin x = 1$

$\sin^n x = 1$

$\therefore \sin^n x + \text{cosec}^n x$

$\Rightarrow \sin^n x + \frac{1}{\sin^n x} = 1 + 1$

$\Rightarrow 2$.

6. Question

If $\sin x + \sin^2 x = 1$, then write the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$.

Answer

Given: $\sin x + \sin^2 x = 1$

To find the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$.

$\Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin x = \cos^2 x$

⇒ $\cos^{12}x = \sin^6x$, $\cos^{10}x = \sin^5x$, $\cos^8x = \sin^4x$, $\cos^6x = \sin^3x$.

Substituting above values in given equation we get

⇒ $\sin^6x + 3\sin^5x + 3\sin^8x + \sin^3x$ $[(a+b)^3 = a^3+3a^2b+3ab^2+b^3]$

⇒ $(\sin x + \sin^2 x)^3 = (1)^3$

⇒ 1.

7. Question

If $\sin x + \sin^2 x = 1$, then write the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$.

Answer

Given: $\sin x + \sin^2x = 1$

To find the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$.

⇒ $\sin x = 1 – \sin^2x$

⇒ $\sin x = \cos^2x$

⇒ $\cos^8x = \sin^4x$, $\cos^6x = \sin^3x$, $\cos^4x = \sin^2x$.

Substituting above values in given equation we get

⇒ $\sin^4x+2\sin^3x+\sin^2x$ $[(a+b)^2 = a^2+2ab+b^2]$

⇒ $(\sin x + \sin^2 x)^2 = (1)^2$

⇒ 1

8. Question

If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then write the value of $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$.

Answer

Given that $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

We know that in general the maximum value of $\sin\theta = 1$ when $\theta = \pi/2$

As $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

⇒ $\theta_1 = \theta_2 = \theta_3 = \pi/2$.

The above case is the only possible condition for the given condition to satisfy.

∴ $\cos\theta_1 + \cos\theta_2 + \cos\theta_3$

⇒ $\cos\pi/2 + \cos\pi/2 + \cos\pi/2$

⇒ 0+0+0

⇒ 0.

9. Question

Write the value of sin 10° + sin 20° + sin 30° + ... + sin 360°.

Answer

We know sin(180+θ) = -sin θ

Also, sin(360-θ) = -sin θ

Given all angles are complementary in nature.

sin350 = sin(360-10) = -sin10°

so finally each of them cancel each other and finally we get

the sum equal to 0.

10. Question

A circular wire of radius 15 cm is cut and bent so as to lie along the circumference of a loop of radius 120 cm. Write the measure of the angle subtended by it at the centre of the loop.

Answer

Let the angle subtended be θ.

For calculating we have the formula $\frac{\text{Radius}}{\text{Circumfernce}} = \frac{\theta}{360}$

$\Rightarrow \frac{15}{120} = \frac{\theta}{360}$

$\Rightarrow \theta = \frac{15 \times 360}{120}$

$\Rightarrow \theta = 45°$

11. Question

Write the value of $2(\sin^6 x + \cos^6 x) - 3(\sin^4 x + \cos^4 x) + 1$.

Answer

$\sin^6x + \cos^6x = (\sin^2x)^3 + (\cos^2x)^3$

$=(\sin^2x + \cos^2x)(\sin^4x+\cos^4x-\sin^2x\cos^2x)$

$= 1 (\sin^4x + \cos^4x - \sin^2x\cos^2x)$

Substituting above value in given equation

$\Rightarrow 2(\sin^4x + \cos^4x - \sin^2x\cos^2x)-3(\sin^4 x + \cos^4 x)+1$

$\Rightarrow 2\sin^4x + 2\cos^4x - 2\sin^2x\cos^2x - 3\sin^4x-3\cos^4x+1.$

$\Rightarrow -\sin^4x-\cos^4x-2\sin^2x\cos^2x+1$

$\Rightarrow -[(\sin^2x)^2+(\cos^2x)^2-2\sin^2x\cos^2x]+1$

$\Rightarrow -[(\sin^2x + \cos^2x)^2]+1$

$\Rightarrow -1+1$

$\Rightarrow 0.$

12. Question

Write the value of cos 1° + cos 2° + cos 3° + ... + cos 180°.

Answer

The given expression can be rearranged as:

(Cos 1 + cos 179) + (cos2 + cos 178) + (cos3+ cos177) +.... + (cos89 + cos 91) + (cos90) + cos180

We know that: cos(180 - x)= - cos x.

So all the bracket totals except last 2 terms will be zero.

So given expression is: 0 + (cos90) + (cos180)

= 0 + 0 + (-1)

=-1.

13. Question

If cot (α + β) = 0, then write the value of sin (α + 2β).

Answer

Given: cot(α + β) = 0

$\therefore \frac{\cot\alpha.\cot\beta - 1}{\cot\alpha + \cot\beta} = 0$

⇒ cotα.cotβ = 1

$\Rightarrow \cot\alpha = \frac{1}{\cot\beta}$

$\Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\sin\beta}{\cos\beta}$

Now,

Sin(α +2β) = sinα.cos2β + cosα.sin2β

=sin α (2 $\cos^2$ β-1)+cos α.2 sin β.cos β

14. Question

If tan A + cot A = 4, then write the value of $\tan^4$ A + $\cot^4$ A.

Answer

Given: tanA + cotA = 4

$\Rightarrow \tan A + \frac{1}{\tan A} = 4$

Squaring both sides we get

$\Rightarrow \left(\tan A + \frac{1}{\tan A}\right)^2 = 4^2$

$\Rightarrow \tan^2 A + \frac{1}{\tan^2 A} + 2.\tan A.\frac{1}{\tan A} = 16$

$\Rightarrow \tan^2 A + + \frac{1}{\tan^2 A} = 14$

Squaring both sides we get

$\Rightarrow \left(\tan^2 A + \frac{1}{\tan^2 A}\right)^2 = 14^2$

$\Rightarrow \tan^4 A + \frac{1}{\tan^4 A} + 2.\tan^2 A.\frac{1}{\tan^2 A} = 196$

$\Rightarrow \tan^4 A + + \frac{1}{\tan^4 A} = 194$

15. Question

Write the least value of $\cos^2$ x + $\sec^2$ x.

Answer

We know that $\cos^2$x and $\sec^2$x ≥ 0

∴ By applying AM and GM we get,

$\Rightarrow \frac{\cos^2 x + \sec^2 x}{2} \geq \cos^2 x.\sec^2 x$

⇒ $\cos^2$ x + $\sec^2$ x≥2

∴ Least value of the given function is 2.

16. Question

If x = $\sin^{14}x + \cos^{20}x$, then write the smallest interval in which the value of x lie.

Answer

We know the range of sin x is

$-1 \le \sin x \le 1$

$\therefore 0 \le \sin^{14}x \le 1$

We know the range of cos x is

$-1 \le \cos x \le 1$

$\therefore 0 \le \cos^{20}x \le 1$

$0 < \sin^{14}x + \cos^{20}x \le 2$

which means that the value of x lies in the interval [0,2]

But there's a problem, when sine is 0 cosine is 1, they might even be 0 and -1 at particular points (not in this case, since they are even powers), so the minimum we would get should be more than 0. Hence the value of x lies in (0,1]

17. Question

If 3 sin x + 5 cos x = 5, then write the value of 5 sin x− 3 cos x.

Answer

$\Rightarrow 3 \sin x + 5\cos x = 5$

$\Rightarrow 3\sin x = 5-5\cos x$

$\Rightarrow 3\sin x = 5(1-\cos x)$

Squaring both sides we get

$\Rightarrow 9\sin^2x = 25(1-\cos x)^2$

$\Rightarrow 9\sin^2x = 25(1+\cos^2x-2\cos x)$

$\Rightarrow 9\sin^2x+9\cos^2x = 25 + 25\cos^2x - 50\cos x + 9\cos^2x$

$\Rightarrow 9(\sin^2x+ \cos^2x) = 25 +34\cos^2x-50\cos x$

$\Rightarrow 34\cos^2x-50\cos x+16=0$

$\Rightarrow 17\cos^2x-25\cos x+8=0$

$\Rightarrow 17\cos^2x-17\cos x-8\cos x+8=0$

$\Rightarrow 17\cos x(\cos x-1)-8(\cos x-1)=0$

$\Rightarrow \cos x = \frac{8}{17}, \cos x = 1$

When cos x = 1

3sin x + 5cos x = 5

3sin x = 0

Sin x = 0

Substituting the value cos x = 1 and sin x = 0

5(0)-3(1) = 0-3

$\Rightarrow -3.$

$\Rightarrow \cos x = \frac{8}{17}$

$\Rightarrow \sin^2x + \cos^2x = 1$

$\Rightarrow \sin^2x = 1 - \cos^2x$

$$\Rightarrow \sin^2 x = 1 - \frac{64}{289}$$

$$\Rightarrow \sin^2 x = \frac{225}{289}$$

$$\Rightarrow \sin x = \frac{15}{17}$$

$\Rightarrow 5\sin x - 3\cos x$

$$\Rightarrow 5 \times \frac{15}{17} - 3 \times \frac{8}{17}$$

$$\Rightarrow \frac{51}{17} = 3.$$

∴ -3 and 3.

MCQ

1. Question

Mark the correct alternative in the following:

If $\tan x = x - \frac{1}{4x}$, then sec x − tan x is equal to

A. $-2x, \frac{1}{2x}$

B. $-\frac{1}{2x}, 2x$

C. 2x

D. $2x, \frac{1}{2x}$

Answer

$$\Rightarrow \tan^2 x = x^2 + \frac{1}{16x^2} - 2x\frac{1}{4x}$$

$$\Rightarrow \tan^2 x = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \sec^2 x - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\Rightarrow \sec^2 x = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \sec^2 x = \left(x + \frac{1}{4x}\right)^2$$

$$\Rightarrow \sec x = x + \frac{1}{4x}, -x - \frac{1}{4x}$$

$\Rightarrow \sec x - \tan x$

$\Rightarrow\ x+\frac{1}{4x}-\left(x-\frac{1}{4x}\right)$

$\Rightarrow\ \frac{1}{2x}$

⇒ sec x – tan x

$\Rightarrow\ -x-\frac{1}{4x}-x+\frac{1}{4x}$

⇒ -2x

∴ the value of sec x – tan x = -2x, 1/2x.

2. Question

Mark the correct alternative in the following:

If $\sec x = x+\frac{1}{4x}$, then sec x + tan x =

A. $x, \frac{1}{x}$

B. $2x, \frac{1}{2x}$

C. $-2x, \frac{1}{2x}$

D. $-\frac{1}{x}, x$

Answer

$\Rightarrow\ \sec^2 x = x^2+\frac{1}{16x^2}+2x\frac{1}{4x}$

$\Rightarrow\ \sec^2 x = x^2+\frac{1}{16x^2}+\frac{1}{2}$

$\Rightarrow\ \tan^2 x+1 = x^2+\frac{1}{16x^2}+\frac{1}{2}$

$\Rightarrow\ \tan^2 x = x^2+\frac{1}{16x^2}-\frac{1}{2}$

$\Rightarrow\ \tan^2 x = \left(x-\frac{1}{4x}\right)^2$

$\Rightarrow\ \tan x = x-\frac{1}{4x}, -x+\frac{1}{4x}$

⇒ sec x – tan x

$\Rightarrow\ x+\frac{1}{4x}-\left(x-\frac{1}{4x}\right)$

$\Rightarrow\ \frac{1}{2x}$

⇒ sec x – tan x

$\Rightarrow\ x+\frac{1}{4x}+x-\frac{1}{4x}$

⇒ 2x

∴ the value of sec x – tan x = 2x, 1/2x.

3. Question

Mark the correct alternative in the following:

If $\frac{\pi}{2} < x < \frac{3\pi}{2}$, then $\sqrt{\frac{1-\sin x}{1+\sin x}}$ is equal to

A. sec x − tan x

B. sec x + tan x

C. tan x − sec x

D. none of these

Answer

Given:

$\Rightarrow \frac{-\pi}{2} < x < \frac{3\pi}{2}$

Now

$\Rightarrow \sqrt{\frac{1-\sin x}{1+\sin x}}$ (Rationalizing we get)

$\Rightarrow \sqrt{\frac{1-\sin x}{1+\sin x}} \times \sqrt{\frac{1-\sin x}{1-\sin x}}$

$\Rightarrow \sqrt{\frac{(1-\sin x)^2}{1-\sin^2 x}}$

$\Rightarrow \sqrt{\frac{(1-\sin x)^2}{\cos^2 x}}$

$\Rightarrow \frac{1-\sin x}{\cos x}$

⇒ sec x – tan x

In the given range tan x = -tan x and sec x is –sec x

∴ -sec x-(-tan x)

⇒ tan x – sec x

4. Question

Mark the correct alternative in the following:

If π < x < 2 π, then $\sqrt{\frac{1+\cos x}{1-\cos x}}$ is equal to

A. cosec x + cot x

B. cosec x − cot x

C. − cosec x + cot x

D. − cosec x − cot x

Answer

Given:

⇒ $\pi<x<2\pi$

Now

⇒ $\sqrt{\frac{1+\cos x}{1-\cos x}}$ (Rationalizing we get)

⇒ $\sqrt{\frac{1+\cos x}{1-\cos x}} \times \sqrt{\frac{1+\cos x}{1+\cos x}}$

⇒ $\sqrt{\frac{(1+\cos x)^2}{1-\cos^2 x}}$

⇒ $\sqrt{\frac{(1+\cos x)^2}{\sin^2 x}}$

⇒ $\frac{1+\cos x}{\sin x}$

⇒ cosec x + cot x

In the given range cot x = -cot x and cosec x is –cosec x

∴ -cosec x-(+cot x)

⇒ -cosec x – cot x

5. Question

Mark the correct alternative in the following:

If $0<x<\frac{\pi}{2}$, and if $\frac{y+1}{1-y}=\sqrt{\frac{1+\sin x}{1-\sin x}}$, then y is equal to

A. $\cot\frac{x}{2}$

B. $\tan\frac{x}{2}$

C. $\cot\frac{x}{2}+\tan\frac{x}{2}$

D. $\cot\frac{x}{2}-\tan\frac{x}{2}$

Answer

$\frac{y+1}{1-y}=\sqrt{\frac{1+\sin x}{1-\sin x}}$ $\left[\text{Use } 1=\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right]$

$$\frac{y+1}{1-y}=\sqrt{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$\frac{y+1}{1-y}=\sqrt{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}$$

If 0 < x < π/2 then we take $\cos\frac{x}{2}-\sin\frac{x}{2}$. So, that square root is open with positive sign.

$$\frac{y+1}{1-y}=\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Adding 1 on both sides

$$\frac{y+1}{1-y}+1=\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}+1$$

$$\frac{(y+1)+(1-y)}{1-y}=\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

$$\frac{2}{1-y}=\frac{2\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

$$\frac{1-y}{1}=\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}}$$

$$1-y=1-\tan\frac{x}{2}$$

$$y=\tan\frac{x}{2}$$

6. Question

Mark the correct alternative in the following:

If $\frac{\pi}{2}<x<\pi$, then $\sqrt{\frac{1-\sin x}{1+\sin x}}+\sqrt{\frac{1+\sin x}{1-\sin x}}$ is equal to

A. 2 sec x

B. – 2 sec x

C. sec x

D. – sec x

Answer

$$\sqrt{\frac{1-\sin x}{1+\sin x}}+\sqrt{\frac{1+\sin x}{1-\sin x}}\ \left[\text{Use } 1=\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right]$$

$$\Rightarrow\sqrt{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}}+\sqrt{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$\Rightarrow\sqrt{\frac{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}}+\sqrt{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}$$

$$\Rightarrow \frac{\sin\frac{x}{2}-\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}}+\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}$$

$$\Rightarrow \frac{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}$$

$$\Rightarrow \frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}$$

$$\Rightarrow \frac{2}{\sin^2\frac{x}{2}-\cos^2\frac{x}{2}}\left[\text{Use} \cos^2\frac{x}{2}-\sin^2\frac{x}{2}=\cos x\right]$$

$$\Rightarrow \frac{2}{-\cos x}$$

$$\Rightarrow -2\sec x$$

7. Question

Mark the correct alternative in the following:

If x = r sin θ cos ϕ, y = r sin θ sin ϕ and z = r cos θ, then $x^2 + y^2 + z^2$ is independent of

A. θ, ϕ

B. r, θ

C. r, ϕ

D. r

Answer

Given:

X = r sin θ cos ϕ

Y = r sin θ sin ϕ

Z = r cos θ

⇒ $x^2+y^2+z^2$

⇒ $(r \sin\theta \cos\phi)^2 + (r \sin\theta \sin\phi)^2 + (r \cos\theta)^2$

⇒ $r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

⇒ $r^2\sin^2\theta(\cos^2\phi+ \sin^2\phi) + r^2\cos^2\theta$

⇒ $r^2\sin^2\theta + r^2\cos^2\theta$

⇒ $r^2(\sin^2\theta + \cos^2\theta)$

⇒ r^2.

∴ It is independent of θ and ϕ .

8. Question

Mark the correct alternative in the following:

If $\tan x+\sec x=\sqrt{3}, 0x<\pi$, then x is equal to

A. $\frac{5\pi}{6}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{3}$

Answer

Given: tan x + sec x = √3

squaring on both sides

$(\tan x + \sec x)^2 = \sqrt{3}^2$

$\tan^2 x + \sec^2 x + 2 \tan x \sec x = 3$

Also, $\sec^2 x - \tan^2 x = 1$

$\tan^2 x + 1 + \tan^2 x + 2\tan x \sec x = 3$

$2\tan^2 x + 2\tan x \sec x = 3-1$

$\tan^2 x + \tan x \sec x = 2/2$

$\tan^2 x + \tan x \sec x = 1$

$\tan x \sec x = 1 - \tan^2 x$

again, squaring on both sides

$\tan^2 x \sec^2 x = 1 + \tan^4 x - 2 \tan^2 x$

$(1 + \tan^2 x) \tan^2 x = 1 + \tan^4 x - 2 \tan^2 x$

$\text{Tan}^4 x + \tan^2 x = 1 + \tan^4 x - 2 \tan^2 x$

$3 \tan^2 x = 1$

tan x = 1/√3

x = π/6.

9. Question

Mark the correct alternative in the following:

If $\tan x = -\frac{1}{\sqrt{5}}$ and x lies in the IV quadrant, then the value of cos x is

A. $\frac{\sqrt{5}}{\sqrt{6}}$

B. $\frac{2}{\sqrt{6}}$

C. $\frac{1}{2}$

D. $\frac{1}{\sqrt{6}}$

Answer

In IV quadrant cos x is positive

We know

$\tan^2 x + 1 = \sec^2 x$

$\Rightarrow \left(-\frac{1}{\sqrt{5}}\right)^2 + 1 = \sec^2 x$

$\Rightarrow \frac{1}{5} + 1 = \sec^2 x$

$\Rightarrow \sec^2 x = \frac{6}{5}$

$\Rightarrow \cos^2 x = \frac{5}{6}$

$\therefore \cos x = \frac{\sqrt{5}}{\sqrt{6}}$

10. Question

Mark the correct alternative in the following:

If $\frac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$ is equal to

A. $1 - \cot\alpha$

B. $1 + \cot\alpha$

C. $-1 + \cot\alpha$

D. $-1 - \cot\alpha$

Answer

Given:

$\Rightarrow \frac{3\pi}{4} < \alpha < \pi$

$\Rightarrow \sqrt{2\cot\alpha + \frac{1}{\sin^2\alpha}}$

$\Rightarrow \sqrt{2\cot\alpha + \csc^2\alpha}$

We know $\csc^2\alpha = \cot^2\alpha + 1$

$\Rightarrow \sqrt{2\cot\alpha + 1 + \cot^2\alpha}$

$\Rightarrow \sqrt{(\cot\alpha + 1)^2}$

$\Rightarrow \cot\alpha + 1$

In the given range cot is negative

$\therefore -1-\cot\alpha$

11. Question

Mark the correct alternative in the following:

$\sin^6 A + \cos^6 A + 3\sin^2 A\cos^2 A =$

A. 0

B. 1

C. 2

D. 3

Answer

$\sin^6 A + \cos^6 A = (\sin^2 A)^3 + (\cos^2 A)^3$

$=(\sin^2 A+ \cos^2 A)(\sin^4 A+\cos^4 A-\sin^2 A\cos^2 A)$

$= 1 (\sin^4 A + \cos^4 A - \sin^2 A\cos^2 A)$

$\therefore \sin^4 A + \cos^4 A - \sin^2 A\cos^2 A+ 3 \sin^2 A \cos^2 A$

$\Rightarrow \sin^4 A + \cos^4 A + 2\sin^2 A\cos^2 A$

$\Rightarrow (\sin^2 A + \cos^2 A)^2$

$= 1^2$

$=1$

12. Question

Mark the correct alternative in the following:

If $\operatorname{cosec} x - \cot x = \frac{1}{2}, 0 < x < \frac{\pi}{2}$, then cos x is equal to

A. $\frac{5}{3}$

B. $\frac{3}{5}$

C. $-\frac{3}{5}$

D. $-\frac{5}{3}$

Answer

Given:

Let cosec x =a, cot x = b

$\therefore$ According to the question

$\Rightarrow a - b = \frac{1}{2}$

But, $\operatorname{cosec}^2 x - \cot^2 x = 1$

$\Rightarrow a^2 - b^2 = 1$

$\Rightarrow (a-b)(a + b) = 1$

$\Rightarrow \frac{1}{2}(a+b) = 1$

$\Rightarrow a + b=2$

a-b =1/2 …(1)

a + b =2 ...(2)

Adding (1) and (2)

2a = 1/2+2

⇒ a = 5/4

$\therefore \csc x = \frac{5}{4}$

$\Rightarrow \sin x = \frac{4}{5}$

$Cos^2x = 1 - sin^2x$

$\Rightarrow \cos^2 x = 1 - \frac{16}{25}$

$\Rightarrow \cos^2 x = \frac{9}{25}$

$\Rightarrow \cos x = \frac{3}{5}$

13. Question

Mark the correct alternative in the following:

If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then tan x =

A. $\frac{21}{22}$

B. $\frac{15}{16}$

C. $\frac{44}{117}$

D. $\frac{117}{44}$

Answer

Let cosec x =a, cot x = b

∴ According to the question

$\Rightarrow a - b = \frac{11}{2}$

But, $cosec^2x - cot^2x = 1$

$\Rightarrow a^2 - b^2 = 1$

⇒ (a-b)(a + b) = 1

$\Rightarrow \frac{11}{2}(a + b) = 1$

$\Rightarrow a + b = \frac{2}{11}$

$a - b = \frac{11}{2} \ldots (1)$

$$a + b = \frac{2}{11} \ldots (2)$$

Adding (1) and (2)

$$\Rightarrow 2a = \frac{11}{2} + \frac{2}{11}$$

$$\Rightarrow a = \frac{125}{44}$$

$$\Rightarrow \frac{125}{44} - \frac{11}{2} = b$$

$$\Rightarrow b = \frac{-117}{44}$$

$$\therefore \cot x = \frac{-117}{44}$$

$$\Rightarrow \tan x = \frac{44}{117}$$

14. Question

Mark the correct alternative in the following:

$\sec^2 x = \frac{4xy}{(x+y)^2}$ is true if and only if

A. $x + y \neq 0$

B. $x + y, x \neq 0$

C. x = y

D. $x \neq 0, y \neq 0$

Answer

First of all we need to check the condition on x

If x = 0 then sec^{2}x attains to infinity, so that condition must be true i.e x should not be zero

Again if x+y =0 then the RHS part becomes infinity so that condition must be true i.e. x+y should not be zero.

∴ Option B is the correct answer.

15. Question

Mark the correct alternative in the following:

If x is an acute angle and $\tan x = \frac{1}{\sqrt{7}}$, then the value of $\frac{\operatorname{cosec}^2 x - \sec^2 x}{\operatorname{cosec}^2 x + \sec^2 x}$ is

A. 3/4

B. 1/2

C. 2

D. 5/4

Answer

Given x is an acute angle and value of tan x =1/√7.

⇒ We know $\tan^2 x + 1 = \sec^2 x$

⇒ Also, $\cot^2 x + 1 = \text{cosec}^2 x$

$\therefore\ \tan^2 x = \frac{1}{7}$

$\therefore\ \tan^2 x + 1 = \frac{1}{7} + 1 = \frac{8}{7}$

$\therefore\ \sec^2 x = \frac{8}{7}$

⇒ $\cot^2 x = 7$

⇒ $\cot^2 x + 1 = 7 + 1$

$= 8$

⇒ $\text{cosec}^2 x = 8$

$\therefore\ \frac{\csc^2 x - \sec^2 x}{\csc^2 x + \sec^{2b} x} = \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$

⇒ $\frac{\frac{48}{7}}{\frac{64}{7}}$

⇒ $\frac{48}{64} = \frac{3}{4}$

⇒ $\frac{3}{4}$

16. Question

Mark the correct alternative in the following:

The value of $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 85° + \sin^2 90°$ is

A. 7

B. 8

C. 9.5

D. 10

Answer

$= \sin^2 5 + \sin^2 10 + \sin^2 15 + \ldots\ldots \sin^2 45 + \ldots\ldots\ldots\ldots + \sin^2 75 + \sin^2 80 + \sin^2 85 + \sin^2 90$

We know that sin(90-x) =cos x

So $\sin^2(90-x) = \cos^2 x$

$= \sin^2 5 + \sin^2 10 + \sin^2 15 + \ldots\ldots \sin^2 45 + \ldots\ldots\ldots\ldots + \cos^2 15 + \cos^2 10 + \cos^2 5 + \sin^2 90$

And $\sin^2 x + \cos^2 x = 1$

So, in given series on rearranging terms we get 8 cases where $\sin^2 x + \cos^2 x = 1$

So, given changes to

$8 + \sin^2 45 + \sin^2 90$

$= 8 + \frac{1}{2} + 1$

$= 9 + \frac{1}{2}$

= 9.5

17. Question

Mark the correct alternative in the following:

$$\sin^2\frac{\pi}{18}+\sin^2\frac{\pi}{9}+\sin^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}=$$

A. 1

B. 4

C. 2

D. 0

Answer

We know that sin(90-x) =cos x

So $\sin^2(90-x)=\cos^2 x$

$\Rightarrow \sin^2\left(\frac{\pi}{2}-\frac{\pi}{9}\right)$

$\Rightarrow \cos^2\frac{7\pi}{18}$

$\Rightarrow \sin^2\left(\frac{\pi}{2}-\frac{\pi}{18}\right)$

$\Rightarrow \cos^2\frac{4\pi}{9}$

And $\sin^2 x+\cos^2 x=1$

Rearranging we get,

$\Rightarrow \sin^2\frac{7\pi}{18}+\cos^2\frac{7\pi}{18}+\sin^2\frac{4\pi}{9}+\cos^2\frac{4\pi}{9}$

=1+1

=2

18. Question

Mark the correct alternative in the following:

If tan A + cot A = 4, then $\tan^4$ A + $\cot^4$ A is equal to

A. 110

B. 191

C. 80

D. 194

Answer

Given: tan A + cot A = 4

$\Rightarrow \tan A+\frac{1}{\tan A}=4$

Squaring both sides we get

$\Rightarrow \left(\tan A+\frac{1}{\tan A}\right)^2=4^2$

$\Rightarrow \tan^2 A + \frac{1}{\tan^2 A} + 2.\tan A.\frac{1}{\tan A} = 16$

$\Rightarrow \tan^2 A + + \frac{1}{\tan^2 A} = 14$

Squaring both sides we get

$\Rightarrow \left(\tan^2 A + \frac{1}{\tan^2 A}\right)^2 = 14^2$

$\Rightarrow \tan^4 A + \frac{1}{\tan^4 A} + 2.\tan^2 A.\frac{1}{\tan^2 A} = 196$

$\Rightarrow \tan^4 A + + \frac{1}{\tan^4 A}$

=194

19. Question

Mark the correct alternative in the following:

If x sin 45° cos^2 60° $= \frac{\tan^2 60° \operatorname{cosec} 30°}{\sec 45° \cot^{2°} 30°}$, then x =

A. 2

B. 4

C. 8

D. 16

Answer

According to the given question:

$\Rightarrow x = \frac{\tan^2 60 \ \csc 30}{\sec 45 \cot^2 30 \sin 45 \cos^2 60}$

$\Rightarrow x = \frac{(\sqrt{3})^2.2}{\sqrt{2}.(\sqrt{3})^2.\frac{1}{\sqrt{2}}.\left(\frac{1}{2}\right)^2}$

⇒ x =8.

20. Question

Mark the correct alternative in the following:

If A lies in second quadrant and 3 tan A + 4 = 0, then the value of 2 cot A− 5 cos A + sin A is equal to

A. −53/10

B. 23/10

C. 37/10

D. 7/10

Answer

Given:

3tanA +4 =0

$\Rightarrow \tan A = \frac{-4}{3}$

$\tan^2 x + 1 = \sec^2 x$

$\Rightarrow \left(\frac{-4}{3}\right)^2 + 1 = \sec^2 x$

$\Rightarrow \sec^2 A = \frac{16}{9} + 1$

$\Rightarrow \sec^2 A = \frac{25}{9}$

$\Rightarrow \sec A = \frac{5}{3}$

$\Rightarrow \cos A = -\frac{3}{5}$

Because in second quadrant cos is negative.

$\Rightarrow \cot A = \frac{-3}{4}$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x$

$\Rightarrow \sin^2 A = 1 - \left(\frac{9}{25}\right)$

$\Rightarrow \sin^2 A = \frac{16}{25}$

$\Rightarrow \sin A = \frac{4}{5}$

∴ The value of 2 cot A − 5 cos A + sin A =

$\Rightarrow 2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) + \frac{4}{5}$

$\Rightarrow \frac{-6}{4} + \frac{15}{5} + \frac{4}{5}$

$\Rightarrow \frac{-6}{4} + \frac{19}{5}$

$\Rightarrow \frac{-30 + 76}{20}$

$\Rightarrow \frac{23}{10}$

21. Question

Mark the correct alternative in the following:

If $\operatorname{cosec} x + \cot x = \frac{11}{2}$, then tan x =

A. 21/22

B. 15/16

C. 44/117

D. 117/43

Answer

Let cosec x =a, cot x = b

∴ According to the question

$$\Rightarrow\ a + b = \frac{11}{2}$$

But, $cosec^2x - cot^2x = 1$

⇒ $a^2 - b^2 = 1$

⇒ (a-b)(a + b) = 1

$$\Rightarrow\ \frac{11}{2}(a - b) = 1$$

$$\Rightarrow\ a - b = \frac{2}{11}$$

a + b =11/2 ...(1)

a-b =2/11 ...(2)

Adding (1) and (2)

$$\Rightarrow\ 2a = \frac{11}{2} + \frac{2}{11}$$

$$\Rightarrow\ a\ = \frac{125}{44}$$

$$\Rightarrow\ \frac{125}{44} - \frac{2}{11} = b$$

$$\Rightarrow\ b = \frac{117}{44}$$

$$\therefore\ \cot x = \frac{117}{44}$$

$$\therefore\ \tan x = \frac{44}{117}$$

22. Question

Mark the correct alternative in the following:

If tan θ + sec θ = e^x, then cos θ equals

A. $\frac{e^x + e^{-x}}{2}$

B. $\frac{2}{e^x + e^{-x}}$

C. $\frac{e^x - e^{-x}}{2}$

D. $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

Answer

We know $\tan^2x + 1 = \sec^2x$...(1)

Let tan θ be a and sec θ be b

According to question

$a + b = e^x$

Manipulating and Substituting in 1 we get

$\Rightarrow a^2 - b^2 = -1$

$\Rightarrow (a-b)(a + b) = -1$

$\Rightarrow (a-b).e^x = -1$

$\Rightarrow a-b = -e^{-x}$

$a + b = e^x$

$a-b = -e^{-x}$

subtracting above equations we get

$2b = e^x + e^{-x}$

$$\Rightarrow b = \frac{e^x + e^{-x}}{2}$$

$$\Rightarrow \sec\theta = \frac{e^x + e^{-x}}{2}$$

$$\Rightarrow \cos\theta = \frac{2}{e^x + e^{-x}}$$

23. Question

Mark the correct alternative in the following:

If sec x + tan x = k, cos x =

A. $\frac{k^2+1}{2k}$

B. $\frac{2k}{k^2+1}$

C. $\frac{k}{k^2+1}$

D. $\frac{k}{k^2-1}$

Answer

We know $\tan^2 x + 1 = \sec^2 x$...(1)

Let tan x be a and sec x be b

According to question

$a + b = k$

Manipulating and Substituting in 1 we get

$\Rightarrow a^2 - b^2 = -1$

$\Rightarrow (a-b)(a + b) = -1$

$\Rightarrow (a-b).k = -1$

$\Rightarrow a-b = -k^{-1}$

$a + b = k$

$a - b = -k^{-1}$

subtracting above equations we get

$2b = k + k^{-1}$

$$\Rightarrow\ b = \frac{k^2 + 1}{2k}$$

$$\Rightarrow\ \sec\theta = \frac{k^2 + 1}{2k}$$

$$\Rightarrow\ \cos\theta = \frac{2k}{k^2 + 1}$$

24. Question

Mark the correct alternative in the following:

If $f(x) = \cos^2 x + \sec^2 x$, the

A. $f(x) < 1$

B. $f(x) = 1$

C. $2 < f(x) < 1$

D. $f(x) \geq 2$

Answer

$\tan^2 x + 1 = \sec^2 x$

$\sin^2 x + \cos^2 x = 1$

$\therefore \cos^2 x = 1 - \sin^2 x$

Substituting in f(x) we get

$1 - \sin^2 x + \tan^2 x + 1$

$$\Rightarrow\ 2 - \sin^2 x + \frac{\sin^2 x}{\cos^2 x}$$

$$\Rightarrow\ 2 - \frac{\sin^2 x \cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\Rightarrow\ 2 + \frac{\sin^2 x\,(1 - \cos^2 x)}{\cos^2 x}$$

$$\Rightarrow\ 2 + \frac{\sin^4 x}{\cos^2 x}$$

Minimum value of $\frac{\sin^4 x}{\cos^2 x}$ is 0.

$\therefore f(x) \geq 2$.

25. Question

Mark the correct alternative in the following:

Which of the following is incorrect?

A. $\sin x = -1/5$

B. $\cos x = 1$

C. sec x = 1/2

D. tan x = 20

Answer

Sec x = 1/2 is incorrect because for no real value of x sec x attains 1/2.

26. Question

Mark the correct alternative in the following:

The value of cos 1° cos 2° cos 3° ... cos 179° is

A. $1/\sqrt{2}$

B. 0

C. 1

D. −1

Answer

Cos 1 × cos 2 × cos 3 × ......× cos 179

= cos 1 × cos 2 × cos 3 × .....× cos 90 × ..... × cos 179

= cos 1 × cos 2 × cos 3 × .....× 0 × ..... × cos 179

= 0 × cos 1 × cos 2 × cos 3 × .... × cos 179

= 0

27. Question

Mark the correct alternative in the following:

The value of tan 1° tan 2° tan 3° ... tan 89° is

A. 0

B. 1

C. 1/2

D. not defined

Answer

tan 1° tan 2° tan 3° ... tan 89° = tan (90° - 89°) tan(90° - 88°) tan(90° - 87°) ... tan(90° - 46°) tan 45° tan 46° .... tan 89°

= cot 89° cot 88° cot 87° ..... cot 46° tan 45° tan 46° ..... tan 89°

(∵ tan(90° - θ) = cot θ)

$$= \frac{1}{\tan 89°} \times \frac{1}{\tan 88°} \times \frac{1}{\tan 87°} \times .... \frac{1}{\tan 46°} \times \tan 45° \tan 46° \; .....\tan 89°$$

= tan 45°

= 1

28. Question

Mark the correct alternative in the following:

Which of the following is correct?

A. sin 1° > sin 1

B. sin 1° < sin 1

C. sin 1° = sin 1

D. $\sin 1° = \frac{\pi}{180} \sin 1$

Answer

$\Rightarrow\ 1° = \frac{\pi}{180} \text{rad}$

$\Rightarrow\ 1\text{rad} = \frac{180°}{\pi}$

∴ 1rad = 57.32°

In Range 0 to π/2 sin x is an increasing function

∴ sin1 will always be greater than sin1°

Because sin1 = sin57.32°