5. Factorization of Algebraic Expressions

Exercise 5.1

1. Question

Factorize:

x^3+x-3x^2-3

Answer

Given,

$x^3 + x - 3x^2 - 3$

$= x(x^2 + 1) - 3(x^2 + 1)$

$= (x - 3)(x^2 + 1)$

2. Question

Factorize:

$a(a+b)3-3a^2b(a+b)$

Answer

Given,

$a(a + b)^3 - 3a^2b(a + b)$

$= a(a + b)\{(a + b)^2 - 3ab\}$ use the identity:$(a + b)^2 = a^2 + b^2 + 2ab$

$= a(a + b)(a^2 + b^2 + 2ab - 3ab)$

$= a(a + b)(a^2 + b^2 - ab)$

3. Question

Factorize:

$x(x^3–y^3)+3xy-(x–y)$

Answer

Given,

$x(x^3–y^3)+3xy-(x–y)$

As $(x^3 - y^3) = (x – y)(x^2 + xy + y^2)$

$x(x^3–y^3)+3xy-(x–y) = x [(x – y)(x^2 + xy + y^2)] + 3xy-(x–y)$

Take x(x-y) common to get,

$x(x-y)[(x^2 + xy + y^2) + 3y]$

4. Question

Factorize:

$a^2x^2+(ax^2+1)x+a$

Answer

Given,

$a^2x^2 + (ax^2 + 1)x + a = a^2x^2 + ax^3 + x + a$

$= ax^2(a+x)+1(x+a)$

$= (ax^2+1)(x+a)$

5. Question

Factorize:

x^2+y-xy-x

Answer

Given,

$x^2+y-xy-x = x^2-x+y-xy$

$= x(x-1)+y(1-x)$

$= x(x-1)-y(x-1)$

$= (x-y)(x-1)$

6. Question

Factorize:

x^3-2x^2y+3xy^2-6y^3

Answer

Given,

$x^3-2x^2y+3xy^2-6y^3$

$= x^2(x-2y)+3y^2(x-2y)$

$= (x-2y)(x^2+3y^2)$

7. Question

Factorize:

6ab-b^2+12ac-2bc

Answer

Given,

6ab-b^2+12ac-2bc = b(6a-b)+2c(6a-b)

= (b+2c)(6a-b)

8. Question

Factorize:

$\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$

Answer

Given,

$\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$

$= \left(x^2+\frac{1}{x^2}+2\right)-4\left(x+\frac{1}{x}\right)+4$

$= \left(x+\frac{1}{x}\right)^2-4\left(x+\frac{1}{x}\right)+4$

$= \left\{\left(x+\frac{1}{x}\right)-2\right\}^2$ [BY applying $(a^2 - 2ab + b^2) = (a-b)^2$]

9. Question

Factorize:

$x(x–2)(x–4)+4x-8$

Answer

Given,

$X(x-2)(x-4)+4x-8 = x(x-2)(x-4)+4(x-2)$

$= (x-2)(x(x-4)+4)$

$= (x-2)(x^2 - 4x + 4)$

$= (x-2)(x-2)^2$

$= (x-2)^3$

10. Question

Factorize:

$(x+2)(x^2+25)-10x^2-20x$

Answer

Given,

$(x+2)(x^2+25) - 10x^2 - 20x = (x+2)(x^2+25)-10x(x+2)$

$= (x+2)(x^2+25-10x)$

$= (x+2)(x-5)^2$

11. Question

Factorize:

$2a^2+2\sqrt{6}ab+3b^2$

Answer

Given,

$2a^2\sqrt{6}ab+3b^2 = (\sqrt{2}a)^2 +2(\sqrt{3}\times\sqrt{2})ab + (\sqrt{3}b)^2$

$= (\sqrt{2}a+\sqrt{3}b)^2$

12. Question

Factorize:

$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

Answer

Given,

$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

$= (a-(b-c))^2 + (a+(b-c))^2 + 2(a-(b-c))(a+(b-c))$

[Applying identity: $x^2 + y^2 +2xy = (x + y)^2$, where x= a-(b-c), y= a+(b-c)]

$=[(a-(b-c)) + (a+(b-c))]^2=(2a)^2=4a^2$

13. Question

Factorize:

$a^2+b^2+2(ab+bc+ca)$

Answer

Given,

$a^2+b^2+2(ab+bc+ca)$

$= a^2+b^2+2ab+2bc+2ca$

$= (a+b)^2+2c(a+b)$

$= (a+b)(a+b+2c)$

14. Question

Factorize:

$4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$

Answer

Given,

$4(x-y)^2 -12(x-y)(x+y)+9(x+y)^2 = 4(x^2-2xy+y^2)-12(x^2 - y^2)+9(x^2+y^2+2xy)$

$= 4x^2-8xy+4y^2-12x^2+12y^2+9x^2+9y+18xy$

$= x^2+25y^2+10xy$

$= (x)^2+(5y)^2+2\times x\times 5y$

$= (x+5y)^2$

15. Question

Factorize:

$a^2-b^2+2ab-c^2$

Answer

Given,

$a^2-b^2+2ab-c^2 = a^2-(b^2-2bc+c^2)$

$= a - (b-c)^2$

$= (a + (b-c)) (a-(b-c))$

$= (a+b-c) (a-b+c)$

16. Question

Factorize:

$a^2+2ab+b^2-c^2$

Answer

Given,

$a^2+2ab+b^2-c^2 = (a+b)^2 - c^2$

$= (a+b-c) (a+b+c)$

17. Question

Factorize:

$a^2+4b^2-4ab-4c^2$

Answer

Given,

$a^2 +4b^2-4ab-4c^2 = (a)^2+ (2b)^2 – 2×a×2b-4c^2$

$= (a-2b)^2 – (2c)^2$

$= (a-2b-2c)(a-2b+2c)$

18. Question

Factorize:

$xy^9–yx^9$

Answer

Given,

$xy^9 - yx^9 = xy (y^8-x^8)$

$= xy ((y^4)^2 – (x^4)^2)$

$= xy (y^4+x^4) (y^4 - x^4)$

$= xy (y^4+x^4) (y^2-x^2) (y^2+x^2)$

$= xy (y^4+x^4) (y^2+x^2) (y – x) (y+x)$

19. Question

Factorize:

$x^4+x^2y^2+y^4$

Answer

Given,

$x^4+x^2y^2+y^4 = x^4+2x^2y^2+y^4 – x^2y^2$

$= (x^2y^2)^2 – (xy)^2$

$= (x^2+ y^2 – xy) (x^2+ y^2 + xy)$

20. Question

Factorize:

$x^2-y^2-4xz+4z^2$

Answer

Given,

$x^2 – y^2 - 4xz + 4z^2 = x^2 – 4xz +4z^2 – y^2$

$= (x)^2 - 2×x×2z+(2z)^2 – y^2$

$= (x-2z)^2 – y^2$

$= (x - 2z - y) (x – 2z + y)$

$= (x – y – 2z) (x + y – 2z)$

21. Question

Factorize:

$x^2+6\sqrt{2}x+10$

Answer

Given,

$x^2+6\sqrt{2}x+10 = x^2+\sqrt{2}x + 10$

$= x(x+\sqrt{2}) + 5\sqrt{2}(x+\sqrt{2})$

$= (x+\sqrt{2})(x+5\sqrt{2})$

22. Question

Factorize:

$x^2-2\sqrt{2}x-30$

Answer

Given,

$x^2 - 2\sqrt{2}x - 30 = x^2 - 5\sqrt{2}x+3\sqrt{2}x-30$

$= x(x-5\sqrt{2}) +3\sqrt{2}(x - 5\sqrt{2})$

$= (x-5\sqrt{2})(x+3\sqrt{2})$

23. Question

Factorize:

$x^2-\sqrt{3}x-6$

Answer

Given,

$X^2 - \sqrt{3}x - 6 = x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$

$= x(x - 2\sqrt{3}) + \sqrt{3}(x-2\sqrt{3})$

$= (x + \sqrt{3})(x - 2\sqrt{3})$

24. Question

Factorize:

$x^2+5\sqrt{5}x+30$

Answer

Given,

$x^2 + 5\sqrt{5}x + 30 = x^2 + 3\sqrt{5}x + 2\sqrt{5}x + 30$

$= x(x + 3\sqrt{5}) + 2\sqrt{5}(x + 3\sqrt{5})$

$= (x + 3\sqrt{5})(x + 2\sqrt{5})$

25. Question

Factorize:

$x^2+2\sqrt{3}x-24$

Answer

Given,

$x^2 + 2\sqrt{3}x - 24 = x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$

$= x(x + 4\sqrt{3}) - 2\sqrt{3}(x + 4\sqrt{3})$

$= (x + 4\sqrt{3})(x - 2\sqrt{3})$

26. Question

Factorize:

$2x^2 - \frac{5}{6}x + \frac{1}{12}$

Answer

Given,

$$2x^2 - \frac{5}{6}x + \frac{1}{12}$$

$$= 2x^2 - \frac{1}{3}x - \frac{1}{2}x + \frac{1}{12}$$

$$= x\left(2x - \frac{1}{3}\right) - \frac{1}{4}\left(2x - \frac{1}{3}\right)$$

$$= \left(2x - \frac{1}{3}\right)\left(x - \frac{1}{4}\right)$$

27. Question

Factorize:

$x^2 + \frac{12}{35}x + \frac{1}{35}$

Answer

Given,

$$x^2 - \frac{12}{36}x + \frac{1}{36} = x^2 - \frac{1}{7}x - \frac{1}{5}x + \frac{1}{35}$$

$$x\left(x + \frac{1}{7}\right) + \frac{1}{5}\left(x + \frac{1}{7}\right)$$

$$= \left(x - \frac{1}{7}\right)\left(x + \frac{1}{5}\right)$$

28. Question

Factorize:

$21x^2 - 2x + \frac{1}{21}$

Answer

Given,

$$21x^2 - 2x + \frac{1}{21} = 21x^2 - x - x + \frac{1}{21}$$

$$= x(21x - 1) - \frac{1}{21}(21x - 1)$$

$$= \left(x - \frac{1}{21}\right)(21x - 1)$$

29. Question

Factorize:

$5\sqrt{5}x^2+20x+3\sqrt{5}$

Answer

Given,

$5\sqrt{5}x^2+20x+3\sqrt{5} = 5\sqrt{5}x^2 + 15x + 5x + 3\sqrt{5}$

$= 5x(\sqrt{5}x+3) + \sqrt{5}(\sqrt{5}x+3)$

$= (5x +\sqrt{5})(\sqrt{5}x + 3)$

30. Question

Factorize:

$2x^2+3\sqrt{5}x+531.9(2a-b)^2-4(2a-b)-13$

Answer

Given,

$2x^2+3\sqrt{5}x+5 = 2x^2 + 2\sqrt{5}x+ \sqrt{5}x+5$

$= 2x(x + \sqrt{5}) +\sqrt{5}(x+\sqrt{5})$

$= (2x+\sqrt{5})(x+\sqrt{5})$

31. Question

Factorize:

$9(2a – b)^2 –4(2a – b) – 13$

Answer

Given,

$9(2a – b)^2 –4(2a – b) – 13$

Let us assume $(2a – b) = x$

$9x^2 – 4x – 13$

$9x^2 – 13x + 9x – 13$

$x(9x – 13)+1(9x + 3)$

$(9x – 13)(x + 1)$

$[9(2a – b) – 13][2a – b + 1]$

$(18a – 9b – 13)(2a – b + 1)$

32. Question

Factorize:

$7(x-2y)^2-25(x-2y)+12$

Answer

Given,

$7(x – 2y)^2 – 25(x – 2y) + 12$

Let $a = (x – 2y)$,

So we have,

$= 7a^2 – 25a + 12$

$= 7a^2 – 21a – 4a + 12$

$= 7a(a - 3) -4 (a - 3)$

$= (7a – 4) (a - 3)$

Put $a = (x – 2y)$

$= \{7(x – 2y) – 4\} (x – 2y - 3)$

$= (7x – 14y -4) (x – 2y - 3)$

33. Question

Factorize:

$2(x+y)^2-9(x+y)-5$

Answer

Given,

$2(x+y)^2 – 9(x+y) – 5,$

$= 2a^2 -9a – 5$

Let $(x+y) = a$

$= 2a^2 – 10a+a – 5$

$= 2a (a-5) +1(a-5)$

$= (2a+1) (a-5)$

$= \{2(x+y)+1\}(x+y-5)$

$(2x+2y+1) (x+y-5)$

34. Question

Give possible expressions for the length and breadth of the rectangle having $35y^2+13y-12$ as its area.

Answer

We know that,

Area of rectangle = length × breadth

Given,

$35y^2 + 13y – 12 = 35y^2 + 28y – 15y -12$

$= 7y(5y+4) -3 (5y + 4)$

$= (7y – 3) (5y + 4)$

Thus,

Length = (7y – 3), then breadth = (5y + 4)

Length = (5y + 4), then breadth = (7y – 3)

35. Question

What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2-12x$.

Answer

Given,

We know that,

Volume of cuboids = length × breadth × height

Given,

$3x^2 - 12x = 3x\ (x-4)$

Thus,

Dimensions of cuboids are –

Length	breadth	height
3	x	(x-4)
X	(x-4)	3
(x-4)	3	x

Exercise 5.2

1. Question

Factorize each of the following expressions:

p^3+27

Answer

Given,

P^3+27,

$= p^3 + (3)^3$ [$\because a^3+b^3 = (a+b)(a^2 - 2ab + b^2)$

$= (p + 3)\ (p^2 + 9 - 3p)$

2. Question

Factorize each of the following expressions:

y^3+125

Answer

Given,

y^3+125,

$= y^3 + (5)^3$ [$\because a^3+b^3 = (a+b)(a^2 - 2ab + b^2)$

$= (y + 5)\ (y^2 - 5y + 25)$

3. Question

Factorize each of the following expressions:

$1-27a^3$

Answer

Given,

$1 - 27a^3$,

$= 1 - (3a)^3$

$= (1 - 3a)\ (1+9a^2+3a)$

4. Question

Factorize each of the following expressions:

$8x^3y^3+27a^3$

Answer

Given,

$8x^3 y^3 + 27a^3$,

$= (2xy)^3 + (3a)^3$

$= (2xy + 3a)(4x^2 y^2 + 9a^2 - 6axy)$

5. Question

Factorize each of the following expressions:

$64a^3 - b^3$

Answer

Given,

$64a^3 - b^3$,

$= (4a)^3 - (b)^3$

$= (4a - b)(16a^2 + b^2 + 4ab)$

6. Question

Factorize each of the following expressions:

$\frac{x^3}{216} - 8y^3$

Answer

Given,

$$\frac{x^3}{216} - 8y^3 = \left(\frac{x}{6}\right)^3 - (2y)^3$$

$$= \left(\frac{x}{6} - 2y\right)\left(\frac{x^2}{36} + 4y^2 + \frac{xy}{3}\right)$$

7. Question

Factorize each of the following expressions:

$10x^4 y - 10xy^4$

Answer

Given,

$10x^4 y - 10xy^4$,

$= 10xy (x^3 - y^3)$

$= 10xy (x - y)(x^2 + xy - y^2)$

8. Question

Factorize each of the following expressions:

$54x^6 y + 2x^3 y^4$

Answer

Given,

$54x^6 y + 2x^3 y^4$,

$= 2x^3 y (27x^3 + y^3)$

$= 2x^3 y \{(3x)^3 + (y)^3\}$

$= 2x^3 y (3x + y) (9x^2 + y^2 – 3xy)$

9. Question

Factorize each of the following expressions:

$32a^2+108b^3$

Answer

Given,

$32a^3+108b^3$,

$= 4 (8a^3+27b^3)$

$= 4 \{ (2a)^3 + (3b)^3 \}$

$= 4 (2a + 3b) (4a^2 +9b^2 – 6ab)$

10. Question

Factorize each of the following expressions:

$(a-2b)^3-512b^3$

Answer

Given,

$(a – 2b)^3 – 512b^3$

$= (a – 2b)^3 – (8b)^3$

$= (a – 2b – 8b) \{(a – 2b)^2 +(8b)^2 +(a – 2b) 8b\}$

$= (a – 10b) (a^2 +4b^2 – 4ab +64b^2 +8ab – 16b^2)$

$= (a – 10b) (a^2 +52b^2 +4ab)$

11. Question

Factorize each of the following expressions:

$(a+b)^3-8(a-b)^3$

Answer

Given,

$(a + b)^3 – \{2(a – b)\}^3$

$= \{(a + b) –2(a - b)\} \{ (a+b)^2 +4(a-b)^2 +2(a+b)(a - b)\}$ [By using: $x^3 - y^3 = (x - y)(x^2 + y^2 +xy)$

$= (a + b – 2a + 2b)(a^2 + b^2 + 2ab + 4a^2 +4b^2 – 8ab + 2a^2 – 2b^2)$

$= (3b – a) (7a^2 +3b^2 - 6ab)$

12. Question

Factorize each of the following expressions:

$(x+2)^3+(x-2)^3$

Answer

Given,

$(x+2)^3 + (x - 2)^3 = (x + 2 + x - 2) \{ (x + 2)^2 + (x - 2)^2 – (x + 2)(x - 2) \}$

= 2x (x^2 + 4 + 4x + x^2 + 4 – 4x – x^2 + 4)

= 2x (x^2 +12)

13. Question

Factorize each of the following expressions:

$8x^2y^3$-x^5

Answer

Given,

$8x^2y^3$ – x^5

= x^2 ($8y^3$ – x^3)

= x^2 { $(2y)^2$ – $(x)^3$}

= x^2 (2y - x) ($4y^2$ + x^2 + 2xy)

14. Question

Factorize each of the following expressions:

1029-$3x^3$

Answer

Given,

1029 – $3x^3$

= 3 (343 – x^3)

= 3 { $(7)^3$ – $(x)^3$}

= 3 (7 - x) (49 +x^2 +7x)

15. Question

Factorize each of the following expressions:

x^6+y^6

Answer

Given,

X^6 + y^6 = $(x^2)^3$ + $(y^2)^3$

= (x^2 + y^2) (x^4 +y^4 – x^2y^2)

16. Question

Factorize each of the following expressions:

x^3y^3+1

Answer

Given,

X^3y^3 + 1 = $(xy)^3$ +$(1)^3$

= (xy + 1) (x^2 y^2 +1 - xy)

17. Question

Factorize each of the following expressions:

x^4y^4-xy

Answer

Given,

$X^4y^4 – xy = xy (x^3y^3 -1)$

$= xy \{ (xy)^3 – (1)^3\}$

$= xy (xy -1) (x^2y^2 +1+xy)$

18. Question

Factorize each of the following expressions:

$a^{12}+b^{12}$

Answer

Given,

$a^{12} +b^{12} = (a^4)^3 + (b^4)^3$

$= (a^4 + b^4)(a^3 +b^3 – a^4b^4)$

19. Question

Factorize each of the following expressions:

$x^3+6x^2+12x+16$

Answer

Given,

$X^3 +6x^212x+16 = (x^3 + 6x^2+12x+8)+8$

$= (x+2)^3 + 8$ [$\because (a+b)^3 = a^3 + b^3 +3ab(a+b)$]

$= (x+2)^3 + (2)^3$

$= (x+2+2) \{(x+2)^2 +4 – 2 (x+2)\}$

$= (x+4)(x^2+4+4x+4 - 2x - 4)$

$= (x+4)(x^2+2x+4)$

20. Question

Factorize each of the following expressions:

a^3+b^3+a+b

Answer

Given,

$a^3+b^3+a+b = (a^3 + b^3) + (a+b)$

$= (a+b) (a^2 – ab +b^2) +a+b$

$= (a+b)(a^2 – ab +b^2+1)$

21. Question

Factorize each of the following expressions:

$a^3-\frac{1}{a^3}-2a\frac{2}{a}$

Answer

Given,

$$a^3-\frac{1}{a^3}-2a+\frac{2}{a}$$

$$=a^3-\frac{1}{a^3}-2\left(a-\frac{1}{a}\right)$$

$$=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}+1\right)-2\left(a-\frac{1}{a}\right)$$

$$=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}+1-2\right)$$

$$=\left(a-\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}-1\right)$$

22. Question

Factorize each of the following expressions:

$a^3+3a^2b^3+3ab^2+b^3-8$

Answer

Given,

$A^3+3a^2\ b+3ab^2+b^3-8 = (a^3+3a^2b+3ab^2+b^3) - 8$

$= (a+b)^3 - 8$

$= (a+b)^3 - (2)^3$

$= (a+b-2)\ \{ (a+b)^2 + (2)^2 + 2(a+b) \}$

$= (a+b-2)\ (a^2+b^2 + 2ab+4+2a+2b)$

23. Question

Factorize each of the following expressions:

$8a^3-b^3-4ax+2bx$

Answer

Given,

$8a^3 - b^3 - 4\ ax + 2bx = (2a)^3 - (b)^3 - 2x(2a - b)$

$= (2a - b)\ (4a^2 + b^2 + 2ab)-2x(2a - b)$

$= (2a - b)\ (4a^2 + b^2 + 2ab - 2x)$

24. Question

Simplify

(i) $\frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$

(ii) $\frac{155\times155\times155-55\times55\times55}{155\times155+155\times55+55\times55}$

(iii) $\frac{1.2\times1.2\times1.2-0.2\times0.2\times0.2}{1.2\times1.2+1.2\times0.2+0.2\times0.2}$

Answer

(i) Given,

$$= \frac{173\times173\times173 + 127\times127\times127}{173\times173 - 173\times127 + 127\times127}$$

$$= \frac{(173)^3 + (127)^3}{(173)^2 - 173\times127 + (127)^2}$$

$$= \frac{(173+127)\{(173)^2 + (127)^2 - 173\times127\}}{(173)^2 - 173\times127 + (127)^2} = 300$$

(ii) Given,

$$\frac{155\times155\times155 - 55\times55\times55}{155\times155 + 155\times55\times55\times55}$$

$$= \frac{(155)^3 - (55)^3}{(155)^2 + 155\times55 + (55)^2}$$

$$= \frac{(155-55)\{(155)^2 + 155\times55 + (55)^2\}}{(155)^2 + 155\times55 + (55)^2}$$

$$= (155 - 55) = 100$$

(ii) Given,

$$= \frac{1.2\times1.2\times1.2 - 0.2\times0.2\times0.2}{1.2\times1.2 + 1.2\times0.2 + 0.2\times0.2}$$

$$= \frac{(1.2)^3 - (0.2)^3}{(1.2)^2 + 1.2\times0.2 + (0.2)^2}$$

$$= \frac{(1.2-0.2)\{(1.2)^2 + (0.2)^2 + 1.2\times0.2\}}{(1.2)^2 + 1.2\times0.2\times(0.2)^2}$$

$$= (1.2 - 0.2) = 1$$

Exercise 5.3

1. Question

Factorize:

$64a^3+125b^3+240a^2b+300ab^2$

Answer

Given,

$64a^3 + 125b^3 + 240a^2b + 300ab^2$,

$= (4a)^3 + (5b)^3 + 60ab\ (4a + 5b)$

$= (4a)^3 + (5b)^3 + 3\times4a\times5b\ (4a + 5b)$

$= (4a+5b)^3$

$= (4a+5b)(4a+5b)(4a+5b)$

2. Question

Factorize:

$125x^3-27y^3-225x^2y+125xy^2$

Answer

Given,

$125x^3-27y^3-225x^2y+125xy^2$,

$= (5x)^3 - (3y)^3 - 45xy\ (5x - 3y)$

$= (5x)^3 - (3y)^3 - 3\times5x\times3y\ (5x - 3y)$

$= (5x - 3y)^3$

$= (5x - 3y)\ (5x - 3y)\ (5x - 3y)$

3. Question

Factorize:

$\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$

Answer

Given,

$$\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x,$$

$$=\left(\frac{2}{3}x\right)^3+(1)^3+2x\left(\frac{2}{3}x+1\right)$$

$$=\left(\frac{2}{3}x\right)^3+(1)^3+3\times\frac{2}{3}x\times1\left(\frac{2}{3}x+1\right)$$

$$=\left(\frac{2}{3}x+1\right)^3$$

$$=\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)\left(\frac{2}{3}x+1\right)$$

4. Question

Factorize:

$8x^3+27y^3+36x^2y+54xy^2$

Answer

Given,

$8x^3+27y^3+36x^2y+54xy^2$,

$= (2x)^3+(3y)^3 +18xy\ (2x +3y)$

$= (2x)^3+(3y)^3 +3\times2x\times3y\ (2x +3y)$

$= (2x+3y)^3$

$= (2x +3y)\ (2x +3y)\ (2x +3y)$

5. Question

Factorize:

$a^3-3a^2b+3ab^2-b^3+8$

Answer

Given,

$a^3 - 3a^2b + 3ab^2 - b^3 + 8$,

$= \{(a)^3 - (b)^3 -3ab\ (a-b)\} +8$

$= (a-b)^3 + (2)^3$

= (a – b +2) {(a - b)^2 +(2)^2 +2(a - b)}

= (a - b +2)(a^2 -2ab +b^2 +4+2a – 2b)

6. Question

Factorize:

$x^3+8y^3+6x^2y+12xy^2$

Answer

Given,

$x^3+8y^3+6x^2y+12xy^2$,

= $(x)^3+(2y)^3$ +6xy (x +2y)

= $(x)^3+(2y)^3$ +3×x×2y (x +2y)

= $(x+2y)^3$

= (x +2y) (x +2y) (x +2y)

7. Question

Factorize:

$8x^3+y^3+12x^2y+6xy^2$

Answer

Given,

$8x^3+y^3+12x^2y+6xy^2$,

= $(2x)^3+(y)^3$ +6xy (2x +y)

= $(2x)^3+(y)^3$ +3×2x×y (2x +y)

= $(2x+y)^3$

= (2x +y) (2x +y) (2x +y)

8. Question

Factorize:

$8a^3+27b^3+36a^2b+54ab^2$

Answer

Given,

$8a^3+27b^3+36a^2b+54ab^2$,

= $(2a)^3+(3b)^3$ +18ab (2a + 3b)

= $(2a)^3$ + $(3b)^3$ +3×2a×3b (2a +3b)

= $(2a +3b)^3$

= (2a +3b) (2a + 3b) (2a +3b)

9. Question

Factorize:

$8a^3-27b^3-36a^2b+54ab^2$

Answer

Given,

$8a^3 - 27b^3 - 36a^2b+54ab^2$,

$= (2a)^3 - (3b)^3 - 18ab\ (2a - 3b)$

$= (2a)^3 - (3b)^3 - 3\times2a\times3b\ (2a - 3b)$

$= (2a - 3b)^3$

$= (2a - 3b)\ (2a - 3b)\ (2a - 3b)$

10. Question

Factorize:

x^3-12x(x-4)-64

Answer

Given,

x^3-12x(x-4)-64,

$= (x)^3$ -12x (x - 4) – $(4)^3$

$= (x)^3$ –$(4)^3$ - 3×x×4 (x - 4)

$= (x - 4)^3$

$= (x - 4)\ (x - 4)\ (x - 4)$

11. Question

Factorize:

$a^3x^3-3a^2bx^2+3ab^2x-b^3$

Answer

Given,

$a^3x^3-3a^2bx^2+3ab^2x-b^3$,

$= (ax)^3$ – 3abx(ax-b) – $(b)^3$

$= (ax)^3$ – $(b)^3$ – 3abx (ax-b)

$= (ax-b)^3$

$= (ax-b)\ (ax-b)\ (ax-b)$

Exercise 5.4

1. Question

Factorize each of the following expressions:

$a^3+8b^3+64c^3-24abc$

Answer

Given,

$= a^3 + 8b^3 + 64c^3 - 24abc$

This can be written in form

$= a^3 + 8b^3 + 64c^3 - 24abc = (a)^3 + (2b)^3 + (4c)^3 - 3\times a\times2b\times4c$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Hence,

$= a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)\{(a)^2 + (2b)^2 + (4c)^2 - a\times 2b - 2b\times 4c - 4c\times a)$

$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

Thus the required factors of $a^3 + 8b^3 + 64c^3 - 24abc$ is $(a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

2. Question

Factorize each of the following expressions:

x^3-$8y^3$+$27z^3$+18xyz

Answer

Given,

$= x^3 - 8y^3 + 27z^3 + 18xyz$

This can be written in form,

$= x^3 - 8y^3 + 27z^3 + 18xyz = (x)^3 + (-2y)^3 + (3z)^3 - 3\times x\times(-2y)\times 3z$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= x^3 - 8y^3 + 27z^3 + 18xyz = \{x + (-2y) + 3z\}\{(x)^2 + (-2y)^2 + (3z)^2 - (x)(-2y) - (-2y)(3z) - (3z)(x)\}$

$= (x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

Thus the required factors of $x^3 - 8y^3 + 27z^3 + 18xyz$ is $(x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

3. Question

Factorize each of the following expressions:

$27x^3$-y^3-z^3-9xyz

Answer

Given,

$= 27x^3 - y^3 - z^3 - 9xyz$

This can be written in form ,

$= 27x^3 - y^3 - z^3 - 9xyz = (3x)^3 + (-y)^3 + (-z)^3 - 3(3x)(-y)(-z)$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

So ,

$= 27x^3 - y^3 - z^3 - 9xyz = \{3x + (-y) + (-z)\}\{(3x)^2 + (-y)^2 + (-z)^2 - (3x)(-y) - y(-z) - z(3x)\}$

$= (3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

Thus the factors of $27x^3 - y^3 - z^3 - 9xyz$ is $(3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

4. Question

Factorize:

$\frac{1}{27}$$x^3$-$y^3$+$125z^3$+5xyz

Answer

Given,

$= \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

This can be written in form ,

$= \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{1}{3}x\right)^3 + (-y)^3 + (5z)^3 - 3\left(\frac{1}{3}x\right)(-y)(5z)$

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= \frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz = \left\{\left(\frac{1}{3}x\right) + (-y) + (5z)\right\}\left\{\left(\frac{1}{3}x\right)^2 + (-y)^2 + (5z)^2 - \left(\frac{1}{3}x\right)(-y) - y\right)(5z) - (5z)\left(\frac{1}{3}x\right)$

$= \left(\frac{1}{3}x - y + 5z\right)\left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$

Thus the factors of $\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$ is $\left(\frac{1}{3}x - y + 5z\right)\left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$

5. Question

Factorize each of the following expressions:

$8x^3+27y^3-216z^3+108xyz$

Answer

Given,

$= 8x^3 + 27y^3 - 216z^3 + 108xyz$

This can be written in form ,

$= 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= 8x^3 + 27y^3 - 216z^3 + 108xyz = \{(2x) + (3y) + (-6z)\}\{(2x)^2 + (3y)^2 + (-6z)^2 - (2x)(3y) - (3y)((-6z) - (-6z)(2x)\}$

$= (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

Thus the factors of

$8x^3 + 27y^3 - 216z^3 + 108xyz$ is $(2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

6. Question

Factorize each of the following expressions:

$125+8x^3-27y^3+90xy$

Answer

Given,

$= 125 + 8x^3 - 27y^3 + 90xy$

This can be written in form ,

$= 125 + 8x^3 - 27y^3 + 90xy = (5)^3 + (2x)^3 + (-3y)^3 - 3(5)(2x)(-3y)$

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= 125 + 8x^3 - 27y^3 + 90xy = \{5 + 2x - 3y\}\{(5)^2 + (2x)^2 + (-3y)^2 - (5)(2x) - (2x)(-3y) - (-3y)(5)\}$

$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

Thus the factors of $125 + 8x^3 - 27y^3 + 90xy$ is $(5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$

7. Question

Factorize:

$(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Answer

Given,

$= (3x-2y)^3+(2y-4z)^3+(4z-3x)^3$

Let $a=(3x-2y), b=(2y-4z), c=(4z-3x)$

$= (3x-2y)^3+(2y-4z)^3+(4z-3x)^3 = a^3+b^3+c^3$

Here ,

$= a+b+c=(3x-2y+2y-4z+4z-3x)=0$

$= a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence ,

$= a^3+b^3+c^3-3abc=0\times(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3=3abc$

$= (3x-2y)^3+(2y-4z)^3+(4z-3x)^3 = 3(3x-2y)(2y-4z)(4z-3x)$

8. Question

Factorize each of the following expressions:

$(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Answer

Given,

$= (2x-3y)^3+(4z-2x)^3+(3y-4z)^3$

Let $a=(2x-3y), b=(4z-2x), c=(3y-4z)$

Then ,

$= (2x-3y)^3+(4z-2x)^3+(3y-4z)^3 = a^3+b^3+c^3$

Here ,

$= a+b+c= 2x-3y+4z-2x+3y-4z = 0$

Hence ,

$= a^3+b^3+c^3-3abc=0\times(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3=3abc$

$= (2x-3y)^3+(4z-2x)^3+(3y-4z)^3=3(2x-3y)(4z-2x)(3y-4z)$.

9. Question

Factorize each of the following expressions:

$\left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+y\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3$

Answer

Given,

$= \left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3$

Let $a=\left(\frac{x}{2}+y+\frac{z}{3}\right), b=\left(\frac{x}{3}-\frac{2y}{3}+z\right), c=\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)$

Then ,

$= \left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3 = a^3+b^3+c^3$

Here ,

$= a+b+c= \left(\frac{x}{2}+y+\frac{z}{3}\right)+\left(\frac{x}{3}-\frac{2y}{3}+z\right)+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right) = 0$

$= a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence ,

$= a^3+b^3+c^3-3abc = 0\times(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3 = 3abc$

$= \left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3 = 3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right).$

10. Question

Factorize each of the following expressions:

$(a-3b)^3+(3b-c)^3+(c-a)^3$

Answer

Given,

$=(a-3b)^3+(3b-c)^3+(c-a)^3$

Let $x= (a-3b), y= (3b-c), z= (c-a)$

Then,

$= (a-3b)^3+(3b-c)^3+(c-a)^3 = x^3+y^3+z^3$

Here ,

$= x+y+z= a-3b+3b-c+c-a = 0$

$= a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence,

$= a^3+b^3+c^3-3abc = 0\times(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3 = 3abc$

$= (a-3b)^3+(3b-c)^3+(c-a)^3 = 3(a-3b)(3b-c)(c-a)$

11. Question

Factorize each of the following expressions:

$2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

Answer

Given,

$= 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc$

This can be written in form ,

$=2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=(\sqrt{2}a)^3+(\sqrt{3}b)^3+(c)^3-3(\sqrt{2}a)(\sqrt{3}b)(c)$

And , $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence,

$= 2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=\{(\sqrt{2}a)+(\sqrt{3}b)+c)\}\{(\sqrt{2}a)^2+(\sqrt{3}b)^2+(c)^2-(\sqrt{2}a)(\sqrt{3}b)-(\sqrt{3}b)(c)-(c)(\sqrt{2}a)\}$

$=\sqrt{2}a+\sqrt{3}b+c)(2a^2+3b^2+c^2-\sqrt{6}ab-\sqrt{3}\,bc-\sqrt{2}ca$

12. Question

Factorize each of the following expressions:

$3\sqrt{3}\,a^3$-b^3-$5\sqrt{5}\,c^3$-$3\sqrt{15}\,abc$

Answer

Given,

$=3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc$

This can be written in form .

$=3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc=(\sqrt{3}a)^3+(-b)^3+(-\sqrt{5}c)^3-3(\sqrt{3}a)(-b)(-\sqrt{5}c)$

And , $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= 3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc=\{(\sqrt{3}a)+(-b)+(-\sqrt{5}c)\}\{(\sqrt{3}a)^2+(-b)^2+(-\sqrt{5}c)^2-(\sqrt{3}a)(-b)—b)(-\sqrt{5}c)—(-\sqrt{5}c)(\sqrt{3}a)$

$=(\sqrt{3}a-b-\sqrt{5}c)\,(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ca$

13. Question

Factorize each of the following expressions:

$8x^3$-$125y^3$+180xy+216

Answer

Given,

$=8x^3-125y^3+180xy+216$

This can be written in form ,

$=8x^3-125y^3+180xy+216=(2x)^3+(-5y)^3+6^3-3(2x)(-5y)(6)$

And , $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Hence ,

$=8x^3-125y^3+180xy+216=\{2x+(-5y)+6\}\{(2x)^2+(-5y)^2+6^2-(2x)(-5y)-(-5y)(6)-(6)(2x)\}$

$=(2x-5y+6)(4x^2+25y^2+36+10xy+30y-12x)$

Thus the factors of $8x^3-125y^3+180xy+216$ is

$(2x-5y+6)(4x^2+25y^2+36+10xy+30y-12x)$

14. Question

Factorize each of the following expressions:

$2\sqrt{2}a^3+16\sqrt{2}b^3+c^3-12abc$

Answer

Given,

$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$

This can be written in form ,

$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3(\sqrt{2}a)(2\sqrt{2}b)(c)$

And , $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Hence ,

$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = \{\sqrt{2}a + 2\sqrt{2}b + c\}\{(\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b - (2\sqrt{2}b)(c) - (c)(\sqrt{2}a)\}$

$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca)$

Thus the factors of $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$ is $(\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca)$

15. Question

Find the value of $x^3+y^3-12xy+64$,when$x+y=-4$.

Answer

Given,

$= x^3 + y^3 - 12xy + 64$

= x + y = -4 Given

= x+y+4 = 0

This can be written in form ,

$= x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3(x)(y)(4)$

And , $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= x^3 + y^3 - 12xy + 64 = (x + y + 4)\{(x)^2 + (y)^2 + 4^2 - xy - 4y - 4x\}$

$= 0 \times (x^2 + y^2 + 16 - xy - 4y - 4x)$

= 0

16. Question

Multiply:

(i) $x^2+y^2+z^2-xy+xz+yzbyx+y-z$

(ii) $x^2+4y^2+z^2+2xy+xz-2yzbyx-2y-z$

(iii) $x^2+4y^2+2xy+-3x+6y+9byx-2y+3$

(iv) $9x^2+25y^2+15xy+12x-20y+16by3x-5y+4$

Answer

(i) Given,

$= x^2 + y^2 + z^2 - xy + xz + yz$

Multiply the above expression by (x+ y – z)

$= (x + y - z)(x^2 + y^2 + z^2 - xy + xz + yz)$

$=\{x + y + (-z)\}\{(x)^2 + (y)^2 + (-z)^2 - xy + yz + zx\}$

$= x^3 + y^3 + (-z)^3 - 3.x.y(-z)$

$= x^3 + y^3 - z^3 + 3xyz$

(ii) Given,

$= x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$

Multiply above expression by (x-2y-z)

Then ,

$=(x - 2y - z)(x^2 + 4y^2 + z^2 + 2xy + xz - 2yz)$

$= \{x + (-2y) + (-z)\}\{x^2 + 4y^2 + z^2 + 2xy - 2yz + zx)$

By formula...

$= x^3 - 8y^3 - z^3 - 6xyz$

iii) we have

$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$

(iii) Given,

$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$

Multiply above equation by (x- 2y +3)

$= (x - 2y + 3)(x^2 + 4y^2 + 2xy - 3x + 6y + 9)$

$= \{x - 2y + 3)\{x^2 + 4y^2 + 9 + 2xy + 6y - 3x\}$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= x^3 - 8y^3 + 27 + 18xy$

(iv) Given,

$= 9x^2 + 25y^2 + 15xy + 12x - 20y + 16$

Multiply above equation by (3x – 5y +4)

We got,

$=(3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16) = \{3x + (-5y) + 4\}\{(3x)^2 + (-5y)^2 + 4^2 - 3x(-5y) - (-5y)(4) - (4)(3x)\}$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

$= (3x)^3 + (-5y)^3 + (4)^3 - 3.3x(-5y)(4)$

$= 27x^3 - 125y^3 + 64 + 180xy$

CCE - Formative Assessment

1. Question

Factorize: x^4+x^2+25.

Answer

We have,

First we rewrite the question,

$x^4 + x^2 + 25 = (x^2)^2 + 2.x^2.5 + 52 - 9x^2$

$= \{(x^2)^2 + 2.x^2.5 + 52\} - (3x)^2$ [By using $a^2 + 2ab + b^2 = (a + b)^2$]

$= \{x^2 + 5\}^2 - (3x)^2$ [By using $a^2 - b^2 = (a + b)(a - b)$

$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$

Thus , the factors of $x^4 + x^2 + 25$ are $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$.

2. Question

Factorize: $x^2-1-2a-a^2$.

Answer

We have ,

$= x^2 - 1 - 2a - a^2$

Taking -1 as common from last three terms

$= x^2 - (1 + 2a + a^2)$

$= x^2 - (1^2 + 2 \times 1 \times a + a^2)$

$= x^2 - (1 + a)^2$ $[a^2 - b^2 = (a + b)(a - b)]$

$= (x + 1 + a)(x - 1 - a)$

Thus the factors of $x^2 - 1 - 2a - a^2$ are $(x + 1 + a)(x - 1 - a)$.

3. Question

If a + b + c = 0, then write the value of $a^3+b^3+c^3$.

Answer

We have,

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

When (a + b + c) = 0 Given

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$

$= a^3 + b^3 + c^3 - 3abc = 0$

$= a^3 + b^3 + c^3 = 3abc.$

4. Question

If$a^2+b^2+c^2=20$,and$a+b+c=0$,find$ab+bc+ca$.

Answer

We have ,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ By formula

$= a^2 + b^2 + c^2 = 20$ Given

$= (a + b + c) = 0$ Given

Then,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= (0)^2 = 20 + 2(ab + bc + ca)$

$= 2(ab + bc + ca) = -20$

$= (ab + bc + ca) = -10$

5. Question

If a + b + c = 9 and ab + bc + ca = 40, find $a^2 + b^2 + c^2$.

Answer

We have ,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ By formula

$= a + b + c = 9$ Given

$= ab + bc + ca = 40$ Given

Then,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= (9)^2 = a^2 + b^2 + c^2 + 2 \times 40$

$= a^2 + b^2 + c^2 = 81 - 80$

$= a^2 + b^2 + c^2 = 1$

6. Question

If $a^2 + b^2 + c^2 = 250$ and ab + bc + ca = 3, find a + b + c.

Answer

We have,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ By formula

$= a^2 + b^2 + c^2 = 250$ Given

$= ab + bc + ca = 3$ Given

Then,

$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$= (a + b + c)^2 = 250 + 2 \times 3$

$= (a + b + c)^2 = 256$

$= (a + b + c) = \sqrt{256} = \pm 16$.

7. Question

Write the value of: $25^3 - 75^3 + 50^3$.

Answer

We have,

$= 25^3 - 75^3 + 50^3$

Let a = 25 , b = -75 , c = 50 ,

Then the expression becomes as ,

$= 25^3 - 75^3 + 50^3 = a^3 + b^3 + c^3$

$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ By formula

Here , $(a+b+c) = 25+(-75)+50 = 0$

Hence,

$= a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= a^3+b^3+c^3-3abc = 0\times(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3 = 3abc$

$= 25^3-75^3+50^3 = 3\times25\times-75\times50$

$= 25^3-75^3+50^3 = -281250.$

8. Question

Write the value of: 48^3–30^3-18^3.

Answer

We have,

$= 48^3-30^3-18^3$ Given

Let a = 48 , b = -30 , c = -18

Then the expression becomes ,

$= 48^3-30^3-18^3 = a^3+b^3+c^3$

$= a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

Here,

$= (a+b+c) = 48+(-30)+(-18) = 0$

Hence,

$= a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= a^3+b^3+c^3-3abc = 0\times(a^2+b^2+c^2-ab-bc-ca)$

$= a^3+b^3+c^3 = 3abc$

$= 48^3-30^3-18^3 = 3\times48\times-30\times-18$

$= 48^3-30^3-18^3 = 77760.$

9. Question

Write the value of: $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$

Answer

We have ,

$= \left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$ Given

Let $a = \frac{1}{2}, b = \frac{1}{3}, c = -\frac{5}{6}$, then the expression becomes

$= \left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3 = a^3+b^3+c^3$

Here,

$= a+b+c = \frac{1}{2}+\frac{1}{3}-\frac{5}{6} = 0$

Hence ,

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = 3 \times \frac{1}{2} \times \frac{1}{3} \times -\frac{5}{6}$

$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = -\frac{5}{12}$

10. Question

Write the value of: $30^3+20^3-50^3$.

Answer

We have ,

$= 30^3 + 20^3 - 50^3$ given

Let $a = 30, b = 20, c = -50$, then the expression becomes,

$= 30^3 + 20^3 - 50^3 = a^3 + b^3 + c^3$

Here ,

$= a + b + c = 30 + 20 - 50 = 0$

Hence,

$= a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$ By formula

$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2+b^2+c^2-ab-bc-ca)$

$= a^3 + b^3 + c^3 = 3abc$

$= 30^3 + 20^3 - 50^3 = 3 \times 30 \times 20 \times -50$

$= 30^3 + 20^3 - 50^3 = -90000$

1. Question

The factors of $a^2-1-2x-x^2$ are

A. (a-x+1)(a-x-1)

B. (a+x-1)(a-x+1)

C. (a+x+1)(a-x-1)

D. none of these

Answer

We have,

$= a^2 - 1 - 2x - x^2$

$= a^2 - (1 + 2x + x^2) = a^2 - (1+x)^2$

$= (a - 1 - x)(a + 1 + x)$ $[a^2 - b^2 = (a+b)(a-b)]$

Thus, the factors of $a^2 - 1 - 2x - x^2$ are $(a - 1 - x)(a + 1 + x)$.

2. Question

The factors of $x^4+x^2=25$ are

A. $(x^2+3x+5)(x^2-3x+5)$

B. $(x^2+3x+5)(x^2+3x-5)$

C. $(x^2+x+5)(x^2-x+5)$

D. none of these

Answer

We have,

$= x^4 + x^2 + 25$

Adding and subtracting $9x^2$ in the equation

$= x^4 + 25 + x^2 + 9x^2 - 9x^2$

$= x^4 + 25 + 10x^2 - 9x^2 = (x^2+5)^2 - (3x)^2$

$= (x^2 + 5 - 3x)(x^2 + 5 + 3x)$

Thus the factors of (x^4+x^2+25) are $(x^2 + 5 - 3x)(x^2 + 5 - 3x)$.

3. Question

The factors of $x^2+4y^2+4y-4xy-2x-8$ are

A. $(x-2y-4)(x-2y+2)$

B. $(x-y+2)(x-4y-4)$

C. $(x+2y-4)(x+2y+2)$

D. none of these

Answer

We have ,

$= x^2 + 4y^2 + 4y - 4xy - 2x - 8$

$= x^2 + 4y^2 - 4xy + 4y - 2x - 8 = (x-2y)^2 - 2(x-2y) - 8$

Let a = (x-2y) , then the expression becomes ,

$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = a^2 - 2a - 8$

$= a^2 - 4a + 2a - 8$

= a(a-4) + 2(a -4)

$=(a-4)(a+2)$

Put a = (x -2y)

$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = (x-2y-4)(x-2y+2)$

Thus the factors of $x^2 + 4y^2 + 4y - 4xy - 2x - 8$ are $(x-2y-4)(x-2y+2)$

4. Question

The factors of $x^3-x^2y-xy^2+y^3$ are

A. $(x+y)(x^2-xy+y^2)$

B. $(x+y)(x^2+xy+y^2)$

C. $(x+y)^2(x-y)$

D. $(x-y)^2(x+y)$

Answer

We have,

$= x^3 - x^2y - xy^2 + y^3$

$= x^3 + y^3 - x^2y - xy^2 = (x+y)^3 - 3x^2y - 3xy^2 - x^2y - xy^2$

As $x^3 + y^3 = (x+y)^3 - 3x^2y - 3xy^2$

$= (x+y)^3 - 4xy(x+y)$

$=(x+y)\{(x+y)^2 - 4xy)$

$= (x+y)(x^2 + y^2 + 2xy - 2xy)$

$=(x+y)(x^2 + y^2 - 2xy) = (x+y)(x-y)^2$

5. Question

The factors of x^3-1+y^3+3xyare

A. (x-1+y)(x^2+1+y^2+x+y-xy)

B. (x+y+1)(x^2+y^2+1-xy-x-y)

C. (x-1+y)(x^2-1-y^2+x+y+xy)

D. 3(x+y-1)(x^2+y^2-1)

Answer

We have ,

$= x^3 - 1 + y^3 + 3xy$

$= (x)^3 + (-1)^3 + (y)^3 - 3(x)(-1)(y)$

$= (x-1+y)(x^2 + 1 + y^2 + x + y - xy)$ $\{(a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)\}$

Thus the factors of $x^3 - 1 + y^3 + 3xy$ are $(x-1+y)(x^2 + 1 + y^2 + x + y - xy)$

6. Question

The factors of 8a^2+b^3-6ab+1are

A. (2a+b-1)(4a^2+b^2+1-3ab-2a)

B. (2a-b+1)(4a^2+b^2-4ab+1-2a+b)

C. (2a+b+1)(4a^2+b^2+1-2ab-b-2a)

D. (2a-1+b)(4a^2+1-4a-b-2ab)

Answer

We have ,

$= 8a^3 + b^3 - 6ab + 1$

$= (2a)^3 + (b)^3 - 3(2a)(b)(1) + (1)^3$ try to make $(a^3 + b^3 + c^3 - 3abc)$

$= (2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$

Thus the factors of $8a^3 + b^3 - 6ab + 1$ are $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$.

7. Question

$(x+y)^3$-$(x-y)^3$ can be factorized as:

A. $2y(3x^2+y^2)$

B. $2x(3x^2+y^2)$

C. $2y(3y^2+x^2)$

D. $2x(x^2+3y^2)$

Answer

We have ,

$=(x+y)^3-(x-y)^3$

Applying formulas,

$=x^3+y^3+3x^2y+3xy^2-(x^3-y^3-3x^2y+3xy^2)$

$=x^3+y^3+3x^2y+3xy^2-x^3+y^3+3x^2y-3xy^2$

$=2y^3+6x^2y$

$=2y(y^2+3x^2)$

Thus the factors of $(x+y)^3-(x-y)^3$ are $2y(y^2+3x^2)$

8. Question

The factors of x^2-7x+6are

A. x(x-6)(x-1)

B. (x^2-6)(x-1)

C. (x+1)(x+2)(x-3)

D. (x-1)(x+3)(x-2)

Answer

We have,

$=x^3-7x-6$

Adding and subtracting 1 in the equation

$=x^3-7x-6-1+1=x^3+1^3-7x-7$

$=(x+1)(x^2+1+x)-7(x+1)$

$=(x+1)(x^2-6+x)$

$=(x+1)(x^3-3x+2x+x)=(x+1)(x+2)(x-3)$

Thus the factors of x^3-7x-6 are $(x+1)(x+2)(x-3)$.

9. Question

The expression $(a-b)^3+(b-c)^3+(c-a)^3$ can be factorized as:

A. (a-b)(b-c)(c-a)

B. 3(a-b)(b-c)(c-a)

C. -3(a-b)(b-c)(c-a)

D. $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Answer

We have,

Let $x = (a-b), y = (b-c), z(c-a)$

So, $x+y+z = a-b+b-c+c-a$

If a+b+c = 0 , then,

$= a^3 + b^3 + c^3 = 3abc$

$= (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a).$

10. Question

The expression x^4+4 can be factorized as

A. $(x^2+2x+2)(x^2-2x+2)$

B. $(x^2+2x+2)(x^2+2x-2)$

C. $(x^2-2x-2)(x^2-2x+2)$

D. $(x^2+2)(x^2-2)$

Answer

We have ,

$= x^4 + 4$

$= x^4 + 4x^2 + 4 - 4x^2$

$=(x^2+2)^2 - 4x^2$

$= (x^2+2)^2 - (2x)^2 \; [x^2 - y^2 = (x-y)(x+y)]$

$= (x^2+2+2x)(x^2+2-2x)$

11. Question

If $3x=a+b+c$, then the value of $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$ is

A. a+b+c

B. (a-b)(b-c)(c-a)

D. 0

D. none of these

Answer

We have,

$= 3x = a+b+c$

Let $a_1 = x-a, b_1 = x-b, c_1 = x-c$

So, $a_1 + b_1 + c_1 = x-a+x-b+x-c = 3x-(a+b+c)$

$= 3x - 3x = 0$ [a +b + c = 3x] given

$= (x-a)^3 + (x-b)^3 + (x-c)^3 = 3(x-a)(x-b)(x-c)$

Now, $(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c) = 0.$

12. Question

If $(x+y)^3-(x-y)^3-6y(x^2-y^2)=ky^2$ then k=

A. 1

B. 2

C. 4

D. 8

Answer

We have,

$=(x+y)^3-(x-y)^3-6y(x^2-y^2)=\ ky^3$

$=\ (x+y-x+y)^3+3(x+y)(x-y)(x+y-x+y)-6y(x^2-y^2)=\ ky^3$

$=\ 2y^3+6y(x^2-y^2)-6y(x^2-y^2)=\ ky^3$

$=\ 8y^3=ky^3$

= k = 8 .

13. Question

Ifx^3-3x^2+3x-7=(x+1)(ax^2+bx+c), then a+b+c=

A. 4

B. 12

C. -10

D. 3

Answer

We have,

$=\ x^3-3x^2+3x-7=(x+1)(ax^2+bx+c)$

$=x^3-3x^2+3x-7=ax^3+bx^2+cx+ax^2+bx+c$

$=\ x^3-3x^2+3x-7=ax^3+(a+b)x^2+(b+c)x+c$

By compairing both sides ,

= a = 1

= a + b = -3

= b + c = 3

= c = -7

Thus , a + (b + c) = 1+3 = 4.

14. Question

The value of $\frac{(2.3)^3-0.027}{(2.3)^2+0.69+0.09}$ is

A. 2

B. 3

C. 2.327

D. 2.273

Answer

We have,

$=\frac{2.3^3-0.027}{2.3^2+0.69+0.09}$

$= \frac{2.3^3 - 0.027}{2.3^2 + 0.69 + 0.09} = \frac{2.3^3 - 0.3^3}{2.3^2 + 0.3^2 + 2.3 \times 0.3}$ $[a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$

Hence,

$= \frac{\{(a-b)(a^2 + b^2 + ab)\}}{a^2 + b^2 + ab} = a - b$

= (2.3 – 0.3) = 2 .

15. Question

The value of $\frac{(0.013)^3 + (0.007)^3}{(0.013)^2 - 0.013 \times 0.013 \times 0.007 + (0.007)^2}$ is

A. 0.006

B. 0.02

C. 0.0091

D. 0.00185

Answer

We have,

$= \frac{0.013^3 + 0.007^3}{0.013^2 - 0.013 \times 0.007 + 0.007^2}$ $[a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

Hence,

$= \frac{\{(a+b)(a^2 + b^2 - ab)\}}{a^2 + b^2 - ab} = a + b$

= 0.013 + 0.007 = 0.020