4. Cubes and Cube Roots

Exercise 4.1

1. Question

Find the cubes of the following numbers:

(i) 7 (ii) 12

(iii) 16 (iv) 21

(v) 40 (vi) 55

(vii) 100 (viii) 302

(ix) 301

Answer

(i) 7

Cube of 7 = 7 × 7 × 7 = 343

(ii) 12

Cube of 12 = 12 × 12 × 12 = 1728

(iii) 16

Cube of 16 = 16 × 16 ×16 = 4096

(iv) 21

Scube of 21 = 21 × 21 × 21 = 9261

(v) 40

Cube of 40 = 40 × 40 × 40 = 64000

(vi) 55

Cube of 55 = 55 × 55 × 55 = 166375

(vii) 100

Cube of 100 = 100 × 100 × 100 = 1000000

(viii) 302

To find cube of 302 we make it in form $(a+b)^3, which\ make\ caluclation\ easier.$

$= (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 = (300+2)^3 = 300^3 + 2^3 + 3\times300^2\times2 + 3\times300\times2^2$

= 27000000 + 8 + 540000 + 3600 = 27362408.

(ix) 301

To find cube of 301 we make it in form $(a+b)^3, which\ make\ caluclation\ easier.$

$= (a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2 = (300+1)^3 = 300^3 + 1^3 + 3\times300^2\times1 + 3\times300\times1^2$

= 27000000 + 1 + 270000 + 900 = 27180601.

2. Question

Write the cubes of all natural numbers between 1 and 10 and verify the following statements:

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

Answer

Cube of natural numbers upto 10 are as follows.

$1^3 = 1 \times 1 \times 1 = 1$

$2^3 = 2 \times 2 \times 2 = 8$

$3^3 = 3 \times 3 \times 3 = 27$

$4^3 = 4 \times 4 \times 4 = 64$

$5^3 = 5 \times 5 \times 5 = 125$

$6^3 = 6 \times 6 \times 6 = 216$

$7^3 = 7 \times 7 \times 7 = 343$

$8^3 = 8 \times 8 \times 8 = 512$

$9^3 = 9 \times 9 \times 9 = 729$

$10^3 = 10 \times 10 \times 10 = 1000$

From above results we can see that,

(i) Cubes of all odd natural numbers are odd.

(ii) Cubes of all even natural numbers are even.

3. Question

Observe the following pattern:

$1^3 = 1$
$1^3 + 2^3 = (1 + 2)^2$
$1^3 + 2^3 + 3^3 = (1 + 2 + 3)^2$

Write the next three rows and calculate the value of $1^3 + 2^3 + 3^3 + \ldots\ldots + 9^3 + 10^3$ by the above pattern.

Answer

According to given pattern,

$= 1^3 + 2^3 + 3^3 + \cdots + n^3) = (1 + 2 + 3 + \cdots + n)^2$

Here n = 10, so ,

$= (1^3 + 2^3 + 3^3 + \cdots + 9^3 + 10^3) = (1 + 2 + 3 + \cdots \ldots. +9 + 10)^2$

$= (1^3 + 2^3 + 3^3 + \cdots + 9^3 + 10^3) = (55)^2 = 55 \times 55 = 3025.$

4. Question

Write the cubes of 5 natural numbers which are multiples of 3 and verify the followings:

“The cube of a natural number which is a multiple of 3 is a multiple of 27’

Answer

First 5 natural numbers which are multiple of 3 are = 3 , 6 , 9 , 12 , 15

Now, cube of them are,

$= 3^3 = 3 \times 3 \times 3 = 27$

$= 6^3 = 6 \times 6 \times 6 = 216$

$= 9^3 = 9 \times 9 \times 9 = 729$

$= 12^3 = 12 \times 12 \times 12 = 1728$

$= 15^3 = 15 \times 15 \times 15 = 3375$

We find that all the cubes are divisible by 27,

Therefore, “The cube of a natural number which is a multiple of 3 is a multiple of 27’

5. Question

Write the cubes of 5 natural numbers which are of the form $3n + 1\left(e.g. 4, 7, 10, ...\right)$ and verify the following:

“The cube of a natural number of the form 3n+1 is a natural number of the same from i.e. when divided by 3 it leaves the remainder 1’

Answer

First 5 natural numbers in the form of (3n + 1) are = 4 , 7 , 10 , 13 , 16

Cube of these numbers are,

$= 4^3 = 4 \times 4 \times 4 = 64$

$= 7^3 = 7 \times 7 \times 7 = 343$

$= 10^3 = 10 \times 10 \times 10 = 1000$

$= 13^3 = 13 \times 13 \times 13 = 2197$

$= 16^3 = 16 \times 16 \times 16 = 4096$

We find that all these cubes gives remainder 1 when divided by ‘3’

Hence, statement is true.

6. Question

Write the cubes 5 natural numbers of the from 3n+2(i.e.5,8,11....) and verify the following:

“The cube of a natural number of the form 3n+2 is a natural number of the same form i. e. when it is dividend by 3 the remainder is 2’

Answer

First 5 natural numbers in form (3n + 2) are = 5 , 8 , 11 , 14 , 17

Cubes of these numbers are,

$= 5^3 = 5 \times 5 \times 5 = 125$

$= 8^3 = 8 \times 8 \times 8 = 512$

$= 11^3 = 11 \times 11 \times 11 = 1331$

$= 14^3 = 14 \times 14 \times 14 = 2744$

$= 17^3 = 17 \times 17 \times 17 = 4313$

We find that all these cubes give remainder 2 when divided by 3..

Hence statement is true.

7. Question

Write the cubes 5 natural numbers of which are multiples of 7 and verity the following:

“The cube of a multiple of 7 is a multiple of 7^3.

Answer

First 5 natural numbers which are multiple of 7 are = 7 , 14 , 21 , 28 , 35

Cube of these numbers are,

$= 7^3 = 7 \times 7 \times 7 = 343$

$= 14^3 = 14 \times 14 \times 14 = 2744$

$= 21^3 = 21\times 21\times 21 = 9261$

$= 28^3 = 28 \times 28 \times 28 = 21952$

$= 35^3 = 35 \times 35 \times 35 = 42875$

We find that all these cubes are multiple of 7^3(343) as well.

8. Question

Which of the following are perfect cubes?

(i) 64 (ii) 216

(iii) 243 (iv) 1000

(v) 1728 (vi) 3087

(vii) 4608 (viii) 106480

(ix) 166375 (x) 456533

Answer

(i) 64

Making factors of $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = (2^2)^3 = 4^3$

Hence, it's a perfect cube.

(ii) 216

Factors of $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$

Hence, it's a perfect cube.

(iii) 243

Factors of $243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 3^3 \times 3^2$

Hence, it's not a perfect cube.

(iv) 1000

Factors of $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 = 2^3 \times 5^3 = 10^3$

Hence, it's a perfect cube.

(v) 1728

Factors of $1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^6 \times 3^3 = (4 \times 3)^3 = 12^3$

Hence, it's a perfect cube.

(vi) 3087

Factors of $3087 = 3 \times 3 \times 7 \times 7 \times 7 = 3^2 \times 7^3$

Hence, it's not a perfect cube.

(vii) 4608

Factors of $4608 = 2 \times 2 \times 3 \times 113$

Hence, it's not a perfect cube.

(viii) 106480

Factors of 106480 = 2 × 2 × 2 × 2 × 5 × 11 × 11 × 11

Hence, it's not a perfect cube.

(ix) 166375

Factors of 166375 = 5 × 5 × 5 × 11 × 11 × 11 = $5^3 \times 11^3 = 55^3$

Hence, it's a perfect cube.

(x) 456533

Factors of 456533 = 11 × 11 × 11 × 7 × 7 × 7 = $11^3 \times 7^3 = 77^3$

Hence, it's a perfect cube.

9. Question

Which of the following are cubes of even natural numbers?

216, 512, 729, 1000, 3375, 13824

Answer

i) 216 = $2^3 \times 3^3 = 6^3$

It's a cube of even natural number.

ii) 512 = $2^9 = (2^3)^3 = 8^3$

It's a cube of even natural number.

iii) 729 = $3^3 \times 3^3 = 9^3$

It's not a cube of even natural number.

iv) 1000 = 10^3

It's a cube of even natural number.

v) 3375 = $3^3 \times 5^3 = 15^3$

It's not a cube of even natural number.

vi) 13824 = $2^2 \times 3^4 \times 41$

Its not even a cube.

10. Question

Which of the following are cubes of odd natural numbers?

125, 343, 1728, 4096, 32768, 6859

Answer

i) 125 = 5 × 5 × 5 × 5 = 5^3

It's a cube of odd natural number.

ii) 343 = 7 × 7 × 7 = 7^3

It's a cube of odd natural number.

iii) 1728 = $2^6 \times 3^3 = 4^3 \times 3^3 = 12^3$

It's not a cube of odd natural number. As 12 is even number.

iv) 4096 = $2^{12} = (2^6)^2 = 64^2$

Its not even a cube.

v) $32768 = 2^{15} = (2^5)^3 = 32^3$

It's a cube of odd natural number. As 32 is an even number.

vi) $6859 = 19 \times 19 \times 19 = 19^3$

It's a cube of odd natural number.

11. Question

What is the smallest number by which the following numbers must be multiplied, so that the products are perfect cubes?

(i) 675 (ii) 1323

(iii) 2560 (iv) 7803

(v) 107811 (vi) 35721

Answer

(i) 675

Factors of $675 = 3 \times 3 \times 3 \times 5 \times 5 = 3^3 \times 5^2$

Hence, to make a perfect cube we need to multiply the product by 5.

(ii) 1323

Factors of $1323 = 3 \times 3 \times 3 \times 7 \times 7 = 3^3 \times 7^2$

Hence, to make a perfect cube we need to multiply the product by 7.

(iii) 2560

Factors of $2560 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^3 \times 2^3 \times 2^3 \times 5$

Hence, to make a perfect cube we need to multiply the product by $5 \times 5 = 25$.

(iv) 7803

Factors of $7803 = 3 \times 3 \times 3 \times 17 \times 17 = 3^3 \times 17^2$

Hence, to make a perfect cube we need to multiply the product by 17.

(v) 107811

Factors of $107811 = 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 = 3^3 \times 3 \times 11^3$

Hence, to make a perfect cube we need to multiply the product by $3 \times 3 = 9$.

(vi) 35721

Factors of $35721 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 7 \times 7 = 3^3 \times 3^3 \times 7^2$

Hence, to make a perfect cube we need to multiply the product by 7.

12. Question

By which smallest number must the following numbers be divided so that the quotient is a perfect, cube?

(i) 675 (ii) 8640

(iii) 1600 (iv) 8788

(v) 7803 (vi) 107811

(vii) 35721 (viii) 243000

Answer

(i) 675

Prime factors of 675 = 3 × 3 × 3 × 5 × 5 = $3^3 \times 5^2$

We find that 675 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 5^2 = 25, which gives 27 as quotient and we know that 27 is a perfect cube .

(ii) 8640

Prime factors of 8640 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 = $2^3 \times 2^3 \times 3^3 \times 5$

We find that 8640 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 5 , which gives 1728 as quotient and we know that 1728 is a perfect cube.

(iii) 1600

Prime factors of 1600 = 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5 = $2^3 \times 2^3 \times 5^2$

We find that 1600 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 5^2 = 25, which gives 64 as quotient and we know that 64 is a perfect cube

(iv) 8788

Prime factors of 8788 = 2 × 2 × 13 × 13 × 13 = $2^2 \times 13^3$

We find that 8788 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 4, which gives 2197 as quotient and we know that 2197 is a perfect cube

(v) 7803

Prime factors of 7803 = 3 × 3 × 3 × 17 × 17 = $3^3 \times 17^2$

We find that 7803 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 17^2 = 289 , which gives 27 as quotient and we know that 27 is a perfect cube

(vi) 107811

Prime factors of 107811 = 3 × 3 × 3 × 3 × 11 × 11 × 11 = $3^3 \times 11^3 \times 3$

We find that 107811 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 3, which gives 35937 as quotient and we know that 35937 is a perfect cube.

(vii) 35721

Prime factors of 35721 = 3 × 3 × 3 × 3 × 3 × 3 × 7 × 7 = $3^3 \times 3^3 \times 7^2$

We find that 35721 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 7^2 = 49, which gives 729 as quotient and we know that 729 is a perfect cube

(viii) 243000

Prime factors of 243000 = 2 × 2 × 2 × 3 × 3 × 3 × 3 × 3 × 5 × 5 × 5 = $2^3 \times 3^3 \times 5^3 \times 3^2$

We find that 243000 is not a perfect cube.

Hence, for making the quotient a perfect cube we divide it by 3^2 = 9, which gives 27000 as quotient and we know that 27000 is a perfect cube

13. Question

Prove that if a number is trebled then its cube is 27 time the cube of the given number.

Answer

Let the number is = a

Cube of number will be = a^3

Now, the number is trebled = 3 × a = 3a

Cube of new number = $(3a)^3 = 27a^3$

Hence, new cube is 27 times of original cube. Hence, proved.

14. Question

What happens to the cube of a number if the number is multiplied by

(i) 3?

(ii) 4?

(iii) 5?

Answer

(i) 3?

Let the number is = a

Its cube will be = a^3

According to the question, the number is multiplied by 3

New number become = 3a

Its new cube will be = $(3a)^3 = 27a^3$

Hence, cube will become $27\ times$

(ii) 4?

Let the number is = a

Its cube will be = a^3

According to the question, the number is multiplied by 4

New number become = 4a

Its new cube will be = $(4a)^3 = 64a^3$

Hence, cube will become $64\ times$

(iii) 5?

Let the number is = a

Its cube will be = a^3

According to the question, the number is multiplied by 5

New number become = 5a

Its new cube will be = $(5a)^3 = 125a^3$

Hence, cube will become $125\ times$

15. Question

Find the value of a cube, one face of which has an area of $64m^2$.

Answer

Area of one face of cube = 64 m^2 (Given)

Let length of edge edge of cube = a metre

$= a^2 = 64$

$= a = \sqrt{64} = 8\,m$

Now, volume of cube = $a^3 = 8^3 = 8 \times 8 \times 8 = 512\,m^3$

16. Question

Find the volume of a cube whose surface area is $384m^2$.

Answer

Surface area of cube = 384 m^2 (Given)

Let length of each edge of cube = a metre

$6a^2 = 384$

$a^2 = \frac{384}{6} = 64$

$a = \sqrt{64} = 8\,m$

Volume of cube = $a^3 = (8)^3 = 512m^3$

17. Question

Evaluate the following:

(i) $\left\{(5^2 + 12^2)^{1/2}\right\}^3$

(ii) $\left\{(6^2 + 8^2)^{1/2}\right\}^3$

Answer

(i) $\left\{(5^2 + 12^2)^{1/2}\right\}^3$

After solving we get,

$\left\{(25 + 144)^{\frac{1}{2}}\right\}^3 = \left\{(13^2)^{\frac{1}{2}}\right\}^3 = \{13\}^3 = 2197$

(ii) $\left\{(6^2 + 8^2)^{1/2}\right\}^3$

After solving we get,

$\left\{(36 + 64)^{\frac{1}{2}}\right\}^3 = \left\{(100)^{\frac{1}{2}}\right\}^3 = \left\{(10^2)^{\frac{1}{2}}\right\}^3 = \{10\}^3 = 1000$

18. Question

Write the units digit of the cube of each of the following numbers:

31, 109, 388, 4276, 5922, 77774, 44447, 125125125

Answer

i) 31

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 31 is = 1

Cube of 1 = 1^3 = 1

Therefore, unit digit of cube of 31 is always be 1.

ii) 109

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 109 is = 9

Cube of 9 = 9^3 = 729

Therefore, unit digit of cube of 109 is always be 9.

iii) 388

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 388 is = 8

Cube of 8 = 8^3 = 512

Therefore, unit digit of cube of 388 is always be 2.

iv) 4276

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 4276 is = 6

Cube of 6 = 6^3 = 216

Therefore, unit digit of cube of 4276 is always be 6.

v) 5922

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 5922 is = 2

Cube of 2 = 2^3 = 8

Therefore, unit digit of cube of 5922 is always be 8.

vi) 77774

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 77774 is = 4

Cube of 4 = 4^3 = 64

Therefore, unit digit of cube of 77774 is always be 4.

vii) 44447

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 44447 is = 7

Cube of 7 = 7^3 = 343

Therefore, unit digit of cube of 44447 is always be 3.

viii) 125125125

To find unit digit of cube of a number we do the cube of unit digit only.

Here, unit digit of 125125125 is = 5

Cube of 5 = 5^3 = 125

Therefore, unit digit of cube of 125125125 is always be 5.

19. Question

Find the cubes of the following numbers by column method:

(i) 35

(ii) 56

(iii) 72

Answer

(i) 35

we have , a = 3 and b = 5

Column I a^3	Column II $3\times a^2\times b$	Column III $3\times a\times b^2$	Column IV b^3
$3^3 = 27$	$3\times9\times5 = 135$	$3\times3\times25 = 225$	$5^3 = 125$
+15	+23	+12	12 $\underline{5}$
$\underline{42}$	15 $\underline{8}$	23 $\underline{7}$	
42	8	7	5

Thus cube of 35 is 42875.

(ii) 56

we have , a = 5 and b = 6

Column I a^3	Column II $3\times a^2\times b$	Column III $3\times a\times b^2$	Column IV b^3
$5^3=125$	$3\times25\times6=450$	$3\times5\times36=540$	$6^3=216$
+50	+56	+21	21 6
175	50 6	56 1	
175	6	1	6

Thus cube of 56 is 175616.

(iii) 72

we have , a = 7 and b = 2

Column I a^3	Column II $3\times a^2\times b$	Column III $3\times a\times b^2$	Column IV b^3
$7^3=343$	$3\times49\times2=294$	$3\times7\times4=84$	$2^3=8$
+30	+8	+0	8
373	30 2	8 4	
373	2	4	8

Thus cube of 72 is 373248.

20. Question

Which of the following numbers are not perfect cubes?

(i) 64

(ii) 216

(iii) 243

(iv) 1728

Answer

(i) 64

Prime factors of 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 = 4^3$

Hence, it’s a perfect cube.

(ii) 216

Prime factors of 216 = $2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 3^3 = 6^3$

Hence, it’s a perfect cube.

(iii) 243

Prime factors of 243 = $3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$

Hence, its not a perfect cube.

(iv) 1728

Prime factors of 1728 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^3 \times 2^3 \times 3^3 = 12^3$

Hence, it’s a perfect cube.

21. Question

For each of the non-pefectcubbes in Q. No 20 find the smallest number by which it must be

(a) Multiplied so that the product is a perfect cube.

(b) Divided so that the quotient is a perfect cube.

Answer

Only non-perfect cube in previous question was = 243

(a) Multiplied so that the product is a perfect cube.

Prime factors of 243 = $3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$

Hence, to make it a perfect cube we should multiply it by 3.

(b) Divided so that the quotient is a perfect cube.

Prime factors of 243 = $3 \times 3 \times 3 \times 3 \times 3 = 3^3 \times 3^2$

Hence, to make it a perfect cube we have to divide it by 9.

22. Question

By taking three different, values of n verify the truth of the following statements:

(i) If n is even, then n^3 is also even.

(ii) If n is odd, then n^3 is also odd.

(ii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.

(iv) If a natural number n is of the form 3p+2 then n^3 also a number of the same type.

Answer

(i) If n is even, then n^3 is also even.

Let the three even natural numbers be 2 , 4 , 6

Cubes of these numbers ,

= 2^3 = 8

= 4^3 = 64

= 6^3 = 216

Hence, we can see that all cubes are even in nature.

Statement verified.

(ii) If n is odd, then n^3 is also odd.

Let three odd natural numbers are = 3 , 5 , 7

Cubes of these numbers =

= 3^3 = 27

= 5^3 = 125

= 7^3 = 343

Hence, we can see that all cubes are odd in nature.

Statement verified.

(iii) If n leaves remainder 1 when divided by 3, then n^3 also leaves 1 as remainder when divided by 3.

Let three natural numbers of the form (3n+1) are = 4, 7 , 10

Cube of numbers = 4^3 = 64 , 7^3 = 343 , 10^3 = 1000

We can see that if we divide these numbers by 3 , we get 1 as remainder in each case.

Statement verified.

(iv) If a natural number n is of the form 3p+2 then n^3 also a number of the same type.

Let three natural numbers of the form (3p+2) are = 5 , 8 , 11

Cube of these numbers = 5^3 = 125 , 8^3 = 512 , 11^3 = 1331

Now, we try to write these cubes in form of (3p + 2)

= 125 = 3 × 41 + 2

= 512 = 3 × 170 + 2

= 1331 = 3 × 443 + 2

Hence, statement verified.

23. Question

Write true (T) or false (F) for the following statements:

(i) 392 is a perfect cube.

(ii) 8640 is not a perfect cube.

(iii) No cube can end with exactly two zeros.

(iv) There is no perfect cube which ends in 4.

(v) For an integer a, a^3 is always greater than a^2.

(vi) If a and b are integers such that $a^2>b^2$, then $a^3>b^3$.

(vii) If a divides b, then a^3 divides b^3.

(viii) If a^2 ends in 9, then a^3 ends in 7.

(ix) If a^2 ends in an even number of zeros, then a^3 ends in 25.

(x) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

Answer

(i) 392 is a perfect cube.

False.

Prime factors of 392 = 2 × 2 × 2 × 7 × 7 = $2^3 \times 7^2$

(ii) 8640 is not a perfect cube.

True

Prime factors of 8640 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 = $2^3 \times 2^3 \times 3^3 \times 5$

(iii) No cube can end with exactly two zeros.

True

Beause a perfect cube always have zeros in multiple of 3.

(iv) There is no perfect cube which ends in 4.

False

64 is a perfect cube = 4 × 4 × 4 and it ends with 4.

(v) For an integer a, a^3 is always greater than a^2.

False

In case of negative integers ,

$= (-2)^2 = 4\ and\ (-2)^3 = -8$

(vi) If a and b are integers such that $a^2>b^2$, then $a^3>b^3$.

False

In case of negative integers,

$= (-5)^2 > (-4)^2 = 25 > 16$

But , $(-5)^3 > (-4)^3 = -125 > -64\ is\ not\ true.$

(vii) If a divides b, then a^3 divides b^3.

True

If a divides b $= \frac{b}{a} = k, so\ b = ak$

$= \frac{b^3}{a^3} = \frac{(ak)^3}{a^3} = \frac{a^3 k^3}{a^3} = k^3$,

For each value of b and a its true.

(viii) If a^2 ends in 9, then a^3 ends in 7.

False

Let a = 7

$7^2 = 49$ and $7^3 = 343$

(ix) If a^2 ends in an even number of zeros, then a^3 ends in 25.

False

Let a = 20

$= a^2 = 20^2 = 400$ and $a^3 = 8000$

(x) If a^2 ends in an even number of zeros, then a^3 ends in an odd number of zeros.

False

Let a = 100

$= a^2 = 100^2 = 10000$ and $a^3 = 100^3 = 1000000$

Exercise 4.2

1. Question

Find the cubes of:

(i) -11

(ii) -12

(iii) -21

Answer

(i) -11

$= (-11)^3 = -11 \times -11 \times -11 = -1331$

(ii) -12

$= (-12)^3 = -12 \times -12 \times -12 = -1728$

(iii) -21

$= (-21)^3 = -21 \times -21 \times -21 = -9261$

2. Question

Which of the following integers are cubes of negative integers

(i) -64

(ii) -1056

(iii) -2197

(iv) -2744

(v) -42875

Answer

(i) -64

Prime factors of 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 = 4^3$

We find that 64 is a perfect cube of negative integer – 4 .

(ii) -1056

Prime factors of 1056 = $2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 11$

We find that 1056 is not a perfect cube.

Hence, - 1056 is not a cube of negative integer

(iii) -2197

Prime factors of 2197 = $13 \times 13 \times 13 = 13^3$

We find that 2197 is a perfect cube.

Hence, - 2197 is a cube of negative integer – 13 .

(iv) -2744

Prime factors of 2744 = $2 \times 2 \times 2 \times 7 \times 7 \times 7 = 2^3 \times 7^3 = 14^3$

We find that 2744 is a perfect cube.

Hence, - 2744 is a cube of negative integer – 14 .

(v) -42875

Prime factors of 42875 = $5 \times 5 \times 5 \times 7 \times 7 \times 7 = 5^3 \times 7^3 = 35^3$

We find that 42875 is a perfect cube.

Hence, - 42875 is a cube of negative integer – 35.

3. Question

Show that the following integers are cubes of negative integers. Also, find the integer whose cube is the given integer.

(i) -5832

(ii) -2744000

Answer

(i) -5832

Prime factors of 5832 = $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 2^3 \times 3^3 \times 3^3 = 18^3$

We find that 5832 is a perfect cube.

Hence, - 5832 is a cube of negative integer – 18 .

(ii) -2744000

Prime factors of 2744000 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7 = 2^3 \times 2^3 \times 5^3 \times 7^3$

We find that 2744000 is a perfect cube.

Hence, - 2744000 is a cube of negative integer – 140 .

4. Question

Find the cube of:

(i) $\frac{7}{9}$ (ii) $-\frac{8}{11}$

(iii) $\frac{12}{7}$ (iv) $-\frac{13}{8}$

(v) $2\frac{2}{5}$ (vi) $3\frac{1}{4}$

(vii) 0.3 (viii) 1.5

(ix) 0.08 (x) 2.1

Answer

(i) $\frac{7}{9}$

$= \left(\frac{7}{9}\right)^3 = \frac{7^3}{9^3} = \frac{343}{729}$

(ii) $-\frac{8}{11}$

$= \left(-\frac{8}{11}\right)^3 = -\left(\frac{8^3}{11^3}\right) = -\frac{512}{1331}$

(iii) $\frac{12}{7}$

$= \left(\frac{12}{7}\right)^3 = \frac{1728}{343}$

(iv) $-\frac{13}{8}$

$= \left(-\frac{13}{8}\right)^3 = -\left(\frac{13^3}{8^3}\right) = -\frac{2197}{512}$

(v) $2\frac{2}{5}$

$= \left(2\frac{2}{5}\right)^3 = \left(\frac{12}{5}\right)^3 = \frac{1728}{125}$

(vi) $3\frac{1}{4}$

$= \left(3\frac{1}{4}\right)^3 = \left(\frac{13}{4}\right)^3 = \frac{2197}{64}$

(vii) 0.3

$= (0.3)^3 = 0.3\times0.3\times0.3 = 0.027$

(viii) 1.5

$= (1.5)^3 = 1.5\times1.5\times1.5 = 3.375$

(ix) 0.08

$= (0.08)^3 = 0.08\times0.08\times0.08 = 0.000512$

(x) 2.1

$= (2.1)^3 = 2.1\times2.1\times2.1 = 9.261$

5. Question

Find which of the following numbers are cubes of rational numbers:

(i) $\frac{27}{64}$

(ii) $\frac{125}{128}$

(iii) 0.001331

(iv) 0.04

Answer

(i) $\frac{27}{64}$

We have,

$= \frac{27}{64} = \frac{3\times3\times3}{4\times4\times4} = \frac{3^3}{4^3} = \left(\frac{3}{4}\right)^3$

Hence, $\frac{27}{64}$ *is a cube of* $\frac{3}{4}$.

(ii) $\frac{125}{128}$

We have,

$= \frac{125}{128} = \frac{5\times5\times5}{2\times2\times2\times2\times2\times2\times2} = \frac{5^3}{2^3\times2^3\times2}$

Hence, $\frac{125}{128}$ *is not a perfect cube.*

(iii) 0.001331

We have,

$= \frac{1331}{1000000} = \frac{11\times11\times11}{100\times100\times100} = \left(\frac{11}{100}\right)^3$

Hence, 0.001331 is a perfect cube of $\frac{11}{100}$.

(iv) 0.04

We have,

$= \frac{4}{10} = \frac{2\times2}{2\times5} = \frac{2^2}{2\times5}$

Hence, 0.04 is not a perfect cube.

Exercise 4.3

1. Question

Find the cube roots of the following numbers by successive subtraction of numbers:

1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397,….

(i) 64

(ii) 512

(iii) 1728

Answer

(i) 64

We have,

64 – 1 = 63

63 – 7 = 56

56 – 19 =37

37 – 37 = 0

∵ Subtraction is performed 4 times.

Hence, cube root of 64 is 4.

(ii) 512

We have,

512 – 1 = 511

511 – 7 = 504

504 – 19 = 485

485 – 37 = 448

448 – 61 = 387

387 – 91 = 296

296 – 127 = 169

169 – 169 = 0

∵ Subtraction is performed 8 times.

Hence, cube root of 512 is 8.

(iii) 1728

We have,

1728 – 1 = 1727

1727 – 7 = 1720

1720 – 19 = 1701

1701 – 37 = 1664

1664 – 91 = 1512

1512 – 127 = 1385

1385 – 169 = 1216

1216 – 217 = 999

999 – 271 = 728

728 – 331 = 397

397 – 397 = 0

∵ Subtraction is performed 12 times.

Hence, cube root of 1728 is 12.

2. Question

Using the method of successive subtraction examine whether or not the following numbers are perfect cubes:

(i) 130

(ii) 345

(iii) 792

(iv) 1331

Answer

(i) 130

Applying subtraction method, We have,

130 – 1 = 129

129 – 7 = 122

122 – 19 = 103

103 – 37 = 66

66 – 61 = 5

∵ Next number to be subtracted is 91, which is greter than 5

Hence, 130 is not a perfect cube.

(ii) 345

Applying subtraction method, We have,

345 – 1 = 344

344 – 7 = 337

337 – 19 = 318

318 – 37 = 281

281 – 61 = 220

220 – 91 = 129

129 – 127 = 2

∵ Next number to be subtracted is 169, which is greter than 2

Hence, 345 is not a perfect cube

(iii) 792

Applying subtraction method, We have,

792 – 1 = 791

791 – 7 = 784

784 – 19 = 765

765 – 37 = 728

728 – 61 = 667

667 – 91 = 576

576 – 127 = 449

449 – 169 = 280

280 – 217 = 63

∵ Next number to be subtracted is 271, which is greter than 63

Hence, 792 is not a perfect cube

(iv) 1331

Applying subtraction method, We have,

1331 – 1 = 1330

1330 – 7 = 1323

1323 – 19 = 1304

1304 – 37 = 1267

1267 – 61 = 1206

1206 – 91 = 1115

1115 – 127 = 988

988 – 169 = 819

819 – 217 = 602

602 – 271 = 331

331 – 331 = 0

∵ Subtraction is performed 11 times.

Hence, 1331 is a perfect cube

3. Question

Find the smallest number that must be subtracted from those of the numbers in question 2 which are not perfect cubes, to make them perfect cubes. What are the corresponding cube roots?

Answer

In previous question there are three numbers which are not perfect cubes.

i) 130

Apply subtraction method,

130 – 1 = 129

129 – 7 = 122

122 – 19 = 103

103 – 37 = 66

66 – 61 = 5

∵ Next number to be subtracted is 91, which is greter than 5

Hence, 130 is not a perfect cube. So, to make it perfect cube we subtract 5 from it.

130 – 5 = 125 (which is a perfect cube of 5)

ii) 345

Apply subtraction method,

345 – 1 = 344

344 – 7 = 337

337 – 19 = 318

318 – 37 = 281

281 – 61 = 220

220 – 91 = 129

129 – 127 = 2

∵ Next number to be subtracted is 169, which is greter than 2

Hence, 345 is not a perfect cube. So, to make it a perfect cube we subtract 2 from it.

345 – 2 = 343 (which is a perfect cube of 7)

iii) 792

Apply subtraction method,

792 – 1 = 791

791 – 7 = 784

784 – 19 = 765

765 – 37 = 728

728 – 61 = 667

667 – 91 = 576

576 – 127 = 449

449 – 169 = 280

280 – 217 = 63

∵ Next number to be subtracted is 271, which is greter than 63

Hence, 792 is not a perfect cube. So, to make it a perfect cube we subtract 63 from it.

792 – 63 = 729 (which is a perfect cube of 9)

4. Question

Find the cube root of each of the following natural numbers:

(i) 343 (ii) 2744

(iii) 4913 (iv) 1728

(v) 35937 (vi) 17576

(vii) 134217728 (viii) 48228544

(ix) 74088000 (x) 157464

(xi) 1157625 (xii) 33698267

Answer

(i) 343

By prime factorization method,

$= \sqrt[3]{343} = \sqrt[3]{7\times7\times7} = 7.$

(ii) 2744

By prime factorization method,

$= \sqrt[3]{2744} = \sqrt[3]{2\times2\times2\times7\times7\times7} = \sqrt[3]{2^3\times7^3} = 2\times7 = 14.$

(iii) 4913

By prime factorization method,

$= \sqrt[3]{4913} = \sqrt[3]{17\times17\times17} = 17.$

(iv) 1728

By prime factorization method,

$= \sqrt[3]{1728} = \sqrt[3]{2\times2\times2\times2\times2\times2\times3\times3\times3} = \sqrt[3]{2^3\times2^3\times3^3} = 2\times2\times3 = 12.$

(v) 35937

By prime factorization method,

$= \sqrt[3]{35937} = \sqrt[3]{3\times3\times3\times11\times11\times11} = \sqrt[3]{3^3\times11^3} = 3\times11 = 33.$

(vi) 17576

By prime factorization method,

$= \sqrt[3]{17576} = \sqrt[3]{2\times2\times2\times13\times13\times13} = \sqrt[3]{2^3\times13^3} = 2\times13 = 26.$

(vii) 134217728

By prime factorization method,

$= \sqrt[3]{134217728} = \sqrt[3]{2^{27}} = 2^9 = 512.$

(viii) 48228544

By prime factorization method,

$= \sqrt[3]{48228544} = \sqrt[3]{2\times2\times2\times2\times2\times2\times7\times7\times7\times13\times13\times13} = \sqrt[3]{2^3\times2^3\times7^3\times13^3}$

= 2 × 2 × 7 × 13 = 364.

(ix) 74088000

By prime factorization method,

$= \sqrt[3]{2\times2\times2\times2\times2\times2\times3\times3\times3\times5\times5\times5\times7\times7\times7} = \sqrt[3]{2^3\times2^3\times3^3\times5^3\times7^3}$

= 2 × 2 × 3 × 5 × 7 = 420.

(x) 157464

By prime factorization method,

$= \sqrt[3]{157464} = \sqrt[3]{(2\times2\times2\times3\times3\times3\times3\times3\times3\times3\times3\times3)} = \sqrt[3]{2^3\times3^3\times3^3\times3^3}$

= 2 × 3 × 3 × 3 = 54.

(xi) 1157625

By prime factorization method,

$= \sqrt[3]{1157625} = \sqrt[3]{3\times3\times3\times5\times5\times5\times7\times7\times7} = \sqrt[3]{3^3\times5^3\times7^3} = 3\times5\times7 = 105.$

(xii) 33698267

By prime factorization method,

$= \sqrt[3]{33698267} = \sqrt[3]{17\times17\times17\times19\times19\times19} = \sqrt[3]{17^3\times19^3} = 17\times19 = 323.$

5. Question

Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.

Answer

First we find out the prime factors of 3600,

3600 = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5 = $2^3 \times 3^2 \times 5^2 \times 2$

∵ only one triples is formed and three factors remained ungrouped in triples.

Hence, 3600 is not a perfect cube.

To make it a perfect cube we have to multiply it by (2 × 2 × 3 × 5) = 60

3600 × 60 = 216000 (which is a perfect cube of 60)

6. Question

Multiply 210125 by the smallest number so that the product is a perfect cube. Also, find out the cube root of the product.

Answer

First we find out the prime factors of 210125,

210125 = 5 × 5 × 5 × 41 × 41

∵ one triples remained incomplete, hence 210125 is not a perfect cube.

We see that if we multiply the factors by 41, we will get 2 triples as 2^3 and 41^3.

And the product become:

210125 × 41 = 8615125 = 5 × 5 × 5 × 41 × 41 × 41

Cube root of product = $\sqrt[3]{8615125} = \sqrt[3]{5^3 \times 41^3} = 5 \times 41 = 205.$

7. Question

What is the smallest number by which 8192 must be divided so that quotient is a perfect cube? Also, find the cube root of the quotient so obtained.

Answer

First we find out prime factors of 8192,

8192 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 \times 2^3 \times 2$

∵ one triples remain incomplete, hence 8192 is not a perfect cube.

So, we divide 8192 by 2 to make its quotient a perfect cube.

$\frac{8192}{2} = 4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^3 \times 2^3 \times 2^3 \times 2^3$

Cube root of 4096 = $\sqrt[3]{4096} = \sqrt[3]{2^3 \times 2^3 \times 2^3 \times 2^3} = 2 \times 2 \times 2 \times 2 = 16.$

8. Question

Three numbers are in the ratio 1:2:3. The sum of their cubes is 98784. Find the numbers.

Answer

Let the numbers are = x, 2x and 3x

According to the question,

$x^3 + (2x)^3 + (3x)^3 = 98784x^3 + 8x^3 + 27x^3 = 9878436x^3 = 98784$

$x^3 = \frac{98784}{36} = 2744$

$x = \sqrt[3]{2744} = \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7} = 2 \times 7 = 14$

So, the numbers are ,

x = 14

2x = 2 × 14 = 28

3x = 3 × 14 = 42

9. Question

The volume of a cube is 9261000 m^3. Find the side of the cube.

Answer

Volume of cube = 9261000 m^3

Let the side of cube = a metre

So,

$= a^3 = 9261000$

$= a = \sqrt[3]{9261000} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 7 \times 7 \times 7} = \sqrt[3]{2^3 \times 3^3 \times 5^3 \times 7^3}$

$= a = 2 \times 3 \times 5 \times 7 = 210.$

Hence, the side of cube = 210 metre

Exercise 4.4

1. Question

Find the cube roots of each of the following integers:

(i)-125 (ii) -5832

(iii)-2744000 (iv) -753571

(v) -32768

Answer

(i) We have,

Cube root of -125 = $\sqrt[3]{-125} = -\sqrt[3]{125} = -\sqrt[3]{5\times5\times5} = -5$

(ii) We have,

Cube root of -5832 = $\sqrt[3]{-5832} = -\sqrt[3]{5832}$

So to find out the cube root of 5832, we will use the mehod of unit digits.

Let's take number 5832.

Unit digit = 2

So unit digit in the cube root of 5832 = 8

After striking out the units, tens and hundreds digits of 5832,

Now we left with 5 only.

As we know that 1 is the Largest number whose cube is less than or equals to 5.

So,

The tens digit of the cube root of 5832 is 1.

$\sqrt[3]{5832} = 18$

$\sqrt[3]{-5832} = -\sqrt[3]{5832} = -18$

(iii) We have,

$\sqrt[3]{-2744000} = -\sqrt[3]{2744000}$

We will use the method of factorization to find out the cube root of 2744000

Factorizing 2744000 into prime factors,

We get,

2744000 = 2×2×2×2×2×2×5×5×5×7×7×7

Now group the factors into triples of equal factors, we get,

2744000 = (2×2×2) ×(2×2×2) ×(5×5×5) ×(7×7×7)

As we can see that all the prime factors of 2744000 can be grouped in to triples of equal factors and no factor is left over.

Now take one factor from each group and by multiplying we get,

2×2×5×7 = 140

So we can say that 2744000 is a cube of 140

Hence,

$\sqrt[3]{-2744000} = -\sqrt[3]{2744000} = -140$

(iv) We have,

$\sqrt[3]{-753571} = -\sqrt[3]{753571}$

By using unit digit method,

Let's take Number = 753571

Unit digit = 1

So unit digit in the cube root of 753571 = 1

After striking out the units, tens and hundreds digits of 753571,

Now we left with 753.

As we know that 9 is the Largest number whose cube is less than or equals to 753($9^3<753<10^3$).

So,

The tens digit of the cube root of 753571 is 9.

$\sqrt[3]{753571} = 91$

$\sqrt[3]{-753571} = -\sqrt[3]{753571} = -91$

(v) We have,

$\sqrt[3]{-32768} = -\sqrt[3]{32768}$

By using unit digit method, we will find out the cube root of 32768,

Let's take Number = 32768

Unit digit = 8

So unit digit in the cube root of 32768 = 2

After striking out the units, tens and hundreds digits of 32768,

Now we left with 32.

As we know that 9 is the Largest number whose cube is less than or equals to 32($3^3<32<4^3$).

So,

The tens digit of the cube root of 32768 is 3.

$\sqrt[3]{32768} = 32$

$\sqrt[3]{-32768} = -\sqrt[3]{32768} = -32$

2. Question

Show that:

(i) $\sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{27 \times 64}$

(ii) $\sqrt[3]{64 \times 729} = \sqrt[3]{64} = \sqrt[3]{729}$

(iii) $\sqrt[3]{-125 \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$

(iv) $\sqrt[3]{-125 \times -1000} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$

Answer

(i) Given,

$\sqrt[3]{27} \times \sqrt[3]{64} = LHS$

$\sqrt[3]{27\times64} = RHS$

$LHS = \sqrt[3]{27} \times \sqrt[3]{64} = \sqrt[3]{3\times3\times3} \times \sqrt[3]{4\times4\times4} = 3\times4 = 12$

$RHS = \sqrt[3]{27\times64} = \sqrt[3]{3\times3\times3\times4\times4\times4} = \sqrt[3]{\{3\times3\times3\}\times\{4\times4\times4\}} = 3\times4 = 12$

As we know LHS = RHS, so the equation is true.

(ii) Given,

$\sqrt[3]{64\times729} = LHS$

$\sqrt[3]{64} \times \sqrt[3]{729} = RHS$

$LHS = \sqrt[3]{64\times729} = \sqrt[3]{4\times4\times4\times9\times9\times9} = \sqrt[3]{\{4\times4\times4\}\times\{9\times9\times9\}} = 4\times9 = 36$

$RHS = \sqrt[3]{64} \times \sqrt[3]{729} = \sqrt[3]{4\times4\times4} \times \sqrt[3]{9\times9\times9} = 4\times9 = 36$

LHS = RHS

(iii) Given,

$LHS = \sqrt[3]{-125\times216} = \sqrt[3]{-5\times -5 \times -5\times\{2\times2\times2\times3\times3\times3\}}$

$= \sqrt[3]{\{-5\times-5\times-5\}\times\{2\times2\times2\}\times\{3\times3\times3\}} = -5\times2\times3 = -30$

$RHS = \sqrt[3]{-125} \times \sqrt[3]{216} = \sqrt[3]{-5\times-5\times-5}\times\sqrt[3]{\{2\times2\times2\}\times\{3\times3\times3\}} = -5\times(2\times3) = -30$

LHS = RHS

(iv) Given,

LHS = $\sqrt[3]{-125\times -1000} = \sqrt[3]{-5\times-5\times-5\times-10\times-10\times-10}$

$= \sqrt[3]{\{-5\times-5\times-5\}\times\{-10\times-10\times-10\}} = -5\times-10 = 50$

RHS = $\sqrt[3]{-125}\times \sqrt[3]{-1000} = \sqrt[3]{-5\times-5\times-5} \times\sqrt[3]{-10\times-10\times-10} = -5\times-10 = 50$

LHS = RHS

3. Question

Find the cube root of each of the following numbers:

(i) 8×125 (ii) -1728×216

(iii) -27×2744 (iv) -729×-15625

Answer

(i) We know that for any two integers a and b, $\sqrt[3]{ab} = \sqrt[3]{a} \times\sqrt[3]{b}$

So from this property, we have:

$\sqrt[3]{8\times125} = \sqrt[3]{8}\times\sqrt[3]{125} = \sqrt[3]{2\times2\times2}\times\sqrt[3]{5\times5\times5} = 2\times5 = 10$

(ii) By Applying a and b, $\sqrt[3]{ab} = \sqrt[3]{a} \times\sqrt[3]{b}$, we have

$\sqrt[3]{-1728\times216}$

$= \sqrt[3]{-1728} \times \sqrt[3]{216}$

$= -\sqrt[3]{1728} \times \sqrt[3]{216}$

To find out cube root by using units digit:

Let’s take the number 1728.

So,

Unit digit = 8

The unit digit in the cube root of 1728 = 2

After striking out the units, tens and hundreds digits of the given number, we are left with the 1.

As we know 1 is the largest number whose cube is less than or equals to 1.

So,

The tens digit of the cube root of 1728 = 1

$\therefore \sqrt[3]{1728} = 12$

Prime factors of 216 = 2×2×2×3×3×3

On grouping the factors in triples of equal factor,

We have,

216 = {2×2×2}×{3×3×3}

Taking one factor from each group we get,

$\sqrt[3]{216} = 2\times3 = 6$

So,

$\sqrt[3]{-1728\times216} = -\sqrt[3]{1728}\times \sqrt[3]{216} = -12\times6 = -72$

(iii) By Applying a and b propertise, $\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$, we have

$\sqrt[3]{-27\times2744}$

$= \sqrt[3]{-27} \times \sqrt[3]{2744}$

$= -\sqrt[3]{27} \times \sqrt[3]{2744}$

To find out cube root by using units digit:

Let’s take the number 2744.

So,

Unit digit = 4

The unit digit in the cube root of 2744= 4

After striking out the units, tens and hundreds digits of the given number, we are left with the 2.

As we know 1 is the largest number whose cube is less than or equals to 2.

So,

The tens digit of the cube root of 2744 = 1

$\therefore \sqrt[3]{2744} = 14$

Prime factors of 216 = 2×2×2×3×3×3

On grouping the factors in triples of equal factor,

We have,

216 = {2×2×2}×{3×3×3}

Taking one factor from each group we get,

$\sqrt[3]{216} = 2\times3 = 6$

So,

$\sqrt[3]{-1728\times216} = -\sqrt[3]{1728}\times\sqrt[3]{216} = -12\times6 = -72$

(iv) By Applying a and b properties, $\sqrt[3]{ab} = \sqrt[3]{a}\times\sqrt[3]{b}$, we have

$\sqrt[3]{-729\times-15625}$

$= \sqrt[3]{-729}\times\sqrt[3]{-15625}$

$= -\sqrt[3]{729}\times-\sqrt[3]{15625}$

To find out cube root by using units digit:

Let's take the number 15625.

So,

Unit digit = 5

The unit digit in the cube root of 15625 = 5

After striking out the units, tens and hundreds digits of the given number, we are left with the 15.

As we know 2 is the largest number whose cube is less than or equals to 15($2^3<15<3^3$).

So,

The tens digit of the cube root of 15625 = 2

$\therefore \sqrt[3]{15625} = 25$

Also

$\sqrt[3]{729} = 9,$

As we know 9×9×9 = 729

Thus,

$\sqrt[3]{-729\times-15625} = -\sqrt[3]{729}\times-\sqrt[3]{15625} = -9\times-25 = 225$

4. Question

Evaluate:

(i) $\sqrt[3]{4^3\times6^3}$

(ii) $\sqrt[3]{8\times17\times17\times17}$

(iii) $\sqrt[3]{700\times2\times49\times5}$

(iv) $125\sqrt[3]{a^3} - \sqrt[3]{125a^6}$

Answer

(i) $\sqrt[3]{4^3\times6^3}$

We have,

$= \sqrt[3]{4^3\times6^3} = \sqrt[3]{4^3}\times\sqrt[3]{6^3} = 4\times6 = 24.$

(ii) $\sqrt[3]{8\times17\times17\times17}$

We have,

$= \sqrt[3]{8\times17\times17\times17} = \sqrt[3]{8}\times\sqrt[3]{17^3} = \sqrt[3]{2^3}\times\sqrt[3]{17^3} = 2\times17 = 34.$

(iii) $\sqrt[3]{700\times2\times49\times5}$

We have,

$= \sqrt[3]{700\times2\times49\times5}$

Getting prime factors of numbers,

$= \sqrt[3]{700\times2\times49\times5} = \sqrt[3]{2\times2\times5\times5\times7\times2\times7\times7\times5} = \sqrt[3]{2^3\times5^3\times7^3} = \sqrt[3]{2^3}\times\sqrt[3]{5^3}\times\sqrt[3]{7^3}$

$= 2 \times 5 \times 7 = 70.$

(iv) $125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$

We have,

$= 125\sqrt[3]{a^6} - \sqrt[3]{125a^6}$

$= 125\sqrt[3]{(a^2)^3} - \sqrt[3]{5^3(a^2)^3} = 125a^2 - \sqrt[3]{5^3}\times\sqrt[3]{(a^2)^3} = 125a^2 - 5a^2 = 120a^2.$

5. Question

Find the cube root of each of the following rational numbers:

(i) $\frac{-125}{729}$

(ii) $\frac{10648}{12167}$

(iii) $\frac{-19683}{24384}$

(iv) $\frac{686}{-3456}$

(v) $\frac{-39304}{-42875}$

Answer

(i) $\frac{-125}{729}$

We have,

$= -\frac{125}{729} = -\frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{9\times9\times9}} = -\frac{\sqrt[3]{5^3}}{\sqrt[3]{7^3}} = -\frac{5}{7}.$

(ii) $\frac{10648}{12167}$

By getting prime factors of given problems. We have,

$= \sqrt[3]{\frac{10648}{12167}} = \frac{\sqrt[3]{2\times2\times2\times11\times11\times11}}{\sqrt[3]{23\times23\times23}} = \frac{\sqrt[3]{2^3\times11^3}}{\sqrt[3]{23^3}} = \frac{2\times11}{23} = \frac{22}{23}.$

(iii) $\frac{-19683}{24384}$

By getting prime factors of given problems. We have,

$$= -\frac{19683}{24384} = \frac{-\sqrt[3]{3\times3\times3\times3\times3\times3\times3\times3\times3}}{\sqrt[3]{29\times29\times29}} = \frac{-\sqrt[3]{3^3\times3^3\times3^3}}{\sqrt[3]{29^3}} = \frac{-(3\times3\times3)}{29} = -\frac{27}{29}$$

(iv) $\frac{686}{-3456}$

By getting prime factors of given problems. We have,

$$= \frac{686}{-3456} = \frac{-\sqrt[3]{2\times7\times7\times7}}{\sqrt[3]{2^7\times2^3}} = \frac{-\sqrt[3]{2\times7^3}}{\sqrt[3]{2^7\times2^3}} = \frac{-\sqrt[3]{7^3}}{\sqrt[3]{2^6\times2^3}} = \frac{-7}{2\times2\times2} = -\frac{7}{8}.$$

(v) $\frac{-39304}{-42875}$

By getting prime factors of given problems. We have,

$$= \sqrt[3]{\frac{-39304}{-42875}} = \frac{-\sqrt[3]{2\times2\times2\times17\times17\times17}}{-\sqrt[3]{5\times5\times5\times7\times7\times7}} = \frac{-\sqrt[3]{2^3\times17^3}}{-\sqrt[3]{5^3\times7^3}} = \frac{-\sqrt[3]{2^3}\times\sqrt[3]{17^3}}{-\sqrt[3]{5^3}\times\sqrt[3]{7^3}} = \frac{-(2\times17)}{-(5\times7)} = \frac{-34}{-35} = \frac{34}{35}$$

6. Question

Find the cube root of each of the following rational numbers:

(i) 0.001728

(ii) 0.003375

(iii) 0.001

(iv) 1.331

Answer

(i) 0.001728

Given,

$0.001728 = \frac{1728}{1000000}$

$$\therefore \sqrt[3]{0.001728} = \frac{1728}{1000000} = \frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}}$$

Getting prime factors of 1728,

$1728 = 2\times2\times2\times2\times2\times2\times3\times3\times3 = 2^3\times2^3\times3^3$

$$\sqrt[3]{1728} = \sqrt[3]{2^3\times2^3\times3^3} = \sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{3^3} = 2\times2\times3 = 12$$

also,

$$\sqrt[3]{1000000} = \sqrt[3]{100\times100\times100} = \sqrt[3]{100^3} = 100$$

$$\therefore \sqrt[3]{0.001728} = \frac{\sqrt[3]{1728}}{\sqrt[3]{1000000}} = \frac{12}{100} = 0.12$$

(ii) 0.003375

Given,

$0.003375 = \frac{3375}{1000000}$

$$\therefore \sqrt[3]{0.003375} = \sqrt[3]{\frac{3375}{1000000}} = \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$$

Getting prime factors of 3375 ,

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$,

$\sqrt[3]{3375} = \sqrt[3]{3^3 \times 5^3} = \sqrt[3]{3^3} \times \sqrt[3]{5^3} = 3 \times 5 = 15$

Also,

$\sqrt[3]{1000000} = \sqrt[3]{100 \times 100 \times 100} = \sqrt[3]{100^3} = 100$

$\therefore \sqrt[3]{0.003375} = \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}} = \frac{15}{100} = 0.15$

(iii) 0.001

Given,

$\because 0.001 = \frac{1}{1000}$

$\therefore \sqrt[3]{0.001} = \sqrt[3]{\frac{1}{1000}} = \frac{\sqrt[3]{1}}{\sqrt[3]{1000}} = \frac{1}{10} = 0.1$

(iv) 1.331

Given

$\because 1.331 = \frac{1331}{1000}$

$\therefore \sqrt[3]{1.331} = \sqrt[3]{\frac{1331}{1000}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{1000}} = \frac{\sqrt[3]{11 \times 11 \times 11}}{\sqrt[3]{1000}} = \frac{11}{10} = 1.1$

7. Question

Evaluate each of the following:

(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$

(iii) $\sqrt[3]{\frac{729}{216}} \times \frac{6}{9}$

(iv) $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1$

(v) $\sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13}$

Answer

(i) $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$

By prime factorization of terms, We have,

$= \sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064} = = \sqrt[3]{3 \times 3 \times 3} + \sqrt[3]{0.2 \times 0.2 \times 0.2} + \sqrt[3]{0.4 \times 0.4 \times 0.4}$

$= \sqrt[3]{3^3} + \sqrt[3]{0.2^3} + \sqrt[3]{0.4^3} = 3 + 0.2 + 0.4 = 3.6.$

(ii) $\sqrt[3]{1000} + \sqrt[3]{0.008} - \sqrt[3]{0.125}$

By prime factorization of terms, We have,

$= \sqrt[3]{10 \times 10 \times 10} + \sqrt[3]{0.2 \times 0.2 \times 0.2} - \sqrt[3]{0.5 \times 0.5 \times 0.5}$

$= \sqrt[3]{10^3} + \sqrt[3]{0.2^3} - \sqrt[3]{0.5^3} = 10 + 0.2 - 0.5 = 9.7.$

(iii) $\sqrt[3]{\frac{729}{216}} \times \frac{6}{9}$

By prime factorization of terms, We have,

$$= \sqrt[3]{\frac{729}{216}} \times \frac{6}{9} = \sqrt[3]{\frac{9\times9\times9}{6\times6\times6}} \times \frac{6}{9} = \frac{\sqrt[3]{9^3}}{\sqrt[3]{6^3}} \times \frac{6}{9} = \frac{9}{6} \times \frac{6}{9} = 1$$

(iv) $\sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} - 1$

By prime factorization of terms, We have,

$$= = \sqrt[3]{\frac{0.027}{0.008}} \div \sqrt{\frac{0.09}{0.04}} = \sqrt[3]{\frac{0.3\times0.3\times0.3}{0.2\times0.2\times0.2}} \div \sqrt{\frac{0.3\times0.3}{0.2\times0.2}} = \frac{\sqrt[3]{0.3^3}}{\sqrt[3]{0.2^3}} \div \frac{\sqrt{0.3^2}}{\sqrt{0.2^2}}$$

$$= \frac{0.3}{0.2} \div \frac{0.3}{0.2} = \frac{0.3}{0.2} \times \frac{0.2}{0.3} = 1.$$

(v) $\sqrt[3]{0.1 \times 0.1 \times 0.1 \times 13 \times 13 \times 13}$

By prime factorization of terms, We have,

$$= \sqrt[3]{0.1\times0.1\times0.1\times13\times13\times13} = \sqrt[3]{0.1^3\times13^3} = \sqrt[3]{0.1^3}\times\sqrt[3]{13^3} = 0.1\times13 = 1.3.$$

8. Question

Show that:

(i) $\frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \sqrt[3]{\frac{729}{1000}}$

(ii) $\frac{\sqrt[3]{-512}}{\sqrt[3]{343}} = \sqrt[3]{\frac{-512}{343}}$

Answer

(i) $\frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \sqrt[3]{\frac{729}{1000}}$

We have,

$$\text{LHS} = \frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \frac{\sqrt[3]{9\times9\times9}}{\sqrt[3]{10\times10\times10}} = \frac{\sqrt[3]{9^3}}{\sqrt[3]{10^3}} = \frac{9}{10}$$

$$\text{RHS} = \sqrt[3]{\frac{729}{1000}} = \sqrt[3]{\frac{9\times9\times9}{10\times10\times10}} = \sqrt[3]{\frac{9^3}{10^3}} = \frac{\sqrt[3]{9^3}}{\sqrt[3]{10^3}} = \frac{9}{10}$$

∵ LHS = RHS

Hence, equation is true.

(ii) $\frac{\sqrt[3]{-512}}{\sqrt[3]{343}} = \sqrt[3]{\frac{-512}{343}}$

We have,

$$\text{LHS} = \frac{\sqrt[3]{-512}}{\sqrt[3]{343}} = \frac{-\sqrt[3]{8\times8\times8}}{\sqrt[3]{7\times7\times7}} = \frac{-\sqrt[3]{8^3}}{\sqrt[3]{7^3}} = \frac{-8}{7}$$

$$\text{RHS} = \sqrt[3]{\frac{-512}{343}} = \sqrt[3]{\frac{-(8\times8\times8)}{7\times7\times7}} = \sqrt[3]{-\frac{8^3}{7^3}} = \frac{\sqrt[3]{-8^3}}{\sqrt[3]{7^3}} = \frac{-\sqrt[3]{8^3}}{\sqrt[3]{7^3}} = -\frac{8}{7}$$

∵ LHS = RHS

Hence, equation is true.

9. Question

Fill in the blanks:

(i) $\sqrt[3]{125 \times 27} = 3 \times ....$

(ii) $\sqrt[3]{8 \times ....} = 8$

(iii) $\sqrt[3]{1728} = 4 \times ....$

(iv) $\sqrt[3]{480} = \sqrt[3]{3} \times 2 \times \sqrt[3]{.....}$

(v) $\sqrt[3]{....} = \sqrt[3]{7} \times \sqrt[3]{8}$

(vi) $\sqrt[3]{....} = \sqrt[3]{4} \times \sqrt[3]{5} \times \sqrt[3]{6}$

(vii) $\sqrt[3]{\frac{27}{125}} = \frac{....}{5}$

(viii) $\sqrt[3]{\frac{729}{1331}} = \frac{9}{.....}$

(ix) $\sqrt[3]{\frac{512}{...}} = \frac{8}{13}$

Answer

(i) $\sqrt[3]{125 \times 27} = 3 \times ....$

We have,

$= \sqrt[3]{125 \times 27} = \sqrt[3]{5 \times 5 \times 5 \times 3 \times 3 \times 3} = \sqrt[3]{5^3 \times 3^3} = \sqrt[3]{5^3} \times \sqrt[3]{3^3} = 5 \times 3 \ or \ 3 \times 5.$

Hence,

$= \sqrt[3]{125 \times 27} = 3 \times \underline{5}$

(ii) $\sqrt[3]{8 \times ....} = 8$

We have,

$= \sqrt[3]{8 \times 8 \times 8} = \sqrt[3]{8^3} = 8$

Hence,

$= \sqrt[3]{8 \times 8 \times 8} = 8$

(iii) $\sqrt[3]{1728} = 4 \times ....$

We have,

$= \sqrt[3]{1728} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3} = \sqrt[3]{2^3 \times 2^3 \times 3^3} = \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{3^3} = 2 \times 2 \times 3$

$= 4 \times 3$

Hence,

$= \sqrt[3]{1728} = 4 \times \underline{3}$

(iv) $\sqrt[3]{480} = \sqrt[3]{3} \times 2 \times \sqrt[3]{.....}$

We have,

$= \sqrt[3]{480} = \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5} = \sqrt[3]{2^3 \times 2^2 \times 3 \times 5} = \sqrt[3]{2^3} \times \sqrt[3]{3} \times \sqrt[3]{2 \times 2 \times 5}$

$= 2 \times \sqrt[3]{3} \times \sqrt[3]{20}$

Hence,

$= \sqrt[3]{480} = 2\times\sqrt[3]{3}\times\sqrt[3]{20}.$

(v) $\sqrt[3]{\ldots} = \sqrt[3]{7}\times\sqrt[3]{8}$

We have,

$= \sqrt[3]{7\times8} = \sqrt[3]{7}\times\sqrt[3]{8}$

$= \sqrt[3]{56} = \sqrt[3]{7}\times\sqrt[3]{8}$

Hence,

$= \sqrt[3]{56} = \sqrt[3]{7}\times\sqrt[3]{8}.$

(vi) $\sqrt[3]{\ldots} = \sqrt[3]{4}\times\sqrt[3]{5}\times\sqrt[3]{6}$

We have,

$=\sqrt[3]{4\times5\times6} = \sqrt[3]{4}\times\sqrt[3]{5}\times\sqrt[3]{6}$

$= \sqrt[3]{120} = \sqrt[3]{4}\times\sqrt[3]{5}\times\sqrt[3]{6}$

Hence,

$= \sqrt[3]{120} = \sqrt[3]{4}\times\sqrt[3]{5}\times\sqrt[3]{6}.$

(vii) $\sqrt[3]{\frac{27}{125}} = \frac{\ldots}{5}$

We have,

$= \sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\sqrt[3]{3\times3\times3}}{\sqrt[3]{5\times5\times5}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{5^3}} = \frac{3}{5}$

Hence,

$= \sqrt[3]{\frac{27}{125}} = \frac{3}{5}.$

(viii) $\sqrt[3]{\frac{729}{1331}} = \frac{9}{\ldots}$

We have,

$= \sqrt[3]{\frac{729}{1331}} = \sqrt[3]{\frac{9\times9\times9}{11\times11\times11}} = \sqrt[3]{\frac{9^3}{11^3}} = \frac{\sqrt[3]{9^3}}{\sqrt[3]{11^3}} = \frac{9}{11}$

Hence,

$= \sqrt[3]{\frac{729}{1331}} = \frac{9}{11}$

(ix) $\sqrt[3]{\frac{512}{\ldots}} = \frac{8}{13}$

$\sqrt[3]{512}\ \sqrt[3]{\sqrt{2\times2\times2}\times\sqrt{2\times2\times2}\times\sqrt{2\times2\times2}}$

$\sqrt[3]{8\times8\times8}$

$= 8$

$\frac{8}{\sqrt[3]{\ldots}} = \frac{8}{13}$

$\sqrt[3]{\ldots} = 13$

$(\ldots) = (13)^2$

$= 2197$

10. Question

The volume of a cubical box is 474. 552 cubic metres. Find the length of each side of the box.

Answer

Given,

Volume of a cube = 474.552 cubic metres

V = 8^3,

S = side of the cube

So,

8^3 = 474.552 cubic metres

$= 8 = \sqrt[3]{474.552} = \sqrt[3]{\frac{474552}{1000}} = \frac{\sqrt[3]{474552}}{\sqrt[3]{1000}}$

On factorising 474552 into prime factors, we get:

474552 = 2×2×2×3×3×3×13×13×13

On grouping the factors in triples of equal factors, we get:

474552 = {2×2×2}×{3×3×3}×{13×13×13}

Now taking 1 factor from each group we get,

$\sqrt[3]{474.552} = \sqrt[3]{\{2\times2\times2\}\times\{3\times3\times3\}\times\{13\times13\times13\}} = 2\times3\times13 = 78$

Also,

$\sqrt[3]{1000} = 10$

$\therefore 8 = \frac{\sqrt[3]{474552}}{\sqrt[3]{1000}} = \frac{78}{10} = 7.8$

So, length of the side is 7.8m.

11. Question

Three numbers are to one another 2:3:4. The sum of their cubes is 0.334125. Find the numbers.

Answer

Lest assume the numbers be 2a, 3a, and 4a.

According to the question:

$(2a)^3 + (3a)^3 + (4a)^3 = 0.334125$

$= 8a^3 + 27\,a^3 + 64\,a^3 = 0.334125$

$= 99\,a^3 = 0.334125$

$= a^3 = \frac{334125}{1000000\times99} = \frac{3375}{1000000}$

$= a = \sqrt[3]{\frac{3375}{1000000}}$

$= a = \frac{\sqrt[3]{3375}}{\sqrt[3]{1000000}}$

$= a = \frac{15}{100} = 0.15$

Thus the numbers are:

$2 \times 0.15 = 0.30$

$3 \times 0.15 = 0.45$

$4 \times 0.15 = 0.60$

12. Question

Find the side of a cube whose volume is $\frac{24389}{216} m^3$.

Answer

Given,

Volume of the side s $= \frac{24389}{216} = v$

$v = 8^3$

$\therefore 8 = \sqrt[3]{v}$

$= \sqrt[3]{\frac{24389}{216}}$

$= \frac{\sqrt[3]{24389}}{\sqrt[3]{216}}$

$= \frac{\sqrt[3]{29 \times 29 \times 29}}{\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}}$ (by prime factorisation)

$= \frac{29}{2 \times 3} = \frac{29}{6}$

thus, $\frac{29}{6}$ is the length of the side.

13. Question

Evaluate:

(i) $\sqrt[3]{36} \times \sqrt[3]{384}$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$

Answer

(i) $\sqrt[3]{36} \times \sqrt[3]{384}$

We have,

$= \sqrt[3]{36} \times \sqrt[3]{384} = \sqrt[3]{36 \times 384} \because (\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab})$

Now by prime factorization method,

$= \sqrt[3]{36 \times 384} = \sqrt[3]{(2 \times 2 \times 3 \times 3) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3)} = \sqrt[3]{2^3 \times 2^3 \times 2^3 \times 3^3}$

$= \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{2^3} \times \sqrt[3]{3^3} = 2 \times 2 \times 2 \times 3 = 24.$

(ii) $\sqrt[3]{96} \times \sqrt[3]{144}$

We have,

$=\sqrt[3]{96}\times\sqrt[3]{122} = \sqrt[3]{96\times122} \because (\sqrt[3]{a}\times\sqrt[3]{b} = \sqrt[3]{ab})$

Now by prime factorization method,

$= \sqrt[3]{96\times122} = \sqrt[3]{(2\times2\times2\times2\times2\times3)\times(2\times2\times2\times2\times3\times3)} = \sqrt[3]{2^3\times2^3\times2^3\times3^3}$

$= \sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{2^3}\times\sqrt[3]{3^3} = 2\times2\times2\times3 = 24.$

(iii) $\sqrt[3]{100} \times \sqrt[3]{270}$

We have,

$= \sqrt[3]{100}\times\sqrt[3]{270} = \sqrt[3]{100\times270} \because (\sqrt[3]{a}\times\sqrt[3]{b} = \sqrt[3]{ab})$

Now by prime factorization method,

$= \sqrt[3]{100\times270} = \sqrt[3]{(2\times2\times5\times5)\times(2\times3\times3\times3\times5)} = \sqrt[3]{2^3\times3^3\times5^3}$

$= \sqrt[3]{2^3}\times\sqrt[3]{3^3}\times\sqrt[3]{5^3} = 2\times3\times5 = 30.$

(iv) $\sqrt[3]{121} \times \sqrt[3]{297}$

We have,

$= \sqrt[3]{121}\times\sqrt[3]{297} = \sqrt[3]{121\times297} \because (\sqrt[3]{a}\times\sqrt[3]{b} = \sqrt[3]{ab})$

Now by prime factorization method,

$= \sqrt[3]{121\times297} = \sqrt[3]{(11\times11)\times(3\times3\times3\times11)} = \sqrt[3]{11^3\times3^3} = \sqrt[3]{11^3}\times\sqrt[3]{3^3} = 11\times3 = 33.$

14. Question

Find the cube roots of the numbers 3048625, 20346417, 210644875, 57066625 using the fact that

(i) $3048625 = 3375 \times 729$

(ii) $20346417 = 9261 \times 2197$

(iii) $210644875 = 42875 \times 4913$

(iv) $57066625 = 166375 \times 343$

Answer

(i) $3048625 = 3375 \times 729$

Taking cube root of the whole, we get,

$= \sqrt[3]{3048625} = \sqrt[3]{3375\times729}$

We know that,

$= \sqrt[3]{ab} = \sqrt[3]{a}\times\sqrt[3]{b}$

$= \sqrt[3]{3375\times729} = \sqrt[3]{3375}\times\sqrt[3]{729}$

Now by prime factorization,

$= \sqrt[3]{3\times3\times3\times5\times5\times5}\times\sqrt[3]{9\times9\times9}$

$= \sqrt[3]{3^3\times5^3}\times\sqrt[3]{9^3} = \sqrt[3]{3^3}\times\sqrt[3]{5^3}\times\sqrt[3]{9^3} = 3\times5\times9 = 135.$

(ii) $20346417 = 9261 \times 2197$

Taking cube root of the whole,

$=\sqrt[3]{20346417} = \sqrt[3]{9261 \times 2197}$

We know that,

$= \sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$

$= \sqrt[3]{9261 \times 2197} = \sqrt[3]{9261} \times \sqrt[3]{2197}$

Now by prime factorization,

$= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7} \times \sqrt[3]{13 \times 13 \times 13} = \sqrt[3]{3^3 \times 7^3} \times \sqrt[3]{13^3} = \sqrt[3]{3^3} \times \sqrt[3]{7^3} \times \sqrt[3]{13^3}$

= 3 × 7 × 13 = 273.

(iii) $210644875 = 42875 \times 4913$

Taking cube root of the whole,

$= \sqrt[3]{210644875} = \sqrt[3]{42875 \times 4913}$

We know that,

$= \sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$

$= \sqrt[3]{42875 \times 4913} = \sqrt[3]{42875} \times \sqrt[3]{4913}$

Now by prime factorization,

$= \sqrt[3]{5 \times 5 \times 5 \times 7 \times 7 \times 7} \times \sqrt[3]{17 \times 17 \times 17} = \sqrt[3]{5^3 \times 7^3} \times \sqrt[3]{13^3} = \sqrt[3]{5^3} \times \sqrt[3]{7^3} \times \sqrt[3]{17^3}$

= 5 × 7 × 17 = 595.

(iv) $57066625 = 166375 \times 343$

Taking cube root of the whole, we get,

$= \sqrt[3]{57066625} = \sqrt[3]{166375 \times 343}$

We know that,

$= \sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$

$= \sqrt[3]{166375 \times 343} = \sqrt[3]{166375} \times \sqrt[3]{343}$

Now by prime factorization method,

$= \sqrt[3]{5 \times 5 \times 5 \times 11 \times 11 \times 11} \times \sqrt[3]{7 \times 7 \times 7} = \sqrt[3]{5^3 \times 11^3} \times \sqrt[3]{7^3} = \sqrt[3]{5^3} \times \sqrt[3]{11^3} \times \sqrt[3]{7^3}$

= 5 × 7 ×11 = 385.

15. Question

Find the unit of the cube root of the following numbers:

(i) 226981

(ii) 13824

(iii) 571787

(iv) 175616

Answer

(i) 226981

Let's consider the number 226981.

Unit digit = 1

The unit digit of the cube root of 226981 = 1

(ii) 13824

Let's consider the number 13824.

Unit digit = 4

The unit digit of the cube root of 13824 = 4

(iii) 571787

Let's consider the number 571787.

Unit digit = 7

The unit digit of the cube root of 571787 = 3

(iv) 175616

Let's consider the number 175616.

Unit digit = 6

The unit digit of the cube root of 175616 = 6

16. Question

Find the tens digit of the cube root of each of the numbers in Q.No.15.

(i) 226981

(ii) 13824

(iii) 571787

(iv) 175616

Answer

(i) 226981

Let's take number 226981.

Unit digit = 1

So unit digit in the cube root of 226981 = 1

After striking out the units, tens and hundreds digits of 226981,

Now we left with 226 only.

As we know that 6 is the Largest number whose cube root is less than or equals to 226(6^3<226<7^3).

So,

The tens digit of the cube root of 226981 is 6.

(ii) 13824

Let's take number 13824.

Unit digit = 4

So unit digit in the cube root of 13824 = 4

After striking out the units, tens and hundreds digits of 13824, Now we left with 13 only.

As we know that 2 is the Largest number whose cube root is less than or equals to 13($2^3<13<3^3$).

So,

The tens digit of the cube root of 13824 is 2.

(iii) 571787

Let's take number 571787.

Unit digit = 7

So unit digit in the cube root of 571787 = 3

After striking out the units, tens and hundreds digits of 571787,

Now we left with 571 only.

As we know that 8 is the Largest number whose cube root is less than or equals to 571($8^3<571<9^3$).

So,

The tens digit of the cube root of 571787 is 8.

(iv) 175616

Let's take number 175616.

Unit digit = 6

So unit digit in the cube root of 175616 = 6

After striking out the units, tens and hundreds digits of 175616,

Now we left with 175 only.

As we know that 5 is the Largest number whose cube root is less than or equals to 175($5^3<175<6^3$).

So,

The tens digit of the cube root of 175616 is 5.

Exercise 4.5

1. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

7

Answer

As we know that 7 lies between 1 and 100 so by using cube root table we have,

$\sqrt[3]{7} = 1.913$

So, Answer is 1.913.

2. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

70

Answer

As we know that 70 lies between 1 and 100 so by using cube root table from column x

we have,

$\sqrt[3]{70} = 4.121$

So, Answer is 4.121

3. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

700

Answer

Given,

700 = 70×10

By using cube root table 700 will be in the column $\sqrt[3]{10x}$ against 70.

So we have,

$\sqrt[3]{700} = 8.879$

4. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

7000

Answer

7000 = 70×100

$\therefore \sqrt[3]{7000} = \sqrt[3]{7\times1000} = \sqrt[3]{7}\times\sqrt[3]{1000}$

By using cube root table,

We get,

$\sqrt[3]{7} = 1.913 \ and \ \sqrt[3]{1000} = 10$

$\therefore \sqrt[3]{7000} = \sqrt[3]{7}\times\sqrt[3]{1000} = 1.913\times10 = 19.13$

5. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

1100

Answer

1100 = 11×100

$\therefore \sqrt[3]{1100} = \sqrt[3]{11\times100} = \sqrt[3]{11}\times\sqrt[3]{100}$

By using cube root table,

We get,

$\sqrt[3]{7} = 1.913 \ and \ \sqrt[3]{1000} = 10$

$\therefore \sqrt[3]{1100} = \sqrt[3]{11}\times\sqrt[3]{100} = 2.224\times4.642 = 10.323$

6. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

780

Answer

780 = 78×10

By using cube root table 780 would be in column $\sqrt[3]{10x}$ against 78.

So we get,

$\sqrt[3]{780} = 9.205$

7. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

7800

Answer

7800 = 78×100

$\therefore \sqrt[3]{7800} = \sqrt[3]{78\times100} = \sqrt[3]{78}\times\sqrt[3]{100}$

By using cube root table,

We get,

$\sqrt[3]{78} = 4.273 \ and \ \sqrt[3]{100} = 4.642$

$\therefore \sqrt[3]{7800} = \ \sqrt[3]{78}\times\sqrt[3]{100} = 4.273\times4.642 = 19.835$

8. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

1346

Answer

By primefactorisation method,

We get,

1346 = 2×673

$\sqrt[3]{1346} = \ \sqrt[3]{2}\times\sqrt[3]{673}$

Also,

670<673<680 => $\sqrt[3]{670} < \sqrt[3]{673} < \sqrt[3]{680}$

By using cube root table,

$\sqrt[3]{670} = 8.750 \ and \ \sqrt[3]{680} = 8.794$

For the difference (680-670) which is 10.

The difference in the values,

= 8.794 - 8.750 = 0.044

For the difference (673-670) which is 3.

The difference in the values,

$= \frac{0.044}{10}\times3 = 0.0132 = 0.013$

$\sqrt[3]{673} = 8.750 + 0.013 = 8.763$

So,

$\sqrt[3]{1346} = \ \sqrt[3]{2}\times\sqrt[3]{673} = 1.260\times8.763 = 11.041$

9. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

250

Answer

250 = 25×100

By using cube root table 250 would be in column $\sqrt[3]{10x}$ against 25.

So we get,

$\sqrt[3]{250} = 6.3$

10. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

5112

Answer

$= \sqrt[3]{5112} = \sqrt[3]{2\times2\times2\times3\times3\times71} = \sqrt[3]{2^3\times3^2\times71} = 2\times\sqrt[3]{9}\times\sqrt[3]{71}$

From cube root table we get,

$= \sqrt[3]{9} = 2.080 \; and \; \sqrt[3]{71} = 4.141$

Hence,

$= \sqrt[3]{5112} = 2\times2.080\times4.141 = 17.227$

Thus, the required cube root is = 17.227.

11. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

9800

Answer

$= \sqrt[3]{9800} = \sqrt[3]{98}\times\sqrt[3]{100}$

From cube root table we get,

$= \sqrt[3]{98} = 4.610 \; and \; \sqrt[3]{100} = 4.642$

Hence,

$= \sqrt[3]{9800} = \sqrt[3]{98}\times\sqrt[3]{100} = 4.610\times4.642 = 21.40$

Thus, the required cube root is = 21.40.

12. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

732

Answer

$= \sqrt[3]{732}$

We know that value of $\sqrt[3]{732} \; will \; lie \; between \; \sqrt[3]{730} \; and \; \sqrt[3]{740}.$

From cube root table we get,

$= \sqrt[3]{730} = 9.004$ *and* $\sqrt[3]{740} = 9.045$

So by unitary method,

∵ For difference (740 – 730 = 10) difference in cube root values = 9.045 – 9.004 = 0.041

∴ For difference (732 – 730 = 2) difference in cube root values = $\frac{0.041}{10} \times 2 = 0.008$

$= \sqrt[3]{732} = 9.004 + 0.008 = 9.012.$

Thus, the required cube root is = = 9.012.

13. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

7342

Answer

$= \sqrt[3]{7342}$

We know that value of $\sqrt[3]{7342}$ *will lie between* $\sqrt[3]{7300}$ *and* $\sqrt[3]{7400}.$

From cube root table we get,

$= \sqrt[3]{7300} = 19.39$ *and* $\sqrt[3]{7400} = 19.48$

So by unitary method,

∵ For difference (7400 – 7300 = 100) difference in cube root values = 19.48 – 19.39 = 0.09

∴ For difference (7342 – 7300 = 42) difference in cube root values = $\frac{0.09}{100} \times 42 = 0.037$

$= \sqrt[3]{7342} = 19.39 + 0.037 = 19.427$

Thus, the required cube root is = 19.427.

14. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

133100

Answer

$= \sqrt[3]{133100} = \sqrt[3]{1331 \times 100} = \sqrt[3]{1331} \times \sqrt[3]{100} = \sqrt[3]{11^3} \times \sqrt[3]{100} = 11 \times \sqrt[3]{100}$

From cube root table we get,

$= \sqrt[3]{100} = 4.462$

Hence,

$= \sqrt[3]{133100} = 11 \times \sqrt[3]{100} = 11 \times 4.462 = 51.062.$

Thus, the required cube root is = 51.062.

15. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

37800

Answer

$= \sqrt[3]{37800} = \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 175} = \sqrt[3]{2^3 \times 3^3 \times 175} = 6 \times \sqrt[3]{175}$

We know that value of $\sqrt[3]{175}$ *will lie between* $\sqrt[3]{170}$ *and* $\sqrt[3]{180}$.

From cube root table we get,

$= \sqrt[3]{170} = 5.540 \text{ and } \sqrt[3]{180} = 5.646$

So by unitary method,

∵ For difference (180 – 170 = 10) difference in cube root values = 5.646 – 5.540 = 0.106

∴ For difference (175 – 170 = 5) difference in cube root values = $\frac{0.106}{10} \times 5 = 0.053$

$= \sqrt[3]{175} = 5.540 + 0.053 = 5.593$

Hence,

$= \sqrt[3]{37800} = 6 \times \sqrt[3]{175} = 6 \times 5.593 = 33.558.$

Thus, the required cube root is 33.558.

16. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

0.27

Answer

$= \sqrt[3]{0.27} = \sqrt[3]{\frac{27}{100}} = \frac{\sqrt[3]{27}}{\sqrt[3]{100}}$

From cube root table we get,

$= \sqrt[3]{27} = 3 \text{ and } \sqrt[3]{100} = 4.642$

Hence,

$= \sqrt[3]{0.27} = \frac{\sqrt[3]{27}}{\sqrt[3]{100}} = \frac{3}{4.642} = 0.646.$

Thus the required cube root is = 0.646.

17. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

8.6

Answer

$= \sqrt[3]{8.6} = \sqrt[3]{\frac{86}{10}} = \frac{\sqrt[3]{86}}{\sqrt[3]{10}}$

From cube root table we get,

$= \sqrt[3]{86} = 4.414 \text{ and } \sqrt[3]{10} = 2.154$

Hence,

$= \sqrt[3]{8.6} = \frac{\sqrt[3]{86}}{\sqrt[3]{10}} = \frac{4.414}{2.514} = 2.049.$

Thus the required cube root is = 2.049.

18. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

0.86

Answer

$= \sqrt[3]{0.86} = \sqrt[3]{\frac{86}{100}} = \frac{\sqrt[3]{86}}{\sqrt[3]{100}}$

From cube root table we get,

$= \sqrt[3]{86} = 4.414 \; and \; \sqrt[3]{100} = 4.642$

Hence,

$= \sqrt[3]{0.86} = \frac{\sqrt[3]{86}}{\sqrt[3]{100}} = \frac{4.414}{4.642} = 0.951.$

Thus the required cube root is = 0.951.

19. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

8.65

Answer

$= \sqrt[3]{8.65} = \sqrt[3]{\frac{865}{100}} = \frac{\sqrt[3]{865}}{\sqrt[3]{100}}$

We know that value of $\sqrt[3]{865} \; will \; lie \; between \; \sqrt[3]{860} \; and \; \sqrt[3]{870}.$

From cube root table we get,

$= \sqrt[3]{860} = 9.510 \; and \; \sqrt[3]{870} = 9.546$

So by unitary method,

∵ For difference (870 – 860 = 10) difference in cube root values = 9.546 – 9.510 = 0.036

∴ For difference (865 – 860 = 5) difference in cube root values = $\frac{0.036}{10} \times 5 = 0.018$

$= \sqrt[3]{865} = 9.510 + 0.018 = 9.528$

We also have, $\sqrt[3]{100} = 4.642 \; (from \; table)$

$\therefore \sqrt[3]{8.65} = \frac{\sqrt[3]{865}}{\sqrt[3]{100}} = \frac{9.528}{4.642} = 2.053.$

Thus the required cube root is = 2.053.

20. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

7532

Answer

$= \sqrt[3]{7532}$

We know that value of $\sqrt[3]{7532} \; will \; lie \; between \; \sqrt[3]{7500} \; and \; \sqrt[3]{7600}.$

From cube root table we get,

$= \sqrt[3]{7500} = 19.57 \; and \; \sqrt[3]{7600} = 19.66$

So by unitary method,

∵ For difference (7600 – 7500 = 100) difference in cube root values = 19.66 – 19.57 = 0.09

∴ For difference (7532 – 7500 = 32) difference in cube root values = $\frac{0.09}{100}\times 32 = 0.029$

$= \sqrt[3]{7532} = 19.57 + 0.029 = 19.599.$

Thus the required cube root is = 19.599.

21. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

833

Answer

$= \sqrt[3]{833}$

We know that value of $\sqrt[3]{833}$ *will lie between* $\sqrt[3]{830}$ *and* $\sqrt[3]{840}$.

From cube root table we get,

$= \sqrt[3]{830} = 9.398$ *and* $\sqrt[3]{840} = 9.435$

So by unitary method,

∵ For difference (840 – 830 = 10) difference in cube root values = 9.435 – 9.398 = 0.037

∴ For difference (833 – 830 = 3) difference in cube root values = $\frac{0.037}{10}\times 3 = 0.011$

$= \sqrt[3]{833} = 9.398 + 0.011$ = 9.409

Thus the required cube root is = 9.409.

22. Question

Making use of the cube root table, find the cube root of the following (currect to three decimal places):

34.2

Answer

$= \sqrt[3]{34.2} = \sqrt[3]{\frac{342}{10}} = \frac{\sqrt[3]{342}}{\sqrt[3]{10}}$

We know that value of $\sqrt[3]{342}$ *will lie between* $\sqrt[3]{340}$ *and* $\sqrt[3]{350}$.

From cube root table we get,

$= \sqrt[3]{340} = 6.980$ *and* $\sqrt[3]{350} = 7.047$

So by unitary method,

∵ For difference (350 – 340 = 10) difference in cube root values = 7.047 – 6.980 = 0.067

∴ For difference (342 – 340 = 2) difference in cube root values = $\frac{0.067}{10}\times 2 = 0.013$

$= \sqrt[3]{342} = 6.980 + 0.013 = 6.993$

We also have, $\sqrt[3]{10} = 2.154\ (from\ table)$

$\therefore \sqrt[3]{34.2} = \frac{\sqrt[3]{342}}{\sqrt[3]{10}} = \frac{6.993}{2.154} = 3.246.$

Thus the required cube root is = 3.246.

23. Question

What is the length of the side of a cube whose volume is 275 cm^3. Make use of the table for the cube root.

Answer

Volume of cube = 275 cm^3 (Given)

Let side of cube = a cm

So,

$= a^3 = 275$

$= a = \sqrt[3]{275}$

We know that value of $\sqrt[3]{275}$ *will lie between* $\sqrt[3]{270}$ *and* $\sqrt[3]{280}$.

From cube root table we get,

$= \sqrt[3]{270} = 6.463$ *and* $\sqrt[3]{280} = 6.542$

So by unitary method,

∵ For difference (280 – 270 = 10) difference in cube root values = 6.542 – 6.463 = 0.079

∴ For difference (275 – 270 = 5) difference in cube root values = $\frac{0.079}{10} \times 5 = 0.0395 \simeq 0.04$

Hence, $\sqrt[3]{275} = 6.463 + 0.04 = 6.503.$

Thus the required cube root is = 6.503.