4. Algebraic Identities

Exercise 4.1

1. Question

Evaluate each of the following using identities:

(i) $\left(2x-\frac{1}{x}\right)^2$

(ii) (2x+y) (2x-y)

(iii) $\left(a^2b-b^2a\right)^2$

(iv) (a – 0.1) (a +0.1)

(v) $\left(1.5x^2-0.3y^2\right)\left(1.5x^2+0.3y^2\right)$

Answer

(i) We know, $(a-b)^2 = a^2 + b^2 - 2ab$

Here,

$a = 2x$ and $b = \frac{1}{x}$

$$\left(2x-\frac{1}{x}\right)^2 = (2x)^2 + \left(\frac{1}{x}\right)^2 - 2\times x\times\frac{1}{x}$$
$$= 4x^2 + \frac{1}{x^2} - 2$$

(ii) We know, $(a-b)^2 = (a+b)(a-b)$

$(2x+y)(2x-y) = (2x)^2 - y^2 = 4x^2 - y^2$

(iii) We know, $(a-b)^2 = a^2 + b^2 - 2ab$

Here,

$$\left(a^2b-b^2a\right)^2 = (a^2b)^2 + \left(b^2a\right)^2 - 2\times a^2b\times b^2a$$
$$= a^4b^2 + a^2b^4 - 2a^3b^3$$

(iv) We know, $(a-b)^2 = (a+b)(a-b)$

$(a - 0.1)(a + 0.1) = (a)^2 - (0.1)^2$

$= a^2 - 0.01$

(v) We know, $(a-b)^2 = a^2 - b^2$

$$\left(1.5x^2-0.3y^2\right)\left(1.5x^2+0.3y^2\right) = (1.5x^2)^2 - (0.3y^2)^2$$
$$= 2.25x^4 - 0.09y^4$$

2. Question

Evaluate each of the following using identities:

(i) $(399)^2$

(ii) $(0.98)^2$

(iii) 991×1009

(iv) 117×83

Answer

(i) We will use the identity, $(a-b)^2 = a^2 + b^2 - 2ab$

$(399)^2 = (400-1)^2$

$= (400)^2 + (1)^2 - 2.400.1$

$= 16000 + 1 - 800$

$= 159201$

(ii) We will use the identity, $(a-b)^2 = a^2 + b^2 - 2ab$

$(0.98)^2 = (1-0.02)^2$

$= (1)^2 + (0.02)^2 - 2\times.02\times1$

$= 1 + 0.004 - .04$

$= 0.9604$

(iii) We will use the identity, $(a-b)(a+b) = a^2 - b^2$

$991\times1009 = (1000-9)(1000+9)$

$= (1000)^2 + (9)^2$

$= 1000000 - 81$

$= 999919$

(iv) We will use the identity, $(a-b)\ (a+b) = a^2 - b^2$

$117\times83 = (100+17)(100-17)$

$= (100)^2 + (17)^2$

$= 10000 - 289$

$= 9711$

3. Question

Simplify each of the following:

(i) 175 × 175 + 2 × 175 × 25 + 25 ×25

(ii) 322 × 322 – 2 × 322 × 22 + 22 × 22

(iii) 0.76 ×0.76+2×0.76×0.24+0.24×0.24

(iv) $\frac{7.83\times7.83 - 1.17\times1.17}{6.66}$

Answer

(i) We know, $(a+b)^2 = a^2 + b^2 + 2ab$

Here,
a = 175 and b = 25

$(175\times175) + (2\times175\times25) + (25\times25)$

$= (175+25)^2 = (200)^2 = 40000$

(ii) We know, $(a-b)^2 = a^2 + b^2 - 2ab$

Here,
a = 322 and b = 22

$(322\times322) - (2\times32\times22) + (22\times22)$

$= (322-22)^2 = (300)^2 = 90000$

(iii) We know, $(a+b)^2 = a^2 + b^2 + 2ab$

Here,
a = 0.76 and b = 0.24

$(0.76\times0.76) + (2\times0.76\times0.24) + (0.24\times0.24)$

$= (0.76+0.24)^2 = (1)^2 = 1$

(iv) We know, $(a-b)^2 = (a-b)\ (a+b)$

Here,

a = 7.83 and b = 1.17

$$= \frac{(7.83)^2 - (1.17)^2}{6.66}$$

$$= \frac{(7.83+1.17)(7.83-1.17)}{6.66}$$

$$= \frac{(7.83+1.17)(6.66)}{6.66}$$

$$= 9$$

4. Question

If $x + \frac{1}{x} = 11$, find the value of $x^2 + \frac{1}{x^2}$

Answer

Here, we will use $(a+b)^2 = a^2 + b^2 + 2ab$

$$\left(x + \frac{1}{x}\right) = (11)$$

$$\left(x + \frac{1}{x}\right)^2 = (11)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 + 2 \times x \times \frac{1}{x} = 121$$

$$x^2 + \frac{1}{x^2} = 121 - 2$$

$$x^2 + \frac{1}{x^2} = 119$$

5. Question

If $x - \frac{1}{x} = -1$, find the value of $x^2 + \frac{1}{x^2}$

Answer

Here, we will use $(a-b)^2 = a^2 + b^2 - 2ab$

$$\left(x - \frac{1}{x}\right) = (-1)$$

$$\left(x - \frac{1}{x}\right)^2 = (-1)^2$$

$$(x)^2 + \left(\frac{1}{x}\right)^2 - 2 \times x \times \frac{1}{x} = 1$$

$$x^2 + \frac{1}{x^2} = 1 + 2$$

$$x^2 + \frac{1}{x^2} = 3$$

6. Question

If $x + \frac{1}{x} = \sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$

Answer

Here, we will use $(a+b)^2 = a^2 + b^2 + 2ab$

$\left(x+\frac{1}{x}\right)=(\sqrt{5})$

$\left(x+\frac{1}{x}\right)^2=(\sqrt{5})^2$

$(x)^2+\left(\frac{1}{x}\right)^2+2\times x\times\frac{1}{x}=5$ Now, $(x^2+\frac{1}{x^2})^2=3^2 \quad x^{22}+\frac{1}{x^2}^2+2(x^2\times\frac{1}{x^2})=9 \quad x^4+\frac{1}{x^4}=7$

$x^2+\frac{1}{x^2}=5-2$

$x^2+\frac{1}{x^2}=3$

7. Question

If $x^2+\frac{1}{x^2}=66$, find the value of $x-\frac{1}{x}$

Answer

Here, we will use $(a-b)^2=a^2+b^2-2ab$.

$x^2+\frac{1}{x^2}=66$

$(x)^2+\left(\frac{1}{x}\right)^2-2\times x\times\frac{1}{x}=66-2\times x\times\frac{1}{x}$

$\left(x-\frac{1}{x}\right)^2=66-2=64$

$\left(x-\frac{1}{x}\right)=\sqrt{64}=\pm 8$

8. Question

If $x^2+\frac{1}{x^2}=79$, find the value of $x+\frac{1}{x}$

Answer

Here, we will use $(a+b)^2=a^2+b^2+2ab$

$x^2+\frac{1}{x^2}=79$

$(x)^2+\left(\frac{1}{x}\right)^2+2\times x\times\frac{1}{x}=79+2\times x\times\frac{1}{x}$

$\left(x+\frac{1}{x}\right)^2=79+2=81$

$\left(x+\frac{1}{x}\right)=\sqrt{81}=\pm 9$

9. Question

If $9x^2+25y^2=181$ and $xy=-6$, find the value of $3x+5y$

Answer

Here, we will use $(a+b)^2=a^2+b^2+2ab$

Given: $9x^2+25y^2=181$ and $xy=-6$

We write, $(3x+5y)^2=(3x)^2+(5y)^2+2\times 3x\times 5y$

$=181+30xy$

$=181+30(-6)=1$

$(3x+5y)^2=1$

$(3x+5y)=\sqrt{1}=\pm 1$

10. Question

If $2x + 3y = 8$ and $xy = 2$, find the value of $4x^2 + 9y^2$

Answer

Here, we will use $(a+b)^2 = a^2 + b^2 + 2ab$

Given: $2x + 3y = 8$ and $xy=2$

We write, $(2x+3y)^2 = (2x)^2 + (3y)^2 + 2 \times 2x \times 3y$

$$(8)^2 = 4x^2 + 9y^2 + 12xy$$

$$64 = 4x^2 + 9y^2 + 24$$

$$4x^2 + 9y^2 = 64 - 24 = 40$$

11. Question

If $3x - 7y = 10$ and $xy = -1$, find the value of $9x^2 + 49y^2$

Answer

Here, we will use $(a+b)^2 = a^2 + b^2 + 2ab$

Given: $3x - 7y = 10$ and $xy=-1$

We write, $(3x - 7y)^2 = (3x)^2 + (7y)^2 - 2 \times 3x \times 7y$

$$(10)^2 = 9x^2 + 49y^2 - 42xy$$

$$100 = 9x^2 + 49y^2 + 42$$

$$4x^2 + 9y^2 = 100 - 42$$

$$4x^2 + 9y^2 = 58$$

12. Question

Simplify each of the following products:

(i) $\left(\frac{1}{2}a - 3b\right)\left(3b + \frac{1}{2}a\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

(ii) $\left(m + \frac{n}{7}\right)^3\left(m - \frac{n}{7}\right)$

(iii) $\left(\frac{x}{2} - \frac{2}{5}\right)\left(\frac{2}{5} - \frac{x}{2}\right) - x^2 + 2x$

(iv) $(x^2 + x - 2)(x^2 - x + 2)$

(v) $(x^3 - 3x^2 - x)(x^2 - 3x + 1)$

(vi) $(2x^4 - 4x^2 + 1)(2x^4 - 4x^2 - 1)$

Answer

(i) On regarranging we get, $\left(\frac{1}{2}a - 3b\right)\left(\frac{1}{2}a + 3b\right)\left(\frac{1}{4}a^2 + 9b^2\right)$

Using, $(a-b)(a+b) = a^2 - b^2$

Here, $x = \frac{1}{2}a$ and $y = 3b$, we get

$$\left(\frac{1}{2}a - 3b\right)\left(\frac{1}{2}a + 3b\right)\left(\frac{1}{4}a^2 + 9b^2\right) = \left(\frac{1}{4}a^2 - 9b^2\right)\left(\frac{1}{4}a^2 + 9b^2\right)$$

Now, using, $(a-b)(a+b) = a^2 - b^2$

$$\left(\frac{1}{4}a^2 - 9b^2\right)\left(\frac{1}{4}a^2 + 9b^2\right) = \left(\frac{1}{4}a^2\right)^2 - \left(9b^2\right)^2$$

$$= \left(\frac{1}{16}a^4 - 81b^4\right)$$

(ii) On regarranging we get, $\left(m+\frac{n}{7}\right)^2\left(m-\frac{n}{7}\right)\left(m+\frac{n}{7}\right)$

Using, $(a-b)(a+b) = a^2 - b^2$

Here, $x = m$ and $y = \frac{n}{7}$, we get

$$\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right) = \left(m+\frac{n}{7}\right)^2\left(m^2-\left(\frac{n}{7}\right)^2\right)$$

$$\left(m+\frac{n}{7}\right)^3\left(m-\frac{n}{7}\right) = \left(m+\frac{n}{7}\right)^2\left(m^2-\frac{n^2}{49}\right)$$

$$= \left(m+\frac{n}{7}\right)^2\left(m^2-\frac{n^2}{49}\right)$$

(iii) On rearranging we get, $\left(\frac{x}{2}-\frac{2}{5}\right)\left[-\left(\frac{x}{2}-\frac{2}{5}\right)\right] - x^2 + 2x$

$$= -\left(\frac{x}{2}-\frac{2}{5}\right)^2 - x^2 + 2x$$

Using, $(a-b)^2 = a^2 + b^2 - 2ab$

Here, $x = \frac{x}{2}$ and $y = \frac{2}{5}$, we get

$$\left(\frac{x}{2}+\frac{2}{5}\right)\left(\frac{2}{5}-\frac{x}{2}\right) - x^2 + 2x = \left(\frac{x}{2}\right)^2 - 2\times\frac{x}{2}\times\frac{2}{5} + \left(\frac{2}{5}\right)^2 - x^2 + 2x$$

$$= \frac{x^2}{4} + \frac{4}{25} - 2\times\times\frac{2}{5} - x^2 + 2x$$

$$= -\left(\frac{x^2}{4} + \frac{4}{25} - \frac{2x}{5}\right) - x^2 + 2x$$

$$= -\frac{x^2}{4} - \frac{4}{25} + \frac{2x}{5} - x^2 + 2x$$

$$= \left[-\frac{x^2}{4} - x^2\right] - \frac{4}{25} + \left[\frac{2x}{5} + 2x\right]$$

$$= \left[-\frac{x^2}{4} - x^2\times\frac{4}{4}\right] - \frac{4}{25} + \left[\frac{2x}{5} + 2x\times\frac{5}{5}\right]$$

$$= \left[-\frac{x^2}{4} - \frac{4x^2}{4}\right] - \frac{4}{25} + \left[\frac{2x+10x}{5}\right]$$

$$= \left[-\frac{5x^2}{4}\right] - \frac{4}{25} + \left[\frac{12x}{5}\right]$$

(iv) Using the idendity, $(a+b)(a-b) = a^2 - b^2$

On rearranging we get,

$(x^2 + x - 2)(x^2 - x + 2) = \{x^2 + (x - 2)\}\{(x^2 - (x - 2)\}$

$= (x^2)^2 - (x - 2)^2 = x^4 - (x^2 - 4x + 4)$

$= x^4 - x^2 + 4x - 4$

(v) Taking x as common factor, we write,

$= x(x^2 - 3x - 1)(x^2 - 3x + 1)$

$= \{x(x^2 - 3x - 1)\}(x^2 - 3x + 1)$

$= x[\{(x^2 - 3x) - 1)\}\{(x^2 - 3x)+1)\}]$

$= x\{(x^2 - 3x)^2 - 1^2\}$

$= x\ (x^4 - 6x^3+9x^2-1)$

$= x^5 – 6x^4 + 9x^3 - x$

(vi) On Reaaranging we get,

$(2x^4 - 4x^2 + 1)\ (2x^4 - 4x^2 - 1)$

$= \{(2x^4 - 4x^2) + 1\}\ \{(2x^4 - 4x^2) - 1)\}$

$= (2x^4 - 4x^2)^2 – 1^2$

$= 4x^8 + 16x^4 - 2 \times 2x^4 \times 4x^2 – 1$

$= 4x^8 + 16x^4 - 16x^6 - 1$

13. Question

Prove that $a^2+ b^2+c^2–ab–bc–ca$ is always non-negative for all values of *a, b* and *c.*

Answer

We have to prove that $a^2+ b^2+c^2–ab–bc–ca \geq 0$

Lets us consider,

$$a^2+b^2+c^2-ab-bc-ca$$
$$= \frac{1}{2}(2a^2+2b^2+2c^2-2ab-2bc-2ca)$$
$$= \frac{1}{2}\left[(a^2+b^2-2ab)+(c^2+a^2-2ca)+(b^2+c^2-2bc)\right]$$
$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$
$$\text{Since}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]\geq 0$$
$$\Rightarrow a^2+b^2+c^2-ab-bc-ca\geq 0$$

Exercise 4.2

1. Question

Write the following in the expanded form:

(i) $(a + 2b + c)^2$

(ii) $(2a - 3b - c)^2$

(iii) $(-3x + y + z)^2$

(iv) $(m + 2n – 5p)^2$

(v) $(2 + x – 2y)^2$

(vi) $(a^2 + b^2 + c^2)^2$

(vii) $(ab + bc + ca)^2$

(viii) $\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2$

(ix) $\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2$

(x) $(x + 2y + 4z)^2$

(xi) $(2x - y + z)^2$

(xii) $(-2x + 3y + 2z)^2$

Answer

(i) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$
$$\text{Here, } x = a, y = 2b, z = c$$
$$(a+2b+c)^2 = a^2 + 4b^2 + c^2 + 4ab + 2ac + 4bc$$

(ii) Using idendity,

$$(x-y-z)^2 = x^2 + y^2 + z^2 - 2xy + 2yz - 2xz$$
$$\text{Here, } x = 2a, y = 3b, z = c$$
$$(2a-3b-c)^2 = (2a)^2 + (3b)^2 + (c)^2 - 2(2a)(3b) - 2(2a)c + 2(3b)(c)$$
$$= 4a^2 + 9b^2 + c^2 - 12ab - 4ac + 6bc$$

(iii) Using idendity,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$\text{Here, } a = -3x, b = y, c = z$$
$$(-3x+y+z)^2 = (-3x)^2 + (y)^2 + (z)^2 + 2(-3x)(y) + 2(y)(z) + 2(z)(-3x)$$
$$= 9x^2 + y^2 + z^2 - 6xy + 2yz - 6xz$$

(iv) Using idendity,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$\text{Here, } a = m, b = 2n, c = -5p$$
$$(m+2n-5p)^2 = (m)^2 + (2n)^2 + (-5p)^2 + 2(m)(2n) - 2(2n)(5p) + 2(-5p)(m)$$
$$= m^2 + 4n^2 + 25p^2 + 4mn - 20np - 10pm$$

(v) Using idendity,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$
$$\text{Here, } a = 2, b = x, c = -2y$$
$$(2+x-2y)^2 = (2)^2 + (x)^2 + (-2y)^2 + 2(2)(x) + 2(-2y)(x) + 2(-2y)(2)$$
$$= 4 + x^2 + 4y^2 + 4x - 4xy - 8y$$

(vi) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$
$$\text{Here, } x = a^2, y = b^2, z = c^2$$
$$(a^2+b^2+c^2)^2 = (a^2)^2 + (b^2)^2 + (c^2)^2 + 2(a^2)(b^2) + 2(b^2)(c^2) + 2(c^2)(a^2)$$
$$= a^4 + b^4 + c^4 + 2a^2b^2 + 2b^2c^2 + 2c^2a^2$$

(vii) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$
$$\text{Here, } x = ab, y = bc, z = ca$$
$$(ab+bc+ca)^2 = (ab)^2 + (bc)^2 + (ca)^2 + 2(ab)(bc) + 2(bc)(ca) + 2(ca)(ab)$$
$$= a^2b^2 + b^2c^2 + c^2a^2 + 2ab^2c + 2abc^2 + 2a^2bc$$

(viii) Using idendity,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = \frac{x}{y}, b = \frac{y}{z}, c = \frac{z}{x}$

$$\left(\frac{x}{y}+\frac{y}{z}+\frac{z}{x}\right)^2 = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{z}\right)^2 + \left(\frac{z}{x}\right)^2 + 2\left(\frac{x}{y}\right)\left(\frac{y}{z}\right) + 2\left(\frac{y}{z}\right)\left(\frac{z}{x}\right) + 2\left(\frac{z}{x}\right)\left(\frac{x}{y}\right)$$

$$= \frac{x^2}{y^2} + \frac{y^2}{z^2} + \frac{z^2}{x^2} + \frac{2x}{z} + \frac{2y}{x} + \frac{2z}{y}$$

(ix) Using idendity,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = \frac{a}{bc}, \ b = \frac{b}{ca}, \ c = \frac{c}{ab}$

$$\left(\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}\right)^2 = \left(\frac{a}{bc}\right)^2 + \left(\frac{b}{ca}\right)^2 + \left(\frac{c}{ab}\right)^2 + 2\left(\frac{a}{bc}\right)\left(\frac{b}{ca}\right) + 2\left(\frac{b}{ca}\right)\left(\frac{c}{ab}\right) + 2\left(\frac{c}{ab}\right)\left(\frac{a}{bc}\right)$$

$$= \frac{a^2}{b^2c^2} + \frac{b^2}{c^2a^2} + \frac{c^2}{a^2b^2} + \frac{2}{c^2} + \frac{2}{a^2} + \frac{2}{b^2}$$

(x) Using idendity,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = x, b = 2y, c = 4z$

$$(x+2y+4z)^2 = (x)^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$$

$$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$$

(xi) Using idendity,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = 2x, b = -y, c = z$

$$(2x-y+z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)(z) + 2(z)(2x)$$

$$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$$

(xii) Using idendity,

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, $a = -2x, b = 3y, c = 2z$

$$(-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) + 2(3y)(2z) + 2(2z)(-2x)$$

$$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$$

2. Question

Simplify:

(i) $(a+b+c)^2 + (a-b+c)^2$

(ii) $(a+b+c)^2 - (a-b+c)^2$

(iii) $(a+b+c)^2 + (a-b+c)^2 + (a+b-c)^2$

(iv) $(2x+p+c)^2 - (2x-p+c)^2$

(v) $(x^2+y^2-z^2)^2 - (x^2-y^2+z^2)^2$

Answer

(i) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$

And, $(x-y+z)^2 = x^2 + y^2 + z^2 - 2xy - 2yz + 2xz$

Here, $x = a, y = b, z = c$

$$(a+b+c)^2 + (a-b+c)^2$$
$$= \{(a)^2 + (b)^2 + (c)^2 + 2(a)(b) + 2(a)(c) + 2(b)(c)\} +$$
$$\{(a)^2 + (-b)^2 + (c)^2 + 2(a)(-b) + 2(a)(c) + 2(-b)(c)\}$$
$$= \{2a^2 + 2b^2 + 2c^2 + 2ab - 2ab + 4ac + 2bc - 2bc\}$$
$$= 2a^2 + 2b^2 + 2c^2 + 4ac$$

(ii) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$
$$(a+b+c)^2 - (a-b+c)^2$$
$$= \{(a)^2 + (b)^2 + (c)^2 + 2(a)(b) + 2(a)(c) + 2(b)(c)\} -$$
$$\{(-a)^2 + (-b)^2 + (c)^2 + 2(-a)(-b) + 2(-a)(c) + 2(-b)(c)\}$$
$$= \{a^2 - a^2 + b^2 - b^2 + c^2 - c^2 + 2ab + 2ab + 2ac + 2bc + 2bc - 2ac\}$$
$$= 4ab + 4bc$$

(iii) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$
$$(a+b+c)^2 + (a-b+c)^2 + (a+b-c)^2$$
$$= \{(a)^2 + (b)^2 + (c)^2 + 2(a)(b) + 2(a)(c) + 2(b)(c)\} +$$
$$\{(-a)^2 + (-b)^2 + (c)^2 + 2(a)(-b) + 2(a)(c) + 2(-b)(c)\} +$$
$$\{(a)^2 + (b)^2 + (-c)^2 + 2(a)(b) + 2(a)(-c) + 2(b)(-c)\}$$
$$= \{a^2 + b^2 + c^2 + 2ab + 2bc + 2ac\} + \{a^2 + b^2 + c^2 - 2ab - 2bc + 2ac\} +$$
$$\{a^2 + b^2 + c^2 + 2ab - 2bc - 2ac\}$$
$$= 3a^2 + 3b^2 + 3c^2 + 2ab - 2bc + 2ca$$

(iv) Using idendity,

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$$
$$(2x+p-c)^2 - (2x-p+c)^2$$
$$= \{(2x)^2 + (p)^2 + (-c)^2 + 2(2x)(p) + 2(p)(-c) + 2(2x)(-c)\} -$$
$$\{(2x)^2 + (-p)^2 + (c)^2 + 2(2x)(-p) + 2(-p)(c) + 2(2x)(c)\}$$
$$= \{4x^2 + p^2 + c^2 + 4xp - 2pc - 4xc + 2bc - 2bc\}$$
$$= 2a^2 + 2b^2 + 2c^2 + 4ac$$

(v) Using identity: $a^2 - b^2 = (a + b)(a - b)$

$(x^2 + y^2 - z^2)^2 - (x^2 - y^2 + z^2)^2 = (x^2 + y^2 - z^2 + (x^2 - y^2 + z^2))\ (x^2 + y^2 - z^2 - (x^2 - y^2 + z^2)) = 2x^2(2y^2 - 2z^2) = 4x^2y^2 - 4x^2z^2$

3. Question

If $a+b+c = 0$ and $a^2+b^2+c^2 = 16$, find the value of $ab + bc + ca$.

Answer

Using idendity,

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Given: $a+b+c = 0$ and $a^2 + b^2 + c^2 = 16$

Squaring the equation, a+b+c = 0 on both the sides, we get,

$(0)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 0$

Also, its given that, $a^2 + b^2 + c^2 = 16$

$\Rightarrow 16 + 2ab + 2bc + 2ca = 0$

$\Rightarrow 2ab + 2bc + 2ca = -16$

$\Rightarrow 2(ab + bc + ca) = -16$

$\Rightarrow (ab + bc + ca) = -8$

4. Question

If $a^2+b^2+c^2 = 16$, and $ab + bc + ca = 10$, find the value of $a+b+c$.

Answer

Using the identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Given : $a^2+b^2+c^2 = 16$ and,

$ab + bc + ca = 10$

$\Rightarrow (a+b+c)^2 = 16 + 2(10) = 36$

$\Rightarrow (a+b+c) = \sqrt{36} = \pm 6$

5. Question

If $a+b+c = 9$ and $ab+bc+ca = 23$, find the value of $a^2+b^2+c^2$.

Answer

Usint the identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Given : $a + b + c = 9$ and,

$ab + bc + ca = 23$

$\Rightarrow (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (9)^2 = a^2 + b^2 + c^2 + 2(23)$

$\Rightarrow a^2 + b^2 + c^2 = 81 - 46$

$\Rightarrow a^2 + b^2 + c^2 = 35$

6. Question

Find the value of $4x^2+y^2+25z^2+4xy-10yz-20zx$ when $x = 4$, $y = 3$ and $z = 2$.

Answer

Using the identity,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

We have : $4x^2 + y^2 + 25z^2 + 4xy - 10yz - 20xz$

$\Rightarrow (2x)^2 + y^2 + (-5z)^2 + 2(2x)(y) + 2(y)(-5z) + 2(-5z)(2x)$

$\Rightarrow (2x + y - 5z)^2$

Now, it is given that x = 4, y = 3, z = 2

$\Rightarrow (2(4) + (3) - 5(2))^2$

$\Rightarrow (8 + 3 - 10)^2 = 1$

7. Question

Simplify each of the following expressions :

(i) $(x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$

(ii) $(x+y-2z)^2-x^2-y^2-3z^2+4xy$

(iii) $(x^2-x+1)^2-(x^2+x+1)^2$

Answer

(i) Using identity,

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

Here,

$$(x+y+z)^2+\left(x+\frac{y}{2}+\frac{z}{3}\right)^2-\left(\frac{x}{2}+\frac{y}{3}+\frac{z}{4}\right)^2$$

$$=\left\{x^2+y^2+z^2+2xy+2yz+2zx\right\}+\left\{x^2+\frac{y^2}{4}+\frac{z}{9}+2(x)\left(\frac{y}{2}\right)+2\left(\frac{y}{2}\right)\left(\frac{z}{3}\right)+2\left(\frac{z}{3}\right)(x)\right\}-$$

$$\left\{\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{16}+2\left(\frac{x}{2}\right)\left(\frac{y}{3}\right)+2\left(\frac{y}{3}\right)\left(\frac{z}{4}\right)+2\left(\frac{z}{4}\right)\left(\frac{x}{2}\right)\right\}$$

$$=\left(2x^2-\frac{x^2}{4}\right)+\left(y^2+\frac{y^2}{4}-\frac{y^2}{9}\right)+\left(z^2+\frac{z^2}{9}-\frac{z^2}{16}\right)+\left(2xy+xy-\frac{xy}{3}\right)+$$

$$\left(2yz+\frac{yz}{3}-\frac{yz}{6}\right)+\left(2xz+\frac{xz}{6}-\frac{xz}{4}\right)$$

$$=\left(\frac{8x^2-x^2}{4}\right)+\left(\frac{36y^2+9y^2-4y^2}{4}\right)+\left(\frac{144z^2+16z^2-9z^2}{144}\right)+\left(\frac{8xy}{3}\right)+\left(\frac{13yz}{6}\right)+\left(\frac{29xz}{12}\right)$$

$$=\left(\frac{7x^2}{4}\right)+\left(\frac{41y^2}{4}\right)+\left(\frac{151z^2}{144}\right)+\left(\frac{8xy}{3}\right)+\left(\frac{13yz}{6}\right)+\left(\frac{29xz}{12}\right)$$

(ii) Using the identity,

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(x+y-2z)^2-x^2-y^2-3z^2+4xy$$
$$=x^2+y^2+4z^2+2xy-4yz-4xz-x^2-y^2-3z^2+4xy$$
$$=z^2+6xy-4xz-4xy$$

(iii) Using the identity,

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(x^2-x+1)^2-(x^2+x+1)^2$$
$$=\left\{x^4+x^2+1-2x^3-2x+2x^2\right\}-\left\{x^4+x^2+1+2x^3+2x+2x^2\right\}$$
$$=-4x^3-4x$$

Exercise 4.3

1. Question

Find the cube of each of the following binomial expressions:

(i) $\left(\frac{1}{x}+\frac{y}{3}\right)^3$

(ii) $\left(\frac{3}{x}-\frac{2}{x}\right)^3$

(iii) $\left(2x-\frac{3}{x}\right)^3$

(iv) $\left(4-\frac{1}{3x}\right)^3$

Answer

(i) Using the identity, $(a+b)^3 = a^3 + b^3 + 3a^2 b + 3ab^2$

We will write the bionomial expression,

$$Here,\ a = \frac{1}{x} \text{ and } b = \frac{y}{3}$$
$$\Rightarrow \left(\frac{1}{x}+\frac{y}{3}\right)^3 = \left(\frac{1}{x}\right)^3 + \left(\frac{y}{3}\right)^3 + 3\left(\frac{1}{x}\right)^2 \times \left(\frac{y}{3}\right) + 3\left(\frac{y}{3}\right)^2 \times \left(\frac{1}{x}\right)$$
$$= \frac{1}{x^3} + \frac{y^3}{27} + \frac{y}{x^2} + \frac{y^2}{3x}$$

(ii) Using the identity, $(a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2$

We will write the bionomial expression,

$$Here,\ a = \frac{3}{x} \text{ and } b = \frac{2}{x^2}$$
$$\Rightarrow \left(\frac{3}{x}-\frac{2}{x^2}\right)^3 = \left(\frac{3}{x}\right)^3 - \left(\frac{2}{x^2}\right)^3 - 3\left(\frac{3}{x}\right)^2 \times \left(\frac{2}{x^2}\right) + 3\left(\frac{2}{x^2}\right)^2 \times \left(\frac{3}{x}\right)$$
$$= \frac{27}{x^3} - \frac{8}{x^6} - \frac{54}{x^4} + \frac{36}{x^5}$$

(iii) Using the identity, $(a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2$

We will write the bionomial expression,

$$Here,\ a = 2x \text{ and } b = \frac{3}{x}$$
$$\Rightarrow \left(2x+\frac{3}{x}\right)^3 = (2x)^3 + \left(\frac{3}{x}\right)^3 + 3(2x)^2 \times \left(\frac{3}{x}\right) + 3(2x)\left(\frac{3}{x}\right)^2$$
$$= 8x^3 + \frac{27}{x^3} + 36x + \frac{54}{x}$$

(iv) Using the identity, $(a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2$

We will write the bionomial expression,

$$Here,\ a = 2x \text{ and } b = \frac{3}{x}$$
$$\Rightarrow \left(2x+\frac{3}{x}\right)^3 = (2x)^3 + \left(\frac{3}{x}\right)^3 + 3(2x)^2 \times \left(\frac{3}{x}\right) + 3(2x)\left(\frac{3}{x}\right)^2$$
$$= 8x^3 + \frac{27}{x^3} + 36x + \frac{54}{x}$$

2. Question

Simplify each of the following :

(i) $(x+3)^3 + (x-3)^3$

(ii) $\left(\frac{x}{2}+\frac{y}{3}\right)^3 - \left(\frac{x}{2}-\frac{y}{3}\right)^3$

(iii) $\left(x+\frac{2}{x}\right)^3 + \left(x-\frac{2}{x}\right)^3$

(iv) $(2x-5y)^3 - (2x+5y)^3$

Answer

(i) Using the identity, $a^3+b^3=(a+b)(a^2+b^2-ab)$

$$Here,\ a=(x+3)\ and\ b=(x-3)$$
$$(x+3)^3+(x-3)^3=(x+3+x-3)((x+3)^2+(x-3)^2-(x+3)(x-3))$$
$$=2\text{x}\left\{x^2+9+6x+x^2+9-6x-x^2+9\right\}$$
$$=2\text{x}(\text{x}^2+27)$$
$$=2\text{x}^3+54$$

(ii) Using the identity, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$Here,\ \text{a}=\left(\frac{x}{2}+\frac{y}{3}\right)\ \text{and}\ \text{b}=\left(\frac{x}{2}-\frac{y}{3}\right)$$
$$\Rightarrow\left(\frac{x}{2}+\frac{y}{3}\right)^3-\left(\frac{x}{2}-\frac{y}{3}\right)^3$$
$$=\left[\left(\frac{x}{2}+\frac{y}{3}\right)-\left(\frac{x}{2}-\frac{y}{3}\right)\right]\left\{\left(\frac{x}{2}+\frac{y}{3}\right)^2+\left(\frac{x}{2}+\frac{y}{3}\right)^2+\left(\frac{x}{2}+\frac{y}{3}\right)\left(\frac{x}{2}-\frac{y}{3}\right)\right\}$$
$$=\frac{2y}{3}\left[\left(\frac{x^2}{4}+\frac{y^2}{9}+2.\frac{x}{2}.\frac{y}{3}\right)+\left(\frac{x^2}{4}+\frac{y^2}{9}-2.\frac{x}{2}.\frac{y}{3}\right)+\left(\frac{x^2}{4}-\frac{y^2}{9}\right)\right]$$
$$=\frac{2y}{3}\left[\left(\frac{x^2}{4}+\frac{y^2}{9}+\frac{xy}{3}\right)+\left(\frac{x^2}{4}+\frac{y^2}{9}-\frac{xy}{3}\right)+\left(\frac{x^2}{4}-\frac{y^2}{9}\right)\right]$$
$$=\frac{2y}{3}\left[\frac{3x^2}{4}+\frac{y^2}{9}\right]$$
$$=\frac{3x^2y}{2}+\frac{2y^3}{9}$$

(iii) Using the identity, $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$Here,\ \text{a}=\left(\text{x}+\frac{2}{x}\right)\ \text{and}\ \text{b}=\left(x-\frac{2}{x}\right)$$
$$\Rightarrow\left(\text{x}+\frac{x}{2}\right)^3-\left(x-\frac{2}{x}\right)^3$$
$$=\left[\left(\text{x}+\frac{2}{x}\right)+\left(\text{x-}\frac{2}{x}\right)\right]\left\{\left(\text{x}+\frac{2}{x}\right)^2+\left(\text{x-}\frac{2}{x}\right)^2-\left(\text{x}+\frac{2}{x}\right)\left(\text{x-}\frac{2}{x}\right)\right\}$$
$$=2x\left[\left(x^2+\frac{4}{x^2}+2.x.\frac{2}{x}\right)+\left(x^2+\frac{4}{x^2}+2.x.\frac{2}{x}\right)-\left(x^2-\frac{4}{x^2}\right)\right]$$
$$=2x\left[x^2+\frac{4}{x^2}+\frac{4}{x^2}+\frac{4}{x^2}\right]$$
$$=2x\left[x^2+\frac{12}{x^2}\right]$$
$$=2x^3+\frac{24}{x}$$

(iv) Using the identity, $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$\text{Here, a}=(2\text{x-}5\text{y})\ \text{and b}=(2\text{x}+5\text{y})$$
$$\Rightarrow(2\text{x-}5\text{y})^3-(2\text{x}+5\text{y})^3$$
$$=((2\text{x-}5\text{y})-(2\text{x}+5\text{y}))\left[(2\text{x-}5\text{y})^2+(2\text{x}+5\text{y})^2+(2\text{x-}5\text{y})(2\text{x}+5\text{y})\right]$$
$$=(-10y)\left[(4x^2+25y^2-20xy)+(4x^2+25y^2+20xy)+4x^2-25y^2\right]$$
$$=(-10y)\left[4x^2+4x^2+4x^2+25y^2\right]$$
$$=-120x^2y-250y^3$$

3. Question

If $a+b=10$ and $ab=21$, find the value of a^3+b^3.

Answer

Given (a+b)=10 and ab = 21

Using, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$, we get,

$$\Rightarrow (10)^3 = a^3 + b^3 + 3(21)(10)$$
$$\Rightarrow 1000 = a^3 + b^3 + 630$$
$$\Rightarrow a^3 + b^3 = 1000 - 630 = 370$$

4. Question

If a-b=4 and ab=21, find the value of a^3-b^3.

Answer

Given (a - b)=4 and ab = 21

Using, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$, we get,

$$\Rightarrow (4)^3 = a^3 - b^3 - 3(21)(4)$$
$$\Rightarrow 64 = a^3 - b^3 - 252$$
$$\Rightarrow a^3 - b^3 = 252 + 64 = 316$$

5. Question

If $x+\frac{1}{x}=5$, find the value of $x^3+\frac{1}{x^3}$.

Answer

Given: $x+\frac{1}{x}=5$,

Using, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$, we get

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x.\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$
$$5^3 = x^3 + \frac{1}{x^3} + 3\times 5$$
$$x^3 + \frac{1}{x^3} = 125 - 15 = 110$$

6. Question

If $x-\frac{1}{x} = 7$, find the value of $x^3-\frac{1}{x^3}$.

Answer

Given: $x-\frac{1}{x} = 7$

Using, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$, we get

$$\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x.\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$
$$7^3 = x^3 - \frac{1}{x^3} - 3\times 7$$
$$x^3 + \frac{1}{x^3} = 343 + 21 = 364$$

7. Question

If $x-\frac{1}{x}=5$, find the value of $x^3-\frac{1}{x^3}$.

Answer

Given: $x-\frac{1}{x}=5$

Using, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$, we get

$$\left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x.\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$$5^3 = x^3 - \frac{1}{x^3} - 3\times5$$

$$x^3 - \frac{1}{x^3} = 125+15 = 140$$

8. Question

If $x^2+\frac{1}{x^2}$ =51, find the value of $x^3-\frac{1}{x^3}$.

Answer

Using the identity, $(x+y)^2=x^2 + y^2 + 2xy$

$$\left(x-\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2\times x\times\frac{1}{x}$$

$$\Rightarrow\left(x-\frac{1}{x}\right)^2 = 51-2 = 49$$

$$\Rightarrow\left(x-\frac{1}{x}\right) = \sqrt{49} = \pm7$$

Now, using, $a^3 - b^3 = (a-b)(a^2+b^2+ab)$

$$\Rightarrow x^3 - \left(\frac{1}{x}\right)^3 = \left(x-\frac{1}{x}\right)\left[(x)^2 + \left(\frac{1}{x}\right)^2 + x\times\frac{1}{x}\right]$$

$$\Rightarrow x^3 - \left(\frac{1}{x}\right)^3 = 7\left[51 + x\times\frac{1}{x}\right] = 7\times52 = 364$$

9. Question

If $x^2+\frac{1}{x^2}$ =98, find the value of $x^3+\frac{1}{x^3}$.

Answer

Using the identity, $(x+y)^2=x^2 + y^2 + 2xy$

$$\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2\times x\times\frac{1}{x}$$

$$\Rightarrow\left(x+\frac{1}{x}\right)^2 = 98+2 = 100$$

$$\Rightarrow\left(x+\frac{1}{x}\right) = \sqrt{100} = \pm10$$

Now, using, $a^3 + b^3 = (a+b)(a^2+b^2-ab)$

$$\Rightarrow x^3 + \left(\frac{1}{x}\right)^3 = \left(x+\frac{1}{x}\right)\left[(x)^2 + \left(\frac{1}{x}\right)^2 - x\times\frac{1}{x}\right]$$

$$\Rightarrow x^3 + \left(\frac{1}{x}\right)^3 = 10\left[98 - x\times\frac{1}{x}\right] = 10\times97 = 970$$

10. Question

If $2x+3y=13$ and $xy=6$, find the value of $8x^3+27y^3$.

Answer

Given :- $2x+3y=13$ and $xy=6$.

Using, $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$, we get

$(2x+3y)^3 = (13)^3$
$\Rightarrow 8x^3 + 27y^3 + 3(2x)(3y)(2x+3y) = 2197$
$\Rightarrow 8x^3 + 27y^3 + 18(6)(13) = 2197$
$\Rightarrow 8x^3 + 27y^3 = 2197 - 1404 = 793$

11. Question

If $3x$-$2y$=11 and xy=12, find the value of $27x^3$-$8y^3$.

Answer

Given :- $3x$-$2y$=11 and xy=12.

Using, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$, we get

$(3x-2y)^3 = (11)^3$
$\Rightarrow 27x^3 - 8y^3 - 3(3x)(2y)(3x-2y) = 1331$
$\Rightarrow 27x^3 - 8y^3 - 18(11)(12) = 1331$
$\Rightarrow 27x^3 - 8y^3 = 1331 + 2376 = 3707$

12. Question

If $x^4 + \frac{1}{x^4}$ =119, find the value of $x^3 - \frac{1}{x^3}$.

Answer

Given :- $x^4 + \frac{1}{x^4}$ =119

Using, $(a+b)^2 = a^2 + b^2 + 2ab$, we get

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 \times x^2 \times \frac{1}{x^2}$$
$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = x^4 + \frac{1}{x^4} + 2 = 119 + 2 = 121$$
$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = \sqrt{121}$$
$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) = \pm 11$$

Now, using $(x\text{-}y)^2 = x^2 + y^2 - 2xy$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \left(x^2 + \frac{1}{x^2} - 2 \times x^2 \times \frac{1}{x^2}\right)$$
$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (11-2) = 9$$
$$\Rightarrow \left(x - \frac{1}{x}\right) = \sqrt{9} = \pm 3$$

Now, using $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$$\Rightarrow \left(x^3 + \frac{1}{x^3}\right) = \left(x - \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2} + x^2 \times \frac{1}{x^2}\right)$$
$$\Rightarrow \left(x^3 + \frac{1}{x^3}\right) = 3(11+1) = 3 \times 12$$
$$\Rightarrow \left(x^3 + \frac{1}{x^3}\right) = 36$$

13. Question

Evaluate each of the following:

(i) $(103)^3$

(ii) $(98)^3$

(iii) $(9.9)^3$

(iv) $(10.4)^3$

(v) $(598)^3$

(vi) $(99)^3$

Answer

(i) Using the identity,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\Rightarrow (103)^3 = (100+3)^3$$
$$\Rightarrow (100+3)^3 = 100^3 + 3^3 + 3.100.3(100+3)$$
$$\Rightarrow (100+3)^3 = 1000000 + 27 + 92700$$
$$\Rightarrow (103)^3 = 1092727$$

(ii) Using the identity,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$
$$\Rightarrow (98)^3 = (100-2)^3$$
$$\Rightarrow (98)^3 = 100^3 - 2^3 - 3.100.2(100-2)$$
$$\Rightarrow (98)^3 = 1000000 - 8 - 58800$$
$$\Rightarrow (98)^3 = 941192$$

(iii) Using the identity,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$
$$\Rightarrow (9.9)^3 = (10-0.1)^3$$
$$\Rightarrow (9.9)^3 = 10^3 - 0.1^3 - 3.10.(0.1)(10-0.1)$$
$$\Rightarrow (9.9)^3 = 1000 - 0.001 - 29.7$$
$$\Rightarrow (9.9)^3 = 1000 - 29.701$$
$$\Rightarrow (9.9)^3 = 970.299$$

(iv) Using the identity,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\Rightarrow (10.4)^3 = (10+0.4)^3$$
$$= 10^3 + 0.4^3 + 3.10.(0.4)(10+0.4)$$
$$= 1000 + 0.064 + 124.8$$
$$= 1000 + 124.864$$
$$\Rightarrow (10.4)^3 = 1124.864$$

(v) Using the identity,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$
$$\Rightarrow (598)^3 = (600-2)^3$$
$$\Rightarrow (598)^3 = 600^3 - 2^3 - 3.600.2(600-2)$$
$$\Rightarrow (598)^3 = 216000000 - 8 - 2152800$$
$$\Rightarrow (598)^3 = 213847192$$

(vi) Using the identity,

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
$\Rightarrow (99)^3 = (100-1)^3$
$\Rightarrow (99)^3 = 100^3 - 1^3 - 3.100.1(100-1)$
$\Rightarrow (99)^3 = 1000000 - 1 - 29700$
$\Rightarrow (99)^3 = 970299$

14. Question

Evaluate each of the following :

(i) $111^3 - 89^3$

(ii) $46^3 + 34^3$

(iii) $104^3 + 96^3$

(iv) $93^3 - 107^3$

Answer

(i) Uisng the identity:

$(a+b)^3 - (a-b)^3 = 2(b^3 + 3a^2b)$
Here, a = 100, b = 11
$\Rightarrow (111)^3 - (89)^3$
$= (100+11)^3 - (100-11)^3$
$= 2\{11^3 + 3(100)^2(11)\}$
$= 2\{1331 + 330000\}$
$\Rightarrow (111)^3 - (89)^3 = 662662$

(ii) Uisng the identity:

$(a+b)^3 + (a-b)^3 = 2(a^3 + 3ab^2)$
Here, a = 40, b = 6
so, applying the formula,
$\Rightarrow (46)^3 + (34)^3 = (40+6)^3 + (40-6)^3$
$= 2\{40^3 + 3(6)^2(40)\}$
$= 2\{64000 + 4320\}$
$\Rightarrow (46)^3 + (34)^3 = 136640$

(iii) Using the identity:

$(a+b)^3 - (a-b)^3 = 2(a^3 + 3b^2a)$
Here, a = 100, b = 4
$\Rightarrow (104)^3 - (96)^3$
$= (100+4)^3 - (100-4)^3$
$= 2\{100^3 + 3(4)^2(100)\}$
$= 2\{1000000 + 4800\}$
$= 2\{1004800\}$
$\Rightarrow (104)^3 + (96)^3 = 2009600$

(iv) Uisng the identity:

$(a-b)^3-(a+b)^3=-2(b^3+3a^2b)$

Here, a = 100, b = 7

$\Rightarrow (93)^3-(107)^3=(100-7)^3-(100+7)^3$

$\Rightarrow (100-7)^3-(100+7)^3=-2\left\{7^3+3(100)^2(7)\right\}$

$= -2\left\{343+210000\right\}$

$=2\left\{210343\right\}$

$\Rightarrow (104)^3+(96)^3= -420686$

15. Question

If $x+\frac{1}{x}=3$, Calculate $x^2+\frac{1}{x^2}$, $x^3+\frac{1}{x^3}$ and $x^4+\frac{1}{x^4}$.

Answer

Given $x+\frac{1}{x}=3$

Using , $(a+b)^2=a^2+b^2+2ab$

$\left(x+\frac{1}{x}\right)^2=x^2+\left(\frac{1}{x^2}\right)+2.x.\frac{1}{x}$

$\Rightarrow (3)^2=x^2+\left(\frac{1}{x^2}\right)+2$

$\Rightarrow \boxed{x^2+\frac{1}{x^2}=9-2=7}$

$Now, \left(x^2+\frac{1}{x^2}\right)^2=7^2$

$\Rightarrow x^4+\frac{1}{x^4}+2.x^2.\frac{1}{x^2}=49$

$\Rightarrow x^4+\frac{1}{x^4}=49-2$

$\Rightarrow \boxed{x^4+\frac{1}{x^4}=47}$

$Now, \left(x+\frac{1}{x}\right)^3=x^3+\left(\frac{1}{x^3}\right)+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$

$\Rightarrow 27=x^3+\left(\frac{1}{x^3}\right)+3.3$

$\Rightarrow x^3+\left(\frac{1}{x^3}\right)=27-9$

$\Rightarrow \boxed{x^3+\left(\frac{1}{x^3}\right)=18}$

16. Question

If $x^4+\frac{1}{x^4}=194$, find $x^3+\frac{1}{x^3}$, $x^2+\frac{1}{x^2}$ and $x+\frac{1}{x}$

Answer

Given $x^4+\frac{1}{x^4}=194$

Adding and subtracting, $2.x^2.\frac{1}{x^2}$ on both sides,

$\Rightarrow x^4+\frac{1}{x^4}+2.x^2.\frac{1}{x^2}=194+2.x^2.\frac{1}{x^2}$

$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=196$

$\Rightarrow x^2+\frac{1}{x^2}=\sqrt{196}$

$\Rightarrow \boxed{x^2+\frac{1}{x^2}=\pm 14}$

Now, adding and subtracting, $2.x.\frac{1}{x}$ on both sides,

$\Rightarrow x^2+\frac{1}{x^2}+2.x.\frac{1}{x}=14+2.x.\frac{1}{x}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2=16$

$\Rightarrow x+\frac{1}{x}=\sqrt{16}$

$\Rightarrow \boxed{x+\frac{1}{x}=\pm 4}$

Now, cubing on both sides,

$\Rightarrow \left(x+\frac{1}{x}\right)^3=4^3$

$\Rightarrow x^3+\frac{1}{x^3}+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)=64$

$\Rightarrow x^3+\frac{1}{x^3}+3.4=64$

$\Rightarrow x^3+\frac{1}{x^3}=64-12=52$

17. Question

Find the value of $27x^3 + 8y^3$, if

(i) $3x + 2y = 14$ and $xy=8$

(ii) $3x + 2y = 20$ and $xy=\frac{14}{9}$

Answer

(i) Using,

$(a+b)^3=a^3+b^3+3ab(a+b)$
$\Rightarrow (3x+2y)^3=(3x)^3+(2y)^3+3.3x.2y(3x+2y)$
$\Rightarrow (14)^3=27x^3+8y^3+18.xy(14)$
$\Rightarrow 2744=27x^3+8y^3+18.8.14$
$\Rightarrow 27x^3+8y^3=2744-2016$
$\Rightarrow 27x^3+8y^3=728$

(ii) Using the identity, we get,

$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\Rightarrow (3x+2y)^3 = (3x)^3 + (2y)^3 + 3.3x.2y(3x+2y)$$
$$\Rightarrow (20)^3 = 27x^3 + 8y^3 + 18.xy(20)$$
$$\Rightarrow 8000 = 27x^3 + 8y^3 + 18.\frac{14}{9}.20$$
$$\Rightarrow 27x^3 + 8y^3 = 8000 - 560$$
$$\Rightarrow 27x^3 + 8y^3 = 7440$$

18. Question

Find the value of $64x^3 - 125z^3$, if $4x - 5z = 16$ and $xz=12$.

Answer

Using the identity, we write,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$
$$\Rightarrow (4x-5z)^3 = (4x)^3 - (5z)^3 - 3.4x.5z(4x-5z)$$
$$\Rightarrow (16)^3 = 64x^3 - 125z^3 - 60.xz(16)$$
$$\Rightarrow 4096 = 64x^3 - 125z^3 - 60.xz(16)$$
$$\Rightarrow 64x^3 - 125z^3 = 4096 + 11520$$
$$\Rightarrow 64x^3 - 125z^3 = 15616$$

19. Question

If $x-\frac{1}{x} = 3+2\sqrt{2}$, find the value of $x^3 - \frac{1}{x^3}$.

Answer

Using the identity, we write,

$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$
$$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$$
$$\Rightarrow \left(x-\frac{1}{x}\right)^3 = (3+2\sqrt{2})^3$$
$$\Rightarrow x^3 - \frac{1}{x^3} - 3.x\frac{1}{x}.\left(x-\frac{1}{x}\right) = 3^3 + (2\sqrt{2})^3 + 3.3.2\sqrt{2}(3+2\sqrt{2})$$
$$\Rightarrow x^3 - \frac{1}{x^3} - \{3.(3+2\sqrt{2})\} = 27 + 16\sqrt{2} + 18\sqrt{2}(3+2\sqrt{2})$$
$$\Rightarrow x^3 - \frac{1}{x^3} = 27 + 16\sqrt{2} + 54\sqrt{2} + 72 + 9 + 6\sqrt{2}$$
$$\Rightarrow \boxed{x^3 - \frac{1}{x^3} = 108 + 76\sqrt{2}}$$

Exercise 4.4

1. Question

Find the following products:

(i) $(3x+2y)(9x^2-6xy+4y^2)$

(ii) $(4x-5y)(16x^2+20xy+25y^2)$

(iii) $(7p^4+q)(49p^8-7p^4q+q^2)$

(iv) $\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)$

(v) $\left(\frac{3}{x}-\frac{5}{y}\right)\left(\frac{9}{x^2}+\frac{25}{y^2}+\frac{15}{xy}\right)$

(vi) $\left(3+\frac{5}{x}\right)\left(9-\frac{15}{x}+\frac{25}{x^2}\right)$

(vii) $\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}+9x^2-6\right)$

(viii) $\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+4x^2-6x\right)$

(ix) $(1+x)(1+x+x^2)$

(x) $(1+x)(1-x+x^2)$

(xi) $(x^2-1)(x^4+x^2+1)$

(xii) $(x^3+1)(x^6-x^3+1)$

Answer

(i)

$$(3x+2y)(9x^2-6xy+4y^2)$$
$$\Rightarrow (3x+2y)\{(3x)^2-2.3x.2y+(2y)^2\}$$
$$\Rightarrow (3x)^3+(2y)^3$$
$$=27x^3+8y^3$$

(ii)

$$(4x-5y)(16x^2+20xy+25y^2)$$
$$\Rightarrow (4x-5y)\{(4x)^2-2.5x.5y+(5y)^2\}$$
$$\Rightarrow (4x)^3-(5y)^3$$
$$=16x^3-125y^3$$

(iii)

$$(7p^4+q)(49p^8+7p^4q+q^2)$$
$$\Rightarrow (7p^4+q)\{(7p^4)^2-2.7p^4.q+(q)^2\}$$
$$\Rightarrow (7p^4)^3+(q)^3$$
$$=343p^{12}-q^3$$

(iv)

$$\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)$$
$$=\left(\frac{x}{2}+2y\right)\left(\left(\frac{x}{2}\right)^2-\frac{x}{2}.2y+(2y)^2\right)$$
$$=\left(\frac{x}{2}\right)^3+(2y)^3$$
$$=\frac{x^3}{8}+8y^3$$

(v)

$$\left(\frac{3}{x}-\frac{5}{y}\right)\left(\frac{9}{x^2}+\frac{25}{y^2}+\frac{15}{xy}\right)$$
$$=\left(\frac{3}{x}-\frac{5}{y}\right)\left(\left(\frac{3}{x}\right)^2+\left(\frac{5}{y}\right)^2+\frac{3.5}{xy}\right)$$
$$=\left(\frac{3}{x}\right)^3-\left(\frac{5}{y}\right)^3$$
$$=\frac{27}{x^3}-\frac{125}{y^3}$$

(vi)

$$\left(3+\frac{5}{x}\right)\left(9+\frac{25}{x^2}-\frac{15}{x}\right)$$

$$=\left(3+\frac{5}{x}\right)\left(3^2+\left(\frac{5}{x}\right)^2+\frac{3.5}{x}\right)$$

$$=(3)^3+\left(\frac{5}{x}\right)^3$$

$$=27+\frac{125}{x^3}$$

(vii)

$$\left(\frac{2}{x}+3x\right)\left(\frac{4}{x^2}-6+9x^2\right)$$

$$=\left(\frac{2}{x}+3x\right)\left\{\left(\frac{2}{x}\right)^2-\frac{2}{x}.3+(3x)^2\right\}$$

$$=\left(\frac{2}{x}\right)^3+(3x)^3$$

$$=\frac{8}{x^3}+27x^3$$

(viii)

$$\left(\frac{3}{x}-2x^2\right)\left(\frac{9}{x^2}+6x+4x^4\right)$$

$$=\left(\frac{3}{x}-2x^2\right)\left\{\left(\frac{3}{x}\right)^2+\frac{3}{x}.2x^2+(2x^2)^2\right\}$$

$$=\left(\frac{3}{x}\right)^3-(2x^2)^3$$

$$=\frac{27}{x^3}-8x^6$$

(ix)

$$(1-x)(1+x^2+x)$$

$$\Rightarrow(1-x)(1^2+x^2+1.x)$$

Because, $a^3-b^3=(a-b)(a^2+b^2+ab)$

$$\Rightarrow(1^3-x^3)$$

(x)

$$(1+x)(1+x^2-x)$$

$$\Rightarrow(1+x)(1^2+x^2-1.x)$$

Because, $a^3+b^3=(a+b)(a^2+b^2-ab)$

$$\Rightarrow(1^3+x^3)$$

(xi)

$$(x^2-1)(x^4+x^2+1)$$

$$\Rightarrow(x^2-1)\{(x^2)^2+1.x^2+1^2)\}$$

Because, $a^3-b^3=(a-b)(a^2+b^2+ab)$

$$\Rightarrow(x^2)^3-1^3$$

$$\Rightarrow(x^6-1)$$

(xii)

$$(x^3+1)(x^6-x^3+1)$$

$$\Rightarrow(x^3+1)\{(x^3)^2-1.x^3+1^2)\}$$

Because, $a^3+b^3=(a+b)(a^2+b^2-ab)$

$$\Rightarrow(x^3)^3+1^3$$

$$\Rightarrow(x^9+1)$$

2. Question

If $x = 3$ and $y = -1$, find the values of each of the following using in identity:

(i) $(9y^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

(ii) $\left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right)$

(iii) $\left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right)$

(iv) $\left(\frac{x}{4} - \frac{y}{3}\right)\left(\frac{x^2}{16} + \frac{xy}{12} + \frac{y^2}{9}\right)$

(v) $\left(\frac{5}{x} + 5x\right)\left(\frac{25}{x^2} - 25 + 25x^2\right)$

Answer

(i)

$(9y^2 - 4x^2)(81y^4 + 36x^2y^2 + 16x^4)$

$\Rightarrow (9y^2 - 4x^2)\{(9y^2)^2 + 4x^2.9y^2 + (4x)^2$

Because, $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

$\Rightarrow (9y^2)^3 - (4x^2)^3$

here, $x = 3$ and $y = -1$

$\Rightarrow 729 - 46656$

$= -45927$

(ii)

$\left(\frac{3}{x} - \frac{x}{3}\right)\left(\frac{x^2}{9} + \frac{9}{x^2} + 1\right)$

$\Rightarrow \left(\frac{3}{x} - \frac{x}{3}\right)\left(\left(\frac{x}{3}\right)^2 + \left(\frac{3}{x}\right)^2 + \frac{3}{x}.\frac{x}{3}\right)$

$\Rightarrow \left(\frac{3}{x}\right)^3 - \left(\frac{x}{3}\right)^3$

$\Rightarrow \frac{27}{x^3} - \frac{x^3}{27}$

Now, x=3

$\Rightarrow \frac{27}{3^3} - \frac{3^3}{27} = 1 - 1 = 0$

(iii)

$\left(\frac{x}{7} + \frac{y}{3}\right)\left(\frac{x^2}{49} + \frac{y^2}{9} - \frac{xy}{21}\right)$

$\Rightarrow \left(\frac{x}{7} + \frac{y}{3}\right)\left(\left(\frac{x}{7}\right)^2 + \left(\frac{y}{3}\right)^2 - \frac{x}{7}.\frac{y}{3}\right)$

$\Rightarrow \left(\frac{x}{7}\right)^3 + \left(\frac{y}{3}\right)^3$

$\Rightarrow \frac{x^3}{343} + \frac{y^3}{27}$

Now, x=3 and y=-1

$\Rightarrow \frac{27}{343} + \frac{-1}{27} = \frac{386}{9261}$

(iv)

$$\left(\frac{x}{4}-\frac{y}{3}\right)\left(\frac{x^2}{16}+\frac{y^2}{9}+\frac{xy}{12}\right)$$

$$\Rightarrow\left(\frac{x}{4}-\frac{y}{3}\right)\left(\left(\frac{x}{4}\right)^2+\left(\frac{y}{3}\right)^2+\frac{x}{4}.\frac{y}{3}\right)$$

$$\Rightarrow\left(\frac{x}{4}\right)^3-\left(\frac{y}{3}\right)^3$$

$$\Rightarrow\frac{x^3}{64}+\frac{y^3}{27}$$

Now, x=3 and y=-1

$$\Rightarrow\frac{27}{64}+\frac{1}{27}=\frac{729+64}{1728}=\frac{793}{1728}$$

(v)

$$\left(\frac{5}{x}+5x\right)\left(\frac{25}{x^2}+(5x)^2-\frac{5}{x}.5x\right)$$

$$\Rightarrow\left(\frac{5}{x}+5x\right)\left(\left(\frac{5}{x}\right)^2+(5x)^2-\frac{5}{x}.5x\right)$$

$$\Rightarrow\left(\frac{5}{x}\right)^3+(5x)^3$$

$$=\frac{125}{x^3}+125x^3 \quad (x=3)$$

$$=\frac{125}{3^3}+125.3^3$$

$$=\frac{125}{27}+3375$$

$$=\frac{91250}{27}$$

3. Question

If $a+b=10$ and $ab=16$, find the value of a^2-ab+b^2 and a^2+ab+b^2.

Answer

Given: a+b=10 and ab =16

To find: a^2-ab+b^2

$$a^2-ab+b^2=a^2+b^2-ab$$

$$\Rightarrow a^2-ab+b^2=a^2+b^2-ab+2ab-2ab$$

$$\Rightarrow a^2-ab+b^2=(a^2+b^2+2ab)\ -2ab-ab$$

$$\Rightarrow a^2-ab+b^2=(a+b)^2\ -3ab$$

$$\Rightarrow a^2-ab+b^2=(10)^2\ -3\times16=52$$

To find: a^2+ab+b^2

$$a^2+ab+b^2=a^2+ab+b^2+ab-ab$$

$$\Rightarrow a^2+ab+b^2=(a^2+b^2+2ab)-ab$$

$$\Rightarrow a^2+ab+b^2=(a+b)^2-ab$$

$$\Rightarrow a^2+ab+b^2=(10)^2-16=84$$

4. Question

If $a+b=8$ and $ab=6$, find the value of a^3+b^3.

Answer

Given: a+b=8 and ab =6

To find: a^3+b^3

$$a^3+b^3=(a+b)(a^2+b^2-ab)$$
$$=(a+b)(a^2+b^2-ab+2ab-2ab)$$
$$=(a+b)\left[(a^2+b^2+2ab)\ -3ab\right]$$
$$=(a+b)\left[(a+b)^2\ \ -3ab\right]$$
$$=8\left[8^2-3\times 6\right]$$
$$=368$$

5. Question

If a-b=6 and ab=20, find the value of a^3-b^3.

Answer

Given: a-b=6 and ab =20

To find: a^3-b^3

$$a^3-b^3=(a+b)(a^2+b^2+ab)$$
$$=(a-b)(a^2+b^2+ab+2ab-2ab)$$
$$=(a-b)\left[(a^2+b^2-2ab)\ +3ab\right]$$
$$=(a-b)\left[(a-b)^2\ +3ab\right]$$
$$=6\left[6^2-3\times 20\right]$$
$$=576$$

6. Question

If x = -2 and y = 1, by using an identity find the value of the following:

(i) $(4y^2-9x^2)\ (16y^4+36x^2y^2+81x^4)$

(ii) $\left(\frac{2}{x}-\frac{x}{2}\right)\left(\frac{4}{x^2}+\frac{x^2}{4}+1\right)$

(iii) $\left(5y+\frac{15}{y}\right)\left(25y^2-75+\frac{225}{y^2}\right)$

Answer

(i)

$$\left(4y^2-9x^2\right)\left(16y^4+36x^2y^2+81x^4\right)$$
$$=\left(4y^2-9x^2\right)\left((4y^2)^2+9x^2\times 4y^2+(9x^2)^2\right)$$
$$=\left(4y^2\right)^3-\left(9x^2\right)^3$$
$$=64y^6-729x^6$$
$$=64(1)^6-729(-2)^6$$
$$=64-729\times 64$$
$$=64-46656$$
$$=\boxed{-46592}$$

(ii)

$$\left(\frac{2}{x}-\frac{x}{2}\right)\left(\frac{4}{x^2}+\frac{x^2}{4}+1\right)$$
$$\Rightarrow\left(\frac{2}{x}-\frac{x}{2}\right)\left(\left(\frac{2}{x}\right)^2+\left(\frac{x}{2}\right)^2+\frac{2}{x}.\frac{x}{2}\right)$$
$$\Rightarrow\left(\frac{2}{x}\right)^3-\left(\frac{x}{2}\right)^3$$
$$\Rightarrow\frac{8}{x^3}-\frac{x^3}{8}$$
Now, x=-2
$$\Rightarrow\frac{8}{(-2)^3}-\frac{(-2)^3}{8}=-1+1=\boxed{0}$$

(iii)

$$\left(5y+\frac{15}{y}\right)\left(25y^2-75+\frac{225}{y^2}\right)$$
$$\Rightarrow\left(5y+\frac{15}{y}\right)\left((5y)^2-5y\times\frac{15}{y}+\left(\frac{15}{y}\right)^2\right)$$
$$\Rightarrow(5y)^3+\left(\frac{15}{y}\right)^3$$
$$\Rightarrow125y^3+\frac{3375}{y^3}$$
Now, y=1
$$\Rightarrow125+3375=\boxed{3500}$$

Exercise 4.5

1. Question

Find the following product:

(i) $(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$

(ii) $(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$

(iii) $(2a-3b-2c)(4a^2+9b^2+4c^2+6ab-6bc+4ca)$

(iv) $(3x-4y+5z)(9x^2+16y^2+25z^2+12xy-15zx+20yz)$

Answer

(i) Using the identity,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
Therefore
$$\Rightarrow(3x+2y+2z)(9x^2+4y^2+4z^2-6xy-4yz-6zx)$$
$$\Rightarrow(3x)^3+(2y)^3+(2z)^3-3(3x)(2y)(2z)$$
$$\Rightarrow\boxed{27x^3+8y^3+8z^3-36xyz}$$

(ii) Using the identity,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
Therefore
$$\Rightarrow(4x-3y+2z)(16x^2+9y^2+4z^2+12xy+6yz-8zx)$$
$$\Rightarrow(4x-3y+2z)((4x)^2+(3y)^2+(2z)^2-(4x)(-3y)-(-3y)(2z)-(2z)(4x)$$
$$\Rightarrow(4x)^3+(-3y)^3+(2z)^3-3(4x)(-3y)(2z)$$
$$\Rightarrow\boxed{64x^3-27y^3+8z^3+72xyz}$$

(iii) Using the identity,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Therefore

$\Rightarrow (2a - 3b - 2c)(4a^2 + 9b^2 + 4c^2 + 6ab + 6bc + 4ca)$

$\Rightarrow (2a - 3b - 2c)((2a)^2 + (-3b)^2 + (-2c)^2 - (2a)(-3b) - (-3b)(-2c) - (-2c)(2a)$

$\Rightarrow (2a)^3 + (-3b)^3 + (-2c)^3 - 3(2a)(-3b)(-2c)$

$\Rightarrow \boxed{8a^3 - 27b^3 - 8c - 36abc}$

(iv) Using the identity,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

Therefore

$\Rightarrow (3x - 4y + 5z)\left(9x^2 + 16y^2 + 25z^2 + 12xy - 15zx + 20yz\right)$

$\Rightarrow (3x - 4y + 5z)((3x)^2 + (-4y)^2 + (5z)^2 - (3x)(-4y) - (5z)(3x) - (-4y)(5z)$

$\Rightarrow (3x)^3 + (-4y)^3 + (5z)^3 - 3(3x)(-4y)(5z)$

$\Rightarrow \boxed{27x^3 - 64y^3 + 125z^3 + 180xyz}$

2. Question

If x+y+z = 8 and xy+yz+zx =20, find the value of $x^3 + y^3 + z^3 - 3xyz$.

Answer

In $x^3 + y^3 + z^3 - 3xyz$,

Using the identity

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$ we get,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[x^2 + y^2 + z^2 - (xy + yz + zx)]$ .... (1)

We also know,

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

For $x^2 + y^2 + z^2$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$\Rightarrow 8^2 = x^2 + y^2 + z^2 + 2(20)$

$\Rightarrow 64 = x^2 + y^2 + z^2 + 40$

$\Rightarrow x^2 + y^2 + z^2 = 24$

From (1) we get,

$x^3 + y^3 + z^3 - 3xyz = 8[24 - 20]$

$= 8(4)$

$= 32$

3. Question

If $a+b+c=9$ and $ab+bc+ca=26$, find the value of $a^3 + b^3 + c^3 - 3abc$.

Answer

Using the identity,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$\Rightarrow a^3+b^3+c^3-3abc=(a+b+c)\left[(a^2+b^2+c^2)-(ab+bc+ca)\right] \_\_\_\_(1)$$
$$\text{Now, } (a+b+c)^2=(a^2+b^2+c^2)+2(ab+bc+ca)$$
$$\Rightarrow (9)^2=(a^2+b^2+c^2)+2(26)$$
$$\Rightarrow a^2+b^2+c^2=81-52=29$$
Now, from (1),
$$\Rightarrow a^3+b^3+c^3-3abc=(9)\left[(29)-(26)\right]=9\times 3=\boxed{27}$$

4. Question

If $a+b+c=9$ and $a^2+b^2+c^2=35$, find the value of $a^3+b^3+c^3-3abc$.

Answer

Using the identity,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$\Rightarrow a^3+b^3+c^3-3abc=(a+b+c)\left[(a^2+b^2+c^2)-(ab+bc+ca)\right] \_\_\_\_(1)$$
$$\text{Now, } (a+b+c)^2=(a^2+b^2+c^2)+2(ab+bc+ca)$$
$$\Rightarrow (9)^2=35+2(ab+bc+ca)$$
$$\Rightarrow 2(ab+bc+ca)=81-35=46$$
$$\Rightarrow (ab+bc+ca)=23$$
Now, from (1),
$$\Rightarrow a^3+b^3+c^3-3abc=(9)\left[35-23\right]=9\times 12=\boxed{108}$$

5. Question

Evaluate :

(i) $25^3 - 75^3 + 50^3$

(ii) $48^3 - 30^3 - 18^3$

(iii) $\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$

(iv) $(0.2)^3 - (0.3)^3 + (0.1)^3$

Answer

(i)

We know,
$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$\text{Now, } (a+b+c)=25-75+50=0$$
$$\Rightarrow (a)^3+(b)^3+(c)^3=3abc$$
$$=3\times 25\times(-75)\times 50=$$
$$=\boxed{-281250}$$

(ii)

We know,
$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$
$$\text{Now, } (a+b+c)=48-30-18=0$$
$$\Rightarrow (a)^3+(b)^3+(c)^3=3abc$$
$$=3\times 48\times(-30)\times(-18)$$
$$=\boxed{77760}$$

(iii)

We know,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

Now, $(a+b+c)=\frac{1}{2}+\frac{1}{3}-\frac{5}{6}=\frac{5}{6}+\frac{5}{6}0$

$$\Rightarrow (a)^3+(b)^3+(c)^3=3abc$$

$$=3\times\frac{1}{2}\times\frac{1}{3}\times(-\frac{5}{6})$$

$$=\boxed{-\frac{5}{12}}$$

(iv)

We know,

$$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$$

Now, $(a+b+c)=0.2-0.3+0.1=0$

$$\Rightarrow (a)^3+(b)^3+(c)^3=3abc$$

$$=3\times 0.2\times(-0.3)\times 0.1$$

$$=\boxed{-0.018}$$

CCE - Formative Assessment

1. Question

If $x+\frac{1}{x}=3$, then find the value of $x^2+\frac{1}{x^2}$.

Answer

Given: $x+\frac{1}{x}=3$

Using $,(a+b)^2=a^2+b^2+2ab$

$$\left(x+\frac{1}{x}\right)^2=x^2+\left(\frac{1}{x^2}\right)+2.x.\frac{1}{x}$$

$$\Rightarrow (3)^2=x^2+\left(\frac{1}{x^2}\right)+2$$

$$\Rightarrow \boxed{x^2+\frac{1}{x^2}=9-2=7}$$

2. Question

If $x+\frac{1}{x}=3$, then find the value of $x^6+\frac{1}{x^6}$.

Answer

We are given that $x+\frac{1}{x}=3$

On cubing we get

$$\left(x+\frac{1}{x}\right)^3 = x^3+\left(\frac{1}{x^3}\right)+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow 27 = x^3+\left(\frac{1}{x^3}\right)+3.3$$

$$\Rightarrow x^3+\left(\frac{1}{x^3}\right)=27-9$$

$$\Rightarrow \boxed{x^3+\left(\frac{1}{x^3}\right)=18}$$

$$Now, \left(x^3+\frac{1}{x^3}\right)^2 = x^6+\left(\frac{1}{x^6}\right)+2.x^3.\frac{1}{x^3}$$

$$\Rightarrow 18^2 = x^6+\left(\frac{1}{x^6}\right)+2$$

$$x^6+\left(\frac{1}{x^6}\right)=324-2=\boxed{322}$$

3. Question

If $a+b=7$ and $ab=12$, find the value of a^2+b^2.

Answer

Using the identity,

$$(a+b)^2 = a^2+b^2+2ab$$
$$\Rightarrow (7)^2 = a^2+b^2+2(12)$$
$$\Rightarrow a^2+b^2 = 49-24 = 25$$

4. Question

If a-b=5 and ab=12, find the value of a^2+b^2.

Answer

Using the identity,

$$(a-b)^2 = a^2+b^2-2ab$$
$$\Rightarrow (5)^2 = a^2+b^2+2(12)$$
$$\Rightarrow a^2+b^2 = 25-24 = 1$$

5. Question

If $x-\frac{1}{x}=\frac{1}{2}$, then write the value of $4x^2+\frac{4}{x^2}$.

Answer

Given: $x-\frac{1}{x}=\frac{1}{2}$

$$\Rightarrow 2\left(x-\frac{1}{x}\right)=1$$

$$\Rightarrow \left(2x-\frac{2}{x}\right)=1$$

Using , $(a+b)^2 = a^2+b^2+2ab$

$$\left(2x-\frac{2}{x}\right)^2 = 4x^2+\left(\frac{4}{x^2}\right)-2.2x.\frac{2}{x}$$

$$\Rightarrow (1)^2 = 4x^2+\left(\frac{4}{x^2}\right)-8$$

$$\Rightarrow \boxed{4x^2+\frac{4}{x^2}=9}$$

6. Question

If $a^2+\frac{1}{a^2}$ =102, find the value of $a-\frac{1}{a}$.

Answer

Here, we will use $(a+b)^2 = a^2 + b^2 + 2ab$

$$a^2+\frac{1}{a^2}=102$$
$$\Rightarrow a^2+\frac{1}{a^2}-2\left(a^2\times\frac{1}{a^2}\right)=102-2\left(a^2\times\frac{1}{a^2}\right)$$
$$\Rightarrow \left(a-\frac{1}{a}\right)^2=100$$
$$\Rightarrow \left(a-\frac{1}{a}\right)=\sqrt{100}=\pm10$$

7. Question

If $a+b+c=0$ then write the value of $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$.

Answer

Given: $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$

Taking LCM we get,

$$\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{a^3+b^3+c^3}{abc} \quad \_\_\_\_(1)$$

We know, $a^3+b^3+c^3-3abc=(a+b+c)\left[a^2+b^2+c^2-ab-bc-ca\right]$ ____(1)

If $a+b+c=0$,

Then, $a^3+b^3+c^3=3abc$

Now, from (1),

$$\Rightarrow \frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{3abc}{abc}=3$$

1. Question

If $x+\frac{1}{x}$ =5, then $x^2+\frac{1}{x^2}$ =

A. 25

B. 10

C. 23

D. 27

Answer

Using , $(a+b)^2=a^2+b^2+2ab$

$$\left(x+\frac{1}{x}\right)^2=x^2+\left(\frac{1}{x^2}\right)+2.x.\frac{1}{x}$$
$$\Rightarrow (5)^2=x^2+\left(\frac{1}{x^2}\right)+2$$
$$\Rightarrow x^2+\frac{1}{x^2}=25-2$$
$$\boxed{x^2+\frac{1}{x^2}=23}$$

2. Question

If x+ $\frac{1}{x}$ =2, then $x^3+\frac{1}{x^3}$ =

A. 64

B. 14

C. 8

D. 2

Answer

On cubing we get

$$\left(x+\frac{1}{x}\right)^3 = x^3+\left(\frac{1}{x^3}\right)+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow 2^3 = x^3+\left(\frac{1}{x^3}\right)+3.2$$

$$\Rightarrow x^3+\left(\frac{1}{x^3}\right) = 8-6$$

$$\Rightarrow \boxed{x^3+\left(\frac{1}{x^3}\right)=2}$$

3. Question

If x+ $\frac{1}{x}$ =4, then $x^4+\frac{1}{x^4}$ =

A. 196

B. 194

C. 192

D. 190

Answer

Using , $(a+b)^2 = a^2+b^2+2ab$

$$\left(x+\frac{1}{x}\right)^2 = x^2+\left(\frac{1}{x^2}\right)+2.x.\frac{1}{x}$$

$$\Rightarrow (4)^2 = x^2+\left(\frac{1}{x^2}\right)+2$$

$$\Rightarrow x^2+\frac{1}{x^2} = 16-2$$

$$\boxed{x^2+\frac{1}{x^2}=14}$$

Again, squaring both sides,

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2 = (14)^2$$

$$\Rightarrow x^4+\left(\frac{1}{x^4}\right)+2.x^2.\frac{1}{x^2} = 196$$

$$\Rightarrow x^4+\left(\frac{1}{x^4}\right) = 196-2$$

$$\Rightarrow \boxed{x^4+\left(\frac{1}{x^4}\right)=194}$$

4. Question

If x+ $\frac{1}{x}$ =3, then $x^6+\frac{1}{x^6}$ =

A. 927

B. 414

C. 364

D. 322

Answer

On cubing we get

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \left(\frac{1}{x^3}\right) + 3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow 27 = x^3 + \left(\frac{1}{x^3}\right) + 3.3$$

$$\Rightarrow x^3 + \left(\frac{1}{x^3}\right) = 27 - 9$$

$$\Rightarrow \boxed{x^3 + \left(\frac{1}{x^3}\right) = 18}$$

$$Now, \left(x^3 + \frac{1}{x^3}\right)^2 = x^6 + \left(\frac{1}{x^6}\right) + 2.x^3.\frac{1}{x^3}$$

$$\Rightarrow 18^2 = x^6 + \left(\frac{1}{x^6}\right) + 2$$

$$x^6 + \left(\frac{1}{x^6}\right) = 324 - 2 = \boxed{322}$$

5. Question

If $x^4 + \frac{1}{x^4}$ = 623, then x+$\frac{1}{x}$ =

A. 27

B. 25

C. 3 $\sqrt{3}$

B. -3 $\sqrt{3}$

Answer

$$\left(x^4 + \frac{1}{x^4}\right) = 623$$

$$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right) + 2 \times x^2 \times \frac{1}{x^2} = 623 + 2 \times x^2 \times \frac{1}{x^2}$$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 625$$

$$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{625} = 25$$

Now,

$$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) + 2 \times x \times \frac{1}{x} = 25 + 2 \times x \times \frac{1}{x}$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 27$$

$$\Rightarrow x + \frac{1}{x} = \sqrt{27} = 3\sqrt{3}$$

6. Question

If $x^2 + \frac{1}{x^2}$ = 102, then x-$\frac{1}{x}$ =

A. 8

B. 10

C. 12

D. 13

Answer

$$x^2+\frac{1}{x^2}=102$$

Now,

$$\Rightarrow (x^2)+\left(\frac{1}{x^2}\right)-2\times x\times\frac{1}{x}=102-2\times x\times\frac{1}{x}$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2=100$$

$$\Rightarrow x+\frac{1}{x}=\sqrt{100}=10$$

7. Question

If $x^3+\frac{1}{x^3}$=110, then x +$\frac{1}{x}$=?

A. 5

B. 10

C. 15

D. none of these

Answer

$$x^3+\left(\frac{1}{x^3}\right)=110$$

$$x^3+\left(\frac{1}{x^3}\right)+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)=110+3.x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^3=110+3\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^3-3\left(x+\frac{1}{x}\right)-110=0$$

$$Let\ x+\frac{1}{x}=a$$

$$\Rightarrow a^3-3a-110=0$$

$$\Rightarrow a^3-5a^2+5a^2-25a+22a-110=0$$

$$\Rightarrow a^2(a-5)+5a(a-5)+22(a-5)=0$$

$$\Rightarrow (a-5)(a^2+5a+22)=0$$

$$\Rightarrow a-5=0 \text{ or } a^2+5a+22=0 \text{ neglected)} \quad \Rightarrow a=5$$

$$\Rightarrow x+\frac{1}{x}=5$$

8. Question

If $x^3-\frac{1}{x^3}$=14, then x -$\frac{1}{x}$=

A. 5

B. 4

C. 3

D. 2

Answer

Given : $x^3 + \left(\frac{1}{x^3}\right) = 14$

$Let\ x = a$ and $\frac{1}{x} = b$

Say, $x - \frac{1}{x} = A$

Then, $a^3 - b^3 = 14$

$\Rightarrow (a-b)(a^2 + ab + b^2) = 14$

$\Rightarrow (a-b)(\{(a-b)^2 + 2ab\} + 2ab) = 14$

$\Rightarrow (a-b)\{(a-b)^2 + 3ab\} = 14$

$\Rightarrow (a-b)\{(a-b)^2 + 3\} = 14$

$\Rightarrow A(A^2 + 3) = 14$

$\Rightarrow A(A^2 + 3) = 14$

$\Rightarrow A^3 + 3A - 14 = 0$

$\Rightarrow A^3 - 2A^2 + 2A^2 - 4A + 7A - 14 = 0$

$\Rightarrow A^2(A-2) + 2Y(Y-2) + 7(Y-2) = 0$

$\Rightarrow (A-2)(A^2 + 2A + 7) = 0$

$\Rightarrow A - 2 = 0, \Rightarrow A = 2$

$\Rightarrow x - \frac{1}{x} = 2$

9. Question

If $x^4 + \frac{1}{x^4} = 194$, then $x^3 - \frac{1}{x^3} =$

A. 76

B. 52

C. 64

D. none of these

Answer

$\left(x^{4} + \frac{1}{x^4}\right) = 194$

$\Rightarrow (x^2)^2 + \left(\frac{1}{x^2}\right) + 2 \times x^2 \times \frac{1}{x^2} = 194 + 2 \times x^2 \times \frac{1}{x^2}$

$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 196$

$\Rightarrow x^2 + \frac{1}{x^2} = \sqrt{196} = 14$

Now,

$\Rightarrow (x^2) + \left(\frac{1}{x^2}\right) + 2 \times x \times \frac{1}{x} = 14 + 2 \times x \times \frac{1}{x}$

$\Rightarrow \left(x - \frac{1}{x}\right)^2 = 16$

$\Rightarrow x + \frac{1}{x} = \sqrt{16} = 4$

Now, $\left(x + \frac{1}{x}\right)^3 = (4)^3$

$\Rightarrow (x)^3 + \left(\frac{1}{x}\right)^3 + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 64$

$\Rightarrow (x^3) + \left(\frac{1}{x^3}\right) + 3(4) = 64$

$\Rightarrow (x^3) + \left(\frac{1}{x^3}\right) = 64 - 12 = 52$

10. Question

If x- $\frac{1}{x}$ = $\frac{15}{4}$, then x +$\frac{1}{x}$ =

A. 4

B. $\frac{17}{4}$

C. $\frac{13}{4}$

D. $\frac{1}{4}$

Answer

$\Rightarrow x-\frac{1}{x}=\frac{15}{4}$

Now, $\left(x-\frac{1}{x}\right)^2=\left(\frac{15}{4}\right)^2$

$\Rightarrow (x^2)+\left(\frac{1}{x^2}\right)-2\times x\times\frac{1}{x}=\frac{225}{16}$

$\Rightarrow (x^2)+\left(\frac{1}{x^2}\right)=\frac{225}{16}+2$

$\Rightarrow (x^2)+\left(\frac{1}{x^2}\right)=\frac{257}{16}$

Now, $\Rightarrow (x^2)+\left(\frac{1}{x^2}\right)+2\times x\times\frac{1}{x}=\frac{257}{16}+2\times x\times\frac{1}{x}$

$\Rightarrow \left(x+\frac{1}{x}\right)^2=\frac{257+32}{16}=\frac{289}{16}$

$\Rightarrow \left(x+\frac{1}{x}\right)=\sqrt{\frac{289}{16}}=\frac{17}{4}$

11. Question

If 3*x*+ $\frac{2}{x}$ =7, then $\left(9x^2-\frac{4}{x^2}\right)$ =

A. 25

B. 35

C. 49

D. 30

Answer

$\Rightarrow 3x+\frac{2}{x}=7$

Now, $3x^2+2-7x=0$

$\Rightarrow 3x^2-6x-x+2=0$

$\Rightarrow (x-2)(3x-1)=0$

$\Rightarrow x=2 \text{ or } x=\frac{1}{3}(neglected)$

$\Rightarrow 3x-\frac{2}{x}=6-1=5$

$\Rightarrow 3x+\frac{2}{x}=6+1=7$

Now,

$(3x)^2-\left(\frac{2}{x}\right)^2=\left(3x-\frac{2}{x}\right)\left(3x+\frac{2}{x}\right)$

$\Rightarrow (3x)^2-\left(\frac{2}{x}\right)^2=5(7)=35$

12. Question

If $a^2+b^2+c^2-ab-bc-ca=0$, then

A. $a+b=c$

B. $b+c=a$

C. $c+a=b$

D. $a=b=c$

Answer

$Given: a^2+b^2+c^2-ab-bc-ca=0$

$\Rightarrow 2\left(a^2+b^2+c^2-ab-bc-ca\right)=0$

$\Rightarrow \left(2a^2+2b^2+2c^2-2ab-2bc-2ca\right)=0$

$\Rightarrow \left(\{a^2+b^2-2ab\}+\{b^2+c^2-2bc\}+\{c^2+a^2-2ca\}\right)=0$

$\Rightarrow (a-b)^2+(b-c)^2+(c-a)^2=0$

Now, since the sum of all squares is zero

$\Rightarrow a-b=0 \Rightarrow a=b$

$\Rightarrow b-c=0 \Rightarrow b=c$

$\Rightarrow c-a=0 \Rightarrow c=a$

$\Rightarrow a=b=c$

13. Question

If a + b + c =0, then, $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$ =

A. 0

B. 1

C. -1

D. 3

Answer

Given: $\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}$

Taking LCM we get,

$\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{a^3+b^3+c^3}{abc}$ ____(1)

We know, $a^3+b^3+c^3-3abc=(a+b+c)\left[a^2+b^2+c^2-ab-bc-ca)\right]$ ____(1)

If $a+b+c=0$,

Then, $a^3+b^3+c^3=3abc$

Now, from (1),

$\Rightarrow \frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}=\frac{3abc}{abc}=3$

14. Question

If $a^{1/3}+b^{1/3}+c^{1/3}=0$, then

A. $a+b+c=0$

B. $(a+b+c)^3=27abc$

C. $a+b+c=3abc$

D. $a^3+b^3+c^3=0$

Answer

$a^{1/3}+b^{1/3}+c^{1/3}=0$

$\Rightarrow a^{1/3}+b^{1/3}=-c^{1/3}$ ______(1)

$\Rightarrow (a^{1/3})^3+(b^{1/3})^3=(-c^{1/3})^3$

$\Rightarrow a+b+3.a^{1/3}.b^{1/3}(a^{1/3}+b^{1/3})=-c$

$\Rightarrow a+b+3.a^{1/3}.b^{1/3}(-c^{1/3})=-c$

$\Rightarrow a+b+c=3.a^{1/3}.b^{1/3}c^{1/3}$

$\Rightarrow (a+b+c)^3=(3.a^{1/3}.b^{1/3}c^{1/3})^3$

$\Rightarrow (a+b+c)^3=27abc$

15. Question

If $a+b+c=9$ and ab + bc + ca = 23, then $a^2+b^2+c^2=$

A. 35

B. 58

B. 127

D. none of these

Answer

$(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$

Hence, $9^2 = a^2 + b^2 + c^2 + 2 \times 23$

$\Rightarrow a^2 + b^2 + c^2 = 35$

16. Question

If $a+b+c=9$, then $ab+bc+ca=23$, then $a^3+b^3+c^3-3abc=$

A. 108

B. 207

C. 669

D. 729

Answer

$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$

$\Rightarrow (9)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

$\Rightarrow (9)^2 = a^2+b^2+c^2+2(23)$

$\Rightarrow a^2+b^2+c^2 = 81-46 = 35$

as we know that $a^3+b^3+c^3-3abc = (a+b+c)\left(a^2+b^2+c^2-ab-bc-ca\right)$

$\Rightarrow a^3+b^3+c^3-3abc = 9\times(35-23)$

$\Rightarrow a^3+b^3+c^3-3abc = 108$

17. Question

$(a-b)^3+(b-c)^3+(c-a)^3=$

A. $(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

B. $(a-b)(b-c)(c-a)$

C. $3(a-b)(b-c)(c-a)$

D. none of these

Answer

Let (a-b) = A

(b-c) = B

and (c-a) = C

$\Rightarrow A+B+C=0$

$\Rightarrow A+B = -C$ ______(1)

$\Rightarrow (A+B)^3 = -C^3$

$\Rightarrow A^3+B^3+3.AB(A+B) = -C^3$

$\Rightarrow A^3+B^3+3.AB(-C) = -C^3$

$\Rightarrow A^3+B^3+C^3 = 3.A.B.C$

$\boxed{\Rightarrow (a-b)^3+(b-c)^3+(c-a)^3 = 3.(a-b)(b-c).(c-a)}$

18. Question

Solve the equation and choose the correct answer: $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2+a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3} =$

A. $3(a+b)(b+c)(c+a)$

B. $3(a-b)(b-c)(c-a)$

C. $(a-b)(b-c)(c-a)$

D. None of these

Answer

We know, $a^3+b^3+c^3 = 3abc$

$\Rightarrow (a-b)^3+(b-c)^3+(c-a)^3 = 3.(a-b)(b-c).(c-a)$

$\Rightarrow \frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

$= \frac{3(a-b)(a+b)(b-c)(b+c)(c-a)(c+a)}{3(a-b)(b-c)(c-a)}$

$= (a+b)(b+c)(c+a)$

19. Question

The product $(a+b)(a-b)(a^2-ab+b^2)(a^2+ab+b^2)$ is equal to

A. a^6+b^6

B. a^6-b^6

C. a^3-b^3

D. a^3+b^3

Answer

$$\text{Given: } (a+b)(a-b)(a^2+b^2+ab)(a^2+b^2-ab)$$
$$\Rightarrow \{(a+b)(a^2+b^2+ab)\}\{(a-b)(a^2+b^2+ab)\}$$
$$\Rightarrow (a^3+b^3)(a^3-b^3)$$
$$\Rightarrow (a^6-b^6)$$

20. Question

If $\frac{a}{b}+\frac{b}{a}$= -1, then a^3-b^3 =

A. 1

B. -1

C. $\frac{1}{2}$

D. 0

Answer

$$\text{Here, } \frac{a}{b}+\frac{b}{a} = -1$$
$$\Rightarrow \frac{a^2+b^2}{ab} = -1$$
$$\Rightarrow a^2+b^2 = -ab$$
$$\Rightarrow a^2+b^2+ab = 0 \_\_\_\_\_(1)$$
$$\text{Using, } a^3-b^3 = (a-b)(a^2+b^2+ab)$$
$$= (a+b)(0)$$
$$= 0$$

21. Question

The product (x^2-1) (x^4+x^2+1) is equal to=

A. x^8-1

B. x^8+1

C. x^6-1

D. x^6+1

Answer

$$\text{Using, } (x^2-1)(x^4+x^2+1)$$
$$\Rightarrow (x^8+x^4+x^2-x^4-x^2-1)$$
$$\Rightarrow x^8-1$$

22. Question

If a-b=-8, and ab =-12, then a^3 – b^3 =

A. -244

B. -240

C. -224

D. -260

Answer

Using, $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

$\Rightarrow a^3 - b^3 = (a-b)^3 + 3ab(a-b)$

$\Rightarrow a^3 - b^3 = (-8)^3 + 3(-12)(-8)$

$\Rightarrow a^3 - b^3 = -512 + 288 = -224$

23. Question

If the volume of a cuboid is $3x^2$-27, then its possible dimensions are

A. 3, x^2, -27x

B. 3, x-3, x+3

C. 3, x^2, 27x

D. 3, 3, 3

Answer

Given: $3x^2 - 27$

We will break down in factors,

$= 3(x^2 - 9)$

Using, $(x^2 - 9) = (x+3)(x-3)$

$\Rightarrow 3(x+3)(x-3)$

Thus, the possible dimensions are 3, (x+3)(x-3)

24. Question

If $\frac{a}{b} + \frac{b}{a}$ = 1, then a^3+b^3 =

A. 1

B. -1

C. $\frac{1}{2}$

D. 0

Answer

Here, $\frac{a}{b} + \frac{b}{a} = 1$

$\Rightarrow \frac{a^2 + b^2}{ab} = 1$

$\Rightarrow a^2 + b^2 = ab$

$\Rightarrow a^2 + b^2 - ab = 0$ ______(1)

Using, $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

$= (a+b)(0)$

$= 0$

25. Question

75×75+2×75×25+25×25 equal to

A. 10000

B. 6250

C. 7500

D. 3750

Answer

We know, $(a+b)^2 = a^2 + b^2 + 2ab$

Here, a = 75 and b = 25

$\Rightarrow (75\times 75)+(2\times 75\times 25)+(25\times 25)$

$= 75^2 + 2\times 75\times 25 + 25^2$

$= (75+25)^2$

$= (100)^2$

$= 10000$

26. Question

$(x-y)(x+y)(x^2+y^2)(x^4+y^4)$ is equal to

A. $x^{16} - y^{16}$

B. $x^8 - y^8$

C. $x^8 + y^8$

D. $x^{16} + y^{16}$

Answer

Given: $(x-y)(x+y)(x^2+y^2)(x^4+y^4)$

Using, $a^2 - b^2 = (a-b)(a+b)$,

$\Rightarrow (x-y)(x+y)(x^2+y^2)(x^4+y^4)$

$= (x^2-y^2)(x^2+y^2)(x^4+y^4)$

$= (x^4-y^4)(x^4+y^4)$

$= (x^8-y^8)$

27. Question

If $48a^2 - b = \left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)$, then the value of b is

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{2}$

Answer

Given: $49a^2 - b = \left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right)$

Using, $a^2 - b^2 = (a-b)(a+b)$,

$\Rightarrow \left(7a+\frac{1}{2}\right)\left(7a-\frac{1}{2}\right) = 49a^2 - \frac{1}{4}$

$\Rightarrow 49a^2 - b = 49a^2 - \frac{1}{4}$

$\Rightarrow b = \frac{1}{4}$