33. Probability

Exercise 33.1

1. Question

A coin is tossed once. Write its sample space.

Answer

Given: A coin is tossed once.

To Find: Write its sample space?

Explanation: Here, the coin is tossed only once,

Then, there are two probability either Head(H) or Tail(T)

So, Sample will be

S = {H, T}

Where, H denotes Head and T denotes Tail

Hence, The sample is {H, T}

2. Question

If a coin is tossed two times, describe the sample space associated to this experiment.

Answer

Given: If Coin is tossed twice times.

To Find: Write the sample space associated to this experiment.

Explanation: Here, two coins are tossed, that means two probability will occur at same time

So, The sample space will be

S={HT, TH, HH, TT}

Hence, Sample space is {HT, HH, TT, TH}

3. Question

If a coin is tossed three times (or three coins are tossed together), then describe the sample space for this experiment.

Answer

Given: If a coin is tossed three times .

To Find: Write the sample space for the given experiment.

Explanation: Here, the coins is tossed three time, then the no. of samples

$2^3=8$

So, The sample space will be

S={HHH, TTT, HHT, HTH, THH, HTT, THT, TTH}

Hence, The sample space is {HHH, TTT, HHT, HTH, THH, HTT, THT, TTH}

4. Question

Write the sample space for the experiment of tossing a coin four times.

Answer

Given: A coin is tossed four times.

To Find: Write the sample space for the given experiment.

Explanation: Here, The coins is tossed four time, then the no. of samples

$2^4=16$

So, The sample space will be

S={HHHH, TTTT, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, HTHT, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH}

Hence, The sample space is {HHHH, TTTT, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, HTHT, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH}

5. Question

Two dice are thrown. Describe the sample space of this experiment.

Answer

Given: Two dice are thrown.

To Find: Write the sample space for the given experiment.

Explanation: We know there are 6 faces on a dice. Contains (1, 2, 3, 4, 5, 6).

But, Here two dice are thrown, then we have two faces of dice (one of each)

So, The total sample space will be $6^2 = 36$

Now, the sample space is:

S={(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1)(3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), 5, 4), (5, 6), (5, 5), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

6. Question

What is the total number of elementary events associated to the random experiment of throwing three dice together?

Answer

Given: Three dice is rolled together.

To Find: What is the total number of elementary events.

Explanation: Here, three dice are thrown together,

And, There are 6 faces on die,

So, The total number of elementary event on throwing three dice are

$6\times6\times6=216$

Hence, The total number is 216

7. Question

A coin is tossed and then a die is thrown. Describe the sample space for this experiment.

Answer

Given: A coin is tossed and a die is thrown.

To Find: Write the sample space for the given experiment.

Explanation: Here, The coin is tossed and die is thrown.

We know, when coin is tossed there will be 2 events either Head or Tail.

And, when die is thrown then there will be 6 faces (1, 2, 3, 4, 5, 6)

SO, The total number of Sample space together is 2×6 = 12

S={(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

Hence, Sample space are {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

8. Question

A coin is tossed and then a die is rolled only in case a head is shown on the coin. Describe the sample space for this experiment.

Answer

Given: A coin is tossed and the a die is rolled.

To Find: Write the sample space for the given experiment.

Explanation: Here, we have a coin and a die,

We know, when coin is tossed there will be 2 event Head and tail,

According to question, If Head occurs on coin then Die will rolled out otherwise not.

So, the sample spaces are :

S={(T, (H, 1)(H, 2), (H, 3), (H, 4), (H, 5), (H, 6)}

Hence, Sample space is (T, (H, 1)(H, 2), (H, 3), (H, 4), (H, 5), (H, 6)

9. Question

A coin is tossed twice. If the second throw results I a tail, a die is thrown. Describe the sample space for this experiment.

Answer

Given: A coin is tossed twice. If the second throw results I a tail, A die is thrown.

To Find: Write the sample space for the given experiment.

Explanation: When a coin tossed twice, Then sample spaces for only coin will be: {HH, TT, HT , TH}

Now, According to question , when we get Tail in second throw, then a dice is thrown.

So, The total number of elementary events are 2+(2×6)=14

And sample space will be

S={HH, TH, (HT, 1), (HT, 2), (HT, 3), (HT, 4), (HT, 5), (HT, 6), (TT, 1), (TT, 2), (TT, 3), (TT, 4), (TT, 5), (TT, 6)}

Hence, this is the sample space for given experiment.

10. Question

An experiment consists of tossing a coin and then tossing it second time if head occurs. If a tail occurs on the first toss, then a die is tossed once. Find the sample space.

Answer

Given: A coin is tossed and a die is rolled.

To Find: Write the sample space for the given experiment.

Explanation: In the given experiment, coin is tossed and if the outcome is tail then, die will be rolled.

The possible outcome for coin is 2 = {H, T}

And, The possible outcome for die is 6 = {1, 2, 3, 4, 5, 6}

If the outcome for the coin is tail then sample space is S1={(T, 1)(T, 2)(T, 3)(T, 4)(T, 5)(T, 6)}

If the outcome is head then the sample space is S2={(H, H)(H, T)}

So, The required outcome sample space is S=S1⋃S2

$S=\{(T, 1)(T, 2)(T, 3)(T, 4)(T, 5)(T, 6)(H, H)(H, T)\}$

Hence, The sample space for the given experiment.

11. Question

A coin is tossed. If it shows tail, we draw a ball from a box which contains 2 red 3 black balls; it shows head, we throw a die. Find the sample space of this experiment.

Answer

Given: A coin is tossed and there is box which contain 2 red and 3 black balls.

To Find: Write the sample space for the given experiment.

Explanation: when coin is tossed , there are 2 outcome {H, T}

According to question, If tail turned up, the a ball is drawn from a box.

So, Sample for This experiment $S_1 = \{(T, R_1)(T, R_2)(T, B_1)(T, B_2)(T, B_3)\}$

Now, If Head is turned up, then die is rolled

So, Sample space for this experiment $S_2=\{(H, 1)(H, 2)(H, 3)(H, 4)(H, 5)(H, 6)\}$

The required sample space will be $S = S_1 \cup S_2$

So, $S=\{(T, R_1), (T, R_2), (T, B_1), (T, B_2), (T, B_3), (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$

Hence, S is the elementary events associated with the given experiment.

12. Question

A coin is tossed repeatedly until a tail comes up for the first time. Write the sample space for this experiment.

Answer

Given: A coin is tossed repeatedly until comes up fro the first time.

To Find: Write the sample space for the given experiment.

Explanation: In the given Experiment, a coin is tossed and if the outcome is tail the experiment is over ,

And, if the outcome is Head then the coin is tossed again.

In the second toss also if the outcome is tail then experiment is over, otherwise coin is tossed again.

This process continues indefinitely

SO, The sample space for this experiment is

S={T, HT, HHT, HHHT, HHHHT…}

Hence, S is th sample space for the given experiment.

13. Question

A box contains 1 red and 3 black balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Answer

Given: A box contains 1 red and 3 black balls.

To Find: Write the sample space for the given experiment.

Explanation: We have 1 red and 3 black balls in a box.

Let Assume Red = R

Let Assume Blue = B

Since, two balls are drawn at random without replacement,

So, The sample spaces for this experiment is:

S={(R, B_1), (R, B_2), (R, B_3)(B_1, B_2)(B_1, B_3)(B_1, R)(B_2, R)(B_2, B_1)(B_2, B_3)(B_3, R)(B_3, B_1)(B_3, B_2)}

Hence, S is the sample spaces for given experiment.

14. Question

A pair of dice is rolled. If the outcome is a doublet, a coin is tossed. Determine the total number of elementary events associated to this experiment.

Answer

Given: A pair of dice is rolled , a coin is tossed.

To Find: Write the sample space for the given experiment.

Explanation: A pair of dice is rolled,

Then, No. of elementary events are 6^2=36

Now, If outcomes is doublet means (1, 1)(2, 2)(3, 3)(4, 4)(5, 5)(6, 6), then a coin is tossed.

If coin is tossed then no. of sample spaces is 2

So, The total no. of elementary events including doublet = 6×2=12

Thus, The Total number of elementary events are 30+12 =42

Hence, 42 events will occur for this experiments.

15. Question

A coin is tossed twice. If the second draw results in a head, a die is rolled. Write the sample space for this experiment.

Answer

Given: There is a coin which tossed twice.

To Find: Write the sample space for the given experiment.

Explanation: A coin is tossed twice , So the outcomes are

S_1={HH, HT, TH, TT}

Now, If the second drawn result is head , the a die is rolled then the elementary events is

S_2={(HH, 1), (HH, 2)(HH, 3), (HH, 4)(HH, 5), (HH, 6), (HH, 1), (TH, 2)(TH, 3), (TH, 4)(TH, 5), (TH, 6)}

Thus, The total sample space for the experiment is S=$S_1 \cup S_2$

S={(HH), (HT), (TH), (TT)(HH, 1), (HH, 2)(HH, 3), (HH, 4)(HH, 5), (HH, 6), (HH, 1), (TH, 2)(TH, 3), (TH, 4)(TH, 5), (TH, 6)}

Hence, S is the sample space for given experiment.

16. Question

A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. What are the possible outcomes of the experiment?

Answer

Given: A bag contains 4 identical red balls and 3 identical black balls.

To Find: Write the sample space for the given experiment.

Explanation: A bag contains 4 red balls and 3 black balls.

Let us Assume Red = R

Let us Assume Black = B

Now, A ball is drawn in first attempt, So elementary events is

S_1={R, B}

And, The bag will put into the bag and draw again, then elementary events are

S_2={R, B}

Thus, The total sample space associated is $S=S_1S_2$

S={RR, RB, BR, BB}

Hence, S is the sample space for given experiment.

17. Question

In a random sampling three items are selected from a lot. Each item is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

Answer

Given: There are three sampling item.

To Find: Write the sample space for the given experiment.

Explanation: In the random sampling , three items are selected so it could be

(a) All defective (D)

(b) All non-defective(N)

(c) Combination of both defective and non defective

We have 2 category (N and D) and we have three condition

So, The number of sample is $2^3 = 8$

Sample space associated with this experiment is

S={DDD, NDN, DND, DNN, NDD, DDN, NND, NNN)

Hence, S is the sample space for given experiment

18. Question

An experiment consists of boy-girl composition of families with 2 children.

(i) What is the sample space if we are interested in knowing whether it is boy or girl in the order of their births?

(ii) What is the sample space if we are interested in the number of boys in a family?

Answer

(i) To Find: Write the sample space for the given experiment.

Explanation: Here, The family has 2 children

Let us Assume Boy = B

Let us Assume Girl = G

So, The number of sample spaces for 2 children is $2^2=4$

Sample space are, S={(B_1, B_2), (G_1, G_2), (G_1, B_2), (B_1, G_2)}

Where , number 1 and 2 are represent elder and younger.

Hence, S is the sample space for given experiment.

(ii) Explanation: Here, The family has 2 children, So the possibility of boys in a family is:

(a) No boys only girl

(b) One boy and one girl

(c) Two boys only

So, The Sample space for the given condition is:

S={0, 1, 2}

Hence, S is the sample spaces for the given experiment.

19. Question

There are three coloured dice of red, white and black colour. These dice are placed in a bag. One die is drawn at random from the bag and rolled, its colour and the number on its uppermost face is noted. Describe the sample space for this experiment.

Answer

Given: There are three colored dice of red , white and black color. These dice are placed in bag.

To Find: Write the sample space for the given experiment.

Explanation: A dice has 6 faces containing numbers (1, 2, 3, 4, 5, 6)

Let us Assume Red = R

Let us Assume White = W

Let us Assume Black = B

According to question, when Dice is selected , then rolled.

Firstly, selected Red Dice then, Possible samples are:

S_R={(R, 1), (R, 2)(R, 3), (R, 4), (R, 5), (R, 6)}

Then, White Dice will be selected , So the sample spaces are:

S_W={(W, 1), (W, 2), (W, 3), (W, 4), (W, 5), (W, 6)}

Lastly, Black Dice will be selected and rolled , So the sample spaces for Black die

S_B= {(B, 1)(B, 2)(B, 3)(B, 4)(B, 5), (B, 6)}

Thus, The total sample for the given experiment is

S=S_RU S_WU S_B

$$\{(B,1)(B,2)(B,3)(B,4)(B,5),(B,6)(W,1),(W,2),(W,3),(W,4),(W,5),(W,6)(R,1),(R,2)(R,3),(R,4),(R,5),(R,6)\}$$

Hence, B is the sample space for the given experiment.

20. Question

2 boys and 2 girls are in room P and 1 boy 3 girls are in room Q. Write the sample space for the experiment in which a room is selected and then a person.

Answer

Given: 2 boys and 2 girls are in room P and 1 boy 3 girls are in room Q.

To Find: Write the sample space for the given experiment.

Explanation: Let us denote two boys and girls in room P as B_1 , B_2 and G_1 , G_2 respectively

Let us denote 1 boy and 3 girls in room Q as B_3 and G_3, G_4, G_5 respectively.

According to the question,

Sample spaces for room S_p are

S_p={(P, B_1), (P, B_2), (P, G_1), (P, G_2)}

And, Sample spaces for room S_q are

S_q={(Q, B_3), (Q, G_1), (Q, G_2), (Q, G_3)}

Now, The total sample spaces for the given experiment:

S=$S_P \cup S_Q$

S={(P, B_1), (P, B_2), (P, G_1), (P, G_2), (Q, B_3), (Q, G_1), (Q, G_2), (Q, G_3)}

Hence, S is the sample space for the given experiment

21. Question

A bag contains one white and one red ball. A ball is drawn from the bag. If the ball drawn is white it is replaced in the bag and again a ball is drawn. Otherwise, a die is tossed. Write the sample space for this experiment.

Answer

Given: A bag contains one white and one red ball.

To Find: Write the sample space for the given experiment.

Explanation: Here, There are 1 white ball , 1 red ball in a bag

Let us denote White ball with W and Red Ball with W

According to question, when one ball is drawn, it may be a white or Red.

So, The sample space of drawing on white ball with replacement

S_W={(W, W), (W, R)}

Again, If red ball is drawn , a dice is rolled

So, The sample space for red ball with dice

S_R={(R, 1), (R, 2), (R, 3), (R, 4), (R, 5), (R, 6)}

Thus, The required sample space for the experiment is:

S=$S_W \cup S_R$

So, S= {(W, W), (W, R), (R, 1), (R, 2), (R, 3), (R, 4), (R, 5), (R, 6)}

Hence, This is the sample space for given experiment

22. Question

A box contains 1 white and 3 identical black balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Answer

Given: A box contain1 white and 3 identical black balls.

To Find: Write the sample space for the given experiment.

Explanation: Here, only 1 white ball and 3 Black identical Balls.

Let us denote the white ball with W and Black ball with B

When two balls are drawn at random in succession without replacement,

So, The sample space for this experiment is

S={WB, BW, BB}

23. Question

An experiment consists of rolling a die and then tossing a coin once it number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Answer

Given: A dice is rolled and then a coin is tossed once.

To Find: Write the sample space for the given experiment.

Explanation: A dice has 6 faces that are numbered from 1 to 6.

And, we know, 2, 4, 6 are even numbers and 1, 3, 5 are odd numbers.

Now, A coin has two face, Head(H) and Tail(T)

So, we can find the total sample spaces by combining both experiments as

S={(2, H), (2, T), (4, H), (4, T), (6, H), (6, T), (1, HH), (1, HT), (1, TH), (1, TT), (3, HH), (3, HT) , (3, TH), (3, TT), (5, HH), (5, HT), (5, TH), (5, TT)}

Hence, S is the required sample space for given experiment.

24. Question

A die is thrown repeatedly until a six comes up. What is the sample space for this experiment.

Answer

Given: A die is thrown repeatedly until a six comes up.

To Find: Sample space for the given experiment

Explanation: Here, The die is thrown repeatedly until a six comes up. It means

Since, 6 may come upon the first throw , second throw , third throw and so on until is obtained.

So, This process will continues indefinitely, then the sample space will be:

S={6, (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (1, 1, 6), (1, 2, 6), (1, 3, 6), (1, 4, 6), (1, 5, 6), (2, 1, 6), (2, 2, 6), (2, 3, 6), (2, 4, 6), (2, 5, 6)........}

Hence, S is the sample space for given experiment.

Exercise 33.2

1. Question

A coin is tossed. Find the total number of elementary events and also the total number of events associated with the random experiment.

Answer

Given: A coin is tossed.

To Find: Find the total number of elementary events and a total number of events associated with the random experiment.

Explanation: When a coin is tossed, there will be two possible outcomes, Head(H) and Tail(T).

Since, the no. of elementary events is 2 {H, T}

But, we know, if there are n elements in a set, then the number of total element in its subset is 2n.

Therefore, the total number of the experiment is 4,

So, there are 4 subset of S = {H}, {T}, {H, T} and Φ

Hence, there are 4 total events in a given experiment.

2. Question

List all events associated with the random experiment of tossing of two coins. How many of them are elementary events?

Answer

Given: Two coins are tossed once.

To find: How many events are elementary events

Explanation: We know, when Two coins are tossed then the no. of possible outcomes are $2^2 = 4$

So, The Sample spaces are {HH, HT, TT, TH}

Hence, there are total 4 events associated with the given experiment.

3. Question

Three coins are tossed once. Describe the following events associated with this random experiment:

A = Getting three heads, B = Getting two heads and one tail, C = Getting three tails, D = Getting a head on the first coin.

(i) Which pairs of events are mutually exclusive?

(ii) Which events are elementary events?

(iii) Which events are compound events?

Answer

Given: There are three coins tossed once.

To Find: Describe the events according to the subparts?

Explanation: when three coins are tossed, then the sample spaces are:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

According to the question,

A = {HHH}

B = {HHT, HTH, THH}

C = {TTT}

D = {HHH, HHT, HTH, HTT}

Now, $A \cap B = \emptyset, A \cap C = \emptyset, A \cap D = \{HHH\}$

$B \cap C = \emptyset, B \cap D = \{HHT, HTH\}, C \cap D = \emptyset$

Since, If the intersection of two sets are null or empty it means both the sets are Mutually Exclusive.

(i) Events A and B, Events A and C, Events B and C and events C and D are mutually exclusive.

(ii) Here, We know, If an event has only one sample point of a sample space, then it is called elementary events.

So, A and C are elementary events.

(iii) If There is an event that has more than one sample point of a sample space, it is called a compound event,

Since, $B \cap D = \{HHT, HTH\}$

So, B and D are compound events.

4. Question

In a single throw of a die describe the following events:

(i) A = Getting a number less than 7

(ii) B = Getting a number greater than 7

(iii) C = Getting a multiple of 3

(iv) D = Getting a number less than 4

(v) E = Getting an even number greater than 4.

(vi) F = Getting a number not less than 3.

Also, find A ∪ B, A ∩ B, B ∩ C, E ∩ F, D ∩ F and $\overline{F}$.

Answer

Given: A dice is thrown once.

To Find: Find the given events, and also find the Also, find A ∪ B, A ∩ B, B ∩ C, E ∩ F, D ∩ F and $\overline{F}$

Explanation: In a single throw of a die, the possible events are:

S = {1, 2, 3, 4, 5, 6}

Now, According to the subparts of the question, we have certain events as:

(i) A = getting a number below 7

So, The sample spaces for A are:

A = {1, 2, 3, 4, 5, 6}

(ii) B = Getting a number greater than 7

So, the sample spaces for B are:

B = {Φ}

(iii) C = Getting multiple of 3

So, The Sample space of C is

C = {3, 6}

(iv) D = Getting a number less than 4

So, The sample space for D is

D = {1, 2, 3}

(v) E = Getting an even number greater than 4.

The sample space for E is

E = {6}

(vi) F = Getting a number not less than 3.

The sapmle space for F is

F = {3, 4, 5, 6}

Now,

A = {1, 2, 3, 4, 5, 6} and B = {Φ}

$\Rightarrow A \cup B = \{1, 2, 3, 4, 5, 6\}$

A = {1, 2, 3, 4, 5, 6} and B = {Φ}

$\Rightarrow A \cap B = \{\emptyset\}$

B = {Φ} and C = {3, 6}

$\Rightarrow B \cap C = \{\emptyset\}$

F = {3, 4, 5, 6}and E = {6}

$\Rightarrow E \cap F = \{6\}$

E = {6} and D = {1, 2, 3}

$\Rightarrow D \cap F = \{3\}$

And, For $\bar{F} = S - F$

S = {1, 2, 3, 4, 5, 6} and F = {3, 4, 5, 6}

$\Rightarrow \bar{F} = \{1, 2\}$

Hence, These are the events for given ecperiment

5. Question

Three coins are tossed. Describe

(i) two events A and B which are mutually exclusive.

(ii) three events A, B and C which are mutually exclusive and exhaustive.

(iii) two events A and B which are not mutually exclusive.

(iv) two events A and B which are mutually exclusive but not exhaustive.

Answer

When three coins are tossed, then the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Now, The subparts are:

(i) The two events which are mutually exclusive are when,

A: getting no tails

B: getting no heads

Then, A = {HHH} and B = {TTT}

SO, The intersection of this set will be null.

Or, The sets are disjoint.

(ii) Three events which are mutually exclusive and exhaustive are:

A: getting no heads

B: getting exactly one head

C:getting at least two head

So, A = {TTT} B = {TTH, THT, HTT} and C = {HHH, HHT, HTH, THH}

Since, $A \cup B = B \cap C = C \cap A = \emptyset$ and $A \cup B \cup C = S$

(iii) The two events that are not mutually exclusive are:

A:getting three heads

B:getting at least 2 heads

So, A = {HHH} B = {HHH, HHT, HTH, THH}

Since $A \cap B = \{HHH\} \neq \emptyset$

(iv) The two events which are mutually exclusive but not exhaustive are:

A:getting exactly one head

B: getting exactly one tail

So, A = {HTT, THT, TTH} and B = {HHT, HTH, THH}

It is because $A \cap B = \emptyset$ but $A \cup B \neq S$

6. Question

A die is thrown twice. Each time the number appearing on it is recorded. Describe the following events:

(i) A = Both numbers are odd.

(ii) B = Both numbers are even

(iii) C = sum of the numbers is less than 6.

Also, find A ∪ B, A ∩ B, A ∪ C, A ∩ C. Which pairs of events are mutually exclusive.

Answer

Given: A dice is thrown twice. And each time number appearing on it is recorded.

To Find: Describe the given events.

Explanation: when the dice is thrown twice then the number of sample spaces are $6^2 = 36$

Now,

The possibility both odd numbers are:

A = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

Since, Possibility of both even numbers are:

B = {(2, 2)(2, 4)(2, 6)(4, 2)(4, 4)(4, 6)(6, 2)(6, 4)(6, 6)}

And, Possible outcome of sum of the numbers is less than 6

C = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}

Therefore,

(AUB) = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) (2, 2)(2, 4)(2, 6)(4, 2)(4, 4)(4, 6)(6, 2)(6, 4)(6, 6)}

(A∩B) = {Φ}

(AUC) = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) (1, 2)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}

(A∩C) = {(1, 1)(1, 3)(3, 1)}

Hence, (A∩B) = Φ and (A∩C)≠Φ, A and B are mutually exclusive, but A and C are not.

7. Question

Two dice are thrown. The events A, B, C, D, E and F are describes as follows.

A = Getting an even number on the first die.

B. Getting an odd number on the first die.

C = Getting at most 5 as sum of the numbers on the two dice.

D = Getting the sum of the numbers on the dice greater than 5 but less than 10.

E = Getting at least 10 as the sum of the numbers on the dice.

F = Getting an odd number on one of the dice.

(i) Describe the following events: A and B, B or C, B and C, A and E, A or F, A and F

(ii) State true or false:

(a) A and B are mutually exclusive.

(b) A and B are mutually exclusive and exhaustive events

(c) A and C are mutually exclusive events

(d) C and D are mutually exclusive and exhaustive events

(e) C, d and E are mutually exclusive and exhaustive events

(f) A' and B' are mutually exclusive events

(g) A, B, F are mutually exclusive and exhaustive events

Answer

Given: Two dice are thrown.

To Find: Describe the given events.

Explanation: when two dice are thrown then the no. of possible outcomes are $6^2 = 36$

Now, According to the question,

A = Getting an even number of the first die.

A = {(2, 1)(2, 2)(2, 3)(2, 4)(2, 5)(2, 6)(4, 1)(4, 2)(4, 3)(4, 4)(6, 1)(6, 2)(6, 3)(6, 4)(6, 5)(6, 6)}

B = Getting an odd number on the first dice

B = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5)(5, 6)}

C = Getting at most 5 as the sum of numbers on the two dices.

C = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}

D = Getting a sum greater than 5 but less than 10

D = {(1, 5)(1, 6) (2, 4)(2, 5) (2, 6)(3, 3)(3, 4)(3, 5)(3, 6)(4, 2)(4, 3)(4, 4)(4, 5)(5, 1) (5, 2)(5, 3) (5, 4)(6, 1) (6, 2) (6, 3)}

E = Getting at least 10 as the sum of numbers on the dices

{(4, 6)(5, 5)(5, 6)(6, 4)(6, 5)(6, 6)}

F = Getting an odd number on one of the dices

{(1, 2)(1, 4)(1, 6)(2, 1)(2, 3)(2, 5)(3, 2)(3, 4)(3, 6)(4, 1)(4, 3)(4, 5)(5, 2)(5, 4)(5, 6)(6, 1)(6, 3)(6, 5)

Now,

(i) A and B = A∩B = Φ

Since, There is no common events in Both A and B so the intersection is Null (Φ).

⇒B or C = BUC

BUC = {(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) (5, 1) (5, 2) (5, 3) (5, 4) (5, 5)(5, 6)(2, 1)(2, 2)(2, 3)(4, 1)}

⇒B and C = B∩C

B∩C = {(1, 1), (1, 2)(1, 3)(1, 4)(3, 1)(3, 2)}

⇒A and E = A∩E

A∩E = {(4, 6)(6, 4)(6, 5)(5, 6)(6, 6)}

⇒A or F = AUF

AUF = {(2, 1)(2, 2)(2, 3)(2, 4)(2, 5)(2, 6)(4, 1)(4, 2)(4, 3)(4, 4)(6, 1)(6, 2)(6, 3)(6, 4)(6, 5)(6, 6)(1, 2)(1, 4)(1, 6)(3, 2)(3, 4)(3, 6)(4, 5)(5, 2)(5, 4)(5, 6)}

(ii)

(a) True, because A∩B = Φ

(b) True, because A∩B = Φ and AUB = S

(c) False, because A∩C = Φ

(d) False, because C∩D = Φ but CUD≠S

(e) True, because C∩D∩E = Φ and CUDUE = S

(f) True, because A'∩B' = Φ

(g) False, because A∩B∩F = Φ and AUBUF = S

8. Question

The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are then put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the following events:

A = The number on the first slip is larger than the one on the second slip.

B = The number on the second slip is greater than 2

C = The sum of the numbers on the two slips is 6 or 7

D = The number on the second slips is twice that on the first slip.

Which pair (s) of events is (are) mutually exclusive?

Answer

Given: There are 4 slips in the box and mixed thoroughly.

To Find: Describe the given events .

Explanation: Here, Four slips of paper 1, 2, 3, and 4 are put in a box.

If Two slips are drawn from it one after the other without replacement. Then

The sample space for the experiment is:

S = {(1, 2)(1, 3), (1, 4)(2, 1)(2, 3)(2, 4)(3, 1)(3, 2)(3, 4)(4, 1)(4, 2)(4, 3)}

(i) A = number on the first slip is larger than the one on the second slip,

So, The sample space for A is:

{(2, 1)(3, 1)(3, 2)(4, 1)(4, 2)(4, 3)}

(ii) B = number on the second slip is greater than 2

So, The sample space for B is

{(1, 3)(2, 3)(1, 4)(2, 4)(3, 4)(4, 3)}

(iii) C = sum of the numbers on the two slips in 6 or 7

The sample space for C is

{(2, 4)(3, 4)(4, 2)(4, 3)}

(iv) D = number on the second slip is two times the number on the first slip

The sample space if:

{(1, 2)(2, 4)}

Now,

We can see, A∩D = Φ

Therefore, A and D are mutually exclusive events

Hence, A and D are mutually exclusive events.

9. Question

A card is picked up from a deck of 52 playing cards.

(i) What is the sample space of the experiment?

(ii) What is the event that the chosen card is a black faced card?

Answer

Given: There is a deck of 52 playing card.

To Find: What is the sample space for the experiment and what is the event that the chosen card is black faced red.

Explanation: If a card is picked up from a deck of 52 playing cards, then the sample space of this experiment S is

(i) S = 52 cards in a deck

(ii) Let E is the event, and a black face card is chosen,

Then The possible outcomes in E are jack, queen, and king of clubs and the jack, queen and king of spades.

E = {J4, Q4, K4, Q*, KAM)

Hence, E = {J4, Q4, K4, Q*, KAM)

Exercise 33.3

1. Question

Which of the following cannot be valid assignment of probability for elementary events or outcomes of sample space S = {w_1, w_2, w_3, w_4, w_5, w_6, w_7}:

Elementary events:	w_1	w_2	w_3	w_4	w_5	w_6	w_7
(i)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(ii)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(iii)	0.7	0.6	0.5	0.4	0.3	0.2	0.1
(iv)	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{15}{14}$

Answer

for each event to be a valid assignment of probability, the probability of each event in sample space should be less than 1 and the sum of probability of all the events should be exactly equal to 1

(i) it is valid as each $P(w_i)$ (for i=1 to 7) lies between 0 to 1 and sum of $P(w_1)$ =1

(ii) it is valid as each $P(w_i)$ (for i=1 to 7) lies between 0 to 1 and sum of $P(w_1)$ =1

(iii) it is not valid as sum of $P(w_i)$=2.8 which is greater than 1

(iv) it is not valid as $P(w_7) = \frac{15}{14}$ which is greater than 1

2. Question

A die is thrown. Find the probability of getting:

(i) a prime number

(ii) 2 or 4

(iii) a multiple of 2 or 3

Answer

given: die is thrown

Therefore, the total number of outcomes is six

n(S)=6

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

(i) let E be the event of getting a prime number

E= {2,3,5}

n(E)=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{6} = \frac{1}{2}$$

(ii) let E be the event of getting 2 or 4

E= {2,4}

n(E)=2

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{2}{6} = \frac{1}{3}$$

(iii) let E be the event of getting a multiple of 2 or 3

E= {2,3,4,6}

n(E)=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

3. Question

In a simultaneous throw of a pair of dice, find the probability of getting:

(i) 8 as the sum

(ii) a doublet

(iii) a doublet of prime numbers

(iv) an even number on first

(v) a sum greater than 9

(vi) an even number on first

(vii) an even number on one and a multiple of 3 on the other

(viii) neither 9 nor 11 as the sum of the numbers on the faces

(ix) a sum less than 6

(x) a sum less than 7

(xi) a sum more than 7

(xii) neither a doublet nor a total of 10

(xiii) odd number on the first and 6 on the second

(xiv) a number greater than 4 on each die

(xv) a total of 9 or 11

(xvi) a total greater than 8

Answer

given: a pair of dice have been thrown, so the number of elementary events in sample space is $6^2=36$

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

therefore n(S)=36

(i) let E be the event that the sum 8 appears

E= {(2,6) (3,5) (4,4) (5,3) (6,2)}

n(E)=5

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{5}{36}$$

(ii) let E be the event of getting a doublet

E= {(1,1) (2,2) (3,3) (4,4) (5,5) (6,6)}

n(E)=6

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{6}{36}=\frac{1}{6}$$

(iii) let E be the event of getting a doublet of prime numbers

E= {((2,2) (3,3) (5,5)}

n(E)=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{36} = \frac{1}{12}$$

(iv) let E be the event of getting a doublet of odd numbers

E= {(1,1) (3,3) (5,5)}

n(E)=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{36} = \frac{1}{12}$$

(v) let E be the event of getting sum greater than 9

E= {(4,6) (5,5) (5,6) (6,4) (6,5) (6,6)}

n(E)=6

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

(vi) let E be the event of getting even on first die

E= {(2,1) (2,2) (2,3) (2,4) (2,5) (2,6) (4,1) (4,2) (4,3) (4,4) (4,5) (4,6) (6,1) (6,2) (6,3) (6,4) (6,5) (6,6)}

n(E)=18

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{18}{36} = \frac{1}{2}$$

(vii) let E be the event of getting even on one and multiple of three on other

E= {(2,3) (2,6) (4,3) (4,6) (6,3) (6,6) (3,2) (3,4) (3,6) (6,2) (6,4)}

n(E)=11

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{11}{36}$$

(viii) let E be the event of getting neither 9 or 11 as the sum

E= {(3,6) (4,5) (5,4) (5,6) (6,3) (6,5)}

n(E)=6

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

(ix) let E be the event of getting sum less than 6

E= {(1,1) (1,2) (1,3) (1,4) (2,1) (2,2) (2,3) (3,1) (3,2) (4,1)}

n(E)=10

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{10}{36} = \frac{5}{18}$$

(x) let E be the event of getting sum less than 7

E= {(1,1) (1,2) (1,3) (1,4) (1,5) (2,1) (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (4,1) (4,2) (5,1)}

n(E)=15

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{15}{36} = \frac{5}{12}$$

(xi) let E be the event of getting more than 7

E= {(2,6) (3,5) (3,6) (4,4) (4,5) (4,6) (5,3) (5,4) (5,5) (5,6) (6,2) (6,3) (6,4) (6,5) (6,6)}

n(E)=15

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{15}{36} = \frac{5}{12}$$

(xii) let E be the event of getting neither a doublet nor a total of 10

E' be the event that either a doublet or a sum of ten appears

E'= {(1,1) (2,2) (3,3) (4,6) (5,5) (6,4) (6,6) (4,4)}

n(E') =8

$$P(E') = \frac{n(E')}{n(S)}$$

$$P(E') = \frac{8}{36} = \frac{2}{9}$$

Therefore P(E)=1-P(E')

$$P(E) = 1 - \frac{2}{9} = \frac{7}{9}$$

(xiii) let E be the event of getting odd number on first and 6 on second

E= {(1,6) (5,6) (3,6)}

n(E)=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{36} = \frac{1}{12}$$

(xiv) let E be the event of getting greater than 4 on each die

E= {(5,5) (5,6) (6,5) (6,6)}

n(E)=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{36} = \frac{1}{9}$$

(xv) let E be the event of getting total of 9 or 11

E= {(3,6) (4,5) (5,4) (5,6) (6,3) (6,5)}

n(E)=6

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

(xvi) let E be the event of getting total greater than 8

E= {(3,6) (4,5) (4,6) (5,4) (5,5) (5,6) (6,3) (6,4) (6,5) (6,6)}

n(E)=10

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{10}{36} = \frac{5}{18}$$

4. Question

In a single throw of three dice, find the probability of getting a total of 17 or 18

Answer

given: three dices are thrown

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

Total number of possible outcomes are 6^3=216

Therefore n(S)=216

Let E be the event of getting total of 17 or 18

E= {(6,6,5) (6,5,6) (5,6,6) (6,6,6)}

n(E)=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{216} = \frac{1}{54}$$

5. Question

Three coins are tossed together. Find the probability of getting:

(i) exactly two heads

(ii) at least two heads

(iii) at least one head and one tail

Answer

given: three coins are tossed together

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

Total number of possible outcomes are $2^3=8$

(i) let E be the event of getting exactly two heads

E= {(H, H, T) (H, T, H) (T, H, H)}

n(E)=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{8}$$

(ii) let E be the event of getting at least two heads

E= {(H, H, T) (H, T, H) (T, H, H) (H, H, H)}

n(E)=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{8} = \frac{1}{2}$$

(iii) let E be the event of getting at least one head and one tail

E= {(H, T, T) (T, H, T) (T, T, H) (H, H, T) (H, T, H) (T, H, H)}

n(E)=6

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{8} = \frac{3}{4}$$

6. Question

What is the probability that an ordinary year has 53 Sundays?

Answer

given: an ordinary year which includes 52 weeks and one day

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

so, we have to determine the probability of that one day being Sunday

Total number of possible outcomes are 7

Therefore n(S)=7

E= {M, T, W, T, F, S, SU}

n(E)=1

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{7}$$

7. Question

What is the probability that a leap year has 53 Sundays and 53 Mondays?

Answer

given: a leap year which includes 52 weeks and two days

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

so, we have to determine the probability of that remaining two days are Sunday and Monday

S= {MT, TW, WT, TF, FS, SSu, SuM}

Therefore n(S)=7

E= {SuM}

n(E)=1

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{7}$$

8. Question

A bag contains 8 red and 5 white balls. Three balls are drawn at random. Find the probability that:

(i) All the three balls are white

(ii) All the three balls are red

(iii) One ball is red and two balls are white

Answer

given: bag which contains 8 red and 5 white balls

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

total number of ways of drawing three balls at random is ${}^{13}C_3$

therefore n(S)=286

(i) let E be the event of getting all white balls

E= {(W) (W) (W)}

n(E)= ${}^{5}C_3$=10

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{10}{286} = \frac{5}{143}$$

(ii) let E be the event of getting all red balls

E= {(R) (R) (R)}

n(E)= ${}^{8}C_3$=56

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{56}{286} = \frac{28}{143}$$

(iii) let E be the event of getting one red and two white balls

n(E)= ${}^8C_1{}^5C_2$=80

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{80}{286} = \frac{40}{143}$$

9. Question

In a single throw of three dice, find the probability of getting the same number on all the three dice

Answer

given: three dice are rolled

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

so, we have to determine the probability of getting the same number on all the three dice

total number of possible outcomes are 6^3=216

therefore n(S)=216

let E be the event of getting same number on all the three dice

E= {(1,1,1) (2,2,2) (3,3,3) (4,4,4) (5,5,5) (6,6,6)}

n(E)=6

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{216} = \frac{1}{36}$$

10. Question

Two unbiased dice are thrown. Find the probability that the total of the numbers on the dice is greater than 10

Answer

given: three dice are rolled

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

so, we have to determine the probability of getting the sum of digits on dice greater than 10

total number of possible outcomes are 6^2=36

therefore n(S)=36

let E be the event of getting same number on all the three dice

E= {(5,6) (6,5) (6,6)}

n(E)=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{36} = \frac{1}{12}$$

11. Question

A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is ;

(i) a black king

(ii) either a black card or a king

(iii) black and a king

(iv) a jack, queen or a king

(v) neither an ace nor a king

(vi) spade or an ace

(vii) neither an ace nor a king

(viii) a diamond card

(ix) not a diamond card

(x) a black card

(xi) not an ace

(xii) not a black card

Answer

given: pack of 52 cards

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

since a card is drawn from a pack of 52 cards, therefore number of elementary events in the sample space is

$n(S) = {}^{52}C_1 = 52$

(i) let E be the event of drawing a black king

$n(E) = {}^{2}C_1 = 2$ (there are two black kings one of spade and other of club)

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{2}{52} = \frac{1}{26}$$

(ii) let E be the event of drawing a black card or a king

$n(E) = {}^{26}C_1 + {}^{4}C_1 - {}^{2}C_1 = 28$

we are subtracting 2 from total because there are two black king which are already counted and to avoid the error of considering it twice

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{28}{52} = \frac{7}{13}$$

(iii) let E be the event of drawing a black card and a king

$n(E) = {}^{2}C_1 = 2$ (there are two black kings one of spade and other of club)

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{2}{52}=\frac{1}{26}$$

(iv) let E be the event of drawing a jack, queen or king

$n(E)={}^{4}C_{1}+{}^{4}C_{1}+{}^{4}C_{1}=12$

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{12}{52}=\frac{3}{13}$$

(v) let E be the event of drawing neither a heart nor a king

now consider E’ as the event that either a heart or king appears

$n(E')={}^{6}C_{1}+{}^{4}C_{1}-1=16$ (there is a heart king so it is deducted)

$$P(E')=\frac{n(E')}{n(S)}$$

$$P(E')=\frac{16}{52}=\frac{4}{13}$$

P(E)=1-P(E’)

$$P(E)=1-\frac{4}{13}=\frac{9}{13}$$

(vi) let E be the event of drawing a spade or king

$n(E)={}^{13}C_{1}+{}^{4}C_{1}-1=16$

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{16}{52}=\frac{4}{13}$$

(vii) let E be the event of drawing neither an ace nor a king

now consider E’ as the event that either an ace or king appears

$n(E')={}^{4}C_{1}+{}^{4}C_{1}=8$

$$P(E')=\frac{n(E')}{n(S)}$$

$$P(E')=\frac{8}{52}=\frac{2}{13}$$

P(E)=1-P(E’)

$$P(E)=1-\frac{2}{13}=\frac{11}{13}$$

(viii) let E be the event of drawing a diamond card

$n(E)={}^{13}C_{1}=13$

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{13}{52}=\frac{1}{4}$$

(ix) let E be the event of drawing not a diamond card

now consider E’ as the event that diamond card appears

n(E’) $={}^{13}C_1=13$

$$P(E')=\frac{n(E')}{n(S)}$$

$$P(E')=\frac{13}{52}=\frac{1}{4}$$

P(E)=1-P(E’)

$$P(E)=1-\frac{1}{4}=\frac{3}{4}$$

(x) let E be the event of drawing a black card

n(E)$={}^{26}C_1=26$ (spades and clubs)

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{26}{52}=\frac{1}{2}$$

(xi) let E be the event of drawing not an ace

now consider E’ as the event that ace card appears

n(E’) $={}^{4}C_1=4$

$$P(E')=\frac{n(E')}{n(S)}$$

$$P(E')=\frac{4}{52}=\frac{1}{13}$$

P(E)=1-P(E’)

$$P(E)=1-\frac{1}{13}=\frac{12}{13}$$

(xii) let E be the event of not drawing a black card

n(E)$={}^{26}C_1=26$ (red cards of hearts and diamonds)

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{26}{52}=\frac{1}{2}$$

12. Question

In shutting a pack of 52 playing cards, four are accidently dropped; find the chance that the missing cards should be one from each suit

Answer

given: pack of 52 cards from which 4 are dropped

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability that the missing cards should be one from each suit

since from well shuffled pack of cards, 4 cards missed out total possible outcomes are

$n(S)= {}^{52}C_4=270725$

let E be the event that four missing cards are from each suite

$n(E)= {}^{13}C_1\times{}^{13}C_1\times{}^{13}C_1\times{}^{13}C_1=13^4$

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{13^4}{270725} = \frac{2197}{20825}$

13. Question

From a deck of 52 cards, four cards are drawn simultaneously, find the chance that they will be the four honors of the same suit

Answer

given: deck of 52 cards

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability that all the face cards of same suits are drawn

total possible outcomes are

$n(S)={}^{52}C_4$

let E be the event that all the cards drawn are face cards of same suit

$n(E)=4\times{}^{4}C_4=4$

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{4}{270725}$

14. Question

Tickets numbered from 1 to 20 are mixed up together and then a ticket is drawn at random. What is the probability that the ticket has a number which is a multiple of 3 or 7?

Answer

given: numbered tickets from 1 to 20

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

to find the probability of the ticket drawn having a number which is a multiple of 3 or 7

Since one ticket is drawn from a lot of mixed number, total possible outcomes are

$n(S)={}^{20}C_1=20$

let E be the event of getting ticket which has number that is multiple of 3 or 7

E= {3,6,9,12,15,18,7,14}

n(E)=8

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{8}{20}=\frac{2}{5}$$

15. Question

A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that one is red, one is white and one is blue

Answer

given: 6 red, 4 white and 8 blue balls

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

three balls are drawn so, we have to find the probability that one is red, one is white and one is blue

total number of outcomes for drawing 3 balls are ${}^{18}C_3$

Therefore n(S)= ${}^{18}C_3$=816

Let E be the event that one red, one white and one blue ball is drawn

n(E)= ${}^{6}C_1{}^{4}C_1{}^{8}C_1$=192

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{192}{816}=\frac{4}{17}$$

16. Question

A bag contains 7 white, 5 black and 4 red balls. If two balls are drawn at random, find the probability that:

(i) both the balls are white

(ii) one ball is black and the other red

(iii) both the balls are of the same colour

Answer

given: bag which contains 4 red, 5 black and 7 white balls

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two balls are drawn at random, therefore

total possible outcomes are ${}^{16}C_2$

therefore n(S)=120

(i) let E be the event of getting both white balls

E= {(W) (W)}

n(E)= ${}^{7}C_2$=21

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{21}{120}=\frac{7}{40}$$

(ii) let E be the event of getting one black and one red ball

E= {(B) (R)}

n(E)= ${}^{5}C_{1}{}^{4}C_{1}=20$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{20}{120} = \frac{1}{6}$$

(iii) let E be the event of getting both balls of same colour

E= {(B) (B)} or {(W) (W)} or {(R) (R)}

n(E)= ${}^{7}C_{2}+{}^{5}C_{2}+{}^{4}C_{2}=37$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{37}{120}$$

17. Question

A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that:

(i) one is red and two are white

(ii) two are blue and one is red

(iii) one is red

Answer

given: bag which contains 6 red, 8 blue and 4 white balls

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two balls are drawn at random, therefore

total possible outcomes are ${}^{18}C_{3}$

therefore n(S)=816

(i) let E be the event of getting one red and two white balls

E= {(W) (W) (R)}

n(E)= ${}^{6}C_{1}{}^{4}C_{2}=36$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{36}{816} = \frac{3}{68}$$

(ii) let E be the event of getting two blue and one red

E= {(B) (B) (R)}

n(E)= ${}^{8}C_{2}{}^{6}C_{1}=168$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{168}{816} = \frac{7}{34}$$

(iii) let E be the event that one of the balls must be red

E= {(R) (B) (B)} or {(R) (W) (W)} or {(R) (B) (W)}

n(E)= ${}^6C_1{}^4C_1{}^8C_1+{}^6C_1{}^4C_2+{}^6C_1{}^8C_2=396$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{396}{816} = \frac{33}{68}$$

18. Question

Five cards are drawn from a pack of 52 cards. What is the chance that these 5 will contain:

(i) just one ace

(ii) at least one ace?

Answer

given: pack of 52 playing cards

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

five cards are drawn at random, therefore

total possible outcomes are ${}^{52}C_5$

therefore n(S)=2598960

(i) let E be the event that exactly only one ace is present

n(E)= ${}^4C_1{}^{48}C_4=778320$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{778320}{2598960} = \frac{3243}{10829}$$

(ii) let E be the event that at least one ace is present

E= {1 or 2 or 3 or 4 ace(s)}

n(E)= ${}^4C_1{}^{48}C_4+{}^4C_2{}^{48}C_3+{}^4C_3{}^{48}C_2+{}^4C_4{}^{48}C_1=886656$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{886656}{2598960} = \frac{18472}{54145}$$

19. Question

The face cards are removed from a full pack. Out of the remaining 40 cards, 4 are drawn at random. What is the probability that they belong to different suits?

Answer

given: pack of 52 cards from which face cards are removed

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

four cards are drawn from the remaining 40 cards, so we have to find the probability that all of them belong to different suit

total possible outcomes of drawing four cards are ${}^{40}C_4$

therefore n(S)= ${}^{40}C_4$

let E be the event that 4 cards belong to different suit

n(E)= ${}^{10}C_1{}^{10}C_1{}^{10}C_1{}^{10}C_1$=10000

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{10000}{91390} = \frac{1000}{9139}$$

20. Question

There are four men and six women on the city councils. If one council member is selected for a committee at random, how likely is that it is a women?

Answer

given: four men and six women

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

from the city council one person is selected as a council member so, we have to find the probability that it is a woman

total possible outcomes of selecting a person is ${}^{10}C_1$

therefore n(S)= ${}^{10}C_1$

let E be the event that it is a woman

n(E)= ${}^{6}C_1$=6

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{10} = \frac{3}{5}$$

21. Question

A box contains 100bulbs, 20 of which are defective. 10 bulbs are selected for inspection. Find the probability that:

(i) all 10 are defective

(ii) all 10 are good

(iii) at least one is defective

(iv) none is defective

Answer

given: box with 100 bulbs of which, 20 are defective

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

ten bulbs are drawn at random for inspection, therefore

total possible outcomes are ${}^{100}C_{10}$

therefore n(S)= ${}^{100}C_{10}$

(i) let E be the event that all ten bulbs are defective

n(E)= ${}^{20}C_{10}$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{20_{C_{10}}}{100_{C_{10}}}$$

(ii) let E be the event that all ten good bulbs are selected

n(E)= ${}^{80}C_{10}$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{80_{C_{10}}}{100_{C_{10}}}$$

(iii) let E be the event that at least one bulb is defective

E= {1,2,3,4,5,6,7,8,9,10} where 1,2,3,4,5,6,7,8,9,10 are the number of defective bulbs

Let E' be the event that none of the bulb is defective

n(E') = ${}^{80}C_{10}$

$$P(E') = \frac{n(E')}{n(S)}$$

$$P(E') = \frac{80_{C_{10}}}{100_{C_{10}}}$$

Therefore,

P(E)=1-P(E')

$$P(E) = 1 - P(E')$$

$$P(E) = 1 - \frac{80_{C_{10}}}{100_{C_{10}}}$$

(iv) let E be the event that none of the selected bulb is defective

n(E)= ${}^{80}C_{10}$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{80_{C_{10}}}{100_{C_{10}}}$$

22. Question

Find the probability that in a random arrangement of the letters of the word 'SOCIAL' vowels come together

Answer

given: word "SOCIAL"

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

In the random arrangement of the alphabets of word “SOCIAL” we have to find the probability that vowels come together

total possible outcomes of arranging the alphabets are 6!

therefore n(S)=6!

let E be the event that vowels come together

number of vowels in SOCIAL is A, I, O

therefore, number of ways to arrange them so (A, I, O) come together

n(E)= 4! × 3!

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4!\,3!}{6!} = \frac{1}{5}$$

23. Question

The letters of the word ‘CLIFTON’ are placed at random in a row. What is the chance that two vowels come together?

Answer

given: word “CLIFTON”

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

In the random arrangement of the alphabets of word “CLIFTON” we have to find the probability that vowels come together

total possible outcomes of arranging the alphabets are 7!

therefore n(S)=7!

let E be the event that vowels come together

number of vowels in SOCIAL is I, O

therefore, number of ways to arrange them so (I, O) come together

n(E)= 6! × 2!

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6!\,2!}{7!} = \frac{2}{7}$$

24. Question

The letters of the word “FORTUNATES’ are arranged at random in a row. What is the chance that the two ‘T’ come together?

Answer

given: word “FORTUNATES”

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

In the random arrangement of the alphabets of word “FORTUNATES” we have to find the probability that two T’s come together

total possible outcomes of arranging the alphabets are 10!

therefore n(S)=10!

let E be the event that T's come together

therefore, number of ways to arrange them so (T, T) come together

n(E)= 9! × 2!

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{9!\,2!}{10!} = \frac{1}{5}$$

25. Question

A committee of two persons is selected from two men and two women. What is the probability that the committee will have (i) no man? (ii) one man? (iii) two men?

Answer

given: two men and two women

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

committee of two persons is to be formed from two men and two women, therefore

total possible outcomes of selecting two persons is ${}^4C_2$

therefore n(S)=6

(i) let E be the event that no man is in the committee

E= {W, W}

n(E)= ${}^2C_2$=1 (only woman)

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{6}$$

(ii) let E be the event that one man is present in committee

E= {M W}

n(E)= ${}^2C_1{}^2C_1$=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

(iii) let E be the event that two men is in the committee

E= {M, M}

n(E)= ${}^2C_2$=1 (only men)

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{6}$$

26. Question

If odds in favour of an event be 2: 3, find the probability of occurrence of this event

Answer

given: odds in favour of event is 2:3

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability of occurrence of this event

total possible outcomes is 2k+3k=5k

therefore n(S)= 5k

let E be the event that it occurs

n(E)=2k

probability of occurrence is

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{2k}{5k}=\frac{2}{5}$$

27. Question

If odds against an events be 7: 9, find the probability of non-occurrence of this event.

Answer

given: odds against of event is 7:9

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability of non-occurrence of this event

total possible outcomes are 7k+9k=16k

therefore n(S)= 16k

let E be the event that it occurs

n(E)=9k

probability of occurrence is

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{9k}{16k}=\frac{9}{16}$$

Therefore, the probability of non-occurrence of the event is

$$p(E')=1-P(E)$$

$$p(E')=1-\frac{9}{16}=\frac{7}{16}$$

28. Question

Two balls are drawn at random from a bag containing 2 white, 3 red, 5 green and 4 black balls, one by one without, replacement. Find the probability that both the balls are of different colours.

Answer

given: 2 white, 3 red, 5 green, 4 black

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two balls are drawn one by one, we have to find the probability that they are of different colours

total possible outcomes are $^{14}C_2$

therefore n(S)= $^{14}C_2$=91

let E be the event that all balls are of same colour

E= {WW, RR, GG, BB}

n(E)= $^{2}C_2+^{3}C_2+^{5}C_2+^{4}C_2$=20

probability of occurrence is

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{20}{91}$$

Therefore, the probability of non-occurrence of the event (all balls are different) is

$$p(E') = 1 - P(E)$$

$$p(E') = 1 - \frac{20}{91} = \frac{71}{91}$$

29. Question

Two unbiased dice are thrown. Find the probability that:

(i) neither a doublet nor a total of 8 will appear

(ii) the sum of the numbers obtained on the two dice is neither a multiple of 2 nor a multiple of 3

Answer

given: two unbiased dice are thrown

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

total possible outcomes of from the dice is $^{6}C_1{}^{6}C_1$

therefore n(S)=36

(i) let E be the event that neither a doublet nor a total of 8 will appear

E’ be the event that a doublet or a total of 8 occurs

E’= {(1,1) (2,2) (3,3) (4,4) (5,5) (6,6) (2,6) (6,2) (3,5) (5,3)}

n(E’) = 10

$$P(E') = \frac{n(E')}{n(S)}$$

$$P(E') = \frac{10}{36}$$

Therefore P(E) is

$$p(E') = 1 - P(E)$$

$$p(E') = 1 - \frac{10}{36} = \frac{26}{36} = \frac{13}{18}$$

(ii) let E be the event that sum of number obtain on the dice is neither a multiple of 2 or 3

E’ be the event that sum of number obtain on the dice is either a multiple of 2 or 3, that is total should be 2,3,4,6,8,9,10,12

E’= {(1,1) (1,2) (2,1) (1,3) (2,2) (3,1) (1,5) (2,4) (3,3) (4,2) (5,1) (2,6) (3,5) (4,4) (5,3) (6,2) (3,6) (4,5) (5,4) (6,3) (4,6) (5,5) (6,4) (6,6)}

n(E’) = 24

$$P(E') = \frac{n(E')}{n(S)}$$

$$P(E') = \frac{24}{36}$$

Therefore P(E) is

$$p(E') = 1 - P(E)$$

$$p(E') = 1 - \frac{24}{36} = \frac{12}{36} = \frac{1}{3}$$

30. Question

A bag contains 8 red, 3 white and 9 blue balls. If three balls are drawn at random, determine the probability that (i) all the three balls are blue balls 9ii) all the balls are of different colours

Answer

given: bag which contains 8 red, 3 white, 9 blue balls

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

three balls are drawn at random therefore

total possible outcomes of selecting two persons is ${}^{20}C_3$

therefore n(S)=${}^{20}C_3$=1140

(i) let E be the event that all the balls are blue

E= {B, B, B}

n(E)= ${}^{9}C_3$=84

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{84}{1140} = \frac{7}{95}$$

(ii) let E be the event that all balls are of different colour

E= {B W R}

n(E)= ${}^{8}C_1{}^{3}C_1{}^{9}C_1$=216

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{216}{1140} = \frac{18}{95}$$

31. Question

A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both balls are red or both are black?

Answer

given: bag which contains 5 red, 6 white, 7 black balls

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two balls are drawn at random

total possible outcomes are ${}^{18}C_2$

therefore n(S)= ${}^{18}C_2=153$

let E be the event that both balls are either red or black

$n(E)={}^{5}C_2+{}^{7}C_2=31$

probability of occurrence of the event is

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{31}{153}$$

32. Question

If a letter is chosen at random from the English alphabet, find the probability that the letter is (i) a vowel (ii) a constant

Answer

given: a letter is chosen at random from English alphabet

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

total possible outcomes of selecting an alphabet is ${}^{26}C_1$

therefore $n(S)={}^{26}C_1=26$

(i) let E be the event that a vowel has been chosen

E= {a, e, i, o, u}

n(E)= ${}^{5}C_1=5$

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{5}{26}$$

(ii) let E be the event that a consonant is chosen

E= all alphabets - {a, e, i, o, u}

n(E)= ${}^{21}C_1=21$

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{21}{26}$$

33. Question

In a lottery, a person chooses six different numbers at random from 1 to 20, and if these six numbers match with six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?

Answer

given: six numbers are chosen from 1-20

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability of winning the prize

total possible outcomes of selecting six numbers from 1-20 is ${}^{20}C_6$

therefore n(S)= ${}^{20}C_6$=38760

let E be the event that all six numbers match with the given number (as winning number is fixed)

n(E)=1

probability of occurrence of the event is

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{1}{38760}$$

34. Question

20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number on the cards is:

(i) a multiple of 4?

(ii) not a multiple of 4?

(iii) odd?

(iv) greater than 12?

(v) divisible by 5?

(vi) not a multiple of 6?

Answer

given: 20 cards numbered from 1-20

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

one card is drawn at random therefore total possible outcomes are ${}^{20}C_1$

therefore n(S)=${}^{20}C_1$=20

(i) let E be the event that the number on the drawn card is a multiple of 4

E= {4, 8, 12, 16, 20}

n(E)= ${}^{5}C_1$=5

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{5}{20} = \frac{1}{4}$$

(ii) let E be the event that the number on the drawn card is not a multiple of 4

E' be the event that the number on the drawn card is a multiple of 4

E'= {4, 8, 12, 16, 20}

n(E)= $^{5}C_{1}$=5

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{5}{20} = \frac{1}{4}$$

P(E)=1-P(E')

$$P(E) = 1 - \frac{1}{4} = \frac{3}{4}$$

(iii) let E be the event that the number on the drawn card is odd

E= {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

n(E)= $^{10}C_{1}$=10

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{10}{20} = \frac{1}{2}$$

(iv) let E be the event that the number on the drawn card is greater than 12

E= {13, 14, 15, 16, 17, 18, 19, 20}

n(E)= $^{8}C_{1}$=8

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{8}{20} = \frac{2}{5}$$

(v) let E be the event that the number on the drawn card is a multiple of 5

E= {5, 10, 15, 20}

n(E)= $^{4}C_{1}$=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{20} = \frac{1}{5}$$

(vi) let E be the event that the number on the drawn card is not divisible by 6

let E' be the event that number on the drawn card is divisible by 6

E'= {6, 12, 18}

n(E')= $^{3}C_{1}$=3

$$P(E') = \frac{n(E')}{n(S)}$$

$$P(E') = \frac{3}{20}$$

P(E)=1-P(E’)

$$P(E) = 1 - \frac{3}{20} = \frac{17}{20}$$

35. Question

Two dice are thrown. Find the odds in favour of getting the sum

(i) 4 (b) 5

(iii) What are the odds against getting the sum 6?

Answer

given: two dices are thrown

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

total possible outcomes are ${}^6C_1{}^6C_1$

therefore n(S)=6^2=36

(i) let E be the event that total sum is 4 on dice

E= {(1,3) (3,1) (2,2)}

n(E)= ${}^3C_1$=3

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{3}{36} = \frac{1}{12}$$

Therefore, probability of event E’ is

P(E’) =1-P(E)

$$P(E') = 1 - \frac{1}{12} = \frac{11}{12}$$

Odds in favour of getting sum as 4 is P(E):P(E’) =1:11

(ii) let E be the event that total sum is 5 on dice

E= {(1,4) (2,3) (3,2) (4,1)}

n(E)= ${}^4C_1$=4

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{36} = \frac{1}{9}$$

Therefore, probability of event E’ is

P(E’) =1-P(E)

$$P(E') = 1 - \frac{1}{9} = \frac{8}{9}$$

Odds in favour of getting sum as 5 is P(E):P(E’) =1:8

(iii) let E be the event that total sum is 6 on dice

E= {(1,5) (2,4) (3,3) (4,2) (5,1)}

n(E)= $^5C_1$=5

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{5}{36}$$

Therefore, probability of event E’ is

P(E’) =1-P(E)

$$P(E')=1-\frac{5}{36}=\frac{31}{36}$$

Odds against of getting sum as 6 is P(E’):P(E) =31:5

36. Question

What are the odds in favour of getting a spade if a card is drawn from a well-shuffled deck of cards? What are the odd in favour of getting a king?

Answer

given: pack of 52 playing cards

formula: $P(E)=\frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

one card is drawn from the given pack of cards, total possible outcomes are $^{52}C_1$

therefore n(S)=$^{52}C_1$=52

(i) let E be the event of getting a spade

n(E)= $^{13}C_1$=13

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{13}{52}=\frac{1}{4}$$

Therefore, probability of event E’ is

P(E’) =1-P(E)

$$P(E')=1-\frac{1}{4}=\frac{3}{4}$$

Odds in favour of getting a spade is P(E):P(E’) =1:3

(ii) let E be the event of getting a king

n(E)= $^4C_1$=4

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{4}{52}=\frac{1}{13}$$

Therefore, probability of event E’ is

P(E’) =1-P(E)

$$P(E')=1-\frac{1}{13}=\frac{12}{13}$$

Odds in favour of getting a king is P(E):P(E') =1:12

37. Question

A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn at random. From the box, what is the probability that:

(i) al are blue?

(ii) at least one is green?

Answer

given: box containing 10 red, 20 blue and 30 green marbles

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

five marbles are drawn from the given box, total possible outcomes are $^{60}C_5$

therefore n(S)=$^{60}C_5$

(i) let E be the event of getting all blue balls

n(E)= $^{20}C_5$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{20_{C_5}}{60_{C_5}} = \frac{34}{11977}$$

(ii) let E be the event of getting at least one green

let E' be the event of getting no green

E'= {(5B) (1R 4B) (2R 3B) (3R 2B) (4R 1B) (5R)}

5B=$^{20}C_5$

1R 4B=$^{10}C_1{}^{20}C_4$

2R 3B=$^{10}C_2{}^{20}C_3$

3R 2B=$^{10}C_3{}^{20}C_2$

4R 1B=$^{10}C_4{}^{20}C_1$

5R= $^{10}C_5$

P(E) =1-P(E')

$$P(E') = \frac{20_{C_5} + 10_{C_1}20_{C_4} + 10_{C_2}20_{C_3} + 10_{C_3}20_{C_2} + 10_{C_4}20_{C_1} + 10_{C_5}}{60_{C_5}}$$

$$P(E') = \frac{117}{4484}$$

$$P(E) = 1 - \frac{117}{4484} = \frac{4367}{4484}$$

38. Question

A box contains 6 red marbles numbered 1 through 6 and 4 white marbles numbered from 12 through 15. Find the probability that a marble drawn is (i) white (ii) white and odd numbered (iii) even numbered (iv) red or even numbered.

Answer

given: box containing 6 red marbles numbered 1-6, 4 white marbles numbered 12-15

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

one marble is drawn from the given box, total possible outcomes are $^{10}C_1$

therefore $n(S)={}^{10}C_1=10$

(i) let E be the event of getting white marble

$n(E)= {}^4C_1=4$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{4}{10} = \frac{2}{5}$$

(ii) let E be the event of getting white marble with odd numbered

E= {13,15}

n(E)= 2

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{2}{10} = \frac{1}{5}$$

(iii) let E be the event of getting even numbered marble

E= {2, 4, 6, 12, 24}

n(E)= 5

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{5}{10} = \frac{1}{2}$$

(iv) let E_1 be the event of getting red marble

$P(E_1) = \frac{6}{10}$ (from (i))

Let E_2 be the event of getting even numbered marble

$P(E_2) = \frac{5}{10}$ (from (ii))

Therefore $(E_1 \cap E_2)$ = red coloured and even numbered

$n(E_1 \cap E_2) = 3$

$$P(E_1 \cap E_2) = \frac{3}{10}$$

By law of addition $P(E_1 \cup E_2) = P(E_1)+P(E_2)- P(E_1 \cap E_2)$

$$P(E_1 \cup E_2) = \frac{6}{10} + \frac{5}{10} - \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$$

39. Question

A class consists of 10 boys and 8 girls. Thee students are selected at random. What is the probability that the selected group has (i) all boys? (ii) all girls? (iii) 1 boy and 2 girls? (iv) at least one girl? (v) at most one girl?

Answer

given: class consisting of 10 boys and 8 girls

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

three students are selected at random, total possible outcomes are $^{18}C_3$

therefore $n(S)={}^{18}C_3=816$

(i) let E be the event that all are boys

$n(E)= {}^{10}C_3=120$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{120}{816} = \frac{5}{34}$$

(ii) let E be the event that all are girls

$n(E)= {}^{8}C_3=56$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{56}{816} = \frac{7}{102}$$

(iii) let E be the event that one boy and two girls are selected

$n(E)= {}^{8}C_1{}^{10}C_2=360$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{360}{816} = \frac{35}{102}$$

(iv) let E be the event that at least one girl is in the group

E= {1,2,3}

$n(E)= {}^{8}C_1{}^{10}C_2+{}^{8}C_2{}^{10}C_1+{}^{8}C_3{}^{10}C_0=696$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{693}{816} = \frac{29}{34}$$

(v) let E be the event that at most one girl is in the group

E= {0, 1}

$n(E)= {}^{8}C_0{}^{10}C_3+{}^{8}C_1{}^{10}C_2=480$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{480}{816} = \frac{10}{17}$$

40. Question

Five cards are drawn from a well-shuffled pack of 52 cards. Find the probability that all the five cards are hearts.

Answer

given: pack of 52 playing cards

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

five cards are drawn from the given deck

we have to find the probability that all of them are hearts

total possible outcomes of one card from the pack is $^{52}C_1$

therefore n(S)= $^{52}C_5$=2598960

let E be the event that all cards belong to hearts

n(E)=$^{13}C_5$=1287

probability of occurrence of the event is

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{33}{66640}$$

41. Question

A bag contains tickets numbered from 1 to 20. Two tickets are drawn. Find the probability that (i) both the tickets have prime numbers on them (ii) on one there is a prime number and on the other, there is a multiple of 4.

Answer

given: bag containing tickets numbered 1-20

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two tickets are drawn at random, total possible outcomes are $^{20}C_2$

therefore n(S)=$^{20}C_2$=190

(i) let E be the event that both tickets have prime number

E= {2,3,5,7,11,13,17,19}

n(E)= $^{8}C_2$=28

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{28}{190} = \frac{14}{95}$$

(ii) let E be the event that one ticket has prime number and other is a multiple of 4

E= {2,3,5,7,11,13,17,19} for prime number

E= {4,8,12,16,20} for multiple of 4

n(E)= $^{8}C_1{}^{5}C_1$=40

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{40}{190} = \frac{4}{19}$

42. Question

An urn contains 7 white, 5 black and 3 red balls. Two balls are drawn at random. Find the probability that 9i) both the balls are red (ii) one ball is red and the other is black (iii) one ball is white.

Answer

given: urn containing 7 white, 5 black and 3 red

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

two balls are drawn at random, total possible outcomes are $^{15}C_2$

therefore n(S)=$^{18}C_2$=105

(i) let E be the event that both balls are red

n(E)= $^{3}C_2$=3

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{3}{105} = \frac{1}{35}$

(ii) let E be the event that one is red and other is black

n(E)= $^{3}C_1{}^{5}C_1$=15

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{15}{105} = \frac{1}{7}$

(iii) let E be the event that one ball is white

n(E)= $^{8}C_1{}^{7}C_1$=56

$P(E) = \frac{n(E)}{n(S)}$

$P(E) = \frac{56}{105} = \frac{8}{15}$

43. Question

A and B throw a pair of dice. If A throws 9, find B’s chance of throwing a higher number

Answer

given: pair of dice

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

A and B throw a pair of dice

we have to find the probability that B throw’s a higher number

total possible outcomes are $^{6}C_1{}^{6}C_1$

therefore n(S)= 6^2=36

let E be the event that A throws 9, and B throws greater than 9

E= {(4,6) (5,5) (5,6) (6,4) (6,5) (6,6)}

n(E)= 6

probability of occurrence of the event is

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

44. Question

In a hand at Whist, what is the probability that four kings are held by a specified player?

Answer

given: game of whist is being played

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability that all four kings are held by a specific player

since in one hand at whist a player has 13 cards

total possible outcomes are $^{52}C_{13}$

therefore n(S)= $^{52}C_{13}$

let E be the event that a player has 4 kings

n(E)= $^{48}C_9{}^4C_4$

probability of occurrence of this event is

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(E) = \frac{48_{C_9}}{52_{C_{13}}} = \frac{11}{4165}$$

45. Question

Find the probability that in a random arrangement of the letters of the word ‘UNIVERSITY’, the two I’s do not come together.

Answer

given: word “UNIVERSITY”

formula: $P(E) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$

we have to find the probability that two I’s do not come together

total possible outcomes for arrangement of alphabets are 10!

therefore n(S)= 10!

let E be the event that both I’s come together

n(E)= 2! × 9!

probability of occurrence of this event is

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)=\frac{2!\,9!}{10!}=\frac{2}{10}=\frac{1}{5}$$

Let E’ be the event that both I’s do not come together

Therefore, the probability that two I’s do not come together is

P(E’) =1-P(E)

$$P(E')=1-\frac{1}{5}=\frac{4}{5}$$

Exercise 33.4

1 A. Question

If A and B be mutually exclusive events associated with a random experiment such that P(A) = 0.4 and P(B) = 0.5, then find :

(i) P(A ∪ B)

(ii) $P(\overline{A}\cap\overline{B})$

(iii) $P(\overline{A}\cap B)$

(iv) $P(A\cap\overline{B})$

Answer

Given A and B are two mutually exclusive events

And, P(A) = 0.4 P(B) = 0.5

By definition of mutually exclusive events we know that:

P(A ∪ B) = P(A) + P(B)

We have to find-

i) P(A ∪ B) = P(A) + P(B) = 0.5 + 0.4 = 0.9

ii) P(A’ ∩ B’) = P(A ∪ B)’ {using De Morgan’s Law}

⇒ P(A’ ∩ B’) = 1 – P(A ∪ B) = 1 – 0.9 = 0.1

iii) P(A’ ∩ B) = This indicates only the part which is common with B and not A ⇒ This indicates only B.

P(only B) = P(B) – P(A ∩ B)

As A and B are mutually exclusive So they don’t have any common parts ⇒ P(A ∩ B) = 0

∴ P(A’ ∩ B) = P(B) = 0.5

iv) P(A ∩ B’) = This indicates only the part which is common with A and not B ⇒ This indicates only A.

P(only A) = P(A) – P(A ∩ B)

As A and B are mutually exclusive So they don’t have any common parts ⇒ P(A ∩ B) = 0

∴ P(A ∩ B’) = P(A) = 0.4

1 B. Question

A and B are two events such that P(A) = 0.54, P(B) = 0.69 and P(A ∩ B) =0.35. Find (i) P(A ∪ B), (ii) $P(\overline{A}\cap\overline{B})$ (iii) $P(A\cap\overline{B})$ (iv) $P(B\cap\overline{A})$

Answer

Given A and B are two events

And, P(A) = 0.54 P(B) = 0.69 P(A ∩ B) = 0.35

By definition of P(A or B) under axiomatic approach we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

We have to find-

i) P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= 0.54 + 0.69 – 0.35 = 0.88

ii) P(A' ∩ B') = P(A ∪ B)' {using De Morgan's Law}

⇒ P(A' ∩ B') = 1 – P(A ∪ B) = 1 – 0.88 = 0.12

iii) P(A ∩ B') = This indicates only the part which is common with A and not B ⇒ This indicates only A.

P(only A) = P(A) – P(A ∩ B)

∴ P(A ∩ B') = P(A) - P(A ∩ B) = 0.54 – 0.35 = 0.19

iv) P(A' ∩ B) = This indicates only the part which is common with B and not A ⇒ This indicates only B.

P(only B) = P(B) – P(A ∩ B)

∴ P(A' ∩ B) = P(B) – P(A ∩ B) = 0.69 – 0.35 = 0.34

1 C. Question

Fill in the blanks in the following table :

	P(A)	P(B)	P(A∩B)	P(A∪B)
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	....
(ii)	0.35	....	0.25	0.6
(iii)	0.5	0.35	....	0.7

Answer

We need to fill the table:

	P(A)	P(B)	P(A∩B)	P(A∪B)
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	....
(ii)	0.35	....	0.25	0.6
(iii)	0.5	0.35	....	0.7

i) By definition of P(A or B) under axiomatic approach we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

Using data from table, we get:

∴ P(A ∪ B) $= \frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{8}{15} - \frac{1}{15} = \frac{7}{15}$

ii) By definition of P(A or B) under axiomatic approach we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

⇒ P(B) = P(A ∪ B) + P(A ∩ B) – P(A)

Using data from table, we get:

∴ P(B) = 0.6 + 0.25 – 0.35 = 0.5

iii) By definition of P(A or B) under axiomatic approach we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

⇒ P(A ∩ B) = P(B) + P(A) - P(A ∪ B)

Using data from table, we get:

∴ P(A ∩ B) = 0.5 + 0.35 – 0.7 = 0.15

Filled table is:

	P(A)	P(B)	P(A∩B)	P(A∪B)
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	$\mathbf{\frac{7}{15}}$
(ii)	0.35	**0.5**	0.25	0.6
(iii)	0.5	0.35	**0.15**	0.7

2. Question

If A and B are two events associated with a random experiment such that P(A) = 0.3, P(B) = 0.4 and P(A ∪ B) = 0.5, find P(A ∩ B).

Answer

Given A and B are two events

And, P(A) = 0.3 P(B) = 0.5 P(A ∪ B) = 0.5

We need to find P(A ∩ B).

By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(A ∩ B) = P(A) + P(B) – P(A ∪ B)

⇒ P(A ∩ B) = 0.3 + 0.4 – 0.5 = 0.7 – 0.5 = 0.2

∴ P(A ∩ B) = 0.2

3. Question

If A and B are two events associated with a random experiment such that P(A) = 0.5, P(B) = 0.3 and P(A ∩ B) = 0.2, find P(A ∪ B).

Answer

Given A and B are two events

And, P(A) = 0.5 P(B) = 0.3 P(A ∩ B) = 0.2

We need to find P(A ∪ B).

By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

⇒ P(A ∪ B) = 0.5 + 0.3 – 0.2 = 0.8 – 0.2 = 0.6

∴ P(A ∪ B) = 0.6

4. Question

If A and B are two events associated with a random experiment such that P(A ∪ B) = 0.8, P(A ∩ B) = 0.3 and $P(\overline{A})$ = 0.5 , find P(B).

Answer

Given A and B are two events

And, P(A’) = 0.5 P(A ∩ B) = 0.3 P(A ∪ B) = 0.8

∵ P(A’) = 1 – P(A) ⇒ P(A) = 1 – 0.5 = 0.5

We need to find P(B).

By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(B) = P(A ∪ B) + P(A ∩ B) – P(A)

⇒ P(B) = 0.8 + 0.3 – 0.5 = 1.1 – 0.5 = 0.6

∴ P(B) = 0.6

5. Question

Given two mutually exclusive events A and B such that P(A) = 1/2 and P(B) = 1/3, find P(A or B).

Answer

Given A and B are two mutually exclusive events

And, P(A) = 1/2 P(B) = 1/3

We need to find P(A ‘or’ B).

P(A or B) = P(A ∪ B)

By definition of mutually exclusive events we know that:

P(A ∪ B) = P(A) + P(B)

∴ P(A ∪ B) = P(A) + P(B)

= 1/2 + 1/3 = 5/6

6. Question

There are three events A, B, C one of which must and only one can happen, the odds are 8 to 3 against A, 5 to 2 against B, fins the odds against C.

Answer

As, out of 3 events A,B and C only one can happen at a time which means no event have anything common.

∴ We can say that A , B and C are mutually exclusive events.

By definition of mutually exclusive events we know that:

P(A ∪ B ∪ C) = P(A) + P(B) + P(C)

According to question one event must happen.

This implies A or B or C is a sure event.

∴ P(A ∪ B ∪ C) = 1 ...Equation 1

We need to find odd against C

Given,

Odd against A = 8/3

⇒ $\frac{P(\bar{A})}{P(A)} = \frac{8}{3}$

⇒ $\frac{1 - P(A)}{P(A)} = \frac{8}{3}$

⇒ 8 P(A) = 3 – 3 P(A)

⇒ 11 P(A) = 3

∴ P(A) = $\frac{3}{11}$ …Equation 2

Similarly, we are given with: Odd against B = 5/2

⇒ $\frac{P(\bar{B})}{P(B)} = \frac{5}{2}$

⇒ $\frac{1-P(B)}{P(B)} = \frac{5}{2}$

⇒ 5 P(B) = 2 – 2 P(B)

⇒ 7 P(B) = 2

∴ P(B) = $\frac{2}{7}$ …Equation 3

From equation 1,2 and 3 we get:

P(C) = 1 - $\frac{3}{11} - \frac{2}{7} = \frac{77-21-22}{77} = \frac{34}{77}$

∴ P(C') = 1 – (34/77) = 43/77

∴ Odd against C = $\frac{P(\bar{C})}{P(C)} = \frac{\frac{43}{77}}{\frac{34}{77}} = \frac{43}{34}$

7. Question

One of the two events must happen. Given that the chance of one is two-third of the other, find the odds in favour of the other.

Answer

Let A and B are two events.

As, out of 2 events A and B only one can happen at a time which means no event have anything common.

∴ We can say that A and B are mutually exclusive events.

By definition of mutually exclusive events we know that:

P(A ∪ B) = P(A) + P(B)

According to question one event must happen.

This implies A or B is a sure event.

∴ P(A ∪ B) = P(A) + P(B) = 1 …Equation 1

Given, P(A) = (2/3)P(B)

To find: odds in favour of B

∴ $P(B) + \frac{2}{3} P(B) = 1$

⇒ $\frac{5}{3}P(B) = 1 \Rightarrow P(B) = \frac{3}{5}$

∴ P(B') = 1 – 3/5 = 2/5

∴ Odd in favour of B = $\frac{P(B)}{P(\bar{B})} = \frac{3/5}{2/5} = \frac{3}{2}$

8. Question

A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.

Answer

As a card is drawn from a deck of 52 cards

Let S denotes the event of card being a spade and K denote the event of card being King.

As we know that a deck of 52 cards contains 4 suits (Heart ,Diamond ,Spade and Club) each having 13 cards. The deck has 4 king cards one from each suit.

We know that probability of an event E is given as-

P(E) = $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$

Where n(E) = numbers of elements in event set E

And n(S) = numbers of elements in sample space.

Hence,

P(S) = $\frac{n(\text{spade})}{\text{total number of cards}} = \frac{13}{52} = \frac{1}{4}$

P(K) = $\frac{4}{52} = \frac{1}{13}$

And P(S ∩ K) = $\frac{1}{52}$

We need to find the probability of card being spade or king, i.e.

P(Spade 'or' King) = P(S ∪ K)

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(S ∪ K) = P(S) + P(K) - P(S ∩ K)

⇒ P(S ∪ K) = $\frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{17}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

∴ P(S ∪ K) = 4/13

9. Question

In a single throw of two dice, find the probability that neither a doublet nor a total of 9 will appear.

Answer

In a single throw of 2 die, we have total 36(6 × 6) outcomes possible.

Say, n(S) = 36 where S represents Sample space

Let A denotes the event of getting a doublet.

∴ A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

And B denotes the event of getting a total of 9

∴ B = {(3,6), (6,3), (4,5), (5,4)}

P(B) = $\frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

We need to find probability of the event of getting neither a doublet nor a total of 9.

P(A' ∩ B') = ?

As, P(A' ∩ B') = P(A ∪ B)' {using De Morgan's theorem}

P(A' ∩ B') = 1 – P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

$\therefore$ P(A ∪ B) = $\frac{1}{6}+\frac{1}{9}+0=\frac{5}{18}$ {As P(A ∩ B) = 0 since nothing is common in set A and B ⇒ n(A ∩ B) = 0 }

Hence,

P(A' ∩ B') = 1 – (5/18) = 13/18

10. Question

A natural number is chosen at random from amongst first 500. What is the probability that the number so chosen is divisible by 3 or 5?

Answer

Given, Sample space is the set of first 500 natural numbers.

$\therefore$ n(S) = 500

Let A be the event of choosing the number such that it is divisible by 3

$\therefore$ n(A) = [500/3] = [166.67] = 166 {where [.] represents Greatest integer function}

$\therefore$ P(A) = $\frac{n(A)}{n(S)}=\frac{166}{500}=\frac{83}{250}$

Let B be the event of choosing the number such that it is divisible by 5

$\therefore$ n(B) = [500/5] = [100] = 100 {where [.] represents Greatest integer function}

$\therefore$ P(B) = $\frac{n(B)}{n(S)}=\frac{100}{500}=\frac{1}{5}$

We need to find the P(such that number chosen is divisible by 3 or 5)

$\because$ P(A or B) = P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

$\therefore$ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

We don't have value of P(A ∩ B) which represents event of choosing a number such that it is divisible by both 3 and 5 or we can say that it is divisible by 15.

n(A ∩ B) = [500/15] = [33.34] = 33

$\therefore$ P(A ∩ B) = $\frac{n(A\cap B)}{n(S)}=\frac{33}{500}$

$\therefore$ P(A ∪ B) = $\frac{83}{250}+\frac{1}{5}-\frac{33}{500}=\frac{166+100-33}{500}=\frac{233}{500}$

11. Question

A die is thrown twice. What is the probability that at least one of the two throws come up with the number 3?

Answer

If a dice is thrown twice , it has a total of(6 × 6) 36 possible outcomes.

If S represents the sample space then,

n(S) = 36

Let A represent events the event such that 3 comes in the first throw.

$\therefore$ A = {(1,3), (2,3), (3,3), (4,3), (5,3), (6,3)}

⇒ P(A) = $\frac{n(A)}{n(S)}=\frac{6}{36}=\frac{1}{6}$

Let B represent events the event such that 3 comes in the second throw.

∴ B = {(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)}

⇒ $P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

Clearly (3,3) is common in both events-

∴ $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{36}$

We need to find the probability of event such that at least one of the 2 throws give 3 i.e. P(A or B) = P(A∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

⇒ $P(A \cup B) = \frac{1}{6} + \frac{1}{6} - \frac{1}{36} = \frac{1}{3} - \frac{1}{36} = \frac{11}{36}$

Hence,

P(at least one of the two throws comes to be 3) $= \frac{11}{36}$

12. Question

A card is drawn from a deck of 52 cards. Find the probability of getting an ace or a spade card.

Answer

As a card is drawn from a deck of 52 cards

Let S denotes the event of card being a spade and K denote the event of card being Ace.

As we know that a deck of 52 cards contains 4 suits (Heart ,Diamond ,Spade and Club) each having 13 cards. The deck has 4 Ace cards one from each suit.

We know that probability of an event E is given as-

$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$

Where n(E) = numbers of elements in event set E

And n(S) = numbers of elements in sample space.

Hence,

$P(S) = \frac{n(\text{spade})}{\text{total number of cards}} = \frac{13}{52} = \frac{1}{4}$

Similarly,

$P(K) = \frac{4}{52} = \frac{1}{13}$

And $P(S \cap K) = \frac{1}{52}$

We need to find the probability of card being spade or king, i.e.

P(Spade ‘or’ Ace) = P(S ∪ K)

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(S ∪ K) = P(S) + P(K) - P(S ∩ K)

⇒ $P(S \cup K) = \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{17}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

∴ P(S ∪ K) = 4/13

13. Question

The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75. What is the probability of passing the Hindi examination?

Answer

Let E denotes the event that student passes in English examination.

And H be the event that student passes in Hindi exam.

Given,

P(E) = 0.75

P(passing both) = P(E ∩ H) = 0.5

P(passing neither) = P(E' ∩ H') = 0.1

P(H) = ?

As, we know that P(A' ∩ B') = P(A ∪ B)' {using De Morgan's law}

∴ P(E' ∩ H') = P(E ∪ H)'

⇒ 0.1 = 1 – P(E ∪ H)

⇒ P(E ∪ H) = 1 – 0.1 = 0.9

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(E ∪ H) = P(E) + P(H) – P(E ∩ H)

⇒ 0.9 = 0.75 + P(H) – 0.5

⇒ 1.4 – 0.75 = P(H)

∴ P(H) = 0.65

14. Question

One number is chosen from numbers 1 to 100. Find the probability that it is divisible by 4 or 6?

Answer

Given, Sample space is the set of first 100 natural numbers.

∴ n(S) = 100

Let A be the event of choosing the number such that it is divisible by 4

∴ n(A) = [100/4] = [25] = 25 {where [.] represents Greatest integer function}

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{25}{100} = \frac{1}{4}$

Let B be the event of choosing the number such that it is divisible by 6

∴ n(B) = [100/6] = [16.67] = 16 {where [.] represents Greatest integer function}

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{16}{100} = \frac{4}{25}$

We need to find the P(such that number chosen is divisible by 4 or 6)

∵ P(A or B) = P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

We don't have value of P(A ∩ B) which represents event of choosing a number such that it is divisible by both 4 and 6 or we can say that it is divisible by 12.

n(A ∩ B) = [100/12] = [8.33] = 8

∴ P(A ∩ B) = $\frac{n(A\cap B)}{n(S)} = \frac{8}{100} = \frac{2}{25}$

∴ P(A ∪ B) = $\frac{1}{4} + \frac{4}{25} - \frac{2}{25} = \frac{1}{4} + \frac{2}{25} = \frac{33}{100}$

15. Question

From a well shuffled deck of 52 cards, 4 cards are drawn at random. What is the probability that all the drawn cards are of the same colour.

Answer

In a deck of 52 cards there are 2 colours. Each colour having 26 cards.

As we need to choose 4 cards out of 52. Let S represents the sample space.

∴ n(S) = $^{52}C_4$

Let A represents the event that all 4 cards drawn are black.

∴ n(A) = ways in which 4 cards can be selected from 26 black cards.

⇒ n(A) = $^{26}C_4$

∴ P(A) = $^{26}C_4/^{52}C_4 = \frac{26\times25\times24\times23}{52\times51\times50\times49} = \frac{46}{833}$

Let B represents the event that all 4 cards drawn are red.

∴ n(B) = ways in which 4 cards can be selected from 26 red cards.

⇒ n(B) = $^{26}C_4$

∴ P(B) = $^{26}C_4/^{52}C_4 = \frac{26\times25\times24\times23}{52\times51\times50\times49} = \frac{46}{833}$

As we need to find the probability of event such that all drawn cards are from same colour. This means we need to find

P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

As both events A and B have no common elements or we can say that they are mutually exclusive

∴ P(A ∩ B) = 0

Hence,

P(A ∪ B) = P(A) + P(B) = $\frac{46}{833} + \frac{46}{833} = \frac{92}{833}$

16. Question

100 student appeared for two examinations. 60 passed the first, 50 passed the second and 30 passed both. Find the probability that a student selected at random has passed at least one examination.

Answer

Let E denotes the event that student passed in first examination.

And H be the event that student passed in second exam.

S is the sample space containing the students who appeared for the exam.

Given,

n(S) = 100

n(E) = 60

n(H) = 50

also no of students who passed both exam = n(E ∩ H) = 30

∴ P(E) = $\frac{n(E)}{n(S)} = \frac{60}{100} = \frac{3}{5}$

Similarly, P(H) = $\frac{n(H)}{n(S)} = \frac{50}{100} = \frac{1}{2}$

And, P(E ∩ H) = $\frac{n(E \cap H)}{n(S)} = \frac{30}{100} = \frac{3}{10}$

We need to find the probability of event such that a student selected at random has passed at least one examination.

This can be given as – P(E or H) = P(E ∪ H)

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(E ∪ H) = P(E) + P(H) – P(E ∩ H)

⇒ P(E ∪ H) = $\frac{3}{5} + \frac{1}{2} - \frac{3}{10} = \frac{11}{10} - \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$

∴ P(E ∪ H) = 4/5

17. Question

A box contains 10 white, 6 red and 10 black balls. A ball is drawn at random from the ox. What is the probability that the ball drawn is either white or red?

Answer

As box contains 26 balls. Let S represents the sample space.

∴ n(S) = 26

Let W denotes the event of drawing a white ball and R represents event of drawing red ball.

As, n(W) = 10 and n(R) = 6

∴ P(W) = 10/26 = 5/13

And P(R) = 6/26 = 3/13

As both events have nothing in common so we can say that they are mutually exclusive.

We need to find the probability of the event such that ball drawn is red or white.

P(Red or White) = P(R ∪ W)

As events are mutually exclusive

∴ P(R ∪ W) = P(R) + P(W)

⇒ P(R ∪ W) = 3/13 + 5/13 = 8/13

18. Question

In a race, the odds in favour of horses A,B,C,D are 1:3, 1:4, 1:5 and 1:6 respectively. Find probability that one of the wins the race.

Answer

Given, odds in favour of A is $\frac{P(A)}{P(\bar{A})} = \frac{1}{3}$

$\Rightarrow \frac{P(A)}{1-P(A)} = \frac{1}{3}$

⇒ 1 – P(A) = 3P(A)

⇒ 4P(A) = 1 ⇒ P(A) = 1/4

Odds in favour of horse B is $\frac{P(B)}{P(\bar{B})} = \frac{1}{4}$

$\Rightarrow \frac{P(B)}{1-P(B)} = \frac{1}{4}$

⇒ 1 – P(B) = 4P(B)

⇒ 5P(B) = 1 ⇒ P(B) = 1/5

Odds in favour of horse C is $\frac{P(C)}{P(\bar{C})} = \frac{1}{5}$

$\Rightarrow \frac{P(C)}{1-P(C)} = \frac{1}{5}$

⇒ 1 – P(C) = 5P(C)

⇒ 6P(C) = 1 ⇒ P(C) = 1/6

Odds in favour of horse D is $\frac{P(D)}{P(\bar{D})} = \frac{1}{6}$

$\Rightarrow \frac{P(D)}{1-P(D)} = \frac{1}{6}$

⇒ 1 – P(D) = 6P(D)

⇒ 7P(D) = 1 ⇒ P(D) = 1/7

We have to find the probability that one of the horses win the race.

∵ only one horse can win the race ⇒ A ,B,C and D are mutually exclusive events.

We need to find P(A ∪ B ∪ C ∪ D).

∵ A ,B,C and D are mutually exclusive events.

∴ P(A ∪ B ∪ C ∪ D) = P(A) + P(B) + P(C) + P(D)

$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} = \frac{319}{420}$

Hence,

probability that one of the horses win the race = 319/420

19. Question

The probability that a person will travel by plane is 3/5 and that he will travel by train is 1/4. What is the probability that he (she) will travel by plane or train?

Answer

Let T denotes the event that person travels by train and A denotes that event that person travels by plane

Given, P(T) = 3/5 and P(A) = 1/4

We need to find the probability that person travels by plane or train i.e. P(T or A) = P(T ∪ A)

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(T ∪ A) = P(T) + P(A) – P(A ∩ T)

∵ A person can never travel with both plane and train simultaneously.

∴ P(A ∩ T) = 0

Hence,

P(T ∪ A) = P(T) + P(A) = 3/5 + 1/4 = 17/20

20. Question

Two cards are drawn from a well shuffled pack of 52 cards. Find the probability that either both are black or both are kings.

Answer

As 2 cards are drawn from a deck of 52 cards. This can be done in $^{52}C_2$ ways. If S represents the sample space,

n(S) = $^{52}C_2$

Let B represents the event that both drawn cards are black.

∵ A deck of 52 cards has 26 black cards. So 2 cards can be selected out of those 26 in $^{26}C_2$ ways

∴ n(B) = $^{26}C_2$

∴ P(B) = $\frac{n(B)}{n(S)} = \frac{26C_2}{52C_2} = \frac{26\times25}{52\times51} = \frac{25}{102}$

Let K represents the event that both drawn cards are king.

∵ A deck of 52 cards has 4 king cards. So 2 cards can be selected out of those 4 in $^{4}C_2$ ways

∴ n(K) = $^{4}C_2$

∴ P(K) = $\frac{n(K)}{n(S)} = \frac{4C_2}{52C_2} = \frac{4\times3}{52\times51} = \frac{1}{221}$

We need to find the probability that either both are black or both are kings i.e. P(B or K) = P(B∪ K)

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(B ∪ K) = P(B) + P(K) – P(B ∩ K)

As we don't have value of P(B ∩ K) so we will find it first.

As there is a common element among the events B and K as both the cards can be a king and can be black 2.

∵ 2 black king cards are present so we need to select 2 cards oout of them only. This can be done in $^{2}C_2$ ways = 1

∴ P(B ∩ K) = $\frac{2C_2}{52C_2} = \frac{2}{52\times51} = \frac{1}{1326}$

∴ P(B ∪ K) = $\frac{25}{102} + \frac{1}{221} - \frac{1}{1326} = \frac{55}{221}$

21. Question

In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of a the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Answer

Let E denotes the event that student passed in first examination.

And H be the event that student passed in second exam.

Given, P(E) = 0.8 and P(H) = 0.7

Also probability of passing atleast one exam i.e P(E or H) = 0.95

Or, $P(E \cup H) = 0.95$

We have to find the probability of the event in which students pass both the examinations i.e. $P(E \cap H)$

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(E \cup H) = P(E) + P(H) - P(E \cap H)$

$\Rightarrow P(E \cap H) = P(E) + P(H) - P(E \cup H)$

$\Rightarrow P(E \cap H) = 0.7 + 0.8 - 0.95 = 1.5 - 0.95 = 0.55$

$\therefore$ Probability of passing both the exams = $P(E \cap H) = 0.55$

22. Question

A box contains 30 bolts and 40 nuts. Half of the bolts and half of the nuts are rusted. If two items are drawn at random, what is the probability that either both are rusted or both are bolts?

Answer

Let S represents the sample space containing all possible ways of selecting 2 items (out 0f 70 nuts and bolts)

Let R represents the event of drawing 2 rusted item.

And B be the event of drawing 2 bolts

According to question-

n(S) = total ways of selecting 2 items out of 70 = ${}^{70}C_2$

$\because$ half of bolts (30/2 = 15) and half of nuts(40/2 = 20) are rusted

$\therefore$ n(R) = no of ways in which 2 rusted items can be drawn.

$\Rightarrow n(R) = {}^{35}C_2$

And n(B) = no of ways in which 2 bolts can be drawn.

$\therefore n(B) = {}^{30}C_2$

Also, $n(B \cap R)$ = no of ways of selecting 2 items such that they are rusted bolts

$\therefore n(B \cap R) = {}^{15}C_2$

Hence, we have –

$P(R) = \frac{n(R)}{n(S)} = \frac{35C_2}{70C_2} = \frac{35\times34}{70\times69} = \frac{17}{69}$

$P(B) = \frac{n(B)}{n(S)} = \frac{30C_2}{70C_2} = \frac{30\times29}{70\times69} = \frac{87}{483}$

And $P(R \cap B) = \frac{n(R\cap B)}{n(S)} = \frac{15C_2}{70C_2} = \frac{15\times14}{70\times69} = \frac{1}{23}$

As we have to find probability of the events such that drawn items are rusted or bolts i.e. $P(R \cup B)$

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(R \cup B) = P(R) + P(B) - P(R \cap B)$

$\Rightarrow P(R \cup B) = \frac{17}{69} + \frac{87}{483} - \frac{1}{23} = \frac{185}{483}$

23. Question

An integer is chosen at random from first 200 positive integers. Find the probability that the integer is divisible by 6 or 8.

Answer

Given, Sample space is the set of first 200 natural numbers.

∴ n(S) = 200

Let A be the event of choosing the number such that it is divisible by 6

∴ n(A) = [200/6] = [33.334] = 33 {where [.] represents Greatest integer function}

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{33}{200}$

Let B be the event of choosing the number such that it is divisible by 8

∴ n(B) = [200/8] = [25] = 25 {where [.] represents Greatest integer function}

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{25}{200}$

We need to find the P(such that number chosen is divisible by 6 or 8)

∵ P(A or B) = P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

We don't have value of P(A ∩ B) which represents event of choosing a number such that it is divisible by both 4 and 6 or we can say that it is divisible by 24.

n(A ∩ B) = [200/24] = [8.33] = 8

$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{8}{200}$

$\therefore P(A \cup B) = \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$

24. Question

Find the probability of getting 2 or 3 tails when a coin is tossed four times.

Answer

When a coin is tossed 4 times. A total of 2^4 = 16 outcomes are possible.

Let S be the set consisting of all such outcomes.

∴ n(S) = 16

Let A be the event of getting 2 tails.

∴ A = { TTHH,THTH,THHT,HTTH,HTHT,HHTT}

∴ n(A) = 6

∴ P(A) = 6/16 = 3/8

Let B be the event of getting 3 tails.

∴ B = { TTTH ,TTHT, THTT,HTTT }

⇒ n(B) = 4

∴ P(B) = 4/16 = 1/4

We need to find the probability of getting 2 tails or 3 tails i.e.

P(A ∪ B) = ?

As we can’t get 2 and 3 tails at the same time. So A and B are mutually exclusive events.

∴ P(A ∪ B) = P(A) + P(B) = 3/8 + 1/4 = 5/8

25. Question

Suppose an integer from 1 through 1000 is chosen at random, fins the probability that the integer is a multiple of 2 or a multiple of 9.

Answer

Given, Sample space is the set of first 1000 natural numbers.

∴ n(S) = 1000

Let A be the event of choosing the number such that it is multiple of 2

∴ n(A) = [1000/2] = [500] = 500 {where [.] represents Greatest integer function}

∴ P(A) $= \frac{n(A)}{n(S)} = \frac{500}{1000}$

Let B be the event of choosing the number such that it is multiple of 9

∴ n(B) = [1000/9] = [111.11] = 111 {where [.] represents Greatest integer function}

∴ P(B) $= \frac{n(B)}{n(S)} = \frac{111}{1000}$

We need to find the P(such that number chosen is multiple of 2 or 9)

∵ P(A or B) = P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

We don’t have value of P(A ∩ B) which represents event of choosing a number such that number is a multiple of both 2 and 9 or we can say that it is a multiple of 18.

n(A ∩ B) = [1000/18] = [55.55] = 55

∴ P(A ∩ B) $= \frac{n(A \cap B)}{n(S)} = \frac{55}{1000}$

∴ P(A ∪ B) $= \frac{500}{1000} + \frac{111}{1000} - \frac{55}{1000} = \frac{556}{1000} = \frac{139}{250}$

26. Question

In a large metropolitan area, the probabilities are 0.87, 0.36, 0.30 that a family (randomly chosen for a sample survey) owns a colour television set, a black and white television set, or both kinds of sets. What is the probability that a family owns either any one or both kinds of sets?

Answer

Let C represents the event that random family has colour television, W represents the event that family has black & white set and C ∩ W represents that they own both kind of sets.

According to question:

P(C) = 0.87

P(W) = 0.36 and P(C ∩ W) = 0.30

We need to find the probability of the event that a family owns either anyone kind of set.

i.e we need to find P(C or W) = P(C ∪ W) = ?

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(C ∪ W) = P(C) + P(W) – P(C ∩ W)

⇒ P(C ∪ W) = 0.87 + 0.36 – 0.30 = 0.93

27. Question

If A and B are mutually exclusive events such that P(A) = 0.35 and P(B) = 0.45, find

(i) P (A ∪ B) (ii) P (A ∩ B)

(iii) $P(A \cap \overline{B})$ (iv) $P(\overline{A} \cap \overline{B})$

Answer

Given A and B are two mutually exclusive events

And, P(A) = 0.35 P(B) = 0.45

By definition of mutually exclusive events we know that:

P(A ∪ B) = P(A) + P(B)

We have to find-

i) P(A ∪ B) = P(A) + P(B) = 0.35 + 0.45 = 0.8

ii) P(A ∩ B) = 0 {∵ nothing is common between A and B}

iii) P(A ∩ B') = This indicates only the part which is common with A and not B ⇒ This indicates only A.

P(only A) = P(A) – P(A ∩ B)

As A and B are mutually exclusive So they don't have any common parts ⇒ P(A ∩ B) = 0

∴ P(A ∩ B') = P(A) = 0.35

iv) P(A' ∩ B') = P(A ∪ B)' {using De Morgan's Law}

⇒ P(A' ∩ B') = 1 – P(A ∪ B) = 1 – 0.8 = 0.2

28. Question

A sample space consists of 9 elementary event E_1, E_2, E_3, ……E_8, E_9 whose probabilities are $P(E_1) = P(E_2) = 0.08$, $P(E_3) = P(E_4) = 0.1$, $P(E_6) = P(E_7) = 0.2$, $P(E_8) = P(E_9) = 0.07$

Suppose A = {E_1, E_5, E_8}, B = {E_2, E_8, E_9}

(i) Compute P(A), P(B) and P(A ∩ B).

(ii) Using the addition law of probability, find P (A ∪ B).

(iii) List the composition of the event A ∪ B, and calcite P(A ∪ B) by adding the probabilities of the elementary events.

(iv) Calculate $P(\overline{B})$ from P(B), also calculate $P(\overline{B})$ directly from the elementary events of $(\overline{B})$.

Answer

Clearly according to questions sample space contains 9 elementary events(events with single outcome)

Let S represents the sample space.

∴ S = E_1∪ E_2∪ E_3 ∪ ……∪ E_8∪ E_9

Given A = {E_1, E_5, E_8}

Or $A = E_1 \cup E_5 \cup E_8$

$P(E_1) = P(E_2) = 0.08$, $P(E_3) = P(E_4) = 0.1$, $P(E_6) = P(E_7) = 0.2$, $P(E_8) = P(E_9) = 0.07$

$\therefore P(A) = P(E_1 \cup E_5 \cup E_8) = P(E_1)+P(E_5) + P(E_8)$

$\Rightarrow P(A) = 0.08 + P(E_5) + 0.07 = 0.15 + P(E_5)$ ...(1)

$P(E_5)$ is missing, so we need to find it.

Given,

$B = \{E_2, E_8, E_9\}$ or $B = E_2 \cup E_8 \cup E_9$

$\therefore P(B) = P(E_2 \cup E_8 \cup E_9) = P(E_2)+P(E_8) + P(E_9) = = 0.08+0.07+0.07 = 0.21$

$\therefore P(B) = 0.21$ ....ans(i)

$\therefore B' = \{E_1, E_3, E_4, E_5, E_6, E_7\}$ or $B' = E_1 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7$

$\therefore P(B') = P(E_1) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7)$

$1 – 0.21 = 0.08 + 0.1 + 0.1 + P(E_5) + 0.2 + 0.2$

$\Rightarrow 0.79 = 0.68 + P(E_5)$

$\therefore P(E_5) = 0.79 – 0.68 = 0.11$

$\therefore$ from equation 1, we get-

$P(A) = 0.15 + P(E_5) = 0.15 + 0.11 = 0.26$ (i)

Clearly $A \cap B = \{E_8\}$

$\therefore P(A \cap B) = P(E_8) = 0.07$ (i)

Using addition law of probability we know that-

$P(A \cup B) = P(A) + P(B) – P(A \cap B)$

$\Rightarrow P(A \cup B) = 0.26 + 0.21 – 0.07$

$\therefore P(A \cup B) = 0.4$ (ii)

As, $A \cup B = \{E_1, E_5, E_8\} \cup \{E_2, E_8, E_9\}$

$\Rightarrow A \cup B = \{E_1, E_5, E_8, E_2, E_9\}$

$\therefore P(A \cup B) = P(E_1)+ P(E_5)+ P(E_8)+ P(E_2)+ P(E_9)$

$\Rightarrow P(A \cup B) = 0.08 + 0.11 + 0.07 + 0.08 + 0.07 = 0.41$

$\therefore P(A \cup B) = 0.41$ (iii)

As , $P(B) = 0.21$

$\therefore P(B') = 1 – 0.21 = 0.79$ (iv)

Calculation of $P(B')$ using sets –

$B' = \{E_1, E_3, E_4, E_5, E_6, E_7\}$ or $B' = E_1 \cup E_3 \cup E_4 \cup E_5 \cup E_6 \cup E_7$

$\therefore P(B') = P(E_1) + P(E_3) + P(E_4) + P(E_5) + P(E_6) + P(E_7)$

$P(B') = 0.08 + 0.1 + 0.1 + 0.11 + 0.2 + 0.2$

$= 0.79$ (iv)

Clearly through both the ways we get the same answer.

Very Short Answer

1. Question

Three numbers are chosen at random from numbers 1 to 30. Write the probability that the chosen numbers are consecutive.

Answer

Let E denote the event that the chosen numbers are consecutive.

No of ways in which 3 numbers can be chosen out of 30 = $^{30}C_3$

As we have to select 3 consecutive numbers, if we select 1 number other two are already selected.

As 29,30 can’t be selected because if they are selected we won’t be able to get 3 consecutive numbers.

∴ number of ways in which 3 consecutive numbers can be selected = number of ways in which 1 number can be chosen out of numbers from 1 to 28 = $^{28}C_1$ ways

∴ $P(E) = \frac{28}{30C_3} = \frac{28 \times 3 \times 2 \times 1}{30 \times 29 \times 28} = \frac{1}{145}$

Thus, $P(E) = \frac{1}{145}$

2. Question

n (> 3) persons are sitting in a row. Two of them are selected. Write the probability that they are together.

Answer

Let E denote the event that the selected persons are sitting together.

As 2 persons can be selected out of n in $^{n}C_2$ ways

Out of n persons we can select two persons sitting together in (n-1) ways.

Because we have to select only one person next person is going to be automatically selected.

We can’t select last person because no one is sitting next to him.

∴ 1 person out of n-1 persons can be selected in (n-1) ways.

∴ $P(E) = \frac{n-1}{nC_2} = \frac{2(n-1)}{n(n-1)} = \frac{2}{n}$

Thus, $P(E) = \frac{2}{n}$

3. Question

A single letter is selected at random from the word ‘PROBABILITY’. What is the probability that it is a vowel?

Answer

As there are 9 distinct letters in “PROBABILITY” (P,R,O,B,A,I,L,T,Y).

∴ A single letter can be selected in 9 ways.

As there are 3 vowels in “PROBABILITY”.

∴ 1 vowel can be selected in 3 ways

∴ P(selected letter is vowel) = 3/9 = 1/3

4. Question

What is the probability that a leap year will have 53 Fridays or 53 Saturday?

Answer

In a leap year we have 366 days. So everyday of a week comes 52 times in 364 days.

Now we have 2 days remaining.

These 2 days can be-

S = {MT, TW, WTh, ThF, FSa, SaSu, SuM}

Where S is the sample space.

As there are total 7 possibilities.

∴ n(S) = 7

Let A denote the event of getting 53 Fridays and B denote event of getting 53 Saturdays.

We have to find P(A ∪ B)

Clearly,

P(A) = 2/7

P(B) = 2/7

And P(A ∩ B) = 1/7

∴ P(A ∪ B) = 2/7 + 2/7 – 1/7 = 3/7

5. Question

Three dice are thrown simultaneously. What is the probability of getting 15 as the sum?

Answer

As 3 dice are thrown. So total possible outcomes = 6×6×6 = 216

For getting 15 as the sum we need this combination-

(5, 5, 5) or (6,4,5) or (6,6,3)

(6, 4, 5) can be arranged in 3! = 6 ways.

(6, 4, 5) can be arranged in (3!)/(2!) = 3 ways

∴ 10 favourable outcomes are possible.

∴ P(E) = 10/216

6. Question

If the letters of the word 'MISSISSIPPI' are written down at random in a row, what is the probability that four S's come together.

Answer

Letters of MISSISSIPPI has 11 letters. Out of which 4 S's are repeating, 4 I's are repeating and 2 P's are also repeating.

By Permutation theory.

Letters of MISSISSIPPI can be arranged in $\frac{11!}{4!4!2!}$

Take 4 S's together and treat them as single letter. Now we have

8 letters in together – MI(SSSS)IIPPI → MI(X)IIPPI

Letters of MI(X)IIPPI can be arranged in $\frac{8!}{4!2!}$

∴ P(4'S come together) = $\frac{\frac{8!}{4!2!}}{\frac{11!}{4!4!2!}} = \frac{8!\times 4!}{11!} = \frac{24}{9\times 10\times 11} = \frac{4}{165}$

7. Question

What is the probability that the 13^{th} days of a randomly chosen month is Friday?

Answer

13^{th} day is going to be any of the day from Monday to Sunday. So probability that 13^{th} day of a month being Friday is 1/7.

But here the month is not declared. So the probability of selecting a random month is 1/12

By multiplication theory of Probability -

∴ probability that the 13^{th} days of a randomly chosen month is Friday = (1/7)×(1/12) = 1/84

8. Question

Three of the six vertices of a regular hexagon are chosen at random. What is the probability that the triangle with these vertices is equilateral.

Answer

3 vertices out of 6 can be chosen in ${}^6C_3$ = 20 ways

In a hexagon only 2 equilateral triangles are possible.

∴ probability that the triangle with these vertices is equilateral = 2/20 = 1/10

9. Question

If E and E_2 are independent events, write the value of $P(E_1 \cup E_2) \cap (\overline{E} \cap \overline{E}_2)$

Answer

As E_1 and E_2 are independent events.

∴ $P((E_1 \cup E_2) \cap (E_1' \cap E_2'))$

As by De Morgan's law.

$E_1' \cap E_2' = (E_1 \cup E_2)'$

∴ $P((E_1 \cup E_2) \cap (E_1' \cap E_2')) = P((E_1 \cup E_2) \cap (E_1 \cup E_2)')$

⇒ $P((E_1 \cup E_2) \cap (E_1 \cup E_2)')$ = P(empty set) = 0

10. Question

If A and B are two independent events such that $P(A \cap B) = \frac{1}{6}$ and $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$, then write the values of P(A) and P(B).

Answer

As A and B are independent events.

∴ P(A ∩ B) = P(A)P(B) = 1/6

Also, P(A' ∩ B') = P(A')P(B') = 1/3

⇒ (1 – P(A))(1 – P(B)) = 1/3

⇒ 1 + P(A)P(B) – P(A) – P(B) = 1/3

⇒ 1 + 1/6 – 1/3 = P(A) + P(B)

⇒ 1 – 1/6 = P(A) + P(B)

∴ P(A) = 5/6 – P(B)

Let $P(B) = x$

$\therefore (5/6 - x)x = 1/6$

$\Rightarrow 5x - 6x^2 = 1$

$\Rightarrow 6x^2 - 5x + 1 = 0$

$\Rightarrow 6x^2 - 3x - 2x + 1 = 0$

$\Rightarrow 3x(2x - 1) - (2x - 1) = 0$

$\Rightarrow (3x - 1)(2x - 1)=0$

$\therefore x = 1/3$ or $x = 1/2$

$\therefore P(B) = 1/3$ or $1/2$

$\therefore P(A) = 1/2$ or $1/3$

MCQ

1. Question

One card is drawn from a pack of 52 cards. The probability that it is the card of a king or spade is

A. $\frac{1}{26}$

B. $\frac{3}{26}$

C. $\frac{4}{13}$

D. $\frac{3}{13}$

Answer

As a card is drawn from a deck of 52 cards

Let S denotes the event of card being a spade and K denote the event of card being King.

As we know that a deck of 52 cards contains 4 suits (Heart ,Diamond ,Spade and Club) each having 13 cards. The deck has 4 king cards one from each suit.

We know that probability of an event E is given as-

$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$

Where n(E) = numbers of elements in event set E

And n(S) = numbers of elements in sample space.

Hence,

$P(S) = \frac{n(\text{spade})}{\text{total number of cards}} = \frac{13}{52} = \frac{1}{4}$

$P(K) = \frac{4}{52} = \frac{1}{13}$

And $P(S \cap K) = \frac{1}{52}$

We need to find the probability of card being spade or king, i.e.

P(Spade ‘or’ King) = P(S ∪ K)

Note: By definition of P(A or B) under axiomatic approach(also called addition theorem) we know that:

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(S ∪ K) = P(S) + P(K) - P(S ∩ K)

⇒ P(S ∪ K) $= \frac{1}{4} + \frac{1}{13} - \frac{1}{52} = \frac{17}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

∴ P(S ∪ K) = 4/13

∴ Option (c) is the correct choice.

2. Question

Two dice are thrown together. The probability that at least one will show its digit greater than 3 is

A. $\frac{1}{4}$

B. $\frac{3}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{8}$

Answer

If a dice is thrown twice, it has a total of (6 × 6) 36 possible outcomes.

If S represents the sample space then,

n(S) = 36

Let A represent events the event such that digit greater than 3 comes in the second throw.

∴ A = {(1,4), (2,4), (3,4), (4,4), (5,4), (6,4), (1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (1,6), (2,6), (3,6), (4,6), (5,6), (6,6)}

⇒ P(A) $= \frac{n(A)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

Let B represent events the event such digit greater than 3 comes in the first throw.

Similarly 18 outcomes are possible as were present for event A

⇒ P(B) $= \frac{n(B)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

Clearly (4,4), (5,4), (6,4), (4,5), (5,5), (6,5) (4,6), (5,6) and (6,6) are common in both events-

∴ P(A ∩ B) $= \frac{n(A \cap B)}{n(S)} = \frac{9}{36} = \frac{1}{4}$

We need to find the probability of event such that at least one of the 2 throws give digit greater than 33 i.e. P(A or B) = P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

⇒ P(A ∪ B) $= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = 1 - \frac{1}{4} = \frac{3}{4}$

Hence,

P(at least one of the two throws shows digit >3) = $3/4$

∴ Option (b) is correct choice.

3. Question

Two dice are thrown simultaneously. The probability of obtaining a total score of 5 is

A. $\frac{1}{18}$

B. $\frac{1}{12}$

C. $\frac{1}{9}$

D. none of these

Answer

As 2 dice are thrown so there are 6×6 = 36 possibilities.

Let E denote the event of getting a total score of 5.

E = {(1,4),(2,3),(3,2),(4,1)}

∴ n(E)=4

Hence,

P(E) = 4/36 = 1/9

As our answer matches only with option (c)

∴ Option (c) is the only correct choice.

4. Question

Two dice are thrown simultaneously. The probability of obtaining total score of seven is

A. $\frac{5}{36}$

B. $\frac{6}{36}$

C. $\frac{7}{36}$

D. $\frac{8}{36}$

Answer

As 2 dice are thrown so there are 6×6 = 36 possibilities.

Let E denote the event of getting a total score of 5.

E = {(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)}

∴ n(E)= 6

Hence,

P(E) = 6/36

As our answer matches only with option (b)

∴ Option (b) is the only correct choice.

5. Question

The probability of getting a total of 10 in a single throw of two dice is

A. $\frac{1}{9}$

B. $\frac{1}{12}$

C. $\frac{1}{6}$

D. $\frac{5}{36}$

Answer

As 2 dice are thrown so there are 6×6 = 36 possibilities.

Let E denote the event of getting a total score of 5.

E = {(4,6),(5,5),(6,4)}

∴ n(E)= 3

Hence,

P(E) = 3/36 = 1/12

As our answer matches only with option (b)

∴ Option (b) is the only correct choice.

6. Question

A card is drawn at random from a pack of 100 cards numbered 1 to 100. The probability of drawing a number which is a square is

A. $\frac{1}{5}$

B. $\frac{2}{5}$

C. $\frac{1}{10}$

D. none of these

Answer

As a card is to be drawn from 1 to 100.

∴ n(S) = 100

As we know that largest no less that or equal to 100 which is a perfect square = 100

And, √100 = 10

∴ there are total 10 numbers from 1 to 100 which are square.

∴ n(E) = 10

Hence,

P(E) = 10/100 = 1/10

As our answer matches only with option (c)

∴ Option (c) is the only correct choice.

7. Question

A bag contains 3 red, 4 white and 5 blue balls. All balls are different. Two balls are drawn at random. The probability that they are of different colour is

A. $\frac{47}{66}$

B. $\frac{10}{33}$

C. $\frac{1}{3}$

D. 1

Answer

As there are 3 +4+5 = 12 balls.

So, 2 balls out of 12 can be drawn in $^{12}C_2$ ways = 66

Let E denote the event that balls drawn are of different colours

Either the balls will be red-white or white-blue or red-blue.

∴ P(E) = $\frac{3C_1 \times 4C_1}{66} + \frac{4C_1 \times 5C_1}{66} + \frac{3C_1 \times 5C_1}{66} = \frac{12}{66} + \frac{20}{66} + \frac{15}{66} = \frac{47}{66}$

Hence,

P(E) = 47/66

As our answer matches only with option (a)

∴ Option (a) is the only correct choice.

8. Question

Two dice are thrown together. The probability that neither they show equal digits nor the sum of their digits is 9 will be

A. $\frac{13}{15}$

B. $\frac{13}{18}$

C. $\frac{1}{9}$

D. $\frac{8}{9}$

Answer

In a single throw of 2 die, we have total 36(6 × 6) outcomes possible.

Say, n(S) = 36 where S represents Sample space

Let A denotes the event of getting a doublet(equal number)

∴ A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}

∴ P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

And B denotes the event of getting a total of 9

∴ B = {(3,6), (6,3), (4,5), (5,4)}

P(B) = $\frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

We need to find probability of the event of getting neither a doublet nor a total of 9.

P(A' ∩ B') = ?

As, P(A' ∩ B') = P(A ∪ B)' {using De Morgan's theorem}

P(A' ∩ B') = 1 – P(A ∪ B)

Note: By definition of P(E or F) under axiomatic approach(also called addition theorem) we know that:

P(E ∪ F) = P(E) + P(F) – P(E ∩ F)

∴ P(A ∪ B) = $\frac{1}{6} + \frac{1}{9} + 0 = \frac{5}{18}$ {As P(A ∩ B) = 0 since nothing is common in set A and B ⇒ n(A ∩ B) = 0 }

Hence,

P(A' ∩ B') = 1 – (5/18) = 13/18

Hence,

P(required event) = 13/18

As our answer matches only with option (b)

∴ Option (b) is the only correct choice.

9. Question

Four persons are selected at random out of 3 men, 2 women and 4 children. The probability that there are exactly 2 children in the selection is

A. $\frac{11}{21}$

B. $\frac{9}{21}$

C. $\frac{10}{21}$

D. none of these

Answer

As there are 3 +2+4 = 9 persons

So, 4 persons out of 9 can be drawn in ${}^9C_4$ ways = 126

Let E denote the event that there are exactly 2 children in the selection.

2children out of 4 can be selected in ${}^4C_2$ = 6 ways

And rest two persons can be male or female. So we will select 2 persons out of remaining 5.

$\therefore$ P(E) $= \frac{4C_2 \times 5C_2}{126} = \frac{6\times10}{126} = \frac{10}{21}$

Hence,

P(E) = 10/21

As our answer matches only with option (c)

∴ Option (c) is the only correct choice.

10. Question

The probabilities of happening of two events. A and B are 0.25 and 0.50 respectively. If the probability of happing of A and B together is 0.14, then probability that neither A nor B happens is

A. 09.39

B. 0.25

C. 0.11

D. none of these

Answer

Given, P(A) = 0.25 and P(B) = 0.5

Also P(A ∩ B) = 0.14

We have to find P(A’ ∩ B’)

By De Morgan’s theorem we know that:

P(A’ ∩ B’) = P(A ∪ B)’

We know that P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(A ∪ B) = 0.25 + 0.5 – 0.14 = 0.61

∴ P(A’ ∩ B’) = P(A ∪ B)’ = 1 -P(A ∪ B) = 1 – 0.61 = 0.39

As our answer does not match with any other option

But option of none of these is there. So most suitable option is option (d)

∴ Option (d) is the only correct choice.

11. Question

A die is rolled, then the probability that a number 1 or 6 may appear is

A. $\frac{2}{3}$

B. $\frac{5}{6}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

Answer

When a die is rolled there are in total 6 possibilities.

∴ n(S) = 6

Let E denote event of getting 1 or 6

∴ n(E) = 2

∴ P(E) = 2/6 = 1/3

Hence,

P(E) = 1/3

As our answer matches only with option (c)

∴ Option (c) is the only correct choice.

12. Question

Six boys and six girls sit in a row randomly. The probability that all girls sit together is

A. $\frac{1}{122}$

B. $\frac{1}{112}$

C. $\frac{1}{102}$

D. $\frac{1}{132}$

Answer

As 6 boys and 6 girls are sitting in a row. So these 12 persons can sit in 12! Ways

Now group all 6 girls together and treat them as 1.

Now, all girls together can sit in 7! Ways

And girls can sit among self in 6! Ways.

∴ total ways in which all 6 girls sit together = 7! × 6!

∴ P(E) = $\frac{7! \times 6!}{12!} = \frac{720}{8 \times 9 \times 10 \times 11 \times 12} = \frac{1}{132}$

Hence,

P(E) = 1/132

As our answer matches only with option (d)

∴ Option (d) is the only correct choice.

13. Question

The probabilities of three mutually exclusive events A, B and C are given by 2/3, 1/4 and 1/6 respectively. The statement

A. is true

B. is false

C. nothing can be said

D. could be either

Answer

As we know that 3 events are said to be mutually exclusive events if –

P(A ∪ B ∪ C) = P(A) + P(B) + P(C)

⇒ P(A ∪ B ∪ C) = $\frac{2}{3}+\frac{1}{4}+\frac{1}{6}=\frac{8+3+2}{12}=\frac{13}{12}>1$

As probability can never be > 1

∴ A,B and C are not mutually exclusive events.

Statement is false

∴ Option (b) is the only correct choice.

14. Question

If $\frac{(1-3p)}{2},\frac{(1+4p)}{3},\frac{(1+p)}{6}$ are the probabilities of three mutually exclusive and exhaustive events, then the set of all values of p is

A. (0, 1)

B. (-1/4, 1/3)

C. (0, 1/3)

D. (0, ∞)

Answer

As 3 events are mutually exclusive and exhaustive.

∴ P(A ∪ B ∪ C) = P(A) + P(B) + P(C) = 1

⇒ $\frac{1-3p}{2}+\frac{1+4p}{3}+\frac{1+p}{6}=1$

⇒ $\frac{2-6p+2+8p+1+p}{6}=1$

⇒ 5 + 3p = 6

∴ p = 1/3

As 1/3 lies in both intervals given in option (a) and (d)

But a larger value like 100 which comes if we choose option d can make P(A) < 0 which is not possible.

∴ Option (a) is the most suitable choice here.

15. Question

A pack of cards contains 4 aces, 4 kings, 4 queens and 4 jacks. Two cards are drawn at random. The probability that at least one of them is an ace is

A. $\frac{1}{5}$

B. $\frac{3}{16}$

C. $\frac{19}{20}$

D. $\frac{1}{9}$

Answer

As total number of cards = 16

2 cards can be drawn in $^{16}C_2$ ways = 120 ways

As we need to find the probability for the event E such that at least one of them is an ace.

We can solve this problem using negation.

We will find the probability for event that both cards are ace (E’)

Automatically 1 – P(E’) will give P(E)

For finding P(E’) we will select both cards out of 4 ace cards

∴ P(E’) = $\frac{^4C_2}{120} = \frac{6}{120} = \frac{1}{20}$

∴ P(E) = 1 – 1/20 = 19/20

Hence,

P(E) = 19/20

As our answer matches only with option (c)

∴ Option (c) is the only correct choice.

16. Question

If three dice are thrown simultaneously, then the probability of getting a score of 5 is

A. 5/216

B. 1/6

C. 1/36

D. none of these

Answer

As 3 dice are thrown so there are 6×6×6 = 216 possibilities.

Let E denote the event of getting a total score of 5.

{(1,1,3),(1,2,2)}

As (1,1,3) can be arranged in $\frac{3!}{2!}$ ways = 3

and (1,2,2) can be arranged in $\frac{3!}{2!}$ ways = 3

∴ n(E) = 6

Hence,

P(E) = 6/36 = 1/6

As our answer matches only with option (b)

∴ Option (b) is the only correct choice.

17. Question

One of the two events must occur. If the chance of one is 2/3 of the other, then odds in favour of the other are

A. 1 : 3

B. 3 : 1

C. 2 : 3

D. 3 : 2

Answer

Let E and F be the two events such that one must occur.

Given,

P(E) = 2/3 P(F)

Also, P(E ∪ F) = 1

P(E) + P(F) = 1

⇒ P(F){2/3 + 1} = 1

∴ P(F) = 3/5

And P(F’) = 1 – 3/5 = 2/5

We have to find $\frac{P(F)}{P(\overline{F})} = \frac{3/5}{2/5} = \frac{3}{2}$

∴ Odds in favour of F = 3/2

As our answer matches only with option (d)

∴ Option (d) is the only correct choice.

18. Question

The probability that a leap year will have 53 Fridays or 53 Saturdays is

A. 2/7

B. 3/7

C. 4/7

D. 1/7

Answer

In a leap year we have 366 days. So, every day of a week comes 52 times in 364 days.

Now we have 2 days remaining.

These 2 days can be-

S = {MT, TW, WTh, ThF, FSa, SaSu, SuM}

Where S is the sample space.

As there are total 7 possibilities.

∴ n(S) = 7

Let A denote the event of getting 53 Fridays and B denote event of getting 53 Saturdays.

We have to find P(A ∪ B)

Clearly,

P(A) = 2/7

P(B) = 2/7

And P(A ∩ B) = 1/7

∴ P(A ∪ B) = 2/7 + 2/7 – 1/7 = 3/7

As our answer matches only with option (b)

∴ Option (b) is the only correct choice.

19. Question

A person write 4 letters and addresses 4 envelopes. If the letters are placed in the envelopes at random, then the probability that all letters are not placed in the right envelopes, is

A. 1/4

B. 11/24

C. 15/24

D. 3/8

Answer

As we have 4 letters and 4 envelopes.

These 4 letters can be arranged in 4! = 24 ways.

∴ n(S) = 24

Let E denotes the event that all letters are not placed in the right envelopes

The number of ways in which 4 letters can be placed in wrong envelopes is given by the number of ways in which N objects can be dearranged.

Numbers of ways of in which N objects can be dearranged is given by –

$$N!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\cdots(-1)^{N}\frac{1}{N!}\right)$$

∴ n(E) = $4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)=24\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right)=24\left(\frac{12-4+1}{24}\right)=9$

∴ P(E) = 9/24 = 3/8

20. Question

A and B are two events such that P(A) = 0.25 and P(B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is

A. 0.39

B. 0.25

C. 0.11

D. none of these

Answer

Given, P(A) = 0.25 and P(B) = 0.5

Also P(A ∩ B) = 0.14

We have to find P(A’ ∩ B’)

By De Morgan’s theorem we know that:

P(A’ ∩ B’) = P(A ∪ B)’

We know that P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

∴ P(A ∪ B) = 0.25 + 0.5 – 0.14 = 0.61

∴ P(A’ ∩ B’) = P(A ∪ B)’ = 1 -P(A ∪ B) = 1 – 0.61 = 0.39

As our answer matches with option (a)

∴ Option (a) is the only correct choice.

21. Question

If the probability of A to fail in an examination is $\frac{1}{5}$ and that of B is $\frac{3}{10}$. Then, the probability that either A or B fails is

A. 1/2

B. 11/25

C. 19/50

D. none of these

Answer

Let A be the event that student 'A' fails in exam and B be the event that student 'B' fails in exam.

Given, P(A) = 1/5 and P(B) = 3/10

We have to find $P(A \cup B)$

We know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

As $P(A \cap B)$ not given, so assuming its value 0

$\therefore P(A \cup B) = 1/5 + 3/10 = 5/10 = 1/2$

As our answer matches with option (a)

∴ Option (a) is the only correct choice.

22. Question

A box contains 10 good articles and 6 defective articles. One item is drawn at random. The probability that it is either good or has a defect, is

A. 64/64

B. 49/64

C. 40/64

D. 24/64

Answer

As total articles = 10 + 6 = 16

P(good) =P(G) = 10/16

And P(defective)= P(D) = 6/16

As there it is not possible to select an item that is both good and defective.

P(good or Defective) = $P(G \cup D) = P(G) + P(D) = 16/16 = 1$

As our answer does not directly seems to match with any option.

But option (a) is 64/64 = 1

∴ option (a) is the only correct choice.

23. Question

Three integers are chosen at random from the first 20 integers. The probability that their product is even is

A. 2/19

B. 2/29

C. 17/19

D. 4/19

Answer

3 integers out of 20 can be chosen in $^{20}C_3$ ways.

As in first 20 integers, 10 are even.

To have the product of 3 integer even the chosen integers must be even.

∴ 3 integers that are even can be chosen from first 20 integers in ${}^{10}C_3$ ways.

∴ Probability $= \frac{10C_3}{20C_3} = \frac{10\times9\times8}{20\times19\times18} = \frac{2}{19}$

Our answer matches with option (a)

∴ Option (a) is the only correct choice

24. Question

Out of 30 consecutive integers, 2 are chosen at random. The probability that their sum is odd, is

A. 14/29

B. 16/29

C. 15/29

D. 10/29

Answer

2 integers out of 30 can be chosen in ${}^{30}C_2$ ways.

As in first 30 integers, 15 are even and 15 are odd.

To have the sum of 2 integer odd one integer must be even and other must be odd.

∴ integers can be chosen in ${}^{15}C_1 \times {}^{15}C_1$ ways

∴ Probability $= \frac{15C_1\times15C_1}{30C_2} = \frac{15\times15\times2}{30\times29} = \frac{15}{29}$

Our answer matches with option (c)

∴ Option (c) is the only correct choice

25. Question

A bag contains 5 black balls, 4 white balls and 3 red balls. If a ball is selected randomly. The probability that it is black or red ball is

A. 1/3

B. 1/4

C. 5/12

D. 2/3

Answer

According to question bag contains 5+4+3 = 12 balls

∴ 1 ball can be chosen in ${}^{12}C_1$ = 12 ways

Let B denotes the event of drawing a black ball and R denotes event of drawing red ball

∴ P(B) = 5/12 and P(R) = 3/12

We have to find P(B ∩ R)

As, we need to find P(B ∪ R)

∵ Black and red balls can’t be drawn simultaneously.

∴ P(B ∩ R) = 0

∴ P(B ∪ R) = P(B) + P(R) = 5/12 + 3/12 = 8/12 = 2/3

Our answer matches with option (d)

∴ Option (d) is the only correct choice

26. Question

Two dice are thrown simultaneously. The probability of getting a pair of sixes is

A. 1/36

B. 1/3

C. 1/6

D. none of these

Answer

Sample Space = 36

Out of 36 possible events, only one event is favourable = (6, 6)

Probability $= \frac{1}{36}$

27. Question

An urn contains 9 balls two of which are red, three blue and four black. Three balls are drawn at random. The probability that they are of the same colour is

A. 5/84

B. 3/9

C. 3/7

D. 7/17

Answer

Balls in urn = 9

3 balls can be drawn in $^9C_3$ ways.

To draw the balls of same colour, we can select balls all three red, all blue or all black.

P(ball of same colour) = P(all blue) or P(all black) or P(all red)

P(all red) = not possible as we have to select 3 balls but there are only 2 red balls.

∴ P(All red) = 0

P(all black) = $\frac{4C_3}{9C_3} = \frac{4\times3\times2}{9\times8\times7} = \frac{1}{21}$

And P(all blue) = $\frac{3C_3}{9C_3} = \frac{3\times2\times1}{9\times8\times7} = \frac{1}{84}$

∴ P(ball of same colour) = P(all blue) or P(all black) or P(all red)

⇒ P(ball of same colour) = P(all blue) + P(all black) + P(all red)

= 1/84 + 1/21 + 0 = 5/84

Our answer matches with option (a)

∴ Option (a) is the only correct choice

28. Question

Five persons entered the lift cabin on the ground floor of an 8 floored house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first, then the probability of all 5 persons leaving at different floor is

A. $\frac{{}^{7}P_{5}}{7^{5}}$

B. $\frac{7^{5}}{{}^{7}P_{5}}$

C. $\frac{6}{{}^{6}P_{5}}$

D. $\frac{{}^{5}P_{5}}{5^{5}}$

Answer

As building has 8 floors including ground.

So, they have 7 options to leave the lift.

∴ total ways in which persons can leave the lift = 7^5

As the number of ways in which all persons leave in a different floor can be given by: 7× 6× 5× 4× 3 = ${}^{7}P_{5}$

∴ P(that all leave lift in different floor) = $\frac{{}^{7}P_{5}}{7^{5}}$

Our answer matches with option (a)

∴ Option (a) is the only correct choice

29. Question

A box contains 10 good articles and 6 with defects. One item is drawn at random. The probability that it is either good or has a defect is

A. 64/64

B. 49/64

C. 40/64

D. 24/64

Answer

As total articles = 10 + 6 = 16

P(good) =P(G) = 10/16

And P(defective)= P(D) = 6/16

As there it is not possible to select an item that is both good and defective.

P(good or Defective) = P(G ∪ D) = P(G) + P(D) = 16/16 = 1

As our answer does not directly seems to match with any option.

But option (a) is 64/64 = 1

∴ option (a) is the only correct choice.

30. Question

A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, the probability that it is rusted or is a nail is

A. 3/16

B. 5/16

C. 11/16

D. 14/16

Answer

As total articles(nuts + nails) = 10 + 6 = 16

Number of rusted nails = 3

Number of rusted nuts = 5

Number of rusted articles = 8

Let N denotes the event that the article drawn is a nails and R be the event denoting drawn article is rusted

P(N) = 6/16

And P(R) = 8/16

Also, we have possibility that drawn nail is rusted.

∴ P(N ∩ R) = 3/16

P(Nail or Rusted) = P(N ∪ R) = P(N) + P(R) – P(N ∩ R) = 6/16 + 8/16 – 3/16 = 11/16

∴ option (c) is the only correct choice.

31. Question

If S is the sample space and $P(A)=\frac{1}{3}P(B) \text{ and } S=A\cup B$, where A and B are two mutually exclusive events, then P(A) =

A. 1/4

B. 1/2

C. 3/4

D. 3/8

Answer

As , S = A ∪ B

∴ P(A ∪ B) = P(S) = 1

As A and B are mutually exclusive events.

∴ P(A ∪ B) = P(A) + P(B)

⇒ 1 = P(A) + 3P(A) [∵ P(A) = 1/3 P(B)]

⇒ 4P(A) = 1

∴ P(A) = 1/4

Our answer matches with option (a)

∴ Option (a) is the only correct choice

32. Question

One mapping is selected at random from all the mappings of the set A = {1, 2, 3, ….., n) into itself. The probability that the mapping selected is one to one is

A. $\frac{1}{n^2}$

B. $\frac{1}{n!}$

C. $\frac{(n-1)!}{n^{n-1}}$

D. none of these

Answer

We know that the no of mapping from a set A to same set containing n elements = n^n

As each element have n options to be mapped with n elements.

For mapping selected to be one to one, for first element we have n options ,for second we have n-1 options an so on...

∴ total such mappings = n × (n-1)× (n-2)× … = n!

∴ P(mapping is one to one) = $\frac{n!}{n^n} = \frac{n(n-1)!}{n^n} = \frac{(n-1)!}{n^{n-1}}$

Our answer matches with option (c)

∴ Option (c) is the only correct choice

33. Question

If A, B, C are three mutually exclusive and exhaustive events of an experiment such that 3P(A) = 2P(B) = P(C), then P(A) is equal to

A. 1/11

B. 2/11

C. 5/11

D. 6/11

Answer

Given,

3P(A) = 2P(B) = P(C) = k(say)

∴ P(A) = k/3

P(B) = k/2

And P(C) = k

As events A,B and C are mutually exclusive and exhaustive,

∴ P(A ∪ B ∪ C) = P(A) + P(B) + P(C) = 1

⇒ 1 = k/3 + k/2 + k

⇒ 1 = $\frac{2k+3k+6k}{6} = \frac{11k}{6}$

⇒ k = 6/11

As, P(A) = k/3 = $\frac{1}{3} \times \frac{6}{11} = \frac{2}{11}$

Our answer matches with option (b)

∴ Option (b) is the only correct choice

34. Question

If A and B are mutually exclusive events then

A. $P(A) \leq P(\overline{B})$

B. $P(A) \geq P(\overline{B})$

C. $P(A) < P(\overline{B})$

D. none of these

Answer

As P(A) and P(B) are mutually exclusive event.

∴ P(A ∪ B) = P(A) + P(B)

As P(A ∪ B) ≤ 1

∴ P(A) + P(B) ≤ 1

⇒ P(A) ≤ 1 – P(B)

⇒ P(A) ≤ P(B')

Our answer matches with option (a)

∴ Option (a) is the only correct choice

35. Question

If P (A ∩ B) = P (A ∪ B) for any two events A and B, then

A. P (A) = P (B)

B. P (A) > P (B)

C. P (A) < P (B)

D. None of these

Answer

As, (A ∪ B) = (A ∩ B)

⇒ A and B are same sets.

∴ P(A) = P(B)

Our answer matches with option (a)

∴ Option (a) is the only correct choice

36. Question

Three numbers are chosen from 1 to 20. The probability that they are not consecutive is

A. $\frac{186}{190}$

B. $\frac{187}{190}$

C. $\frac{188}{190}$

D. $\frac{18}{{}^{20}C_3}$

Answer

3 numbers can be chosen out of 20 in ${}^{20}C_3$ ways.

As we have to choose 3 consecutive numbers. So we can't choose 19 and 18.

Choosing consecutive number is same as choosing a single number because other 2 numbers are automatically chosen.

This can be chosen 18 ways.

∴ P(chosen numbers are consecutive) = $\frac{18}{{}^{20}C_3}$

Our answer matches with option (d)

∴ Option (d) is the only correct choice

37. Question

6 boys and 6 girls sit in a row a row at random. The probability that all the girls sit together is

A. $\frac{1}{432}$

B. $\frac{12}{431}$

C. $\frac{1}{132}$

D. none of these

Answer

As 6 boys and 6 girls are sitting in a row. So these 12 persons can sit in 12! Ways

Now group all 6 girls together and treat them as 1.

Now, all girls together can sit in 7! Ways

And girls can sit among self in 6! Ways.

∴ total ways in which all 6 girls sit together = 7! × 6!

∴ P(E) = $\frac{7!\times 6!}{12!} = \frac{720}{8\times 9\times 10\times 11\times 12} = \frac{1}{132}$

Hence,

P(E) = 1/132

As our answer matches only with option (d)

∴ Option (d) is the only correct choice.

38. Question

Without repetition of the numbers, four digit numbers are formed with the numbers 0, 2, 3, 5. The probability of such a number divisible by 5 is

A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{1}{30}$

D. $\frac{5}{9}$

Answer

Total numbers of 4digit that can be formed using 0,2,3,5 without repetition= 3× 3 × 2 × 1 = 18

For number to be divisible by 5 unit digit must end with 5 or 0.

∴ total such numbers(4 digit) ending with zero are = 3× 2× 1(0 is fixed at the unit digit) = 6

∴ total such numbers(4 digit) ending with zero are = 2× 2× 1(5 is fixed at the unit digit) = 4

Total 4 digit numbers using given digit divisible by 5 are 6 + 4 =10

∴ P(a number divisible by 5) = 10/18 = 5/9

As our answer matches only with option (d)

∴ Option (d) is the only correct choice.

39. Question

If the probability for A to fail in an examination is 0.2 and that for B is 0.3, then the probability that either A or B fails is

A. > 0.5

B. 0.5

C. ≤ 0.5

D. 0

Answer

Let A be the event that student ‘A’ fails in exam and B be the event that student ‘B’ fails in exam.

Given, P(A) = 0.2 and P(B) = 0.3

We have to find P(A ∪ B)

We know that P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

As P(A ∩ B) not given, but we know that – 0 ≤ P(A ∩ B) ≤ 1

∴ P(A ∪ B) = 0.2 + 0.3 – P(A ∩ B) = 0.5 - P(A ∩ B)

Clearly a +ve number will be subtracted form 0.5 to make it equal to P(A ∪ B).

∴ P(A ∪ B) ≤ 0.5

As our answer matches with option (c)

∴ Option (c) is the only correct choice.

40. Question

Three digit numbers are formed using the digits 0, 2, 4, 6, 8. A number is chosen at random out of these numbers what is the probability that this number has the same digits?

A. $\frac{1}{16}$

B. $\frac{16}{25}$

C. $\frac{1}{645}$

D. $\frac{1}{25}$

Answer

Total numbers of 3 digit that can be formed using 0,2,4,6,8 = 4× 5 × 5 = 100

3-digit number formed using 0,2,4,6,8 with all digits same are 222,444,666,888

∴ P(number has the same digits) = 4/100 = 1/25

As our answer matches only with option (d)

∴ Option (d) is the only correct choice.