3. Functions

Exercise 3.1

1. Question

Define a function as a set of ordered pairs.

Answer

A function from is defined by a set of ordered pairs such that any two ordered pairs should not have the same first component and the different second component.

This means that each element of a set, say X is assigned exactly to one element of another set, say Y.

The set X containing the first components of a function is called the domain of the function.

The set Y containing the second components of a function is called the range of the function.

For example, f = {(a, 1), (b, 2), (c, 3)} is a function.

Domain of f = {a, b, c}

Range of f = {1, 2, 3}

2. Question

Define a function as a correspondence between two sets.

Answer

A function from a set X to a set Y is defined as a correspondence between sets X and Y such that for each element of X, there is only one corresponding element in Y.

The set X is called the domain of the function.

The set Y is called the range of the function.

For example, X = {a, b, c}, Y = {1, 2, 3, 4, 5} and f be a correspondence which assigns the position of a letter in the set of alphabets.

Therefore, f(a) = 1, f(b) = 2 and f(c) = 3.

As there is only one element of Y for each element of X, f is a function with domain X and range Y.

3. Question

What is the fundamental difference between a relation and a function? Is every relation a function?

Answer

Let f be a function and R be a relation defined from set X to set Y.

The domain of the relation R might be a subset of the set X, but the domain of the function f must be equal to X. This is because each element of the domain of a function must have an element associated with it, whereas this is not necessary for a relation.

In relation, one element of X might be associated with one or more elements of Y, while it must be associated with only one element of Y in a function.

Thus, not every relation is a function. However, every function is necessarily a relation.

4. Question

Let A = {–2, –1, 0, 1, 2} and f : A→ Z be a function defined by $f(x) = x^2 - 2x - 3$. Find:

i. range of f i.e. f(A)

ii. pre-images of 6, –3 and 5

Answer

Given A = {–2, –1, 0, 1, 2}

f : A → Z such that $f(x) = x^2 - 2x - 3$

i. range of f i.e. f(A)

A is the domain of the function f. Hence, range is the set of elements f(x) for all x ∈ A.

Substituting x = –2 in f(x), we get

$f(-2) = (-2)^2 - 2(-2) - 3$

$\Rightarrow f(-2) = 4 + 4 - 3$

$\therefore f(-2) = 5$

Substituting x = –1 in f(x), we get

$f(-1) = (-1)^2 - 2(-1) - 3$

$\Rightarrow f(-1) = 1 + 2 - 3$

$\therefore f(-1) = 0$

Substituting x = 0 in f(x), we get

$f(0) = (0)^2 - 2(0) - 3$

$\Rightarrow f(0) = 0 - 0 - 3$

$\therefore f(0) = -3$

Substituting x = 1 in f(x), we get

$f(1) = 1^2 - 2(1) - 3$

$\Rightarrow f(1) = 1 - 2 - 3$

$\therefore f(1) = -4$

Substituting x = 2 in f(x), we get

$f(2) = 2^2 - 2(2) - 3$

$\Rightarrow f(2) = 4 - 4 - 3$

$\therefore f(2) = -3$

Thus, the range of f is {5, 0, –3, –4}.

ii. pre-images of 6, –3 and 5

Let x be the pre-image of 6 ⇒ f(x) = 6

$\Rightarrow x^2 - 2x - 3 = 6$

$\Rightarrow x^2 - 2x - 9 = 0$

$$\Rightarrow x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2(1)}$$

$$\Rightarrow x = \frac{2 \pm \sqrt{4 + 36}}{2}$$

$$\Rightarrow x = \frac{2 \pm \sqrt{40}}{2}$$

$$\Rightarrow x = \frac{2 \pm 2\sqrt{10}}{2}$$

$$\therefore x = 1 \pm \sqrt{10}$$

However, $1 \pm \sqrt{10} \notin A$

Thus, there exists no pre-image of 6.

Now, let x be the pre-image of –3 ⇒ f(x) = –3

⇒ $x^2 - 2x - 3 = -3$

⇒ $x^2 - 2x = 0$

⇒ x(x – 2) = 0

∴ x = 0 or 2

Clearly, both 0 and 2 are elements of A.

Thus, 0 and 2 are the pre-images of –3.

Now, let x be the pre-image of 5 ⇒ f(x) = 5

⇒ $x^2 - 2x - 3 = 5$

⇒ $x^2 - 2x - 8 = 0$

⇒ $x^2 - 4x + 2x - 8 = 0$

⇒ x(x – 4) + 2(x – 4) = 0

⇒ (x + 2)(x – 4) = 0

∴ x = –2 or 4

However, 4 ∉ A but –2 ∈ A

Thus, –2 is the pre-images of 5.

5. Question

If a function f: R → R be defined by

$$f(x) = \begin{cases} 3x - 2, x < 0 \\ 1, x = 0 \\ 4x + 1, x > 0 \end{cases}$$

Find: f(1), f(–1), f(0), f(2).

Answer

Given $f(x) = \begin{cases} 3x - 2, x < 0 \\ 1, x = 0 \\ 4x + 1, x > 0 \end{cases}$

We need to find f(1), f(–1), f(0) and f(2).

When x > 0, f(x) = 4x + 1

Substituting x = 1 in the above equation, we get

f(1) = 4(1) + 1

⇒ f(1) = 4 + 1

∴ f(1) = 5

When x < 0, f(x) = 3x – 2

Substituting x = –1 in the above equation, we get

f(–1) = 3(–1) – 2

⇒ f(–1) = –3 – 2

∴ f(–1) = –5

When x = 0, f(x) = 1

∴ f(0) = 1

When x > 0, f(x) = 4x + 1

Substituting x = 2 in the above equation, we get

f(2) = 4(2) + 1

⇒ f(2) = 8 + 1

∴ f(2) = 9

Thus, f(1) = 5, f(–1) = –5, f(0) = 1 and f(2) = 9.

6. Question

A function f : R → R is defined by f(x) = x^2. Determine

i. range of f

ii. {x: f(x) = 4}

iii. {y: f(y) = –1}

Answer

Given f : R → R and f(x) = x^2.

i. range of f

Domain of f = R (set of real numbers)

We know that the square of a real number is always positive or equal to zero.

Hence, the range of f is the set of all non-negative real numbers.

Thus, range of f = R^+ ∪ {0}

ii. {x: f(x) = 4}

Given f(x) = 4

⇒ x^2 = 4

⇒ x^2 – 4 = 0

⇒ (x – 2)(x + 2) = 0

∴ x = ±2

Thus, {x: f(x) = 4} = {–2, 2}

iii. {y: f(y) = –1}

Given f(y) = –1

⇒ y^2 = –1

However, the domain of f is R, and for every real number y, the value of y^2 is non-negative.

Hence, there exists no real y for which y^2 = –1.

Thus, {y: f(y) = –1} = ∅

7. Question

Let f: $R^+ \to R$, where R^+ is the set of all positive real numbers, be such that $f(x) = \log_e x$. Determine

i. the image set of the domain of f

ii. {x: f(x) = –2}

iii. whether f(xy) = f(x) + f(y) holds.

Answer

Given f : $R^+ \to R$ and $f(x) = \log_e x$.

i. the image set of the domain of f

Domain of f = R^+ (set of positive real numbers)

We know the value of logarithm to the base e (natural logarithm) can take all possible real values.

Hence, the image set of f is the set of real numbers.

Thus, the image set of f = R

ii. {x: f(x) = –2}

Given f(x) = –2

$\Rightarrow \log_e x = -2$

$\therefore x = e^{-2}$ [$\because \log_b a = c \Rightarrow a = b^c$]

Thus, {x: f(x) = –2} = $\{e^{-2}\}$

iii. whether f(xy) = f(x) + f(y) holds.

We have $f(x) = \log_e x \Rightarrow f(y) = \log_e y$

Now, let us consider f(xy).

$f(xy) = \log_e(xy)$

$\Rightarrow f(xy) = \log_e(x \times y)$ [$\because \log_b(a \times c) = \log_b a + \log_b c$]

$\Rightarrow f(xy) = \log_e x + \log_e y$

$\therefore f(xy) = f(x) + f(y)$

Hence, the equation f(xy) = f(x) + f(y) holds.

8. Question

Write the following relations as sets of ordered pairs and find which of them are functions:

i. {(x, y): y = 3x, x ∈ {1, 2, 3}, y ∈ {3, 6, 9, 12}}

ii. {(x, y): y > x + 1, x = 1, 2 and y = 2, 4, 6}

iii. {(x, y): x + y = 3, x, y ∈ {0, 1, 2, 3}}

Answer

i. {(x, y): y = 3x, x ∈ {1, 2, 3}, y ∈ {3, 6, 9, 12}}

When x = 1, we have y = 3(1) = 3

When x = 2, we have y = 3(2) = 6

When x = 3, we have y = 3(3) = 9

Thus, R = {(1, 3), (2, 6), (3, 9)}

Every element of set x has an ordered pair in the relation and no two ordered pairs have the same first component but different second components.

Hence, the given relation R is a function.

ii. $\{(x, y): y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$

When $x = 1$, we have $y > 1 + 1$ or $y > 2 \Rightarrow y = \{4, 6\}$

When $x = 2$, we have $y > 2 + 1$ or $y > 3 \Rightarrow y = \{4, 6\}$

Thus, $R = \{(1, 4), (1, 6), (2, 4), (2, 6)\}$

Every element of set x has an ordered pair in the relation. However, two ordered pairs (1, 4) and (1, 6) have the same first component but different second components.

Hence, the given relation R is not a function.

iii. $\{(x, y): x + y = 3, x, y \in \{0, 1, 2, 3\}\}$

When $x = 0$, we have $0 + y = 3 \Rightarrow y = 3$

When $x = 1$, we have $1 + y = 3 \Rightarrow y = 2$

When $x = 2$, we have $2 + y = 3 \Rightarrow y = 1$

When $x = 3$, we have $3 + y = 3 \Rightarrow y = 0$

Thus, $R = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$

Every element of set x has an ordered pair in the relation and no two ordered pairs have the same first component but different second components.

Hence, the given relation R is a function.

9. Question

Let $f : R \rightarrow R$ and $g : C \rightarrow C$ be two functions defined as $f(x) = x^2$ and $g(x) = x^2$. Are they equal functions?

Answer

Given $f : R \rightarrow R \ni f(x) = x^2$ and $g : R \rightarrow R \ni g(x) = x^2$

As f is defined from R to R, the domain of f = R.

As g is defined from C to C, the domain of g = C.

Two functions are equal only when the domain and codomain of both the functions are equal.

In this case, the domain of f ≠ domain of g.

Thus, f and g are not equal functions.

10. Question

If f, g, h are three functions defined from R to R as follows:

i. $f(x) = x^2$

ii. $g(x) = \sin x$

iii. $h(x) = x^2 + 1$

Find the range of each function.

Answer

i. $f(x) = x^2$

Domain of f = R (set of real numbers)

We know that the square of a real number is always positive or equal to zero.

Hence, the range of f is the set of all non-negative real numbers.

Thus, range of $f = [0, \infty) = \{y: y \geq 0\}$

ii. $g(x) = \sin x$

Domain of g = R (set of real numbers)

We know that the value of sine function always lies between –1 and 1.

Hence, the range of g is the set of all real numbers lying in the range –1 to 1.

Thus, range of g = $[-1, 1] = \{y: -1 \leq y \leq 1\}$

iii. $h(x) = x^2 + 1$

Domain of h = R (set of real numbers)

We know that the square of a real number is always positive or equal to zero.

Furthermore, if we add 1 to this squared number, the result will always be greater than or equal to 1.

Hence, the range of h is the set of all real numbers greater than or equal to 1.

Thus, range of h = $[1, \infty) = \{y: y \geq 1\}$

11. Question

Let X = {1, 2, 3, 4} and Y = {1, 5, 9, 11, 15, 16}. Determine which of the following sets are functions from X to Y.

i. f_1 = {(1, 1), (2, 11), (3, 1), (4, 15)}

ii. f_2 = {(1, 1), (2, 7), (3, 5)}

iii. f_3 = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

Answer

Given X = {1, 2, 3, 4} and Y = {1, 5, 9, 11, 15, 16}

i. f_1 = {(1, 1), (2, 11), (3, 1), (4, 15)}

Every element of set X has an ordered pair in the relation f_1 and no two ordered pairs have the same first component but different second components.

Hence, the given relation f_1 is a function.

ii. f_2 = {(1, 1), (2, 7), (3, 5)}

In the relation f_2, the element 2 of set X does not have any image in set Y.

However, for a relation to be a function, every element of the domain should have an image.

Hence, the given relation f_2 is not a function.

iii. f_3 = {(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)}

Every element of set X has an ordered pair in the relation f_3. However, two ordered pairs (2, 9) and (2, 11) have the same first component but different second components.

Hence, the given relation f_3 is not a function.

12. Question

Let A = {12, 13, 14, 15, 16, 17} and $f : A \rightarrow Z$ be a function given by f(x) = highest prime factor of x. Find range of f.

Answer

Given A = {12, 13, 14, 15, 16, 17}

$f : A \rightarrow Z$ such that f(x) = highest prime factor of x.

A is the domain of the function f. Hence, the range is the set of elements f(x) for all $x \in A$.

We have f(12) = highest prime factor of 12

The prime factorization of 12 = $2^2 \times 3$

Thus, the highest prime factor of 12 is 3.

∴ f(12) = 3

We have f(13) = highest prime factor of 13

We know 13 is a prime number.

∴ f(13) = 13

We have f(14) = highest prime factor of 14

The prime factorization of 14 = 2 × 7

Thus, the highest prime factor of 14 is 7.

∴ f(14) = 7

We have f(15) = highest prime factor of 15

The prime factorization of 15 = 3 × 5

Thus, the highest prime factor of 15 is 5.

∴ f(15) = 5

We have f(16) = highest prime factor of 16

The prime factorization of 16 = 2^4

Thus, the highest prime factor of 16 is 2.

∴ f(16) = 2

We have f(17) = highest prime factor of 17

We know 17 is a prime number.

∴ f(17) = 17

Thus, the range of f is {3, 13, 7, 5, 2, 17}.

13. Question

If f : R → R be defined by $f(x) = x^2 + 1$, then find $f^{-1}\{17\}$ and $f^{-1}\{-3\}$.

Answer

Given f : R → R and $f(x) = x^2 + 1$.

We need to find $f^{-1}\{17\}$ and $f^{-1}\{-3\}$.

Let $f^{-1}\{17\} = x$

⇒ f(x) = 17

⇒ $x^2 + 1 = 17$

⇒ $x^2 - 16 = 0$

⇒ (x – 4)(x + 4) = 0

∴ x = ±4

Clearly, both –4 and 4 are elements of the domain R.

Thus, $f^{-1}\{17\} = \{-4, 4\}$

Now, let $f^{-1}\{-3\} = x$

$\Rightarrow f(x) = -3$

$\Rightarrow x^2 + 1 = -3$

$\Rightarrow x^2 = -4$

However, the domain of f is R and for every real number x, the value of x^2 is non-negative.

Hence, there exists no real x for which $x^2 = -4$.

Thus, $f^{-1}\{-3\} = \varnothing$

14. Question

Let A = {p, q, r, s} and B = {1, 2, 3}. Which of the following relations from A to B is not a function?

i. R_1 = {(p, 1), (q, 2), (r, 1), (s, 2)}

ii. R_2 = {(p, 1), (q, 1), (r, 1), (s, 1)}

iii. R_3 = {(p, 1), (q, 2), (p, 2), (s, 3)}

iv. R_4 = {(p, 2), (q, 3), (r, 2), (s, 2)}

Answer

Given A = {p, q, r, s} and B = {1, 2, 3}

i. R_1 = {(p, 1), (q, 2), (r, 1), (s, 2)}

Every element of set A has an ordered pair in the relation R_1 and no two ordered pairs have the same first component but different second components.

Hence, the given relation R_1 is a function.

ii. R_2 = {(p, 1), (q, 1), (r, 1), (s, 1)}

Every element of set A has an ordered pair in the relation $R_{2,}$ and no two ordered pairs have the same first component but different second components.

Hence, the given relation R_2 is a function.

iii. R_3 = {(p, 1), (q, 2), (p, 2), (s, 3)}

Every element of set A has an ordered pair in the relation R_3. However, two ordered pairs (p, 1) and (p, 2) have the same first component but different second components.

Hence, the given relation R_3 is not a function.

iv. R_4 = {(p, 2), (q, 3), (r, 2), (s, 2)}

Every element of set A has an ordered pair in the relation $R_{4,}$ and no two ordered pairs have the same first component but different second components.

Hence, the given relation R_4 is a function.

15. Question

Let A = {9, 10, 11, 12, 13} and let f : A → Z be a function given by f(n) = the highest prime factor of n. Find the range of f.

Answer

Given A = {9, 10, 11, 12, 13}

f : A → Z such that f(n) = the highest prime factor of n.

A is the domain of the function f. Hence, the range is the set of elements f(n) for all $n \in A$.

We have f(9) = highest prime factor of 9

The prime factorization of 9 = 3^2

Thus, the highest prime factor of 9 is 3.

∴ f(9) = 3

We have f(10) = highest prime factor of 10

The prime factorization of 10 = 2 × 5

Thus, the highest prime factor of 10 is 5.

∴ f(10) = 5

We have f(11) = highest prime factor of 11

We know 11 is a prime number.

∴ f(11) = 11

We have f(12) = highest prime factor of 12

The prime factorization of 12 = $2^2 \times 3$

Thus, the highest prime factor of 12 is 3.

∴ f(12) = 3

We have f(13) = highest prime factor of 13

We know 13 is a prime number.

∴ f(13) = 13

Thus, the range of f is {3, 5, 11, 13}.

16. Question

The function f is defined by $f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$

The relation g is defined by $g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

Show that f is a function and g is not a function.

Answer

Given $f(x) = \begin{cases} x^2, 0 \le x \le 3 \\ 3x, 3 \le x \le 10 \end{cases}$ and $g(x) = \begin{cases} x^2, 0 \le x \le 2 \\ 3x, 2 \le x \le 10 \end{cases}$

Let us first show that f is a function.

When 0 ≤ x ≤ 3, f(x) = x^2.

The function x^2 associates all the numbers 0 ≤ x ≤ 3 to unique numbers in R.

Hence, the images of {x ∈ Z: 0 ≤ x ≤ 3} exist and are unique.

When 3 ≤ x ≤ 10, f(x) = 3x.

The function x^2 associates all the numbers 3 ≤ x ≤ 10 to unique numbers in R.

Hence, the images of {x ∈ Z: 3 ≤ x ≤ 10} exist and are unique.

When x = 3, using the first definition, we have

$f(3) = 3^2 = 9$

When x = 3, using the second definition, we have

f(3) = 3(3) = 9

Hence, the image of x = 3 is also distinct.

Thus, as every element of the domain has an image and no element has more than one image, f is a function.

Now, let us show that g is not a function.

When $0 \le x \le 2$, $g(x) = x^2$.

The function x^2 associates all the numbers $0 \le x \le 2$ to unique numbers in R.

Hence, the images of $\{x \in Z: 0 \le x \le 2\}$ exist and are unique.

When $2 \le x \le 10$, g(x) = 3x.

The function x^2 associates all the numbers $2 \le x \le 10$ to unique numbers in R.

Hence, the images of $\{x \in Z: 2 \le x \le 10\}$ exist and are unique.

When x = 2, using the first definition, we have

$g(2) = 2^2 = 4$

When x = 2, using the second definition, we have

g(2) = 3(2) = 6

Here, the element 2 of the domain is associated with two elements distinct elements 4 and 6.

Thus, g is not a function.

17. Question

If $f(x) = x^2$, find $\frac{f(1.1) - f(1)}{1.1 - 1}$

Answer

Given $f(x) = x^2$.

We need to find the value of $\frac{f(1.1)-f(1)}{1.1-1}$

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1)^2 - (1)^2}{1.1 - 1}$$

$$\Rightarrow \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(1.1 + 1)(1.1 - 1)}{0.1}$$

$$\Rightarrow \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{(2.1)(0.1)}{0.1}$$

$$\therefore \frac{f(1.1) - f(1)}{1.1 - 1} = 2.1$$

Thus, $\frac{f(1.1)-f(1)}{1.1-1} = 2.1$

18. Question

Express the function f : X → R given by $f(x) = x^3 + 1$ as set of ordered pairs, where X = {–1, 0, 3, 9, 7}.

Answer

Given X = {–1, 0, 3, 9, 7}

$f : X \to R$ and $f(x) = x^3 + 1$

When x = –1, we have $f(-1) = (-1)^3 + 1$

⇒ $f(-1) = -1 + 1$

∴ $f(-1) = 0$

When x = 0, we have $f(0) = 0^3 + 1$

⇒ $f(0) = 0 + 1$

∴ $f(0) = 1$

When x = 3, we have $f(3) = 3^3 + 1$

⇒ $f(3) = 27 + 1$

∴ $f(3) = 28$

When x = 9, we have $f(9) = 9^3 + 1$

⇒ $f(9) = 729 + 1$

∴ $f(9) = 730$

When x = 7, we have $f(7) = 7^3 + 1$

⇒ $f(7) = 343 + 1$

∴ $f(7) = 344$

Thus, f = {(–1, 0), (0, 1), (3, 28), (9, 730), (7, 344)}

Exercise 3.2

1. Question

If $f(x) = x^2 - 3x + 4$, then find the values of x satisfying the equation f(x) = f(2x + 1).

Answer

Given $f(x) = x^2 - 3x + 4$.

We need to find x satisfying f(x) = f(2x + 1).

We have $f(2x + 1) = (2x + 1)^2 - 3(2x + 1) + 4$

⇒ $f(2x + 1) = (2x)^2 + 2(2x)(1) + 1^2 - 6x - 3 + 4$

⇒ $f(2x + 1) = 4x^2 + 4x + 1 - 6x + 1$

∴ $f(2x + 1) = 4x^2 - 2x + 2$

Now, f(x) = f(2x + 1)

⇒ $x^2 - 3x + 4 = 4x^2 - 2x + 2$

⇒ $3x^2 + x - 2 = 0$

⇒ $3x^2 + 3x - 2x - 2 = 0$

⇒ 3x(x + 1) – 2(x + 1) = 0

⇒ (x + 1)(3x – 2) = 0

⇒ x + 1 = 0 or 3x – 2 = 0

⇒ x = –1 or 3x = 2

∴ x = –1 or $\frac{2}{3}$

Thus, the required values of x are –1 and $\frac{2}{3}$.

2. Question

If f(x) = $(x - a)^2(x - b)^2$, find f(a + b).

Answer

Given f(x) = $(x - a)^2(x - b)^2$

We need to find f(a + b).

We have f(a + b) = $(a + b - a)^2(a + b - b)^2$

⇒ f(a + b) = $(b)^2(a)^2$

∴ f(a + b) = a^2b^2

Thus, f(a + b) = a^2b^2

3. Question

If $y = f(x) = \frac{ax - b}{bx - a}$, show that x = f(y).

Answer

Given $y = f(x) = \frac{ax-b}{bx-a} \Rightarrow f(y) = \frac{ay-b}{by-a}$

We need to prove that x = f(y).

We have $y = \frac{ax-b}{bx-a}$

⇒ y(bx – a) = ax – b

⇒ bxy – ay = ax – b

⇒ bxy – ax = ay – b

⇒ x(by – a) = ay – b

$\Rightarrow x = \frac{ay - b}{by - a} = f(y)$

∴ x = f(y)

Thus, x = f(y).

4. Question

If $f(x) = \frac{1}{1 - x}$, show that f[f{f(x)}] = x.

Answer

Given $f(x) = \frac{1}{1-x}$

We need to prove that f[f{f(x)}] = x.

First, we will evaluate f{f(x)}.

$$f\{f(x)\} = f\left\{\frac{1}{1 - x}\right\}$$

$$\Rightarrow f\{f(x)\} = \frac{1}{1 - \left(\frac{1}{1 - x}\right)}$$

$$\Rightarrow f\{f(x)\} = \frac{1}{\frac{1-x-1}{1-x}}$$

$$\Rightarrow f\{f(x)\} = \frac{1}{\frac{-x}{1-x}}$$

$$\Rightarrow f\{f(x)\} = \frac{1-x}{-x}$$

$$\therefore f\{f(x)\} = \frac{x-1}{x}$$

Now, we will evaluate f[f{f(x)}]

$$f[f\{f(x)\}] = f\left[\frac{x-1}{x}\right]$$

$$\Rightarrow f[f\{f(x)\}] = \frac{1}{1-\left(\frac{x-1}{x}\right)}$$

$$\Rightarrow f[f\{f(x)\}] = \frac{1}{\frac{x-(x-1)}{x}}$$

$$\Rightarrow f[f\{f(x)\}] = \frac{1}{\frac{x-x+1}{x}}$$

$$\Rightarrow f[f\{f(x)\}] = \frac{1}{\frac{1}{x}}$$

∴ f[f{f(x)}] = x

Thus, f[f{f(x)}] = x

5. Question

If $f\left(x\right) = \frac{x+1}{x-1}$, show that f[f(x)] = x.

Answer

Given $f(x) = \frac{x+1}{x-1}$

We need to prove that f[f(x)] = x.

$$f[f(x)] = f\left[\frac{x+1}{x-1}\right]$$

$$\Rightarrow f[f(x)] = \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1}$$

$$\Rightarrow f[f(x)] = \frac{\frac{(x+1)+(x-1)}{x-1}}{\frac{(x+1)-(x-1)}{x-1}}$$

$$\Rightarrow f[f(x)] = \frac{(x+1)+(x-1)}{(x+1)-(x-1)}$$

$$\Rightarrow f[f(x)] = \frac{x+1+x-1}{x+1-x+1}$$

$\Rightarrow f[f(x)] = \frac{2x}{2}$

∴ f[f(x)] = x

Thus, f[f(x)] = x

6. Question

If $f(x) = \begin{cases} x^2, \text{when } x < 0 \\ x, \text{when } 0 \le x \le 1 \\ \frac{1}{x}, \text{when } x > 1 \end{cases}$, find:

i. $f\left(\frac{1}{2}\right)$

ii. f(–2)

iii. f(1)

iv. $f\left(\sqrt{3}\right)$

v. $f\left(\sqrt{-3}\right)$

Answer

Given $f(x) = \begin{cases} x^2, \text{when } x < 0 \\ x, \text{ when } 0 \le x < 1 \\ \frac{1}{x}, \text{when } x \ge 1 \end{cases}$

i. $f\left(\frac{1}{2}\right)$

When 0 ≤ x ≤ 1, f(x) = x

$\therefore f\left(\frac{1}{2}\right) = \frac{1}{2}$

ii. f(–2)

When x < 0, f(x) = x^2

⇒ f(–2) = $(-2)^2$

∴ f(–2) = 4

iii. f(1)

When x ≥ 1, $f(x) = \frac{1}{x}$

$\Rightarrow f(1) = \frac{1}{1}$

∴ f(1) = 1

iv. $f(\sqrt{3})$

We have $\sqrt{3} \approx 1.732 > 1$

When x ≥ 1, $f(x) = \frac{1}{x}$

$\therefore f(\sqrt{3}) = \frac{1}{\sqrt{3}}$

v. $f(\sqrt{-3})$

We know $\sqrt{-3}$ is not a real number and the function f(x) is defined only when x ∈ R.

Thus, $f(\sqrt{-3})$ does not exist.

7. Question

If $f(x) = x^3 - \frac{1}{x^3}$, show that $f(x) + f\left(\frac{1}{x}\right) = 0$.

Answer

Given $f(x) = x^3 - \frac{1}{x^3}$

We need to prove that $f(x) + f\left(\frac{1}{x}\right) = 0$

We have, $f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1^3}{x^3} - \frac{1}{\frac{1^3}{x^3}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$

$\Rightarrow f\left(\frac{1}{x}\right) = -\left(-\frac{1}{x^3} + x^3\right)$

$\Rightarrow f\left(\frac{1}{x}\right) = -\left(x^3 - \frac{1}{x^3}\right)$

$\Rightarrow f\left(\frac{1}{x}\right) = -f(x)$

$\therefore f(x) + f\left(\frac{1}{x}\right) = 0$

Thus, $f(x) + f\left(\frac{1}{x}\right) = 0$

8. Question

If $f(x) = \frac{2x}{1+x^2}$, show that f(tanθ) = sin2θ.

Answer

Given $f(x) = \frac{2x}{1+x^2}$

We need to prove that f(tanθ) = sin2θ.

We have $f(\tan\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$

We know $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\Rightarrow f(\tan\theta) = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2}$$

$$\Rightarrow f(\tan\theta) = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$$

$$\Rightarrow f(\tan\theta) = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}$$

However, $\cos^2\theta + \sin^2\theta = 1$

$$\Rightarrow f(\tan\theta) = \frac{2\left(\frac{\sin\theta}{\cos\theta}\right)}{\frac{1}{\cos^2\theta}}$$

$$\Rightarrow f(\tan\theta) = 2\left(\frac{\sin\theta}{\cos\theta}\right) \times \cos^2\theta$$

$\Rightarrow f(\tan\theta) = 2\sin\theta\cos\theta$

$\therefore f(\tan\theta) = \sin 2\theta$

Thus, $f(\tan\theta) = \sin 2\theta$

9. Question

If $f(x) = \frac{x+1}{x-1}$, then show that

i. $f\left(\frac{1}{x}\right) = -f(x)$

ii. $f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$

Answer

Given $f(x) = \frac{x+1}{x-1}$

i. We need to prove that $f\left(\frac{1}{x}\right) = -f(x)$

We have $f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}+1}{\frac{1}{x}-1}$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{\frac{1+x}{x}}{\frac{1-x}{x}}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{1+x}{1-x}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = \frac{x+1}{-(x-1)}$$

$$\Rightarrow f\left(\frac{1}{x}\right) = -\left(\frac{x+1}{x-1}\right)$$

$\therefore f\left(\frac{1}{x}\right) = -f(x)$

Thus, $f\left(\frac{1}{x}\right) = -f(x)$

ii. We need to prove that $f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$

We have $f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x}+1}{-\frac{1}{x}-1}$

$$\Rightarrow f\left(-\frac{1}{x}\right) = \frac{\frac{-1+x}{x}}{\frac{-1-x}{x}}$$

$$\Rightarrow f\left(-\frac{1}{x}\right) = \frac{-1+x}{-1-x}$$

$$\Rightarrow f\left(-\frac{1}{x}\right) = \frac{x-1}{-(x+1)}$$

$$\Rightarrow f\left(-\frac{1}{x}\right) = -\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow f\left(-\frac{1}{x}\right) = -\frac{1}{\left(\frac{x+1}{x-1}\right)}$$

$$\therefore f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$$

Thus, $f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$

10. Question

If $f(x) = (a - x^n)^{\frac{1}{n}}$, a > 0 and n ∈ N, then prove that f[f(x)] = x for all x.

Answer

Given $f(x) = (a - x^n)^{\frac{1}{n}}$, where a > 0 and n ∈ N.

We need to prove that f[f(x)] = x.

$$f[f(x)] = f\left[(a - x^n)^{\frac{1}{n}}\right]$$

$$\Rightarrow f[f(x)] = \left[a - \left((a - x^n)^{\frac{1}{n}}\right)^n\right]^{\frac{1}{n}}$$

$\Rightarrow f[f(x)] = \left[a - (a - x^n)^{\frac{1}{n}\times n}\right]^{\frac{1}{n}}$ [∵ $(a^m)^n = a^{mn}$]

$$\Rightarrow f[f(x)] = [a - (a - x^n)^1]^{\frac{1}{n}}$$

$$\Rightarrow f[f(x)] = [a - (a - x^n)]^{\frac{1}{n}}$$

$$\Rightarrow f[f(x)] = [a - a + x^n]^{\frac{1}{n}}$$

$$\Rightarrow f[f(x)] = [x^n]^{\frac{1}{n}}$$

$\Rightarrow f[f(x)] = x^{n\times\frac{1}{n}}$ [∵ $(a^m)^n = a^{mn}$]

$\Rightarrow f[f(x)] = x^1$

$\therefore f[f(x)] = x$

Thus, f[f(x)] = x for all x.

11. Question

If for non-zero x, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where a ≠ b, then find f(x).

Answer

Given x ≠ 0 and a ≠ b such that

$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$ ..... (1)

Substituting $\frac{1}{x}$ in place of x, we get

$$af\left(\frac{1}{x}\right) + bf\left(\frac{1}{\left(\frac{1}{x}\right)}\right) = \frac{1}{\left(\frac{1}{x}\right)} - 5$$

$\Rightarrow af\left(\frac{1}{x}\right) + bf(x) = x - 5$ .....-- (2)

On adding equations (1) and (2), we get

$$af(x) + bf\left(\frac{1}{x}\right) + af\left(\frac{1}{x}\right) + bf(x) = \frac{1}{x} - 5 + x - 5$$

$$\Rightarrow af(x) + bf(x) + af\left(\frac{1}{x}\right) + bf\left(\frac{1}{x}\right) = x + \frac{1}{x} - 10$$

$$\Rightarrow (a+b)f(x) + (a+b)f\left(\frac{1}{x}\right) = x + \frac{1}{x} - 10$$

$$\Rightarrow (a+b)\left[f(x) + f\left(\frac{1}{x}\right)\right] = x + \frac{1}{x} - 10$$

$\therefore f(x) + f\left(\frac{1}{x}\right) = \frac{1}{a+b}\left(x + \frac{1}{x} - 10\right)$ ..... (3)

On subtracting equations (1) and (2), we get

$$af(x) + bf\left(\frac{1}{x}\right) - \left[af\left(\frac{1}{x}\right) + bf(x)\right] = \frac{1}{x} - 5 - (x - 5)$$

$$\Rightarrow af(x) + bf\left(\frac{1}{x}\right) - af\left(\frac{1}{x}\right) - bf(x) = \frac{1}{x} - 5 - x + 5$$

$$\Rightarrow af(x) - bf(x) - af\left(\frac{1}{x}\right) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - x$$

$$\Rightarrow (a-b)f(x) - (a-b)f\left(\frac{1}{x}\right) = \frac{1}{x} - x$$

$$\Rightarrow (a-b)\left[f(x) - f\left(\frac{1}{x}\right)\right] = \frac{1}{x} - x$$

$\therefore f(x) - f\left(\frac{1}{x}\right) = \frac{1}{a-b}\left(\frac{1}{x} - x\right)$ ..... (4)

On adding equations (3) and (4), we get

$$f(x) + f\left(\frac{1}{x}\right) + f(x) - f\left(\frac{1}{x}\right) = \frac{1}{a+b}\left(x + \frac{1}{x} - 10\right) + \frac{1}{a-b}\left(\frac{1}{x} - x\right)$$

$$\Rightarrow 2f(x)=\frac{(a-b)\left(x+\frac{1}{x}-10\right)+(a+b)\left(\frac{1}{x}-x\right)}{(a+b)(a-b)}$$

$$\Rightarrow 2f(x)=\frac{1}{a^2-b^2}\left[(a-b)x+\frac{(a-b)}{x}-10(a-b)+\frac{(a+b)}{x}-(a+b)x\right]$$

$$\Rightarrow 2f(x)=\frac{1}{a^2-b^2}\left[(a-b-a-b)x+\frac{a-b+a+b}{x}-10(a-b)\right]$$

$$\Rightarrow 2f(x)=\frac{1}{a^2-b^2}\left[-2bx+\frac{2a}{x}-10(a-b)\right]$$

$$\Rightarrow 2f(x)=\frac{2}{a^2-b^2}\left[-bx+\frac{a}{x}-5(a-b)\right]$$

$$\Rightarrow f(x)=\frac{1}{a^2-b^2}\left[-bx+\frac{a}{x}-5(a-b)\right]$$

$$\Rightarrow f(x)=\frac{1}{a^2-b^2}\left[-bx+\frac{a}{x}\right]-\frac{5(a-b)}{a^2-b^2}$$

$$\Rightarrow f(x)=\frac{1}{a^2-b^2}\left[-bx+\frac{a}{x}\right]-\frac{5(a-b)}{(a+b)(a-b)}$$

$$\therefore f(x)=\frac{1}{a^2-b^2}\left[\frac{a}{x}-bx\right]-\frac{5}{a+b}$$

Thus, $f(x)=\frac{1}{a^2-b^2}\left[\frac{a}{x}-bx\right]-\frac{5}{a+b}$

Exercise 3.3

1. Question

Find the domain of each of the following real valued functions of real variable:

i. $f(x)=\frac{1}{x}$

ii. $f(x)=\frac{1}{x-7}$

iii. $f(x)=\frac{3x-2}{x+1}$

iv. $f(x)=\frac{2x+1}{x^2-9}$

v. $f(x)=\frac{x^2+2x+1}{x^2-8x+12}$

Answer

i. $f(x)=\frac{1}{x}$

Clearly, f(x) is defined for all real values of x, except for the case when x = 0.

When x = 0, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {0}

ii. $f(x)=\frac{1}{x-7}$

Clearly, f(x) is defined for all real values of x, except for the case when x – 7 = 0 or x = 7.

When x = 7, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {7}

iii. $f(x) = \frac{3x-2}{x+1}$

Clearly, f(x) is defined for all real values of x, except for the case when x + 1 = 0 or x = –1.

When x = –1, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {–1}

iv. $f(x) = \frac{2x+1}{x^2-9}$

Clearly, f(x) is defined for all real values of x, except for the case when $x^2 - 9 = 0$.

$x^2 - 9 = 0$

$\Rightarrow x^2 - 3^2 = 0$

⇒ (x + 3)(x – 3) = 0

⇒ x + 3 = 0 or x – 3 = 0

⇒ x = ±3

When x = ±3, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {–3, 3}

v. $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

Clearly, f(x) is defined for all real values of x, except for the case when $x^2 - 8x + 12 = 0$.

$x^2 - 8x + 12 = 0$

$\Rightarrow x^2 - 2x - 6x + 12 = 0$

⇒ x(x – 2) – 6(x – 2) = 0

⇒ (x – 2)(x – 6) = 0

⇒ x – 2 = 0 or x – 6 = 0

⇒ x = 2 or 6

When x = 2 or 6, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {2, 6}

2 A. Question

Find the domain of each of the following real valued functions of real variable:

$$f(x) = \sqrt{x-2}$$

Answer

$f(x) = \sqrt{x-2}$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when x – 2 ≥ 0

⇒ x ≥ 2

∴ x ∈ [2, ∞)

Thus, domain of f = [2, ∞)

2 B. Question

Find the domain of each of the following real valued functions of real variable:

$$f(x) = \frac{1}{\sqrt{x^2 - 1}}$$

Answer

$$f(x) = \frac{1}{\sqrt{x^2 - 1}}$$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $x^2 - 1 \geq 0$

$\Rightarrow x^2 - 1^2 \geq 0$

$\Rightarrow (x + 1)(x - 1) \geq 0$

$\Rightarrow x \leq -1$ or $x \geq 1$

$\therefore x \in (-\infty, -1] \cup [1, \infty)$

In addition, f(x) is also undefined when $x^2 - 1 = 0$ because denominator will be zero and the result will be indeterminate.

$x^2 - 1 = 0 \Rightarrow x = \pm 1$

Hence, $x \in (-\infty, -1] \cup [1, \infty) - \{-1, 1\}$

$\therefore x \in (-\infty, -1) \cup (1, \infty)$

Thus, domain of f = $(-\infty, -1) \cup (1, \infty)$

2 C. Question

Find the domain of each of the following real valued functions of real variable:

$$f(x) = \sqrt{9 - x^2}$$

Answer

$$f(x) = \sqrt{9 - x^2}$$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $9 - x^2 \geq 0$

$\Rightarrow 9 \geq x^2$

$\Rightarrow x^2 \leq 9$

$\Rightarrow x^2 - 9 \leq 0$

$\Rightarrow x^2 - 3^2 \leq 0$

$\Rightarrow (x + 3)(x - 3) \leq 0$

$\Rightarrow x \geq -3$ and $x \leq 3$

$\therefore x \in [-3, 3]$

Thus, domain of f = $[-3, 3]$

2 D. Question

Find the domain of each of the following real valued functions of real variable:

$$f(x) = \sqrt{\frac{x-2}{3-x}}$$

Answer

$$f(x) = \sqrt{\frac{x-2}{3-x}}$$

We know the square root of a real number is never negative.

Clearly, f(x) takes real values only when x – 2 and 3 – x are both positive or negative.

(a) Both x – 2 and 3 – x are positive

$x - 2 \geq 0 \Rightarrow x \geq 2$

$3 - x \geq 0 \Rightarrow x \leq 3$

Hence, $x \geq 2$ and $x \leq 3$

$\therefore x \in [2, 3]$

(b) Both x – 2 and 3 – x are negative

$x - 2 \leq 0 \Rightarrow x \leq 2$

$3 - x \leq 0 \Rightarrow x \geq 3$

Hence, $x \leq 2$ and $x \geq 3$

However, the intersection of these sets in null set. Thus, this case is not possible.

In addition, f(x) is also undefined when 3 – x = 0 because the denominator will be zero and the result will be indeterminate.

$3 - x = 0 \Rightarrow x = 3$

Hence, $x \in [2, 3] - \{3\}$

$\therefore x \in [2, 3)$

Thus, domain of f = [2, 3)

3 A. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \frac{ax+b}{bx-a}$$

Answer

$$f(x) = \frac{ax+b}{bx-a}$$

Clearly, f(x) is defined for all real values of x, except for the case when bx – a = 0 or $x = \frac{a}{b}$.

When $x = \frac{a}{b}$, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – $\left\{\frac{a}{b}\right\}$

Let f(x) = y

$$\Rightarrow \frac{ax+b}{bx-a} = y$$

$\Rightarrow ax + b = y(bx - a)$

⇒ ax + b = bxy – ay

⇒ ax – bxy = –ay – b

⇒ x(a – by) = –(ay + b)

$\therefore x = -\frac{(ay + b)}{a - by}$

Clearly, when a – by = 0 or $y = \frac{a}{b}$, x will be undefined as the division result will be indeterminate.

Hence, f(x) cannot take the value $\frac{a}{b}$.

Thus, range of f = R – $\left\{\frac{a}{b}\right\}$

3 B. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \frac{ax - b}{cx - d}$$

Answer

$f(x) = \frac{ax - b}{cx - d}$

Clearly, f(x) is defined for all real values of x, except for the case when cx – d = 0 or $x = \frac{d}{c}$.

When $x = \frac{d}{c}$, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – $\left\{\frac{d}{c}\right\}$

Let f(x) = y

$\Rightarrow \frac{ax - b}{cx - d} = y$

⇒ ax – b = y(cx – d)

⇒ ax – b = cxy – dy

⇒ ax – cxy = b – dy

⇒ x(a – cy) = b – dy

$\therefore x = \frac{b - dy}{a - cy}$

Clearly, when a – cy = 0 or $y = \frac{a}{c}$, x will be undefined as the division result will be indeterminate.

Hence, f(x) cannot take the value $\frac{a}{c}$.

Thus, range of f = R – $\left\{\frac{a}{c}\right\}$

3 C. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \sqrt{x - 1}$$

Answer

$f(x) = \sqrt{x - 1}$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when x – 1 ≥ 0

⇒ x ≥ 1

∴ x ∈ [1, ∞)

Thus, domain of f = [1, ∞)

When x ≥ 1, we have x – 1 ≥ 0

Hence, $\sqrt{x-1} \geq 0 \Rightarrow f(x) \geq 0$

∴ f(x) ∈ [0, ∞)

Thus, range of f = [0, ∞)

3 D. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \sqrt{x-3}$$

Answer

$f(x) = \sqrt{x-3}$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when x – 3 ≥ 0

⇒ x ≥ 3

∴ x ∈ [3, ∞)

Thus, domain of f = [3, ∞)

When x ≥ 3, we have x – 3 ≥ 0

Hence, $\sqrt{x-3} \geq 0 \Rightarrow f(x) \geq 0$

∴ f(x) ∈ [0, ∞)

Thus, range of f = [0, ∞)

3 E. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \frac{x-2}{2-x}$$

Answer

$f(x) = \frac{x-2}{2-x}$

Clearly, f(x) is defined for all real values of x, except for the case when 2 – x = 0 or x = 2.

When x = 2, f(x) will be undefined as the division result will be indeterminate.

Thus, domain of f = R – {2}

We have $f(x) = \frac{x-2}{2-x}$

$\Rightarrow f(x) = \frac{-(2-x)}{2-x}$

∴ f(x) = –1

Clearly, when x ≠ 2, f(x) = –1

Thus, range of f = {–1}

3 F. Question

Find the domain and range of each of the following real valued functions:

f(x) = |x – 1|

Answer

f(x) = |x – 1|

We know $|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$

Now, we have $|x - 1| = \begin{cases} -(x - 1), x - 1 < 0 \\ x - 1, x - 1 \geq 0 \end{cases}$

$\therefore f(x) = |x - 1| = \begin{cases} 1 - x, x < 1 \\ x - 1, x \geq 1 \end{cases}$

Hence, f(x) is defined for all real numbers x.

Thus, domain of f = R

When x < 1, we have x – 1 < 0 or 1 – x > 0.

Hence, |x – 1| > 0 ⇒ f(x) > 0

When x ≥ 1, we have x – 1 ≥ 0.

Hence, |x – 1| ≥ 0 ⇒ f(x) ≥ 0

∴ f(x) ≥ 0 or f(x) ∈ [0, ∞)

Thus, range of f = [0, ∞)

3 G. Question

Find the domain and range of each of the following real valued functions:

f(x) = –|x|

Answer

f(x) = –|x|

We know $|x| = \begin{cases} -x, x < 0 \\ x, x \geq 0 \end{cases}$

Now, we have $-|x| = \begin{cases} -(-x), x < 0 \\ -x, x \geq 0 \end{cases}$

$\therefore f(x) = -|x| = \begin{cases} x, x < 0 \\ -x, x \geq 0 \end{cases}$

Hence, f(x) is defined for all real numbers x.

Thus, domain of f = R

When x < 0, we have –|x| < 0

Hence, f(x) < 0

When x ≥ 0, we have –x ≤ 0.

Hence, –|x| ≤ 0 ⇒ f(x) ≤ 0

∴ f(x) ≤ 0 or f(x) ∈ (–∞, 0]

Thus, range of f = [0, ∞)

3 H. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \sqrt{9 - x^2}$$

Answer

$$f(x) = \sqrt{9 - x^2}$$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $9 - x^2 \geq 0$

$\Rightarrow 9 \geq x^2$

$\Rightarrow x^2 \leq 9$

$\Rightarrow x^2 - 9 \leq 0$

$\Rightarrow x^2 - 3^2 \leq 0$

$\Rightarrow (x + 3)(x - 3) \leq 0$

$\Rightarrow x \geq -3$ and $x \leq 3$

$\therefore x \in [-3, 3]$

Thus, domain of f = [–3, 3]

When $x \in [-3, 3]$, we have $0 \leq 9 - x^2 \leq 9$

Hence, $0 \leq \sqrt{9 - x^2} \leq 3 \Rightarrow 0 \leq f(x) \leq 3$

$\therefore f(x) \in [0, 3]$

Thus, range of f = [0, 3]

3 I. Question

Find the domain and range of each of the following real valued functions:

$$f(x) = \frac{1}{\sqrt{16 - x^2}}$$

Answer

$$f(x) = \frac{1}{\sqrt{16 - x^2}}$$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $16 - x^2 \geq 0$

$\Rightarrow 16 \geq x^2$

$\Rightarrow x^2 \leq 16$

$\Rightarrow x^2 - 16 \leq 0$

$\Rightarrow x^2 - 4^2 \leq 0$

$\Rightarrow (x + 4)(x - 4) \leq 0$

$\Rightarrow x \geq -4$ and $x \leq 4$

$\therefore x \in [-4, 4]$

In addition, f(x) is also undefined when $16 - x^2 = 0$ because denominator will be zero and the result will be indeterminate.

$16 - x^2 = 0 \Rightarrow x = \pm 4$

Hence, $x \in [-4, 4] - \{-4, 4\}$

$\therefore x \in (-4, 4)$

Thus, domain of f = (–4, 4)

Let f(x) = y

$\Rightarrow \frac{1}{\sqrt{16 - x^2}} = y$

$\Rightarrow \left(\frac{1}{\sqrt{16 - x^2}}\right)^2 = y^2$

$\Rightarrow \frac{1}{16 - x^2} = y^2$

$\Rightarrow 1 = (16 - x^2)y^2$

$\Rightarrow 1 = 16y^2 - x^2y^2$

$\Rightarrow x^2y^2 + 1 - 16y^2 = 0$

$\Rightarrow (y^2)x^2 + (0)x + (1 - 16y^2) = 0$

As $x \in R$, the discriminant of this quadratic equation in x must be non-negative.

$\Rightarrow 0^2 - 4(y^2)(1 - 16y^2) \geq 0$

$\Rightarrow -4y^2(1 - 16y^2) \geq 0$

$\Rightarrow 4y^2(1 - 16y^2) \leq 0$

$\Rightarrow 1 - 16y^2 \leq 0$ [$\because y^2 \geq 0$]

$\Rightarrow 16y^2 - 1 \geq 0$

$\Rightarrow (4y)^2 - 1^2 \geq 0$

$\Rightarrow (4y + 1)(4y - 1) \geq 0$

$\Rightarrow 4y \leq -1$ and $4y \geq 1$

$\Rightarrow y \leq -\frac{1}{4}$ and $y \geq \frac{1}{4}$

$\Rightarrow y \in \left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{1}{4}, \infty\right)$

$\Rightarrow f(x) \in \left(-\infty, -\frac{1}{4}\right] \cup \left[\frac{1}{4}, \infty\right)$

However, y is always positive because it is the reciprocal of a non-zero square root.

$\therefore f(x) \in \left[\frac{1}{4}, \infty\right)$

Thus, range of f = $\left[\frac{1}{4}, \infty\right)$

3 J. Question

Find the domain and range of each of the following real valued functions:

$f(x) = \sqrt{x^2 - 16}$

Answer

$f(x) = \sqrt{x^2 - 16}$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $x^2 - 16 \geq 0$

$\Rightarrow x^2 - 4^2 \geq 0$

$\Rightarrow (x + 4)(x - 4) \geq 0$

$\Rightarrow x \leq -4$ or $x \geq 4$

$\therefore x \in (-\infty, -4] \cup [4, \infty)$

Thus, domain of f = $(-\infty, -4] \cup [4, \infty)$

When $x \in (-\infty, -4] \cup [4, \infty)$, we have $x^2 - 16 \geq 0$

Hence, $\sqrt{x^2 - 16} \geq 0 \Rightarrow f(x) \geq 0$

$\therefore f(x) \in [0, \infty)$

Thus, range of f = $[0, \infty)$

Exercise 3.4

1 A. Question

Find f + g, f – g, cf (c ∈ R, c ≠ 0), fg, 1/f and f/g in each of the following:

$f(x) = x^3 + 1$ and $g(x) = x + 1$

Answer

i. $f(x) = x^3 + 1$ and $g(x) = x + 1$

We have f(x) : R → R and g(x) : R → R

(a) f + g

We know (f + g)(x) = f(x) + g(x)

$\Rightarrow (f + g)(x) = x^3 + 1 + x + 1$

$\therefore (f + g)(x) = x^3 + x + 2$

Clearly, (f + g)(x) : R → R

Thus, f + g : R → R is given by $(f + g)(x) = x^3 + x + 2$

(b) f – g

We know (f – g)(x) = f(x) – g(x)

$\Rightarrow (f - g)(x) = x^3 + 1 - (x + 1)$

$\Rightarrow (f - g)(x) = x^3 + 1 - x - 1$

$\therefore (f - g)(x) = x^3 - x$

Clearly, (f – g)(x) : R → R

Thus, f – g : R → R is given by $(f - g)(x) = x^3 - x$

(c) cf (c ∈ R, c ≠ 0)

We know (cf)(x) = c × f(x)

⇒ (cf)(x) = $c(x^3 + 1)$

∴ (cf)(x) = $cx^3 + c$

Clearly, (cf)(x) : R → R

Thus, cf : R → R is given by (cf)(x) = $cx^3 + c$

(d) fg

We know (fg)(x) = f(x)g(x)

⇒ (fg)(x) = $(x^3 + 1)(x + 1)$

⇒ (fg)(x) = $(x + 1)(x^2 - x + 1)(x + 1)$

∴ (fg)(x) = $(x + 1)^2(x^2 - x + 1)$

Clearly, (fg)(x) : R → R

Thus, fg : R → R is given by (fg)(x) = $(x + 1)^2(x^2 - x + 1)$

(e) $\frac{1}{f}$

We know $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$

∴ $\left(\frac{1}{f}\right)(x) = \frac{1}{x^3 + 1}$

Observe that $\frac{1}{f(x)}$ is undefined when f(x) = 0 or when x = – 1.

Thus, $\frac{1}{f}$: R – {–1} → R is given by $\left(\frac{1}{f}\right)(x) = \frac{1}{x^3+1}$

(f) $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

⇒ $\left(\frac{f}{g}\right)(x) = \frac{x^3 + 1}{x + 1}$

Observe that $\frac{x^3+1}{x+1}$ is undefined when g(x) = 0 or when x = –1.

Using $x^3 + 1 = (x + 1)(x^2 - x + 1)$, we have

$$\left(\frac{f}{g}\right)(x) = \frac{(x+1)(x^2 - x + 1)}{x + 1}$$

∴ $\left(\frac{f}{g}\right)(x) = x^2 - x + 1$

Thus, $\frac{f}{g}$: R – {–1} → R is given by $\left(\frac{f}{g}\right)(x) = x^2 - x + 1$

1 B. Question

Find f + g, f – g, cf (c ∈ R, c ≠ 0), fg, 1/f and f/g in each of the following:

$f(x) = \sqrt{x - 1}$ and $g(x) = \sqrt{x + 1}$

Answer

$f(x) = \sqrt{x - 1}$ and $g(x) = \sqrt{x + 1}$

We have $f(x) : [1, \infty) \rightarrow R^+$ and $g(x) : [-1, \infty) \rightarrow R^+$ as real square root is defined only for non-negative numbers.

(a) f + g

We know (f + g)(x) = f(x) + g(x)

$\therefore (f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$

Domain of f + g = Domain of f ∩ Domain of g

⇒ Domain of f + g = [1, ∞) ∩ [–1, ∞)

∴ Domain of f + g = [1, ∞)

Thus, f + g : [1, ∞) → R is given by $(f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$

(b) f – g

We know (f – g)(x) = f(x) – g(x)

$\therefore (f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$

Domain of f – g = Domain of f ∩ Domain of g

⇒ Domain of f – g = [1, ∞) ∩ [–1, ∞)

∴ Domain of f – g = [1, ∞)

Thus, f – g : [1, ∞) → R is given by $(f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$

(c) cf (c ∈ R, c ≠ 0)

We know (cf)(x) = c × f(x)

$\therefore (cf)(x) = c\sqrt{x-1}$

Domain of cf = Domain of f

∴ Domain of cf = [1, ∞)

Thus, cf : [1, ∞) → R is given by $(cf)(x) = c\sqrt{x-1}$

(d) fg

We know (fg)(x) = f(x)g(x)

$\Rightarrow (fg)(x) = \sqrt{x-1}\sqrt{x+1}$

$\therefore (fg)(x) = \sqrt{x^2-1}$

Domain of fg = Domain of f ∩ Domain of g

⇒ Domain of fg = [1, ∞) ∩ [–1, ∞)

∴ Domain of fg = [1, ∞)

Thus, fg : [1, ∞) → R is given by $(fg)(x) = \sqrt{x^2-1}$

(e) $\frac{1}{f}$

We know $\left(\frac{1}{f}\right)(x) = \frac{1}{f(x)}$

$\therefore \left(\frac{1}{f}\right)(x) = \frac{1}{\sqrt{x-1}}$

Domain of $\frac{1}{f}$ = Domain of f

∴ Domain of $\frac{1}{f}$ = [1, ∞)

Observe that $\frac{1}{\sqrt{x-1}}$ is also undefined when x – 1 = 0 or x = 1.

Thus, $\frac{1}{f}$: (1, ∞) → R is given by $\left(\frac{1}{f}\right)(x) = \frac{1}{\sqrt{x-1}}$

(f) $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-1}}{\sqrt{x+1}}$$

$$\therefore \left(\frac{f}{g}\right)(x) = \sqrt{\frac{x-1}{x+1}}$$

Domain of $\frac{f}{g}$ = Domain of f ∩ Domain of g

⇒ Domain of $\frac{f}{g}$ = [1, ∞) ∩ [–1, ∞)

∴ Domain of $\frac{f}{g}$ = [1, ∞)

Thus, $\frac{f}{g}$: [1, ∞) → R is given by $\left(\frac{f}{g}\right)(x) = \sqrt{\frac{x-1}{x+1}}$

2. Question

Let f(x) = 2x + 5 and g(x) = x^2 + x. Describe

i. f + g

ii. f – g

iii. fg

iv. $\frac{f}{g}$

Find the domain in each case.

Answer

Given f(x) = 2x + 5 and g(x) = x^2 + x

Clearly, both f(x) and g(x) are defined for all x ∈ R.

Hence, domain of f = domain of g = R

i. f + g

We know (f + g)(x) = f(x) + g(x)

⇒ (f + g)(x) = 2x + 5 + x^2 + x

∴ (f + g)(x) = x^2 + 3x + 5

Clearly, (f + g)(x) is defined for all real numbers x.

∴ The domain of (f + g) is R

ii. f – g

We know (f – g)(x) = f(x) – g(x)

$\Rightarrow (f - g)(x) = 2x + 5 - (x^2 + x)$

$\Rightarrow (f - g)(x) = 2x + 5 - x^2 - x$

$\therefore (f - g)(x) = 5 + x - x^2$

Clearly, (f – g)(x) is defined for all real numbers x.

∴ The domain of (f – g) is R

iii. fg

We know (fg)(x) = f(x)g(x)

$\Rightarrow (fg)(x) = (2x + 5)(x^2 + x)$

$\Rightarrow (fg)(x) = 2x(x^2 + x) + 5(x^2 + x)$

$\Rightarrow (fg)(x) = 2x^3 + 2x^2 + 5x^2 + 5x$

$\therefore (fg)(x) = 2x^3 + 7x^2 + 5x$

Clearly, (fg)(x) is defined for all real numbers x.

∴ The domain of fg is R

iv. $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{2x + 5}{x^2 + x}$

Clearly, $\left(\frac{f}{g}\right)(x)$ is defined for all real values of x, except for the case when $x^2 + x = 0$.

$x^2 + x = 0$

$\Rightarrow x(x + 1) = 0$

$\Rightarrow x = 0$ or $x + 1 = 0$

$\Rightarrow x = 0$ or -1

When x = 0 or –1, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

Thus, domain of $\frac{f}{g}$ = R – {–1, 0}

3. Question

If f(x) be defined on [–2, 2] and is given by $f(x) = \begin{cases} -1, -2 \le x \le 0 \\ x - 1, 0 \le x \le 2 \end{cases}$ and g(x) = f(|x|) + |f(x)|. Find g(x).

Answer

Given $f(x) = \begin{cases} -1, -2 \le x \le 0 \\ x - 1, 0 \le x \le 2 \end{cases}$ and g(x) = f(|x|) + |f(x)|

Now, we have $f(|x|) = \begin{cases} -1, -2 \le |x| \le 0 \\ |x| - 1, 0 \le |x| \le 2 \end{cases}$

However, |x| ≥ 0 ⇒ f(|x|) = |x| – 1 when 0 ≤ |x| ≤ 2

We also have $|f(x)| = \begin{cases} |-1|, -2 \le x \le 0 \\ |x - 1|, 0 \le x \le 2 \end{cases}$

$$\Rightarrow |f(x)| = \begin{cases} 1, -2 \le x \le 0 \\ |x-1|, 0 \le x \le 2 \end{cases}$$

We know $|x-1| = \begin{cases} -(x-1), x-1 < 0 \\ x-1, x-1 \ge 0 \end{cases}$

$$\Rightarrow |x-1| = \begin{cases} -(x-1), x < 1 \\ x-1, x \ge 1 \end{cases}$$

Here, we are interested only in the range [0, 2].

$$\Rightarrow |x-1| = \begin{cases} -(x-1), 0 \le x < 1 \\ x-1, 1 \le x \le 2 \end{cases}$$

Substituting this value of |x – 1| in |f(x)|, we get

$$|f(x)| = \begin{cases} 1, -2 \le x \le 0 \\ -(x-1), 0 < x < 1 \\ x-1, 1 \le x \le 2 \end{cases}$$

$$\therefore |f(x)| = \begin{cases} 1, -2 \le x \le 0 \\ 1-x, 0 < x < 1 \\ x-1, 1 \le x \le 2 \end{cases}$$

We need to find g(x).

g(x) = f(|x|) + |f(x)|

$$\Rightarrow g(x) = \{|x| - 1, 0 \le |x| \le 2 + \begin{cases} 1, -2 \le x \le 0 \\ 1-x, 0 < x < 1 \\ x-1, 1 \le x \le 2 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} -x-1, -2 \le x \le 0 \\ x-1, 0 < x < 1 \\ x-1, 1 \le x \le 2 \end{cases} + \begin{cases} 1, -2 \le x \le 0 \\ 1-x, 0 < x < 1 \\ x-1, 1 \le x \le 2 \end{cases}$$

$$\Rightarrow g(x) = \begin{cases} -x-1+1, -2 \le x \le 0 \\ x-1+1-x, 0 < x < 1 \\ x-1+x-1\ 1 \le x \le 2 \end{cases}$$

$$\therefore g(x) = \begin{cases} -x, -2 \le x \le 0 \\ 0, 0 < x < 1 \\ 2(x-1), 1 \le x \le 2 \end{cases}$$

Thus, $g(x) = f(|x|) + |f(x)| = \begin{cases} -x, -2 \le x \le 0 \\ 0, 0 < x < 1 \\ 2(x-1), 1 \le x \le 2 \end{cases}$

4. Question

Let f, g be two real functions defined by $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9-x^2}$. Then, describe each of the following functions.

i. f + g

ii. g – f

iii. fg

iv. $\frac{f}{g}$

v. $\frac{g}{f}$

vi. $2f - \sqrt{5}g$

vii. $f^2 + 7f$

viii. $\frac{5}{g}$

Answer

Given $f(x) = \sqrt{x+1}$ and $g(x) = \sqrt{9 - x^2}$

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when $x + 1 \geq 0$

$\Rightarrow x \geq -1$

$\therefore x \in [-1, \infty)$

Thus, domain of f = $[-1, \infty)$

Similarly, g(x) takes real values only when $9 - x^2 \geq 0$

$\Rightarrow 9 \geq x^2$

$\Rightarrow x^2 \leq 9$

$\Rightarrow x^2 - 9 \leq 0$

$\Rightarrow x^2 - 3^2 \leq 0$

$\Rightarrow (x + 3)(x - 3) \leq 0$

$\Rightarrow x \geq -3$ and $x \leq 3$

$\therefore x \in [-3, 3]$

Thus, domain of g = $[-3, 3]$

i. f + g

We know $(f + g)(x) = f(x) + g(x)$

$\therefore (f+g)(x) = \sqrt{x+1} + \sqrt{9 - x^2}$

Domain of f + g = Domain of f ∩ Domain of g

$\Rightarrow$ Domain of f + g = $[-1, \infty) \cap [-3, 3]$

$\therefore$ Domain of f + g = $[-1, 3]$

Thus, f + g : $[-1, 3] \rightarrow R$ is given by $(f+g)(x) = \sqrt{x+1} + \sqrt{9 - x^2}$

ii. f – g

We know $(f - g)(x) = f(x) - g(x)$

$\therefore (f-g)(x) = \sqrt{x+1} - \sqrt{9 - x^2}$

Domain of f – g = Domain of f ∩ Domain of g

$\Rightarrow$ Domain of f – g = $[-1, \infty) \cap [-3, 3]$

$\therefore$ Domain of f – g = $[-1, 3]$

Thus, f – g : $[-1, 3] \rightarrow R$ is given by $(f-g)(x) = \sqrt{x+1} - \sqrt{9 - x^2}$

iii. fg

We know $(fg)(x) = f(x)g(x)$

$\Rightarrow (fg)(x) = \sqrt{x+1}\sqrt{9 - x^2}$

$\Rightarrow (fg)(x) = \sqrt{(x+1)(9-x^2)}$

$\Rightarrow (fg)(x) = \sqrt{x(9-x^2)+(9-x^2)}$

$\Rightarrow (fg)(x) = \sqrt{9x - x^3 + 9 - x^2}$

$\therefore (fg)(x) = \sqrt{9 + 9x - x^2 - x^3}$

As earlier, domain of fg = [–1, 3]

Thus, f – g : [–1, 3] → R is given by $(fg)(x) = \sqrt{9 + 9x - x^2 - x^3}$

iv. $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x+1}}{\sqrt{9-x^2}}$

$\therefore \left(\frac{f}{g}\right)(x) = \sqrt{\frac{x+1}{9-x^2}}$

As earlier, domain of $\frac{f}{g}$ = [–1, 3]

However, $\left(\frac{f}{g}\right)(x)$ is defined for all real values of x ∈ [–1, 3], except for the case when 9 – x^2 = 0 or x = ±3

When x = ±3, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

⇒ Domain of $\frac{f}{g}$ = [–1, 3] – {–3, 3}

∴ Domain of $\frac{f}{g}$ = [–1, 3)

Thus, $\frac{f}{g}$: [–1, 3) → R is given by $\left(\frac{f}{g}\right)(x) = \sqrt{\frac{x+1}{9-x^2}}$

v. $\frac{g}{f}$

We know $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$\Rightarrow \left(\frac{g}{f}\right)(x) = \frac{\sqrt{9-x^2}}{\sqrt{x+1}}$

$\therefore \left(\frac{g}{f}\right)(x) = \sqrt{\frac{9-x^2}{x+1}}$

As earlier, domain of $\frac{g}{f}$ = [–1, 3]

However, $\left(\frac{g}{f}\right)(x)$ is defined for all real values of x ∈ [–1, 3], except for the case when x + 1 = 0 or x = –1

When x = –1, $\left(\frac{g}{f}\right)(x)$ will be undefined as the division result will be indeterminate.

⇒ Domain of $\frac{g}{f}$ = [–1, 3] – {–1}

∴ Domain of $\frac{g}{f}$ = (–1, 3]

Thus, $\frac{g}{f}$: (–1, 3] → R is given by $\left(\frac{f}{g}\right)(x) = \sqrt{\frac{9-x^2}{x+1}}$

vi. $2f - \sqrt{5}g$

We know (f – g)(x) = f(x) – g(x) and (cf)(x) = cf(x)

⇒ $\left(2f - \sqrt{5}g\right)(x) = 2f(x) - \sqrt{5}g(x)$

∴ $\left(2f - \sqrt{5}g\right)(x) = 2\sqrt{x+1} - 5\sqrt{9-x^2}$

As earlier, Domain of $2f - \sqrt{5}g$ = [–1, 3]

Thus, $2f - \sqrt{5}g$: [–1, 3] → R is given by $\left(2f - \sqrt{5}g\right)(x) = 2\sqrt{x+1} - 5\sqrt{9-x^2}$

vii. $f^2 + 7f$

We know $(f^2 + 7f)(x) = f^2(x) + (7f)(x)$

⇒ $(f^2 + 7f)(x) = f(x)f(x) + 7f(x)$

⇒ $(f^2 + 7f)(x) = \sqrt{x+1}\sqrt{x+1} + 7\sqrt{x+1}$

∴ $(f^2 + 7f)(x) = x + 1 + 7\sqrt{x+1}$

Domain of $f^2 + 7f$ is same as domain of f.

∴ Domain of $f^2 + 7f$ = [–1, ∞)

Thus, $f^2 + 7f$: [–1, ∞) → R is given by $(f^2 + 7f)(x) = x + 1 + 7\sqrt{x+1}$

viii. $\frac{5}{g}$

We know $\left(\frac{1}{g}\right)(x) = \frac{1}{g(x)}$ and (cg)(x) = cg(x)

∴ $\left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$

Domain of $\frac{5}{g}$ = Domain of g = [–3, 3]

However, $\left(\frac{5}{g}\right)(x)$ is defined for all real values of x ∈ [–3, 3], except for the case when $9 - x^2 = 0$ or x = ±3

When x = ±3, $\left(\frac{5}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

⇒ Domain of $\frac{5}{g}$ = [–3, 3] – {–3, 3}

∴ Domain of $\frac{5}{g}$ = (–3, 3)

Thus, $\frac{5}{g}$: (–3, 3) → R is given by $\left(\frac{5}{g}\right)(x) = \frac{5}{\sqrt{9-x^2}}$

5. Question

If $f(x) = \log_e(1 - x)$ and g(x) = [x], then determine each of the following functions:

i. f + g

ii. fg

iii. $\frac{f}{g}$

iv. $\frac{g}{f}$

Also, find (f + g)(–1), (fg)(0), $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$ and $\left(\frac{g}{f}\right)\left(\frac{1}{2}\right)$.

Answer

Given $f(x) = \log_e(1 - x)$ and $g(x) = [x]$

Clearly, f(x) takes real values only when $1 - x > 0$

$\Rightarrow 1 > x$

$\Rightarrow x < 1$

$\therefore x \in (-\infty, 1)$

Thus, domain of f = $(-\infty, 1)$

g(x) is defined for all real numbers x.

Thus, domain of g = R

i. f + g

We know (f + g)(x) = f(x) + g(x)

$\therefore (f + g)(x) = \log_e(1 - x) + [x]$

Domain of f + g = Domain of f ∩ Domain of g

⇒ Domain of f + g = $(-\infty, 1) \cap R$

∴ Domain of f + g = $(-\infty, 1)$

Thus, f + g : $(-\infty, 1) \to R$ is given by $(f + g)(x) = \log_e(1 - x) + [x]$

ii. fg

We know (fg)(x) = f(x)g(x)

$\Rightarrow (fg)(x) = \log_e(1 - x) \times [x]$

$\therefore (fg)(x) = [x]\log_e(1 - x)$

Domain of fg = Domain of f ∩ Domain of g

⇒ Domain of fg = $(-\infty, 1) \cap R$

∴ Domain of fg = $(-\infty, 1)$

Thus, f – g : $(-\infty, 1) \to R$ is given by $(fg)(x) = [x]\log_e(1 - x)$

iii. $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{\log_e(1 - x)}{[x]}$

As earlier, domain of $\frac{f}{g} = (-\infty, 1)$

However, $\left(\frac{f}{g}\right)(x)$ is defined for all real values of $x \in (-\infty, 1)$, except for the case when $[x] = 0$.

We have $[x] = 0$ when $0 \le x < 1$ or $x \in [0, 1)$

When $0 \le x < 1$, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

⇒ Domain of $\frac{f}{g}$ = (–∞, 1) – [0, 1)

∴ Domain of $\frac{f}{g}$ = (–∞, 0)

Thus, $\frac{f}{g}$: (–∞, 0) → R is given by $\left(\frac{f}{g}\right)(x) = \frac{\log_e(1-x)}{[x]}$

iv. $\frac{g}{f}$

We know $\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)}$

$$\therefore \left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$$

As earlier, domain of $\frac{g}{f}$ = (–∞, 1)

However, $\left(\frac{g}{f}\right)(x)$ is defined for all real values of x ∈ (–∞, 1), except for the case when $\log_e(1 – x) = 0$.

$\log_e(1 – x) = 0$ ⇒ 1 – x = 1 or x = 0

When x = 0, $\left(\frac{g}{f}\right)(x)$ will be undefined as the division result will be indeterminate.

⇒ Domain of $\frac{g}{f}$ = (–∞, 1) – {0}

∴ Domain of $\frac{g}{f}$ = (–∞, 0) ∪ (0, ∞)

Thus, $\frac{g}{f}$: (–∞, 0) ∪ (0, ∞) → R is given by $\left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$

We have (f + g)(x) = $\log_e(1 – x)$ + [x], x ∈ (–∞, 1)

We need to find (f + g)(–1).

Substituting x = –1 in the above equation, we get

(f + g)(–1) = $\log_e(1 – (–1))$ + [–1]

⇒ (f + g)(–1) = $\log_e(1 + 1)$ + (–1)

∴ (f + g)(–1) = $\log_e 2$ – 1

Thus, (f + g)(–1) = $\log_e 2$ – 1

We have (fg)(x) = [x]$\log_e(1 – x)$, x ∈ (–∞, 1)

We need to find (fg)(0).

Substituting x = 0 in the above equation, we get

(fg)(0) = [0]$\log_e(1 – 0)$

⇒ (fg)(0) = 0 × $\log_e 1$

∴ (fg)(0) = 0

Thus, (fg)(0) = 0

We have $\left(\frac{f}{g}\right)(x) = \frac{\log_e(1-x)}{[x]}$, x ∈ (–∞, 0)

We need to find $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$

However, $\frac{1}{2}$ is not in the domain of $\frac{f}{g}$.

Thus, $\left(\frac{f}{g}\right)\left(\frac{1}{2}\right)$ does not exist.

We have $\left(\frac{g}{f}\right)(x) = \frac{[x]}{\log_e(1-x)}$, x ∈ (–∞, 0) ∪ (0, ∞)

We need to find $\left(\frac{g}{f}\right)\left(\frac{1}{2}\right)$

Substituting $x = \frac{1}{2}$ in the above equation, we get

$$\left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{\left[\frac{1}{2}\right]}{\log_e\left(1-\frac{1}{2}\right)}$$

$$\Rightarrow \left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{[0.5]}{\log_e\left(\frac{1}{2}\right)}$$

$$\Rightarrow \left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = \frac{0}{\log_e\left(\frac{1}{2}\right)}$$

$$\therefore \left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = 0$$

Thus, $\left(\frac{g}{f}\right)\left(\frac{1}{2}\right) = 0$

6. Question

If f, g, h are real functions defined by $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x}$ and h(x) = $2x^2$ – 3, then find the values of (2f + g – h)(1) and (2f + g – h)(0).

Answer

Given $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x}$ and h(x) = $2x^3$ – 3

We know the square of a real number is never negative.

Clearly, f(x) takes real values only when x + 1 ≥ 0

⇒ x ≥ –1

∴ x ∈ [–1, ∞)

Thus, domain of f = [–1, ∞)

g(x) is defined for all real values of x, except for 0.

Thus, domain of g = R – {0}

h(x) is defined for all real values of x.

Thus, domain of h = R

We know (2f + g – h)(x) = (2f)(x) + g(x) – h(x)

⇒ (2f + g – h)(x) = 2f(x) + g(x) – h(x)

$$\Rightarrow (2f+g-h)(x) = 2\sqrt{x+1} + \frac{1}{x} - (2x^2 - 3)$$

$$\therefore (2f+g-h)(x) = 2\sqrt{x+1} + \frac{1}{x} - 2x^2 + 3$$

Domain of 2f + g – h = Domain of f ∩ Domain of g ∩ Domain of h

⇒ Domain of 2f + g – h = [–1, ∞) ∩ R – {0} ∩ R

∴ Domain of 2f + g – h = [–1, ∞) – {0}

i. (2f + g – h)(1)

We have $(2f+g-h)(x) = 2\sqrt{x+1} + \frac{1}{x} - 2x^2 + 3$

$\Rightarrow (2f+g-h)(1) = 2\sqrt{1+1} + \frac{1}{1} - 2(1)^2 + 3$

$\Rightarrow (2f+g-h)(1) = 2\sqrt{2} + 1 - 2 + 3$

$\therefore (2f+g-h)(1) = 2\sqrt{2} + 2$

ii. (2f + g – h)(0)

0 is not in the domain of (2f + g – h)(x).

Hence, (2f + g – h)(0) does not exist.

Thus, $(2f+g-h)(1) = 2\sqrt{2} + 2$ and (2f + g – h)(0) does not exist as 0 is not in the domain of (2f + g – h)(x).

7. Question

The function f is defined by $f(x) = \begin{cases} 1-x, x<0 \\ 1, x=0 \\ x+1, x>0 \end{cases}$. Draw the graph of f(x).

Answer

Given $f(x) = \begin{cases} 1-x, x<0 \\ 1, x=0 \\ x+1, x>0 \end{cases}$

When x < 0, we have f(x) = 1 – x

f(–4) = 1 – (–4) = 1 + 4 = 5

f(–3) = 1 – (–3) = 1 + 3 = 4

f(–2) = 1 – (–2) = 1 + 2 = 3

f(–1) = 1 – (–1) = 1 + 1 = 2

When x = 0, we have f(x) = f(0) = 1

When x > 0, we have f(x) = 1 + x

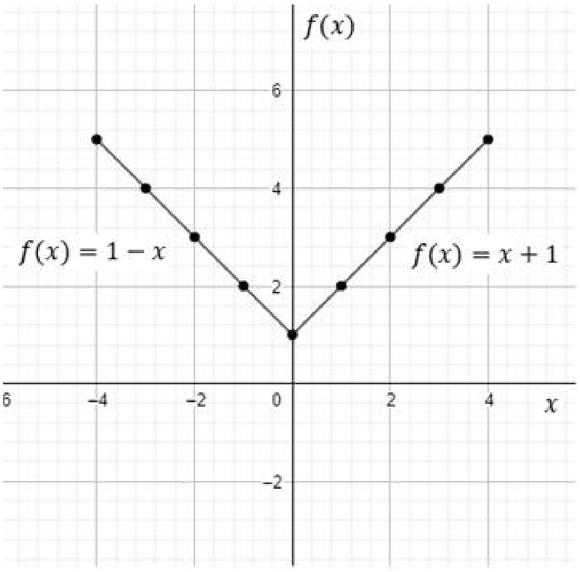
f(1) = 1 + 1 = 2

f(2) = 1 + 2 = 3

f(3) = 1 + 3 = 4

f(4) = 1 + 4 = 5

Plotting these points on a graph sheet, we get



8. Question

Let f, g : R → R be defined, respectively by f(x) = x + 1 and g(x) = 2x – 3. Find f + g, f – g and $\frac{f}{g}$.

Find the domain in each case.

Answer

Given f(x) = x + 1 and g(x) = 2x – 3

Clearly, both f(x) and g(x) exist for all real values of x.

Hence, Domain of f = Domain of g = R

Range of f = Range of g = R

i. f + g

We know (f + g)(x) = f(x) + g(x)

⇒ (f + g)(x) = x + 1 + 2x – 3

∴ (f + g)(x) = 3x – 2

Domain of f + g = Domain of f ∩ Domain of g

⇒ Domain of f + g = R ∩ R

∴ Domain of f + g = R

Thus, f + g : R → R is given by (f + g)(x) = 3x – 2

ii. f – g

We know (f – g)(x) = f(x) – g(x)

⇒ (f – g)(x) = x + 1 – (2x – 3)

⇒ (f – g)(x) = x + 1 – 2x + 3

∴ (f – g)(x) = –x + 4

Domain of f – g = Domain of f ∩ Domain of g

⇒ Domain of f – g = R ∩ R

∴ Domain of f – g = R

Thus, f – g : R → R is given by (f – g)(x) = –x + 4

iii. $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}$

Clearly, $\left(\frac{f}{g}\right)(x)$ is defined for all real values of x, except for the case when 2x – 3 = 0 or $x = \frac{3}{2}$.

When $x = \frac{3}{2}$, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

Thus, domain of $\frac{f}{g}$ = R – $\left\{\frac{3}{2}\right\}$

Thus, $\frac{f}{g}$: R – $\left\{\frac{3}{2}\right\}$ → R is given by $\left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}$

9. Question

Let f : [0, ∞) → R and g : R → R be defined by $f(x) = \sqrt{x}$ and g(x) = x. Find f + g, f – g, fg and $\frac{f}{g}$

Answer

Given $f(x) = \sqrt{x}$ and g(x) = x

Domain of f = [0, ∞)

Domain of g = R

i. f + g

We know (f + g)(x) = f(x) + g(x)

$\therefore (f+g)(x) = \sqrt{x} + x$

Domain of f + g = Domain of f ∩ Domain of g

⇒ Domain of f + g = [0, ∞) ∩ R

∴ Domain of f + g = [0, ∞)

Thus, f + g : [0, ∞) → R is given by $(f+g)(x) = \sqrt{x} + x$

ii. f – g

We know (f – g)(x) = f(x) – g(x)

$\therefore (f-g)(x) = \sqrt{x} - x$

Domain of f – g = Domain of f ∩ Domain of g

⇒ Domain of f – g = [0, ∞) ∩ R

∴ Domain of f – g = [0, ∞)

Thus, f – g : [0, ∞) → R is given by $(f-g)(x) = \sqrt{x} - x$

iii. fg

We know (fg)(x) = f(x)g(x)

$\Rightarrow (fg)(x) = \sqrt{x} \times x$

$\Rightarrow (fg)(x) = x^{\frac{1}{2}} \times x$

$\therefore (fg)(x) = x^{\frac{3}{2}}$

Clearly, (fg)(x) is also defined only for non-negative real numbers x as square of a real number is never negative.

Thus, fg : [0, ∞) → R is given by $(fg)(x) = x^{\frac{3}{2}}$

iv. $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x}$

$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{(\sqrt{x})^2}$

$\therefore \left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}}$

Clearly, $\left(\frac{f}{g}\right)(x)$ is defined for all positive real values of x, except for the case when x = 0.

When x = 0, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

⇒ Domain of $\frac{f}{g}$ = [0, ∞) – {0}

∴ Domain of $\frac{f}{g}$ = (0, ∞)

Thus, $\frac{f}{g}$: (0, ∞) → R is given by $\left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}}$

10. Question

Let f(x) = x^2 and g(x) = 2x + 1 be two real functions. Find (f + g)(x), (f – g)(x), (fg)(x) and $\left(\frac{f}{g}\right)(x)$.

Answer

Given f(x) = x^2 and g(x) = 2x + 1

Both f(x) and g(x) are defined for all x ∈ R.

Hence, domain of f = domain of g = R

i. f + g

We know (f + g)(x) = f(x) + g(x)

⇒ (f + g)(x) = x^2 + 2x + 1

∴ (f + g)(x) = $(x + 1)^2$

Clearly, (f + g)(x) is defined for all real numbers x.

∴ Domain of (f + g) is R

Thus, f + g : R → R is given by (f + g)(x) = $(x + 1)^2$

ii. f – g

We know (f – g)(x) = f(x) – g(x)

⇒ (f – g)(x) = x^2 – (2x + 1)

∴ (f – g)(x) = x^2 – 2x – 1

Clearly, (f – g)(x) is defined for all real numbers x.

∴ Domain of (f – g) is R

Thus, f – g : R → R is given by $(f - g)(x) = x^2 - 2x - 1$

iii. fg

We know (fg)(x) = f(x)g(x)

⇒ $(fg)(x) = x^2(2x + 1)$

∴ $(fg)(x) = 2x^3 + x^2$

Clearly, (fg)(x) is defined for all real numbers x.

∴ Domain of fg is R

Thus, fg : R → R is given by $(fg)(x) = 2x^3 + x^2$

iv. $\frac{f}{g}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

∴ $\left(\frac{f}{g}\right)(x) = \frac{x^2}{2x + 1}$

Clearly, $\left(\frac{f}{g}\right)(x)$ is defined for all real values of x, except for the case when 2x + 1 = 0.

2x + 1 = 0

⇒ 2x = –1

⇒ $x = -\frac{1}{2}$

When $x = -\frac{1}{2}$, $\left(\frac{f}{g}\right)(x)$ will be undefined as the division result will be indeterminate.

Thus, the domain of $\frac{f}{g}$ = R – $\left\{-\frac{1}{2}\right\}$

Very Short Answer

1. Question

Write the range of the real function f(x) = |x|.

Answer

f(x) = |x|

f(-x) = |-x|

therefore, f(x) will always be 0 or positive.

Thus, range of f(x) ϵ [0,∞).

2. Question

If f is a real function satisfying $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ for all x ∈ R – {0}, then write the expression for f (x).

Answer

$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$

$= x^2 + \frac{1}{x^2} + 2.x.\frac{1}{x} - 2$

{since, $(a + b)^2 = a^2 + b^2 + 2ab$}

$= \left(x + \frac{1}{x}\right)^2 - 2$

Let $x + \frac{1}{x} = y$

$f(y) = y^2 - 2$

$x + \frac{1}{x} = y$

$x + \frac{1}{x} = y$

x + 1 = xy

$x^2 - yx + 1 = 0$

$x = \frac{y \pm \sqrt{y^2 - 4.1.1}}{2.1}$

for x to be real

$y^2 - 4 \geq 0$

$y \in (-\infty, 2] \cup [2, \infty)$

$|y| > 2$ Ans.

3. Question

Write the range of the function f (x) = sin [x] where $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$.

Answer

$F(x) = \sin[x]$

$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$

$\sin\left[-\frac{\pi}{4}\right] = \sin(-1)$

=-sin 1

$\sin 0 = 0$

$\sin\frac{\pi}{4} = \sin 0$

= 0

Using properties of greatest integer function:

[1] = 1; [0.5] = 0; [-0.5] = -1

Therefore, R(f) = {$-\sin 1, 0$}

4. Question

If f(x) = cos2[π^2]x + cos [-π^2] x, where [x] denotes the greatest integer less than or equal to x, then write the value of f(π).

Answer

$f(x) = \cos 2[\pi^2]x + \cos[-\pi^2]x$

$\pi^2 \approx 9.8596$

So, we have $[\pi^2]=9$ and $[-\pi^2]=-10$

f(x)=cos 18x+ cos (-10)x

=cos 18x + cos 10x

$$= 2\cos\left(\frac{18x + 10x}{2}\right)\cos\left(\frac{18x - 10x}{2}\right)$$

=2 cos 14x cos4x

f(π)=2 cos 14π cos 4π

=2×1×1

Therefore, f(π)= 2

5. Question

Write the range of the function f (x) = cos [x], where $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Answer

for $-\frac{\pi}{2} < x < -1$

[x]= -2

f(x)= cos [x]= cos (-2)

= cos 2

because cos(-x) = cos(x)

for-1 ≤x<0

[x]=-1

f(x)= cos[x]=cos (-1)

= cos 1

for 0 ≤x< 1

[x]=0

f(x)=cos 0 =1

for 1 ≤x<π/2

[x]=1

f(x)=cos1

Therefore, R(f) = {1, cos 1, cos 2}

6. Question

Write the range of the function f(x) = $e^{x - [x]}$, X ∈R.

Answer

$f(x) = e^{x-[x]}$

0 ≤x-[x]<1

$e^0 \leq e^{x-[x]} < e^1$

$1 \le e^{x-[x]} < e$

Therefore, R(f) = [1, e)

7. Question

Let $f(x)=\frac{\alpha x}{x+1}, x \neq -1$. Then write the value of α satisfying f(x)) = x for allx ≠ - 1.

Answer

$f(x) = \frac{ax}{x+1}, x \neq -1$

If $f(f(x)) = x$

$a\frac{\frac{ax}{x+1}}{\frac{ax}{x+1}+1} = x$

$\frac{\frac{a^2x}{x+1}}{\frac{ax+x+1}{x+1}} = x$

$\frac{a^2x}{ax+x+1} = x$

$a^2x=ax^2+x^2+x$

$x^2(a+1)+x(1-a^2)=0$

$x^2(a+1)+x(1-a)(1+a)=0$

$(a+1)(x^2+x(1-a))=0$

a+1=0

Therefore, a=-1

8. Question

If $f(x)=1-\frac{1}{x}$, then write the value of $f\left(f\left(\frac{1}{x}\right)\right)$.

Answer

$f(x) = 1-\frac{1}{x}$

$\text{replace} x \text{by} \frac{1}{x}$

$f\left(\frac{1}{x}\right) = 1-\frac{1}{\frac{1}{x}} = 1-x$

$\text{now}, f\left(f\left(\frac{1}{x}\right)\right) = 1-\frac{1}{f\left(\frac{1}{x}\right)}$

$= 1-\frac{1}{1-x} = \frac{1-x-1}{1-x}$

$f\left(f\left(\frac{1}{x}\right)\right) = \frac{-x}{1-x} = \frac{x}{x-1}$

9. Question

Write the domain and range of the function $f(x)=\frac{x-2}{2-x}$.

Answer

For function to be defined, $2-x\neq 0$

$x\neq 2$

Therefore, D(f)= R-{2}.

Let $y=\frac{x-2}{2-x}$

y=-1

Therefore, R(f) = {-1}.

10. Question

If f(x) = 4x – x^2, x ∈ R, then write the value of f(a + 1) – f(a – 1).

Answer

$f(x)=4x-x^2$

f(a+1)-f(a-1) =[4(a+1) -$(a+1)^2$]-[4(a-1)-$(a-1)^2$]

=4[(a+1) -(a-1)]- [$(a+1)^2$ -$(a+1)^2$]

=4(2)-[(a+1+a-1) (a+1-a+1)]

Using: a^2 -b^2= (a + b)(a-b)

f(a+1)-f(a-1)=4(2)-2a(2)

=4(2-a)

11. Question

If f, g, h are real functions given by f(x) = x^2, g(x) = tan x and h(x) = $\log_e$ x, then write the value of (hogof) $\left(\sqrt{\frac{\pi}{4}}\right)$.

Answer

$f(x)=x^2\ ;\ g(x)=\tan x\ ;\ h(x)=\log_e x$

$f\left(\sqrt{\frac{\pi}{4}}\right)=\left(\sqrt{\frac{\pi}{4}}\right)^2=\frac{\pi}{4}$

$g\left(f\left(\sqrt{\frac{\pi}{4}}\right)\right)=g\left(\frac{\pi}{4}\right)=\tan\frac{\pi}{4}=1$

$(hogof)\left(\sqrt{\frac{\pi}{4}}\right)=h(1)=\log_e 1=0$

Therefore, answer = 0.

12. Question

Write the domain and range of function f(x) given by $f(x)=\frac{1}{\sqrt{x-|x|}}$.

Answer

For f(x) to be defined,

x-|x|>0

But x-|x|≤0

So, f(x) does not exist..

Therefore, $D(f) = R(f) = \phi$

13. Question

Write the domain and range of $f(x) = \sqrt{x - [x]}$

Answer

For f(x) to be defined,

x-[x]≥0

We know that, $\{x\} + [x] = x$ where {x} is fractional part function and [x] is greatest integer function.

{x}≥0

Also, $0 \leq \{x\} < 1$

Therefore, D(f) = R and range = [0, 1).

14. Question

Write the domain and range of function f(x) given by $f(x) = \sqrt{[x] - x}$.

Answer

For function to be defined,

[x]-x≥0

-{x}≥0

Therefore, domain of f(x) is integers.

D(f)∈I

Range = {0}.

15. Question

Let A and B be two sets such that n(A) = p and n(B) = q, write the number of functions from A to B.

Answer

For each value of set A, we can have q functions as each value of A pair up with all the values of B.

So, total number of functions from A to B =q× q× q.....{p times}

$=q^p$

16. Question

Let f and g be two functions given by

f = {(2, 4), (5, 6), (8, -1), (10, -3)} and g = {(2, 5), (7, 1), (8, 4), (10, 13), (11, -5)}.

Find the domain of f + g.

Answer

D(f) = {2, 5, 8, 10}

D(g) = {2, 7, 8, 10, 11}

Therefore, D(f+g) = {2, 8, 10}

17. Question

Find the set of values of x for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal.

Answer

$f(x)=3x^2-1; g(x)=3+x$

For $f(x) = g(x)$

$3x^2-1=3+x$

$3x^2-x-4=0$

$(3x-4)(x+1)=0$

$3x-4=0$ or $x+1=0$

$$x = \frac{4}{3}, -1$$

18. Question

Let f and g be two real functions given by

f = {(0, 1), (2, 0), (3, -4), (4, 2), (5, 1)} and g = {(1, 0), (2, 2), (3, -1), (4, 4), (5, 3)}.

Find the domain of fg.

Answer

D(f) = {0, 2, 3, 4, 5}

D(g) = {1, 2, 3, 4, 5}

So, D(fg) = {2, 3, 4, 5}

MCQ

1. Question

Mark the correct alternative in the following:

Let A = {1, 2, 3}, B = {2, 3, 4}, then which of the following is a function from A to B?

A.{(1, 2), (1, 3), (2, 3), (3, 3)}

B. {(1, 3), (2, 4)}

C. {(1, 3), (2, 2), (3, 3)}

D. {(1, 2), (2, 3), (3, 2), (3, 4)

Answer

A function is said to be defined from A to B if each element in set A has an unique image in set B. Not all the elements in set B are the images of any element of set A.

Therefore, option C is correct.

2. Question

Mark the correct alternative in the following:

If f : Q → Q is defined as $f(x) = x^2$, then $f^{-1}(9)$s is equal to

A. 3

B. -3

C. {-3, 3}

D. ϕ

Answer

$f(x) = x^2$

Replace f(x) by y,

$y = x^2$

$x = \sqrt{y}$

Replace x by $f^{-1}x$ and y by x.

$f^{-1}x = \sqrt{x}$

So, $f^{-1}(9) = \sqrt{9}$

$= \pm 3$

Option C is correct.

3. Question

Mark the correct alternative in the following:

Which one of the following is not a function?

A. $\{(x, y) : x, y \in R, x^2 = y\}$

B. $\{(x, y) : x, y \in R, y^2 = x\}$

C. $\{(x, y) : x, y \in R, x = y^3\}$

D. $\{(x, y) : x, y \in R, y = x^3\}$

Answer

A function is said to exist when we get a unique value for any value of x..

Therefore, option B is correct..$y^2 = x$ is not a function as for each value of x, we will get 2 values of y..which is not as per the definition of a function..

4. Question

Mark the correct alternative in the following:

If f(x) = cos (log x), then $f\left(x^2\right)f\left(y^2\right)-\frac{1}{2}\left\{f\left(\frac{x^2}{y^2}\right)+f\left(x^2\ y^2\right)\right\}$ has the value

A. -2

B. -1

C. 1/2

D. None of these

Answer

$f(x) = \cos(\log x)$

Now, $f(x^2)f(y^2) - \frac{1}{2}\left\{f\left(\frac{x^2}{y^2}\right) + f(x^2y^2)\right\}$

$= \cos(\log x^2)\cos(\log y^2) - \frac{1}{2}\{\cos\left(\log\left(\frac{x^2}{y^2}\right)\right) + \cos(\log x^2y^2)\}$

$$= \cos(2\log x)\cos(2\log y) - \frac{1}{2}\{\cos(\log x^2 - \log y^2) + \cos(\log x^2 + \log y^2)\}$$

$$= \cos(2\log x)\cos(2\log y) - \frac{1}{2}\{\cos(2\log x - 2\log y) + \cos(2\log x + 2\log y)\}$$

Using: $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

=cos(2 log x)□ cos(2 log y)-cos(2 log x) cos(2 log y)

=0

5. Question

Mark the correct alternative in the following:

If f(x) = cos (log x), then $f(x)f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\}$ has the value

A. -1

B. 1/2

C. -2

D. None of these

Answer

$f(x) = \cos(\log x)$

Now, $f(x)f(y) - \frac{1}{2}\left\{f\left(\frac{x}{y}\right) + f(xy)\right\}$

$$= \cos(\log x)\cos(\log y) - \frac{1}{2}\left\{\cos\left(\log\left(\frac{x}{y}\right)\right) + \cos(\log xy)\right\}$$

$$= \cos(\log x)\cos(\log y) - \frac{1}{2}\{\cos(\log x - \log y) + \cos(\log x + \log y)\}$$

Using: $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

=cos(log x) cos(log y)-cos(log x) cos(log y)

=0

6. Question

Mark the correct alternative in the following:

Let f(x) = |x – 1|. Then,

A. $f(x^2) = [f(x)]^2$

B. f(x + y) = f(x) f(y)

C. f(|x|) = |f(x)|

D. None of these

Answer

f(x)=|x-1|

$f(x^2) = |x^2 - 1|$

$[f(x)^2 = (x-1)^2$

$= x^2 + 1 - 2x$

So, $f(x^2) \neq [f(x)]^2$

$f(x + y) = |x+y-1|$

$f(x)f(y) = (x-1)(y-1)$

So, $f(x + y) \neq f(x)f(y)$

$f(|x|) = ||x|-1|$

Therefore, option D is correct.

7. Question

Mark the correct alternative in the following:

The range of f(x) = cos [x], for -π/2< x <π/2 is

A. {-1, 1, 0}

B. {cos 1, cos 2, 1}

C. {cos 1, -cos 1, 1}

D. [-1, 1]

Answer

for $-\frac{\pi}{2} < x < -1$

[x]= -2

f(x)= cos[x]= cos(-2)

= cos2

because cos(-x)= cos(x)

for-1 ≤x<0

[x]=-1

f(x)= cos[x]

=cos(-1)

= cos1

for 0 ≤x< 1

[x]=0

f(x) = cos 0

=1

for $1 \leq x < \frac{\pi}{2}$

[x]=1

f(x)=cos 1

Therefore, R(f) = {1, cos 1,cos 2}

Option B is correct.

8. Question

Mark the correct alternative in the following:

Which of the following are functions?

A. {(x, y) : y2 = x, x, y ∈ R}

B. {(x, y) : y = |x|, x, y, ∈ R}

C. {(x, y) : $X^2 + y^2 = 1$, x, y ∈ R}

D. {(x, y) : $x^2 – y^2 = 1$, x, y ∈ R}

Answer

A function is said to exist when we get a unique value of y for any value of x..If we get 2 values of y for any value of x, then it is not a function..

Therefore, option B is correct .

NOTE: To check if a given curve is a function or not, draw the curve and then draw a line parallel to y-axis..If it intersects the curve at only one point, then it is a function, else not..

9. Question

Mark the correct alternative in the following:

If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then f(g(x) is equal to

A. f (3x)

B. $\{f(x)\}^3$

C. 3f (x)

D. -f (x)

Answer

$$f(g(x)) = \log\left(\frac{1+g(x)}{1-g(x)}\right)$$

$$= \log\left(\frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}\right)$$

$$= \log\left(\frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}\right)$$

Using: $(1+x)^3=1+3x+3x^2+x^3$

And $(1-x)^3=1-3x+3x^2-x^3$

$$= \log\left(\frac{1+x}{1-x}\right)^3 = 3\log\left(\frac{1+x}{1-x}\right)$$

f(g(x))=3f(x)

Option C is correct.

10. Question

Mark the correct alternative in the following:

If A = {1, 2, 3}, B = {x, y}, then the number of functions that can be defined from A into B is

A. 12

B. 8

C. 6

D. 3

Answer

Since A has 3 elements and B has 2..then number of functions from A to B $= 2 * 2 * 2 = 2^3 = 8$

Option B is correct.

11. Question

Mark the correct alternative in the following:

If $f(x) = \log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

A. $\{f(x)\}^2$

B. $\{f(x)\}^3$

C. 2f (x)

D. 3f (x)

Answer

$f\left(\frac{2x}{1+x^2}\right) = \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$

$= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$

$= \log\left(\frac{1+x}{1-x}\right)^2$

$f\left(\frac{2x}{1+x^2}\right) = 2\log\left(\frac{1+x}{1-x}\right)$

=2f(x)

Option C is correct..

12. Question

Mark the correct alternative in the following:

If f(x) = cos (log x), then value of $f(x)f(4) - \frac{1}{2}\left\{f\left(\frac{x}{4}\right) + f(4x)\right\}$ is

A. 1

B. -1

C. 0

D. ± 1

Answer

$f(x) = \cos(\log x)$

Now, $f(x)f(4) - \frac{1}{2}\left\{f\left(\frac{x}{4}\right) + f(4x)\right\}$

$= \cos(\log x)\cos(\log 4) - \frac{1}{2}\{\cos\left(\log\left(\frac{x}{4}\right)\right) + \cos(\log 4x)\}$

$= \cos(\log x)\cos(\log 4) - \frac{1}{2}\{\cos(\log x - \log 4) + \cos(\log x + \log 4)\}$

Using: $\cos x \cos y = \frac{1}{2}[\cos(x+y) + \cos(x-y)]$

=cos(log x) cos(log 4)-cos(log x)cos(4)

=0

Option C is correct..

13. Question

Mark the correct alternative in the following:

If $f(x)=\frac{2^x+2^{-x}}{2}$, then f(x + y) f(x – y) is equals to

A. $\frac{1}{2}\{f(2x)+f(2y)\}$

B. $\frac{1}{2}\{f(2x)-f(2y)\}$

C. $\frac{1}{4}\{f(2x)+f(2y)\}$

D. $\frac{1}{4}\{f(2x)-f(2y)\}$

Answer

$$f(x+y)f(x-y)=\left(\frac{2^{x+y}+2^{-(x+y)}}{2}\right)\left(\frac{2^{x-y}+2^{-(x-y)}}{2}\right)$$

$$=\left(\frac{2^{x+y}+\frac{1}{2^{x+y}}}{2}\right)\left(\frac{2^{x-y}+\frac{1}{2^{x-y}}}{2}\right)$$

$$=\left(\frac{2^{2(x+y)}+1}{2.2^{(x+y)}}\right)\left(\frac{2^{2(x-y)}+1}{2.2^{(x-y)}}\right)$$

$$=\left(\frac{2^{2(x+y)}2^{2(x-y)}+2^{2(x+y)}+2^{2(x-y)}+1}{4.2^{(x+y)}2^{(x-y)}}\right)$$

$$=\left(\frac{2^{4x}+2^{2(x+y)}+2^{2(x-y)}+1}{4.2^{2x}}\right)$$

$$=\left(\frac{2^{2x}+2^{2y}+2^{-2y}+2^{-2x}}{4}\right)$$

$$=\frac{1}{2}\left(\frac{2^{2x}+2^{-2x}}{2}+\frac{2^{2y}+2^{-2y}}{2}\right)$$

$$=\frac{1}{2}\{f(2x)+f(2y)\}$$

Option A is correct.

14. Question

Mark the correct alternative in the following:

If $2f(x)-3f\left(\frac{1}{x}\right)=x^2\,(x\neq 0)$, then f(2) is equal to

A. $-\frac{7}{4}$

B. $\frac{5}{2}$

C. -1

D. None of these

Answer

$2f(x) - 3f\left(\frac{1}{x}\right) = x^2$ eqn.1

Replace x by 1/x in eqn.1;

$2f\left(\frac{1}{x}\right) - 3f(x) = \frac{1}{x^2}$ eqn.2

Multiply eqn.1 by 2 and eqn.2 by 3 and add them..

On adding, we get

$$-5f(x) = 2x^2 + \frac{3}{x^2}$$

$$f(x) = \frac{-1}{5}(2x^2 + \frac{3}{x^2})$$

$$f(2) = \frac{-1}{5}\left(2 \times 2^2 + \frac{3}{2^2}\right) = \frac{-1}{5}\left(8 + \frac{3}{4}\right)$$

$$= \frac{-1}{5}\left(\frac{35}{4}\right) = \frac{-7}{4}$$

Option A is correct.

15. Question

Mark the correct alternative in the following:

Let f : R → r be defined by f(x) = 2x + |x|. Then f(2x) + f(-x) – f(x) =

A. 2x

B. 2 |x|

C. -2 x

D. -2 |x|

Answer

$f(x) = 2x + |x|$

f(2x)=2(2x)+|2x|=4x+2|x|

f(-x)=2(-x)+|-x|

f(2x)+f(-x)-f(x)=4x+2|x|-2x+|-x|-(2x+|x|)

=4x+2|x|-2x+|x|-2x-|x|=2|x|

Option B is correct..

16. Question

Mark the correct alternative in the following:

The range of the function $f(x)=\frac{x^2-x}{x^2+2x}$ is

A. R

B. R –{1}

C. R – {-1/2, 1}

D. None of these

Answer

Let $y=\frac{x^2-x}{x^2+2x}$

$y(x^2+2x)= x^2-x$

yx(x+2)=x(x-1)

y(x+2)=x-1

$x(y-1)=-(1+2y)$

$x=-\frac{(1+2y)}{y-1}$

Value of x can’t be zero or it cannot be not defined..

y≠1, -1/2

So, range= R-{-1/2, 1}

17. Question

Mark the correct alternative in the following:

If x ≠ 1 and $f(x)=\frac{x+1}{x-1}$ is a real function, the f(f(f(2)) is

A. 1

B. 2

C. 3

D. 4

Answer

$f(x)=\frac{x+1}{x-1}$

$f(f(x))=\frac{f(x)+1}{f(x)-1}=\frac{\frac{x+1}{x-1}+1}{\frac{x+1}{x-1}-1}$

$=\frac{x+1+x-1}{x+1-x+1}=\frac{2x}{2}=x$

$f\left(f(f(x))\right)=f(x)=\frac{x+1}{x-1}$

$f\left(f(f(2))\right)=\frac{2+1}{2-1}$

=3

Option C is correct..

18. Question

Mark the correct alternative in the following:

If f(x) = cos ($\log_e$ x), then $f\left(\frac{1}{x}\right)f\left(\frac{1}{y}\right)-\frac{1}{2}\left\{f(xy)+f\left(\frac{x}{y}\right)\right\}$ is equal to

A. cos (x – y)

B. log (cos (x – y))

C. 1

D. cos (x + y)

Answer

$f(x) = \cos(\log_e x)$

Now, $f\left(\frac{1}{x}\right)f\left(\frac{1}{y}\right)-\frac{1}{2}\left\{f(xy)+f\left(\frac{x}{y}\right)\right\}$

$= \cos\left(\log_e\frac{1}{x}\right)\cos\left(\log_e\frac{1}{y}\right)-\frac{1}{2}\{\cos(\log_e xy)+\cos(\log_e\frac{x}{y})\}$

$= \cos(\log_e x^{-1})\cos(\log_e y^{-1})-\frac{1}{2}\{\cos(\log_e x+\log_e y)+\cos(\log_e x-\log_e y)\}$

$= \cos(-\log_e x)\cos(-\log_e y)-\{\cos(\log_e x)+\cos(\log_e y)\}$

Using: $\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$

=cos($\log_e$ x) cos($\log_e$ x y)-{cos($\log_e$ x x) cos($\log_e$ x y)}

=0

19. Question

Mark the correct alternative in the following:

Let $f(x)=x, g(x)=\frac{1}{x}$ and h(x) = f(x) g(x). Then, h(x) = 1 for

A. x ∈ R

B. x ∈ Q

C. x ∈ R – Q

D. x ∈ R, x ≠ 0

Answer

$f(x) = x;\ g(x) = \frac{1}{x};\ h(x) = f(x)g(x)$

h(x)=1

f(x)g(x)=1

$= x\left(\frac{1}{x}\right)$

x≠0

Option D is correct.

20. Question

Mark the correct alternative in the following:

If $f(x)=\frac{\sin^4 x+\cos^2 x}{\sin^2 x+\cos^4 x}$ for x ∈ R, then f (2002) =

A. 1

B. 2

C. 3

D. 4

Answer

$f(x)=\frac{(\sin^2 x)^2+\cos^2 x}{1-\cos^2 x+(\cos^2 x)^2}$

$=\frac{(1-\cos^2 x)^2+\cos^2 x}{1-\cos^2 x+\cos^4 x}$

$=\frac{1+\cos^4 x-2\cos^2 x+\cos^2 x}{1-\cos^2 x+\cos^4 x}$

$=\frac{1+\cos^4 x-\cos^2 x}{1-\cos^2 x+\cos^4 x}=1$

Now, f(2002)=1

Option A is correct..

21. Question

Mark the correct alternative in the following:

The function f : R → R is defined by f(x) = $\cos^2 x + \sin^4 x$. Then, f(R) =

A. [3/4, 1)

B. (3/4, 1]

C. [3/4, 1]

D. (3/4, 1)

Answer

$f(x)=\sin^4 x+1-\sin^2 x$

$f(x)=\sin^4 x-\sin^2 x+\frac{1}{4}-\frac{1}{4}+1$

$f(x)=\left(\sin^2 x-\frac{1}{2}\right)^2+\frac{3}{4}$

$\left(\sin^2 x-\frac{1}{2}\right)^2\geq 0$

Minimum value of f(x)=3/4

$0\leq\sin^2 x\leq 1$

So, maximum value of $f(x)=\left(1-\frac{1}{2}\right)^2+\frac{3}{4}$

$=\frac{1}{4}+\frac{3}{4}$

=1

R(f)=[3/4, 1]

Answer is C.

22. Question

Mark the correct alternative in the following:

Let A = {x ∈R : x ≠ 0, -4 ≤ x ≤ 4} and f : A→ R be defined by $f(x)=\frac{|x|}{x}$ for x ∈ A. Then A is

A. {1, -1}

B. {x : 0 ≤ x ≤ 4}

C. {1}

D. {x : -4 ≤ x ≤ 0}

Answer

When -4≤x<0

$f(x)=-\frac{x}{x}$

=-1

When 0<x≤4

$f(x)=\frac{x}{x}$

=1

R(f)={-1, 1}

Option A is correct..

23. Question

Mark the correct alternative in the following:

If f : R → R and g : R → R are defined by f(x) = 2x + 3 and g(x) = x^2 + 7, then the values of x such that g(f (x)) = 8 are

A. 1, 2

B. -1, 2

C. -1, -2

D. 1, -2

Answer

g(f(x))=8

$(f(x))^2+7=8$

$(2x+3)^2=1$

$4x^2+12x+9=1$

$4x^2+12x+8=0$

$x^2+3x+2=0$

(x+1)(x+2)=0

x+1=0 or x+2=0

x=-1 or x=-2

Option C is correct..

24. Question

Mark the correct alternative in the following:

If f : [-2, 2] → R is defined by $f(x)=\begin{cases}-1, \text{ for } -2\le x\le 0\\ x-1, \text{ for } 0\le x\le 2\end{cases}$, then

{x [-2, 2] : x ≤ 0 and f(|x|) = x} =

A. {-1}

B. {0}

C. {-1/2}

D. ϕ

Answer

f(|x|)=|x|-1

f(|x|)=x

We have, |x|=x ;x≥0

And |x|=-x ;x≤0

So, -x-1=x

2x=-1

$x=-\frac{1}{2}$

Option C...

25. Question

Mark the correct alternative in the following:

If $e^{f(x)}=\frac{10+x}{10-x}, x\in(-10,10)$ and $f(x)=kf\left(\frac{200x}{100+x^2}\right)$, then k =

A. 0.5

B. 0.6

C. 0.7

D. 0.8

Answer

$e^{f(x)}=\frac{10+x}{10-x}$

$f(x)=\ln\left(\frac{10+x}{10-x}\right)$

$f(x)=kf\left(\frac{200x}{100+x^2}\right)$

$\ln\left(\frac{10+x}{10-x}\right)=k\ \ln\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)$

$\ln\left(\frac{10+x}{10-x}\right) = k\ \ln\left(\frac{1000+10x^2+200x}{1000+10x^2-200x}\right)$

$= k\ \ln\left(\frac{100+x^2+20x}{100+x^2-20x}\right)$

$\ln\left(\frac{10+x}{10-x}\right) = k\ \ln\left(\frac{10+x}{10-x}\right)^2$

$\ln\left(\frac{10+x}{10-x}\right) = \ln\left(\frac{10+x}{10-x}\right)^{2k}$

2k=1;

$k = \frac{1}{2}$

=0.5

Option A is correct.

26. Question

Mark the correct alternative in the following:

If f is a real valued function given by $f(x) = 27x^3 + \frac{1}{x^3}$ and α, β are roots of $3x + \frac{1}{x} = 12$. Then,

A. f(α) ≠ f(β)

B. f(α) = 10

C. f(β) = - 10

D. None of these

Answer

There is a mistake in the question...

$3x + \frac{1}{x} = 2$

Now, $f(x) = \left(3x + \frac{1}{x}\right)^3 - 3(3x)\left(\frac{1}{x}\right)\left(3x + \frac{1}{x}\right)$

Since, α, β are roots of $3x + \frac{1}{x} = 12$.

So, f(α)=f(β)

=$(2)^3$-9(2)

=8-18

=-10

Option C...

27. Question

Mark the correct alternative in the following:

If $f(x) = 64x^3 + \frac{1}{x^3}$ and α, β are the roots of $4x + \frac{1}{x} = 3$. Then,

A. f (α) = f (β) = - 9

B. f (α) = f (β) = 63

C. f (α) ≠ f (β)

D. None of these

Answer

$f(x) = 64x^3 + \frac{1}{x^3}$

$= \left(4x + \frac{1}{x}\right)^3 - 3(4x)\left(\frac{1}{x}\right)\left(4x + \frac{1}{x}\right)$

Since, $4x + \frac{1}{x} = 3$ and α, β are its roots,

$f(x)=3^3-12(3)$

=27-36

=-9

So, f (α) = f (β) = - 9

Option A is correct..

28. Question

Mark the correct alternative in the following:

If $3f(x) + 5f\left(\frac{1}{x}\right) = \frac{1}{x} - 3$ for all non-zero x, then f(x) =

A. $\frac{1}{14}\left(\frac{3}{x} + 5x - 6\right)$

B. $\frac{1}{14}\left(-\frac{3}{x} + 5x - 6\right)$

C. $\frac{1}{14}\left(-\frac{3}{x} + 5x + 6\right)$

D. None of these

Answer

$3f(x) + 5f\left(\frac{1}{x}\right) = \frac{1}{x} - 3$ eqn. 1

Replacing x by 1/x;

$3f\left(\frac{1}{x}\right) + 5f(x) = x - 3$ eqn. 2

Multiply eqn. 1 by 3 and eqn. 2 by 5, and then subtract them

We get,

$9f(x) + 15f\left(\frac{1}{x}\right) - 15f\left(\frac{1}{x}\right) - 25f(x) = \frac{3}{x} - 9 - 5x + 15$

$-16f(x) = \frac{3}{x} - 5x + 6$

$f(x) = \frac{1}{16}\left(-\frac{3}{x} + 5x - 6\right)$

29. Question

Mark the correct alternative in the following:

If f : R → R be given by $f(x)=\frac{4^x}{4^x+2}$ for all x ∈ R. Then,

A. f(x) = f(1 – x)

B. f(x) + f(1 – x) = 0

C. f(x) + f(1 – x) = 1

D. f(x) + f(x – 1) = 1

Answer

$$f(x) = \frac{4^x}{4^x+2}$$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2}$$

$$= \frac{4.4^{-x}}{4.4^{-x}+2}$$

$$= \frac{\frac{2}{4^x}}{\frac{2}{4^x}+1}$$

$$= \frac{2}{2+4^x}$$

$$f(x-1) = \frac{4^{x-1}}{4^{x-1}+2}$$

$$= \frac{4^x}{4^x+8}$$

$$f(x)+f(1-x) = \frac{4^x}{4^x+2} + \frac{2}{2+4^x} = \frac{4^x+2}{4^x+2} = 1$$

$$f(x)+f(x-1) = \frac{4^x}{4^x+2} + \frac{4^x}{4^x+8} \neq 1$$

30. Question

Mark the correct alternative in the following:

If f(x) = sin [π^2] x + sin [-π^2] x, where [x] denotes the greatest integer less than or equal to x, then

A. f (π /2) = 1

B. f(π) = 2

C. f (π /4) = - 1

D. None of these

Answer

$\pi^2 \approx 9.8596$

[π^2]=9 and [-π^2]=-10

Now, f(x)=sin[π^2] x + sin[-π^2]x

=sin 9x-sin 10x

Now, checking values of f(x) at given points..

$$f\left(\frac{\pi}{2}\right) = \sin 9\left(\frac{\pi}{2}\right) - \sin 10\left(\frac{\pi}{2}\right)$$

=1-0

=1

Option A is correct..

f(π)=sin 9π-sin 10π

=0-0

=0

$f\left(\frac{\pi}{4}\right) = \sin 9\left(\frac{\pi}{4}\right) - \sin 10\left(\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}} - 1$

Option B & C are incorrect..

31. Question

Mark the correct alternative in the following:

The domain of the function $f(x) = \sqrt{2 - 2x - x^2}$ is

A. $\left[-\sqrt{3}, \sqrt{3}\right]$

B. $\left[-1-\sqrt{3}, -1+\sqrt{3}\right]$

C. [-2, 2]

D. $\left[-2-\sqrt{3}, -2+\sqrt{3}\right]$

Answer

for f(x) to be defined,

$2-2x-x^2 \geq 0$

$x^2+2x-2 \leq 0$

(x-(1-√3))(x-(-1+√3))≤0

x∈[-1-√3,-1+√3]

Option B is correct..

32. Question

Mark the correct alternative in the following:

The domain of definition of $f(x) = \sqrt{\frac{x+3}{(2-x)(x-5)}}$ is

A. $(-\infty, -3] \cup (2, 5)$

B. $(-\infty, -3) \cup (2, 5)$

C. $(-\infty, -3] \cup [2, 5]$

D. None of these

Answer

for given function,

$$\frac{x+3}{(2-x)(x-5)} \geq 0$$

$$\frac{x+3}{(x-2)(x-5)} \leq 0$$

x≠2, 5

Therefore, x∈(-∞,-3]∪(2, 5)

Option B is correct..

33. Question

Mark the correct alternative in the following:

The domain of the function $f(x)=\sqrt{\frac{(x+1)(x-3)}{x-2}}$ is

A. [-1, 2) ∪ [3, ∞)

B. (-1, 2) ∪ [3, ∞)

C. [-1, 2] ∪ [3, ∞)

D. None of these

Answer

Here, $\frac{(x+1)(x-3)}{(x-2)} \geq 0$

But $x \neq 2$

So, $x \in [-1,2) \cup [3,\infty)$

Option A is correct..

34. Question

Mark the correct alternative in the following:

The domain of definition of the function $f(x)=\sqrt{x-1}+\sqrt{3-x}$ is

A. [1, ∞)

B. (-∞, 3)

C. (1, 3)

D. [1, 3]

Answer

Here, x-1≥0 and 3-x≥0

So, x≥1 and x≤3

Therefore, x∈[1, 3]option D is correct..

35. Question

Mark the correct alternative in the following:

The domain of definition of the function $f(x)=\sqrt{\frac{x-2}{x+2}}+\sqrt{\frac{1-x}{1+x}}$ is

A. (-∞, -2] ∪ [2, ∞)

B. [-1, 1]

C. ϕ

D. None of these

Answer

For function to be defined,

$\frac{x-2}{x+2} \geq 0, x \neq -2$

x∈(-∞,-2)∪[2, ∞) ...(1)

And $\frac{1-x}{1+x} \geq 0, x \neq -1$

$\frac{x-1}{x+1} \leq 0$

So, $x \in (-1, 1]$ ...(2)

Taking common of both the solutions, we get $x \in \phi$.

Option C is correct..

36. Question

Mark the correct alternative in the following:

The domain of definition of the function $f(x) = \log|x|$ is

A. R

B. (-∞, 0)

C. (0, ∞)

D. R – {0}

Answer

For f(x) =log|x|;

It is defined at all positive values of x except 0..

But since we have |x|;

So, |x|>0;

x ∈ R-{0}

37. Question

Mark the correct alternative in the following:

The domain of definition of the function $f(x) = \sqrt{4x - x^2}$ is

A. R – [0, 4]

B. R – (0, 4)

C. (0, 4)

D. [0, 4]

Answer

Here, $4x-x^2 \geq 0$

$x^2-4x\le 0$

$x(x-4)\le 0$

So, $x\in[0,4]$

Option D is correct..

38. Question

Mark the correct alternative in the following:

The domain of definition of $f(x)=\sqrt{x-3-2\sqrt{x-4}}-\sqrt{x-3+2\sqrt{x-4}}$ is

A. [4, ∞)

B. (-∞, 4]

C. (4, ∞)

D. (-∞, 4)

Answer

Here, $x-3-2\sqrt{x-4}\ge 0$

$\left(\sqrt{x-4}\right)^2+1-2\sqrt{x-4}\ge 0$

$\left(\sqrt{x-4}-1\right)^2\ge 0$

$x-4\ge 0$; $x\ge 4$ .....(1)

Also, $x-3+2\sqrt{x-4}\ge 0$

$\left(\sqrt{x-4}\right)^2+1+2\sqrt{x-4}\ge 0$

$\left(\sqrt{x-4}+1\right)^2\ge 0$

$x\ge 4$

Option A is correct..

39. Question

Mark the correct alternative in the following:

The domain of definition of the function $f(x)=\sqrt{5|x|-x^2-6}$ is

A. (-3, -2) ∪ (2, 3)

B. [-3, -2) ∪ [2, 3)

C. [-3, -2] ∪ [2, 3]

D. None of these

Answer

$5|x|-x^2-6\ge 0$

$x^2-5|x|+6\le 0$

$(|x|-2)(|x|-3)\le 0$

So, $|x|\in[2, 3]$

Therefore, $x\in[-3, -2]\cup[2, 3]$

Option C is correct.

40. Question

Mark the correct alternative in the following:

The range of the function $f(x)=\frac{x}{|x|}$ is

A. R – {0}

B. R – {-1, 1}

C. {-1, 1}

D. None of these

Answer

We know that

|x| = -x in (-∞, 0) and |x| = x in [0, ∞)

So, $f(x)=\frac{x}{-x}=-1$ in (-∞, 0)

And $f(x)=\frac{x}{x}=1$ in (0, ∞)

As clearly shown above f(x) has only two values 1 and -1

So, range of f(x) = {-1, 1}

41. Question

Mark the correct alternative in the following:

The range of the function $f(x)=\frac{x+2}{|x+2|}, x \neq -2$ is

A. {-1, 1}

B. {-1, 0, 1}

C. {1}

D. (0, ∞)

Answer

$f(x)=\frac{x+2}{|x+2|}$

When x>-2,

We have $f(x)=\frac{x+2}{x+2}$

=1

When x<-2,

We have $f(x)=\frac{x+2}{-(x+2)}$

=-1

R(f)={-1, 1}

Option A is correct..

42. Question

Mark the correct alternative in the following:

The range of the function f(x) = |x – 1| is

A. (-∞, 0)

B. [0, ∞)

C. (0, ∞)

D. R

Answer

A modulus function always gives a positive value..

$R(f) = [0, \infty)$

Option B..

43. Question

Mark the correct alternative in the following:

Let $f(x) = \sqrt{x^2 + 1}$. Then, which of the following is correct?

A. f(xy) = f(x) f(y)

B. f(xy) ≥ f(x) f(y)

C. f(xy) ≤ f(x) f(y)

D. None of these

Answer

$f(xy) = \sqrt{x^2y^2 + 1}$

$f(x)f(y) = \left(\sqrt{x^2+1}\right)\left(\sqrt{y^2+1}\right)$

$= \sqrt{x^2y^2 + 1 + x^2 + y^2}$

So, comparing, f(xy) and f(x)f(y);

We get f(xy)≤f(x)f(y)

Option C..

44. Question

Mark the correct alternative in the following:

If $[x]^2$ – 5[x] + 6 = 0, where [•] denotes the greatest integer function, then

A. x ∈ [3, 4]

B. x ∈ (2, 3]

C. x ∈ [2, 3]

D. x ∈ [2, 4]

Answer

$[x]^2$-5[x]+6=0

([x]-2)([x]-3)=0

if [x]=2

2≤x<3

and if [x]=3

3≤x<4

Therefore, $x \in [2, 4]$

Option D..

45. Question

Mark the correct alternative in the following:

The range of $f(x) = \frac{1}{1 - 2\cos x}$ is

A. [1/3, 1]

B. [-1, 1/3]

C. (-∞, -1) ∪ [1/3, ∞)

D. [-1,3, 1]

Answer

we know, -1≤cosx≤1

-2≤-2cosx≤2

-1≤(1-2cosx)≤3

$-1 \leq \left(\frac{1}{1 - 2cosx}\right) \leq \frac{1}{3}$

So, R(f)=[-1, 1/3]

Option ..B