28. Straight Line in Space

Exercise 28.1

1. Question

Find the vector and Cartesian equations of the line through the point (5, 2, –4) and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.

Answer

vector equation of a line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

here $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$ & $\vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$

so the vector equation of the line is

$\vec{r} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

the Cartesian equation of the line is

$$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$$

where x_1, y_1, z_1 are the coordinates of the fixed-point 'a', and b_1, b_2, b_3 are coordinates of $\vec{b}$.

so, the cartesian equation becomes as follows

$$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z-(-4)}{-8}$$

2. Question

Find the vector equation of the line passing through the points (–1, 0, 2) and (3, 4, 6).

Answer

the direction ratios of the required line are

(3–(–1), 4–0, 6–2) = (4, 4, 4)

$\Rightarrow \vec{b} = 4\hat{i} + 4\hat{j} + 4\hat{k}$

since the line passes through (–1, 0, 2)

$\Rightarrow \vec{a} = -1\hat{i} + 0\hat{j} + 2\hat{k}$

the vector equation of the line,

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = \left(-1\hat{i} + 0\hat{j} + 2\hat{k}\right) + \lambda\left(4\hat{i} + 4\hat{j} + 4\hat{k}\right)$

3. Question

Find the vector equation of a line which is parallel to the vector $2\hat{i}-\hat{j}+3\hat{k}$ and which passes through the point (5, –2, 4). Also, reduce it to Cartesian form.

Answer

the vector equation of a line that passes through a fixed point $\vec{a} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ and is parallel to the vector $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ is

$\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = (5\hat{i} - 2\hat{j} + 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$

the Cartesian equation of the line is

$$\frac{x - x_1}{b_1} = \frac{y - y_1}{b_2} = \frac{z - z_1}{b_3}$$

where x_1, y_1, z_1 are the coordinates of the fixed-point 'a', and b_1, b_2, b_3 are coordinates of $\vec{b}$.

so, the cartesian equation becomes as follows

$$\frac{x - 5}{2} = \frac{y - (-2)}{-1} = \frac{z - 4}{3}$$

4. Question

A line passes through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and is the direction of $3\hat{i} + 4\hat{j} - 5\hat{k}$. Find equations of the line in vector and Cartesian form.

Answer

The vector equation of a line passing through a fixed-point 'a' and having directions parallel to $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda \vec{b}$

Here $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ & $\vec{b} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

$\Rightarrow \vec{r} = (2\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(3\hat{i} + 4\hat{j} - 5\hat{k})$

the Cartesian equation of the line is

$$\frac{x - x_1}{b_1} = \frac{y - y_1}{b_2} = \frac{z - z_1}{b_3}$$

where x_1, y_1, z_1 are the coordinates of the fixed-point 'a', and b_1, b_2, b_3 are coordinates of $\vec{b}$.

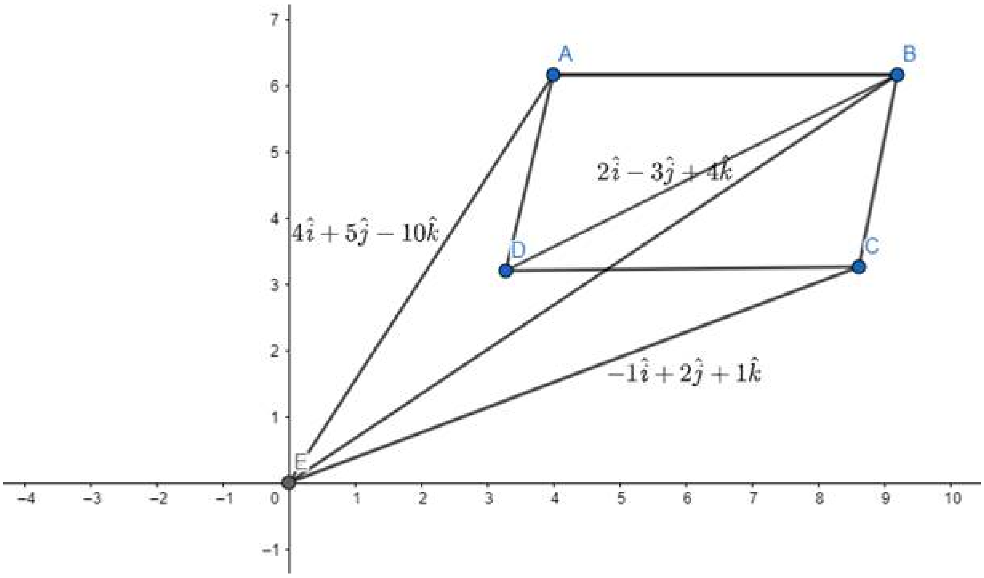
so, the cartesian equation becomes as follows

$$\frac{x - 2}{3} = \frac{y - (-3)}{4} = \frac{z - 4}{-5}$$

5. Question

ABCD is a parallelogram. The position vectors of the points A, B and C are respectively, $4\hat{i} + 5\hat{j} - 10\hat{k}$, $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-\hat{i} + 2\hat{j} + \hat{k}$. Find the vector equation of the line BD. Also, reduce it to Cartesian form.

Answer



Given: the vectors of point A = $4\hat{\imath} + 5\hat{\jmath} - 10\hat{k}$, B = $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and

C = $-\hat{\imath} + 2\hat{\jmath} + \hat{k}$

this means that the vector equation of line AB is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

where $\vec{a} = 4\hat{\imath} + 5\hat{\jmath} - 10\hat{k}$ and $\vec{b} = (2-4)\hat{\imath} + (-3-5)\hat{\jmath} + \left(4-(-10)\right)\hat{k}$

$\Rightarrow \vec{b} = -2\hat{\imath} - 8\hat{\jmath} + 14\hat{k}$

so the vector equation of AB is

$\vec{r_1} = 4\hat{\imath} + 5\hat{\jmath} - 10\hat{k} + \lambda\left(-2\hat{\imath} - 8\hat{\jmath} + 14\hat{k}\right)$

now the vector equation of BC is given as

$\vec{r} = \vec{a} + \lambda\vec{b}$

where $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = (-1-2)\hat{\imath} + \left(2-(-3)\right)\hat{\jmath} + (1-4)\hat{k}$

$\Rightarrow \vec{b} = -3\hat{\imath} + 5\hat{\jmath} - 3\hat{k}$

therefore the vector equation of BC is given as

$\vec{r_2} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k} + \lambda\left(-3\hat{\imath} + 5\hat{\jmath} - 3\hat{k}\right)$

<u>The concept of the question:</u>

the vector equation of the diagonal BD is given by

$\vec{r} = \vec{r_2} - \vec{r_1}$

and the vector equation of diagonal AC is given by

$\vec{r} = \vec{r_2} + \vec{r_1}$

$$\Rightarrow \vec{r} = (2-4)\hat{\imath} + (-3-5)\hat{\jmath} + \left(4-(-10)\right)\hat{k} + \lambda\left\{\left(-3-(-2)\right)\hat{\imath} + \left(5-(-8)\right)\hat{\jmath} + (-3-14)\hat{k}\right\}$$

$\Rightarrow \vec{r} = -2\hat{\imath} - 8\hat{\jmath} + 14\hat{k} + \lambda\left(-\hat{\imath} + 13\hat{\jmath} - 17\hat{k}\right)$......(1)

$\vec{r}$ can be written as

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.....(2)

Comparing 1&2

x = –2–λ, y = –8 + 13λ, z = 14–17λ

$$\lambda = \frac{x+2}{-1} = \frac{y+8}{13} = \frac{z-14}{-17}$$

Therefore, a cartesian equation of the required line is

$$\frac{x+2}{-1} = \frac{y+8}{13} = \frac{z-14}{-17}$$

6. Question

Find in vector form as well as in Cartesian form, the equation of the line passing through the points A(1, 2, –1) and B(2, 1, 1).

Answer

the vector of point A can be written as $\hat{i} + 2\hat{j} - \hat{k}$ and that of point B can be written as $2\hat{i} + j + \hat{k}$

vector equation of line AB is given by

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

Here $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = \{(2-1)\hat{i} + (1-2)\hat{j} + (1-(-1))\hat{k}\}$

$\Rightarrow \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

$$\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k})$$

Here x = 1 + λ, $y = 2 - \lambda, z = -1 + 2\lambda$

$$\Rightarrow \lambda = \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$$

Hence the cartesian equation of the line is

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z+1}{2}$$

7. Question

Find the vector equation for the line which passes through the point (1, 2, 3) and parallel to the vector $\hat{i} - 2\hat{j} + 3\hat{k}$. Reduce the corresponding equation in Cartesian from.

Answer

here the vector for point A(1, 2, 3) is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and the vector parallel to which the line is required is $\hat{i} - 2\hat{j} + 3\hat{k}$

vector equation of a line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

here $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ & $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$

so the vector equation of the line is

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

Here x = 1 + λ, y = 2–2λ, z = 3 + 3λ

$$\lambda = \frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{3}$$

Hence the cartesian equation of the above line is

$$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{3}$$

8. Question

Find the vector equation of a line passing through (2, –1, 1) and parallel to the line whose equations are $\frac{x-3}{2} = \frac{y+1}{7} = \frac{z-2}{-3}$.

Answer

the direction ratios of the line $\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$

Are $\langle b_1, b_2, b_3 \rangle$

So in this question the direction ratios of the given line are ‹2, 7, –3›

So the vector equation of $\vec{b} = 2\hat{i} + 7\hat{j} - 3\hat{k}$

Here $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

vector equation of a line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

so the vector equation of the line is

$\vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \lambda(2\hat{i} + 7\hat{j} - 3\hat{k})$

Here x = 2 + 2λ, y = –1 + 7λ, z = 1-3λ

$$\lambda = \frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3}$$

Hence the cartesian equation of the above line is

$$\frac{x-2}{2} = \frac{y+1}{7} = \frac{z-1}{-3}$$

9. Question

The Cartesian equations of a line are $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Find a vector equation for the line.

Answer

Cartesian equations of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$

Let this be equal to λ

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} = \lambda$$

$x = 3\lambda + 5, y = -4 + 7\lambda, z = 6 + 2\lambda$....(1)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ....(2)

Hence comparing 1&2

$\vec{r} = 5\hat{i} - 4\hat{j} + 6\hat{k} + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$

10. Question

Find the Cartesian equation of a line passing through (1, –1, 2) and parallel to the line whose equations are

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+1}{-2}$. Also, reduce the equation obtained in vector form.

Answer

cartesian equation of the given line is $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{-2}$

Hence its direction ratios are‹1, 2, –2›

The cartesian equation of the line is given by

$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$the point is (1, –1, 2) and the direction ratios are ‹1, 2, –2›

The cartesian equation of the line is

$$\frac{x-1}{1} = \frac{y-(-1)}{2} = \frac{z-2}{-2}$$

Let this be equal to λ

$$\frac{x-1}{1} = \frac{y-(-1)}{2} = \frac{z-2}{-2} = \lambda$$

$x = 1 + \lambda, y = -1 + 2\lambda, z = 2 - 2\lambda$....(1)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ....(2)

Hence comparing 1&2

The vector equation of the line is given as

$$\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} - 2\hat{k})$$

11. Question

Find the direction cosines of the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$. Also, reduce it to vector form.

Answer

the equation of the line is

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$$

But to make it cartesian equation the coefficient of x, y, z must be one so the above equation becomes as

$$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$$

Now the direction ratios of this line is ‹–2, 6, –3›

Concept of the question

Direction cosines of line

$$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3}$$

Are given as$\left(\frac{b_1}{\sqrt{b_1^2 + b_2^2 + b_3^2}}, \frac{b_2}{\sqrt{b_1^2 + b_2^2 + b_3^2}}, \frac{b_3}{\sqrt{b_1^2 + b_2^2 + b_3^2}}\right)$

Here

$$b_1^2 + b_2^2 + b_3^2 = 2^2 + 6^2 + (-3)^2 = 49$$

$\sqrt{b_1^2 + b_2^2 + b_3^2} = \sqrt{49} = 7$

Hence direction cosines of the above line are

$\left(\frac{2}{7}, \frac{6}{7}, \frac{-3}{7}\right)$

cartesian equation of the line is

$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3}$

Let this be equal to λ

$\frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$

Hence $x = 4 - 2\lambda, y = 6\lambda, z = 1 - 3\lambda$....(1)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ....(2)

Hence comparing 1&2

$\vec{r} = 4\hat{i} + \hat{k} + \lambda\left(-2\hat{i} + 6\hat{j} - 3\hat{k}\right)$

12. Question

The Cartesian equations of a line are x = ay + b, z = cy + d. Find its direction ratios and reduce it to vector form.

Answer

the cartesian equation of the line can be written as

$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$

We have written the above equation ion this way because the coefficients of x, y, z must be 1

Therefore the direction ratios of the above line are ‹a, 1, c›

Let $\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} = \lambda(\text{say})$

Therefore x = b + aλ, y = λ, z = d + cλ....(1)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ ....(2)

Hence comparing 1&2

The vector equation of the line is given as

$\vec{r} = b\hat{i} + d\hat{k} + \lambda\left(a\hat{i} + \hat{j} + c\hat{k}\right)$

13. Question

Find the vector equation of a line passing through the point with position vector $\hat{i} - 2\hat{j} - 3\hat{k}$ and parallel to the line joining the points with position vectors $\hat{i} - \hat{j} + 4\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$. Also, find the Cartesian equivalent of this equation.

Answer

direction ratios of the line joining the points with position vector $\hat{i} - \hat{j} + 4\hat{k}$ and $\hat{i} + \hat{j} + 2\hat{k}$ is ‹(1–1), (–1–1), (4–2)›

i. e. ‹0, –2, 2›

the vector equation of the line passing through the point with position vector $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ and having direction ratio ‹0, –2, 2› is given as

$$\vec{r} = \hat{\imath} - 2\hat{\jmath} - 3\hat{k} + \lambda(-2\hat{\jmath} + 2\hat{k})$$

Here x = 1, y = –2–2λ, z = –3 + 2λ

$$\Rightarrow \frac{x-1}{0} = \frac{y+2}{-2} = \frac{z+3}{2} = \lambda$$

hence the cartesian equation of the line is

$$\frac{x-1}{0} = \frac{y+2}{-2} = \frac{z+3}{2}$$

14. Question

Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point P(1, 3, 3).

Answer

let Q be the point on the given line

It is of the form (3λ–2, 2 λ–1, 2 λ + 3)

By letting $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$

distance of point P from PQ = $\sqrt{(3\lambda - 2 - 1)^2 + (2\lambda - 1 - 3)^2 + (2\lambda + 3 - 3)^2}$

$$\Rightarrow 5^2 = (3\lambda - 3)^2 + (2\lambda - 4)^2 + (2\lambda)^2$$

$$\Rightarrow 25 = 9\lambda^2 + 9 - 18\lambda + 4\lambda^2 + 16 - 16\lambda + 4\lambda^2$$

$$\Rightarrow 17\lambda^2 - 34\lambda = 0$$

$$\Rightarrow 17\lambda(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 0, 2$$

So the points on the line are ((3(0)–2), (2(0)–1), (2(0) + 3)) and (3(2)–2, 2(2)–1, 2(2) + 3)

i.e. (–2, –1, 3) and (4, 3, 7)

15. Question

Show that the points whose position vectors are $-2\hat{\imath} + 3\hat{\jmath}$, $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $7\hat{\imath} + 9\hat{k}$ are collinear.

Answer

let the given points are A, B, C with position vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively so

$\vec{a} = -2\hat{\imath} + 3\hat{\jmath}, \vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{c} = 7\hat{\imath} + 9\hat{k}$

We know that equation of line passing through $\vec{a}$ and $\vec{b}$ is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$\vec{r} = -2\hat{\imath} + 3\hat{\jmath} + \lambda\{(1 - (-2))\hat{\imath} + (2 - 3)\hat{\jmath} + (0 - 3)\hat{k}\}$$

$$\vec{r} = -2\hat{\imath} + 3\hat{\jmath} + \lambda(3\hat{\imath} - \hat{\jmath} - 3\hat{k}) \ldots (1)$$

If A, B, C are collinear then C must satisfy eq. (1)

$$7\hat{\imath} + 9\hat{k} = -2\hat{\imath} + 3\hat{\jmath} + \lambda(3\hat{\imath} - \hat{\jmath} - 3\hat{k})$$

On comparing the coefficient of $\hat{\imath}, \hat{\jmath}, \hat{k}$ on both sides of the equation $\lambda = 3$ for all $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$

Hence the points A, B, C are collinear.

16. Question

Find the Cartesian and vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

Answer

Firstly, we need to write the cartesian form of the given line i.e. of $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$

Cartesian equation is

$$\frac{x+2}{-1} = \frac{y+3}{7} = \frac{z-3}{\frac{3}{2}}$$

So the direction ratios of the given line are ‹–1, 7, 3/2›

Vector Equation of line passing through the point (1, 2, 3) i.e. position vector $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and having direction ratios ‹–1, 7, 3/2› is

$$\vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda\left(-\hat{\imath} + 7\hat{\jmath} + \frac{3}{2}\hat{k}\right)$$

And the cartesian equation is

$$\frac{x-1}{-1} = \frac{y-2}{7} = \frac{z-3}{\frac{3}{2}}$$

17. Question

The Cartesian equations of a line are 3x + 1 = 6y – 2 = 1 – z. Find the fixed point through which it passes, its direction ratios and also its vector equation.

Answer

The equation of line is

3x + 1 = 6y – 2 = 1 – z

This can be written as

$$\frac{3x+1}{1} = \frac{6y-2}{1} = \frac{1-z}{1}$$

To make it a cartesian equation we need to make the coefficient of x, y, z to be 1

Therefore, the cartesian equation is

$$\frac{x+1/3}{1/3} = \frac{y-1/3}{1/6} = \frac{z-1}{-1} \text{....(1)}$$

the Cartesian equation of the line is

$$\frac{x-x_1}{b_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{b_3} \text{....(2)}$$

Therefore on comparing (1)&(2)

The fixed point through which it passes is $\left(\frac{-1}{3}, \frac{1}{3}, 1\right)$

And the direction ratios are $< \frac{1}{3}, \frac{1}{6}, -1 >$

Let $\frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1} = \lambda(\text{say})$

$x = \frac{-1}{3} + \frac{1}{3}\lambda, y = \frac{1}{3} + \frac{1}{6}\lambda, z = 1 - \lambda$...(3)

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ ....(4)

Hence, comparing 3&4 we get,

$$\vec{r} = \frac{-1}{3}\hat{\imath} + \frac{1}{3}\hat{\jmath} + \hat{k} + \lambda\left(\frac{1}{3}\hat{\imath} + \frac{1}{6}\hat{\jmath} - \hat{k}\right)$$

18. Question

Find the vector equation of the line passing through the point A(1, 2, –1) and parallel to the line 5x – 25 = 14 – 7y = 35z.

Answer

firstly we need to resolve the given line

The given line can be written as

$$\frac{5x-25}{1} = \frac{14-7y}{1} = \frac{35z}{1}$$

We need to make the coefficients of x, y, z equal to 1

Therefore the line becomes as follows

$$\frac{x-5}{1/5} = \frac{y-2}{-7} = \frac{z-0}{1/35}$$

Therefore the direction ratios of the given line are $< \frac{1}{5}, -7, \frac{1}{35} >$

Vector equation of line passing through the point A(1, 2, –1) and having direction ratios

$$< \frac{1}{5}, -7, \frac{1}{35} >$$

$$\vec{r} = \hat{\imath} + 2j - \hat{k} + \lambda\left(\frac{1}{5}\hat{\imath} - 7\hat{\jmath} + \frac{1}{35}\hat{k}\right)$$

Exercise 28.2

1. Question

Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ are mutually perpendicular.

Answer

l, m, n are generally written as direction cosines or the direction ratios of unit vector-*,

Let $l_1 = \frac{12}{13}, m_1 = \frac{-3}{13}, n_1 = \frac{-4}{13}$

$l_2 = \frac{4}{13}, m_2 = \frac{12}{13}, n_2 = \frac{3}{13}$

$l_3 = \frac{3}{13}, m_3 = \frac{-4}{13}, n_3 = \frac{12}{13}$

For the lines or vectors to be perpendicular their dot product or scalar product should be zero and for the lines or vectors to be parallel their cross product or vector product should be zero.

So we will use scalar product to prove these lines perpendicular to each other.

$$l_1l_2 + m_1m_2 + n_1n_2 = \left(\frac{12}{13}\times\frac{4}{13}\right) + \left(\frac{-3}{13}\times\frac{12}{13}\right) + \left(\frac{-4}{13}\times\frac{3}{13}\right) = \frac{48-36-12}{169} = 0$$

$$l_2l_3 + m_2m_3 + n_2n_3 = \left(\frac{4}{13}\times\frac{3}{13}\right) + \left(\frac{12}{13}\times\frac{-4}{13}\right) + \left(\frac{12}{13}\times\frac{3}{13}\right) = \frac{12-48+36}{169} = 0$$

$$l_1l_3 + m_1m_3 + n_1n_3 = \left(\frac{12}{13}\times\frac{3}{13}\right) + \left(\frac{-4}{13}\times\frac{-3}{13}\right) + \left(\frac{12}{13}\times\frac{-4}{13}\right) = \frac{36+12-48}{169} = 0$$

Therefore, all three lines or vectors are mutually perpendicular to each other.

2. Question

Show that the line through the points (1, –1, 2) and (3, 4, –2) is perpendicular to the through the point (0, 3, 2) and (3, 5, 6).

Answer

The direction ratios of a line can be found by subtracting the corresponding coordinates of two points through which the line passes i.e. (subtract x coordinates, subtract y coordinates, subtract z coordinates), this is the direction ratio of the line. There can be no direction ratio of a line passing through only one point, there should be at least two points.

The direction ratios of a line passing through the points (1,–1,2) and (3,4,–2) are,

(3–1,4–{–1},–2–2) = (2,5,–4)

Or it can also be the other way you can choose the first and the second point of your own choice.

The direction ratios of a line passing through the points (0,3,2) and (3,5,6) are,

(3–0,5–3,6–2) = (3,2,4)

The direction ratios of lines are,

(a_1,b_1,c_1) = (2,5,–4)

(a_2,b_2,c_2) = (3,2,4)

By using dot product.

$$\cos\theta = \frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{[2\times3 + 5\times2 + (-4)\times4]}{\sqrt{2^2+5^2+(-4)^2}\sqrt{3^2+2^2+4^2}}$$

$$\cos\theta = \frac{0}{\sqrt{2^2+5^2+(-4)^2}\sqrt{3^2+2^2+4^2}}$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

Therefore,, the lines are perpendicular.

3. Question

Show that the line through the points (4, 7, 8) and (2, 3, 4) is parallel to the line through the points (–1, –2, 1) and (1, 2, 5).

Answer

The direction ratios of a line can be found by subtracting the corresponding coordinates of two points through which the line passes i.e. (subtract x coordinates, subtract y coordinates, subtract z coordinates), this is the direction ratio of the line. There can be no direction ratio of a line passing through only one point, there should be at least two points.

For the two lines to be parallel or anti parallel to each other the fraction of their corresponding direction ratios should be equal.

The direction ratios of a line passing through the points (4,7,8) and (2,3,4) are (4–2,7–3,8–4) = (2,4,4)

The direction ratios of a line passing through the points (–1,–2,1) and (1,2,5) are (–1–1,–2–2,1–5) = (–2,–4,–4)

The direction ratios are proportional.

$$\frac{2}{-2} = \frac{4}{-4} = \frac{4}{-4} = -1(\text{constant})$$

Hence the lines are mutually parallel and even overlapping each other because of the constant –1 or 1.

4. Question

Find the Cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

Answer

The **Cartesian equation or the symmetrical form** of equation is the one which is of the form $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \mu(\text{constant})$

Where (x_1,y_1,z_1) is the point through which the line passes and a,b,c are the directional ratios of the line or the directional ratios of the line are proportional to them.

The Cartesian equation of a line passing through the point (–2,4,–5) and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$.

5. Question

Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Answer

The Cartesian equation of the lines are $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z-0}{1}$ and $\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3}$, as we know their direction ratios we can find weather they are perpendicular or not, for proving the lines to be perpendicular we can only consider the numerator of the dot product when we use **cos θ = numerator**

cos θ = 7×1 + 2×(–5) + 1×3

cos θ = 7 – 10 + 3

cos θ = 0

$\theta = \frac{\pi}{2}$

Therefore, the lines are perpendicular.

6. Question

that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3,5,–1) and (4, 3, –1).

Answer

The direction ratios of a line can be found by subtracting the corresponding coordinates of two points through which the line passes i.e. (subtract x coordinates, subtract y coordinates, subtract z coordinates), this is the direction ratio of the line. There can be no direction ratio of a line passing through only one point, there should be at least two points.

The direction ratios of a line joining the origin to the point (2,1,1) are (2–0,1–0,1–0) = (2,1,1)

The direction ratios of a line joining the points (3,5,–1) and (4,3,–1) are (4–3,3–5,–1–{–1}) = (1,–2,0)

By using the dot product we can find the angle between the two lines,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{2\times 1 + 1\times(-2) + 1\times 0}{\sqrt{2^2 + 1^2 + 1^2}\sqrt{1^2 + (-2)^2 + 0^2}}$$

$$\cos\theta = \frac{0}{\sqrt{2^2+1^2+1^2}\sqrt{1^2+(-2)^2+0^2}}$$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

Therefore, the lines are mutually perpendicular.

7. Question

Find the equation of a line parallel to x-axis and passing through the origin.

Answer

Vector equation of a line is $\vec{r} = \vec{a} + \lambda\vec{b}$ where $\vec{a}$ is the position vector of the point a through which our line passes through and $\vec{b}$ is the vector parallel to our line and $\vec{r}$ is the general vector of a line satisfying these conditions and λ is a constant.

The direction cosines of the x-axis are (1,0,0), direction cosines are the direction ratios of a unit vector.

The equation of a line parallel to x-axis and passing through the origin is,

$\vec{r} = (0\hat{i}+0\hat{j}+0\hat{k}) + \lambda(1\hat{i}+0\hat{j}+0\hat{k})$

$\vec{r} = \lambda\hat{i}$

Therefore, this is the family of lines satisfying the condition of the question.

8 A. Question

Find the angle between the following pairs of line :

$\vec{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$ and $\vec{r} = \hat{i} - \hat{j} + 2\hat{k} - \mu\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)$

Answer

We know that angle between two lines given with their vector equation is the angle between their parallel vectors always.

By using dot product to find the angle between the lines using their parallel vectors $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$.

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$\vec{r} = (4\hat{i}-\hat{j}) + \lambda(\hat{i}+2\hat{j}-2\hat{k})$

$\vec{r} = (\hat{i}-\hat{j}+2\hat{k}) - \mu(2\hat{i}+4\hat{j}-4\hat{k})$

$\overrightarrow{a_1} = 4\hat{i} - \hat{j}$, $\overrightarrow{b_1} = \hat{i}+2\hat{j}-2\hat{k}$

$\overrightarrow{a_2} = \hat{i} - \hat{j} + 2\hat{k}$, $\overrightarrow{b_2} = 2\hat{i} + 4\hat{j} - 4\hat{k}$

$|\overrightarrow{b_1}| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$

$|\overrightarrow{b_2}| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$

Let θ be the angle between given lines. So using dot product we can find the angle.

$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

$= \frac{(\hat{i}+2\hat{j}-2\hat{k})(2\hat{i}+4\hat{j}-4\hat{k})}{3\times 6}$

$= \frac{2+8+8}{18} = 1$

$\cos\theta = 1$

$\theta = 0^o$

8 B. Question

Find the angle between the following pairs of line :

$\vec{r} = \left(3\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+2\hat{k}\right)$ and $\vec{r} = \left(5\hat{j}-2\hat{k}\right)+\mu\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$

Answer

We know that angle between two lines given with their vector equation is the angle between their parallel vectors always.

By using dot product to find the angle between the lines using their parallel vectors $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$.

$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

$\vec{r} = (3\hat{i}+2\hat{j}-4\hat{k})+\lambda(\hat{i}+2\hat{j}+2\hat{k})$

$\vec{r} = (5\hat{j}-2\hat{k})+\mu(3\hat{i}+2\hat{j}+6\hat{k})$

$\overrightarrow{a_1} = 3\hat{i}+2\hat{j}-4\hat{k}$, $\overrightarrow{b_1} = \hat{i}+2\hat{j}+2\hat{k}$

$\overrightarrow{a_2} = 5\hat{j}-2\hat{k}$, $\overrightarrow{b_2} = 3\hat{i}+2\hat{j}+6\hat{k}$

$|\overrightarrow{b_1}| = \sqrt{1^2+2^2+2^2} = 3$

$|\overrightarrow{b_2}| = \sqrt{3^2+2^2+6^2} = 7$

Let θ be the angle between given lines, so using the dot product,

$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

$= \frac{(\hat{i}+2\hat{j}+2\hat{k})(3\hat{i}+2\hat{j}+6\hat{k})}{3\times 7}$

$= \frac{3+4+12}{21} = 19$

$\theta = \cos^{-1}\left(\frac{19}{21}\right)$

8 C. Question

Find the angle between the following pairs of line :

$\vec{r} = \lambda\left(\hat{i}+\hat{j}+2\hat{k}\right)$ and $\vec{r} = 2\hat{j}+\mu\left\{\left(\sqrt{3}-1\right)\hat{i}-\left(\sqrt{3}+1\right)\hat{j}+4\hat{k}\right\}$

Answer

We know that angle between two lines given with their vector equation is the angle between their parallel vectors always.

By using dot product to find the angle between the lines using their parallel vectors $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$.

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$\vec{r} = \lambda(\hat{i}+\hat{j}+2\hat{k})$

$\vec{r} = 2\hat{j}+\mu[(\sqrt{3} - 1)\hat{i}-(\sqrt{3} + 1)\hat{j}+4\hat{k}]$

$\overrightarrow{b_1} = \hat{i}+\hat{j}+2\hat{k}$, $\overrightarrow{b_2} = (\sqrt{3} - 1)\hat{i}-(\sqrt{3} + 1)\hat{j}+4\hat{k}$

$|\overrightarrow{b_1}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$

$|\overrightarrow{b_2}| = \sqrt{(\sqrt{3} - 1)^2 + [-(\sqrt{3} + 1)]^2 + 4^2} = \sqrt{24} = 2\sqrt{6}$

Let θ be the angle between given lines, so using the dot product equation,

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$$= \frac{(\hat{i}+\hat{j}+2\hat{k})((\sqrt{3} - 1)\hat{i}-(\sqrt{3} + 1)\hat{j}+4\hat{k})}{\sqrt{6}\times 2\sqrt{6}}$$

$= \frac{-2+8}{2\times 6} = \frac{6}{2\times 6} = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

9 A. Question

Find the angle between the following pairs of lines :

$$\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4} \text{ and } \frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x+4}{3} = \frac{y-1}{5} = \frac{z+3}{4}$ and $\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

$a_1 = 3$, $b_1 = 5$, $c_1 = 4$; $a_2 = 1$, $b_2 = 1$, $c_2 = 2$

now to find the angle between two lines we use cross product equation,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\cos\theta = \frac{3\times 1+5\times 1+4\times 2}{\sqrt{3^3+5^2+4^2}\sqrt{1^2+1^2+2^2}}$

$\cos\theta = \frac{16}{10\sqrt{3}}$

$\cos\theta = \frac{8}{5\sqrt{3}}$

$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

9 B. Question

Find the angle between the following pairs of lines :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{-3} \text{ and } \frac{x+3}{-1} = \frac{y-5}{8} = \frac{z-1}{4}$$

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{-3}$ and $\frac{x+3}{-1} = \frac{y-5}{8} = \frac{z-1}{4}$

$a_1 = 2$, $b_1 = 3$, $c_1 = -3$; $a_2 = -1$, $b_2 = 8$, $c_2 = 4$

now to find the angle between two lines we use cross product equation,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\cos\theta = \frac{2\times(-1)+3\times8+(-3)\times4}{\sqrt{2^2+3^2+(-3)^2}\sqrt{(-1)^2+8^2+4^2}}$

$\cos\theta = \frac{10}{9\sqrt{22}}$

$\theta = \cos^{-1}\left(\frac{10}{9\sqrt{22}}\right)$

9 C. Question

Find the angle between the following pairs of lines :

$$\frac{5-x}{-2} = \frac{y+3}{1} = \frac{1-z}{3} \text{ and } \frac{x}{3} = \frac{1-y}{-2} = \frac{z+5}{-1}$$

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{5-x}{-2} = \frac{y+3}{1} = \frac{1-z}{3}$ and $\frac{x}{3} = \frac{1-y}{-2} = \frac{z+5}{-1}$ these are not in the standard form but after converting them, we get,

$\frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$ and $\frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$

$a_1 = 2$, $b_1 = 1$, $c_1 = -3$; $a_2 = 3$, $b_2 = 2$, $c_2 = -1$

now to find the angle between two lines we use cross product equation,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{(2)\times 3+1\times(2)+(-3)\times(-1)}{\sqrt{2^2+1^2+3^2}\sqrt{3^2+2^2+(-1)^2}}$$

$$\cos\theta = \frac{6+2+3}{14}$$

$$\theta = \cos^{-1}\left(\frac{11}{14}\right)$$

9 D. Question

Find the angle between the following pairs of lines :

$$\frac{x-2}{3} = \frac{y+3}{-2}, z = 5 \text{ and } \frac{x+1}{1} = \frac{2y-3}{3} = \frac{z-5}{2}$$

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-2}{3} = \frac{y+3}{-2}$, $z = 5$ and $\frac{x+1}{1} = \frac{2y-3}{3} = \frac{z-5}{2}$ these are not in the standard form but after converting them, we get,

$$\frac{x-2}{3} = \frac{y+3}{-2} = \frac{z-5}{0} \text{ and } \frac{x+1}{1} = \frac{y-\frac{3}{2}}{\frac{3}{2}} = \frac{z-5}{2}$$

$a_1 = 3$, $b_1 = -2$, $c_1 = 0$; $a_2 = 1$, $b_2 = \frac{3}{2}$, $c_2 = 2$

now to find the angle between two lines we use cross product equation,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{3\times 1+(-2)\times\frac{3}{2}+0\times 2}{\sqrt{3^2+(-2)^2+0^2}\sqrt{1^2+(\frac{3}{2})^2+2^2}}$$

$$\cos\theta = \frac{0}{\sqrt{3^2+(-2)^2+0^2}\sqrt{1^2+(\frac{3}{2})^2+2^2}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

9 E. Question

Find the angle between the following pairs of lines :

$$\frac{x-5}{1} = \frac{2y+6}{-2} = \frac{z-3}{1} \text{ and } \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$$

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-5}{1} = \frac{2y+6}{-2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$ these are not in the standard form but after converting them, we get,

$\frac{x-5}{1} = \frac{y+3}{-1} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$

Given there are two vectors which are parallel to these lines,

$\vec{a} = 1\hat{i} - 1\hat{j} + 1\hat{k}$, $\vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

They are parallel because they have same direction ratios.

So angle between the lines is the angle between these vectors which are parallel to the lines.

By using dot product equation,

$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta = \frac{(1\hat{i}-1\hat{j}+1\hat{k})(3\hat{i}+4\hat{j}+5\hat{k})}{|1\hat{i}-1\hat{j}+1\hat{k}||3\hat{i}+4\hat{j}+5\hat{k}|}$

$\cos\theta = \frac{1\times(3)+(-1)\times 4+1\times 5}{\sqrt{1^2+(-1)^2+1^2}\sqrt{3^2+4^2+5^2}}$

$\cos\theta = \frac{3-4+5}{\sqrt{3}\,5\sqrt{2}}$

$\cos\theta = \frac{4}{5\sqrt{6}}$

$\theta = \cos^{-1}(\frac{4}{5\sqrt{6}})$

9 F. Question

Find the angle between the following pairs of lines :

$\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-2}{2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{2} = \frac{z-5}{4}$

Given there are two vectors which are parallel to these lines,

$\vec{a} = 2\hat{i} + 7\hat{j} - 3\hat{k}$, $\vec{b} = -1\hat{i} + 2\hat{j} + 4\hat{k}$

They are parallel because they have same direction ratios.

So angle between the lines is the angle between these vectors which are parallel to the lines.

By using dot product equation,

$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos\theta = \frac{(2\hat{i}+7\hat{j}-3\hat{k})(-1\hat{i}+2\hat{j}+4\hat{k})}{|2\hat{i}+7\hat{j}-3\hat{k}||-1\hat{i}+2\hat{j}+4\hat{k}|}$

$\cos\theta = \frac{-2+14-12}{\sqrt{2^2+7^2+(-3)^2}\sqrt{(-1)^2+2^2+4^2}}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

10 A. Question

Find the angle between the pairs of lines with direction ratios proportional to

5, –12, 13 and –3, 4, 5

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

We are given with the direction ratios and we have to find the angle between the lines having these direction ratios.

$a_1 = 5$, $b_1 = -12$, $c_1 = 13$; $a_2 = -3$, $b_2 = 4$, $c_2 = 5$

By using dot product equation, we get

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\cos\theta = \frac{5\times(-3)+(-12)\times4+13\times5}{\sqrt{5^2+(-12)^2+13^2}\sqrt{(-3)^2+4^2+5^2}}$

$\cos\theta = \frac{-15-48+65}{13\sqrt{2}\,5\sqrt{2}}$

$\cos\theta = \frac{1}{65}$

$\theta = \cos^{-1}(\frac{1}{65})$

10 B. Question

Find the angle between the pairs of lines with direction ratios proportional to

2, 2, 1 and 4, 1, 8

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

We are given with the direction ratios and we have to find the angle between the lines having these direction ratios.

$a_1 = 2$, $b_1 = 2$, $c_1 = 1$; $a_2 = 4$, $b_2 = 1$, $c_2 = 8$

By using dot product equation, we get

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$\cos\theta = \frac{2\times4+2\times1+1\times8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$

$\cos\theta = \frac{8+2+8}{3\times9}$

$\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}(\frac{2}{3})$

10 C. Question

Find the angle between the pairs of lines with direction ratios proportional to

1, 2, –2 and –2, 2, 1

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$

We are given with the direction ratios and we have to find the angle between the lines having these direction ratios.

$a_1 = 1, b_1 = 2, c_1 = -2$; $a_2 = -2, b_2 = 2, c_2 = 1$

By using dot product equation, we get

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$\cos\theta = \frac{1\times(-2)+2\times2+(-2)\times1}{\sqrt{1^2+2^2+(-2)^2}\sqrt{(-2)^2+2^2+1^2}}$

$\cos\theta = \frac{-2+4-2}{3\times3}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

10 D. Question

Find the angle between the pairs of lines with direction ratios proportional to

a, b, c and b – c, c – a, a – b

Answer

We are given with the direction ratios and the vector equations of two lines, we have to find the angle between the two lines, and weather they are parallel or perpendicular to each other.

The direction ratios are a,b,c and b–c, c–a, a–b .

The vectors with these direction ratios are,

$\vec{x} = a\hat{i}+b\hat{j}+c\hat{k}$ and $\vec{y} = (b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k}$

By using dot product equation to find the angle between them, we get,

$\cos\theta = \frac{\vec{x}\vec{y}}{|\vec{x}||\vec{y}|}$

$\cos\theta = \frac{(a\hat{i}+b\hat{j}+c\hat{k})\{(b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k}\}}{|a\hat{i}+b\hat{j}+c\hat{k}||(b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k}|}$

$\cos\theta = \frac{a(b-c)+b(c-a)+c(a-b)}{|a\hat{i}+b\hat{j}+c\hat{k}||(b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k}|}$

$\cos\theta = \frac{ab-ac+bc-ab+ca-cb}{|a\hat{i}+b\hat{j}+c\hat{k}||(b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k}|}$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

11. Question

Find the angle between two lines, one of which has direction ratios 2, 2, 1 while the other one is obtained by joining the points (3, 1, 4) and (7, 2, 12).

Answer

In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation. $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ in this equation a,b,c are the direction ratios of this equation.

Direction ratios of the first line are $a_1 = 2$, $b_1 = 2$, $c_1 = 1$ which corresponds to 2,2,1.

Direction ratios of the line joining (3,1,4) and (7,2,12) .

= (7–3,2–1,12–4) = (4,1,8) .

$a_2 = 4$, $b_2 = 1$, $c_2 = 8$

now to find the angle between two lines we use cross product equation,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$\cos\theta = \frac{2\times4+2\times1+1\times8}{\sqrt{2^2+2^2+1^2}\sqrt{4^2+1^2+8^2}}$$

$\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}(\frac{2}{3})$

12. Question

Find the equation of the line passing through the point (1, 2, –4) and parallel to the line

$$\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}.$$

Answer

The Cartesian equation of a line passing through a point (x_1, y_1, z_1) and having directional ratios proportional to a,b,c is given by,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

Now the point (x_1, y_1, z_1) = (1,2,–4) and the required line is parallel to a given line $\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}$, now as we know that if two lines are parallel and direction ratios of one line are a,b,c then the direction ratios of other lines will be ka,kb,kc where k is a constant and which gets cancelled when we put these direction ratios in the equation of the required line.

So the direction ratios of the required line are ;

$a = 4\lambda$, $b = 2\lambda$, $c = 3\lambda$

hence the equation of the required line is,

$$\frac{x-1}{4\lambda}=\frac{y-2}{2\lambda}=\frac{z+4}{3\lambda}$$

$$\frac{x-1}{4}=\frac{y-2}{2}=\frac{z+4}{3}$$

13. Question

Find the equation of the line passing through the point (–1, 2, 1) and parallel to the line $\frac{2x-1}{4}=\frac{3y+5}{2}=\frac{2-z}{3}$.

Answer

The Cartesian equation of a line passing through a point (x_1, y_1, z_1) and having directional ratios proportional to a,b,c is given by,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

Now the point (x_1, y_1, z_1) = (–1,2,1) and the required line is parallel to a given line $\frac{2x-1}{4}=\frac{3y+5}{2}=\frac{2-z}{3}$ which is not in the general form of the Cartesian equation because the coefficients of x,y,z in the Cartesian equation are 1, so the equation will reduce to the form $\frac{x-\frac{1}{2}}{2}=\frac{z+\frac{5}{3}}{\frac{2}{3}}=\frac{z-2}{-3}$ now as we know that if two lines are parallel and direction ratios of one line are a,b,c then the direction ratios of other lines will be ka,kb,kc where k is a constant and which gets cancelled when we put these direction ratios in the equation of the required line.

So the direction ratios of the required line are ;

a = 2λ, b = $\frac{2}{3}$λ, c = –3λ

hence the equation of the required line is,

$$\frac{x+1}{2\lambda}=\frac{y-2}{\frac{2}{3}\lambda}=\frac{z-1}{-3\lambda}$$

$$\frac{x+1}{2}=\frac{y-2}{\frac{2}{3}}=\frac{z-1}{-3}$$

14. Question

Find the equation of the line passing through the point (2, –1, 3) and parallel to the line $\vec{r}=\left(\hat{i}-2\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-5\hat{k}\right)$.

Answer

Vector equation of a line is $\vec{r}=\vec{a}+\lambda\vec{b}$ where $\vec{a}$ is the position vector of the point a through which our line passes through and $\vec{b}$ is the vector parallel to our line and $\vec{r}$ is the general vector of a line satisfying these conditions and λ is a constant.

Now the point vector through which the line passes is $\vec{a}$ = $2\hat{i}-\hat{j}+3\hat{k}$ and the required line is parallel to a line having vector equation,

$\vec{r}$ = $(\hat{i}-2\hat{j}+\hat{k})$ + λ$(2\hat{i}+3\hat{j}-5\hat{k})$

The parallel vector is,

$\vec{b}$ = $(2\hat{i}+3\hat{j}-5\hat{k})$

So the vector equation of the required line is,

$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} - 5\hat{k})$

Where μ is a constant or a scalar.

15. Question

Find the equation of the line passing through the point (2, 1, 3) and parallel to the line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$

Answer

The Cartesian equation of a line passing through a point (x_1, y_1, z_1) and having directional ratios proportional to a,b,c is given by,

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

The required line passes through the point (2,1,3), now we need to find the direction ratios of the line which are a,b,c . this equation of the required line is,

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$$

We are given with the Cartesian equation of the two lines which are perpendicular to the given equation, as the lines are perpendicular with the required line so the dot product will result in zero.

The first line is $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$, the dot product equation is,

a×1 + b×2 + c×3 = 0

a+2b+3c = 0 ....(i).

The second line is $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$, the dot product equation is,

a×(–3) + b×2 + c×5 = 0

–3a+2b+5c = 0 ....(ii).

Solving equations (i) and (ii), we get, by cross multiplication method,

$$\frac{a}{10-6} = \frac{-b}{5-(-9)} = \frac{c}{2-(-6)} = \lambda$$

$$\frac{a}{4} = \frac{b}{-14} = \frac{c}{8} = \lambda$$

$$\frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda$$

a = 2λ, b = –7λ, c = 4λ

using a,b,c in the required equation we get,

$$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda}$$

$$\frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$$

This is the required equation.

16. Question

Find the equation of the line passing through the point $\hat{i} + \hat{j} - 3\hat{k}$ and perpendicular to the lines

$\vec{r} = \hat{i} + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$ and $\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$.

Answer

The vector equation of a line passing through a point with position vector $\vec{\alpha}$ and perpendicular to two lines with vector equations $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, is given by $\vec{r} = \vec{\alpha} + \lambda(\overrightarrow{b_1} \times \overrightarrow{b_2})$ because as we know that vector product between two vectors will give you a vector whose direction is perpendicular to both the vectors, so the required equation is parallel to this vector and hence this is the equation.

Now we are given with,

$\vec{\alpha} = (\hat{i}+\hat{j}-3\hat{k})$ and the lines perpendicular to required line are,

$\vec{r} = \hat{i}+\lambda(2\hat{i}+\hat{j}-3\hat{k})$ and $\vec{r} = (2\hat{i}+\hat{j}-\hat{k})+\mu(\hat{i}+\hat{j}+\hat{k})$

$\overrightarrow{b_1} = 2\hat{i}+\hat{j}-3\hat{k}$ and $\overrightarrow{b_2} = \hat{i}+\hat{j}+\hat{k}$

Now,

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= \hat{i}(1+3) - \hat{j}(2+3) + \hat{k}(2-1)$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = 4\hat{i} - 5\hat{j} + \hat{k}$

Therefore, the required equation is,

$\vec{r} = \hat{i}+\hat{j}-3\hat{k} + \rho(4\hat{i}-5\hat{j}+\hat{k})$

17. Question

Find the equation of the line passing through the point (1, –1, 1) and perpendicular to the lines joining the points (4,3,2), (1,–1,0) and (1,2,–1), (2, 1, 1).

Answer

The Cartesian equation of a line passing through a point (x_1, y_1, z_1) and having directional ratios proportional to a,b,c is given by,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

The required line passes through the point (1,–1,1), now we need to find the direction ratios of the line which are a,b,c . this equation of the required line is,

$$\frac{x-1}{a} = \frac{y+1}{b} = \frac{z-1}{c}$$

Direction ratios of the line joining A(4,3,2) and B(1,–1,0)

= (4–1,3+1,2–0) = (3,4,2)

Direction ratios are 3,4,2

Direction ratios of the line joining C(1,2,–1) and D(2,1,1)

= (2-1,1-2,1+1) = (1,–1,2)

Direction ratios are 1,–1,2

It is given that the line AB is perpendicular to the required line, so the dot product equation will be equal to zero .

$a\times3 + b\times4 + c\times2 = 0$

3a+4b+2c = 0 ....(i).

It is given that line CD is perpendicular to the required line, so the dot product will be equal to zero .

a×1 + b×(–1) + c×2 = 0

a–b+2c = 0 ....(ii).

Solving equations (i) and (ii) by cross multiplication method, we get

$$\frac{a}{(4)(2)-(-1)(2)}=\frac{-b}{(2)(3)-(2)(1)}=\frac{c}{(-1)(3)-(4)(1)}$$

$$\frac{a}{8+2}=\frac{-b}{6-2}=\frac{c}{-3-4}$$

$$\frac{a}{10}=\frac{-b}{4}=\frac{c}{-7}=\lambda$$

a = 10λ, b = –4λ, c = –7λ

Therefore, the cartesian or symmetry form of equation of the required line is,

$$\frac{x-1}{10\lambda}=\frac{y+1}{-4\lambda}=\frac{z-1}{-7\lambda}$$

$$\frac{x-1}{10}=\frac{y+1}{-4}=\frac{z-1}{-7}$$

18. Question

Determine the equations of the line passing through the point (1, 2, –4) and perpendicular to the two lines $\frac{x-8}{8}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$

Answer

The Cartesian equation of a line passing through a point (x_1, y_1, z_1) and having directional ratios proportional to a,b,c is given by,

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

The required line passes through the point (1,2,–4), now we need to find the direction ratios of the line which are a,b,c . this equation of the required line is,

$$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z+4}{c}$$

It is given that a line having Cartesian equation $\frac{x-8}{8}=\frac{y+9}{-16}=\frac{z-10}{7}$ is perpendicular to the required line, so the dot product equation will be equal to zero.

a×8 + b×(–16) + c×7 = 0

8a–16b+7c = 0 ......(i).

It is given that a line having Cartesian equation $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ is perpendicular to the required line,

So the dot product equation will be equal to zero.

a×3 + b×8 + c×(–5) = 0

3a+8b–5c = 0 ....(ii).

By solving equation (i) and (ii), we get, by using cross multiplication method,

$$\frac{a}{(-5)(-16)-(8)(7)}=\frac{-b}{(-5)(8)-(3)(7)}=\frac{c}{(8)(8)-(3)(-16)}$$

$\frac{a}{24} = \frac{b}{61} = \frac{c}{112} = \lambda$

$a = 24\lambda,\ b = 61\lambda,\ c = 112\lambda$

Put these values in the required equation of line,

$$\frac{x-1}{24\lambda} = \frac{y-2}{61\lambda} = \frac{z+4}{112\lambda}$$

Therefore, this is the required equation of line.

19. Question

Show that the lines $\frac{x-5}{7} = \frac{\psi+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Answer

The Cartesian equation of the lines are $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z-0}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and we need to find that weather the lines are perpendicular or not, so we will use the dot product equation, as we know the direction ratios of both the lines.

$a_1a_2+b_1b_2+c_1c_2 = (7)(1) + (-5)(2) + (1)(3) = 7 - 10 + 3 = 0$

Hence the given lines are perpendicular because,

$\cos\theta = 0$

$\theta = \frac{\pi}{2}$

20. Question

Find the vector equation of the line passing through the point (2, –1, –1) which is parallel to the line $6x-2 = 3y+1 = 2z-2$.

Answer

The Cartesian equation of a line passing through a point (x_1, y_1, z_1) and having directional ratios proportional to a,b,c is given by,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

The required line passes through the point (2,–1,–1), now we need to find the direction ratios of the line which are a,b,c . this equation of the required line is,

$$\frac{x-2}{a} = \frac{y+1}{b} = \frac{z+1}{c}$$

It is given that a line is parallel to the required line and has the Cartesian equation 6x–2 = 3y+1 = 2z–2, which can be further solved to it’s generalized form, which is $\frac{6x-2}{6} = \frac{3y+1}{6} = \frac{2z-2}{6}$, $\frac{x-\frac{1}{3}}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-\frac{1}{3}}{3}$

So we get the direction ratios as, $\frac{x-\frac{1}{3}}{1} = \frac{y+\frac{1}{2}}{2} = \frac{z-\frac{1}{3}}{3} = \lambda$

$a = 1\lambda,\ b = 2\lambda,\ c = 3\lambda$

as we know that two parallel lines have their direction ratios, suppose a line has direction ratios a,b,c and the line parallel to this line will have direction ratios ka,kb,kc .

putting these values in the required line equation, we get,

$$\frac{x-2}{1\lambda} = \frac{y+1}{2\lambda} = \frac{z+1}{3\lambda}$$

$$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+1}{3}$$

To convert this Cartesian form to the vector equation form, first equate the Cartesian form to a scalar,

$\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+1}{3}=\lambda$

Now equate all parts to this scalar individually,

$x-2 = \lambda$, $y+1 = 2\lambda$, $z+1 = 3\lambda$

$x = 2+\lambda$, $y = 2\lambda - 1$, $z = 3\lambda - 1$

we know that $\vec{r} = x\hat{\imath}+y\hat{\jmath}+z\hat{k} = \vec{a}+\lambda\vec{b}$

$x\hat{\imath}+y\hat{\jmath}+z\hat{k} = (2+\lambda)\hat{\imath} + (2\lambda - 1)\hat{\jmath} + (3\lambda - 1)\hat{k}$

$x\hat{\imath}+y\hat{\jmath}+z\hat{k} = (2\hat{\imath}-1\hat{\jmath}-1\hat{k}) + \lambda(1\hat{\imath}+2\hat{\jmath}+3\hat{k})$

21. Question

If the lines $\frac{x-1}{-3}=\frac{y-2}{2\lambda}=\frac{z-3}{2}$ and $\frac{x-1}{3\lambda}=\frac{y-1}{1}=\frac{z-6}{-5}$ are perpendicular, find the value of k.

Answer

We are given with the Cartesian equation of two lines and their direction ratios are in the form of some variable we need to find the value of this variable so that these lines are perpendicular to each other. We can use the dot product equation to solve this problem, which is,

$a_1a_2+b_1b_2+c_1c_2 = 0$

the equation of the lines are,

$$\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$$

$$\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$$

By using the direction ratios of these lines in the dot product equation, we get,

$(-3)3k + 2k(1) + 2(-5) = 0$

$-9k + 2k - 10 = 0$

$k = \frac{-10}{7}$

22. Question

If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), (–4, 3, –6) and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Answer

We need to find the angle between two lines AB and CD but we are not given with the equations this time and we are given with only the points through which these lines start and end.

The coordinates of A, B, C, D are (1,2,3), (4,5,7), (–4,3,–6), (2,9,2).

The direction ratios of line AB are (4–1,5–2,7–3) = (3,3,4)

The direction ratios of line CD are (2+4,9–3,2+6) = (6,6,8)

We can see that the fraction of the corresponding direction ratios will be a constant,

$\frac{3}{6}=\frac{3}{6}=\frac{4}{8}=\frac{1}{2}$ (constant)

Therefore, lines AB and CD are parallel to each other, so the angle between them can be 0^0 or 180^0 .

23. Question

Find the value of so that the following lines are perpendicular to each other.

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1}, \frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

Answer

We are given with the Cartesian equations of two lines and their direction ratios are in the form of variables, we need to find the value of this variable and give the complete equation, it is also given that the lines are perpendicular to each other.

The equations of the lines are,

$$\frac{x-5}{5\lambda+2}=\frac{2-y}{5}=\frac{1-z}{-1}$$

$$\frac{x}{1}=\frac{2y+1}{4\lambda}=\frac{1-z}{-3}$$

These are not in their standard form, so after converting them, we get

$$\frac{x-5}{5\lambda+2}=\frac{y-2}{-5}=\frac{z-1}{1}$$

$$\frac{x-0}{1}=\frac{y+\frac{1}{2}}{2\lambda}=\frac{z-1}{3}$$

Now as we know the direction ratios of both the lines, so by using the dot product equation, we get,

$a_1a_2 + b_1 b_2 + c_1c_2 = 0$

$(5\lambda + 2)1 + (-5)2\lambda + 1(3) = 0$

$5\lambda + 2 - 10\lambda + 3 = 0$

$-5\lambda + 5 = 0$

$\lambda = 1$

24. Question

Find the direction cosines of the line $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$.Also, find the vector equation of the line through the point A(–1, 2, 3) and parallel to the given line.

Answer

In this question we have to convert Cartesian equation to vector equation, we are given with the Cartesian equation of the line which is $\frac{x+2}{2}=\frac{2y-7}{6}=\frac{5-z}{6}$ as we can see that this line is not in the standard form, so after converting it we get,

$$\frac{x+2}{2}=\frac{y-\frac{7}{2}}{3}=\frac{z-5}{-6}$$

Now the direction ratios of this line is 2,3,–6

The direction cosines of the line are,

$l = \frac{2}{\sqrt{2^2+3^2+(-6)^2}}=\frac{2}{7}$

$m = \frac{3}{\sqrt{2^2+3^2+(-6)^2}}=\frac{3}{7}$

$n = \frac{-6}{\sqrt{2^2+3^2+(-6)^2}} = \frac{-6}{7}$

To convert this Cartesian form to the vector equation form, first equate the Cartesian form to a scalar,

$\frac{x+2}{2} = \frac{y-\frac{7}{2}}{3} = \frac{z-5}{-6} = \lambda$

Now equate all parts to this scalar individually,

$x+2 = 2\lambda$, $y - \frac{7}{2} = 3\lambda$, $z-5 = -6\lambda$

$x = 2\lambda-2$, $y = 3\lambda+\frac{7}{2}$, $z = 5-6\lambda$

we know that $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k} = \vec{a}+\lambda\vec{b}$

$x\hat{i}+y\hat{j}+z\hat{k} = (2\lambda-2)\hat{i} + (3\lambda+\frac{7}{2})\hat{j} + (-6\lambda+5)\hat{k}$

$x\hat{i}+y\hat{j}+z\hat{k} = (-2\hat{i}+\frac{7}{2}\hat{j}+5\hat{k}) + \lambda(2\hat{i}+3\hat{j}-6\hat{k})$

Therefore, the vector equation of the line is the mentioned above.

Exercise 28.3

1. Question

Show that the lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect and find their point of intersection.

Answer

Given: - Two lines equation: $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$

To find: - Intersection point

We have,

$\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3} = \lambda \text{ (let)}$

$\Rightarrow x = \lambda$, $y = 2\lambda + 2$ and $z = 3\lambda - 3$

So, the coordinates of a general point on this line are

$(\lambda, 2\lambda + 2, 3\lambda - 3)$

The equation of the 2nd line is

$\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4} = \mu \text{ (let)}$

$\Rightarrow x = 2\mu + 2$, $y = 3\mu + 6$ and $z = 4\mu + 3$

So, the coordinates of a general point on this line are

$(2\mu + 2, 3\mu + 6, 4\mu + 3)$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

$\Rightarrow \lambda = 2\mu + 2$, $2\lambda + 2 = 3\mu + 6$, and $3\lambda - 3 = 4\mu + 3$

$\Rightarrow \lambda = 2\mu + 2$ ......(i)

$\Rightarrow 2\lambda - 3\mu = 4$ ......(ii) and $3\lambda - 4\mu = 6$ ......(iii)

putting value of λ from eq i in eq ii, we get

$\Rightarrow 2(2\mu + 2) - 3\mu = 4$

$\Rightarrow 4\mu + 4 - 3\mu = 4$

$\Rightarrow \mu = 0$

Now putting value of μ in eq i, we get

$\Rightarrow \lambda = 2\mu + 2$

$\Rightarrow \lambda = 2(0) + 2$

$\Rightarrow \lambda = 2$

As we can see by putting value of λ and μ in eq iii, that it satisfy the equation.

Check

$\Rightarrow 3\lambda - 4\mu = 6$

$\Rightarrow 3(2) = 6$;Hence intersection point exist or line do intersects

We can find intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

$\lambda, 2\lambda + 2, 3\lambda - 3$

$\Rightarrow 2, 6, 3$

2. Question

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$ do not intersect.

Answer

Given: - Two lines equation: $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$

To find: - Intersection point

We have,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \text{ (let)}$$

$\Rightarrow x = 3\lambda + 1, y = 2\lambda - 1$ and $z = 5\lambda + 1$

So, the coordinates of a general point on this line are

$(3\lambda + 1, 2\lambda - 1, 5\lambda + 1)$

The equation of the 2nd line is

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \text{ (let)}$$

$\Rightarrow x = 4\mu - 2, y = 3\mu + 1$ and $z = -2\mu - 1$

So, the coordinates of a general point on this line are

$(4\mu - 2, 3\mu + 1, -2\mu - 1)$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

$\Rightarrow 3\lambda + 1 = 4\mu - 2$, $2\lambda - 1 = 3\mu + 1$, and $5\lambda + 1 = -2\mu - 1$

$\Rightarrow \lambda = \frac{4\mu - 3}{3}$ ......(i)

$\Rightarrow 2\lambda - 3\mu = 2$ ......(ii)

and $5\lambda + 2\mu = -2$ ......(iii)

putting value of λ from eq i in eq ii, we get

$$\Rightarrow 2\left(\frac{4\mu - 3}{3}\right) - 3\mu = 2$$

$$\Rightarrow \frac{8\mu - 6 - 9\mu}{3} = 2$$

$$\Rightarrow 8\mu - 6 - 9\mu = 6$$

$\Rightarrow -\mu = 12$

$\Rightarrow \mu = -12$

Now putting value of μ in eq i, we get

$\Rightarrow \lambda = \frac{4\mu - 3}{3}$

$\Rightarrow \lambda = \frac{4(-12) - 3}{3}$

$\Rightarrow \lambda = -17$

As we can see by putting value of λ and μ in eq iii, that it does not satisfy the equation.

Check

LHS

$= 5\lambda + 2\mu$

$= 5(-17) + 2(-12)$

$= -85 - 24$

$= -109$

$\neq$ RHS

Hence intersection point does not exist or line do not intersects

3. Question

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Find their point of intersection.

Answer

Given: - Two lines equation: $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$

To find: - Intersection point

We have,

$$\frac{x + 1}{3} = \frac{y + 3}{5} = \frac{z + 5}{7} = \lambda \text{ (let)}$$

$\Rightarrow x = 3\lambda - 1$, $y = 5\lambda - 3$ and $z = 7\lambda - 5$

So, the coordinates of a general point on this line are

(3λ – 1, 5λ – 3, 7λ – 5)

The equation of the 2nd line is

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \text{ (let)}$$

⇒ x = μ + 2, y = 3μ + 4 and z = 5μ + 6

So, the coordinates of a general point on this line are

(μ + 2, 3μ + 4, 5μ + 6)

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

⇒ 3λ – 1 = μ + 2 , 5λ – 3 = 3μ + 4, and 7λ – 5 = 5μ + 6

$\Rightarrow \lambda = \frac{\mu+3}{3}$ ......(i)

⇒ 5λ – 3μ = 7 ......(ii)

and 7λ – 5μ = 11 ......(iii)

putting value of λ from eq i in eq ii, we get

$$\Rightarrow 5\left(\frac{\mu+3}{3}\right) - 3\mu = 7$$

$$\Rightarrow \frac{5\mu + 15 - 9\mu}{3} = 7$$

$$\Rightarrow 5\mu + 15 - 9\mu = 21$$

$$\Rightarrow -4\mu = 6$$

$$\Rightarrow \mu = -\frac{3}{2}$$

Now putting value of μ in eq i, we get

$$\Rightarrow \lambda = \frac{-\frac{3}{2}+3}{3}$$

$$\Rightarrow \lambda = \frac{-3+6}{6}$$

$$\Rightarrow \lambda = \frac{1}{2}$$

As we can see by putting the value of λ and μ in eq iii, that it satisfy the equation.

Check

⇒ 7λ – 5μ = 11

$$\Rightarrow 7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\Rightarrow \frac{7+15}{2} = 11$$

⇒ 11 = 11

⇒ LHS = RHS ; Hence intersection point exists or line do intersects

We can find an intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

$3\lambda - 1, 5\lambda - 3, 7\lambda - 5$

$$3\left(\frac{1}{2}\right) - 1, 5\left(\frac{1}{2}\right) - 3, 7\left(\frac{1}{2}\right) - 5$$

$$\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$$

4. Question

Prove that the lines through A(0, – 1, – 1) and B(4, 5, 1) intersects the line through C(3, 9, 4) and D(– 4, 4, 4). Also, find their point of intersection.

Answer

Given: - Line joining A(0, – 1, – 1) and B(4, 5, 1).

Line joining C(3, 9, 4) and D(– 4, 4, 4).

To Prove: - Both lines intersects

Proof: - Equation of a line joined by two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Now equation of line joining A(0, – 1, – 1) and B(4, 5, 1)

$$= \frac{x - 0}{4 - 0} = \frac{y + 1}{5 + 1} = \frac{z + 1}{1 + 1}$$

$$= \frac{x}{4} = \frac{y+1}{6} = \frac{z+1}{2} = \lambda \text{ (let)}$$

$\Rightarrow x = 4\lambda, y = 6\lambda - 1$ and $z = 2\lambda - 1$

So, the coordinates of a general point on this line are

$(4\lambda, 6\lambda - 1, 2\lambda - 1)$

And equation of line joining C(3, 9, 4) and D(– 4, 4, 4)

$$= \frac{x - 3}{-4 - 3} = \frac{y - 4}{4 - 9} = \frac{z - 4}{4 - 4}$$

$$= \frac{x-3}{-7} = \frac{y-9}{-5} = \frac{z-4}{0} = \mu \text{ (let)}$$

$\Rightarrow x = -7\mu + 3, y = -5\mu + 9$ and $z = 4$

So, the coordinates of a general point on this line are

$(-7\mu + 3, -5\mu + 9, 4)$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

$\Rightarrow 4\lambda = -7\mu + 3$, $6\lambda - 1 = -5\mu + 9$, and $2\lambda - 1 = 4$

$\Rightarrow \lambda = \frac{-7\mu+3}{4}$ ......(i)

$\Rightarrow 6\lambda + 5\mu = 10$ ......(ii)

and $2\lambda = 5$ ......(iii)

from eq iii, we get

$\Rightarrow \lambda = \frac{5}{2}$

Now putting the value of λ in eq i, we get

$\Rightarrow \frac{5}{2} = \frac{-7\mu + 3}{4}$

$\Rightarrow \mu = -1$

As we can see by putting the value of λ and μ in eq ii, that it satisfy the equation.

Check

⇒ 6λ + 5μ = 10

$\Rightarrow 6\left(\frac{5}{2}\right) + 5(-1) = 10$

$\Rightarrow \frac{30-10}{2} = 10$

⇒ 10 = 10

⇒ LHS = RHS ;Hence intersection point exist or line do intersects

We can find intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

– 7μ + 3, – 5μ + 9, 4

– 7(– 1) + 3, – 5(– 1) + 9, 4

10, 14, 4

5. Question

Prove that the line $\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$ and $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$ intersect and find their point of intersection.

Answer

Given: – Two lines having vector notion $\vec{r} = \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \lambda(3\hat{\imath} - \hat{\jmath})$ and $\vec{r} = \left(4\hat{\imath} - \hat{k}\right) + \mu(2\hat{\imath} + 3\hat{k})$

The position vectors of arbitrary points on the given lines are

1^{st} line

$= \left(\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \lambda(3\hat{\imath} - \hat{\jmath})$

$= (3\lambda + 1)\hat{\imath} + (1 - \lambda)\hat{\jmath} - \hat{k}$

2^{nd} line

$= \left(4\hat{\imath} - \hat{k}\right) + \mu(2\hat{\imath} + 3\hat{k})$

$= (2\mu + 4)\hat{\imath} + (3\mu - 1)\hat{k}$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

⇒ (3λ + 1) = 2μ + 4, 1 – λ = 0, – 1 = 3μ – 1

⇒ 3λ – 2μ = 3 ......(i)

⇒ λ = 1 ......(ii)

and μ = 0 ......(iii)

from eq ii and eq iii we get

⇒ λ = 1 and μ = 0

As we can see by putting the value of λ and μ in eq i, that it satisfy the equation.

Check

⇒ 3λ – 2μ = 3

⇒ 3(1) – 2(0) = 3

⇒ 3 = 3

⇒ LHS = RHS ; Hence intersection point exists or line do intersect

We can find an intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

$\Rightarrow \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$

$\Rightarrow \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + (3\hat{i} - \hat{j})$

$\Rightarrow \vec{r} = 4\hat{i} - \hat{k}$

Hence, Intersection point is (4,0, – 1)

6 A. Question

Determine whether the following pair of lines intersect or not :

$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

Answer

Given: - Two lines having vector notion $\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

The position vectors of arbitrary points on the given lines are

1^{st} line

$= (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$

$= (2\lambda + 1)\hat{i} - \hat{j} + \lambda\hat{k}$

2^{nd} line

$= (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

$= (\mu + 2)\hat{i} + (\mu - 1)\hat{j} - \mu\hat{k}$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

⇒ (2λ + 1) = μ + 2, – 1 = μ – 1, λ = – μ

⇒ 2λ – μ = 1 ......(i)

⇒ μ = 0 ......(ii)

and λ = μ ......(iii)

from eq ii and eq iii we get

⇒ λ = 0 and μ = 0

As we can see by putting the value of λ and μ in eq i, that it does not satisfy the equation.

Check

⇒ 2λ – μ = 1

⇒ 2(0) – (0) = 1

⇒ 0 = 1

⇒ LHS≠RHS ;Hence intersection point exist or line does not intersects

6 B. Question

Determine whether the following pair of lines intersect or not :

$\frac{x-1}{2}=\frac{y+1}{3}=z$ and $\frac{x+1}{5}=\frac{y-2}{1}$; z = 2

Answer

Given: - Two lines equation: $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x-1}{5} = \frac{y-2}{1}$; z = 3

We have,

$$\frac{x-1}{2} = \frac{y\ +1}{3} = z = \lambda\,(\text{let})$$

⇒ x = 2λ + 1, y = 3λ – 1 and z = λ

So, the coordinates of a general point on this line are

(2λ + 1, 3λ – 1, λ)

The equation of the 2nd line is

$$\frac{x-1}{5} = \frac{y-2}{1} = \mu\,(\text{let}), z = 3$$

⇒ x = 5μ + 1, y = μ + 2 and z = 3

So, the coordinates of a general point on this line are

(5μ + 1, μ + 2, 3)

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

⇒ 2λ + 1 = 5μ + 1 , 3λ – 1 = μ + 2, and λ = 3

⇒ 2λ – 5μ = 0 ……(i)

⇒ 3λ – μ = 3 ……(ii)

and λ = 3 ……(iii)

putting the value of λ from eq iii in eq ii, we get

⇒ 3λ – μ = 3

⇒ 3(3) – μ = 3

⇒ μ = 6

As we can see by putting the value of λ and μ in eq i, that it does not satisfy the equation.

Check

⇒ 2λ – 5μ = 0

⇒ 2(3) – 5(6) = 0

⇒ – 24 = 0

⇒ LHS≠RHS; Hence intersection point exists or line does not intersects.

6 C. Question

Determine whether the following pair of lines intersect or not :

$$\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0} \text{ and } \frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}.$$

Answer

Given: - Two lines equation: $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}$

We have,

$$\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}=\lambda\,(\text{let})$$

⇒ x = 3λ + 1, y = – λ + 1 and z = – 1

So, the coordinates of a general point on this line are

(3λ + 1, – λ + 1, – 1)

The equation of the 2^{nd} line is

$$\frac{x-4}{2}=\frac{y-0}{0}=\frac{z+1}{3}=\mu\,(\text{let})$$

⇒ x = 2μ + 4, y = 0 and z = 3μ – 1

So, the coordinates of a general point on this line are

(2μ + 4, 0, 3μ – 1)

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

⇒ 3λ + 1 = 2μ + 4 , – λ + 1 = 0, and – 1 = 3μ – 1

⇒ 3λ – 2μ = 3 ……(i)

⇒ λ = 1 ……(ii)

and μ = 0 ……(iii)

from eq ii and eq iii, we get

⇒ λ = 1

and μ = 0

As we can see by putting the value of λ and μ in eq i, that it satisfy the equation.

Check

⇒ 3λ – 2μ = 3

⇒ 3(1) = 3

⇒ 3 = 3

⇒ LHS = RHS ;Hence intersection point exist or line do intersects

We can find intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

$2\mu + 4, 0, 3\mu - 1$

$4, 0, -1$

6 D. Question

Determine whether the following pair of lines intersect or not :

$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$.

Answer

Given: - Two lines equation: $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$

To find: - Intersection point

We have,

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} = \lambda\,(\text{let})$$

$\Rightarrow x = 4\lambda + 5, y = 4\lambda + 7$ and $z = -5\lambda - 3$

So, the coordinates of a general point on this line are

$(4\lambda + 5, 4\lambda + 7, -5\lambda - 3)$

The equation of the 2nd line is

$$\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} = \mu\,(\text{let})$$

$\Rightarrow x = 7\mu + 8, y = \mu + 4$ and $z = 3\mu + 5$

So, the coordinates of a general point on this line are

$(7\mu + 8, \mu + 4, 3\mu + 5)$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

$\Rightarrow 4\lambda + 5 = 7\mu + 8$, $4\lambda + 7 = \mu + 4$, and $-5\lambda - 3 = 3\mu + 5$

$\Rightarrow 4\lambda - 7\mu = 3$ ......(i)

$\Rightarrow \mu = 4\lambda + 3$ ......(ii)

and $-5\lambda - 3\mu = 8$ ......(iii)

putting the value of μ from eq ii in eq i, we get

$\Rightarrow 4\lambda - 7\mu = 3$

$\Rightarrow 4\lambda - 7(4\lambda + 3) = 3$

$\Rightarrow 4\lambda - 28\lambda - 21 = 3$

$\Rightarrow -24\lambda = 24$

$\Rightarrow \lambda = -1$

Now putting the value of λ in eq ii, we get

$\Rightarrow \mu = 4\lambda + 3$

$\Rightarrow \mu = 4(-1) + 3$

$\Rightarrow \mu = -1$

As we can see by putting the value of λ and μ in eq iii, that it satisfy the equation.

Check

⇒ – 5λ – 3μ = 8

⇒ – 5(– 1) – 3(– 1) = 8

⇒ 5 + 3 = 8

⇒ 8 = 8

⇒ LHS = RHS ;Hence intersection point exist or line do intersects

We can find intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

4λ + 5, 4λ + 7, – 5λ – 3

4(– 1) + 5, 4(– 1) + 7, – 5(– 1) – 3

1, 3, 2

7. Question

Show that the lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ are intersecting. Hence, find their point of intersection.

Answer

Given: – Two lines having vector notion $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

To show: - Lines are intersecting

The position vectors of arbitrary points on the given lines are

1st line

$= 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$= (\lambda + 3)\hat{i} + (2\lambda + 2)\hat{j} + (2\lambda - 4)\hat{k}$

2nd line

$= 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

$= (3\mu + 5)\hat{i} + (2\mu - 2)\hat{j} + 6\mu\hat{k}$

If the lines intersect, then they must have a common point.

Therefore for some value of λ and μ, we have

⇒ (λ + 3) = 3μ + 5, 2λ + 2 = 2μ – 2, 2λ – 4 = 6μ

⇒ λ = 3μ + 2 ...... (i)

⇒ λ – μ = – 2 ......(ii)

and λ – 3μ = 2 ......(iii)

putting value of λ from eq i in eq ii, we get

⇒ λ – μ = – 2

⇒ 3μ + 2 – μ = – 2

⇒ 2μ = – 4

⇒ $\mu = -2$

Now putting value of μ in eq i, we get

⇒ $\lambda = 3\mu + 2$

⇒ $\lambda = 3(-2) + 2$

⇒ $\lambda = -6 + 2$

⇒ $\lambda = -4$

As we can see by putting value of λ and μ in eq iii, that it satisfy the equation.

Check

⇒ $\lambda - 3\mu = 2$

⇒ $-4 - 3(-2) = 2$

⇒ $-4 + 6 = 2$

⇒ $2 = 2$

⇒ LHS = RHS ; Hence intersection point exists or line do intersect

We can find an intersecting point by putting values of μ or λ in any one general point equation

Thus,

Intersection point

⇒ $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

⇒ $\vec{r} = 5\hat{i} - 2\hat{j} + (-2)(3\hat{i} + 2\hat{j} + 6\hat{k})$

⇒ $\vec{r} = -\hat{i} - 6\hat{j} - 12\hat{k}$

Hence, Intersection point is (– 1, – 6, – 12)

Exercise 28.4

1. Question

Find the perpendicular distance of the point (3, –1, 11) from the line $\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4}$.

Answer

Given: - Point P(3, – 1, 11) and the equation of the line $\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4}$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

Thus to find Distance PQ we have to first find coordinates of Q

$$\frac{x}{2} = \frac{y-2}{-3} = \frac{z-3}{4} = \lambda \text{ (let)}$$

⇒ $x = 2\lambda,\ y = -3\lambda + 2,\ z = 4\lambda + 3$

Therefore, coordinates of Q(2λ, $-3\lambda + 2$,$4\lambda + 3$)

Now as we know (TIP) ‘if two points A(x_1,y_1,z_1) and B(x_2,y_2,z_2) on a line, then its direction ratios are proportional to ($x_2 - x_1$,$y_2 - y_1$,$z_2 - z_1$)’

Hence

Direction ratio of PQ is

= (2λ – 3), (– 3λ + 2 + 1), (4λ + 3 – 11)

= (2λ – 3), (– 3λ + 1), (4λ – 8)

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

= (2, – 3,4)

Since PQ is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

⇒ 2(2λ – 3) + (– 3)(– 3λ + 3) + 4(4λ – 8) = 0

⇒ 4λ – 6 + 9λ – 9 + 16λ – 32 = 0

⇒ 29λ – 47 = 0

$$\Rightarrow \lambda = \frac{47}{29}$$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$$= Q\,(2(\frac{47}{29}), -3(\frac{47}{29}) + 2, 4(\frac{47}{29}) + 3)$$

$$= Q\,(\frac{94}{29}, \frac{-83}{29}, \frac{275}{29})$$

Now,

Distance between PQ

Tip: - Distance between two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{\left(\frac{94}{29} - 3\right)^2 + \left(\frac{-83}{29} + 1\right)^2 + \left(\frac{275}{29} - 11\right)^2}$$

$$= \sqrt{\left(\frac{94 - 87}{29}\right)^2 + \left(\frac{-83 + 29}{29}\right)^2 + \left(\frac{275 - 319}{29}\right)^2}$$

$$= \sqrt{\left(\frac{7}{29}\right)^2 + \left(\frac{-54}{29}\right)^2 + \left(\frac{-44}{29}\right)^2}$$

$$= \sqrt{\frac{49}{841} + \frac{2916}{841} + \frac{1936}{841}}$$

$= \sqrt{\frac{4901}{841}}$ unit

2. Question

Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

Answer

Given: - Point P(1, 0, 0) and equation of line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

Thus to find Distance PQ we have to first find coordinates of Q

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda\ (\text{let})$$

⇒ x = 2λ + 1, y = – 3λ – 1, z = 8λ – 10

Therefore, coordinates of Q(2λ + 1, – 3λ – 1,8λ – 10)

Now as we know (TIP) 'if two points A(x_1,y_1,z_1) and B(x_2,y_2,z_2) on a line, then its direction ratios are proportional to ($x_2 - x_1$,$y_2 - y_1$,$z_2 - z_1$)'

Hence

Direction ratio of PQ is

= (2λ + 1 – 1), (– 3λ – 1 – 0), (8λ – 10 – 0)

= (2λ), (– 3λ – 1), (8λ – 10)

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

= (2, – 3,8)

Since PQ is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

⇒ 2(2λ) + (– 3)(– 3λ – 1) + 8(8λ – 10) = 0

⇒ 4λ + 9λ + 3 + 64λ – 80 = 0

⇒ 77λ – 77 = 0

⇒ λ = 1

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$$= Q\,(2(1) + 1, -3(1) - 1, 8(1) - 10)$$

$$= Q\,(3, -4, -2)$$

Now,

Distance between PQ

Tip: – Distance between two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$= \sqrt{(1-3)^2 + (0+4)^2 + (-2-0)^2}$$

$$= \sqrt{(-2)^2 + (4)^2 + (-2)^2}$$

$$= \sqrt{4 + 16 + 4}$$

$$= \sqrt{24}$$

= 2√6 unit

3. Question

Find the foot of the perpendicular drawn from the point A(1, 0, 3) to the joint of the points B(4, 7, 1) and C(3, 5, 3).

Answer

Given: - Perpendicular from A(1, 0, 3) drawn at line joining points B(4, 7, 1) and C(3, 5, 3)

Let D be the foot of the perpendicular drawn from A(1, 0, 3) to line joining points B(4, 7, 1) and C(3, 5, 3).

Now let's find the equation of the line which is formed by joining points B(4, 7, 1) and C(3, 5, 3)

Tip: - Equation of a line joined by two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$= \frac{x-4}{3-4} = \frac{y-7}{5-7} = \frac{z-1}{3-1}$$

$$= \frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2}$$

Now

$$= \frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2} = \lambda \text{ (let)}$$

Therefore,

⇒ $x = -\lambda + 4$, $y = -2\lambda + 7$, $z = 2\lambda + 1$

Therefore, coordinates of $D(-\lambda + 4, -2\lambda + 7, 2\lambda + 1)$

Now as we know (TIP) ‘if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$’

Hence

Direction Ratios of AD

$= (-\lambda + 4 - 1), (-2\lambda + 7 - 0), (2\lambda - 2)$

$= (-\lambda + 3), (-2\lambda + 7), (2\lambda - 2)$

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

= (– 1, – 2,2)

Since AD is perpendicular to given line, therefore by "condition of perpendicularity"

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

⇒ – 1(– λ + 3) + (– 2)(– 2λ + 7) + 2(2λ – 2) = 0

⇒ λ – 3 + 4λ – 14 + 4λ – 4 = 0

⇒ 9λ – 21 = 0

$$\Rightarrow \lambda = \frac{21}{9}$$

$$\Rightarrow \lambda = \frac{7}{3}$$

Therefore coordinates of D

i.e Foot of perpendicular

By putting value of λ in D coordinate equation, we get

$$= D(-\lambda + 4, -2\lambda + 7, 2\lambda + 1)$$

$$= D(-\frac{7}{3} + 4, -2(\frac{7}{3}) + 7, 2(\frac{7}{3}) + 1)$$

$$= D(\frac{5}{3}, \frac{7}{3}, \frac{17}{3})$$

4. Question

A (1, 0, 4), B(0, – 11, 3), C(2, – 3, 1) are three points, and D is the foot of the perpendicular from A on BC. Find the coordinates of D.

Answer

Given: - Perpendicular from A(1, 0, 4) drawn at line joining points B(0, – 11, 3) and C(2, – 3, 1)

and D be the foot of the perpendicular drawn from A(1, 0, 4) to line joining points B(0, – 11, 3) and C(2, – 3, 1).

Now let's find the equation of the line which is formed by joining points B(0, – 11, 3) and C(2, – 3, 1)

Tip: - Equation of a line joined by two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$= \frac{x - 0}{2 - 0} = \frac{y + 11}{-3 + 11} = \frac{z - 3}{1 - 3}$$

$$= \frac{x}{2} = \frac{y + 11}{8} = \frac{z - 3}{-2}$$

Now

$$= \frac{x}{2} = \frac{y + 11}{8} = \frac{z - 3}{-2} = \lambda \text{ (let)}$$

Therefore,

$\Rightarrow x = 2\lambda,\ y = 8\lambda - 11,\ z = -2\lambda + 3$

Therefore, coordinates of $D(2\lambda, 8\lambda - 11, -2\lambda + 3)$

Now as we know (TIP) 'if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$'

Hence

Direction Ratios of AD

$= (2\lambda - 1), (8\lambda - 11 - 0), (-2\lambda + 3 - 4)$

$= (2\lambda - 1), (8\lambda - 11), (-2\lambda - 1)$

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (2, 8, -2)$

Since the AD is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow 2(2\lambda - 1) + (8)(8\lambda - 11) - 2(-2\lambda - 1) = 0$

$\Rightarrow 4\lambda - 2 + 64\lambda - 88 + 4\lambda + 2 = 0$

$\Rightarrow 72\lambda - 88 = 0$

$$\Rightarrow \lambda = \frac{88}{72}$$

$$\Rightarrow \lambda = \frac{11}{9}$$

Therefore coordinates of D

i.e. Foot of perpendicular

By putting the value of λ in D coordinate equation, we get

$$= D(2\lambda, 8\lambda - 11, -2\lambda + 3)$$

$$= D(2\left(\frac{11}{9}\right), 8\left(\frac{11}{9}\right) - 11, -2(\frac{11}{9}) + 3)$$

$$= D(\frac{22}{9}, -\frac{11}{9}, \frac{5}{9})$$

5. Question

Find the foot of perpendicular from the point (2, 3, 4) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line.

Answer

Given: - Point P(2, 3, 4) and the equation of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

Thus to find Distance PQ we have to first find coordinates of Q

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}=\lambda\ (\text{let})$$

⇒ $x = 4 - 2\lambda$, $y = 6\lambda$, $z = 1 - 3\lambda$

Therefore, coordinates of Q$(- 2\lambda + 4, 6\lambda, - 3\lambda + 1)$

Now as we know (TIP) 'if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$'

Hence

Direction ratio of PQ is

$= (- 2\lambda + 4 - 2), (6\lambda - 3), (- 3\lambda + 1 - 4)$

$= (- 2\lambda + 2), (6\lambda - 3), (- 3\lambda - 3)$

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (- 2, 6, - 3)$

Since PQ is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

⇒ $- 2(- 2\lambda + 2) + (6)(6\lambda - 3) - 3(- 3\lambda - 3) = 0$

⇒ $4\lambda - 4 + 36\lambda - 18 + 9\lambda + 9 = 0$

⇒ $49\lambda - 13 = 0$

$$\Rightarrow \lambda = \frac{13}{49}$$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$$= Q\left(-2\left(\frac{13}{49}\right) + 4, 6\left(\frac{13}{49}\right), -3\left(\frac{13}{49}\right) + 1\right)$$

$$= Q\left(\frac{170}{49}, \frac{78}{49}, \frac{10}{49}\right)$$

Now,

Distance between PQ

Tip: - Distance between two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

$$= \sqrt{\left(\frac{170}{49}-2\right)^2 + \left(\frac{78}{49}-3\right)^2 + \left(\frac{10}{49}-4\right)^2}$$

$= \sqrt{\left(\frac{72}{49}\right)^2 + \left(\frac{69}{49}\right)^2 + \left(-\frac{168}{49}\right)^2}$

$= \sqrt{\frac{5184}{2401} + \frac{4761}{2401} + \frac{34596}{2401}}$

$= \sqrt{\frac{44541}{2401}}$

$= \sqrt{\frac{909}{49}}$

$= \frac{3}{7}\sqrt{101}$ units

6. Question

Find the equation of the perpendicular drawn from the point P(2, 4, –1) to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}.$$

Also, write down the coordinates of the foot of the perpendicular from P.

Answer

Given: - Point P(2, 4, – 1) and equation of line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

Thus to find Distance PQ we have to first find coordinates of Q

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda \text{ (let)}$$

⇒ x = λ – 5, y = 4λ – 3, z = – 9λ + 6

Therefore, coordinates of Q(λ – 5,4λ – 3, – 9λ + 6)

Now as we know (TIP) ‘if two points A(x_1,y_1,z_1) and B(x_2,y_2,z_2) on a line, then its direction ratios are proportional to (x_2 – x_1,y_2 – y_1,z_2 – z_1)’

Hence

Direction ratio of PQ is

= (λ – 5 – 2), (4λ – 3 – 4), (– 9λ + 6 + 1)

= (λ – 7), (4λ – 7), (– 9λ + 7)

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

= (1,4, – 9)

Since PQ is perpendicular to given line, therefore by “condition of perpendicularity.”

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow 1(\lambda - 7) + (4)(4\lambda - 7) - 9(-9\lambda + 7) = 0$

$\Rightarrow \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 = 0$

$\Rightarrow 98\lambda - 98 = 0$

$\Rightarrow \lambda = 1$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$= Q\,((1) - 5, 4(1) - 3, -9(1) + 6)$

$= Q\,(-4, 1, -3)$

Now,

So, Equation of perpendicular PQ is

Tip: - Equation of a line joined by two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$= \frac{x - 2}{-4 - 2} = \frac{y - 4}{1 - 4} = \frac{z + 1}{-3 + 1}$$

$$= \frac{x - 2}{-6} = \frac{y - 4}{-3} = \frac{z + 1}{-2}$$

7. Question

Find the length of the perpendicular drawn from the point (5, 4, –1) to the line $\vec{r} = \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k})$.

Answer

Given: - Point (5, 4, – 1) and equation of line $\hat{r} = \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k})$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

As we know position vector is given by

$\hat{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Therefore,

Position vector of point P is

$5\hat{i} + 4\hat{j} - 1\hat{k}$

and, from a given line, we get

$\Rightarrow \hat{r} = \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k})$

$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k})$

$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = (1 + 2\lambda)\hat{i} + 9\lambda\hat{j} + 5\lambda\hat{k}$

On comparing both sides we get,

$\Rightarrow x = 1 + 2\lambda,\ y = 9\lambda,\ z = 5\lambda$

$\Rightarrow \frac{x-1}{2} = \frac{y}{9} = \frac{z}{5} = \lambda$; Equation of line

Thus, coordinates of Q i.e. General point on the given line

$\Rightarrow Q((1 + 2\lambda), 9\lambda, 5\lambda)$

Now as we know (TIP) 'if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$'

Hence

Direction ratio of PQ is

$= (2\lambda + 1 - 5), (9\lambda - 4), (5\lambda + 1)$

$= (2\lambda - 4), (9\lambda - 4), (5\lambda + 1)$

and by comparing with line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (2,9,5)$

Since PQ is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow 2(2\lambda - 4) + (9)(9\lambda - 4) + 5(5\lambda + 1) = 0$

$\Rightarrow 4\lambda - 8 + 81\lambda - 36 + 25\lambda + 5 = 0$

$\Rightarrow 110\lambda - 39 = 0$

$$\Rightarrow \lambda = \frac{39}{110}$$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$$= Q\,(2\left(\frac{39}{110}\right) + 1, 9\left(\frac{39}{110}\right), 5(\frac{39}{110}))$$

$$= Q\,(\frac{188}{110}, \frac{351}{110}, \frac{195}{110})$$

Now,

Distance between PQ

Tip: - Distance between two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{\left(\frac{188}{110} - 5\right)^2 + \left(\frac{351}{110} - 4\right)^2 + \left(\frac{195}{110} + 1\right)^2}$$

$$= \sqrt{\left(-\frac{362}{110}\right)^2 + \left(-\frac{89}{110}\right)^2 + \left(\frac{305}{110}\right)^2}$$

$$= \sqrt{\frac{131044}{12100} + \frac{7921}{12100} + \frac{93025}{12100}}$$

$$= \sqrt{\frac{231990}{12100}}$$

$$= \sqrt{\frac{2109}{110}}$$

$= \sqrt{\frac{2109}{110}}$ units

8. Question

Find the foot of the perpendicular drawn from the point $\hat{i} + 6\hat{j} + 3\hat{k}$ to the line $\vec{r} = \hat{j} + 2\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$. Also, find the length of the perpendicular.

Answer

Given: - Point with position vector $\hat{i} + 6\hat{j} + 3\hat{k}$ and equation of line $\hat{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

Let, PQ be the perpendicular drawn from P $(\hat{i} + 6\hat{j} + 3\hat{k})$ to given line whose endpoint/ foot is Q point.

Q is on line

$\hat{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}$ is the position vector of Q

Hence,

$\overrightarrow{PQ} = \text{position vector of Q} - \text{position vector of P}$

$\Rightarrow \overrightarrow{PQ} = \left(\lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}\right) - (\hat{i} + 6\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{PQ} = \lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k} - \hat{i} - 6\hat{j} - 3\hat{k}$

$\Rightarrow \overrightarrow{PQ} = (\lambda - 1)\hat{i} + (1 + 2\lambda - 6)\hat{j} + (3\lambda - 3 + 2)\hat{k}$

$\Rightarrow \overrightarrow{PQ} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$

Since, PQ is perpendicular on line $\hat{r} = \hat{j} + 2\hat{k} + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

Therefore, their Dot product is zero

Compare given line equation with $\hat{r} = \hat{a} + \lambda\hat{b}$

$\Rightarrow \overrightarrow{PQ}.\hat{b} = 0$

$\Rightarrow \{(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}\}.\{\hat{i} + 2\hat{j} + 3\hat{k}\} = 0$

$\Rightarrow (\lambda - 1)(1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$

$\Rightarrow 14\lambda - 14 = 0$

$\Rightarrow \lambda = 1$

Hence Position vector of Q by putting the value of λ

$\lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k}$

$\Rightarrow \hat{i} + (2 + 1)\hat{j} + (3 + 2)\hat{k}$

$\Rightarrow \hat{i} + 3\hat{j} + 5\hat{k}$; Foot of perpendicular

and Distance PQ, putting the value of λ in PQ vector equation, we get

$\Rightarrow \overrightarrow{PQ} = (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}$

$\Rightarrow \overrightarrow{PQ} = (1 - 1)\hat{i} + (2 - 5)\hat{j} + (3 - 1)\hat{k}$

$\Rightarrow \overrightarrow{PQ} = -3\hat{j} + 2\hat{k}$

Now, Magnitude of PQ

We know that,

$|AB| = \sqrt{x^2 + y^2 + z^2}$; where x,y,z are coefficient of vector

Hence

$\Rightarrow |PQ| = \sqrt{0^2 + (-3)^2 + 2^2}$

$\Rightarrow |PQ| = \sqrt{9 + 4}$

$\Rightarrow |PQ| = \sqrt{13}$ units

9. Question

Find the equation of the perpendicular drawn from the point P(–1, 3, 2) to the line $\vec{r} = \left(2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$. Also, find the coordinates of the foot of the perpendicular from P.

Answer

Given: -

Point P(– 1, 3, 2) and equation of line $\hat{r} = (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

As we know position vector is given by

$\hat{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Therefore,

Position vector of point P is

$-\hat{i} + 3\hat{j} + 2\hat{k}$

and, from a given line, we get

$\Rightarrow \hat{r} = (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = (2\lambda)\hat{i} + (\lambda + 2)\hat{j} + (3\lambda + 3)\hat{k}$

On comparing both sides we get,

$\Rightarrow x = 2\lambda,\ y = \lambda + 2,\ z = 3\lambda + 3$

$\Rightarrow \frac{x}{2} = \frac{y-2}{1} = \frac{z-3}{3} = \lambda$; Equation of line

Thus, coordinates of Q i.e. General point on the given line

$\Rightarrow Q(2\lambda, (\lambda + 2), (3\lambda + 3))$

Now as we know (TIP) 'if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$'

Hence

Direction ratio of PQ is

$= (2\lambda + 1), (\lambda + 2 - 3), (3\lambda + 3 - 2)$

$= (2\lambda + 1), (\lambda - 1), (3\lambda + 1)$

and by comparing with line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (2,1,3)$

Since PQ is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow 2(2\lambda + 1) + (\lambda - 1) + 3(3\lambda + 1) = 0$

$\Rightarrow 4\lambda + 2 + \lambda - 1 + 9\lambda + 3 = 0$

$\Rightarrow 14\lambda + 4 = 0$

$\Rightarrow \lambda = -\frac{4}{14}$

$\Rightarrow \lambda = -\frac{2}{7}$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$2\lambda, (\lambda + 2), (3\lambda + 3)$

$= Q\ (2\left(-\frac{2}{7}\right), \left(-\frac{2}{7}\right) + 2, 3\left(-\frac{2}{7}\right) + 3)$

$= Q\ (-\frac{4}{7}, \frac{12}{7}, \frac{15}{7})$

Position Vector of Q

$-\frac{4}{7}\hat{\imath} + \frac{12}{7}\hat{\jmath} + \frac{15}{7}\hat{k}$

Now,

Equation of line passing through two points with position vector $\hat{a}$ and $\hat{b}$ is given by

$\hat{r} = \hat{a} + \lambda(\hat{b} - \hat{a})$

Here,

$\hat{a} = -\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

and $\hat{b} = -\frac{4}{7}\hat{\imath} + \frac{12}{7}\hat{\jmath} + \frac{15}{7}\hat{k}$

$\Rightarrow \hat{r} = (-\hat{\imath} + 3\hat{\jmath} + 2\hat{k}) + \lambda\left[(-\frac{4}{7}\hat{\imath} + \frac{12}{7}\hat{\jmath} + \frac{15}{7}\hat{k}) - (-\hat{\imath} + 3\hat{\jmath} + 2\hat{k})\right]$

10. Question

Find the foot of the perpendicular from (0, 2, 7) on the line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$.

Answer

Given: - Point P(0, 2, 7) and equation of line $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

Thus to find Distance PQ we have to first find coordinates of Q

$\frac{x + 2}{-1} = \frac{y - 1}{3} = \frac{z - 3}{-2} = \lambda \text{ (let)}$

$\Rightarrow x = -\lambda - 2,\ y = 3\lambda + 1,\ z = -2\lambda + 3$

Therefore, coordinates of Q(– λ – 2, 3λ + 1, – 2λ + 3)

Now as we know (TIP) ‘if two points A(x_1,y_1,z_1) and B(x_2,y_2,z_2) on a line, then its direction ratios are proportional to ($x_2 - x_1$,$y_2 - y_1$,$z_2 - z_1$)’

Hence

Direction ratio of PQ is

= (– λ – 2 – 0), (3λ + 1 – 2), (– 2λ + 3 – 7)

= (– λ – 2), (3λ – 1), (– 2λ – 4)

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

= (– 1,3, – 2)

Since PQ is perpendicular to given line, therefore by “condition of perpendicularity.”

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where ‘a’ terms and ‘b’ terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow -1(-\lambda - 2) + (3)(3\lambda - 1) - 2(-2\lambda - 4) = 0$

$\Rightarrow \lambda + 2 + 9\lambda - 3 + 4\lambda + 8 = 0$

$\Rightarrow 14\lambda + 7 = 0$

$\Rightarrow \lambda = -\frac{1}{2}$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$$= Q\left(-\left(-\frac{1}{2}\right) - 2, 3\left(-\frac{1}{2}\right) + 1, -2\left(-\frac{1}{2}\right) + 3\right)$$

$$= Q\left(-\frac{3}{2}, -\frac{1}{2}, 4\right)$$

11. Question

Find the foot of the perpendicular from(1, 2, –3) to the line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$.

Answer

Given: – Point P(1, 2, – 3) and equation of line $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$

Let, PQ be the perpendicular drawn from P to given line whose endpoint/ foot is Q point.

Thus to find Distance PQ we have to first find coordinates of Q

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda \text{ (let)}$$

$\Rightarrow x = 2\lambda - 1,\ y = -2\lambda + 3,\ z = -\lambda$

Therefore, coordinates of $Q(2\lambda - 1, -2\lambda + 3, -\lambda)$

Now as we know (TIP) ‘if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$’

Hence

Direction ratio of PQ is

$= (2\lambda - 1 - 1), (-2\lambda + 3 - 2), (-\lambda + 3)$

$= (2\lambda - 2), (-2\lambda + 1), (-\lambda + 3)$

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (2, -2, -1)$

Since PQ is perpendicular to given line, therefore by “condition of perpendicularity.”

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow 2(2\lambda - 2) + (-2)(-2\lambda + 1) - 1(-\lambda + 3) = 0$

$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$

$\Rightarrow 9\lambda - 9 = 0$

$\Rightarrow \lambda = 1$

Therefore coordinates of Q

i.e. Foot of perpendicular

By putting the value of λ in Q coordinate equation, we get

$= Q\,(2(1) - 1, -2(1) + 3, -1)$

$= Q\,(1,1,-1)$

12. Question

Find the equation of the line passing through the points A(0, 6, – 9) and B(– 3, – 6, 3). If D is the foot of the perpendicular drawn from a point C(7, 4, – 1) on the line AB, then find the coordinates of the point D and the equation of line CD.

Answer

Given: – Line passing through the points A(0, 6, – 9) and B(– 3, – 6, 3). Point C(7, 4, – 1).

We know that

Tip: - Equation of a line joined by two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ is given by

$$= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Hence equation of line AB

$$= \frac{x - 0}{-3 - 0} = \frac{y - 6}{-6 - 6} = \frac{z + 9}{3 + 9}$$

$$= \frac{x}{-3} = \frac{y - 6}{-12} = \frac{z + 9}{12}$$

$$= \frac{x}{-3} = \frac{y - 6}{-12} = \frac{z + 9}{12} = \lambda\,(\text{let})$$

$\Rightarrow x = -3\lambda,\ y = -12\lambda + 6,\ z = 12\lambda - 9$

Now coordinates of point D

$D(-3\lambda, (-12\lambda + 6), (12\lambda - 9))$

Now as we know (TIP) 'if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$'

Hence

Direction ratio of CD is

$= (-3\lambda - 7), (-12\lambda + 6 - 4), (12\lambda - 9 + 1)$

$= (-3\lambda - 7), (-12\lambda + 2), (12\lambda - 8)$

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (-3, -12, 12)$

Since CD is perpendicular to given line, therefore by "condition of perpendicularity."

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

$\Rightarrow (-3)(-3\lambda - 7) + (-12)(-12\lambda + 2) + 12(12\lambda - 8) = 0$

$\Rightarrow 9\lambda + 21 + 144\lambda - 24 + 144\lambda - 96 = 0$

$\Rightarrow 297\lambda - 99 = 0$

$$\Rightarrow \lambda = \frac{1}{3}$$

Therefore coordinates of D

i.e. Foot of perpendicular

By putting the value of λ in D coordinate equation, we get

$-3\lambda, (-12\lambda + 6), (12\lambda - 9)$

$$= D\left[-3\left(\frac{1}{3}\right), -12\left(\frac{1}{3}\right) + 6, 12\left(\frac{1}{3}\right) - 9\right]$$

$$= D(-1, 2, -5)$$

Hence equation of line CD

$$= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$= \frac{x - 7}{-1 - 7} = \frac{y - 4}{2 - 4} = \frac{z + 1}{-5 + 1}$$

$$= \frac{x - 7}{-8} = \frac{y - 4}{-2} = \frac{z + 1}{-4}$$

$$= \frac{x - 7}{4} = \frac{y - 4}{1} = \frac{z + 1}{2}$$

13. Question

Find the distance of the point (2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

Answer

Given: - Point P(2, 4, – 1) and equation of line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$

Let, Q be a point through which line passes

Thus from given equation of line coordinates of Q is

Q(– 5, – 3,6)

As we know line equation with direction ratio of given line is parallel to given line.

Hence Line is parallel to $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$

Now,

$\Rightarrow \overrightarrow{PQ} = (-5\hat{i} - 3\hat{j} + 6\hat{k}) - (2\hat{i} + 4\hat{j} - \hat{k})$

$\Rightarrow \overrightarrow{PQ} = (-7\hat{i} - 7\hat{j} + 7\hat{k})$

Now let's find cross product of this two vectors

$$\Rightarrow \vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ -7 & -7 & 7 \end{vmatrix}$$

$\Rightarrow \vec{b} \times \overrightarrow{PQ} = -35\hat{i} + 56\hat{j} + 21\hat{k}$

The magnitude of this cross product

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{1225 + 3136 + 441}$

$\Rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{4802}$

And Magnitude of $\vec{b}$

$\Rightarrow |\vec{b}| = \sqrt{1 + 16 + 81}$

$\Rightarrow |\vec{b}| = \sqrt{98}$

Thus distance of point from line is

$\Rightarrow d = \frac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$\Rightarrow d = \frac{\sqrt{4802}}{\sqrt{98}}$

$\Rightarrow d = 7$ units

14. Question

Find the coordinates of the foot of the perpendicular drawn from the point A(1, 8, 4) to the line joining the points B(0, – 1, 3) and C(2, – 3, – 1).

Answer

Given: – Perpendicular from A(1, 8, 4) drawn at line joining points B(0, – 1, 3) and C(2, – 3, – 1)

and D be the foot of the perpendicular drawn from A(1, 8, 4) to line joining points B(0, – 1, 3) and C(2, – 3, – 1).

Now let's find the equation of the line which is formed by joining points B(0, – 1, 3) and C(2, – 3, – 1)

Tip: - Equation of a line joined by two points A(x_1,y_1,z_1) and B(x_2,y_2,z_2) is given by

$$= \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$= \frac{x - 0}{2 - 0} = \frac{y + 1}{-3 + 1} = \frac{z - 3}{-1 - 3}$$

$= \frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4}$

Now

$= \frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} = \lambda \text{ (let)}$

Therefore,

⇒ $x = 2\lambda$, $y = -2\lambda - 1$, $z = -4\lambda + 3$

Therefore, coordinates of $D(2\lambda, -2\lambda - 1, -4\lambda + 3)$

Now as we know (TIP) 'if two points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ on a line, then its direction ratios are proportional to $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$'

Hence

Direction Ratios of AD

$= (2\lambda - 1), (-2\lambda - 1 - 8), (-4\lambda + 3 - 4)$

$= (2\lambda - 1), (-2\lambda - 9), (-4\lambda - 1)$

and by comparing with given line equation, direction ratios of the given line are

(hint: denominator terms of line equation)

$= (2, -2, -4)$

Since AD is perpendicular to given line, therefore by "condition of perpendicularity"

$a_1a_2 + b_1b_2 + c_1c_2 = 0$; where a terms and b terms are direction ratio of lines which are perpendicular to each other.

⇒ $2(2\lambda - 1) + (-2)(-2\lambda - 9) - 4(-4\lambda - 1) = 0$

⇒ $4\lambda - 2 + 4\lambda + 18 + 16\lambda + 4 = 0$

⇒ $24\lambda + 20 = 0$

⇒ $\lambda = -\frac{20}{24}$

⇒ $\lambda = -\frac{5}{6}$

Therefore coordinates of D

i.e Foot of perpendicular

By putting value of λ in D coordinate equation, we get

$D(2\lambda, -2\lambda - 1, -4\lambda + 3)$

$= D(2\left(-\frac{5}{6}\right), -2\left(-\frac{5}{6}\right) - 1, -4(-\frac{5}{6}) + 3)$

$= D(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3})$

Exercise 28.5

1 A. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda\left(3\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu\left(-3\hat{i} + 2\hat{j} + 4\hat{k}\right)$

Answer

Equation of line in vector form

Line I:

$\vec{r} = \left(3\hat{i} + 8\hat{j} + 3\hat{k}\right) + \lambda(3\hat{i} - \hat{j} + \hat{k})$

Line II:

$\vec{r} = \left(-3\hat{i} - 7\hat{j} + 6\hat{k}\right) + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$

Here

$\overrightarrow{a_1} = 3\hat{i} + 8\hat{j} + 3\hat{k}$

$\overrightarrow{a_2} = -3\hat{i} - 7\hat{j} + 6\hat{k}$

$\overrightarrow{b_1} = 3\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{b_2} = -3\hat{i} + 2\hat{j} + 4\hat{k}$

The shortest distance between lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (-3\hat{i} - 7\hat{j} + 6\hat{k}) - (3\hat{i} + 8\hat{j} + 3\hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = -6\hat{i} - 15\hat{j} + 3\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}$$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = (-4 - 2)\,\hat{i} - (12 + 3)\hat{j} + (6 - 3)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -6\hat{i} - 15\hat{j} + 3\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-6)^2 + (-15)^2 + 3^2}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{270}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(-6\hat{i} - 15\hat{j} + 3\hat{k})(-6\hat{i} - 15\hat{j} + 3\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 270$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{270}{\sqrt{270}}$$

$d = \sqrt{270}$

$d = 3\sqrt{30}$ units

1 B. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 7\hat{k}\right)$ and $\vec{r} = -\hat{i} - \hat{j} - \hat{k} + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$

Answer

Equation of line in vector form

Line I:

$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda(\hat{i} - 2\hat{j} + 7\hat{k})$

Line II:

$\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$

Here,

$\overrightarrow{a_1} = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\overrightarrow{a_2} = -\hat{i} - \hat{j} - \hat{k}$

$\overrightarrow{b_1} = \hat{i} - 2\hat{j} + 7\hat{k}$

$\overrightarrow{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$

The shortest distance between lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = -4\hat{i} - 6\hat{j} - 8\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ 7 & -6 & 1 \end{vmatrix}$$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (-2 + 42)\,\hat{i} - (1 - 49)\hat{j} + (-6 + 14)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = 40\hat{i} + 48\hat{j} + 8\hat{k}$

$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{40^2 + (48)^2 + 8^2}$

$\Rightarrow \left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = 8\sqrt{62}$

$\left|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})\right| = |(-4\hat{i} - 6\hat{j} - 8\hat{k})(40\hat{i} + 48\hat{j} + 8\hat{k})|$

$\Rightarrow \left|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})\right| = |-512|$

$\Rightarrow \left|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})\right| = 512$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{512}{8\sqrt{62}}$$

$$d = \frac{64}{\sqrt{62}}$$

$d = \frac{64}{\sqrt{62}}$ units

1 C. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

Answer

Equation of line in vector form

Line I:

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

Line II:

$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 4\hat{j} + 5\hat{k})$

Here,

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a_2} = 2\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{b_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$(\vec{a_2} - \vec{a_1}) = (2\hat{i} + 4\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$(\vec{a_2} - \vec{a_1}) = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\vec{b_1} \times \vec{b_2} = (15 - 16)\,\hat{i} - (10 - 12)\hat{j} + (8 - 9)\hat{k}$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$\Rightarrow |\vec{b_1} \times \vec{b_2}| = \sqrt{6}$

$|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(\hat{i} + 2\hat{j} + 2\hat{k})(-\hat{i} + 2\hat{j} - \hat{k})|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = 1$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$d = \frac{1}{\sqrt{6}}$ units

1 D. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-t)\hat{k}$ and $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

Answer

Equation of line in vector form

Line I:

$\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-t)\hat{k}$

$\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + t(-\hat{\imath} + \hat{\jmath} - \hat{k})$

Line II:

$\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$

$\vec{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + s(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$

Here,

$\vec{a_1} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$\vec{a_2} = \hat{\imath} - \hat{\jmath} - \hat{k}$

$\vec{b_1} = -\hat{\imath} + \hat{\jmath} - \hat{k}$

$\vec{b_2} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$(\vec{a_2} - \vec{a_1}) = (\hat{\imath} - \hat{\jmath} - \hat{k}) - (\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$

$(\vec{a_2} - \vec{a_1}) = 0\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -1 \\ 1 & 2 & -2 \end{vmatrix}$$

$\vec{b_1} \times \vec{b_2} = (-2+2)\,\hat{\imath} - (2+1)\hat{\jmath} + (-2-1)\hat{k}$

$\Rightarrow \vec{b_1} \times \vec{b_2} = 0\hat{\imath} - 3\hat{\jmath} - 3\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(0)^2 + (-3)^2 + (-3)^2}$

$\Rightarrow |\vec{b_1} \times \vec{b_2}| = 3\sqrt{2}$

$|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(0\hat{\imath} + \hat{\jmath} - 4\hat{k})(0\hat{\imath} - 3\hat{\jmath} - 3\hat{k})|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = 9$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$d = \frac{9}{3\sqrt{2}}$$

$d = \frac{3}{\sqrt{2}}$ units

1 E. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$ and $\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

Answer

V. Equation of line in vector form

Line I:

$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$

$\vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$

Line II:

$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overrightarrow{a_1} = -\hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{a_2} = \hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{b_1} = \hat{i} + \hat{j} - \hat{k}$

$\overrightarrow{b_2} = -\hat{i} + 2\hat{j} + \hat{k}$

The shortest distance between lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (\hat{i} - \hat{j} + 2\hat{k}) - (-\hat{i} + \hat{j} - \hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (1 + 2)\,\hat{i} - (1 - 1)\hat{j} + (2 + 1)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = 3\hat{i} + 0\hat{j} + 3\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{3^2 + 0^2 + 3^2}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = 3\sqrt{2}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(2\hat{i} - 2\hat{j} + 3\hat{k})(3\hat{i} + 0\hat{j} + 3\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 15$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{15}{3\sqrt{2}}$$

$$d = \frac{5}{\sqrt{2}} \text{ units}$$

1 F. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 5\hat{j} + 2\hat{k}\right)$ and $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$

Answer

VI. Equation of line in vector form

Line I:

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda(2\hat{i} - 5\hat{j} + 2\hat{k})$$

Line II:

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \mu(\hat{i} - \hat{j} + \hat{k})$$

Here,

$$\vec{a_1} = 2\hat{i} - \hat{j} - \hat{k}$$

$$\vec{a_2} = \hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b_1} = 2\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\vec{b_2} = \hat{i} - \hat{j} + \hat{k}$$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} + 2\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} - \hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\vec{b_1} \times \vec{b_2} = (-5 + 2)\,\hat{i} - (2 - 2)\hat{j} + (-2 + 5)\hat{k}$$

$$\Rightarrow \vec{b_1} \times \vec{b_2} = -3\hat{i} + 0\hat{j} + 3\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-3)^2 + 0^2 + 3^2}$$

$$\Rightarrow \left|\vec{b_1} \times \vec{b_2}\right| = 3\sqrt{2}$$

$$\left|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})\right| = |(-\hat{i} + 3\hat{j} + 2\hat{k})(-3\hat{i} + 0\hat{j} + 3\hat{k})|$$

$$\Rightarrow \left|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})\right| = 9$$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$d = \frac{9}{3\sqrt{2}}$$

$$d = \frac{3}{\sqrt{2}} \text{ units}$$

1 G. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$

Answer

VII. Equation of line in vector form

Line I:

$$\vec{r} = (\hat{i} + 0\hat{j} + \hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

Line II:

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

Here,

$$\vec{a_1} = \hat{i} + \hat{j} + 0\hat{k}$$

$$\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$$

$$\vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$(\vec{a_2} - \vec{a_1}) = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j} + 0\hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = \hat{i} + 0\hat{j} - \hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$\vec{b_1} \times \vec{b_2} = (-2 + 5)\,\hat{i} - (4 - 3)\hat{j} + (-10 + 3)\hat{k}$$

$$\Rightarrow \vec{b_1} \times \vec{b_2} = 3\hat{i} - \hat{j} - 7\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{3^2 + (-1)^2 + (-7)^2}$$

$$\Rightarrow |\vec{b_1} \times \vec{b_2}| = \sqrt{59}$$

$$|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(\hat{i} + 0\hat{j} - \hat{k})(3\hat{i} - \hat{j} - 7\hat{k})|$$

$$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = 10$$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$d = \frac{10}{\sqrt{59}}$$

$$d = \frac{10}{\sqrt{59}} \text{ units}$$

1 H. Question

Find the shortest distance between the following pairs of lines whose vector equations are :

$\vec{r} = (8+3\lambda)\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

Answer

VIII. Equation of line in vector form

Line I:

$$\vec{r} = \left((8+3\lambda)\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}\right)$$

$$\vec{r} = \left(8\hat{i} - 9\hat{j} + 10\hat{k}\right) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$$

Line II:

$$\vec{r} = \left(15\hat{i} + 29\hat{j} + 5\hat{k}\right) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$$

Here,

$$\vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$$

$$\vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$$

$$\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$$

$$\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$(\vec{a_2} - \vec{a_1}) = (15\hat{i} + 29\hat{j} + 5\hat{k}) - (8\hat{i} - 9\hat{j} + 10\hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = 7\hat{i} + 38\hat{j} - 5\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$$\vec{b_1} \times \vec{b_2} = (80 - 56)\,\hat{i} - (-15 - 21)\hat{j} + (24 + 48)\hat{k}$$

$$\Rightarrow \vec{b_1} \times \vec{b_2} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

$$\Rightarrow \vec{b_1} \times \vec{b_2} = 4(6\hat{i} + 9\hat{j} + 18\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = 4\sqrt{6^2 + 9^2 + 18^2}$$

$$|\vec{b_1} \times \vec{b_2}| = 4\sqrt{441}$$

$$|\vec{b_1} \times \vec{b_2}| = 4 \times 21$$

$$\Rightarrow |\vec{b_1} \times \vec{b_2}| = 84$$

$$|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})| = |4(7\hat{\imath}+38\hat{\jmath}-5\hat{k})(6\hat{\imath}+9\hat{\jmath}+18\hat{k})|$$

$$\Rightarrow |(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})| = 4(42+342-90)$$

$$\Rightarrow |(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})| = 1176$$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$$

$$d = \frac{1176}{84}$$

$$d = 14 \text{ units}$$

2 A. Question

Find the shortest distance between the following pairs of lines whose Cartesian equations are :

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$

Answer

Given data:

Pair of lines:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}, \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5}$$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \alpha$$

$$\vec{r} = (\hat{\imath}+2\hat{\jmath}+3\hat{k}) + \alpha(2\hat{\imath}+3\hat{\jmath}+4\hat{k})$$

Line II:

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5} = \beta$$

$$\vec{r} = (2\hat{\imath}+3\hat{\jmath}+5\hat{k}) + \beta(3\hat{\imath}+4\hat{\jmath}+5\hat{k})$$

Here $\overrightarrow{a_1} = \hat{\imath}+2\hat{\jmath}+3\hat{k}$

$$\overrightarrow{a_2} = 2\hat{\imath}+3\hat{\jmath}+5\hat{k}$$

$$\overrightarrow{b_1} = 2\hat{\imath}+3\hat{\jmath}+4\hat{k}$$

$$\overrightarrow{b_2} = 3\hat{\imath}+4\hat{\jmath}+5\hat{k}$$

The shortest distance between two skew lines is

$$d = \frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$$

$$(\overrightarrow{a_2}-\overrightarrow{a_1}) = (2\hat{\imath}+3\hat{\jmath}+5\hat{k}) - (\hat{\imath}+2\hat{\jmath}+3\hat{k})$$

$\Rightarrow (\overrightarrow{a_2} - \overrightarrow{a_1}) = \hat{i} + \hat{j} + 2\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (15 - 16)\,\hat{i} - (10 - 12)\hat{j} + (8 - 9)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{6}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(\hat{i} + \hat{j} + 2\hat{k})(-\hat{i} + 2\hat{j} - \hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(-1 + 2 - 2)|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 1$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{1}{\sqrt{6}}$$

$$d = \frac{1}{\sqrt{6}} \text{ units}$$

2 B. Question

Find the shortest distance between the following pairs of lines whose Cartesian equations are :

$\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{3} = \frac{y-2}{1}$; $z = 2$

Answer

Given data:

Pair of lines:

$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{3} = \frac{y-2}{1}; z = 2$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \alpha$

$\vec{r} = (\hat{i} - \hat{j} + 0\hat{k}) + \alpha(2\hat{i} + 3\hat{j} + \hat{k})$

Line II:

$\frac{x+1}{3} = \frac{y-2}{1} = \beta;\ z = 2$

$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \beta(3\hat{i} + \hat{j} + 0\hat{k})$

Here $\overrightarrow{a_1} = \hat{\imath} - \hat{\jmath} + 0\hat{k}$

$\overrightarrow{a_2} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

$\overrightarrow{b_1} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\overrightarrow{b_2} = 3\hat{\imath} + \hat{\jmath} + 0\hat{k}$

The shortest distance between two skew lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (-\hat{\imath} + 2\hat{\jmath} + 2\hat{k}) - (\hat{\imath} - \hat{\jmath} + 0\hat{k})$

$\Rightarrow (\overrightarrow{a_2} - \overrightarrow{a_1}) = -2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 3 & 1 \\ 3 & 1 & 0 \end{vmatrix}$$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (0 - 1)\,\hat{\imath} - (0 - 3)\hat{\jmath} + (2 - 9)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{\imath} + 3\hat{\jmath} - 7\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 3^2 + (-7)^2}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{59}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(-2\hat{\imath} + 3\hat{\jmath} + 2\hat{k})(-\hat{\imath} + 3\hat{\jmath} - 7\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(2 + 9 - 14)|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 3$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{3}{\sqrt{59}}$$

$$d = \frac{3}{\sqrt{59}} \text{ units}$$

2 C. Question

Find the shortest distance between the following pairs of lines whose Cartesian equations are :

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} \text{ and } \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}$$

Answer

Given data:

Pair of lines:

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}, \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2}$$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$$\frac{x-1}{-1}=\frac{y+2}{1}=\frac{z-3}{-2}=\alpha$$

$$\vec{r}=(\hat{\imath}-2\hat{\jmath}+3\hat{k})+\alpha(-\hat{\imath}+\hat{\jmath}-2\hat{k})$$

Line II:

$$\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+1}{-2}=\beta$$

$$\vec{r}=(\hat{\imath}-\hat{\jmath}-\hat{k})+\beta(\hat{\imath}+2\hat{\jmath}-2\hat{k})$$

Here

$$\vec{a_1}=\hat{\imath}-2\hat{\jmath}+3\hat{k}$$

$$\vec{a_2}=\hat{\imath}-\hat{\jmath}-\hat{k}$$

$$\vec{b_1}=-\hat{\imath}+\hat{\jmath}-2\hat{k}$$

$$\vec{b_2}=\hat{\imath}+2\hat{\jmath}-2\hat{k}$$

The shortest distance between two skew lines is

$$d=\frac{|(\vec{a_2}-\vec{a_1})(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$$

$$(\vec{a_2}-\vec{a_1})=(\hat{\imath}-\hat{\jmath}-\hat{k})-(\hat{\imath}-2\hat{\jmath}+3\hat{k})$$

$$\Rightarrow(\vec{a_2}-\vec{a_1})=0\hat{\imath}+\hat{\jmath}-4\hat{k}$$

$$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 1 & -2\\ 1 & 2 & -2\end{vmatrix}$$

$$\vec{b_1}\times\vec{b_2}=(-2+4)\,\hat{\imath}-(2+2)\hat{\jmath}+(-2-1)\hat{k}$$

$$\Rightarrow\vec{b_1}\times\vec{b_2}=2\hat{\imath}-4\hat{\jmath}-3\hat{k}$$

$$|\vec{b_1}\times\vec{b_2}|=\sqrt{2^2+(-4)^2+(-3)^2}$$

$$\Rightarrow|\vec{b_1}\times\vec{b_2}|=\sqrt{29}$$

$$|(\vec{a_2}-\vec{a_1})(\vec{b_1}\times\vec{b_2})|=|(0\hat{\imath}+\hat{\jmath}-4\hat{k})(2\hat{\imath}-4\hat{\jmath}-3\hat{k})|$$

$$\Rightarrow|(\vec{a_2}-\vec{a_1})(\vec{b_1}\times\vec{b_2})|=|(0-4+12)|$$

$$\Rightarrow|(\vec{a_2}-\vec{a_1})(\vec{b_1}\times\vec{b_2})|=8$$

Putting these values in the expression,

$$d=\frac{|(\vec{a_2}-\vec{a_1})(\vec{b_1}\times\vec{b_2})|}{|\vec{b_1}\times\vec{b_2}|}$$

$$d=\frac{8}{\sqrt{29}}$$

$$d=\frac{8}{\sqrt{29}}\text{ units}$$

2 D. Question

Find the shortest distance between the following pairs of lines whose Cartesian equations are :

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

Answer

Given data:

Pair of lines:

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}, \frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}=\alpha$

$\vec{r}=(3\hat{i}+5\hat{j}+7\hat{k})+\alpha(\hat{i}-2\hat{j}+\hat{k})$

Line II:

$\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}=\beta$

$\vec{r}=(-\hat{i}-\hat{j}-\hat{k})+\beta(7\hat{i}-6\hat{j}+\hat{k})$

Here

$\overrightarrow{a_1}=3\hat{i}+5\hat{j}+7\hat{k}$

$\overrightarrow{a_2}=-\hat{i}-\hat{j}-\hat{k}$

$\overrightarrow{b_1}=\hat{i}-2\hat{j}+\hat{k}$

$\overrightarrow{b_2}=7\hat{i}-6\hat{j}+\hat{k}$

The shortest distance between two skew lines is

$$d=\frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$$

$(\overrightarrow{a_2}-\overrightarrow{a_1})=(-\hat{i}-\hat{j}-\hat{k})-(3\hat{i}+5\hat{j}+7\hat{k})$

$\Rightarrow(\overrightarrow{a_2}-\overrightarrow{a_1})=-4\hat{i}-6\hat{j}-8\hat{k}$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 1\\ 7 & -6 & 1\end{vmatrix}$$

$\overrightarrow{b_1}\times\overrightarrow{b_2}=(-2+6)\,\hat{i}-(1-7)\hat{j}+(-6+14)\hat{k}$

$\Rightarrow\overrightarrow{b_1}\times\overrightarrow{b_2}=4\hat{i}+6\hat{j}+8\hat{k}$

$|\overrightarrow{b_1}\times\overrightarrow{b_2}|=\sqrt{4^2+6^2+8^2}$

$\Rightarrow|\overrightarrow{b_1}\times\overrightarrow{b_2}|=2\sqrt{29}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(-4\hat{i} - 6\hat{j} - 8\hat{k})(4\hat{i} + 6\hat{j} + 8\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(-16 - 36 - 64)|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 116$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{116}{2\sqrt{29}}$$

$$d = \frac{58}{\sqrt{29}} \text{ units}$$

3 A. Question

By computing the shortest distance determine whether the following pairs of lines intersect or not :

$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

Answer

Equation of line in vector form

Line I:

$\vec{r} = (\hat{i} - \hat{j} + 0\hat{k}) + \lambda(2\hat{i} + 0\hat{j} + \hat{k})$

Line II:

$\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$

Here,

$\overrightarrow{a_1} = \hat{i} - \hat{j} + 0\hat{k}$

$\overrightarrow{a_2} = 2\hat{i} - \hat{j}$

$\overrightarrow{b_1} = 2\hat{i} + 0\hat{j} + \hat{k}$

$\overrightarrow{b_2} = \hat{i} + \hat{j} - \hat{k}$

The shortest distance between lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (2\hat{i} - \hat{j}) - (\hat{i} - \hat{j} + 0\hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = \hat{i} + 0\hat{j} + 0\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (0 - 1)\,\hat{i} - (-2 - 1)\hat{j} + (2 - 0)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{i} + 3\hat{j} + 2\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 3^2 + 2^2}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{14}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(\hat{\imath} + 0\hat{\jmath} + 0\hat{k})(-\hat{\imath} + 3\hat{\jmath} + 2\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 1$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{1}{\sqrt{14}}$$

$$d = \frac{1}{\sqrt{14}} \text{ units}$$

Shortest distance d between the lines is not 0. So lines are not intersecting.

3 B. Question

By computing the shortest distance determine whether the following pairs of lines intersect or not :

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

Answer

Equation of line in vector form

Line I:

$\vec{r} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} + 0\hat{k})$

Line II:

$\vec{r} = (4\hat{\imath} + 0\hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + 0\hat{\jmath} + 3\hat{k})$

Here,

$\overrightarrow{a_1} = \hat{\imath} + \hat{\jmath} - \hat{k}$

$\overrightarrow{a_2} = 4\hat{\imath} + 0\hat{\jmath} - \hat{k}$

$\overrightarrow{b_1} = 3\hat{\imath} - \hat{\jmath} + 0\hat{k}$

$\overrightarrow{b_2} = 2\hat{\imath} + 0\hat{\jmath} + 3\hat{k}$

The shortest distance between lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (4\hat{\imath} + 0\hat{\jmath} - \hat{k}) - (\hat{\imath} + \hat{\jmath} - \hat{k})$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) = 3\hat{\imath} - \hat{\jmath} + 0\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (-3 - 0)\,\hat{\imath} - (9 - 0)\hat{\jmath} + (0 + 2)\hat{k}$

$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -3\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-3)^2 + (-9)^2 + 2^2}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{94}$

$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(3\hat{i} - \hat{j} + 0\hat{k})(-3\hat{i} - 9\hat{j} + 2\hat{k})|$

$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 0$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{0}{\sqrt{94}}$$

d = 0

Shortest distance d between the lines is 0. So lines are intersecting.

3 C. Question

By computing the shortest distance determine whether the following pairs of lines intersect or not :

$\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}$; $z = 2$

Answer

Given data:

Pair of lines:

$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{3} = \frac{y-2}{1}; z = 2$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \alpha$$

$\vec{r} = (\hat{i} - \hat{j} + 0\hat{k}) + \alpha(2\hat{i} + 3\hat{j} + \hat{k})$

Line II:

$$\frac{x+1}{5} = \frac{y-2}{1} = \beta;\ z = 2$$

$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \beta(5\hat{i} + \hat{j} + 0\hat{k})$

Here $\overrightarrow{a_1} = \hat{i} - \hat{j} + 0\hat{k}$

$\overrightarrow{a_2} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{b_2} = 5\hat{i} + \hat{j} + 0\hat{k}$

The shortest distance between two skew lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (-\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} - \hat{j} + 0\hat{k})$$

$$\Rightarrow (\overrightarrow{a_2} - \overrightarrow{a_1}) = -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = (0 - 1)\,\hat{i} - (0 - 5)\hat{j} + (2 - 15)\hat{k}$$

$$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-1)^2 + 5^2 + (-13)^2}$$

$$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{195}$$

$$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(-2\hat{i} + 3\hat{j} + 2\hat{k})(-\hat{i} + 5\hat{j} - 13\hat{k})|$$

$$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(2 + 15 - 26)|$$

$$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 9$$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{9}{\sqrt{195}}$$

Shortest distance d between the lines is not 0. So lines are not intersecting.

3 D. Question

By computing the shortest distance determine whether the following pairs of lines intersect or not :

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$$

Answer

Given data:

Pair of lines:

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5}, \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} = \alpha$$

$$\vec{r} = (5\hat{i} + 7\hat{j} - 3\hat{k}) + \alpha(4\hat{i} - 5\hat{j} - 5\hat{k})$$

Line II:

$$\frac{x-8}{7}=\frac{y-7}{1}=\frac{z-5}{3}=\beta$$

$$\vec{r}=\left(8\hat{i}+7\hat{j}+5\hat{k}\right)+\beta(7\hat{i}+\hat{j}+3\hat{k})$$

Here

$$\overrightarrow{a_1}=5\hat{i}+7\hat{j}-3\hat{k}$$

$$\overrightarrow{a_2}=8\hat{i}+7\hat{j}+5\hat{k}$$

$$\overrightarrow{b_1}=4\hat{i}-5\hat{j}-5\hat{k}$$

$$\overrightarrow{b_2}=7\hat{i}+\hat{j}+3\hat{k}$$

The shortest distance between two skew lines is

$$d=\frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$$

$$(\overrightarrow{a_2}-\overrightarrow{a_1})=\left(8\hat{i}+7\hat{j}+5\hat{k}\right)-(5\hat{i}+7\hat{j}-3\hat{k})$$

$$\Rightarrow(\overrightarrow{a_2}-\overrightarrow{a_1})=3\hat{i}-0\hat{j}+8\hat{k}$$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -5 & -5\\ 7 & 1 & 3\end{vmatrix}$$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=(-15+5)\,\hat{i}-(12+35)\hat{j}+(4+35)\hat{k}$$

$$\Rightarrow\overrightarrow{b_1}\times\overrightarrow{b_2}=-10\hat{i}-47\hat{j}+39\hat{k}$$

$$\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{20^2+(-47)^2+39^2}$$

$$\Rightarrow\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{820872}$$

$$\left|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})\right|=|(3\hat{i}-0\hat{j}+8\hat{k})(-10\hat{i}-47\hat{j}+39\hat{k})|$$

$$\Rightarrow\left|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})\right|=|(-30+0+312)|$$

$$\Rightarrow\left|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})\right|=282$$

Putting these values in the expression,

$$d=\frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}$$

$$d=\frac{282}{\sqrt{820872}}$$

Shortest distance d between the lines is not 0. So lines are not intersecting.

4 A. Question

Find the shortest distance between the following pairs of parallel lines whose equations are :

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(-\hat{i}+\hat{j}-\hat{k}\right)$

Answer

Equation of a line in vector form

Line I:

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

Line II:

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(-\hat{i} + \hat{j} - \hat{k})$

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) - \mu(\hat{i} - \hat{j} + \hat{k})$

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu'(\hat{i} - \hat{j} + \hat{k})$

Here,

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

Since lines are parallel therefore they have common normal

$\vec{b} = \hat{i} - \hat{j} + \hat{k}$

$|\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2}$

$|\vec{b}| = \sqrt{3}$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$(\vec{a_2} - \vec{a_1}) = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$(\vec{a_2} - \vec{a_1}) = \hat{i} - 3\hat{j} - 4\hat{k}$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$(\vec{a_2} - \vec{a_1}) \times \vec{b} = (-3 - 4)\,\hat{i} - (1 + 4)\hat{j} + (-1 + 3)\hat{k}$

$\Rightarrow (\vec{a_2} - \vec{a_1}) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$

$|(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{(-7)^2 + (-5)^2 + 2^2}$

$\Rightarrow |(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{78}$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$$d = \sqrt{\frac{78}{3}}$$

$d = \sqrt{26}$

4 B. Question

Find the shortest distance between the following pairs of parallel lines whose equations are :

$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$

Answer

Equation of a line in vector form

Line I:

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$$

Line II:

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu(4\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + 2\mu(2\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu'(2\hat{i} - \hat{j} + \hat{k})$$

Here,

$$\vec{a_1} = \hat{i} + \hat{j} + 0\hat{k}$$

$$\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$$

Since lines are parallel therefore they have common normal

$$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$$

$$|\vec{b}| = \sqrt{2^2 + (-1)^2 + 1^2}$$

$$|\vec{b}| = \sqrt{6}$$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$$(\vec{a_2} - \vec{a_1}) = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j} + 0\hat{k})$$

$$(\vec{a_2} - \vec{a_1}) = \hat{i} - 0\hat{j} - \hat{k}$$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = (0 - 1)\,\hat{i} - (1 + 2)\hat{j} + (-1 + 0)\hat{k}$$

$$\Rightarrow (\vec{a_2} - \vec{a_1}) \times \vec{b} = -\hat{i} - 3\hat{j} - \hat{k}$$

$$|(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{(-1)^2 + (-3)^2 + (-1)^2}$$

$$\Rightarrow |(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{11}$$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$$d = \sqrt{\frac{11}{6}} \text{ units}$$

5. Question

Find the equations of the lines joining the following pairs of vertices and then find the shortest distance between the lines

i. (0, 0, 0) and (1, 0, 2)

ii. (1, 3, 0) and (0, 3, 0)

Answer

We need to write this in vector form

Line I:

Equation of line passing through a (0, 0, 0) and b (1, 0, 2)

$$\vec{\mathbf{r}} = \vec{\mathbf{a}} + \boldsymbol{\lambda}(\vec{\mathbf{b}} - \vec{\mathbf{a}})$$

$$\vec{r} = \left(0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}\right) + \lambda((1-0)\hat{\imath} + (0-0)\hat{\jmath} + (2-0)\hat{k})$$

$$\vec{r} = \left(0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}\right) + \lambda(\hat{\imath} + 0\hat{\jmath} + 2\hat{k})$$

Line II:

Equation of line passing through a (1, 3, 0) and b (0, 3, 0)

$$\vec{\mathbf{r}} = \vec{\mathbf{a}} + \boldsymbol{\lambda}(\vec{\mathbf{b}} - \vec{\mathbf{a}})$$

$$\vec{r} = \left(\hat{\imath} + 3\hat{\jmath} + 0\hat{k}\right) + \mu((0-1)\hat{\imath} + (3-3)\hat{\jmath} + (0-0)\hat{k})$$

$$\vec{r} = \left(\hat{\imath} + 3\hat{\jmath} + 0\hat{k}\right) + \mu(-\hat{\imath} + 0\hat{\jmath} + 0\hat{k})$$

Here

$$\overrightarrow{a_1} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

$$\overrightarrow{a_2} = \hat{\imath} + 3\hat{\jmath} + 0\hat{k}$$

$$\overrightarrow{b_1} = \hat{\imath} + 0\hat{\jmath} + 2\hat{k}$$

$$\overrightarrow{b_2} = -\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$$

The shortest distance between two skew lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) = \left(\hat{\imath} + 3\hat{\jmath} + 0\hat{k}\right) - \left(0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}\right)$$

$$\Rightarrow (\overrightarrow{a_2} - \overrightarrow{a_1}) = \hat{\imath} + 3\hat{\jmath}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 0 & 0 \end{vmatrix}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = (0-0)\,\hat{\imath} - (0+2)\hat{\jmath} + (0-0)\hat{k}$$

$$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -2\hat{\jmath}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-2)^2}$$

$$\Rightarrow \left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = 2$$

$$\left|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})\right| = |(\hat{\imath} + 3\hat{\jmath})(-2\hat{\jmath})|$$

$$\Rightarrow \left|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})\right| = |-6|$$

$$\Rightarrow \left|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})\right| = 6$$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{6}{2}$$

d = 3 units

6. Question

Write the vector equations of the following lines and hence determine the distance between them

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ and $\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

Answer

Given data:

Pair of lines:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}, \frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$$

To find the shortest distance between these two lines

We need to write this in vector form

Line I:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \alpha$$

$$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \alpha(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$

Line II:

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} = \beta$$

$$\vec{r} = (3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) + \beta(4\hat{\imath} + 6\hat{\jmath} + 12\hat{k})$$

$$\vec{r} = (3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) + 2\beta(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$

$$\vec{r} = (3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) + \beta'(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$$

Here the lines are parallel as can be seen from above vector equation

$$\overrightarrow{a_1} = \hat{\imath} + 2\hat{\jmath} - 4\hat{k}$$

$$\overrightarrow{a_2} = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$$

Since lines are parallel therefore they have common normal

$$\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$$

$$|\vec{b}| = \sqrt{2^2 + 3^2 + 6^2}$$

$$|\vec{b}| = \sqrt{49}$$

The shortest distance between lines is

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$$(\overrightarrow{a_2} - \overrightarrow{a_1}) = (3\hat{\imath} + 3\hat{\jmath} - 5\hat{k}) - (\hat{\imath} + 2\hat{\jmath} - 4\hat{k})$$

$(\vec{a_2} - \vec{a_1}) = 2\hat{i} + \hat{j} - \hat{k}$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$(\vec{a_2} - \vec{a_1}) \times \vec{b} = (6 + 3)\,\hat{i} - (12 + 2)\hat{j} + (6 - 2)\hat{k}$

$\Rightarrow (\vec{a_2} - \vec{a_1}) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$|(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$\Rightarrow |(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{293}$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$$d = \sqrt{\frac{293}{49}}$$

$$d = \frac{\sqrt{293}}{7} \text{ units}$$

7 A. Question

Find the shortest distance between the lines

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Answer

Equation of a line in vector form

Line I:

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

Line II:

$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

Here,

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$

$\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$(\vec{a_2} - \vec{a_1}) = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$

$(\vec{a_2} - \vec{a_1}) = \hat{i} - 3\hat{j} - 2\hat{k}$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$$

$$\overrightarrow{b_1} \times \overrightarrow{b_2} = (-2-1)\,\hat{i} - (2-2)\hat{j} + (1+2)\hat{k}$$

$$\Rightarrow \overrightarrow{b_1} \times \overrightarrow{b_2} = -3\hat{i} + 0\hat{j} + 3\hat{k}$$

$$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-3)^2 + 0^2 + 3^2}$$

$$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = 3\sqrt{2}$$

$$|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |(\hat{i} - 3\hat{j} - 2\hat{k})(-3\hat{i} + 0\hat{j} + 3\hat{k})|$$

$$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = |-9|$$

$$\Rightarrow |(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})| = 9$$

Putting these values in the expression,

$$d = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1})(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}$$

$$d = \frac{9}{3\sqrt{2}}$$

$$d = \frac{3}{\sqrt{2}} \text{ units}$$

7 B. Question

Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Answer

Given data:

Pair of lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

To find the shortest distance between these two lines

Solution: We need to write this in vector form

Line I:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} = \alpha$$

$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \alpha(7\hat{i} - 6\hat{j} + \hat{k})$$

Line II:

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \beta$$

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \beta(\hat{i} - 2\hat{j} + \hat{k})$$

Here

$\vec{a_1} = -\hat{i} - \hat{j} - \hat{k}$

$\vec{a_2} = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\vec{b_1} = 7\hat{i} - 6\hat{j} + \hat{k}$

$\vec{b_2} = \hat{i} - 2\hat{j} + \hat{k}$

Shortest distance between two skew lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$(\vec{a_2} - \vec{a_1}) = (3\hat{i} + 5\hat{j} + 7\hat{k}) - (-\hat{i} - \hat{j} - \hat{k})$

$\Rightarrow (\vec{a_2} - \vec{a_1}) = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$\vec{b_1} \times \vec{b_2} = (-6 + 2)\,\hat{i} - (7 - 1)\hat{j} + (-14 + 6)\hat{k}$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -4\hat{i} - 6\hat{j} - 8\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$

$\Rightarrow |\vec{b_1} \times \vec{b_2}| = 2\sqrt{29}$

$|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(4\hat{i} + 6\hat{j} + 8\hat{k})(-4\hat{i} - 6\hat{j} - 8\hat{k})|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(-16 - 36 - 64)|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = 116$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$d = \frac{116}{2\sqrt{29}}$$

$d = 2\sqrt{29}$ units

7 C. Question

Find the shortest distance between the lines

$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and $\vec{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Answer

We need to write this in vector form

Line I:

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$

Line II:

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

Here

$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$

Shortest distance between two skew lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$(\vec{a_2} - \vec{a_1}) = (4\hat{i} + 5\hat{j} + 6\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow (\vec{a_2} - \vec{a_1}) = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$\vec{b_1} \times \vec{b_2} = (-3 - 6)\,\hat{i} - (1 - 4)\hat{j} + (3 + 6)\hat{k}$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -9\hat{i} + 3\hat{j} + 9\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-9)^2 + 3^2 + 9^2}$

$\Rightarrow |\vec{b_1} \times \vec{b_2}| = 3\sqrt{19}$

$|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(3\hat{i} + 3\hat{j} + 3\hat{k})(-9\hat{i} + 3\hat{j} + 9\hat{k})|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(-27 + 9 + 27)|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = 9$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$d = \frac{9}{3\sqrt{19}}$$

$$d = \frac{3}{\sqrt{19}} \text{ units}$$

7 D. Question

Find the shortest distance between the lines

$\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

Answer

We need to write this in vector form

Line I:

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

Line II:

$\vec{r} = (-4\hat{i} + 0\hat{j} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$

Here

$\vec{a_1} = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a_2} = -4\hat{i} + 0\hat{j} - \hat{k}$

$\vec{b_1} = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{b_2} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

Shortest distance between two skew lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$(\vec{a_2} - \vec{a_1}) = (-4\hat{i} + 0\hat{j} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k})$

$\Rightarrow (\vec{a_2} - \vec{a_1}) = -10\hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$\vec{b_1} \times \vec{b_2} = (4 + 4)\hat{i} - (-2 - 6)\hat{j} + (-2 + 6)\hat{k}$

$\Rightarrow \vec{b_1} \times \vec{b_2} = 8\hat{i} + 8\hat{j} + 4\hat{k}$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{8^2 + 8^2 + 4^2}$

$\Rightarrow |\vec{b_1} \times \vec{b_2}| = 4\sqrt{9}$

$\Rightarrow |\vec{b_1} \times \vec{b_2}| = 12$

$|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(-10\hat{i} - 2\hat{j} - 3\hat{k})(8\hat{i} + 8\hat{j} + 4\hat{k})|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = |(-80 - 16 - 12)|$

$\Rightarrow |(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})| = 108$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1})(\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$d = \frac{108}{12}$$

d = 9 units

8. Question

Find the distance between the lines l_1 and l_2 given by

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Answer

We need to write this in vector form

Line I:

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

Line II:

$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$

Since lines are parallel therefore they have common normal

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$|\vec{b}| = \sqrt{2^2 + 3^2 + 6^2}$

$|\vec{b}| = \sqrt{49}$

The shortest distance between lines is

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$(\vec{a_2} - \vec{a_1}) = (3\hat{i} + 3\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} - 4\hat{k})$

$(\vec{a_2} - \vec{a_1}) = 2\hat{i} + \hat{j} - \hat{k}$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$(\vec{a_2} - \vec{a_1}) \times \vec{b} = (6 + 3)\,\hat{i} - (12 + 2)\hat{j} + (6 - 2)\hat{k}$

$\Rightarrow (\vec{a_2} - \vec{a_1}) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$

$|(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{9^2 + (-14)^2 + 4^2}$

$\Rightarrow |(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{293}$

Putting these values in the expression,

$$d = \frac{|(\vec{a_2} - \vec{a_1}) \times \vec{b}|}{|\vec{b}|}$$

$$d = \sqrt{\frac{293}{49}}$$

$$d = \frac{\sqrt{293}}{7} \text{ units}$$

Very Short Answer

1. Question

Write the cartesian and vector equations of X-axis.

Answer

X-axis passes through the point (0, 0, 0).

Position Vector → $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Since, it is also parallel to the vector, $\vec{b} = \hat{i} + 0\hat{j} + 0\hat{k}$ having direction ratios proportional to 1, 0, 0, the

Cartesian equation of x-axis is,

$$\frac{x-0}{1}=\frac{y-0}{0}=\frac{z-0}{0}$$

$$=\frac{x}{1}=\frac{y}{0}=\frac{z}{0}$$

Also, its vector equation is

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$=0\hat{\imath}+0\hat{\jmath}+0\hat{k}+\lambda(\hat{\imath}+0\hat{\jmath}+0\hat{k})$$

$$=\lambda\hat{\imath}$$

2. Question

Write the cartesian and vector equations of Y-axis.

Answer

Y-axis passes through the point (0, 0, 0).

Position Vector → $\vec{a}=0\hat{\imath}+0\hat{\jmath}+0\hat{k}$

Since, it is also parallel to the vector, $\vec{b}=0\hat{\imath}+\hat{\jmath}+0\hat{k}$ having direction ratios proportional to 0, 1, 0, the Cartesian equation of y-axis is,

$$\frac{x-0}{0}=\frac{y-0}{1}=\frac{z-0}{0}$$

$$=\frac{x}{0}=\frac{y}{1}=\frac{z}{0}$$

Also, its vector equation is

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$=0\hat{\imath}+0\hat{\jmath}+0\hat{k}+\lambda(0\hat{\imath}+\hat{\jmath}+0\hat{k})$$

$$=\lambda\hat{\jmath}$$

3. Question

Write the cartesian and vector equation of Z-axis.

Answer

Z-axis passes through the point (0, 0, 0).

Position Vector → $\vec{a}=0\hat{\imath}+0\hat{\jmath}+0\hat{k}$

Since, it is also parallel to the vector, $\vec{b}=0\hat{\imath}+0\hat{\jmath}+\hat{k}$ having direction ratios proportional to 0, 0, 1, the Cartesian equation of z-axis is,

$$\frac{x-0}{0}=\frac{y-0}{0}=\frac{z-0}{1}$$

$$=\frac{x}{0}=\frac{y}{0}=\frac{z}{1}$$

Also, its vector equation is

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$=0\hat{\imath}+0\hat{\jmath}+0\hat{k}+\lambda(0\hat{\imath}+0\hat{\jmath}+\hat{k})$$

$= \lambda\hat{k}$

4. Question

Write the vector equation of a line passing through a point having position vector $\vec{\alpha}$ and parallel to vector $\vec{\beta}$.

Answer

The vector equation of the line passing through the point having position vector $\vec{\alpha}$

And parallel to vector $\vec{\beta}$ is,

$\vec{r} = \vec{a} + \lambda\vec{\beta}$

5. Question

Cartesian equations of a line AB are $\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$. Write the direction ratios of a line parallel to AB

Answer

We have

$$\frac{2x-1}{2} = \frac{4-y}{7} = \frac{z+1}{2}$$

Now, the equation of the line AB can be re-written as,

$$\frac{x-\frac{1}{2}}{1} = \frac{y-4}{-7} = \frac{z+1}{2}$$

The direction ratios of the line parallel to AB proportional to → 1, -7, 2

6. Question

Write the direction cosines of the line whose cartesian equations are 6x – 2 = 3y + 1 = 2z – 4.

Answer

We have

6x – 2 = 3y + 1 = 2z – 4

The equation of the given line can be re-written as,

$$\frac{x-\frac{1}{3}}{\frac{1}{6}} = \frac{y+\frac{1}{3}}{\frac{1}{3}} = \frac{z-2}{\frac{1}{2}}$$

$$= \frac{x-\frac{1}{3}}{1} = \frac{y+\frac{1}{3}}{2} = \frac{z-2}{3}$$

The direction ratios of the line parallel to AB are proportional to → 1,2,3

The direction cosines of the line parallel to AB are proportional to →

$$\frac{1}{\sqrt{1^2+2^2+3^2}}, \frac{2}{\sqrt{1^2+2^2+3^2}}, \frac{3}{\sqrt{1^2+2^2+3^2}}$$

$$= \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$$

7. Question

Write the direction cosines of the line $\frac{x-2}{2} = \frac{2y-5}{-3}, z = 2$.

Answer

We have

$\frac{x-2}{2} = \frac{2y-5}{-3}$, z = 2

The equation of the given line can be re-written as,

$$\frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = \frac{z-2}{0}$$

$$= \frac{x-2}{4} = \frac{y-\frac{5}{2}}{-3} = \frac{z-2}{0}$$

The direction ratios of the line parallel to AB are proportional to → 4, -3, 0

The direction cosines of the line parallel to AB are proportional to →

$$\frac{4}{\sqrt{4^2+(-3)^2+0^2}}, \frac{-3}{\sqrt{4^2+(-3)^2+0^2}}, \frac{0}{\sqrt{4^2+(-3)^2+0^2}}$$

$= \frac{4}{5}, \frac{-3}{5}, 0$

8. Question

Write the coordinate axis to which the line $\frac{x-2}{3} = \frac{y+5}{4} = \frac{z-1}{0}$ is perpendicular.

Answer

We have,

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-1}{0}$$

The given line is parallel to the vector,

$\vec{b} = 3\hat{i} + 4\hat{j} + 0\hat{k}$

Now let,

$x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

3x+4y+0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0,0,1).

Hence, the given line is perpendicular to z-axis.

9. Question

Write the angle between the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z-2}{1}$ and $\frac{x-1}{1} = \frac{y}{2} = \frac{z-2}{3}$.

Answer

We have,

$$\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z-2}{1}$$

$$\frac{x-1}{1} = \frac{y}{2} = \frac{z-1}{3}$$

The given lines are parallel to the vectors

$\rightarrow \overrightarrow{b_1} = 7\hat{i} - 5\hat{j} + \hat{k}$ and $\overrightarrow{b_2} = \hat{i} + 2\hat{j} + 3\hat{k}$

Let θ be the angle between the given lines.

Now,

$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$

$$= \frac{(7\hat{i} - 5\hat{j} + \hat{k}).(\hat{i} + 2\hat{j} + 3\hat{k})}{\sqrt{7^2 + (-5)^2 + 1^2}\sqrt{1^2 + 2^2 + 3^2}}$$

$$= \frac{7 - 10 + 3}{\sqrt{49 + 25 + 1}\sqrt{1 + 4 + 9}}$$

= 0

$\rightarrow \theta = \frac{\pi}{2}$

10. Question

Write the direction cosines of the line whose cartesian equations are 2x = 3y = –z.

Answer

We have

2x = 3y = -z

The equation of the given line can be re-written as,

$$\frac{x}{\frac{1}{2}} = \frac{y}{\frac{1}{3}} = \frac{z}{-1}$$

$$= \frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$$

The direction ratios of the line parallel to AB are proportional to → 3, 2, -6

The direction cosines of the line parallel to AB are proportional to →

$$\frac{3}{\sqrt{3^2+2^2+(-6)^2}}, \frac{2}{\sqrt{3^2+2^2+(-6)^2}}, \frac{-6}{\sqrt{3^2+2^2+(-6)^2}}$$

$$= \frac{3}{7}, \frac{2}{7}, \frac{-6}{7}$$

11. Question

Write the angle between the lines 2x = 3y = –z and 6x = –y = –4z.

Answer

We have,

2x = 3y = -z

6x = -y = -4z

The given lines can be re-written as

These lines are parallel to vectors,

$\overrightarrow{b_1} = 3\hat{i} + 2\hat{j} - 6\hat{k}$ and $\overrightarrow{b_2} = 2\hat{i} - 12\hat{j} - 3\hat{k}$

Let θ be the angle between these lines.

Now,

$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$

$= \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}).(2\hat{i} - 12\hat{j} - 3\hat{k})}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{2^2 + (-12)^2 + (-3)^2}}$

$= \frac{6 - 24 + 18}{\sqrt{9 + 4 + 36}\sqrt{4 + 144 + 9}}$

= 0

$\rightarrow \theta = \frac{\pi}{2}$

12. Question

Write the value of λ for which the lines $\frac{x-3}{-3} = \frac{y+2}{2\lambda} = \frac{z+4}{2}$ and $\frac{x+1}{3\lambda} = \frac{y-2}{1} = \frac{z+6}{-5}$ are perpendicular to each other.

Answer

We have,

$\frac{x-3}{-3} = \frac{y+2}{2\lambda} = \frac{z+4}{2}$

$\frac{x+1}{3\lambda} = \frac{y-2}{1} = \frac{z+6}{-5}$

The given lines are parallel to vectors,

$\vec{b_1} = -3\hat{i} + 2\lambda\hat{j} + 2\hat{k}$ and $\vec{b_2} = 3\lambda\hat{i} + \hat{j} - 5\hat{k}$

Now, for $\vec{b_1} \perp \vec{b_2}$ we must have,

$\vec{b_1}.\vec{b_2} = 0$

$(-3\hat{i} + 2\lambda\hat{j} + 2\hat{k}).(3\lambda\hat{i} + \hat{j} - 5\hat{k}) = 0$

$-7\lambda - 10 = 0$

$\lambda = -\frac{10}{7}$

13. Question

Write the formula for the shortest distance between the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}$.

Answer

The shortest distance d between the parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is given by –

$d = \left|\frac{(\vec{a_2} - \vec{a_1}).\vec{b}}{|\vec{b}|}\right|$

14. Question

Write the condition for the lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ to be intersecting.

Answer

The shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by –

$$d = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

For the lines to be intersecting, d = 0

$$\left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| = 0$$

$$(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2}) = 0$$

15. Question

The cartesian equations of a line AB are $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

Answer

We have,

$$\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$$

The equation of the given line can be re-written as,

$$\frac{x - \frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{y+2}{2} = \frac{z-3}{3}$$

$$= \frac{x - \frac{1}{2}}{\sqrt{3}} = \frac{y+2}{4} = \frac{z-3}{6}$$

The direction ratios of the line parallel to AB are proportional to → $\sqrt{3}$, 4, 6

The direction cosines of the line parallel to AB are proportional to →

$$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{4}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{6}{\sqrt{(\sqrt{3})^2+4^2+6^2}}$$

$$= \frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$$

16. Question

If the equations of a line AB are $\frac{3-x}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$, write the direction ratios of a line parallel to AB.

Answer

We have,

$$\frac{3-x}{1} = \frac{y+2}{-2} = \frac{z-5}{4}$$

The equation of the line AB can be re-written as,

$$\frac{x-3}{-1} = \frac{y+2}{-2} = \frac{z-5}{4}$$

Thus, the direction ratios of the line parallel to AB are proportional to -1, -2, 4.

17. Question

Write the vector equation of a line given by $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$.

Answer

We have,

$$\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$$

The given line passes through the point (5, -4, 6) and has direction ratios proportional to 3, 7, 2.

Vector equation of the given line passing through the point having position vector

$\vec{a}=5\hat{i}-4\hat{j}+6\hat{k}$ and parallel to a vector $\vec{b}=3\hat{i}+7\hat{j}+2\hat{k}$ is →

$$\vec{r}=\vec{a}+\lambda\vec{b}$$

$$\vec{r}=5\hat{i}-4\hat{j}+6\hat{k}+\lambda(3\hat{i}+7\hat{j}+2\hat{k})$$

18. Question

The equations of a line are given by $\frac{4-x}{3}=\frac{y+3}{3}=\frac{z+2}{6}$. Write the direction cosines of a line parallel to this line.

Answer

We have,

$$\frac{4-x}{3}=\frac{y+3}{3}=\frac{z+2}{6}$$

The equation of the given line can be re-written as,

$$\frac{x-4}{-3}=\frac{y+3}{3}=\frac{z+2}{6}$$

The direction ratios of the line parallel to the given line are proportional to → -3, 3, 6

The direction cosines of the line parallel to the given line are proportional to →

$$\frac{-3}{\sqrt{(-3)^2+3^2+6^2}},\frac{3}{\sqrt{(-3)^2+3^2+6^2}},\frac{6}{\sqrt{(-3)^2+3^2+6^2}}$$

$$=\frac{-1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{2}{\sqrt{6}}$$

19. Question

Find the Cartesian equations of the line which passes through the point (–2, 4, –5) and is parallel to the line $\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$.

Answer

The equation of the given line is,

$$\frac{x+3}{3}=\frac{4-y}{5}=\frac{z+8}{6}$$

It can be re-written as,

$$\frac{x+3}{3}=\frac{y-4}{-5}=\frac{z+8}{6}$$

Since the required line is parallel to the given line, the direction ratios of the required line are proportional to 3, -5, 6.

Hence, the Cartesian equations of the line passing through the point (-2, 4, -5) and parallel to a vector having direction

ratios proportional to 3, -5, 6 is →

$$\frac{x+2}{3}=\frac{y-4}{-5}=\frac{z+5}{6}$$

20. Question

Find the angle between the lines $\vec{r}=\left(2\hat{i}-5\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+6\hat{k}\right)$ and $\vec{r}=7\hat{i}-6\hat{k}+\mu\left(\hat{i}+2\hat{j}+2\hat{k}\right)$.

Answer

Let θ be the angle between the given lines.

The given lines are parallel to the vectors $\overrightarrow{b_1}=3\hat{i}+2\hat{j}+6\hat{k}$ and $\overrightarrow{b_2}=\hat{i}+2\hat{j}+2\hat{k}$ respectively.

So, the angle θ between the given lines is given by,

$$\cos\theta=\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$$=\frac{\left(3\hat{i}+2\hat{j}+6\hat{k}\right).\left(\hat{i}+2\hat{j}+2\hat{k}\right)}{\sqrt{3^2+2^2+6^2}\sqrt{1^2+2^2+2^2}}$$

$$=\frac{19}{\sqrt{49}\sqrt{9}}$$

$$=\frac{19}{21}$$

$$\to\theta=\cos^{-1}\frac{19}{21}$$

Thus the angle between the given lines is $\cos^{-1}\frac{19}{21}$.

21. Question

Find the angle between the lines 2x = 3y = –z and 6x = –y = –4z.

Answer

The equations of the given lines can be re-written as –

$\frac{x}{3}=\frac{y}{2}=\frac{z}{-6}$ And $\frac{x}{2}=\frac{y}{-12}=\frac{z}{-3}$

We know that angle between the lines

$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is given by –

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{{a_1}^2+{b_1}^2+{c_1}^2}\times\sqrt{{a_2}^2+{b_2}^2+{c_2}^2}}$$

Let θ be the angle between the given lines,

$$\cos\theta=\frac{3\times2+2\times(-12)+(-6)\times(-3)}{\sqrt{3^2+2^2+(-6)^2}\times\sqrt{2^2+(-12)^2+(-3)^2}}$$

$$=\frac{6-24+18}{\sqrt{49}\sqrt{157}}$$

$$=0$$

$$\to\theta=\frac{\pi}{2}$$

Thus the angle between the given lines is $\frac{\pi}{2}$.

MCQ

1. Question

The angle between the straight lines

$\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$ and $\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$ is

A. 45°

B. 30°

C. 60°

D. 90°

Answer

We have,

$$\frac{x+1}{2}=\frac{y-2}{5}=\frac{z+3}{4}$$

$$\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-3}{-3}$$

The direction ratios of the given lines are proportional to 2,5,4 and 1,2,-3.

The given lines are parallel to the vectors

$\rightarrow \overrightarrow{b_1}=2\hat{\imath}+5\hat{\jmath}+4\hat{k}$ and $\overrightarrow{b_2}=\hat{\imath}+2\hat{\jmath}-3\hat{k}$

Let θ be the angle between the given lines.

Now,

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$$= \frac{(2\hat{\imath}+5\hat{\jmath}+4\hat{k}).(\hat{\imath}+2\hat{\jmath}-3\hat{k})}{\sqrt{2^2+5^2+4^2}\sqrt{1^2+2^2+(-3)^2}}$$

$$= \frac{2+10-12}{\sqrt{45}\sqrt{14}}$$

= 0

$\rightarrow \theta = \frac{\pi}{2} = 90°$

2. Question

The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are

A. coincident

B. skew

C. intersecting

D. parallel

Answer

The equations of the given lines are →

$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \rightarrow (1)$

$\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$

$= \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \rightarrow (2)$

Thus the two lines are parallel to the vector $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and pass through the points

(0,0,0) and (1,2,3).

Now,

$(\vec{a_2} - \vec{a_2}) \times \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \vec{0} \rightarrow (\vec{a} \times \vec{a} = \vec{0})$

Since the distance between the two parallel lines is 0, the given lines are coincident.

3. Question

The direction ratios of the line perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$ are proportional to

A. 4, 5, 7

B. 4, -5, 7

C. 4, -5, -7

D. -4, 5, 7

Answer

We have,

$\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$

$\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-4}{-2}$

The direction ratios of the given lines are proportional to 2,-3,1 and 1,2,-2.

The given lines are parallel to the vectors →

$\vec{b_1} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

Vector perpendicular to the given two lines is →

$\vec{b} = \vec{b_1} \times \vec{b_2}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix}$

$= 4\hat{i} + 5\hat{j} + 7\hat{k}$

Hence, the direction ratios of the line perpendicular to the given two lines are proportional to 4,5,7.

4. Question

The angle between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-1}{2}$ and, $\frac{x-1}{-\sqrt{3}-1} = \frac{y-1}{\sqrt{3}-1} = \frac{z-1}{4}$ is

A. $\cos^{-1}\left(\frac{1}{65}\right)$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer

We have,

$$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{2}$$

$$\frac{x-1}{-\sqrt{3}-1}=\frac{y-1}{\sqrt{3}-1}=\frac{z-1}{4}$$

The direction ratios of the given lines are proportional to 1,1,2 and $-\sqrt{3}$-1, $\sqrt{3}$-1, 4.

The given lines are parallel to the vectors

$\rightarrow \overrightarrow{b_1}=\hat{i}+\hat{j}+2\hat{k}$ and $\overrightarrow{b_2}=\left(-\sqrt{3}-1\right)\hat{i}+\left(\sqrt{3}-1\right)\hat{j}+4\hat{k}$

Let θ be the angle between the given lines.

Now,

$$\cos\theta=\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}$$

$$=\frac{(\hat{i}+\hat{j}+2\hat{k}).\{(-\sqrt{3}-1)\hat{i}+(\sqrt{3}-1)\hat{j}+4\hat{k})\}}{\sqrt{1^2+1+2^2}\sqrt{(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2+4^2}}$$

$$=\frac{-\sqrt{3}-1+\sqrt{3}-1+8}{\sqrt{3}\sqrt{24}}$$

$$=\frac{6}{6\sqrt{2}}$$

$$=\frac{1}{\sqrt{2}}$$

$\rightarrow \theta=\frac{\pi}{3}$

5. Question

The direction ratios of the line x – y + z – 5 = 0 = x – 3y – 6 are proportional to

A. 3, 1, –2

B. 2, –4, 1

C. $\frac{3}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{-2}{\sqrt{14}}$

D. $\frac{2}{\sqrt{41}}, \frac{-4}{\sqrt{41}}, \frac{1}{\sqrt{41}}$

Answer

We have,

x – y + z – 5 = 0 = x – 3y – 6

→ x – y + z – 5 = 0

x – 3y – 6 = 0

→ x – y + z – 5 = 0→ (1)

x = 3y + 6 → (2)

From (1) and (2) we get,

3y + 6 – y + z – 5 = 0

2y + z + 1 = 0

$y = \frac{-z-1}{2}$

Also, $y = \frac{x-6}{3}$ → From (2)

$\therefore \frac{x-6}{3} = y = \frac{-z-1}{2}$

So, the given equation can be re-written as

$\frac{x-6}{3} = \frac{y}{1} = \frac{z+1}{-2}$

Hence the direction ratios of the given line are proportional to 3,1,-2.

6. Question

The perpendicular distance of the point P(1, 2, 3) from the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$ is

A. 7

B. 5

C. 0

D. none of these

Answer

We have,

$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

Let point (1,2,3) be P and the point through which the line passes be Q (6,7,7) → (Given)

Also, the line is parallel to the vector →

$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$ → (Given)

$\overrightarrow{PQ} = 5\hat{i} + 5\hat{j} + 4\hat{k}$

Now,

$$\vec{b} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -2 \\ 5 & 5 & 4 \end{vmatrix}$$

$= 18\hat{i} - 22\hat{j} + 5\hat{k}$

$\rightarrow |\vec{b} \times \overrightarrow{PQ}| = \sqrt{18^2 + (-22)^2 + 5^2}$

$= \sqrt{324 + 484 + 25}$

$= \sqrt{833}$

$\therefore d = \dfrac{|\vec{b} \times \overrightarrow{PQ}|}{|\vec{b}|}$

$= \dfrac{\sqrt{833}}{\sqrt{17}}$

$= \sqrt{49}$

= 7

7. Question

The equation of the line passing through the points $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is

A. $\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

B. $\vec{r} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) - t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

C. $\vec{r} = a_1(1-t)\hat{i} + a_2(1-t)\hat{j} + a_3(1-t)\hat{k} + t\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right)$

D. none of these

Answer

Equation of the line passing through the points having position vectors

$a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is,

$\vec{r} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + t\{(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) - (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\}$ → (Where t is a parameter)

$= (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) - t(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) + t(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$= a_1(1-t)\hat{i} + a_2(1-t)\hat{j} + a_3(1-t)\hat{k} + t(b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

8. Question

If a line makes angle α, β and γ with the axes respectively, then cos 2α + cos 2β + cos 2γ =

A. –2

B. –1

C. 1

D. 2

Answer

If a line makes angles α , β and γ with the axes then,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \rightarrow (1)$

We have,

$\text{Cos}2\alpha + \cos2\beta + \cos2\gamma = 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1 \rightarrow (\cos2\theta = 2\cos^2\theta - 1)$

$= 2(\cos^2\alpha + 2\cos^2\beta + 2\cos^2\gamma) - 3$

$= 2(1) - 3 \rightarrow$ From (1)

$= 2 - 3$

$= -1$

9. Question

If the direction ratios of a line are proportional to 1, –3, 2, the its direction cosines are

A. $\frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

B. $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

C. $-\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

D. $-\frac{1}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}$

Answer

The direction ratios of the line are proportional to 1, -3, 2.

The direction cosines of the lines are →

$\frac{1}{\sqrt{1^2+(-3)^2+2^2}}, \frac{-3}{\sqrt{1^2+(-3)^2+2^2}}, \frac{2}{\sqrt{1^2+(-3)^2+2^2}}$

$= \frac{1}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, \frac{2}{\sqrt{14}}$

10. Question

If a line makes angle $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with x-axis and y-axis respectively, then the angle made by the line with z-axis is

A. π/2

B. π/3

C. π/4

D. 5π/12

Answer

If a line makes angles α , β and γ with the axes then,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \rightarrow (1)$

Here,

$\alpha = \frac{\pi}{3}$

$\beta = \frac{\pi}{4}$

Now,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\gamma = 1$

$\frac{1}{4} + \frac{1}{2} + \cos^2\gamma = 1$

$\frac{3}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = 1 - \frac{3}{4}$

$\cos^2\gamma = \frac{1}{4}$

$\cos\gamma = \frac{1}{2}$

$\gamma = \frac{\pi}{3}$

11. Question

The projections of a line segment on X, Y and Z axes are 12, 4 and 3 respectively. The length and direction cosines of the line segment are

A. $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

B. $19; \frac{12}{19}, \frac{4}{19}, \frac{3}{19}$

C. $11; \frac{12}{11}, \frac{14}{11}, \frac{3}{11}$

D. none of these

Answer

If a line makes angles α , β and γ with the axes then,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \rightarrow (1)$

Let ‘r’ be the length of the line segment.

Then,

$r\cos\alpha = 12,\ r\cos\beta = 4,\ r\cos\gamma = 3 \rightarrow (2)$

Now,

$(r\cos\alpha)^2 + (r\cos\beta)^2 + (r\cos\gamma)^2 = 12^2 + 4^2 + 3^2 \rightarrow$ From (2)

$r^2 (\cos^2\alpha + \cos^2\beta + \cos^2\gamma) = 169$

$r^2 (1) = 169 \rightarrow$ From (1)

$r = \sqrt{169}$

$r = \pm 13$

$r = 13 \rightarrow$ (Since length cannot be negative)

Substituting r = 13 in (2) , we get

$\cos\alpha = \frac{12}{13}$, $\cos\beta = \frac{4}{13}$, $\cos\gamma = \frac{1}{13}$

Thus, the direction cosines of the line are - $\frac{12}{13}, \frac{4}{13}, \frac{1}{13}$.

12. Question

The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are

A. parallel

B. intersecting

C. skew

D. coincident

Answer

The equations of the given lines are

$\frac{x}{1}=\frac{y}{2}=\frac{z}{3}\rightarrow(1)$

$$\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$$

$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}\rightarrow(2)$

Thus, the two lines are parallel to the vector $\vec{b}=\hat{i}+2\hat{j}+3\hat{k}$ and pass through the points (0, 0, 0) and (1, 2, 3).

Now,

$$(\vec{a_1}-\vec{a_2})\times\vec{b}=(\hat{i}+2\hat{j}+3\hat{k})\times(\hat{i}+2\hat{j}+3\hat{k})$$

$$=\vec{0}\rightarrow(\vec{a}\times\vec{a}=\vec{0})$$

Since, the distance between the two parallel lines is 0, the given two lines are coincident lines.

13. Question

The straight line $\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$ is

A. parallel to x-axis

B. parallel to y-axis

C. parallel to z-axis

D. perpendicular to z-axis

Answer

We have,

$$\frac{x-3}{3}=\frac{y-2}{1}=\frac{z-1}{0}$$

Also, the given line is parallel to the vector,

$$\vec{b}=3\hat{i}+\hat{j}+0\hat{k}$$

Let $x\hat{i}+y\hat{j}+z\hat{k}$ be perpendicular to the given line.

Now,

3x + 4y + 0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1).

Hence, the given line is perpendicular to z-axis.

14. Question

The shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is

A. $\sqrt{30}$

B. $2\sqrt{30}$

C. $5\sqrt{30}$

D. $3\sqrt{30}$

Answer

We have,

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \rightarrow (1)$

$\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4} \rightarrow (2)$

We know that line (1) passes through the point (3, 8, 3) and has direction ratios proportional to 3, -1, 1.

It's vector equation is –

$\vec{r}=\vec{a_1}+\lambda\vec{b_1}$, where $\vec{a_1}=3\hat{i}+8\hat{j}+3\hat{k}$ and $\vec{b_1}=3\hat{i}-\hat{j}+\hat{k}$

Also, line (2) passes through the point (-3, -7, 6) and has direction ratios proportional to -3, 2, 4.

It's vector equation is –

$\vec{r}=\vec{a_2}+\nu\vec{b_2}$, where $\vec{a_2}=-3\hat{i}-7\hat{j}+6\hat{k}$ and $\vec{b_2}=-3\hat{i}+2\hat{j}+4\hat{k}$

Now,

$\vec{a_2}-\vec{a_1}=-6\hat{i}-15\hat{j}+3\hat{k}$

$\vec{b_1}\times\vec{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ -3 & 2 & 4\end{vmatrix}$

$=-6\hat{i}-15\hat{j}+3\hat{k}$

$|\vec{b_1}\times\vec{b_2}|=\sqrt{(-6)^2+(-15)^2+3^2}$

$=\sqrt{36+225+9}$

$=\sqrt{270}$

$(\vec{a_2}-\vec{a_1})\times(\vec{b_1}\times\vec{b_2})=(-6\hat{i}-15\hat{j}+3\hat{k})\times(-6\hat{i}-15\hat{j}+3\hat{k})$

= 36 + 225 + 9

= 270

The shortest distance between the lines, $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$ and $\vec{r}=\vec{a_2}+\nu\vec{b_2}$ is given by,

$d=\left|\frac{(\vec{a_2}-\vec{a_1}).(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$

$=\left|\frac{270}{\sqrt{270}}\right|$

$=\sqrt{270}$

$= 3\sqrt{30}$