28. Introduction to 3-D Coordinate Geometry

Exercise 28.1

1. Question

Name the octants in which the following points lie:

(i) (5, 2, 3)

(ii) (-5, 4, 3)

(iii) (4, -3, 5)

(iv) (7, 4, -3)

(v) (-5, -4, 7)

(vi) (-5, -3, -2)

(vii) (2, -5, -7)

(viii) (-7, 2, -5)

Answer

Given: Points are given

To find: name of the octant

Formula used:

<u>**Notation of octants:**</u>

If x, y and z all three are positive, then octant will be XOYZ

If x is negative and y and z are positive, then the octant will be X’OYZ

If y is negative and x and z are positive, then the octant will be XOY’Z

If z is negative and x and y are positive, then the octant will be XOYZ’

If x and y are negative and z is positive, then the octant will be X’OY’Z

If z and y are negative and x is positive, then the octant will be XOY’Z’

If x and z are negative and x is positive, then the octant will be X’OYZ’

If x, y and z all three are negative, then octant will be X’OY’Z’

(i) (5, 2, 3)

In this case, since x, y and z all three are positive then octant will be XOYZ

(ii) (-5, 4, 3)

In this case, since x is negative and y and z are positive then the octant will be X’OYZ

(iii) (4, -3, 5)

In this case, since y is negative and x and z are positive then the octant will be XOY’Z

(iv) (7, 4, -3)

In this case, since z is negative and x and y are positive then the octant will be XOYZ’

(v) (-5, -4, 7)

In this case, since x and y are negative and z is positive then the octant will be X’OY’Z

(vi) (-5, -3, -2)

In this case, since x, y and z all three are negative then octant will be X’OY’Z’

(vii) (2, -5, -7)

In this case, since z and y are negative and x is positive then the octant will be XOY’Z’

(viii) (-7, 2, -5)

In this case, since x and z are negative and x is positive then the octant will be X’OYZ’

2. Question

Find the image of:

(i) (-2, 3, 4) in the yz-plane

(ii) (-5, 4, -3) in the xz-plane

(iii) (5, 2, -7) in the xy-plane

(iv) (-5, 0, 3) in the xz-plane

(v) (-4, 0, 0) in the xy-plane

Answer

(i) **Given:** Point is (-2, 3, 4)

To find: the image of the point in yz-plane

Since we need to find its image in yz-plane, a sign of its x-coordinate will change

So, Image of point (-2, 3, 4) is **(2, 3, 4)**

(ii) **Given:** Point is (-5, 4, -3)

To find: image of the point in xz-plane

Since we need to find its image in xz-plane, sign of its y-coordinate will change

So, Image of point (-5, 4, -3) is **(-5, -4, -3)**

(iii) **Given:** Point is (5, 2, -7)

To find: the image of the point in xy-plane

Since we need to find its image in xy-plane, a sign of its z-coordinate will change

So, Image of point (5, 2, -7) is **(5, 2, 7)**

(iv) **Given:** Point is (-5, 0, 3)

To find: image of the point in xz-plane

Since we need to find its image in xz-plane, sign of its y-coordinate will change

So, Image of point (-5, 0, 3) is **(-5, 0, 3)**

(v) **Given:** Point is (-4, 0, 0)

To find: image of the point in xy-plane

Since we need to find its image in xy-plane, sign of its z-coordinate will change

So, Image of point (-4, 0, 0) is **(-4, 0, 0)**

3. Question

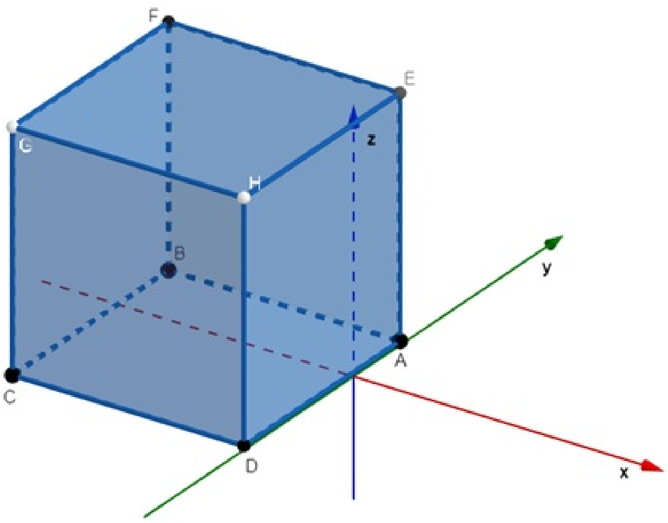
A cube of side 5 has one vertex at the point (1, 0, 1), and the three edges from this vertex are, respectively, parallel to the negative x and y-axes and positive z-axis. Find the coordinates of the other vertices of the cube.

Answer

Given: A cube has side 4 having one vertex at (1, 0, 1)

To find: coordinates of the other vertices of the cube.

Let Point A(1, 0, 1) and AB, AD and AE is parallel to –ve x-axis, -ve y-axis and +ve z-axis respectively



Since side of cube = 5

Point B is (-4, 0, 1)

Point D is (1, -5, 1)

Point E is (1, 0, 6)

Now, EH is parallel to –ve y-axis

⇒ Point H is (1, -5, 6)

HG is parallel to –ve x-axis

⇒ Point G is (-4, -5, 6)

Now, again GC and GF is parallel to –ve z-axis and +ve y-axis respectively

Point C is (-4, -5, 1)

Point F is (-4, 0, 6)

4. Question

Planes are drawn parallel to the coordinates planes through the points (3, 0, -1) and (-2, 5, 4). Find the lengths of the edges of the parallelepiped so formed.

Answer

Given: Points are (3, 0, -1) and (-2, 5, 4)

To find: lengths of the edges of the parallelepiped formed

For point (3, 0, -1)

$x_1 = 3$, $y_1 = 0$ and $z_1 = -1$

For point (-2, 5, 4)

$x_2 = -2$, $y_2 = 5$ and $z_2 = 4$

Plane parallel to coordinate planes of x_1 and x_2 is yz-plane

Plane parallel to coordinate planes of y_1 and y_2 is xz-plane

Plane parallel to coordinate planes of z_1 and z_2 is xy-plane

Distance between planes $x_1 = 3$ and $x_2 = -2$ is 3 – (-2) = 3 + 2 = 5

Distance between planes $x_1 = 0$ and $y_2 = 5$ is 5 – 0 = 5

Distance between planes $z_1 = -1$ and $z_2 = 4$ is 4 – (-1) = 4 + 1 = 5

Hence, **edges of parallelepiped is 5, 5, 5**

5. Question

Planes are drawn through the points (5, 0, 2) and (3, -2, 5) parallel to the coordinate planes. Find the lengths of the edges of the rectangular parallelepiped so formed.

Answer

Given: Points are (5, 0, 2) and (3, -2, 5)

To find: lengths of the edges of the parallelepiped formed

For point (5, 0, 2)

$x_1 = 5$, $y_1 = 0$ and $z_1 = 2$

For point (3, -2, 5)

$x_2 = 3$, $y_2 = -2$ and $z_2 = 5$

Plane parallel to coordinate planes of x_1 and x_2 is yz-plane

Plane parallel to coordinate planes of y_1 and y_2 is xz-plane

Plane parallel to coordinate planes of z_1 and z_2 is xy-plane

Distance between planes $x_1 = 5$ and $x_2 = 3$ is 5 – 3 = 2

Distance between planes $x_1 = 0$ and $y_2 = -2$ is 0 – (-2) = 0 + 2 = 2

Distance between planes $z_1 = 2$ and $z_2 = 5$ is 5 – 2 = 3

Hence, **edges of parallelepiped is 2, 2, 3**

6. Question

Find the distances of the point P (-4, 3, 5) from the coordinate axes.

Answer

Given: Point P(-4, 3, 5)

To find: distances of the point P from coordinate axes

The distance of the point from x-axis will be given by,

$= \sqrt{y^2 + z^2}$

$= \sqrt{3^2 + 5^2}$

$= \sqrt{9 + 25}$

$= \sqrt{34}$

The distance of the point from y-axis will be given by,

$= \sqrt{x^2 + z^2}$

$= \sqrt{(-4)^2 + 5^2}$

$= \sqrt{16 + 25}$

$= \sqrt{41}$

The distance of the point from z-axis will be given by,

$= \sqrt{x^2 + y^2}$

$= \sqrt{(-4)^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

= 5

7. Question

The coordinates of a point are (3, -2, 5). Write down the coordinates of seven points such that the absolute values of their coordinates are the same as those of the coordinates of the given point.

Answer

Given: Point (3, -2, 5)

To find: the coordinates of 7 more points such that the absolute values of all 8 coordinates are the same

Formula used:

Absolute value of any point(x, y, z) is given by,

$\sqrt{x^2 + y^2 + z^2}$

We need to make sure that absolute value to be the same for all points

In the formula of absolute value, there is square of the coordinates. So when we change the sign of any of the coordinates, it will not affect the absolute value.

Let point A(3, -2, 5)

Remaining 7 points are:

Point B(3, 2, 5) (By changing the sign of y coordinate)

Point C(-3, -2, 5) (By changing the sign of x coordinate)

Point D(3, -2, -5) (By changing the sign of z coordinate)

Point E(-3, 2, 5) (By changing the sign of x and y coordinate)

Point F(3, 2, -5) (By changing the sign of y and z coordinate)

Point G(-3, -2, -5) (By changing the sign of x and z coordinate)

Point H(-3, 2, -5) (By changing the sign of x, y and z coordinate)

Exercise 28.2

1 A. Question

Find the distance between the following pairs of points :

P(1, -1, 0) and Q (2, 1, 2)

Answer

Given: P(1, -1, 0) and Q(2, 1, 2)

To find: Distance between given two points

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between (1, -1, 0) and (2, 1, 2) is

$= \sqrt{(1-2)^2+(-1-1)^2+(0-2)^2}$

$= \sqrt{(-1)^2+(-2)^2+(-2)^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

= 3

Hence, **Distance between P and Q is 3 units**

1 B. Question

Find the distance between the following pairs of points :

A(3, 2, -1) and B (-1, -1, -1)

Answer

Given: A(3, 2, -1) and Q(-1, -1, -1)

To find: Distance between given two points

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between (3, 2, -1) and (-1, -1, -1) is

$= \sqrt{(3-(-1))^2+(2-(-1))^2+(-1-(-1))^2}$

$= \sqrt{(3+1)^2+(2+1)^2+(-1+1)^2}$

$= \sqrt{(4)^2+(3)^2+(0)^2}$

$= \sqrt{16+9+0}$

$= \sqrt{25}$

= 5

Hence, **Distance between A and B is 5 units**

2. Question

Find the distance between the points P and Q having coordinates (-2, 3, 1) and (2, 1, 2).

Answer

Given: Points are (-2, 3, 1) and (2, 1, 2)

To find: Distance between given two points

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between (-2, 3, 1) and (2, 1, 2) is

$= \sqrt{(-2-2)^2+(3-1)^2+(1-2)^2}$

$= \sqrt{(-4)^2+(2)^2+(-1)^2}$

$= \sqrt{16+4+1}$

$= \sqrt{21}$

Hence, **Distance between two given points is √21 units**

3 A. Question

Using distance formula prove that the following points are collinear :

A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)

Answer

Given: A(4, -3, -1), B(5, -7, 6) and C(3, 1, -8)

To prove: Points A, B and C are collinear

Formula used:

Points A, B and C are collinear if AB + BC = AC or AB + AC = BC or AC + BC = AB

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between A(4, -3, -1) and B(5, -7, 6) is AB,

$= \sqrt{(4-5)^2+(-3-(-7))^2+(-1-6)^2}$

$= \sqrt{(-1)^2+(-4)^2+(-7)^2}$

$= \sqrt{1+16+49}$

$= \sqrt{66}$

Distance between B(5, -7, 6) and C(3, 1, -8) is BC,

$= \sqrt{(5-3)^2+(-7-1)^2+(6-(-8))^2}$

$= \sqrt{(-2)^2+(-8)^2+(14)^2}$

$= \sqrt{4+64+196}$

$= \sqrt{264}$

$= 2\sqrt{66}$

Distance between A(4, -3, -1) and C(3, 1, -8) is AC,

$= \sqrt{(4-3)^2+(-3-1)^2+(-1-(-8))^2}$

$= \sqrt{(1)^2+(-4)^2+(7)^2}$

$= \sqrt{1 + 16 + 49}$

$= \sqrt{66}$

Clearly,

AB + AC

$= \sqrt{66} + \sqrt{66}$

$= 2\sqrt{66}$

= BC

Hence, **A, B and C are collinear**

3 B. Question

Using distance formula prove that the following points are collinear :

P(0, 7, -7), Q(1, 4, -5) and R(-1, 10, -9)

Answer

Given: P(0, 7, -7), Q(1, 4, -5) and R(-1, 10, -9)

To prove: Points P, Q and R are collinear

Formula used:

Points P, Q and R are collinear if PQ + QR = PR or PQ + PR = QR or PR + QR = PQ

Distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between P(0, 7, -7) and Q(1, 4, -5) is PQ,

$= \sqrt{(0-1)^2 + (7-4)^2 + (-7-(-5))^2}$

$= \sqrt{(-1)^2 + (3)^2 + (-2)^2}$

$= \sqrt{1 + 9 + 4}$

$= \sqrt{14}$

Distance between Q(1, 4, -5) and R(-1, 10, -9) is QR,

$= \sqrt{(1-(-1))^2 + (4-10)^2 + (-5-(-9))^2}$

$= \sqrt{(2)^2 + (-6)^2 + (4)^2}$

$= \sqrt{4 + 36 + 16}$

$= \sqrt{56}$

$= 2\sqrt{14}$

Distance between P(0, 7, -7) and R(-1, 10, -9) is PR,

$= \sqrt{(0-(-1))^2 + (7-10)^2 + (-7-(-9))^2}$

$= \sqrt{(1)^2 + (-3)^2 + (2)^2}$

$= \sqrt{1+9+4}$

$= \sqrt{14}$

Clearly,

PQ + PR

$= \sqrt{14} + \sqrt{14}$

$= 2\sqrt{14}$

= QR

Hence, **P, Q and R are collinear**

3 C. Question

Using distance formula prove that the following points are collinear :

A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

Answer

Given: A(3, -5, 1), B(-1, 0, 8) and C(7, -10, -6)

To prove: Points A, B and C are collinear

Formula used:

Points A, B and C are collinear if AB + BC = AC or AB + AC = BC or AC + BC = AB

Distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between A(3, -5, 1) and B(-1, 0, 8) is AB,

$= \sqrt{(3-(-1))^2 + (-5-0)^2 + (1-8)^2}$

$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$

$= \sqrt{16+25+49}$

$= \sqrt{90}$

$= 3\sqrt{10}$

Distance between B(-1, 0, 8) and C(7, -10, -6) is BC,

$= \sqrt{(-1-7)^2 + (0-(-10))^2 + (8-(-6))^2}$

$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$

$= \sqrt{64+100+196}$

$= \sqrt{360}$

$= 6\sqrt{10}$

Distance between A(3, -5, 1) and C(7, -10, -6) is AC,

$= \sqrt{(3-7)^2 + (-5-(-10))^2 + (1-(-6))^2}$

$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$

$= \sqrt{16 + 25 + 49}$

$= \sqrt{90}$

$= 3\sqrt{10}$

Clearly,

AB + AC

$= 3\sqrt{10} + 3\sqrt{10}$

$= 6\sqrt{10}$

= BC

Hence, **A, B and C are collinear**

4. Question

Determine the points in (i) xy-plane (ii) yz-plane and (iii) zx-plane which are equidistant from the points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1).

Answer

(i) xy-plane

Given: Points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1)

To find: the point on xy-plane which is equidistant from the points

As we know z = 0 in xy-plane.

Let P(x, y, 0) any point in xy-plane

According to the question:

PA = PB = PC

⇒ $PA^2 = PB^2 = PC^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between P(x, y, 0) and A(1, -1, 0) is PA,

$= \sqrt{(x-1)^2 + (y-(-1))^2 + (0-0)^2}$

$= \sqrt{(x-1)^2 + (y+1)^2}$

The distance between P(x, y, 0) and B(2, 1, 2) is PB,

$= \sqrt{(x-2)^2 + (y-1)^2 + (0-2)^2}$

$= \sqrt{(x-2)^2 + (y-1)^2 + 4}$

Distance between P(x, y, 0) and C(3, 2, -1) is PC,

$= \sqrt{(x-3)^2 + (y-2)^2 + (0-(-1))^2}$

$= \sqrt{(x-3)^2 + (y-2)^2 + 1}$

As $PA^2 = PB^2$

$(x - 1)^2 + (y + 1)^2 = (x - 2)^2 + (y - 1)^2 + 4$

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$

$\Rightarrow - 2x + 2 + 2y = 9 - 4x - 2y$

$\Rightarrow - 2x + 2 + 2y - 9 + 4x + 2y = 0$

$\Rightarrow 2x + 4y - 7 = 0$

$\Rightarrow 2x = - 4y + 7$........................(1)

As $PA^2 = PC^2$

$(x - 1)^2 + (y + 1)^2 = (x - 3)^2 + (y - 2)^2 + 1$

$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$

$\Rightarrow - 2x + 2 + 2y = 14 - 6x - 4y$

$\Rightarrow - 2x + 2 + 2y - 14 + 6x + 4y = 0$

$\Rightarrow 4x + 6y - 12 = 0$

$\Rightarrow 2(2x + 3y - 6) = 0$

Put the value of 2x from (1):

$\Rightarrow 7 - 4y + 3y - 6 = 0$

$\Rightarrow - y + 1 = 0$

$\Rightarrow y = 1$

Put this value of y in (1):

$2x = 7 - 4y$

$\Rightarrow 2x = 7 - 4(1)$

$\Rightarrow 2x = 3$

$\Rightarrow x = \frac{3}{2}$

Hence **point** $\mathbf{P\left(\frac{3}{2}, 1, 0\right)}$ in xy-plane is equidistant from A, B and C

(ii) yz-plane

Given: Points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1)

To find: the point on yz-plane which is equidistant from the points

As we know x = 0 in yz-plane.

Let Q(0, y, z) any point in yz-plane

According to the question:

QA = QB = QC

$\Rightarrow QA^2 = QB^2 = QC^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between Q(0, y, z) and A(1, -1, 0) is QA,

$= \sqrt{(0-1)^2+(y-(-1))^2+(z-0)^2}$

$= \sqrt{1+(y+1)^2+z^2}$

The distance between Q(0, y, Z) and B(2, 1, 2) is QB,

$= \sqrt{(0-2)^2+(y-1)^2+(z-2)^2}$

$= \sqrt{(z-2)^2+(y-1)^2+4}$

Distance between Q(0, y, z) and C(3, 2, -1) is QC,

$= \sqrt{(0-3)^2+(y-2)^2+(z-(-1))^2}$

$= \sqrt{(z+1)^2+(y-2)^2+9}$

As $QA^2 = QB^2$

$1 + z^2 + (y + 1)^2 = (z - 2)^2 + (y - 1)^2 + 4$

$\Rightarrow z^2 + 1 + y^2 + 1 + 2y = z^2 + 4 - 4z + y^2 + 1 - 2y + 4$

$\Rightarrow 2 + 2y = 9 - 4z - 2y$

$\Rightarrow 2 + 2y - 9 + 4z + 2y = 0$

$\Rightarrow 4y + 4z - 7 = 0$

$\Rightarrow 4z = -4y + 7$

$\Rightarrow z = \frac{-4y+7}{4} \dots\dots\dots\dots(1)$

As $QA^2 = QC^2$

$1 + z^2 + (y + 1)^2 = (z + 1)^2 + (y - 2)^2 + 9$

$\Rightarrow z^2 + 1 + y^2 + 1 + 2y = z^2 + 1 + 2z + y^2 + 4 - 4y + 9$

$\Rightarrow 2 + 2y = 14 + 2z - 4y$

$\Rightarrow 2 + 2y - 14 - 2z + 4y = 0$

$\Rightarrow -2z + 6y - 12 = 0$

$\Rightarrow 2(-z + 3y - 6) = 0$

Put the value of z from (1):

$\Rightarrow 3y - \frac{(-4y+7)}{4} - 6 = 0$

$\Rightarrow \frac{12y-(-4y+7)-24}{4} = 0$

$\Rightarrow 12y + 4y - 7 - 24 = 0$

$\Rightarrow 16y - 31 = 0$

$\Rightarrow y = \frac{31}{16}$

Put this value of y in (1):

$$z = \frac{-4y + 7}{4}$$

$$\Rightarrow z = \frac{-4\left(\frac{31}{16}\right) + 7}{4}$$

$$\Rightarrow z = \frac{-\frac{124}{16} + 7}{4}$$

$$\Rightarrow z = \frac{\frac{-124+112}{16}}{4}$$

$$\Rightarrow z = \frac{-12}{4 \times 16}$$

$$\Rightarrow z = \frac{-3}{16}$$

Hence **point** $\mathbf{Q\left(0, \frac{31}{16}, -\frac{3}{16}\right)}$ in yz-plane is equidistant from A, B and C

(iii) xz-plane

Given: Points A(1, -1, 0), B(2, 1, 2) and C(3, 2, -1)

To find: the point on xz-plane which is equidistant from the points

As we know y = 0 in xz-plane.

Let R(x, 0, z) any point in xz-plane

According to the question:

RA = RB = RC

$\Rightarrow RA^2 = RB^2 = RC^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

Therefore,

Distance between R(x, 0, z) and A(1, -1, 0) is RA,

$$= \sqrt{(x-1)^2 + (0-(-1))^2 + (z-0)^2}$$

$$= \sqrt{1 + (x-1)^2 + z^2}$$

Distance between R(x, 0, z) and B(2, 1, 2) is RB,

$$= \sqrt{(x-2)^2 + (0-1)^2 + (z-2)^2}$$

$$= \sqrt{(z-2)^2 + (x-2)^2 + 1}$$

Distance between R(x, 0, z) and C(3, 2, -1) is RC,

$$= \sqrt{(x-3)^2 + (0-2)^2 + (z-(-1))^2}$$

$$= \sqrt{(z+1)^2 + (x-3)^2 + 4}$$

As $RA^2 = RB^2$

$1 + z^2 + (x - 1)^2 = (z - 2)^2 + (x - 2)^2 + 1$

$\Rightarrow z^2 + 1 + x^2 + 1 - 2x = z^2 + 4 - 4z + x^2 + 4 - 4x + 1$

$\Rightarrow 2 - 2x = 9 - 4z - 4x$

$\Rightarrow 2 + 4z - 9 + 4x - 2x = 0$

$\Rightarrow 2x + 4z - 7 = 0$

$\Rightarrow 2x = -4z + 7$................................(1)

As $RA^2 = RC^2$

$1 + z^2 + (x - 1)^2 = (z + 1)^2 + (x - 3)^2 + 4$

$\Rightarrow z^2 + 1 + x^2 + 1 - 2x = z^2 + 1 + 2z + x^2 + 9 - 6x + 4$

$\Rightarrow 2 - 2x = 14 + 2z - 6x$

$\Rightarrow 2 - 2x - 14 - 2z + 6x = 0$

$\Rightarrow -2z + 4x - 12 = 0$

$\Rightarrow 2(2x) = 12 + 2z$

Put the value of 2x from (1):

$\Rightarrow 2(-4z + 7) = 12 + 2z$

$\Rightarrow -8z + 14 = 12 + 2z$

$\Rightarrow 14 - 12 = 8z + 2z$

$\Rightarrow 10z = 2$

$$\Rightarrow z = \frac{1}{5}$$

Put this value of z in (1):

$$2x = -4z + 7$$

$$\Rightarrow 2x = -4\left(\frac{1}{5}\right) + 7$$

$$\Rightarrow 2x = -\frac{4}{5} + 7$$

$$\Rightarrow 2x = \frac{-4 + 35}{5}$$

$$\Rightarrow 2x = \frac{31}{5}$$

$$\Rightarrow x = \frac{31}{10}$$

Hence **point** $\mathbf{R}\left(\frac{31}{10}, 0, \frac{1}{5}\right)$ in xz-plane is equidistant from A, B and C

5. Question

Determine the point on z-axis which is equidistant from the points (1, 5, 7) and (5, 1, -4)

Answer

Given: Points are A(1, 5, 7), B(5, 1, -4)

To find: the point on z-axis which is equidistant from the points

As we know x = 0 and y = 0 on z-axis

Let R(0, 0, z) any point on z-axis

According to the question:

RA = RB

⇒ $RA^2 = RB^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between R(0, 0, z) and A(1, 5, 7) is RA,

$= \sqrt{(0-1)^2+(0-5)^2+(z-7)^2}$

$= \sqrt{1+25+(z-7)^2}$

$= \sqrt{26+(z-7)^2}$

Distance between R(0, 0, z) and B(5, 1, -4) is RB,

$= \sqrt{(0-5)^2+(0-1)^2+(z-(-4))^2}$

$= \sqrt{(z+4)^2+25+1}$

$= \sqrt{(z+4)^2+26}$

As $RA^2 = RB^2$

$26 + (z - 7)^2 = (z + 4)^2 + 26$

⇒ $z^2 + 49 - 14z + 26 = z^2 + 16 + 8z + 26$

⇒ 49 – 14z = 16 + 8z

⇒ 49 – 16 = 14z + 8z

⇒ 22z = 33

⇒ $z = \frac{33}{22}$

⇒ $z = \frac{3}{2}$

Hence **point** $\mathbf{R\left(0, 0, \frac{3}{2}\right)}$ on z-axis is equidistant from (1, 5, 7) and (5, 1, -4)

6. Question

Find the point on y-axis which is equidistant from the points (3, 1, 2) and (5, 5, 2).

Answer

Given: Points are A(3, 1, 2) and B(5, 5, 2)

To find: the point on y-axis which is equidistant from the points

As we know x = 0 and z = 0 on y-axis

Let R(0, y, 0) any point on the y-axis

According to the question:

RA = RB

⇒ $RA^2 = RB^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

The distance between R(0, y, 0) and A(3, 1, 2) is RA,

$=\sqrt{(0-3)^2+(y-1)^2+(0-2)^2}$

$=\sqrt{9+4+(y-1)^2}$

$=\sqrt{13+(y-1)^2}$

Distance between R(0, y, 0) and B(5, 5, 2) is RB,

$=\sqrt{(0-5)^2+(y-5)^2+(0-2)^2}$

$=\sqrt{(y-5)^2+25+4}$

$=\sqrt{(y-5)^2+29}$

As $RA^2 = RB^2$

$13 + (y - 1)^2 = (y - 5)^2 + 29$

⇒ $y^2 + 1 - 2y + 13 = y^2 + 25 - 10y + 29$

⇒ 10y – 2y = 54 – 14

⇒ 8y = 40

⇒ $y = \frac{40}{8}$

⇒ y = 5

Hence **point R(0, 5, 0)** on y-axis is equidistant from (3, 1, 2) and (5, 5, 2)

7. Question

Find the points on z-axis which are at a distance $\sqrt{21}$ from the point (1, 2, 3).

Answer

Given: Points A(1, 2, 3)

To find: the point on z-axis which is at distance of $\sqrt{21}$ from the given point

As we know x = 0 and y = 0 on z-axis

Let R(0, 0, z) any point on z-axis

According to question:

$RA = \sqrt{21}$

$\Rightarrow RA^2 = 21$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

Therefore,

Distance between R(0, 0, z) and A(1, 2, 3) is RA,

$$= \sqrt{(0-1)^2 + (0-2)^2 + (z-3)^2}$$

$$= \sqrt{1 + 4 + (z-3)^2}$$

$$= \sqrt{5 + (z-3)^2}$$

As $RA^2 = 21$

$5 + (z - 3)^2 = 21$

$\Rightarrow z^2 + 9 - 6z + 5 = 21$

$\Rightarrow z^2 - 6z = 21 - 14$

$\Rightarrow z^2 - 6z - 7 = 0$

$\Rightarrow z^2 - 7z + z - 7 = 0$

$\Rightarrow z(z - 7) + 1(z - 7) = 0$

$\Rightarrow (z - 7)(z + 1) = 0$

$\Rightarrow (z - 7) = 0$ or $(z + 1) = 0$

$\Rightarrow z = 7$ or $z = -1$

Hence **points (0, 0, 7) and (0, 0, -1)** on z-axis is equidistant from (1, 2, 3)

8. Question

Prove that the triangle formed by joining the three points whose coordinates are (1, 2, 3), (2, 3, 1) and (3, 1, 2) is an equilateral triangle.

Answer

Given: Points are A(1, 2, 3), B(2, 3, 1) and C(3, 1, 2)

To prove: the triangle formed by given points is an equilateral triangle

An equilateral triangle is a triangle whose all sides are equal

So we need to prove AB = BC = AC

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

Therefore,

The distance between A(1, 2, 3) and B(2, 3, 1) is AB,

$$= \sqrt{(1-2)^2 + (2-3)^2 + (3-1)^2}$$

$$= \sqrt{(-1)^2 + (-1)^2 + 2^2}$$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

Distance between B(2, 3, 1) and C(3, 1, 2) is BC,

$= \sqrt{(2-3)^2 + (3-1)^2 + (1-2)^2}$

$= \sqrt{(-1)^2 + 2^2 + (-1)^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

The distance between A(1, 2, 3) and C(3, 1, 2) is AC,

$= \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2}$

$= \sqrt{(-2)^2 + 1^2 + 1^2}$

$= \sqrt{1 + 1 + 4}$

$= \sqrt{6}$

Clearly,

AB = BC = AC

Thus, **Δ ABC is a equilateral triangle**

Hence Proved

9. Question

Show that the points (0, 7, 10), (-1, 6, 6) and (-4, 9, 6) are the vertices of an isosceles right-angled triangle.

Answer

Given: Points are A(0, 7, 10), B(-1, 6, 6) and C(-4, 9, 6)

To prove: the triangle formed by given points is an isosceles right-angled triangle

Isosceles right-angled triangle is a triangle whose two sides are equal and also satisfies Pythagoras Theorem

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between A(0, 7, 10) and B(-1, 6, 6) is AB,

$= \sqrt{(0-(-1))^2 + (7-6)^2 + (10-6)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between B(-1, 6, 6) and C(-4, 9, 6) is BC,

$= \sqrt{(-1-(-4))^2 + (6-9)^2 + (6-6)^2}$

$= \sqrt{3^2 + (-3)^2 + 0^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between A(0, 7, 10) and C(-4, 9, 6) is AC,

$= \sqrt{(0-(-4))^2 + (7-9)^2 + (10-6)^2}$

$= \sqrt{4^2 + (-2)^2 + 4^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

= 6

Since, AB = BC

$AB^2 + BC^2$

$= (3\sqrt{2})^2 + (3\sqrt{2})^2$

= 18 + 18

= 36

$= AC^2$

As, AB = BC and $AB^2 + BC^2 = AC^2$

Thus, **Δ ABC is an isosceles-right angled triangle**

Hence Proved

10. Question

Show that the points A(3, 3, 3), B(0, 6, 3), C(1, 7, 7) and D(4, 4, 7) are the vertices of squares.

Answer

Given: Points are A(3, 3, 3), B(0, 6, 3), C(1, 7, 7) and D(4, 4, 7)

To prove: the quadrilateral formed by these 4 points is a square

All sides of a square are equal

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

The distance between A(3, 3, 3) and B(0, 6, 3) is AB,

$= \sqrt{(3-0)^2 + (3-6)^2 + (3-3)^2}$

$= \sqrt{3^2 + 3^2 + 0^2}$

$= \sqrt{9 + 9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between B(0, 6, 3) and C(1, 7, 7) is BC,

$= \sqrt{(0 - 1)^2 + (6 - 7)^2 + (3 - 7)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between C(1, 7, 7) and D(4, 4, 7) is CD,

$= \sqrt{(1 - 4)^2 + (7 - 4)^2 + (7 - 7)^2}$

$= \sqrt{3^2 + 3^2 + 0^2}$

$= \sqrt{9 + 9 + 0}$

$= \sqrt{18}$

$= 3\sqrt{2}$

The distance between A(3, 3, 3) and D(4, 4, 7) is AD,

$= \sqrt{(3 - 4)^2 + (3 - 4)^2 + (3 - 7)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Clearly,

AB = BC = CD = AD

Thus, **Quadrilateral formed by ABCD is a square**

Hence Proved

11. Question

Prove that the point A(1, 3, 0), B(-5, 5, 2), C(-9, -1, 2) and D(-3, -3, 0) taken in order are the vertices of a parallelogram. Also, show that ABCD is not a rectangle.

Answer

Given: Points are A(1, 3, 0), B(-5, 5, 2), C(-9, -1, 2) and D(-3, -3, 0)

To prove: the quadrilateral formed by these 4 points is a parallelogram but not a rectangle

Opposite sides of both parallelogram and rectangle are equal

But diagonals of a parallelogram are not equal whereas they are equal for rectangle

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between A(1, 3, 0) and B(-5, 5, 2) is AB,

$=\sqrt{(1-(-5))^2+(3-5)^2+(0-2)^2}$

$=\sqrt{6^2+2^2+2^2}$

$=\sqrt{36+4+4}$

$=\sqrt{44}$

$=2\sqrt{11}$

Distance between B(-5, 5, 2) and C(-9, -1, 2) is BC,

$=\sqrt{(-5-(-9))^2+(5-(-1))^2+(2-2)^2}$

$=\sqrt{4^2+6^2+0^2}$

$=\sqrt{16+36+0}$

$=\sqrt{52}$

$=2\sqrt{13}$

Distance between C(-9, -1, 2) and D(-3, -3, 0) is CD,

$=\sqrt{(-9-(-3))^2+(-1-(-3))^2+(2-0)^2}$

$=\sqrt{6^2+2^2+2^2}$

$=\sqrt{36+4+4}$

$=\sqrt{44}$

$=2\sqrt{11}$

Distance between A(1, 3, 0) and D(-3, -3, 0) is AD,

$=\sqrt{(1-(-3))^2+(3-(-3))^2+(0-0)^2}$

$=\sqrt{4^2+6^2+0^2}$

$=\sqrt{16+36+0}$

$=\sqrt{52}$

$=2\sqrt{13}$

Clearly,

AB = CD

BC = AD

Opposite sides are equal

Now, we will find length of diagonals

Distance between A(1, 3, 0) and C(-9, -1, 2) is AC,

$= \sqrt{(1-(-9))^2 + (3-(-1))^2 + (0-2)^2}$

$= \sqrt{10^2 + 4^2 + 2^2}$

$= \sqrt{100 + 16 + 4}$

$= \sqrt{120}$

$= 2\sqrt{30}$

Distance between B(-5, 5, 2) and D(-3, -3, 0) is BD,

$= \sqrt{(-5-(-3))^2 + (5-(-3))^2 + (2-0)^2}$

$= \sqrt{(-2)^2 + 8^2 + 2^2}$

$= \sqrt{4 + 64 + 4}$

$= \sqrt{72}$

$= 6\sqrt{2}$

Clearly,

AC $\neq$ BD

The diagonals are not equal, but opposite sides are equal

Thus, **Quadrilateral formed by ABCD is a parallelogram but not a rectangle**

Hence Proved

12. Question

Show that the points A(1, 3, 4), B(-1, 6, 10), C(-7, 4, 7) and D(-5, 1, 1) are the vertices of a rhombus.

Answer

Given: Points are A(1, 3, 4), B(-1, 6, 10), C(-7, 4, 7) and D(-5, 1, 1)

To prove: the quadrilateral formed by these 4 points is a rhombus

All sides of both square and rhombus are equal

But diagonals of a rhombus are not equal whereas they are equal for square

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between A(1, 3, 4) and B(-1, 6, 10) is AB,

$= \sqrt{(1-(-1))^2 + (3-6)^2 + (4-10)^2}$

$= \sqrt{2^2 + (-3)^2 + (-6)^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

Distance between B(-1, 6, 10) and C(-7, 4, 7) is BC,

$= \sqrt{(-1-(-7))^2 + (6-4)^2 + (10-7)^2}$

$= \sqrt{6^2 + 2^2 + 3^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

= 7

Distance between C(-7, 4, 7) and D(-5, 1, 1) is CD,

$= \sqrt{(-7-(-5))^2 + (4-1)^2 + (7-1)^2}$

$= \sqrt{(-2)^2 + 3^2 + 6^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

= 7

Distance between A(1, 3, 4) and D(-5, 1, 1) is AD,

$= \sqrt{(1-(-5))^2 + (3-1)^2 + (4-1)^2}$

$= \sqrt{6^2 + 2^2 + 3^2}$

$= \sqrt{36+4+9}$

$= \sqrt{49}$

= 7

Clearly,

AB = BC = CD = AD

All sides are equal

Now, we will find length of diagonals

Distance between A(1, 3, 4) and C(-7, 4, 7) is AC,

$= \sqrt{(1-(-7))^2 + (3-4)^2 + (4-7)^2}$

$= \sqrt{8^2 + (-1)^2 + (-3)^2}$

$= \sqrt{64+1+9}$

$= \sqrt{74}$

Distance between B(-1, 6, 10) and D(-5, 1, 1) is BD,

$= \sqrt{(-1-(-5))^2 + (6-1)^2 + (10-1)^2}$

$= \sqrt{4^2 + 5^2 + 9^2}$

$= \sqrt{16 + 25 + 81}$

$= \sqrt{112}$

$= 4\sqrt{7}$

Clearly,

AC $\neq$ BD

The diagonals are not equal but all sides are equal

Thus, **Quadrilateral formed by ABCD is a rhombus but not square**

Hence Proved

13. Question

Prove that the tetrahedron with vertices at the points O(0, 0, 0), A(0, 1, 1), B(1, 0, 1) and C(1, 1, 0) is a regular one.

Answer

Given: Points are O(0, 0, 0), A(0, 1, 1), B(1, 0, 1) and C(1, 1, 0)

To prove: given points are forming a regular tetrahedron

All edges of a regular tetrahedron are equal

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a - m)^2 + (b - n)^2 + (c - o)^2}$

Therefore,

Distance between O(0, 0, 0) and A(0, 1, 1) is OA,

$= \sqrt{(0 - 0)^2 + (0 - 1)^2 + (0 - 1)^2}$

$= \sqrt{0^2 + (-1)^2 + (-1)^2}$

$= \sqrt{0 + 1 + 1}$

$= \sqrt{2}$

Distance between O(0, 0, 0) and B(1, 0, 1) is OB,

$= \sqrt{(0 - 1)^2 + (0 - 0)^2 + (0 - 1)^2}$

$= \sqrt{0^2 + (-1)^2 + (-1)^2}$

$= \sqrt{0 + 1 + 1}$

$= \sqrt{2}$

Distance between O(0, 0, 0) and C(1, 1, 0) is OC,

$= \sqrt{(0 - 1)^2 + (0 - 1)^2 + (0 - 0)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + 0^2}$

$= \sqrt{1 + 1 + 0}$

$= \sqrt{2}$

Distance between A(0, 1, 1) and B(1, 0, 1) is AB,

$= \sqrt{(0-1)^2 + (1-0)^2 + (1-1)^2}$

$= \sqrt{(-1)^2 + 1^2 + 0^2}$

$= \sqrt{1 + 1 + 0}$

$= \sqrt{2}$

Distance between B(1, 0, 1) and C(1, 1, 0) is BC,

$= \sqrt{(1-1)^2 + (0-1)^2 + (1-0)^2}$

$= \sqrt{0^2 + (-1)^2 + 1^2}$

$= \sqrt{0 + 1 + 1}$

$= \sqrt{2}$

Distance between A(0, 1, 1) and C(1, 1, 0) is AC,

$= \sqrt{(0-1)^2 + (1-1)^2 + (1-0)^2}$

$= \sqrt{(-1)^2 + 0^2 + 1^2}$

$= \sqrt{1 + 0 + 1}$

$= \sqrt{2}$

Clearly,

AB = BC = AC = OA = OB = OC

All edges are equal

Thus, **A, B, C and O forms a regular tetrahedron**

Hence Proved

14. Question

Show that the points (3, 2, 2), (-1, 4, 2), (0, 5, 6), (2, 1, 2) lie on a sphere whose centre is (1, 3, 4). Find also its radius.

Answer

Given: Points are A(3, 2, 2), B(-1, 4, 2), C(0, 5, 6), D(2, 1, 2)

To prove: given points lie on sphere whose centre is (1, 3, 4)

To find: radius of sphere

Let Center is O(1, 3, 4)

Since O is centre of sphere and A, B, C, D lie on a sphere

⇒ OA = OB = OC = OD = radius

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

The distance between O(1, 3, 4) and A(3, 2, 2) is OA,

$= \sqrt{(1-3)^2 + (3-2)^2 + (4-2)^2}$

$= \sqrt{(-2)^2 + 1^2 + 2^2}$

$= \sqrt{4+1+4}$

$= \sqrt{9}$

= 3

Distance between O(1, 3, 4) and B(-1, 4, 2) is OB,

$= \sqrt{(1-(-1))^2 + (3-4)^2 + (4-2)^2}$

$= \sqrt{2^2 + (-1)^2 + 2^2}$

$= \sqrt{4+1+4}$

$= \sqrt{9}$

= 3

Distance between O(1, 3, 4) and C(0, 5, 6) is OC,

$= \sqrt{(1-0)^2 + (3-5)^2 + (4-6)^2}$

$= \sqrt{1^2 + (-2)^2 + (-2)^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

= 3

Distance between O(1, 3, 4) and D(2, 1, 2) is OD,

$= \sqrt{(1-2)^2 + (3-1)^2 + (4-2)^2}$

$= \sqrt{(-1)^2 + 2^2 + 2^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

= 3

Clearly,

OA = OB = OC = OD = 3 units

Therefore, **radius of sphere = 3 units and A, B, C, D lie on sphere having centre O**

15. Question

Find the coordinates of the point which is equidistant from the four points O(0, 0, 0), A(2, 0, 0), B(0, 3, 0) and C(0, 0, 8).

Answer

Given: Points are O(0, 0, 0), A(2, 0, 0), B(0, 3, 0) and C(0, 0, 8)

To find: the coordinates of point which is equidistant from the points

Let required point P(x, y, z)

According to question:

PA = PB = PC = PO

⇒ $PA^2 = PB^2 = PC^2 = PO^2$

Formula used:

Distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

The distance between P(x, y, z) and O(0, 0, 0) is PO,

$= \sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$

$= \sqrt{x^2+y^2+z^2}$

Distance between P(x, y, z) and A(2, 0, 0) is PA,

$= \sqrt{(x-2)^2+(y-0)^2+(z-0)^2}$

$= \sqrt{(x-2)^2+y^2+z^2}$

Distance between P(x, y, z) and B(0, 3, 0) is PB,

$= \sqrt{(x-0)^2+(y-3)^2+(z-0)^2}$

$= \sqrt{x^2+(y-3)^2+z^2}$

Distance between P(x, y, z) and C(0, 0, 8) is PC,

$= \sqrt{(x-0)^2+(y-0)^2+(z-8)^2}$

$= \sqrt{x^2+y^2+(z-8)^2}$

As $PO^2 = PA^2$

$x^2 + y^2 + z^2 = (x - 2)^2 + y^2 + z^2$

⇒ $x^2 = x^2 + 4 - 4x$

⇒ 4x = 4

⇒ x = 1

As $PO^2 = PB^2$

$x^2 + y^2 + z^2 = x^2 + (y - 3)^2 + z^2$

⇒ $y^2 = y^2 + 9 - 6y$

⇒ 6y = 9

$\Rightarrow\ y = \frac{9}{6}$

$\Rightarrow\ y = \frac{3}{2}$

As $PO^2 = PC^2$

$x^2 + y^2 + z^2 = x^2 + y^2 + (z - 8)^2$

⇒ $z^2 = z^2 + 64 – 16x$

⇒ $16z = 64$

⇒ $z = 4$

Hence **point** $\mathbf{P\left(1, \frac{3}{2}, 4\right)}$ is equidistant from given points

16. Question

If A(-2, 2, 3) and B(13, -3, 13) are two pints. Find the locus of a point P which moves in such a way that 3PA = 2PB.

Answer

Given: Points are A(-2, 2, 3) and B(13, -3, 13)

To find: the locus of point P which moves in such a way that 3PA = 2PB

Let the required point P(x, y, z)

According to the question:

3PA = 2PB

⇒ $9PA^2 = 4PB^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

Therefore,

The distance between P(x, y, z) and A(-2, 2, 3) is PA,

$$= \sqrt{(x-(-2))^2 + (y-2)^2 + (z-3)^2}$$

$$= \sqrt{(x+2)^2 + (y-2)^2 + (z-3)^2}$$

The distance between P(x, y, z) and B(13, -3, 13) is PB,

$$= \sqrt{(x-13)^2 + (y-(-3))^2 + (z-13)^2}$$

$$= \sqrt{(x-13)^2 + (y+3)^2 + (z-13)^2}$$

As $9PA^2 = 4PB^2$

$9\{(x + 2)^2 + (y – 2)^2 + (z – 3)^2\} = 4\{(x – 13)^2 + (y + 3)^2 + (z – 13)^2\}$

⇒ $9\{x^2 + 4 + 4x + y^2 + 4 – 4y + z^2 + 9 – 6z\} = 4\{x^2 + 169 – 26x + y^2 + 9 + 6y + z^2 + 169 – 26z\}$

⇒ $9\{x^2 + 4x + y^2 – 4y + z^2 – 6z + 17\} = 4\{x^2 – 26x + y^2 + 6y + z^2 – 26z + 347\}$

⇒ $9x^2 + 36x + 9y^2 – 36y + 9z^2 – 54z + 153 = 4x^2 – 104x + 4y^2 + 24y + 4z^2 – 104z + 1388$

⇒ $9x^2 + 36x + 9y^2 – 36y + 9z^2 – 54z + 153 – 4x^2 + 104x – 4y^2 – 24y – 4z^2 + 104z – 1388 = 0$

⇒ $5x^2 + 5y^2 + 5z^2 + 140x – 60y + 50z – 1235 = 0$

Hence **locus of point P is** $\mathbf{5x^2 + 5y^2 + 5z^2 + 140x – 60y + 50z – 1235 = 0}$

17. Question

Find the locus of P if $PA^2 + PB^2 = 2k^2$, where A and B are the points (3, 4, 5) and (-1, 3, -7).

Answer

Given: Points are A(3, 4, 5) and B(-1, 3, -7)

To find: the locus of point P which moves in such a way that $PA^2 + PB^2 = 2k^2$

Let the required point P(x, y, z)

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

Therefore,

The distance between P(x, y, z) and A(3, 4, 5) is PA,

$$=\sqrt{(x-3)^2+(y-4)^2+(z-5)^2}$$

Distance between P(x, y, z) and B(-1, 3, -7) is PB,

$$=\sqrt{(x-(-1))^2+(y-3)^2+(z-(-7))^2}$$

$$=\sqrt{(x+1)^2+(y-3)^2+(z+7)^2}$$

According to question:

$PA^2 + PB^2 = 2k^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 + (z - 5)^2 + (x + 1)^2 + (y - 3)^2 + (z + 7)^2 = 2k^2$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 - 10z + x^2 + 1 + 2x + y^2 + 9 - 6y + z^2 + 49 + 14z = 2k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2$

$\Rightarrow 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - 2k^2 = 0$

Hence **locus of point P is $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - 2k^2 = 0$**

18. Question

Show that the points (a, b, c), (b, c, a) and (c, a, b) are the vertices of an equilateral triangle.

Answer

Given: Points are A(a, b, c), B(b, c, a) and C(c, a, b)

To prove: the triangle formed by given points is an equilateral triangle

An equilateral triangle is a triangle whose all sides are equal

So we need to prove AB = BC = AC

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

Therefore,

The distance between A(a, b, c) and B(b, c, a) is AB,

$$=\sqrt{(a-b)^2+(b-c)^2+(c-a)^2}$$

$$=\sqrt{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac}$$

$$=\sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ac}$$

The distance between B(b, c, a) and C(c, a, b) is AB,

$= \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

$= \sqrt{b^2 + c^2 - 2bc + c^2 + a^2 - 2ac + a^2 + b^2 - 2ab}$

$= \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$

The distance between A(a, b, c) and C(c, a, b) is AB,

$= \sqrt{(a-c)^2 + (b-a)^2 + (c-b)^2}$

$= \sqrt{c^2 + a^2 - 2ac + a^2 + b^2 - 2ab + b^2 + c^2 - 2bc}$

$= \sqrt{2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ac}$

Clearly,

AB = BC = AC

Thus, **Δ ABC is a equilateral triangle**

Hence Proved

19. Question

Are the points A(3, 6, 9), B(10, 20, 30) and C(25, -41, 5), the vertices of a right-angled triangle?

Answer

Given: Points are A(3, 6, 9), B(10, 20, 30) and C(25, -41, 5)

To check: the triangle formed by given points is a right-angled triangle or not

A right-angled triangle is a triangle which satisfies Pythagoras Theorem

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

The distance between A(3, 6, 9) and B(10, 20, 30) is AB,

$= \sqrt{(3-10)^2 + (6-20)^2 + (9-30)^2}$

$= \sqrt{(-7)^2 + (-14)^2 + (-21)^2}$

$= \sqrt{49 + 196 + 441}$

$= \sqrt{686}$

Distance between B(10, 20, 30) and C(25, -41, 5) is BC,

$= \sqrt{(10-25)^2 + (20-(-41))^2 + (30-5)^2}$

$= \sqrt{(-15)^2 + 61^2 + 25^2}$

$= \sqrt{225 + 3721 + 625}$

$= \sqrt{4571}$

Distance between A(3, 6, 9) and C(25, -41, 5) is AC,

$= \sqrt{(3-25)^2 + (6-(-41))^2 + (9-5)^2}$

$= \sqrt{(-22)^2 + 47^2 + 4^2}$

$= \sqrt{484 + 2209 + 16}$

$= \sqrt{2709}$

$AB^2 + BC^2$

$= \left(\sqrt{686}\right)^2 + \left(\sqrt{4571}\right)^2$

= 686 + 4571

= 5257

$\neq AC^2$

$AB^2 + AC^2$

$= \left(\sqrt{686}\right)^2 + \left(\sqrt{2709}\right)^2$

= 686 + 2709

= 3395

$\neq BC^2$

$AC^2 + BC^2$

$= \left(\sqrt{2709}\right)^2 + \left(\sqrt{4571}\right)^2$

= 2709 + 4571

= 7280

$\neq AB^2$

As, $AB^2 + BC^2 \neq AC^2$

$AC^2 + BC^2 \neq AB^2$

$AB^2 + AC^2 \neq BC^2$

Thus, **Δ ABC is not a right angled triangle**

20 A. Question

Verify the following:

(0, 7, -10), (1, 6, -6) and (4, 9, -6) are vertices of an isosceles triangle.

Answer

Given: Points are A(0, 7, -10), B(1, 6, -6) and C(4, 9, -6)

To prove: the triangle formed by given points is an isosceles triangle

Isosceles right-angled triangle is a triangle whose two sides are equal

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between A(0, 7, -10) and B(1, 6, -6) is AB,

$= \sqrt{(0-1)^2 + (7-6)^2 + (-10-(-6))^2}$

$= \sqrt{(-1)^2 + 1^2 + (-4)^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between B(1, 6, -6) and C(4, 9, -6)is BC,

$= \sqrt{(1-4)^2 + (6-9)^2 + (-6-(-6))^2}$

$= \sqrt{(-3)^2 + (-3)^2 + 0^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between A(0, 7, -10) and C(4, 9, -6) is AC,

$= \sqrt{(0-4)^2 + (7-9)^2 + (-10-(-6))^2}$

$= \sqrt{(-4)^2 + (-2)^2 + (-4)^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

= 6

Clearly,

AB = BC

Thus, **Δ ABC is an isosceles triangle**

Hence Proved

20 B. Question

Verify the following:

(0, 7, 10), (-1, 6, 6) and (2, -3, 4) are vertices of a right-angled triangle

Answer

Given: Points are A(0, 7, 10), B(-1, 6, 6) and C(-4, 9, 6)

To prove: the triangle formed by given points is a right-angled triangle

Right-angled triangle satisfies Pythagoras Theorem

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between A(0, 7, 10) and B(-1, 6, 6) is AB,

$= \sqrt{(0-(-1))^2+(7-6)^2+(10-6)^2}$

$= \sqrt{1^2+1^2+4^2}$

$= \sqrt{1+1+16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between B(-1, 6, 6) and C(-4, 9, 6) is BC,

$= \sqrt{(-1-(-4))^2+(6-9)^2+(6-6)^2}$

$= \sqrt{3^2+(-3)^2+0^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Distance between A(0, 7, 10) and C(-4, 9, 6) is AC,

$= \sqrt{(0-(-4))^2+(7-9)^2+(10-6)^2}$

$= \sqrt{4^2+(-2)^2+4^2}$

$= \sqrt{16+4+16}$

$= \sqrt{36}$

= 6

$AB^2 + BC^2$

$= (3\sqrt{2})^2 + (3\sqrt{2})^2$

= 18 + 18

= 36

$= AC^2$

As, $AB^2 + BC^2 = AC^2$

Thus, **Δ ABC is a right angled triangle**

Hence Proved

20 C. Question

Verify the following:

(-1, 2, 1), (1,-2, 5), (4, -7, 8) and (2, -3, 4) are vertices of a parallelogram.

Answer

Given: Points are A(-1, 2, 1), B(1,-2, 5), C(4, -7, 8) and D(2, -3, 4)

To prove: the quadrilateral formed by these 4 points is a parallelogram

Opposite sides of a parallelogram are equal, but diagonals are not equal

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between A(-1, 2, 1) and B(1,-2, 5) is AB,

$= \sqrt{(-1-1)^2+(2-(-2))^2+(1-5)^2}$

$= \sqrt{(-2)^2+4^2+(-4)^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

= 6

Distance between B(1,-2, 5) and C(4, -7, 8) is BC,

$= \sqrt{(1-4)^2+(-2-(-7))^2+(5-8)^2}$

$= \sqrt{(-3)^2+5^2+(-3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Distance between C(4, -7, 8) and D(2, -3, 4) is CD,

$= \sqrt{(4-2)^2+(-7-(-3))^2+(8-4)^2}$

$= \sqrt{2^2+(-4)^2+4^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

= 6

Distance between A(-1, 2, 1) and D(2, -3, 4) is AD,

$= \sqrt{(-1-2)^2+(2-(-3))^2+(1-4)^2}$

$= \sqrt{(-3)^2+5^2+(-3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Clearly,

AB = CD

BC = AD

Opposite sides are equal

Now, we will find the length of diagonals

Distance between A(-1, 2, 1) and C(4, -7, 8) is AC,

$= \sqrt{(-1-4)^2+(2-(-7))^2+(1-8)^2}$

$= \sqrt{(-5)^2 + 9^2 + (-7)^2}$

$= \sqrt{25 + 81 + 49}$

$= \sqrt{155}$

Distance between B(1, -2, 5) and D(2, -3, 4) is BD,

$= \sqrt{(1-2)^2 + (-2-(-3))^2 + (5-4)^2}$

$= \sqrt{(-1)^2 + 1^2 + 1^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

Clearly,

AC $\neq$ BD

The diagonals are not equal, but opposite sides are equal

Thus, **Quadrilateral formed by ABCD is a parallelogram**

Hence Proved

20 D. Question

Verify the following:

(5, -1, 1), (7, -4, 7), (1, -6, 10) and (-1, -3, 4) are the vertices of a rhombus.

Answer

Given: Points are A(5, -1, 1), B(7, -4, 7), C(1, -6, 10) and D(-1, -3, 4)

To prove: the quadrilateral formed by these 4 points is a rhombus

All sides of both square and rhombus are equal

But diagonals of a rhombus are not equal whereas they are equal for square

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

Distance between A(5, -1, 1) and B(7, -4, 7) is AB,

$= \sqrt{(5-7)^2 + (-1-(-4))^2 + (1-7)^2}$

$= \sqrt{(-2)^2 + 3^2 + (-6)^2}$

$= \sqrt{4+9+36}$

$= \sqrt{49}$

= 7

Distance between B(7, -4, 7) and C(1, -6, 10) is BC,

$= \sqrt{(7-1)^2 + (-4-(-6))^2 + (7-10)^2}$

$= \sqrt{6^2 + 2^2 + (-3)^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

= 7

Distance between C(1, -6, 10) and D(-1, -3, 4) is CD,

$= \sqrt{(1 - (-1))^2 + (-6 - (-3))^2 + (10 - 4)^2}$

$= \sqrt{2^2 + (-3)^2 + 6^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

= 7

Distance between A(5, -1, 1) and D(-1, -3, 4) is AD,

$= \sqrt{(5 - (-1))^2 + (-1 - (-3))^2 + (1 - 4)^2}$

$= \sqrt{6^2 + 2^2 + (-3)^2}$

$= \sqrt{36 + 4 + 9}$

$= \sqrt{49}$

= 7

Clearly,

AB = BC = CD = AD

All sides are equal

Now, we will find length of diagonals

Distance between A(5, -1, 1) and C(1, -6, 10) is AC,

$= \sqrt{(5 - 1)^2 + (-1 - (-6))^2 + (1 - 10)^2}$

$= \sqrt{6^2 + 5^2 + (-9)^2}$

$= \sqrt{36 + 25 + 81}$

$= \sqrt{142}$

Distance between B(7, -4, 7) and D(-1, -3, 4) is BD,

$= \sqrt{(7 - (-1))^2 + (-4 - (-3))^2 + (7 - 4)^2}$

$= \sqrt{8^2 + (-1)^2 + 3^2}$

$= \sqrt{64 + 1 + 9}$

$= \sqrt{74}$

Clearly,

AC $\neq$ BD

The diagonals are not equal, but all sides are equal

Thus, **Quadrilateral formed by ABCD is a rhombus**

Hence Proved

21. Question

Find the locus of the points which are equidistant from the points (1, 2, 3) and (3, 2, -1).

Answer

Given: Points are A(1, 2, 3) and B(3, 2, -1)

To find: the locus of points which are equidistant from the given points

Let the required point P(x, y, z)

According to the question:

PA = PB

$\Rightarrow PA^2 = PB^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

The distance between P(x, y, z) and A(1, 2, 3) is PA,

$= \sqrt{(x-1)^2 + (y-2)^2 + (z-3)^2}$

The distance between P(x, y, z) and B(3, 2, -1) is PB,

$= \sqrt{(x-3)^2 + (y-2)^2 + (z-(-1))^2}$

$= \sqrt{(x-3)^2 + (y-2)^2 + (z+1)^2}$

As $PA^2 = PB^2$

$(x - 1)^2 + (y - 2)^2 + (z - 3)^2 = (x - 3)^2 + (y - 2)^2 + (z + 1)^2$

$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$

$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z - x^2 - 9 + 6x - y^2 - 4 + 4y - z^2 - 1 - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow 4(x - 2z) = 0$

$\Rightarrow x - 2z = 0$

Hence **locus of point P is x – 2z = 0**

22. Question

Find the locus of the point, the sum of whose distances from the points A(4, 0, 0) and B(-4, 0, 0) is equal to 10.

Answer

Given: Points are A(4, 0, 0) and B(-4, 0, 0)

To find: the locus of point P, the sum of whose distances from the given points is equal to 10, i.e. PA + PB = 10

Let the required point P(x, y, z)

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

Therefore,

The distance between P(x, y, z) and A(4, 0, 0) is PA,

$$=\sqrt{(x-4)^2+(y-0)^2+(z-0)^2}$$

$$=\sqrt{(x-4)^2+y^2+z^2}$$

Distance between P(x, y, z) and B(-4, 0, 0) is PB,

$$=\sqrt{(x-(-4))^2+(y-0)^2+(z-0)^2}$$

$$=\sqrt{(x+4)^2+y^2+z^2}$$

According to question:

PA + PB = 10

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}+\sqrt{(x+4)^2+y^2+z^2}=10$$

$$\Rightarrow \sqrt{(x-4)^2+y^2+z^2}=10-\sqrt{(x+4)^2+y^2+z^2}$$

Squaring both sides:

$$\Rightarrow (x-4)^2+y^2+z^2=100+(x+4)^2+y^2+z^2-2\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow x^2+16-8x=100+x^2+16+8x-2\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow -8x-8x-100=-20\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow -4(4x-25)=-20\sqrt{(x+4)^2+y^2+z^2}$$

$$\Rightarrow 4x-25=5\sqrt{(x+4)^2+y^2+z^2}$$

Squaring both sides:

$$\Rightarrow 16x^2+625-100x=25\{(x+4)^2+y^2+z^2\}$$

$\Rightarrow 16x^2 + 625 - 100x = 25x^2 + 400 + 200x + 25y^2 + 25z^2$

$\Rightarrow 16x^2 + 625 - 100x - 25x^2 - 400 - 200x - 25y^2 - 25z^2 = 0$

$\Rightarrow -9x^2 - 25y^2 - 25z^2 - 300x + 225 = 0$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 + 300x - 225 = 0$

Hence **locus of point P is $9x^2 + 25y^2 + 25z^2 + 300x - 225 = 0$**

23. Question

Show that the point A(1,2, 3), B(-1, -2, -1), C(2, 3, 2) and D(4, 7, 6) are the vertices of a parallelogram ABCD, but not a rectangle.

Answer

Given: Points are A(1,2, 3), B(-1, -2, -1), C(2, 3, 2) and D(4, 7, 6)

To prove: the quadrilateral formed by these 4 points is a parallelogram but not a rectangle

Opposite sides of both parallelogram and rectangle are equal

But diagonals of a parallelogram are not equal whereas they are equal for rectangle

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between A(1, 2, 3) and B(-1, -2, -1) is AB,

$= \sqrt{(1-(-1))^2+(2-(-2))^2+(3-(-1))^2}$

$= \sqrt{2^2+4^2+4^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

= 6

Distance between B(-1, -2, -1) and C(2, 3, 2) is BC,

$= \sqrt{(-1-2)^2+(-2-3)^2+(-1-2)^2}$

$= \sqrt{(-3)^2+(-5)^2+(-3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Distance between C(2, 3, 2) and D(4, 7, 6) is CD,

$= \sqrt{(2-4)^2+(3-7)^2+(2-6)^2}$

$= \sqrt{(-2)^2+(-4)^2+(-4)^2}$

$= \sqrt{4+16+16}$

$= \sqrt{36}$

= 6

The distance between A(1, 2, 3) and D(4, 7, 6) is AD,

$= \sqrt{(1-4)^2+(2-7)^2+(3-6)^2}$

$= \sqrt{(-3)^2+(-5)^2+(-3)^2}$

$= \sqrt{9+25+9}$

$= \sqrt{43}$

Clearly,

AB = CD

BC = AD

Opposite sides are equal

Now, we will find the length of diagonals

The distance between A(1, 2, 3) and C(2, 3, 2) is AC,

$= \sqrt{(1-2)^2 + (2-3)^2 + (3-2)^2}$

$= \sqrt{(-1)^2 + (-1)^2 + 1^2}$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

Distance between B(-1, -2, -1) and D(4, 7, 6) is BD,

$= \sqrt{(-1-4)^2 + (-2-7)^2 + (-1-6)^2}$

$= \sqrt{(-5)^2 + (-9)^2 + (-5)^2}$

$= \sqrt{25+81+25}$

$= \sqrt{131}$

Clearly,

$AC \neq BD$

The diagonals are not equal, but opposite sides are equal

Thus, **Quadrilateral formed by ABCD is a parallelogram but not a rectangle**

Hence Proved

24. Question

Find the equation of the set of the points P such that its distances from the points A(3, 4, -5) and B(-2, 1, 4) are equal.

Answer

Given: Points are A(3, 4, -5) and B(-2, 1, 4)

To find: the equation of the set of the points, i.e. locus of points which are equidistant from the given points

Let the required point P(x, y, z)

According to the question:

PA = PB

$\Rightarrow PA^2 = PB^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Therefore,

The distance between P(x, y, z) and A(3, 4, -5) is PA,

$= \sqrt{(x-3)^2 + (y-4)^2 + (z-(-5))^2}$

$= \sqrt{(x-3)^2 + (y-4)^2 + (z+5)^2}$

The distance between P(x, y, z) and B(-2, 1, 4) is PB,

$= \sqrt{(x-(-2))^2 + (y-1)^2 + (z-4)^2}$

$= \sqrt{(x+2)^2 + (y-1)^2 + (z-4)^2}$

As $PA^2 = PB^2$

$(x - 3)^2 + (y - 4)^2 + (z + 5)^2 = (x + 2)^2 + (y - 1)^2 + (z - 4)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 + 10z = x^2 + 4 + 4x + y^2 + 1 - 2y + z^2 + 16 - 8z$

$\Rightarrow x^2 + 9 - 6x + y^2 + 16 - 8y + z^2 + 25 + 10z - x^2 - 4 - 4x - y^2 - 1 + 2y - z^2 - 16 + 8z = 0$

$\Rightarrow - 6x - 6y + 18z + 29 = 0$

$\Rightarrow 6x + 6y - 18z - 29 = 0$

Hence **locus of point P is 6x + 6y – 18z – 29 = 0**

Exercise 28.3

1. Question

The vertices of the triangle are A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2). The internal bisector of angle A meets BC at D. Find the coordinates of D and the length AD.

Answer

Given: The vertices of the triangle are A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2)

To find: the coordinates of D and the length AD

Formula used:

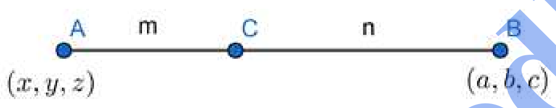
Distance Formula:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



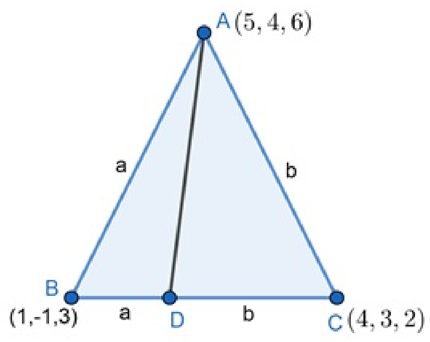
The coordinates of C is given by,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

We know angle bisector divides opposite side in the ratio of the other two sides.

As AD is angle bisector of A and meets BC at D

$\Rightarrow$ BD : DC = AB : BC



Distance between A(5, 4, 6) and B(1, -1, 3) is AB,

$= \sqrt{(5-1)^2 + (4-(-1))^2 + (6-3)^2}$

$= \sqrt{4^2 + 5^2 + 3^2}$

$= \sqrt{16 + 25 + 9}$

$= \sqrt{50}$

$= 5\sqrt{2}$

The distance between A(5, 4, 6) and C(4, 3, 2) is AC,

$= \sqrt{(5-4)^2 + (4-3)^2 + (6-2)^2}$

$= \sqrt{1^2 + 1^2 + 4^2}$

$= \sqrt{1 + 1 + 16}$

$= \sqrt{18}$

$= 3\sqrt{2}$

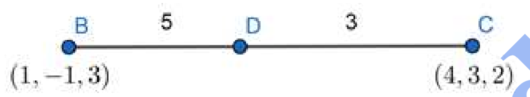
$$\frac{AB}{AC} = \frac{5\sqrt{2}}{3\sqrt{2}} = \frac{5}{3}$$

AB : AC = 5:3

⇒ BD: DC = 5:3

Therefore, m = 5 and n = 3

B(1, -1, 3) and C(4, 3, 2)



Coordinates of D using section formula:

$$= \left(\frac{3(1) + 5(4)}{5+3}, \frac{3(-1) + 5(3)}{5+3}, \frac{3(3) + 5(2)}{5+3}\right)$$

$$= \left(\frac{3 + 20}{8}, \frac{-3 + 15}{8}, \frac{9 + 10}{8}\right)$$

$$= \left(\frac{23}{8}, \frac{12}{8}, \frac{19}{8}\right)$$

$$= \left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8}\right)$$

The distance between A(5, 4, 6) and $D\left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8}\right)$ is AD,

$$= \sqrt{\left(5 - \frac{23}{8}\right)^2 + \left(4 - \frac{3}{2}\right)^2 + \left(6 - \frac{19}{8}\right)^2}$$

$$= \sqrt{\left(\frac{40 - 23}{8}\right)^2 + \left(\frac{8 - 3}{2}\right)^2 + \left(\frac{48 - 19}{8}\right)^2}$$

$$= \sqrt{\left(\frac{17}{8}\right)^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{29}{8}\right)^2}$$

$$= \sqrt{\frac{289}{64} + \frac{25}{4} + \frac{361}{64}}$$

$$= \sqrt{\frac{289 + 400 + 841}{64}}$$

$$= \sqrt{\frac{1530}{64}}$$

$= \sqrt{\frac{765}{32}}$ units

Hence, Coordinates of D are $\left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8}\right)$ and the length of AD is $\sqrt{\frac{765}{32}}$ units

2. Question

A point C with z-coordinate 8 lies on the line segment joining the points A(2, -3, 4) and B(8, 0, 10). Find the coordinates.

Answer

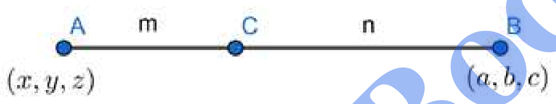
Given: A point C with z-coordinate 8 lies on the line segment joining the points A(2, -3, 4) and B(8, 0, 10)

To find: the coordinates of C

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



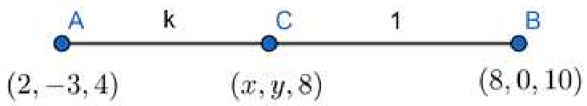
The coordinates of C is given by,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let Point C(x, y, 8), and C divides AB in ratio k: 1

Therefore, m = k and n = 1

A(2, -3, 4) and B(8, 0, 10)



Coordinates of C using section formula:

$$\Rightarrow (x, y, 8) = \left(\frac{k(8) + 1(2)}{k + 1}, \frac{k(0) + 1(-3)}{k + 1}, \frac{k(10) + 1(4)}{k + 1}\right)$$

$\Rightarrow (x,y,8) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$

On comparing:

$\frac{10k+4}{k+1} = 8$

⇒ 10k + 4 = 8(k + 1)

⇒ 10k + 4 = 8k + 8

⇒ 10k – 8k = 8 – 4

⇒ 2k = 4

$\Rightarrow k = \frac{4}{2}$⇒ k = 2

Here C divides AB in ratio 2:1

$x = \frac{8k+2}{k+1}$

$\Rightarrow x = \frac{8(2)+2}{2+1}$

$\Rightarrow x = \frac{16+2}{3}$

$\Rightarrow x = \frac{18}{3}$

⇒ x = 6

$y = \frac{-3}{k+1}$

$\Rightarrow y = \frac{-3}{2+1}$

$\Rightarrow y = \frac{-3}{3}$

⇒ y = -1

Hence, Coordinates of C are (6, -1, 8)

3. Question

Show that the three points A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10) are collinear and find the ratio in which C divides AB.

Answer

Given: A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)

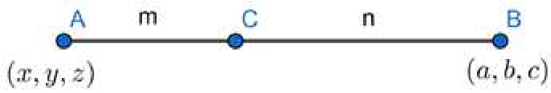
To prove: A, B and C are collinear

To find: the ratio in which C divides AB

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

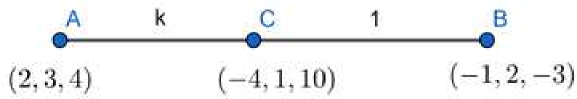
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let C divides AB in ratio k: 1

Three points are collinear if the value of k is the same for x, y and z coordinates

Therefore, m = k and n = 1

A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)



Coordinates of C using section formula:

$$\Rightarrow (-4, 1, -10) = \left(\frac{k(-1) + 1(2)}{k + 1}, \frac{k(2) + 1(3)}{k + 1}, \frac{k(-3) + 1(4)}{k + 1}\right)$$

$$\Rightarrow (-4, 1, -10) = \left(\frac{-k + 2}{k + 1}, \frac{2k + 3}{k + 1}, \frac{-3k + 4}{k + 1}\right)$$

On comparing:

$$\frac{-k + 2}{k + 1} = -4$$

⇒ -k + 2 = -4(k + 1)

⇒ -k + 2 = -4k – 4

⇒ 4k – k = - 2 – 4

⇒ 3k = -6

$\Rightarrow k = \frac{-6}{3}$ ⇒ k = -2

$$\frac{2k + 3}{k + 1} = 1$$

⇒ 2k + 3 = k + 1

⇒ 2k – k = 1 – 3

⇒ k = – 2

$$\frac{-3k + 4}{k + 1} = -10$$

⇒ -3k + 4 = -10(k + 1)

⇒ -3k + 4 = -10k – 10

⇒ -3k + 10k = -10 – 4

⇒ 7k = -14

$\Rightarrow k = \frac{-14}{7}$ ⇒ k = -2

The value of k is the same in all three times

Hence, **A, B and C are collinear**

As k = -2

C divides AB externally in ratio 2:1

4. Question

Find the ratio in which the line joining (2, 4, 5) and (3, 5, 4) is divided by the yz-plane.

Answer

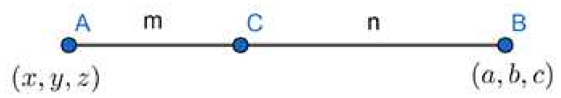
Given: points A(2, 4, 5) and B(3, 5, 4)

To find: the ratio in which the line joining given points is divided by the yz-plane

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

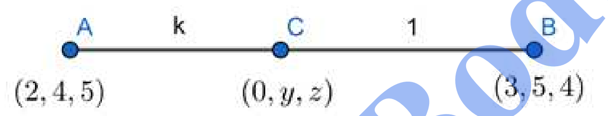
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

X coordinate is always 0 on yz-plane

Let Point C(0, y, z), and C divides AB in ratio k: 1

Therefore, m = k and n = 1

A(2, 4, 5) and B(3, 5, 4)



Coordinates of C using section formula:

$$\Rightarrow (0, y, z) = \left(\frac{k(3) + 1(2)}{k + 1}, \frac{k(5) + 1(4)}{k + 1}, \frac{k(4) + 1(5)}{k + 1}\right)$$

$$\Rightarrow (0, y, z) = \left(\frac{3k + 2}{k + 1}, \frac{5k + 4}{k + 1}, \frac{4k + 5}{k + 1}\right)$$

On comparing:

$$\frac{3k + 2}{k + 1} = 0$$

⇒ 3k + 2 = 0(k + 1)

⇒ 3k + 2 = 0

⇒ 3k = – 2

$$\Rightarrow k = \frac{-2}{3}$$

Hence, **C divides AB externally in ratio 2 : 3**

5. Question

Find the ratio in which the line segment joining the points (2, -1, 3) and (-1, 2, 1) is divided by the plane x + y + z = 5.

Answer

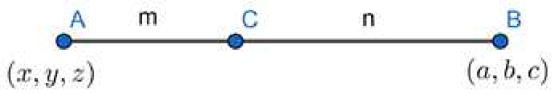
Given: A(2, -1, 3) and B(-1, 2, 1)

To find: the ratio in which the line segment AB is divided by the plane x + y + z = 5

Formula used:

Section Formula:

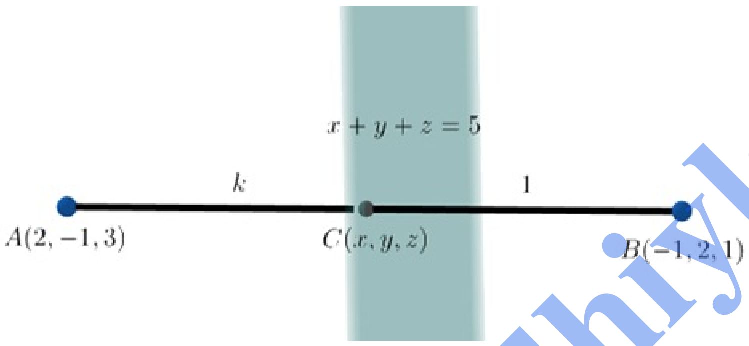
A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let C(x, y, z) be any point on the given plane and C divides AB in ratio k: 1



Therefore, m = k and n = 1

A(2, -1, 3) and B(-1, 2, 1)

Coordinates of C using section formula:

$$\Rightarrow (x, y, z) = \left(\frac{k(-1) + 1(2)}{k + 1}, \frac{k(2) + 1(-1)}{k + 1}, \frac{k(-1) + 1(3)}{k + 1}\right)$$

$$\Rightarrow (x, y, z) = \left(\frac{-k + 2}{k + 1}, \frac{2k - 1}{k + 1}, \frac{-k + 3}{k + 1}\right)$$

On comparing:

$$\frac{-k + 2}{k + 1} = x; \frac{2k - 1}{k + 1} = y; \frac{-k + 3}{k + 1} = z$$

Since, x + y + z = 5

$$\Rightarrow \frac{-k + 2}{k + 1} + \frac{2k - 1}{k + 1} + \frac{-k + 3}{k + 1} = 5$$

$$\Rightarrow \frac{-k + 2 + 2k - 1 - k + 3}{k + 1} = 5$$

$$\Rightarrow \frac{4}{k + 1} = 5$$

⇒ 5(k + 1) = 4

⇒ 5k + 5 = 4

⇒ 5k = 4 – 5

⇒ 5k = – 1

⇒ $k = \frac{-1}{5}$

Hence, the plane divides AB externally in ratio 1:5

6. Question

If the points A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6) are collinear, find the ratio in which C divided AB.

Answer

Given: A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6)

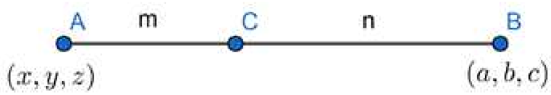
To prove: A, B and C are collinear

To find: the ratio in which C divides AB

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

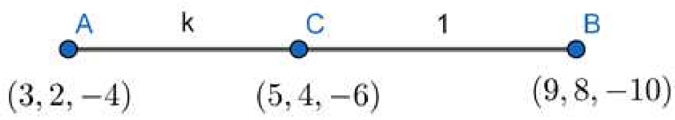
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let C divides AB in ratio k: 1

Three points are collinear if the value of k is the same for x, y and z coordinates

Therefore, m = k and n = 1

A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6)



Coordinates of C using section formula:

$$\Rightarrow (5,4,-6) = \left(\frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1}\right)$$

$$\Rightarrow (5,4,-6) = \left(\frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1}\right)$$

On comparing:

$$\frac{9k + 3}{k + 1} = 5$$

⇒ 9k + 3 = 5(k + 1)

⇒ 9k + 3 = 5k + 5

⇒ 9k – 5k = 5 – 3

⇒ 4k = 2

$\Rightarrow \mathrm{k} = \frac{2}{4} \Rightarrow \mathrm{k} = \frac{1}{2}$

$\frac{8\mathrm{k} + 2}{\mathrm{k} + 1} = 4$

⇒ 8k + 2 = 4(k + 1)

⇒ 8k + 2 = 4k + 4

⇒ 8k – 4k = 4 – 2

⇒ 4k = 2

$\Rightarrow \mathrm{k} = \frac{2}{4} \Rightarrow \mathrm{k} = \frac{1}{2}$

$\frac{-10\mathrm{k} - 4}{\mathrm{k} + 1} = -6$

⇒ -10k – 4 = -6(k + 1)

⇒ -10k – 4 = -6k – 6

⇒ -10k + 6k = 4 – 6

⇒ -4k = -2

$\Rightarrow \mathrm{k} = \frac{-2}{-4} \Rightarrow \mathrm{k} = \frac{1}{2}$

The value of k is the same in all three times

Hence, **A, B and C are collinear**

$\mathrm{As}, \mathrm{k} = \frac{1}{2}$

C divides AB externally in ratio 1:2

7. Question

The mid-points of the sides of a triangle ABC are given by (-2, 3, 5), (4, -1, 7) and (6, 5, 3). Find the coordinates of A, B and C.

Answer

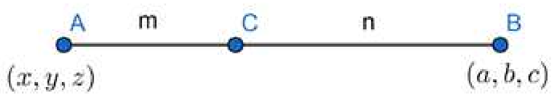
Given: The mid-points of the sides of the triangle are P(-2, 3, 5), Q(4, -1, 7) and R(6, 5, 3).

To find: the coordinates of vertices A, B and C

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

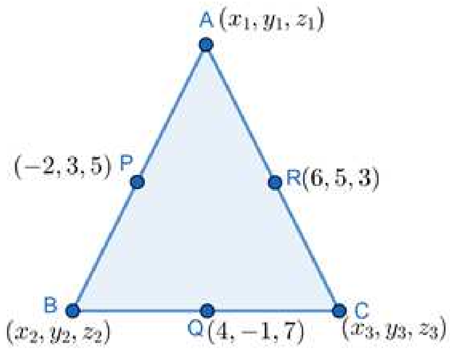
$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$

We know the mid-point divides side in the ratio of 1:1.

Therefore,

The coordinates of C is given by,

$\left(\frac{x+a}{2}, \frac{y+b}{2}, \frac{z+c}{2}\right)$



P(-2, 3, 5) is mid-point of A(x_1, y_1, z_1) and B(x_2, y_2, z_2)

Therefore,

$(-2, 3, 5) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

$\Rightarrow (-4, 6, 10) = (x_1 + x_2, y_1 + y_2, z_1 + z_2) \ldots\ldots\ldots (1)$

Q(4, -1, 7) is mid-point of B(x_2, y_2, z_2) and C(x_3, y_3, z_3)

Therefore,

$(4, -1, 7) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$

$\Rightarrow (8, -2, 14) = (x_3 + x_3, y_3 + y_3, z_3 + z_3) \ldots\ldots\ldots (2)$

R(6, 5, 3) is mid-point of A(x_1, y_1, z_1) and C(x_3, y_3, z_3)

Therefore,

$(6, 5, 3) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$

$\Rightarrow (12, 10, 6) = (x_1 + x_3, y_1 + y_3, z_1 + z_3) \ldots\ldots\ldots (3)$

$x_1 + x_2 = -4$.......................(4)

$x_2 + x_3 = 8$..........................(5)

$x_1 + x_3 = 12$.......................(6)

Adding (4), (5) and (6):

$\Rightarrow x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 8 + 12 - 4$

$\Rightarrow 2x_1 + 2x_2 + 2x_3 = 16$

$\Rightarrow 2(x_1 + x_2 + x_3) = 16$

$\Rightarrow x_1 + x_2 + x_3 = 8$..........................(7)

Subtract (4), (5) and (6) from (7) separately:

$x_1 + x_2 + x_3 - x_1 - x_2 = 8 - (-4)$

$\Rightarrow x_3 = 12$

$x_1 + x_2 + x_3 - x_2 - x_3 = 8 - 8$

$\Rightarrow x_1 = 0$

$x_1 + x_2 + x_3 - x_1 - x_3 = 8 - 12$

$\Rightarrow x_2 = -4$

$y_1 + y_2 = 6$……………………(8)

$y_2 + y_3 = -2$……………………(9)

$y_1 + y_3 = 10$……………………(10)

Adding (8), (9) and (10):

$\Rightarrow y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 10 + 6 - 2$

$\Rightarrow 2y_1 + 2y_2 + 2y_3 = 14$

$\Rightarrow 2(y_1 + y_2 + y_3) = 14$

$\Rightarrow y_1 + y_2 + y_3 = 7$………………………(11)

Subtract (8), (9) and (10) from (11) separately:

$y_1 + y_2 + y_3 - y_1 - y_2 = 7 - 6$

$\Rightarrow y_3 = 1$

$y_1 + y_2 + y_3 - y_2 - y_3 = 7 - (-2)$

$\Rightarrow y_1 = 9$

$y_1 + y_2 + y_3 - y_1 - y_3 = 7 - 10$

$\Rightarrow y_2 = -3$

$z_1 + z_2 = 10$……………………(12)

$z_2 + z_3 = 14$……………………(13)

$z_1 + z_3 = 6$……………………(14)

Adding (12), (13) and (14):

$\Rightarrow z_1 + z_2 + z_2 + z_3 + z_1 + z_3 = 6 + 14 + 10$

$\Rightarrow 2z_1 + 2z_2 + 2z_3 = 30$

$\Rightarrow 2(z_1 + z_2 + z_3) = 30$

$\Rightarrow z_1 + z_2 + z_3 = 15$………………………(15)

Subtract (8), (9) and (10) from (11) separately:

$z_1 + z_2 + z_3 - z_1 - z_2 = 15 - 10$

$\Rightarrow z_3 = 5$

$z_1 + z_2 + z_3 - z_2 - z_3 = 15 - 14$

$\Rightarrow z_1 = 1$

$z_1 + z_2 + z_3 - z_1 - z_3 = 15 - 6$

$\Rightarrow z_2 = 9$

Hence, vertices of sides are A(0, 9, 1) B(-4,-3, 9) and C(12, 1, 5)

8. Question

A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3) are the vertices of a triangle ABC. Find the point in which the bisector of the angle ∠BAC meets BC.

Answer

Given: The vertices of the triangle are A(1, 2, 3), B(0, 4, 1) and C(-1, -1, -3)

To find: the coordinates of D

Formula used:

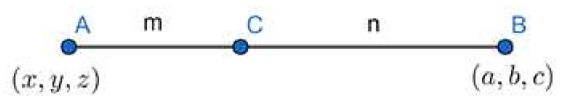
Distance Formula:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



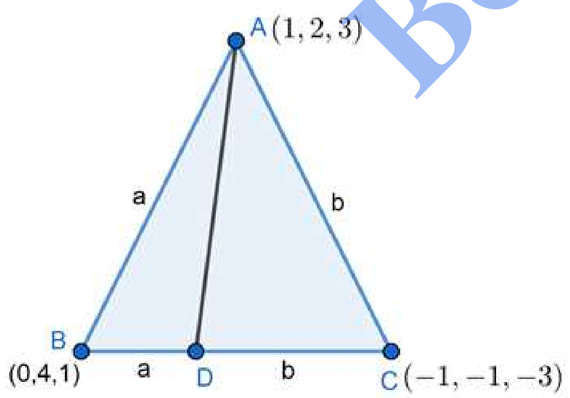
The coordinates of C is given by,

$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$

We know angle bisector divides opposite side in the ratio of the other two sides.

As AD is angle bisector of A and meets BC at D

$\Rightarrow$ BD : DC = AB : BC



Distance between A(1, 2, 3) and B(0, 4, 1) is AB,

$= \sqrt{(1-0)^2 + (2-4)^2 + (3-1)^2}$

$= \sqrt{1^2 + (-2)^2 + 2^2}$

$= \sqrt{1+4+4}$

$= \sqrt{9}$

= 3

Distance between A(1, 2, 3) and C(-1, -1, -3) is AC,

$= \sqrt{(1-(-1))^2 + (2-(-1))^2 + (3-(-3))^2}$

$= \sqrt{2^2 + 3^2 + 6^2}$

$= \sqrt{4 + 9 + 36}$

$= \sqrt{49}$

= 7

$\frac{AB}{AC} = \frac{3}{7}$

AB : AC = 3:7

⇒ BD: DC = 3:7

Therefore, m = 3 and n = 7

B(0, 4, 1) and C(-1, -1, -3)

Coordinates of D using section formula:

$= \left(\frac{7(0)+3(-1)}{7+3}, \frac{7(4)+3(-1)}{7+3}, \frac{7(1)+3(-3)}{7+3}\right)$

$= \left(\frac{0-3}{10}, \frac{28-3}{10}, \frac{7-9}{10}\right)$

$= \left(\frac{-3}{10}, \frac{25}{10}, \frac{-2}{10}\right)$

$= \left(\frac{-3}{10}, \frac{5}{2}, \frac{-1}{5}\right)$

Hence, Coordinates of D are $\left(\frac{-3}{10}, \frac{5}{2}, \frac{-1}{5}\right)$

9. Question

Find the ratio in which the sphere $x^2 + y^2 + z^2 = 504$ divides the line joining the point (12, -4, 8) and (27, -9, 18).

Answer

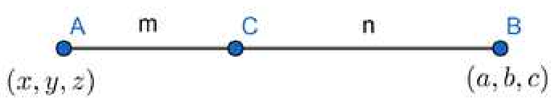
Given: A(12, -4, 8) and B(27, -9, 18)

To find: the ratio in which the line segment AB is divided by the sphere $x^2 + y^2 + z^2 = 504$

Formula used:

Section Formula:

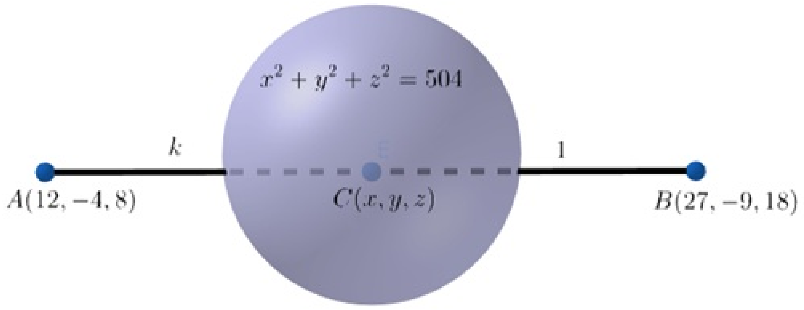
A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let C(x, y, z) be any point on given plane and C divides AB in ratio k: 1



Therefore, m = k and n = 1

A(12, -4, 8) and B(27, -9, 18)

Coordinates of C using section formula:

$$\Rightarrow (x,y,z) = \left(\frac{k(27) + 1(12)}{k + 1}, \frac{k(-9) + 1(-4)}{k + 1}, \frac{k(18) + 1(8)}{k + 1}\right)$$

$$\Rightarrow (x,y,z) = \left(\frac{27k + 12}{k + 1}, \frac{-9k - 4}{k + 1}, \frac{18k + 8}{k + 1}\right)$$

On comparing:

$$\frac{27k + 12}{k + 1} = x\,;\ \frac{-9k - 4}{k + 1} = y\,;\ \frac{18k + 8}{k + 1} = z$$

Since, $x^2 + y^2 + z^2 = 504$

$$\Rightarrow \left(\frac{27k + 12}{k + 1}\right)^2 + \left(\frac{-9k - 4}{k + 1}\right)^2 + \left(\frac{18k + 8}{k + 1}\right)^2 = 504$$

$$\Rightarrow \frac{3^2(9k + 4)^2 + (-1)^2(9k + 4)^2 + 2^2(9k + 4)^2}{(k + 1)^2} = 504$$

$$\Rightarrow \frac{(9 + 1 + 4)(9k + 4)^2}{(k + 1)^2} = 504$$

$$\Rightarrow \frac{14(81k^2 + 16 + 72k)}{(k + 1)^2} = 504$$

$$\Rightarrow \frac{81k^2 + 16 + 72k}{k^2 + 1 + 2k} = \frac{504}{14}$$

$$\Rightarrow \frac{81k^2 + 16 + 72k}{k^2 + 1 + 2k} = 36$$

$$\Rightarrow 81k^2 + 16 + 72k = 36(k^2 + 1 + 2k)$$

$$\Rightarrow 81k^2 + 16 + 72k = 36k^2 + 36 + 72k$$

$$\Rightarrow 81k^2 + 16 + 72k - 36k^2 - 36 - 72k = 0$$

$$\Rightarrow 45k^2 - 20 = 0$$

$$\Rightarrow 45k^2 = 20$$

$\Rightarrow k^2 = \frac{20}{45}$

$\Rightarrow k^2 = \frac{4}{9}$

$\Rightarrow k = \frac{2}{3}$

Hence, the sphere divides AB in ratio 2 : 3

10. Question

Show that the plane ax + by + cz + d = 0 divides the line joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) in the ratio $-\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$.

Answer

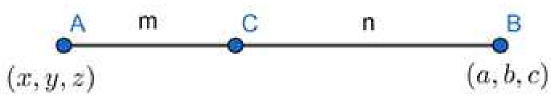
Given: A(x_1, y_1, z_1) and B(x_2, y_2, z_2)

To prove: the ratio in which the line segment AB is divided by the plane ax + by + cz + d = 0 is $-\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$

Formula used:

Section Formula:

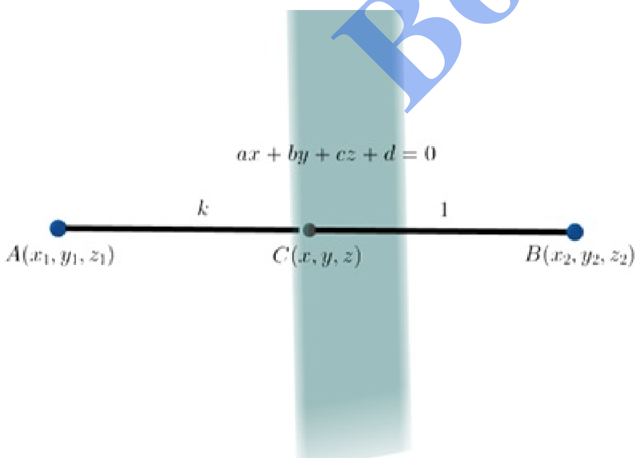
A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$

Let C(x, y, z) be any point on given plane and C divides AB in ratio k: 1



Therefore, m = k and n = 1

A(x_1, y_1, z_1) and B(x_2, y_2, z_2)

Coordinates of C using section formula:

$\Rightarrow (x,y,z) = \left(\frac{k(x_1) + 1(x_2)}{k + 1}, \frac{k(x_2) + 1(y_2)}{k + 1}, \frac{k(z_1) + 1(z_2)}{k + 1}\right)$

$$\Rightarrow (x,y,z) = \left(\frac{x_1k+x_2}{k+1}, \frac{y_1k+y_2}{k+1}, \frac{z_1k+z_2}{k+1}\right)$$

On comparing:

$$\frac{x_1k+x_2}{k+1} = x ; \frac{y_1k+y_2}{k+1} = y ; \frac{z_1k+z_2}{k+1} = z$$

Since, ax + by + cz + d = 0

$$\Rightarrow a\left(\frac{x_1k+x_2}{k+1}\right) + b\left(\frac{y_1k+y_2}{k+1}\right) + c\left(\frac{z_1k+z_2}{k+1}\right) + d = 0$$

$$\Rightarrow \frac{a(x_1k+x_2) + b(y_1k+y_2) + c(z_1k+z_2) + d(k+1)}{k+1} = 0$$

$$\Rightarrow ax_1k + ax_2 + by_1k + by_2 + cz_1k + cz_2 + dk + d = 0$$

$$\Rightarrow k(ax_1 + by_1 + cz_1 + d) = -(ax_2 + by_2 + cz_2 + d)$$

$$\Rightarrow k = -\frac{ax_2 + by_2 + cz_2 + d}{(ax_1 + by_1 + cz_1 + d)}$$

The plane divides AB in the ratio $-\frac{ax_2+by_2+cz_2+d}{(ax_1+by_1+cz_1+d)}$

Hence Provedco

11. Question

Find the centroid of a triangle, mid-points of whose are (1, 2, -3), (3, 0, 1) and (-1, 1, -4).

Answer

Given: The mid-points of the sides of the triangle are P(1, 2, -3), Q(3, 0, 1) and R(-1, 1, -4).

To find: the coordinates of the centroid

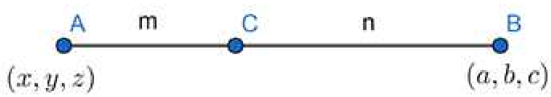
Formula used:

Centroid of triangle ABC whose vertices are A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) is given by,

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

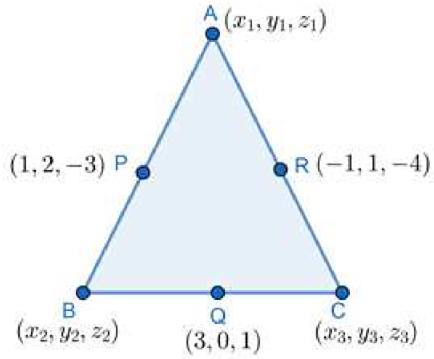
$$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$$

We know the mid-point divides side in the ratio of 1:1.

Therefore,

The coordinates of C is given by,

$$\left(\frac{x+a}{2}, \frac{y+b}{2}, \frac{z+c}{2}\right)$$



P(1, 2, -3) is mid-point of A(x_1, y_1, z_1) and B(x_2, y_2, z_2)

Therefore,

$$(1, 2, -3) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

$$\Rightarrow (2, 4, -6) = (x_1 + x_2, y_1 + y_2, z_1 + z_2)$$

Q(3, 0, 1) is mid-point of B(x_2, y_2, z_2) and C(x_3, y_3, z_3)

Therefore,

$$(3, 0, 1) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}, \frac{z_2 + z_3}{2}\right)$$

$$\Rightarrow (6, 0, 2) = (x_3 + x_3, y_3 + y_3, z_3 + z_3)$$

R(-1, 1, -4) is mid-point of A(x_1, y_1, z_1) and C(x_3, y_3, z_3)

Therefore,

$$(-1, 1, -4) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}, \frac{z_1 + z_3}{2}\right)$$

$$\Rightarrow (-2, 2, -8) = (x_1 + x_3, y_1 + y_3, z_1 + z_3)$$

$x_1 + x_2 = 2$.......................(1)

$x_2 + x_3 = 6$..........................(2)

$x_1 + x_3 = -2$.......................(3)

Adding (1), (2) and (3):

$\Rightarrow x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 6 - 2$

$\Rightarrow 2x_1 + 2x_2 + 2x_3 = 6$

$\Rightarrow 2(x_1 + x_2 + x_3) = 6$

$\Rightarrow x_1 + x_2 + x_3 = 3$

$y_1 + y_2 = 4$.......................(4)

$y_2 + y_3 = 0$.......................(5)

$y_1 + y_3 = 2$.......................(6)

Adding (4), (5) and (6):

$\Rightarrow y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 4 + 0 + 2$

$\Rightarrow 2y_1 + 2y_2 + 2y_3 = 6$

$\Rightarrow 2(y_1 + y_2 + y_3) = 6$

$\Rightarrow y_1 + y_2 + y_3 = 3$

$z_1 + z_2 = -6$……………………(7)

$z_2 + z_3 = 2$……………………(8)

$z_1 + z_3 = -8$……………………(9)

Adding (7), (8) and (9):

$\Rightarrow z_1 + z_2 + z_2 + z_3 + z_1 + z_3 = -6 + 2 - 8$

$\Rightarrow 2z_1 + 2z_2 + 2z_3 = -12$

$\Rightarrow 2(z_1 + z_2 + z_3) = -12$

$\Rightarrow z_1 + z_2 + z_3 = -6$

Centroid of the triangle

$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

$= \left(\frac{3}{3}, \frac{3}{3}, \frac{-6}{3}\right)$

$= (1, 1, -2)$

Hence, the centroid of the triangle is (1, 1, -2)

12. Question

The centroid of a triangle ABC is at the point (1, 1, 1). If the coordinates of A and B are (3, -5, 7) and (-1, 7, -6) respectively, find the coordinates of the point C.

Answer

Given: The coordinates of the A and B of the triangle ABC are (3, -5, 7) and (-1, 7, -6) respectively. The centroid of the triangle is (1, 1, 1)

To find: the coordinates of vertex C

Formula used:

Centroid of triangle ABC whose vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is given by,

$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

Here A(3, -5, 7) and B(-1, 7, -6)

Centroid of the triangle

$(1,1,1) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

$\Rightarrow (1,1,1) = \left(\frac{3 - 1 + x_3}{3}, \frac{-5 + 7 + y_3}{3}, \frac{7 - 6 + z_3}{3}\right)$

$\Rightarrow (1,1,1) = \left(\frac{2 + x_3}{3}, \frac{2 + y_3}{3}, \frac{1 + z_3}{3}\right)$

On comparing:

$\frac{2 + x_3}{3} = 1$

⇒ $2 + x_3 = 3$

⇒ $x_3 = 3 - 2$

⇒ $x_3 = 1$

$\frac{2 + y_3}{3} = 1$

⇒ $2 + y_3 = 3$

⇒ $y_3 = 3 - 2$

⇒ $y_3 = 1$

$\frac{1 + z_3}{3} = 1$

⇒ $1 + z_3 = 3$

⇒ $z_3 = 3 - 1$

⇒ $z_3 = 2$

Hence, coordinates of vertex C(1, 1, 2)

13. Question

Find the coordinates of the points which trisect the line segment joining the points P(4, 2, -6) and Q(10, -16, 6).

Answer

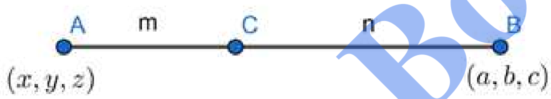
Given: Points P(4, 2, -6) and Q(10, -16, 6)

To find: the coordinates of points which trisect the line PQ

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).

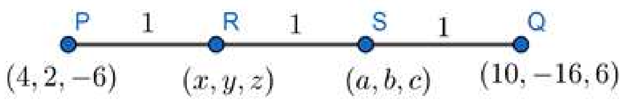


The coordinates of C is given by,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let Point R(x, y, z) and Point S(a, b, c) trisects line PQ

So, PR : RS : SQ = 1 : 1 : 1



Now, we will firstly apply section formula on PQ and find coordinates of R

Therefore, m = 1 and n = 2

P(4, 2, -6) and Q(10, -16, 6)

Coordinates of R using section formula:

$$\Rightarrow (x,y,z) = \left(\frac{2(4)+1(10)}{2+1}, \frac{2(2)+1(-16)}{2+1}, \frac{2(-6)+1(6)}{2+1}\right)$$

$$\Rightarrow (x,y,z) = \left(\frac{8+10}{3}, \frac{4-16}{3}, \frac{-12+6}{3}\right)$$

$$\Rightarrow (x,y,z) = \left(\frac{18}{3}, \frac{-12}{3}, \frac{-6}{3}\right)$$

$$\Rightarrow (x,y,z) = (6,-4,-2)$$

Now, we will apply section formula on PQ and find coordinates of S

Therefore, m = 2 and n = 1

P(4, 2, -6) and Q(10, -16, 6)

Coordinates of R using section formula:

$$\Rightarrow (a,b,c) = \left(\frac{1(4)+2(10)}{2+1}, \frac{1(2)+2(-16)}{2+1}, \frac{1(-6)+2(6)}{2+1}\right)$$

$$\Rightarrow (a,b,c) = \left(\frac{4+20}{3}, \frac{2-32}{3}, \frac{-6+12}{3}\right)$$

$$\Rightarrow (a,b,c) = \left(\frac{24}{3}, \frac{-30}{3}, \frac{6}{3}\right)$$

$$\Rightarrow (a,b,c) = (8,-10,2)$$

Hence, Coordinates of R and S are (6, -4, -2) and (8, -10, 2) respectively

14. Question

Using section formula, show that the points A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2) are collinear.

Answer

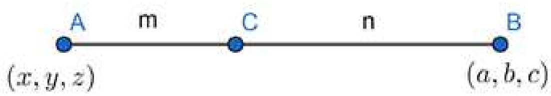
Given: A(2, -3, 4), B(-1, 2, 1) and $C\left(0,\frac{1}{3},2\right)$

To prove: A, B and C are collinear

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

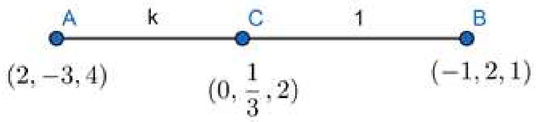
$$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$$

Let C divides AB in ratio k: 1

Three points are collinear if the value of k is the same for x, y and z coordinates

Therefore, m = k and n = 1

A(2, -3, 4), B(-1, 2, 1) and $C\left(0,\frac{1}{3},2\right)$



Coordinates of C using section formula:

$$\Rightarrow (-1,2,1) = \left(\frac{k(0)+1(2)}{k+1}, \frac{k\left(\frac{1}{3}\right)+1(-3)}{k+1}, \frac{k(2)+1(4)}{k+1}\right)$$

$$\Rightarrow (-1,2,1) = \left(\frac{2}{k+1}, \frac{\frac{k}{3}-3}{k+1}, \frac{2k+4}{k+1}\right)$$

On comparing:

$$\frac{2}{k+1} = -1$$

⇒ 2 = -1(k + 1)

⇒ 2 = -k – 1

⇒ k = -1 – 2

⇒ k = -3

$$\frac{\frac{k}{3}-3}{k+1} = 2$$

$$\Rightarrow \frac{k-9}{3} = 2(k+1)$$

⇒ k – 9 = 6(k + 1)

⇒ k – 9 = 6k + 6

⇒ k – 6k = 6 + 9

⇒ -5k = 15

⇒ k = -3

$$\frac{2k+4}{k+1} = 1$$

⇒ 2k + 4 = 1(k + 1)

⇒ 2k + 4 = k + 1

⇒ 2k – k = 1 – 4

⇒ k = -3

The value of k is the same in all three times

Hence, **A, B and C are collinear**

15. Question

Given that P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) are collinear. Find the ratio in which Q divides PR.

Answer

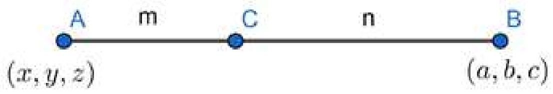
Given: P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10) and P, Q and R are collinear

To find: the ratio in which Q divides PR

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



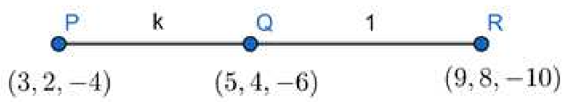
The coordinates of C is given by,

$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let Q divides PR in ratio k : 1

Therefore, m = k and n = 1

P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10)



Coordinates of Q using section formula:

$$\Rightarrow (5,4,-6) = \left(\frac{k(9) + 1(3)}{k + 1}, \frac{k(8) + 1(2)}{k + 1}, \frac{k(-10) + 1(-4)}{k + 1}\right)$$

$$\Rightarrow (5,4,-6) = \left(\frac{9k + 3}{k + 1}, \frac{8k + 2}{k + 1}, \frac{-10k - 4}{k + 1}\right)$$

On comparing:

$$\frac{9k + 3}{k + 1} = 5$$

⇒ 9k + 3 = 5(k + 1)

⇒ 9k + 3 = 5k + 5

⇒ 9k – 5k = 5 – 3

⇒ 4k = 2

$$\Rightarrow k = \frac{2}{4} \Rightarrow k = \frac{1}{2}$$

Q divides PR externally in ratio 1:2

16. Question

Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, -8) is divided by the yz-plane.

Answer

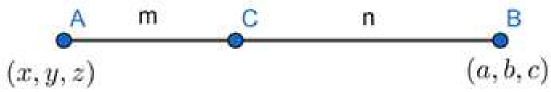
Given: points A(4, 8, 10) and B(6, 10, -8)

To find: the ratio in which the line joining given points is divided by the yz-plane

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

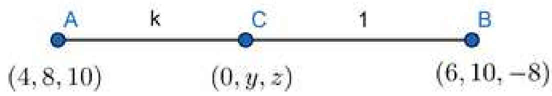
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

the x coordinate is always 0 on yz-plane

Let Point C(0, y, z), and C divides AB in ratio k: 1

Therefore, m = k and n = 1

A(4, 8, 10) and B(6, 10, -8)



Coordinates of C using section formula:

$$\Rightarrow (0, y, z) = \left(\frac{k(6) + 1(4)}{k + 1}, \frac{k(10) + 1(8)}{k + 1}, \frac{k(-8) + 1(10)}{k + 1}\right)$$

$$\Rightarrow (0, y, z) = \left(\frac{6k + 4}{k + 1}, \frac{10k + 8}{k + 1}, \frac{-8k + 10}{k + 1}\right)$$

On comparing:

$$\frac{6k + 4}{k + 1} = 0$$

⇒ 6k + 4 = 0(k + 1)

⇒ 6k + 4 = 0

⇒ 6k = – 4

$$\Rightarrow k = \frac{-4}{6}$$

$$\Rightarrow k = \frac{-2}{3}$$

Hence, **C divides AB externally in ratio 2 : 3**

Very Short Answer

1. Question

Write the distance of the point P(2, 3, 5) from the xy-plane.

Answer

Given: Points P(2, 3, 5)

To find: the distance of the point P from xy-plane

As we know z = 0 in xy-plane.

The shortest distance of the plane will be the z-coordinate of the point

Hence, **the distance of point P from xy-plane is 5 units**

2. Question

Write the distance of the point P(3, 4, 5) from the z-axis.

Answer

Given: point P(3, 4, 5)

To find: distance of the point P from the z-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

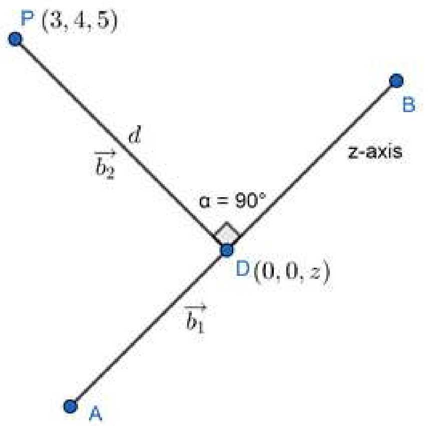
As, x and y coordinate on z-axis are zero

Let point D on z-axis is (0, 0, z)

Direction cosines of z-axis are (0, 0, 1)

Direction cosines of PD are (3 – 0, 4 – 0, 5 – z) = (3, 4, 5 – z)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2}=0$

⇒ 3 × 0 + 4 × 0 + (5 – z) × 1 = 0

⇒ 0 + 0 + 5 – z = 0

⇒ z = 5

Hence point D(0, 0, 5)

Distance between point P(3, 4, 5) and D(0, 0, 5) is d

$=\sqrt{(3-0)^2+(4-0)^2+(5-5)^2}$

$=\sqrt{3^2+4^2+0^2}$

$=\sqrt{9+16+0}$

$=\sqrt{25}$

= 5

Hence, the **distance of the point P from z-axis is 5 units**

3. Question

If the distance between the points P(a, 2, 1) and Q(1, -1, 1) is 5 units, find the value of a.

Answer

Given: distance between the points P(a, 2, 1) and Q(1, -1, 1) is 5 units

To find: the value of a

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

PQ = 5 units

The distance between points P(a, 2, 1) and Q(1, -1, 1) is PQ

$\Rightarrow \sqrt{(a-1)^2+(2-(-1))^2+(1-1)^2}=5$

$\Rightarrow \sqrt{(a-1)^2+3^2+0^2}=5$

$\Rightarrow \sqrt{a^2+1-2a+9+0}=5$

$\Rightarrow \sqrt{a^2-2a+10}=5$

Squaring both sides:

$\Rightarrow a^2 - 2a + 10 = 25$

$\Rightarrow a^2 - 2a + 10 - 25 = 0$

$\Rightarrow a^2 - 2a - 15 = 0$

$\Rightarrow a^2 - 5a + 3a - 15 = 0$

$\Rightarrow a(a - 5) + 3(a - 5) = 0$

$\Rightarrow (a - 5)(a + 3) = 0$

$\Rightarrow a = 5$ or -3

Hence, **the value of a is 5 or -3**

4. Question

The coordinates of the mid-points of sides AB, BC and CA of ΔABC are D(1, 2, -3), E(3, 0, 1) and F(-1, 1, -4) respectively. Write the coordinates of its centroid.

Answer

Given: The mid-points of the sides of the triangle are P(1, 2, -3), Q(3, 0, 1) and R(-1, 1, -4).

To find: the coordinates of the centroid

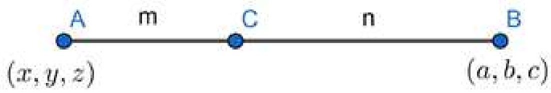
Formula used:

Centroid of triangle ABC whose vertices are $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is given by,

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

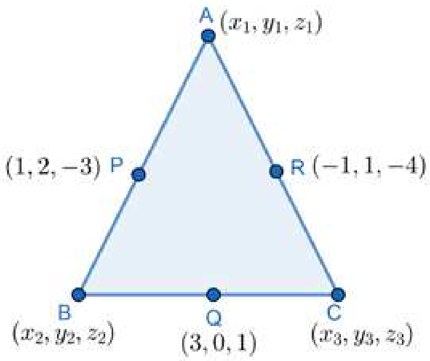
$$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$$

We know the mid-point divides side in the ratio of 1:1.

Therefore,

The coordinates of C is given by,

$$\left(\frac{x+a}{2}, \frac{y+b}{2}, \frac{z+c}{2}\right)$$



P(1, 2, -3) is mid-point of A(x_1, y_1, z_1) and B(x_2, y_2, z_2)

Therefore,

$$(1,2,-3) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$$\Rightarrow (2,4,-6) = (x_1+x_2, y_1+y_2, z_1+z_2)$$

Q(3, 0, 1) is mid-point of B(x_2, y_2, z_2) and C(x_3, y_3, z_3)

Therefore,

$$(3,0,1) = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right)$$

$$\Rightarrow (6,0,2) = (x_3+x_3, y_3+y_3, z_3+z_3)$$

R(-1, 1, -4) is mid-point of A(x_1, y_1, z_1) and C(x_3, y_3, z_3)

Therefore,

$$(-1,1,-4) = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right)$$

$$\Rightarrow (-2,2,-8) = (x_1+x_3, y_1+y_3, z_1+z_3)$$

$x_1 + x_2 = 2$.......................(1)

$x_2 + x_3 = 6$..........................(2)

$x_1 + x_3 = -2$.......................(3)

Adding (1), (2) and (3):

$\Rightarrow x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 6 – 2$

$\Rightarrow 2x_1 + 2x_2 + 2x_3 = 6$

$\Rightarrow 2(x_1 + x_2 + x_3) = 6$

$\Rightarrow x_1 + x_2 + x_3 = 3$

$y_1 + y_2 = 4$……………………(4)

$y_2 + y_3 = 0$……………………(5)

$y_1 + y_3 = 2$……………………(6)

Adding (4), (5) and (6):

$\Rightarrow y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 4 + 0 + 2$

$\Rightarrow 2y_1 + 2y_2 + 2y_3 = 6$

$\Rightarrow 2(y_1 + y_2 + y_3) = 6$

$\Rightarrow y_1 + y_2 + y_3 = 3$

$z_1 + z_2 = -6$……………………(7)

$z_2 + z_3 = 2$……………………(8)

$z_1 + z_3 = -8$……………………(9)

Adding (7), (8) and (9):

$\Rightarrow z_1 + z_2 + z_2 + z_3 + z_1 + z_3 = -6 + 2 - 8$

$\Rightarrow 2z_1 + 2z_2 + 2z_3 = -12$

$\Rightarrow 2(z_1 + z_2 + z_3) = -12$

$\Rightarrow z_1 + z_2 + z_3 = -6$

Centroid of the triangle

$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

$= \left(\frac{3}{3}, \frac{3}{3}, \frac{-6}{3}\right)$

= (1, 1, -2)

Hence, the centroid of the triangle is (1, 1, -2)

5. Question

Write the coordinates of the foot of the perpendicular from the point P(1, 2, 3) on the y-axis.

Answer

Given: point P(1, 2, 3)

To find: coordinates of the foot of the perpendicular from the point on the y-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

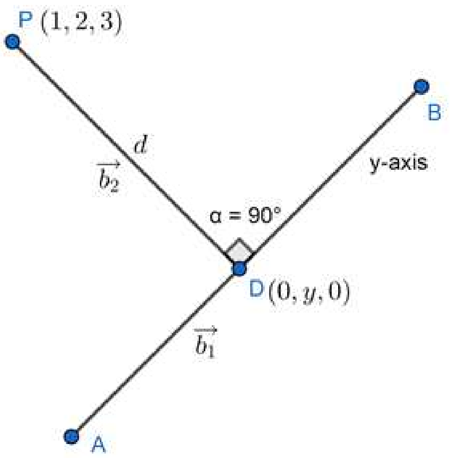
As x and z coordinate on the y-axis is zero

Let point D is the point of the foot of perpendicular on the y-axis from point P be (0, y, 0)

Direction cosines of y-axis are (0, 1, 0)

Direction cosines of PD are (1 – 0, 2 – y, 3 – 0) = (1, 2 – y, 3)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$

⇒ 1 × 0 + (2 – y) × 1 + 3 × 0 = 0

⇒ 0 + 0 + 2 – y= 0

⇒ y = 2

Hence, **coordinates of point D are (0, 2, 0)**

6. Question

Write the length of the perpendicular drawn from the point P(3, 5, 12) on the x-axis.

Answer

Given: point P(3, 5, 12)

To find: length of the perpendicular drawn from the point P from the x-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

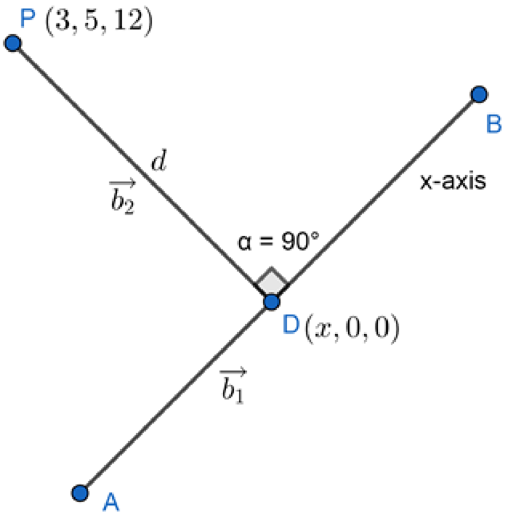
As, y and z coordinate on x-axis are zero

Let point D on x-axis is (x, 0, 0)

Direction cosines of z-axis are (1, 0, 0)

Direction cosines of PD are (3 – x, 5 – 0, 12 – 0) = (3 – x, 5, 12)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$

⇒ (3 – x) × 1 + 5 × 0 + 12 × 0 = 0

⇒ 3 – x + 0 + 0 = 0

⇒ x = 3

Hence point D(3, 0, 0)

Distance between point P(3, 5, 12) and D(3, 0, 0) is d

$= \sqrt{(3-3)^2 + (5-0)^2 + (12-0)^2}$

$= \sqrt{0^2 + 5^2 + 12^2}$

$= \sqrt{0 + 25 + 144}$

$= \sqrt{169}$

= 13

Hence, the **distance of the point P from x-axis is 13 units**

7. Question

Write the coordinates of the third vertex of a triangle having centroid at the origin and two vertices at (3, -5, 7) and (3, 0, 1).

Answer

Given: The coordinates of the A and B of the triangle ABC are (3, -5, 7) and (3, 0, 1) respectively. The centroid of the triangle is (0, 0, 0)

To find: the coordinates of vertex C

Formula used:

Centroid of triangle ABC whose vertices are A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) is given by,

$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

Here A(3, -5, 7) and B(3, 0, 1)

Centroid of the triangle

$$(0,0,0) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

$$\Rightarrow (0,0,0) = \left(\frac{3 + 3 + x_3}{3}, \frac{-5 + 0 + y_3}{3}, \frac{7 + 1 + z_3}{3}\right)$$

$$\Rightarrow (0,0,0) = \left(\frac{6 + x_3}{3}, \frac{-5 + y_3}{3}, \frac{8 + z_3}{3}\right)$$

On comparing:

$$\frac{6 + x_3}{3} = 0$$

$\Rightarrow 6 + x_3 = 0$

$\Rightarrow x_3 = -6$

$$\frac{-5 + y_3}{3} = 0$$

$\Rightarrow -5 + y_3 = 0$

$\Rightarrow y_3 = 5$

$$\frac{8 + z_3}{3} = 0$$

$\Rightarrow 8 + z_3 = 0$

$\Rightarrow z_3 = -8$

Hence, coordinates of vertex C(-6, 5, -8)

8. Question

What is the locus of a point (x, y, z) for which y = 0, z = 0?

Answer

Locus is a moving point which satisfies given conditions

Here, conditions are y = 0 and z = 0

Hence, locus for this is x-axis whose equation is y = z = 0

9. Question

Find the ratio in which the line segment joining the points (2, 4, 5) and (3, -5, 4) is divided by the yz-plane.

Answer

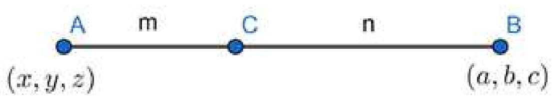
Given: points A(2, 4, 5) and B(3, -5, 4)

To find: the ratio in which the line joining given points is divided by the yz-plane

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

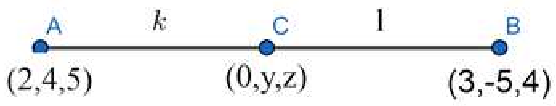
$\left(\frac{nx+ma}{m+n}, \frac{ny+mb}{m+n}, \frac{nz+mc}{m+n}\right)$

the x coordinate is always 0 on yz-plane

Let Point C(0, y, z), and C divides AB in ratio k: 1

Therefore, m = k and n = 1

A(2, 4, 5) and B(3, -5, 4)



Coordinates of C using section formula:

$\Rightarrow (0, y, z) = \left(\frac{k(3)+1(2)}{k+1}, \frac{k(-5)+1(4)}{k+1}, \frac{k(4)+1(5)}{k+1}\right)$

$\Rightarrow (0, y, z) = \left(\frac{3k+2}{k+1}, \frac{-5k+4}{k+1}, \frac{4k+5}{k+1}\right)$

On comparing:

$\frac{3k+2}{k+1} = 0$

⇒ 3k + 2 = 0(k + 1)

⇒ 3k + 2 = 0

⇒ 3k = –2

$\Rightarrow k = \frac{-2}{3}$

Hence, **C divides AB externally in ratio 2 : 3**

10. Question

Find the point on y-axis which is at a distance of $\sqrt{10}$ units from the point (1, 2, 3).

Answer

Given: point P(1, 2, 3)

To find: coordinates of the foot of the perpendicular from the point on the y-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

As x and z coordinate on the y-axis is zero

Let point D any point on y-axis be (0, y, 0)

$PD = \sqrt{10}$

⇒ $PD^2 = 10$

Distance between P(1, 2, 3) and D(0, y, 0) is PD,

$= \sqrt{(1-0)^2 + (2-y)^2 + (3-0)^2}$

$= \sqrt{1^2 + (2 - y)^2 + 3^2}$

$= \sqrt{1 + (2 - y)^2 + 9}$

$= \sqrt{10 + (2 - y)^2}$

Now,

$10 + (2 - y)^2 = 10$

$\Rightarrow (2 - y)^2 = 10 - 10$

$\Rightarrow (2 - y)^2 = 0$

$\Rightarrow 2 - y = 0$

$\Rightarrow y = 2$

Hence, **coordinates of point D are (0, 2, 0)**

11. Question

Find the point on x-axis which is equidistant from the points A(3, 2, 2) and B(5, 5, 4).

Answer

Given: points A(3, 2, 2) and B(5, 5, 4)

To find coordinates of a point on x-axis which is equidistant from given points.

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a - m)^2 + (b - n)^2 + (c - o)^2}$

As, y and z coordinate on x-axis are zero

Let point D any point on x-axis be (x, 0, 0)

AD = BD

Distance between B(5, 5, 4) and D(x, 0, 0) is BD,

$= \sqrt{(5 - x)^2 + (5 - 0)^2 + (4 - 0)^2}$

$= \sqrt{5^2 + (5 - x)^2 + 4^2}$

$= \sqrt{25 + (5 - x)^2 + 16}$

$= \sqrt{41 + (5 - x)^2}$

Distance between A(3, 2, 2) and D(x, 0, 0) is BD,

$= \sqrt{(3 - x)^2 + (2 - 0)^2 + (2 - 0)^2}$

$= \sqrt{2^2 + (3 - x)^2 + 2^2}$

$= \sqrt{4 + (3 - x)^2 + 4}$

$= \sqrt{8 + (3 - x)^2}$

As, AD = BD

$\Rightarrow AD^2 = BD^2$

$8 + (3 - x)^2 = 41 + (5 - x)^2$

$\Rightarrow 8 + 9 + x^2 - 6x = 41 + 25 + x^2 - 10x$

$\Rightarrow 17 - 6x = 66 - 10x$

$\Rightarrow 10x - 6x = 66 - 17$

$\Rightarrow 4x = 49$

$$\Rightarrow x = \frac{49}{4}$$

Hence, **coordinates of point D are** $\left(\frac{49}{4}, 0, 0\right)$

12. Question

Find the coordinates of a point equidistant from the origin and points A(a, 0, 0), B(0, b, 0) and C(0, 0, c).

Answer

Given: Points are O(0, 0, 0), A(a, 0, 0), B(0, b, 0) and C(0, 0, c)

To find: the coordinates of point which is equidistant from the points

Let required point P(x, y, z)

According to question:

PA = PB = PC = PO

$\Rightarrow PA^2 = PB^2 = PC^2 = PO^2$

Formula used:

Distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$$

Therefore,

The distance between P(x, y, z) and O(0, 0, 0) is PO,

$$= \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$= \sqrt{x^2 + y^2 + z^2}$$

Distance between P(x, y, z) and A(a, 0, 0) is PA,

$$= \sqrt{(x-a)^2 + (y-0)^2 + (z-0)^2}$$

$$= \sqrt{(x-a)^2 + y^2 + z^2}$$

Distance between P(x, y, z) and B(0, b, 0) is PB,

$$= \sqrt{(x-0)^2 + (y-b)^2 + (z-0)^2}$$

$$= \sqrt{x^2 + (y-b)^2 + z^2}$$

Distance between P(x, y, z) and C(0, 0, c) is PC,

$$= \sqrt{(x-0)^2 + (y-0)^2 + (z-c)^2}$$

$$= \sqrt{x^2 + y^2 + (z-c)^2}$$

As $PO^2 = PA^2$

$x^2 + y^2 + z^2 = (x - a)^2 + y^2 + z^2$

$\Rightarrow x^2 = x^2 + a^2 - 2ax$

$\Rightarrow 2ax = a^2$

$$\Rightarrow x = \frac{a}{2}$$

As $PO^2 = PB^2$

$x^2 + y^2 + z^2 = x^2 + (y - b)^2 + z^2$

$\Rightarrow y^2 = y^2 + b^2 - 2by$

$\Rightarrow 2by = b^2$

$$\Rightarrow y = \frac{b}{2}$$

As $PO^2 = PC^2$

$x^2 + y^2 + z^2 = x^2 + y^2 + (z - c)^2$

$\Rightarrow z^2 = z^2 + c^2 - 2cz$

$\Rightarrow 2cz = c^2$

$$\Rightarrow z = \frac{c}{2}$$

Hence **point** $\mathbf{P}\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ is equidistant from given points

13. Question

Write the coordinates of the point P which is five-sixth of the way from A(-2, 0, 6) to B(10, -6, -12).

Answer

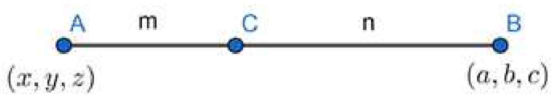
Given: Points A(-2, 0, 6) and B(10, -6, -12)

To find: the coordinates of points P which is five-sixth of AB

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

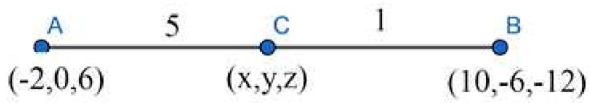
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

Let AB = 6 units and Point C(x, y, z) is fifth-sixth of AB

$$\Rightarrow AC = \frac{5}{6} \times 6 = 5$$

$\Rightarrow CB = 6 - 5 = 1$

Hence, AC : CB = 5 : 1



Now, we will firstly apply section formula on AB and find coordinates of C

Therefore, m = 5 and n = 1

A(-2, 0, 6) and B(10, -6, -12)

Coordinates of R using section formula:

$$\Rightarrow (x,y,z) = \left(\frac{5(10)+1(-2)}{5+1}, \frac{5(-6)+1(0)}{5+1}, \frac{5(-12)+1(6)}{5+1}\right)$$

$$\Rightarrow (x,y,z) = \left(\frac{50-2}{6}, \frac{-30+0}{6}, \frac{-60+6}{6}\right)$$

$$\Rightarrow (x,y,z) = \left(\frac{48}{6}, \frac{-30}{6}, \frac{-54}{6}\right)$$

$$\Rightarrow (x,y,z) = (8,-5,-9)$$

Hence, Coordinates of C are (8, -5, -9)

14. Question

If a parallelepiped is formed by the planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinates planes, then write the lengths of edges of the parallelepiped and length of the diagonal.

Answer

Given: a parallelepiped is formed by the planes drawn through the points (2, 3, 5) and (5, 9, 7) parallel to the coordinates planes.

To find: length of edges of parallelepiped and length of diagonal

Planes parallel to (2, 3, 5) are:

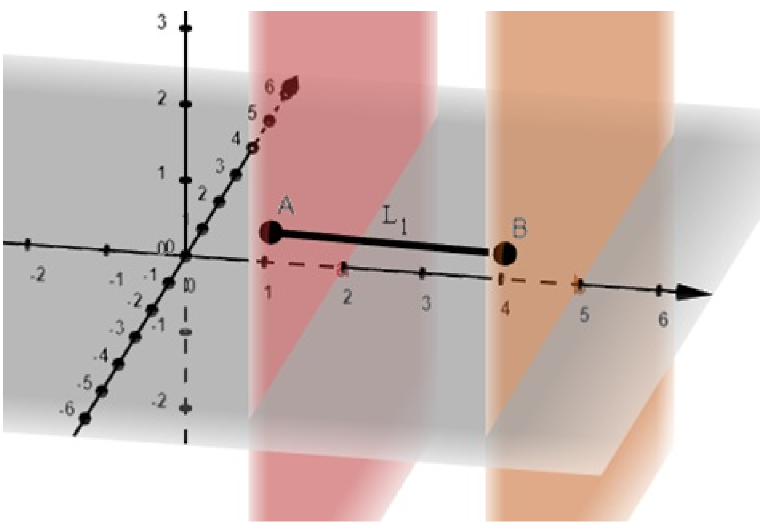
x = 2, y = 3 and z = 5

Similarly, planes parallel to (5, 9, 7) are:

x = 5, y = 9 and z = 7

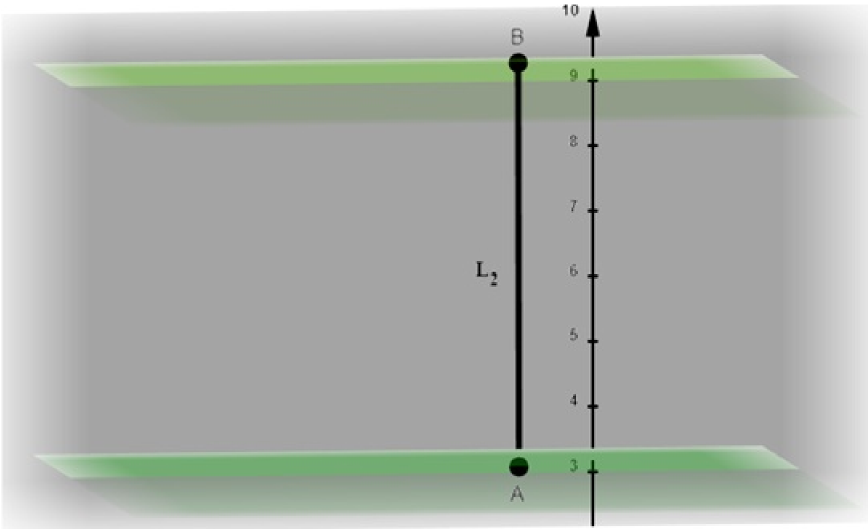
Now, let the length of the parallelepiped are L_1, L_2 and L_3

L_1 is the length of edge between planes x = 2 and x = 5



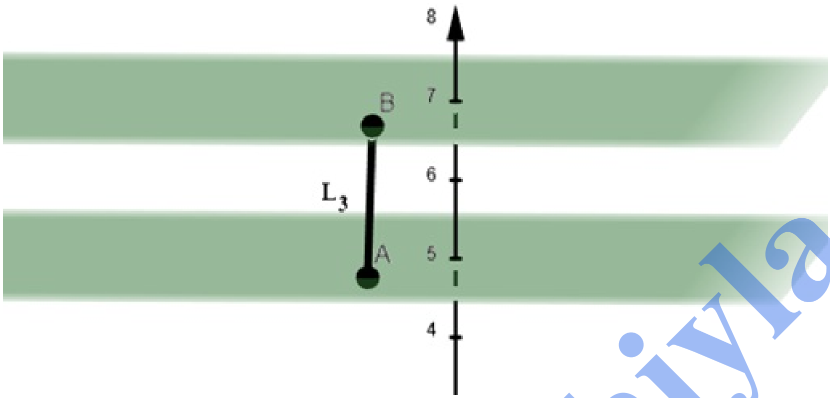
Clearly, L_1 = 5 – 3 = 2

L_2 is the length of an edge between planes y = 3 and y = 9



Clearly, $L_2 = 9 – 3 = 6$

L_3 is the length of an edge between planes z = 5 and z = 7



Clearly, $L_3 = 7 – 5 = 2$

15. Question

Determine the point on yz-plane which is equidistant from points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

Answer

Given: Points A(2, 0, 3), B(0, 3, 2) and C(0, 0, 1)

To find: the point on yz-plane which is equidistant from the points

As we know x = 0 in yz-plane.

Let Q(0, y, z) any point in yz-plane

According to the question:

QA = QB = QC

$\Rightarrow QA^2 = QB^2 = QC^2$

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

The distance between Q(0, y, z) and A(2, 0, 3) is QA,

$= \sqrt{(0-2)^2 + (y-0)^2 + (z-3)^2}$

$= \sqrt{2^2 + y^2 + (z-3)^2}$

$= \sqrt{4 + y^2 + (z-3)^2}$

The distance between Q(0, y, Z) and B(0, 3, 2) is QB,

$= \sqrt{(0-0)^2 + (y-3)^2 + (z-2)^2}$

$= \sqrt{(z-2)^2 + (y-3)^2}$

Distance between Q(0, y, z) and C(0, 0, 1) is QC,

$= \sqrt{(0-0)^2 + (y-0)^2 + (z-1)^2}$

$= \sqrt{(z-1)^2 + y^2}$

As $QA^2 = QB^2$

$4 + (z - 3)^2 + y^2 = (z - 2)^2 + (y - 3)^2$

$\Rightarrow z^2 + 9 - 6z + y^2 + 4 = z^2 + 4 - 4z + y^2 + 9 - 6y$

$\Rightarrow - 6z = - 4z - 6y$

$\Rightarrow 6y - 6z + 4z = 0$

$\Rightarrow 6y - 2z = 0$

$\Rightarrow 6y = 2z$

$\Rightarrow z = \frac{6y}{2}$

$\Rightarrow z = 3y$....................(1)

As $QA^2 = QC^2$

$4 + (z - 3)^2 + y^2 = (z - 1)^2 + y^2$

$\Rightarrow z^2 + 9 - 6z + y^2 + 4 = z^2 + 1 - 2z + y^2$

$\Rightarrow 13 - 6z = 1 - 2z$

$\Rightarrow 13 - 1 = 6z - 2z$

$\Rightarrow 4z = 12$

$\Rightarrow z = 3$

Put the value of z from (1):

$\Rightarrow y = \frac{z}{3}$

$\Rightarrow y = \frac{3}{3}$

$\Rightarrow y = 1$

Hence **point Q(0, 1, 3)** in yz-plane is equidistant from A, B and C

6. Question

If the origin is the centroid of a triangle ABC having vertices A(a, 1, 3), B(-2, b, -5) and C(4, 7, c), find the values of a, b, c.

Answer

Given: The coordinates of the A, B and C of the triangle ABC are (a, 1, 3), (-2, b, -5) and (4, 7, c) respectively. The centroid of the triangle is (0, 0, 0)

To find: the values of a, b, c

Formula used:

Centroid of triangle ABC whose vertices are A(x_1, y_1, z_1), B(x_2, y_2, z_2) and C(x_3, y_3, z_3) is given by,

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

Here A(a, 1, 3), B(-2, b, -5) and C(4, 7, c)

Centroid of the triangle

$$(0,0,0) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

$$\Rightarrow (0,0,0) = \left(\frac{a - 2 + 4}{3}, \frac{-2 + b + 7}{3}, \frac{3 - 5 + c}{3}\right)$$

$$\Rightarrow (0,0,0) = \left(\frac{a + 2}{3}, \frac{b + 5}{3}, \frac{c - 2}{3}\right)$$

On comparing:

$$\frac{a + 2}{3} = 0$$

$\Rightarrow a + 2 = 0$

$\Rightarrow a = -2$

$$\frac{b + 5}{3} = 0$$

$\Rightarrow b + 5 = 0$

$\Rightarrow b = -5$

$$\frac{c - 2}{3} = 0$$

$\Rightarrow c - 2 = 0$

$\Rightarrow c = 2$

Hence, values of a, b and c are -2, -5, 2

MCQ

1. Question

The ratio in which the line joining (2, 4, 5) and (3, 5, -9) is divided by the yz-plane is

A. 2 : 3

B. 3 : 2

C. -2 : 3

D. 4 : -3

Answer

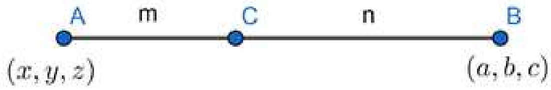
Given: points A(2, 4, 5) and B(3, 5, -9)

To find: the ratio in which the line joining given points is divided by the yz-plane

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

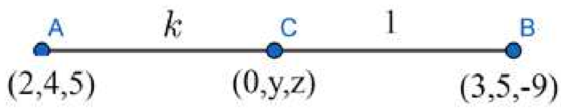
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

x coordinate is always 0 on yz-plane

Let Point C(0, y, z) and C divides AB in ratio k : 1

Therefore, m = k and n = 1

A(2, 4, 5) and B(3, 5, -9)



Coordinates of C using section formula:

$$\Rightarrow (0, y, z) = \left(\frac{k(3) + 1(2)}{k + 1}, \frac{k(5) + 1(4)}{k + 1}, \frac{k(-9) + 1(5)}{k + 1}\right)$$

$$\Rightarrow (0, y, z) = \left(\frac{3k + 2}{k + 1}, \frac{5k + 4}{k + 1}, \frac{-9k + 5}{k + 1}\right)$$

On comparing:

$$\frac{3k + 2}{k + 1} = 0$$

⇒ 3k + 2 = 0(k + 1)

⇒ 3k + 2 = 0

⇒ 3k = –2

$$\Rightarrow k = \frac{-2}{3}$$

Hence, **C divides AB externally in ratio 2 : 3**

2. Question

The ratio in which the line joining the points (a, b, c) and (-1, -c, -b) is divided by the xy-plane is

A. a : b

B. b : c

C. c : a

D. c : b

Answer

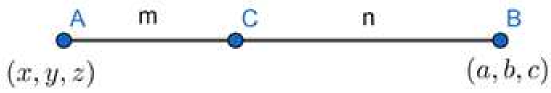
Given: points A(a, b, c) and B(-1, -c, -b)

To find: the ratio in which the line joining given points is divided by the xy-plane

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).



The coordinates of C is given by,

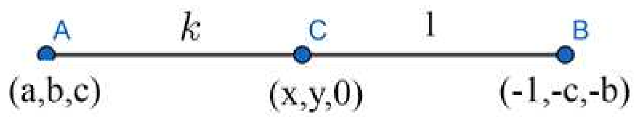
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

z coordinate is always 0 on xy-plane

Let Point C(x, y, 0), and C divides AB in ratio k: 1

Therefore, m = k and n = 1

A(a, b, c) and B(-1, -c, -b)



Coordinates of C using section formula:

$$\Rightarrow (x,y,0) = \left(\frac{k(-1) + 1(a)}{k + 1}, \frac{k(-c) + 1(b)}{k + 1}, \frac{k(-b) + 1(c)}{k + 1}\right)$$

$$\Rightarrow (x,y,0) = \left(\frac{-k + a}{k + 1}, \frac{-ck + b}{k + 1}, \frac{-bk + c}{k + 1}\right)$$

On comparing:

$$\frac{-bk + c}{k + 1} = 0$$

⇒ -bk + c = 0(k + 1)

⇒ -bk = -c

$$\Rightarrow k = \frac{-c}{-b} = \frac{c}{b}$$

Hence, **C divides AB internally in ratio c: b**

3. Question

If P (0, 1, 2), Q(4, -2, 1) and O(0, 0, 0) are three points, then∠POQ =

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer

Given: Points are P(0, 1, 2), Q(4, -2, 1) and O(0, 0, 0)

To check: the value of ∠POQ

Formula used:

Distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between P(0, 1, 2) and Q(4, -2, 1) is PQ,

$=\sqrt{(0-4)^2+(1-(-2))^2+(2-1)^2}$

$=\sqrt{(-4)^2+3^2+1^2}$

$=\sqrt{16+9+1}$

$=\sqrt{26}$

Distance between Q(4, -2, 1) and O(0, 0, 0) is QO,

$=\sqrt{(4-0)^2+(-2-0)^2+(1-0)^2}$

$=\sqrt{4^2+(-2)^2+1^2}$

$=\sqrt{16+4+1}$

$=\sqrt{21}$

Distance between P(0, 1, 2) and O(0, 0, 0) is PO,

$=\sqrt{(0-0)^2+(1-0)^2+(2-0)^2}$

$=\sqrt{0^2+1^2+2^2}$

$=\sqrt{0+1+4}$

$=\sqrt{5}$

Now,

$PO^2 = 5$

$QO^2 = 21$

$PQ^2 = 26$

Clearly,

$PO^2 + QO^2 = PQ^2$

A right-angled triangle is a triangle which satisfies Pythagoras Theorem

These points satisfy Pythagoras Theorem

Thus, $\angle \mathbf{POQ} = \frac{\boldsymbol{\pi}}{\mathbf{2}}$

4. Question

If the extremities of the diagonal of a square are (1, -2, 3) and (2, -3, 5), then the length of the side is

A. $\sqrt{6}$

B. $\sqrt{3}$

C. $\sqrt{5}$ D. $\sqrt{7}$

Answer

Given: Extremities of the diagonal of a square are P(1, -2, 3) and Q(2, -3, 5)

To find: Length of the diagonal

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

Therefore,

Distance between P(1, -2, 3) and Q(2, -3, 5) is

$$=\sqrt{(1-2)^2+(-2-(-3))^2+(3-5)^2}$$

$$=\sqrt{(-1)^2+1^2+(-2)^2}$$

$$=\sqrt{1+1+4}$$

$$=\sqrt{6}$$

Hence, **Length of the diagonal is $\sqrt{6}$ units**

5. Question

The points (5, -4, 2), (4, -3, 1), (7, 6, 4) and (8, -7, 5) are the vertices of

A. a rectangle

B. a square

C. a parallelogram

D. none of these

Answer

Given: Points are A(5, -4, 2), B(4, -3, 1), C(7, 6, 4) and D(8, -7, 5)

To find: name of the quadrilateral formed by these 4 points

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$$

Therefore,

Distance between A(5, -4, 2) and B(4, -3, 1) is AB,

$$=\sqrt{(5-4)^2+(-4-(-3))^2+(2-1)^2}$$

$$=\sqrt{1^2+(-1)^2+1^2}$$

$= \sqrt{1+1+1}$

$= \sqrt{3}$

Distance between B(4, -3, 1) and C(7, 6, 4) is BC,

$= \sqrt{(4-7)^2 + (-3-6)^2 + (1-4)^2}$

$= \sqrt{(-3)^2 + (-9)^2 + (-3)^2}$

$= \sqrt{9+81+9}$

$= \sqrt{99}$

$= 3\sqrt{11}$

The distance between C(7, 6, 4) and D(8, -7, 5) is CD,

$= \sqrt{(7-8)^2 + (6-(-7))^2 + (4-5)^2}$

$= \sqrt{1^2 + 13^2 + (-1)^2}$

$= \sqrt{1+169+1}$

$= \sqrt{171}$

Distance between A(5, -4, 2) and D(8, -7, 5) is AD,

$= \sqrt{(5-8)^2 + (-4-(-7))^2 + (2-5)^2}$

$= \sqrt{(-3)^2 + 3^2 + (-3)^2}$

$= \sqrt{9+9+9}$

$= 3\sqrt{3}$

Clearly,

No two sides are equal

So, **it cannot be square, rectangle or parallelogram**.

6. Question

In a three dimensional space the equation $x^2 - 5x + 6 = 0$ represents

A. points

B. planes

C. curves

D. pair of straight lines

Answer

Given: $x^2 - 5x + 6 = 0$

$x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x - 3) - 2(x - 3) = 0$

$\Rightarrow (x - 3)(x - 2) = 0$

⇒ (x – 3) = 0 or (x – 2) = 0

⇒ x = 3 or x = 2

Both the results represents planes which are parallel to yz-plane

Hence, **x^2 – 5x + 6 = 0 represents planes**

7. Question

Let (3, 4, -1) and (-1, 2, 3) be the endpoints of a diameter of a sphere. Then, the radius of the sphere is equal to

A. 2

B. 3

C. 6

B. 7

Answer

Given: P(3, 4, -1) and Q(-1, 2, 3) represents diameter of sphere

To find: Radius of the sphere

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

Therefore,

Distance between P(3, 4, -1) and Q(-1, 2, 3), PQ is

$=\sqrt{(3-(-1))^2+(4-2)^2+(-1-3)^2}$

$=\sqrt{4^2+2^2+(-4)^2}$

$=\sqrt{16+4+16}$

$=\sqrt{36}$

= 6

So, Diameter = 6

We know that Diameter = 2 Radius

$\Rightarrow \text{ Radius } = \frac{\text{Diameter}}{2}$

$\Rightarrow \text{ Radius } = \frac{6}{2} = 3$

Hence, **Radius of the sphere is 3 units**

8. Question

XOZ-plane divides the join of (2, 3, 1) and (6, 7, 1)

A. 3 : 7

B. 2 : 7

C. -3 : 7

D. -2 : 7

Answer

Given: points A(2, 3, 1) and B(6, 7, 1)

To find: the ratio in which the line joining given points is divided by the XOZ-plane

Formula used:

Section Formula:

A line AB is divided by C in m:n where A(x, y, z) and B(a, b, c).

The coordinates of C is given by,

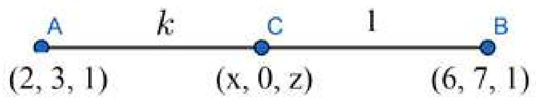
$$\left(\frac{nx + ma}{m + n}, \frac{ny + mb}{m + n}, \frac{nz + mc}{m + n}\right)$$

the y coordinate is always 0 on XOZ-plane

Let Point C(x, 0, z), and C divides AB in ratio k: 1

Therefore, m = k and n = 1

A(2, 3, 1) and B(6, 7, 1)



Coordinates of C using section formula:

$$\Rightarrow (x, 0, z) = \left(\frac{k(6) + 1(2)}{k + 1}, \frac{k(7) + 1(3)}{k + 1}, \frac{k(1) + 1(1)}{k + 1}\right)$$

$$\Rightarrow (x, 0, z) = \left(\frac{6k + 2}{k + 1}, \frac{7k + 3}{k + 1}, \frac{k + 1}{k + 1}\right)$$

On comparing:

$$\frac{7k + 3}{k + 1} = 0$$

⇒ 7k + 3 = 0(k + 1)

⇒ 7k + 3 = 0

⇒ 7k = –3

$$\Rightarrow k = \frac{-3}{7}$$

Hence, **C divides AB externally in ratio 3: 7**

9. Question

What is the locus of a point for which y = 0, z = 0?

A. x-axis

B. y-axis

C. z-axis

D. yz-plane

Answer

Locus is a moving point which satisfies given conditions

Here, conditions are y = 0 and z = 0

Hence, locus for this is x-axis whose equation is y = z = 0

10. Question

the coordinates of the foot of the perpendicular drawn from the point P(3, 4, 5) on the yz-plane are

A. (3, 4, 0)

B. (0, 4, 5)

C. (3, 0, 5)

D. (3, 0, 0)

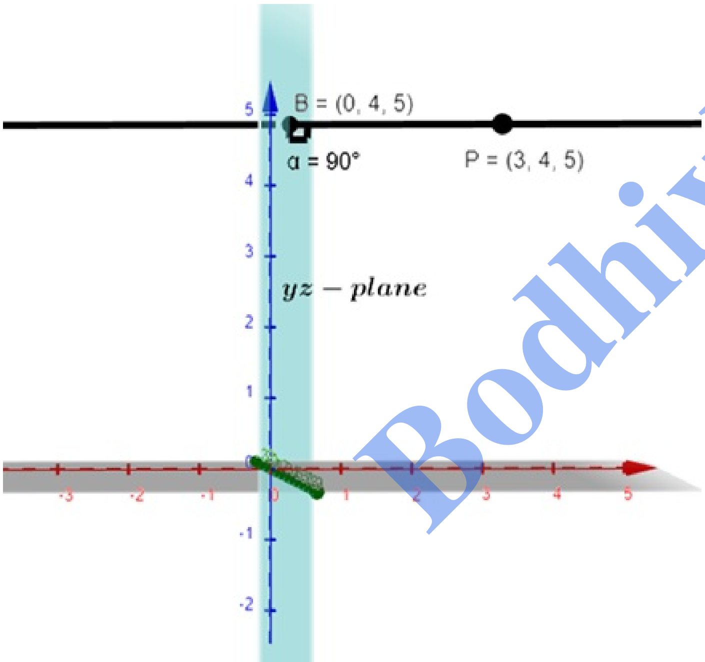
Answer

Given: Points P(3, 4, 5)

To find: the coordinates of the foot of the perpendicular drawn from the point P on yz-plane

As we know x = 0 in yz-plane.

But y and z coordinates will remain the same for the foot of perpendicular.



Hence, **the coordinates of the foot of the perpendicular from point P on yz-plane are (0, 4, 5)**

11. Question

The coordinates of the foot of the perpendicular from a point P(6, 7, 8) on the x-axis are

A. (6, 0, 0)

B. (0, 7, 0)

C. (0, 0, 8)

D. (0, 7, 8)

Answer

Given: point P(6, 7, 8)

To find: coordinates of the foot of the perpendicular from a point P from the x-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

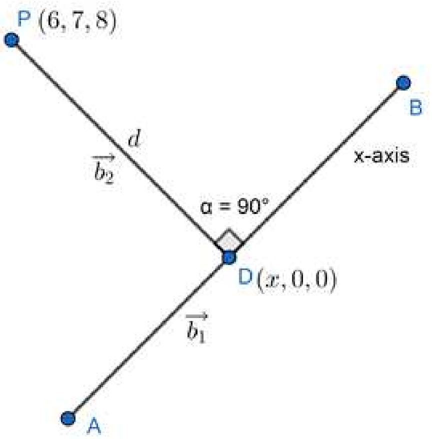
As, y and z coordinate on x-axis are zero

Let point D on x-axis is (x, 0, 0)

Direction cosines of z-axis are (1, 0, 0)

Direction cosines of PD are (6 – x, 7 – 0, 8 – 0) = (6 – x, 7, 8)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$

⇒ (6 – x) × 1 + 7 × 0 + 8 × 0 = 0

⇒ 6 – x + 0 + 0 = 0

⇒ x = 6

Hence, **coordinates of the foot of the perpendicular i.e. point D(6, 0, 0)**

12. Question

The perpendicular distance of the point P(6, 7, 8) from xy-plane is

A. 8

B. 7

C. 6

D. 10

Answer

Given: Points P(6, 7, 8)

To find: the perpendicular distance of the point P from xy-plane

As we know z = 0 in xy-plane.

The shortest distance of the plane will be the z-coordinate of the point

Hence, **the distance of point P from xy-plane is 8 units**

13. Question

The length of the perpendicular drawn from the point P(3, 4, 5) on the y-axis is

A. 10

B. $\sqrt{34}$

C. $\sqrt{113}$

D. $5\sqrt{2}$

Answer

Given: point P(3, 4, 5)

To find: length of the perpendicular from the point on the y-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2+(b-n)^2+(c-o)^2}$

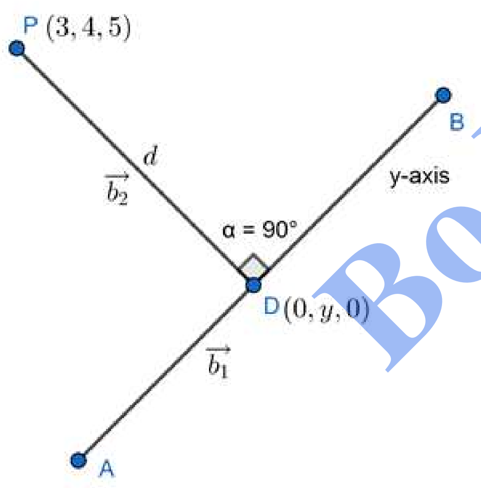
As x and z coordinate on the y-axis is zero

Let point D is the point of the foot of perpendicular on the y-axis from point P be (0, y, 0)

Direction cosines of y-axis are (0, 1, 0)

Direction cosines of PD are (3 – 0, 4 – y, 5 – 0) = (3, 4 – y, 5)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$

⇒ 3 × 0 + (4 – y) × 1 + 5 × 0 = 0

⇒ 0 + 0 + 4 – y= 0

⇒ y = 4

Hence point D(0, 4, 0)

Distance between point P(3, 4, 5) and D(0, 4, 0) is d

$= \sqrt{(3-0)^2+(4-4)^2+(5-0)^2}$

$= \sqrt{3^2 + 0^2 + 5^2}$

$= \sqrt{9 + 0 + 25}$

$= \sqrt{34}$

Hence, the **distance of the point P from y-axis is** $\sqrt{34}$ **units**

14. Question

The perpendicular distance of the point P(3, 3, 4) from the x-axis is

A. $3\sqrt{2}$

B. 5

C. 3

D. 4

Answer

Given: point P(3, 3, 4)

To find: length of the perpendicular drawn from the point P from the x-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

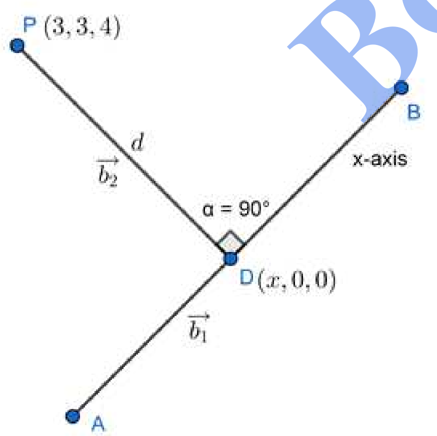
As, y and z coordinate on x-axis are zero

Let point D on x-axis is (x, 0, 0)

Direction cosines of z-axis are (1, 0, 0)

Direction cosines of PD are (3 – x, 3 – 0, 4 – 0) = (3 – x, 3, 4)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$

⇒ (3 – x) × 1 + 3 × 0 + 4 × 0 = 0

⇒ 3 – x + 0 + 0 = 0

⇒ x = 3

Hence point D(3, 0, 0)

Distance between point P(3, 3, 4) and D(3, 0, 0) is d

$= \sqrt{(3-3)^2 + (3-0)^2 + (4-0)^2}$

$= \sqrt{0^2 + 3^2 + 4^2}$

$= \sqrt{0 + 9 + 16}$

$= \sqrt{25}$

= 5

Hence, the **distance of the point P from x-axis is 5 units**

15. Question

The length of the perpendicular drawn from the point P(a, b, c) from z-axis is

A. $\sqrt{a^2 + b^2}$

B. $\sqrt{b^2 + c^2}$

C. $\sqrt{a^2 + c^2}$

D. $\sqrt{a^2 + b^2 + c^2}$

Answer

Given: point P(a, b, c)

To find: distance of the point P from the z-axis

Formula used:

The distance between any two points (a, b, c) and (m, n, o) is given by,

$\sqrt{(a-m)^2 + (b-n)^2 + (c-o)^2}$

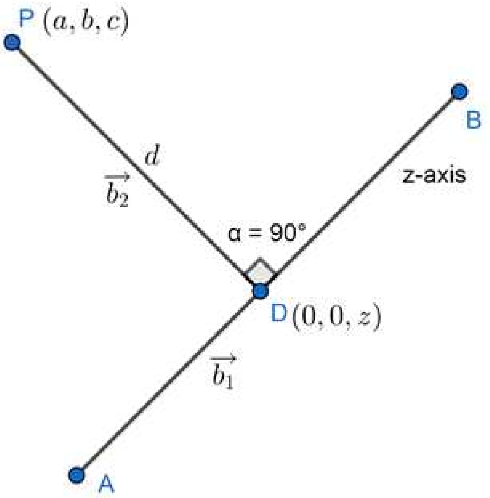
As, x and y coordinate on z-axis are zero

Let point D on z-axis is (0, 0, z)

Direction cosines of z-axis are (0, 0, 1)

Direction cosines of PD are (a – 0, b – 0, c – z) = (a, b, c – z)

Let $\overrightarrow{b_1}$ and $\overrightarrow{b_2}$ are two vectors as shown in the figure:



The dot product of perpendicular vectors is always zero

Therefore, $\overrightarrow{b_1}.\overrightarrow{b_2} = 0$

⇒ a × 0 + b × 0 + (c – z) × 1 = 0

⇒ 0 + 0 + c – z = 0

⇒ z = c

Hence point D(0, 0, c)

Distance between point P(a, b, c) and D(0, 0, c) is d

$= \sqrt{(a-0)^2 + (b-0)^2 + (c-c)^2}$

$= \sqrt{a^2 + b^2 + 0^2}$

$= \sqrt{a^2 + b^2}$

Hence, the **distance of the point P from z-axis is** $\sqrt{\mathbf{a^2 + b^2}}$ **units**