23. Algebra of Vectors

Exercise 23.1

1. Question

Represent the following graphically:

i. a displacement of 40 km, 30° east of north

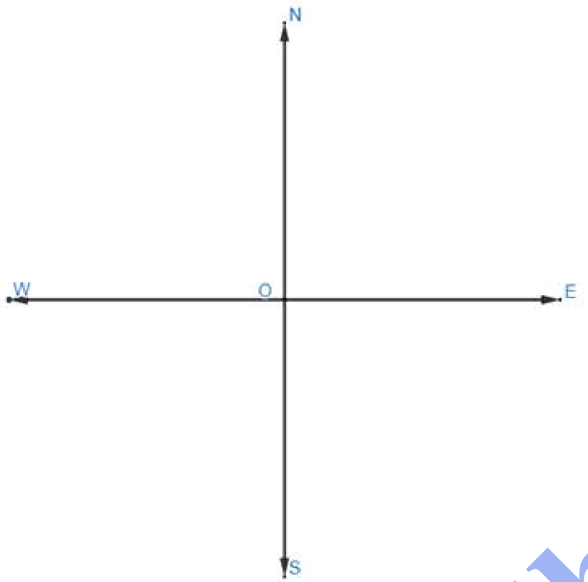
ii. a displacement of 50 km south - east

iii. a displacement of 70 km, 40° north of west.

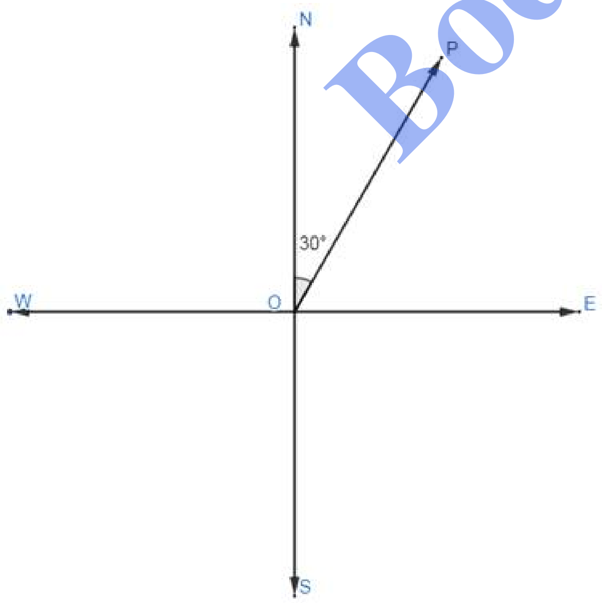
Answer

i. a displacement of 40 km, 30° east of north

Step 1: Draw north, south, east and west as shown below:

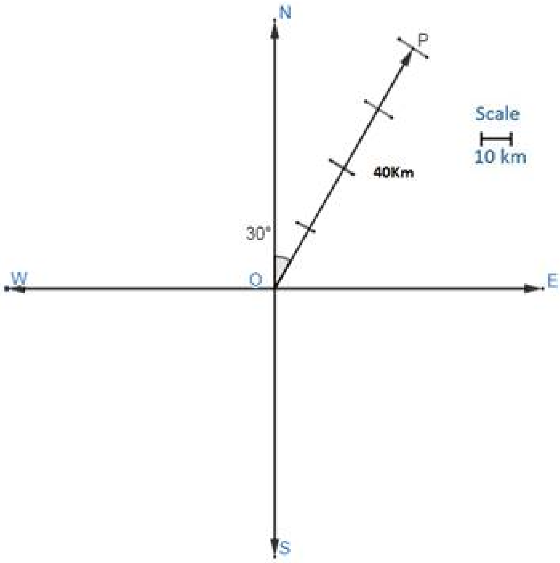


Step 2: Plot a line $\overrightarrow{OP}$ 30° east of north as shown below:



Step 3: Define scale and mark 40km on line $\overrightarrow{OP}$

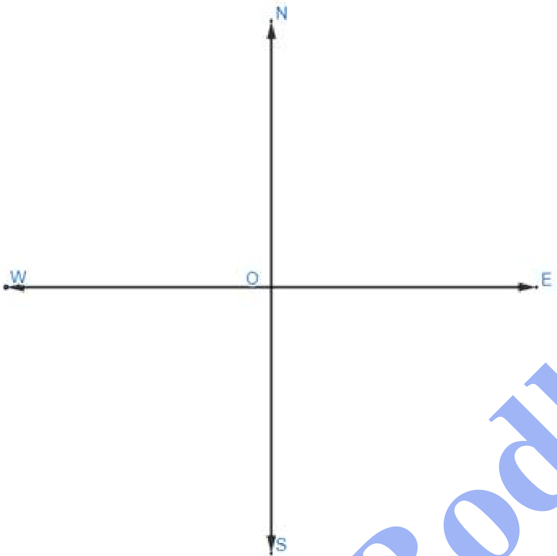
Let the scale be 10km = 1cm



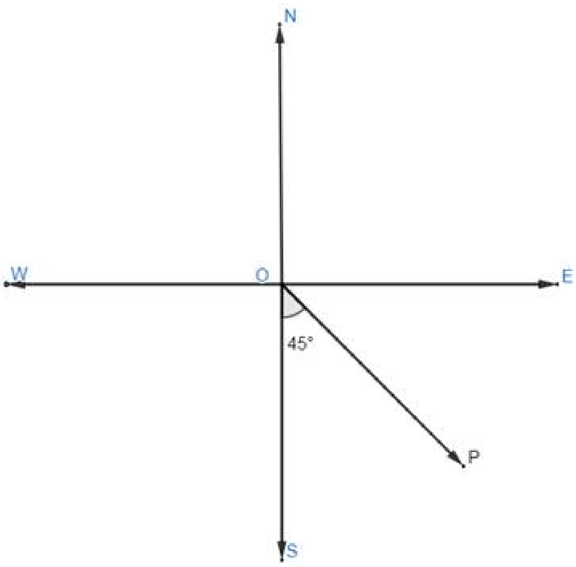
$\therefore \overrightarrow{OP}$ represents the displacement of 40 km, 30^o East of North

ii. a displacement of 50 km south - east

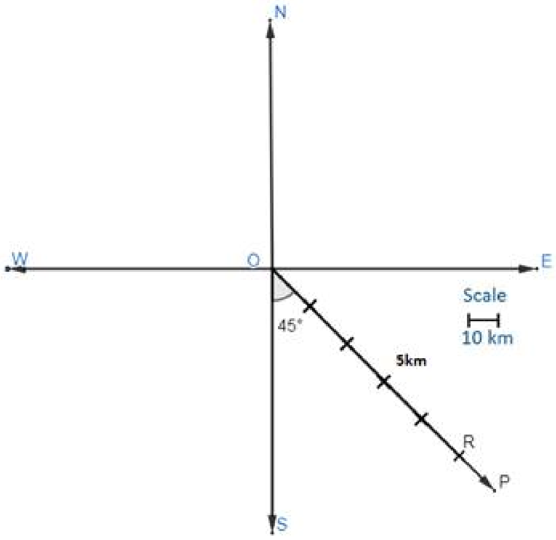
Step 1: Draw north, south, east and west as shown below:



Step 2: As the displacement should be south - east, the angle between the displacement and east (or south) will be 45°. Now, plot a line $\overrightarrow{OP}$ 45° east of south as shown below:



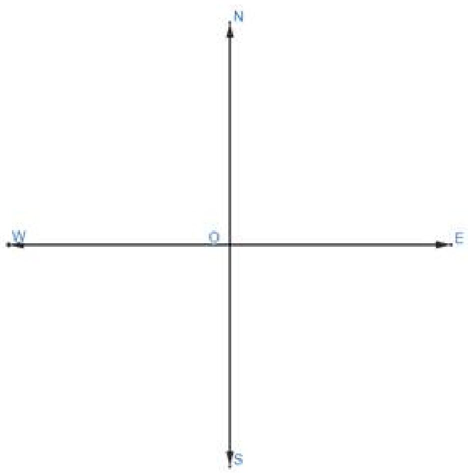
Step 3: Define scale and mark point R such that OR = 50km on line $\overrightarrow{OP}$. Let the scale be 10km = 1cm



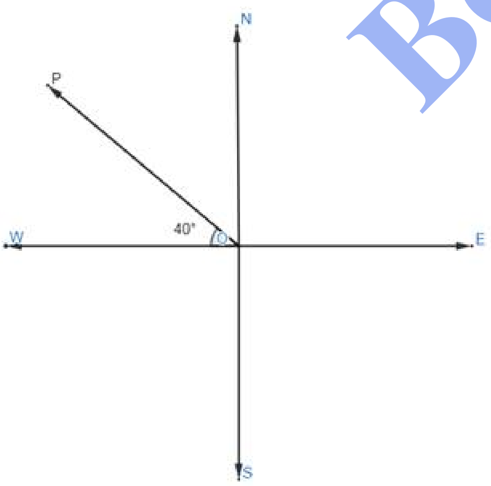
$\therefore \overrightarrow{OR}$ represents the displacement of 50 km south – east

iii. A displacement of 70 km, 40^o north of west.

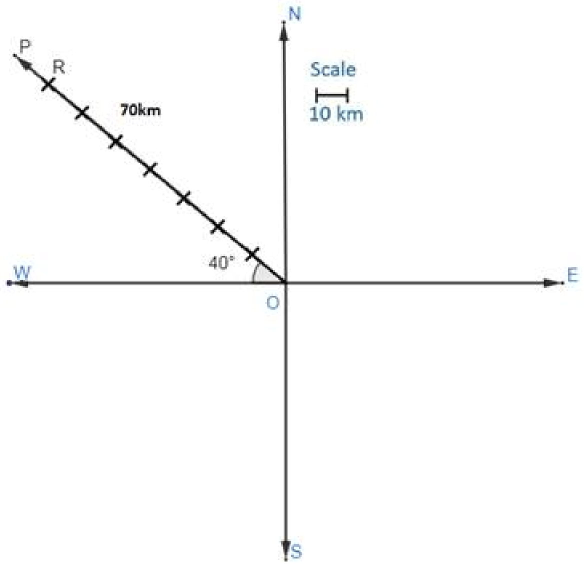
Step 1: Draw north, south, east and west as shown below:



Step 2: Plot a line $\overrightarrow{OP}$ 40° north of west as shown below:



Step 3: Define scale and mark point R such that OR = 70km on line $\overrightarrow{OP}$.Let the scale be 10km = 1cm



∴ $\overrightarrow{OP}$ represents the displacement of 70 km, 40^o north of west

2. Question

Classify the following measures as scalars and vectors :

i. 15 kg

ii. 20 kg weight

iii. 45°

iv. 10 meters south - east

v. 50 m/sec^2

Answer

i. 15 kg - is a scalar quantity as this involves only mass. A scalar quantity is a one - dimensional measurement of a quantity, like temperature, or **mass**.

ii. 20 kg weight - is a vector quantity as it involves both magnitude and direction. **Weight** is a force which is a **vector** and has a magnitude and direction.

iii. 45^o is a scalar quantity as it involves the only magnitude. A scalar quantity is a one - dimensional measurement of a quantity, like temperature, or **mass**.

iv. 10 meters south - east is a vector quantity as it involves both magnitude and direction.

v. 50 m/sec^2 is a scalar quantity as it involves a magnitude of acceleration. A scalar quantity is a one - dimensional measurement of a quantity.

3. Question

Classify the following as scalars and vector quantities:

i. Time period

ii. Distance

iii. Displacement

iv. Force

v. Work

vi. Velocity

vii. Acceleration

Answer

i. Time period - is a scalar quantity as it involves only magnitude. A scalar quantity is a one - dimensional measurement of a quantity. Eg: 10 seconds has only magnitude, i.e., 10 and no direction.

ii. Distance - is a scalar quantity as it involves only magnitude. A scalar quantity is a one dimensional measurement of a quantity. Eg: 5meters has only magnitude 5 and no direction.

iii. Displacement - is vector quantity as it involves both magnitude and direction. Vector quantity has both magnitude and direction.

iv. Force - is a vector quantity as it involves both magnitude and direction. Vector quantity has both magnitude and direction. Eg., 5N downward has magnitude of 5 and direction is downward.

v. Work done - is a scalar quantity as it involves only magnitude and no particular direction. A scalar quantity is a one dimensional measurement of a quantity.

vi. Velocity - is a vector quantity as it involves both magnitude as well as direction. Vector quantity has both magnitude and direction. Eg., 5m/s east has magnitude of 5m/s and also direction towards east.

vii. Acceleration is a vector quantity because it involves both magnitude as well as direction.

4. Question

In fig 23.5 ABCD is a regular hexagon, which vectors are:

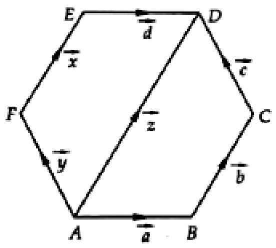


Fig. 23.5

i. Collinear

ii. Equal

iii. Cointitial

iv. Collinear but not equal.

Answer

i. Collinear

Two or more **vectors** that lie on the same line or on a parallel line to this are called **collinear vectors.** Two collinear vectors may point in either same or opposite direction. But, they cannot be inclined at some angle from each other.

Hence FE ($\vec{x}$), AD ($\vec{z}$) and BC ($\vec{b}$) are collinear vectors.

And also AF ($\vec{y}$) and CD ($\vec{c}$) are collinear vectors.

And AB ($\vec{a}$) and ED ($\vec{d}$) are collinear vectors.

ii. Equal

Equal vectors are **vectors** that have the same magnitude and the same direction. **Equal vectors** may start at different positions.

Hence AF ($\vec{y}$) and CD ($\vec{c}$) are equal vectors.

And also FE ($\vec{x}$) and BC ($\vec{b}$) are equal vectors.

And AB ($\vec{a}$) and ED ($\vec{d}$) are equal vectors.

iii. Co - initial

Any given two vectors are called co - initial vectors if both the given vectors have the same initial point.

Hence, AB ($\vec{a}$), AF ($\vec{y}$) and AD ($\vec{z}$) are co - initial vectors.

iv. Collinear but not equal.

And AD ($\vec{z}$) and BC ($\vec{b}$) are collinear but not equal vectors.

And AD ($\vec{z}$) and FE ($\vec{x}$) are collinear but not equal vectors

5. Question

Answer the following as true or false:

i. $\vec{a}$ and $\vec{a}$ are collinear.

ii. Two collinear vectors are always equal in magnitude.

iii. Zero vector is unique.

iv. Two vectors having same magnitude are collinear.

v. Two collinear vectors having the same magnitude are equal.

Answer

i. $\vec{a}$ and $\vec{a}$ are collinear. (True)

Two or more **vectors** that lie on the same line or on a parallel line to this are called **collinear vectors.**

$\vec{a}$ and $\vec{a}$ are collinear.

ii. Two collinear vectors are always equal in magnitude. (False)

Two or more **vectors** that lie on the same line or on a parallel line to this are called **collinear vectors.** Two collinear vectors may point in either same or opposite direction. And they are not necessarily equal in magnitude they can be of different magnitude also.

iii. Zero vector is unique.(True)

There is only one zero - vector in a vector space. Hence zero vector is unique.

iv. Two vectors having same magnitude are collinear. (False)

It is not necessary for two vectors having the same magnitude to be parallel to the same line. Hence two vectors having same magnitude need not be collinear.

v. Two collinear vectors having the same magnitude are equal.(False)

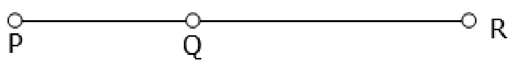
Two vectors are said to be equal if they have the same magnitude and direction, regardless of the positions of their initial points.

Exercise 23.2

1. Question

If P, Q and R are three collinear points such that $\overline{PQ} = \vec{a}$ and $\overline{QR} = \vec{b}$. Find the vector $\overline{PR}$.

Answer



As P, Q and R are three collinear points.

Hence, $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$ as shown in above fig

And given $\overrightarrow{PQ} = \vec{a}$ and $\overrightarrow{QR} = \vec{b}$.

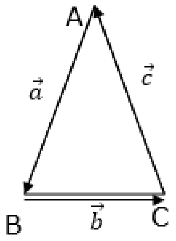
Therefore $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \vec{a} + \vec{b}$

2. Question

Give a condition that three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ form the three sides of a triangle. What are the other possibilities?

Answer

Given that, $\vec{a}, \vec{b}$ and $\vec{c}$ are three sides of a triangle.



Hence from the above figure we get,

AB = $\vec{a}$, BC = $\vec{b}$ and AC = $\vec{c}$

So $\vec{a} + \vec{b} + \vec{c} = AB + BC + CA = AC + CA$

[since $AB + BC = AC$]

$= AC - AC = 0$ [Since CA = - AC]

Triangle law says that, if vectors are represented in magnitude and direction by the two sides of a triangle is same order, then their sum is represented by the third side took in reverse order. Thus,

$\vec{a} + \vec{b} = -\vec{c}$ or $\vec{a} + \vec{c} = -\vec{b}$ or $\vec{b} + \vec{c} = -\vec{a}$

3. Question

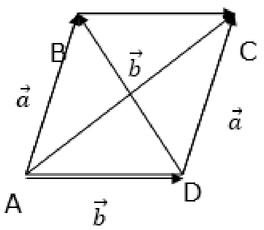
If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors having the same initial point. What are the vectors represented by $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

Answer

Given $\vec{a}$ and $\vec{b}$ are two non - collinear vectors having the same initial point.

Let $AB = \vec{a}$ and $AD = \vec{b}$

Let us draw a parallelogram with AB and AD as any of the two sides of the parallelogram as shown below.



We know in parallelogram opposite sides are equal hence,

$DC = \vec{a}$ and $BC = \vec{b}$

Now consider ΔABC, applying triangles law of vectors, we get

$AB + BC = AC \Rightarrow \vec{a} + \vec{b} = AC \ldots\ldots\ldots (i)$

Similarly in ΔABD, applying triangles law of vectors, we get

$AD + DB = AB \Rightarrow \vec{a} + DB = \vec{b} \Rightarrow \vec{a} - \vec{b} = DB \ldots\ldots\ldots (ii)$

Looking at the two equations (i) and (ii) we can conclude that

$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are the diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$.

4. Question

If is a vector and m is a scalar such that m $\vec{a} = \vec{0}$, then what are the alternatives for m and $\vec{a}$?

Answer

Given $\vec{a}$ is a vector and m is a scalar such that $m\vec{a} = \vec{0}$

Let $\vec{a} = a_1\,\hat{i} + b_1\,\hat{j} + c_1\hat{k}$ then according to the given question

$m\vec{a} = \vec{0}$

$\Rightarrow m(a_1\,\hat{i} + b_1\,\hat{j} + c_1\hat{k}) = 0\,\hat{i} + 0\,\hat{j} + 0\hat{k}$

$\Rightarrow (ma_1\,\hat{i} + mb_1\,\hat{j} + mc_1\hat{k}) = 0\,\hat{i} + 0\,\hat{j} + 0\hat{k}$

Compare the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$ma_1 = 0 \Rightarrow m = 0$ or $a_1 = 0$

Similarly, $mb_1 = 0 \Rightarrow m = 0$ or $b_1 = 0$

And, $mc_1 = 0 \Rightarrow m = 0$ or $c_1 = 0$

From the above three conditions,

$m = 0$ or $a_1 = b_1 = c_1 = 0$

$\Rightarrow m = 0$ or $\vec{a} = a_1\,\hat{i} + b_1\,\hat{j} + c_1\hat{k} = 0\,\hat{i} + 0\,\hat{j} + 0\hat{k} = 0$

Hence the alternatives for m and $\vec{a}$ are $m = 0$ or $\vec{a} = 0$

5 A. Question

If $\vec{a}, \vec{b}$ are two vectors, then write the truth value of the following statements :

$\vec{a} = -\vec{b} \Rightarrow |\vec{a}| = |\vec{b}|$

Answer

Given: $\vec{a} = -\vec{b}$

Let $\vec{a} = a_1\,\hat{i} + b_1\,\hat{j} + c_1\hat{k}$ and $\vec{b} = a_2\,\hat{i} + b_2\,\hat{j} + c_2\hat{k}$

So according to the given criteria,

$\vec{a} = -\vec{b}$

$\Rightarrow a_1\,\hat{i} + b_1\,\hat{j} + c_1\hat{k} = -(a_2\,\hat{i} + b_2\,\hat{j} + c_2\hat{k})$

$\Rightarrow a_1\,\hat{i} + b_1\,\hat{j} + c_1\hat{k} = -a_2\,\hat{i} - b_2\,\hat{j} - c_2\hat{k}$

Compare the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$a_1 = a_2$, $b_1 = b_2$ and $c_1 = c_2$..........(i)

and $|\vec{a}| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

Substitute the values from eqn (i) in above eqn we get

$|\vec{a}| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$

$\Rightarrow |\vec{a}| = \sqrt{(a_2)^2 + (b_2)^2 + (c_2)^2}$

But, $|\vec{b}| = \sqrt{(a_2)^2 + (b_2)^2 + (c_2)^2}$

Hence $|\vec{a}| = |\vec{b}|$

Therefore, $\vec{a} = -\vec{b} \Rightarrow |\vec{a}| = |\vec{b}|$

5 B. Question

If $\vec{a}, \vec{b}$ are two vectors, then write the truth value of the following statements :

$$|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \pm\vec{b}$$

Answer

Given: $|\vec{a}| = |\vec{b}|$

It means the magnitude of the vector $\vec{a}$is equal to the magnitude of the vector $\vec{b}$, but we cannot conclude anything about the direction of the vector.

So it is false that $|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \pm\vec{b}$

5 C. Question

If $\vec{a}, \vec{b}$ are two vectors, then write the truth value of the following statements :

$$|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \vec{b}$$

Answer

Given: $|\vec{a}| = |\vec{b}|$

It means the magnitude of the vector $\vec{a}$is equal to the magnitude of the vector $\vec{b}$, but we cannot conclude anything about the direction of the vector.

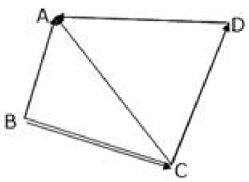
And we know that $\vec{a} = \vec{b}$ means magnitude and same direction. So, it is false that $|\vec{a}| = |\vec{b}| \Rightarrow \vec{a} = \vec{b}$

6. Question

ABCD is a quadrilateral. Find the sum of the vectors $\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{CD}$ and $\overrightarrow{DA}$.

Answer

Given: ABCD is a quadrilateral as shown below



Consider ΔADC and apply triangle law of vector, we get

$\overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{CA} \ldots\ldots (i)$

Similarly, consider ΔABC and apply triangle law of vector, we get

$\overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$ ..... (ii)

Substituting the value of $\overrightarrow{CA}$ from eqn(i) into eqn(ii), we get

$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA}$

Now add $\overrightarrow{BA}$ on both sides, we get

$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{BA} = \overrightarrow{BA} + \overrightarrow{BA}$

$\Rightarrow \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} + \overrightarrow{BA} = 2\overrightarrow{BA}$

Hence sum of the vectors $\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{CD}$ and $\overrightarrow{DA}$ is $2\overrightarrow{BA}$

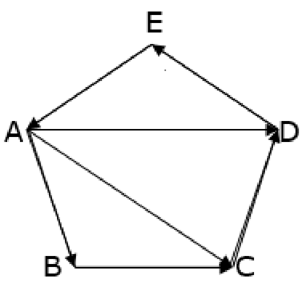
7 A. Question

ABCDE is a pentagon, prove that

$$\overline{AB} + \overline{BC} + \overline{CD} + \overline{DE} + \overline{EA} = \vec{0}$$

Answer

Given: ABCDE is a pentagon as shown below



Consider ΔABC and apply triangle law of vector, we get

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ ..... (i)

Similarly, consider ΔACD and apply triangle law of vector, we get

$\overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}$ ..... (ii)

And, consider ΔADE and apply triangle law of vector, we get

$\overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$ ..... (iii)

Adding (i), (ii) and (iii), we get

$\overrightarrow{AB} + \overrightarrow{BC} + \cancel{\overrightarrow{AC}} + \overrightarrow{CD} + \overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE} + \overrightarrow{AD} + \cancel{\overrightarrow{AC}}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \cancel{\overrightarrow{AD}} + \overrightarrow{DE} = \overrightarrow{AE} + \cancel{\overrightarrow{AD}}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = \overrightarrow{AE}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} = -\overrightarrow{EA}$ [as $\overrightarrow{AE} = -\overrightarrow{EA}$]

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = 0$

Hence proved

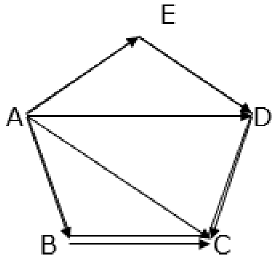
7 B. Question

ABCDE is a pentagon, prove that

$$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC} = 3\overrightarrow{AC}$$

Answer

Given: ABCDE is a pentagon as shown below



Consider ΔABC and apply triangle law of vector, we get

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \ldots..(i)$$

Similarly, consider ΔADE and apply triangle law of vector, we get

$$\overrightarrow{AE} + \overrightarrow{ED} = \overrightarrow{AD} \ldots..(ii)$$

And, consider ΔADC and apply triangle law of vector, we get

$$\overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC} \ldots..(iii)$$

Adding (i), (ii) and (iii), we get

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AE} + \overrightarrow{ED} + \cancel{\overrightarrow{AD}} + \overrightarrow{DC} = \overrightarrow{AC} + \cancel{\overrightarrow{AD}} + \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{AC}$$

Add $\overrightarrow{AC}$ on both sides we get,

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{AC} = \overrightarrow{AC} + \overrightarrow{AC} + \overrightarrow{AC}$$

$$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AE} + \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{AC} = 3\overrightarrow{AC}$$

Or $\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC} = 3\overrightarrow{AC}$

Hence proved.

8. Question

Prove that the sum of all vectors drawn from the centre of a regular octagon to its vertices is the zero vector.

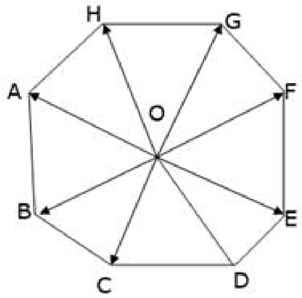
Answer

Given: a regular octagon

To prove the sum of all vectors drawn from the centre of a regular octagon to its vertices is the zero vector

Proof:

Let O be the centre of a regular octagon, we know that the centre of a regular octagon bisects all the diagonals passing through it as shown in figure below



Thus,

$\overrightarrow{OA} = -\overrightarrow{OE}, \overrightarrow{OB} = -\overrightarrow{OF}, \overrightarrow{OC} = -\overrightarrow{OG}, \overrightarrow{OD} = -\overrightarrow{OH} \ldots.(i)$

The sum of all vectors drawn from the centre of a regular octagon to its vertices is

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}$

Substitute the values from eqn(i) in above eqn, we get

$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}$

$= -\overrightarrow{OE} - \overrightarrow{OF} - \overrightarrow{OG} - \overrightarrow{OH} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}$

$= \vec{0}$

Hence, the sum of all vectors drawn from the centre of a regular octagon to its vertices is a zero vector.

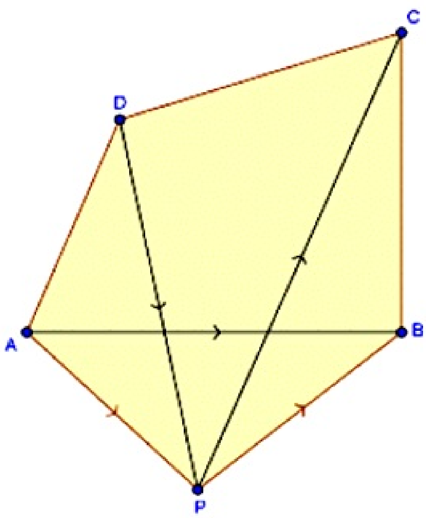
Hence, proved.

9. Question

If P is a point and ABCD is a quadrilateral and $\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PD} = \overrightarrow{PC},$ show that ABCD is a parallelogram.

Answer

Given a quadrilateral ABCD, P is a point outside the quadrilateral and $\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PD} = \overrightarrow{PC}$



$\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PD} = \overrightarrow{PC}$ [given]

Or, $\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PC} - \overrightarrow{PD}$

$\Rightarrow \overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PC} + \overrightarrow{DP}$ ....(i)[as $\overrightarrow{DP} = -\overrightarrow{PD}$]

Consider ΔAPB and apply triangle law of vector, we get

$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{AB} \ldots..(ii)$

And consider ΔDPC and apply triangle law of vector, we get

$\overrightarrow{DP} + \overrightarrow{PC} = \overrightarrow{DC} \ldots..(iii)$

Substitute the values from eqn(ii) an eqn(iii) in eqn(i), we get

$\overrightarrow{AB} = \overrightarrow{DC}$

Therefore, AB is parallel to DC and equal is magnitude.

Hence, ABCD is a parallelogram.

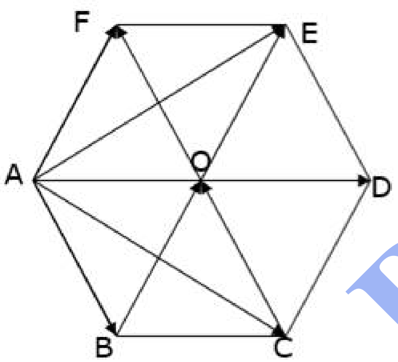
Hence proved

10. Question

Five forces and $\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}$ and $\overline{AF}$ act at the vertex of a regular hexagon ABCDEF. Prove that the resultant is 6 $\overline{AO},$ where O is the centre of hexagon.

Answer

Given a regular hexagon ABCDEF with O as the centre of the hexagon as shown in figure below



To prove $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$

We know that centre O of the hexagon bisects the diagonals

$\therefore 2\overrightarrow{AO} = \overrightarrow{AD}, \overrightarrow{BO} = -\overrightarrow{OE}, \overrightarrow{CO} = -\overrightarrow{OF} \ldots\ldots\ldots(i)$

Consider ΔABO and apply triangle law of vector, we get

$\overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{AO} \Rightarrow \overrightarrow{AB} = \overrightarrow{AO} - \overrightarrow{BO} \ldots..(ii)$

And consider ΔACO and apply triangle law of vector, we get

$\overrightarrow{AC} + \overrightarrow{CO} = \overrightarrow{AO} \Rightarrow \overrightarrow{AC} = \overrightarrow{AO} - \overrightarrow{CO} \ldots..(iii)$

And consider ΔAEO and apply triangle law of vector, we get

$\overrightarrow{AE} + \overrightarrow{EO} = \overrightarrow{AO} \Rightarrow \overrightarrow{AE} = \overrightarrow{AO} - \overrightarrow{EO} \ldots..(iv)$

And consider ΔAFO and apply triangle law of vector, we get

$\overrightarrow{AF} + \overrightarrow{FO} = \overrightarrow{AO} \Rightarrow \overrightarrow{AF} = \overrightarrow{AO} - \overrightarrow{FO} \ldots\ldots (v)$

Now,

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

Substitute the corresponding values from eqn(i) to eqn(v) in above eqn, we get

$= \overrightarrow{AO} - \overrightarrow{BO} + \overrightarrow{AO} - \overrightarrow{CO} + 2\overrightarrow{AO} + \overrightarrow{AO} - \overrightarrow{EO} + \overrightarrow{AO} - \overrightarrow{FO}$

$= \overrightarrow{AO} - \overrightarrow{OE} + \overrightarrow{AO} - \overrightarrow{OF} + 2\overrightarrow{AO} + \overrightarrow{AO} - \overrightarrow{EO} + \overrightarrow{AO} - \overrightarrow{FO}$ [from eqn(i)]

$= \overrightarrow{AO} + \overrightarrow{EO} + \overrightarrow{AO} + \overrightarrow{FO} + 2\overrightarrow{AO} + \overrightarrow{AO} - \overrightarrow{EO} + \overrightarrow{AO} - \overrightarrow{FO}$ [as $\overrightarrow{EO} = -\overrightarrow{OE}$ and $\overrightarrow{FO} = -\overrightarrow{OF}$]

$= \overrightarrow{AO} + \overrightarrow{AO} + 2\overrightarrow{AO} + \overrightarrow{AO} + \overrightarrow{AO}$

$= 6\overrightarrow{AO}$

Hence $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$

Therefore the resultant of the five forces $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AE}$ and $\overrightarrow{AF}$ is $6\overrightarrow{AO}$

Hence proved

Exercise 23.3

1. Question

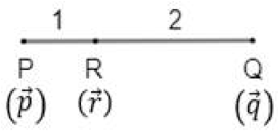
Find the position vector of a point R which divides the line joining the two points P and Q with position vectors $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 2\vec{b}$, respectively in the ratio 1 : 2 internally and externally.

Answer

Let the position vectors of points P, Q and R be $\vec{p}, \vec{q}$ and $\vec{r}$ respectively.

Given $\vec{p} = \overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\vec{q} = \overrightarrow{OQ} = \vec{a} - 2\vec{b}$

(i) R divides PQ internally in the ratio 1:2



Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Here, m = 1 and n = 2.

$$\Rightarrow \vec{r} = \frac{(1)\vec{q} + (2)\vec{p}}{1 + 2}$$

$$\Rightarrow \vec{r} = \frac{\vec{q} + 2\vec{p}}{3}$$

We have $\vec{p} = 2\vec{a} + \vec{b}$ and $\vec{q} = \vec{a} - 2\vec{b}$

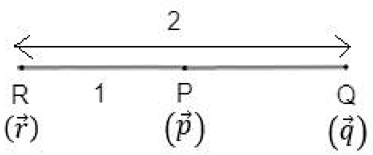
$$\Rightarrow \vec{r} = \frac{(\vec{a} - 2\vec{b}) + 2(2\vec{a} + \vec{b})}{3}$$

$$\Rightarrow \vec{r} = \frac{(1 + 4)\vec{a} + (-2 + 2)\vec{b}}{3}$$

$$\therefore \vec{r} = \frac{5}{3}\vec{a}$$

Thus, the position vector of point R is $\frac{5}{3}\vec{a}$.

(ii) R divides PQ externally in the ratio 1:2



Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, externally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

Here, m = 1 and n = 2.

$$\Rightarrow \vec{r} = \frac{(1)\vec{q} - (2)\vec{p}}{1 - 2}$$

$$\Rightarrow \vec{r} = \frac{\vec{q} - 2\vec{p}}{-1}$$

$$\Rightarrow \vec{r} = 2\vec{p} - \vec{q}$$

We have $\vec{p} = 2\vec{a} + \vec{b}$ and $\vec{q} = \vec{a} - 2\vec{b}$

$$\Rightarrow \vec{r} = 2(2\vec{a} + \vec{b}) - (\vec{a} - 2\vec{b})$$

$$\Rightarrow \vec{r} = (4 - 1)\vec{a} + (2 + 2)\vec{b}$$

$$\therefore \vec{r} = 3\vec{a} + 4\vec{b}$$

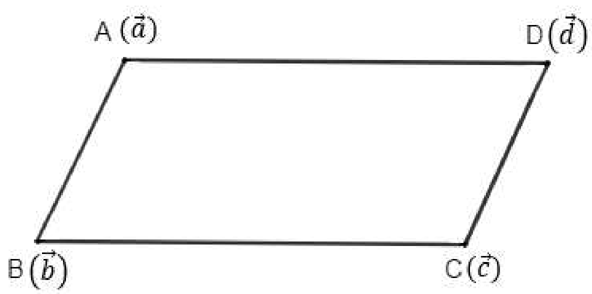
Thus, the position vector of point R is $3\vec{a} + 4\vec{b}$.

32. Question

Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of the four distinct points A, B, C, D. If $\vec{b} - \vec{a} = \vec{c} - \vec{d}$, then show that ABCD is a parallelogram.

Answer

Given the position vectors of points A, B, C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.



Recall the vector $\overrightarrow{AB}$ is given by

$$\overrightarrow{AB} = \text{position vector of B} - \text{position vector of A}$$

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

Similarly, the vector $\overrightarrow{DC}$ is given by

$\overrightarrow{DC}$ = position vector of C − position vector of D

$\Rightarrow \overrightarrow{DC} = \vec{c} - \vec{d}$

But, it is given that $\vec{b} - \vec{a} = \vec{c} - \vec{d}$.

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$

Two vectors are equal only when both their magnitudes and directions are equal.

$\Rightarrow \overrightarrow{AB} \parallel \overrightarrow{DC}$ and $|\overrightarrow{AB}| = |\overrightarrow{DC}|$.

This means that the opposite sides in quadrilateral ABCD are parallel and equal.

Thus, ABCD is a parallelogram.

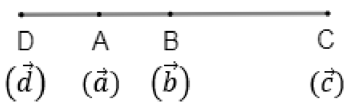
3. Question

If $\vec{a}, \vec{b}$ are the position vectors of A, B respectively, find the position vector of a point C in AB produced such that AC = 3 AB and that a point D in BA produced such that BD = 2 BA.

Answer

Given the position vectors of points A and B are $\vec{a}$ and $\vec{b}$.

Let the position vectors of points C and D be $\vec{c}$ and $\vec{d}$.



We have AC = 3AB.

From the above figure, observe AB = AC – BC

⇒ AC = 3 (AC – BC)

⇒ AC = 3AC – 3BC

⇒ 2AC = 3BC

∴ AC : BC = 3 : 2

So, C divides AB externally in the ratio 3:2.

Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, externally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

Here, m = 3 and n = 2

So, the position vector of C is

$$\vec{c} = \frac{3\vec{b} - 2\vec{a}}{3 - 2}$$

$$\Rightarrow \vec{c} = \frac{-2\vec{a} + 3\vec{b}}{1}$$

$$\therefore \vec{c} = -2\vec{a} + 3\vec{b}$$

We also have BD = 2BA.

From the figure, observe BA = BD – AD

⇒ BD = 2 (BD – AD)

⇒ BD = 2BD – 2AD

⇒ BD = 2AD

∴ BD : AD = 2 : 1

So, D divides BA externally in the ratio 2:1.

We now use the same formula as earlier to find the position vector of D.

Here, m = 2 and n = 1

$$\Rightarrow \vec{d} = \frac{2\vec{a} - \vec{b}}{2 - 1}$$

$$\Rightarrow \vec{d} = \frac{2\vec{a} - \vec{b}}{1}$$

$$\therefore \vec{d} = 2\vec{a} - \vec{b}$$

Thus, the position vector of point C is $-2\vec{a} + 3\vec{b}$ and the position vector of point D is $2\vec{a} - \vec{b}$.

4. Question

Show that the four points A, B, C, D with position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ respectively such that $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$, are coplanar. Also, find the position vector of the point of intersection of the line segments AC and BD.

Answer

Given the position vectors of points A, B, C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

We have $3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = 0$

Rearranging the terms in the above equation,

$$3\vec{a} + 5\vec{c} = 2\vec{b} + 6\vec{d}$$

Observe that the sum of coefficients on the LHS of this equation (3 + 5 = 8) is equal to that on the RHS (2 + 6 = 8).

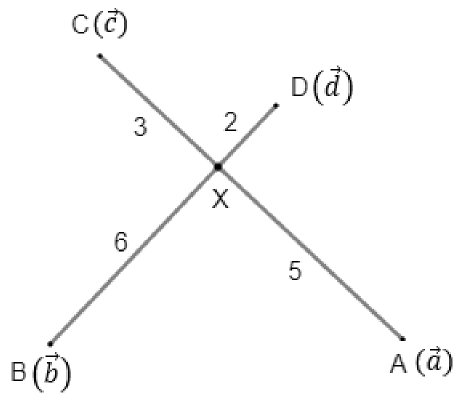
We now divide the equation with 8 on both sides.

$$\Rightarrow \frac{3\vec{a} + 5\vec{c}}{8} = \frac{2\vec{b} + 6\vec{d}}{8}$$

$$\Rightarrow \frac{3\vec{a} + 5\vec{c}}{3 + 5} = \frac{2\vec{b} + 6\vec{d}}{2 + 6}$$

Now, consider the LHS of this equation.

Let $\frac{3\vec{a} + 5\vec{c}}{3 + 5} = \vec{x}$, the position vector of some point X.



Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Here, m = 3 and n = 5

So, X divides CA internally in the ratio 3:5.

Similarly, considering the RHS of this equation, we have the same point X dividing DB in the ratio 2:6.

So, the point X lies on both the line segments AC and BD making it the point of intersection of AC and BD.

As AC and BD are two straight lines having a common point, we have all the points A, B, C and D lying in the same plane.

Thus, the points A, B, C and D are coplanar and in addition, the position vector of the point of intersection of line segments AC and BD is $\frac{3\vec{a}+5\vec{c}}{8}$ or $\frac{2\vec{b}+6\vec{d}}{8}$.

5. Question

Show that the four points P, Q, R, S with position vectors $\vec{p}, \vec{q}, \vec{r}, \vec{s}$ respectively such that $5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = \vec{0}$, are coplanar. Also, find the position vector of the point of intersection of the line segments PR and QS.

Answer

Given the position vectors of points P, Q, R and S are $\vec{p}$, $\vec{q}$, $\vec{r}$ and $\vec{s}$ respectively.

We have $5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = 0$

Rearranging the terms in the above equation,

$$5\vec{p} + 6\vec{r} = 2\vec{q} + 9\vec{s}$$

Observe that the sum of coefficients on the LHS of this equation (5 + 6 = 11) is equal to that on the RHS (2 + 9 = 11).

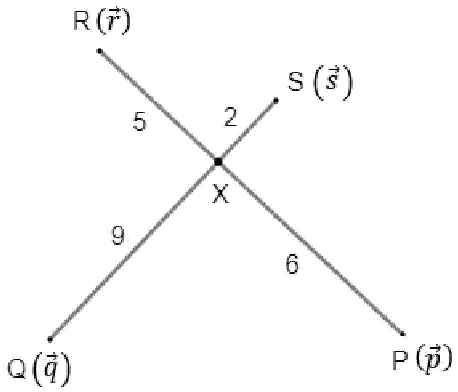
We now divide the equation with 11 on both sides.

$$\Rightarrow \frac{5\vec{p} + 6\vec{r}}{11} = \frac{2\vec{q} + 9\vec{s}}{11}$$

$$\Rightarrow \frac{5\vec{p} + 6\vec{r}}{5 + 6} = \frac{2\vec{q} + 9\vec{s}}{2 + 9}$$

Now, consider the LHS of this equation.

Let $\frac{5\vec{p}+6\vec{r}}{5+6} = \vec{x}$, the position vector of some point X.



Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Here, m = 5 and n = 6

So, X divides RP internally in the ratio 5:6.

Similarly, considering the RHS of this equation, we have the same point X dividing SQ in the ratio 2:9.

So, the point X lies on both the line segments PR and QS making it the point of intersection of PR and QS.

As PR and QS are two straight lines having a common point, we have all the points P, Q, R and S lying in the same plane.

Thus, the points P, Q, R and S are coplanar and in addition, the position vector of the point of intersection of line segments PR and QS is $\frac{5\vec{p}+6\vec{r}}{11}$ or $\frac{2\vec{q}+9\vec{s}}{11}$.

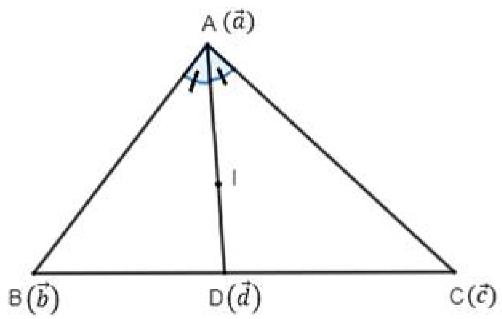
6. Question

The vertices A, B, C of triangle ABC have respectively position vectors $\vec{a}, \vec{b}, \vec{c}$ with respect to a given origin O. Show that the point D where the bisector of ∠A meets BC has position vector $\vec{d} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$, where $\beta = |\vec{c} - \vec{a}|$ and $\gamma = |\vec{a} - \vec{b}|$.

Answer

Given the position vectors of vertices A, B and C of ΔABC are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

D is point on BC with position vector $\vec{d}$ such that AD is the bisector of ∠A. I is the incenter of ΔABC.



Observe from the figure that D divides BC in the ratio BD:DC.

Using the angular bisector theorem, we know that the angle bisector of an angle in a triangle bisects the opposite side in the ratio equal to the ratio of the other two sides.

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC}$

But, $AB = |\overrightarrow{AB}|$ and $AC = |\overrightarrow{AC}|$.

$$\Rightarrow \frac{BD}{DC} = \frac{|\overrightarrow{AB}|}{|\overrightarrow{AC}|}$$

Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{position vector of B} - \text{position vector of A}$

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

Similarly, $\overrightarrow{AC} = \vec{c} - \vec{a}$

So, we have $\frac{BD}{DC} = \frac{|\vec{a} - \vec{b}|}{|\vec{c} - \vec{a}|}$.

Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Here, we have D dividing BC internally in the ratio m:n where m = BD = $|\vec{a} - \vec{b}|$ and n = DC = $|\vec{c} - \vec{a}|$

$$\Rightarrow \vec{d} = \frac{|\vec{a} - \vec{b}|\vec{c} + |\vec{c} - \vec{a}|\vec{b}}{|\vec{a} - \vec{b}| + |\vec{c} - \vec{a}|}$$

$$\Rightarrow \vec{d} = \frac{|\vec{c} - \vec{a}|\vec{b} + |\vec{a} - \vec{b}|\vec{c}}{|\vec{c} - \vec{a}| + |\vec{a} - \vec{b}|}$$

Suppose $|\vec{c} - \vec{a}| = \beta$ and $|\vec{a} - \vec{b}| = \gamma$.

$$\therefore \vec{d} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$$

From angular bisector theorem above, we have $\frac{BD}{DC} = \frac{AB}{AC}$.

Adding 1 to both sides,

$$\frac{BD}{DC} + 1 = \frac{AB}{AC} + 1$$

$$\Rightarrow \frac{BD + DC}{DC} = \frac{AB + AC}{AC}$$

$$\Rightarrow \frac{BC}{DC} = \frac{AB + AC}{AC}$$

$$\Rightarrow \frac{AC}{DC} = \frac{AB + AC}{BC}$$

In addition, as CI is the angular bisector of ∠C in ΔACD, using the angular bisector theorem, we have

$$\frac{ID}{AI} = \frac{CD}{AC} \Rightarrow \frac{AI}{ID} = \frac{AC}{DC}$$

So, we get $\frac{AI}{ID} = \frac{AB + AC}{BC}$

We have $AB = |\vec{a} - \vec{b}|, BC = |\vec{b} - \vec{c}|$ and $AC = |\vec{c} - \vec{a}|$

$$\Rightarrow \frac{AI}{ID} = \frac{AB + AC}{BC} = \frac{|\vec{a} - \vec{b}| + |\vec{c} - \vec{a}|}{|\vec{b} - \vec{c}|}$$

Assume $|\vec{b} - \vec{c}| = \alpha$

$$\Rightarrow \frac{AI}{ID} = \frac{\beta + \gamma}{\alpha}$$

So, I divides AD in the ratio (β + γ):α.

Let the position vector of I be $\vec{x}$.

Using the aforementioned section formula, we can write

$$\vec{x} = \frac{(\beta + \gamma)\vec{d} + \alpha\vec{a}}{\beta + \gamma + \alpha}$$

But, we already found $\vec{d} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$.

$$\Rightarrow \vec{x} = \frac{(\beta + \gamma)\left[\frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}\right] + \alpha\vec{a}}{\beta + \gamma + \alpha}$$

$$\Rightarrow \vec{x} = \frac{\beta\vec{b} + \gamma\vec{c} + \alpha\vec{a}}{\beta + \gamma + \alpha}$$

$$\therefore \vec{x} = \frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

Thus, $\vec{d} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$ and the position vector of the incenter is $\frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$, where $\alpha = |\vec{b} - \vec{c}|$, $\beta = |\vec{c} - \vec{a}|$ and $\gamma = |\vec{a} - \vec{b}|$.

Exercise 23.4

1. Question

If O is a point in space, ABC is a triangle and D, E, F are the mid-points of the sides BC, CA and AB respectively of the triangle, prove that $\overline{OA} + \overline{OB} + \overline{OC} = \overline{OD} + \overline{OE} + \overline{OF}$.

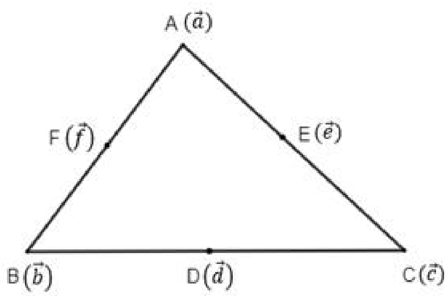
Answer

Let position vectors of the vertices A, B and C of ΔABC with respect to O be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

$\Rightarrow \overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$

Let us also assume the position vectors of the midpoints D, E and F with respect to O are $\vec{d}$, $\vec{e}$ and $\vec{f}$ respectively.

$\Rightarrow \overrightarrow{OD} = \vec{d}, \overrightarrow{OE} = \vec{e}$ and $\overrightarrow{OF} = \vec{f}$



Now, D is the midpoint of side BC.

This means D divides BC in the ratio 1:1.

Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Here, m = n = 1

$$\Rightarrow \vec{d} = \frac{(1)\vec{c} + (1)\vec{b}}{1 + 1}$$

$$\Rightarrow \vec{d} = \frac{\vec{b} + \vec{c}}{2}$$

$$\therefore \vec{b} + \vec{c} = 2\vec{d}$$

Similarly, for midpoint E and side CA, we get $\vec{c} + \vec{a} = 2\vec{e}$ and for midpoint F and side AB, we get $\vec{a} + \vec{b} = 2\vec{f}$.

Adding these three equations, we get

$$\vec{b} + \vec{c} + \vec{c} + \vec{a} + \vec{a} + \vec{b} = 2\vec{d} + 2\vec{e} + 2\vec{f}$$

$$\Rightarrow 2\vec{a} + 2\vec{b} + 2\vec{c} = 2\vec{d} + 2\vec{e} + 2\vec{f}$$

$$\Rightarrow 2(\vec{a} + \vec{b} + \vec{c}) = 2(\vec{d} + \vec{e} + \vec{f})$$

$$\therefore \vec{a} + \vec{b} + \vec{c} = \vec{d} + \vec{e} + \vec{f}$$

Thus, $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF}$.

2. Question

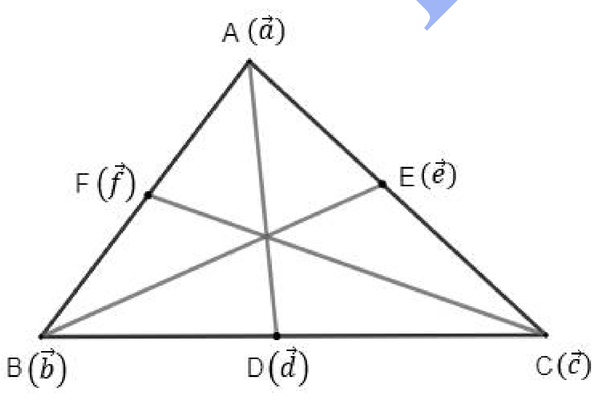
Show that the sum of the three vectors determined by the medians of a triangle directed from the vertices is zero.

Answer

Consider a ΔABC with D, E and F being the midpoints of sides BC, CA and AB respectively.

Let the position vectors of these vertices and midpoints be as shown in the figure.

We need to prove $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \vec{0}$.



As D is the midpoint of BC, using midpoint formula, we have

$$\vec{d} = \frac{\vec{b} + \vec{c}}{2}$$

Similarly, $\vec{e} = \frac{\vec{c} + \vec{a}}{2}$ and $\vec{f} = \frac{\vec{a} + \vec{b}}{2}$.

Recall the vector $\overrightarrow{AD}$ is given by

$\overrightarrow{AD}$ = position vector of D − position vector of A

$\Rightarrow \overrightarrow{AD} = \vec{d} - \vec{a}$

Similarly, $\overrightarrow{BE} = \vec{e} - \vec{b}$ and $\overrightarrow{CF} = \vec{f} - \vec{c}$

Now, consider the vector $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF}$.

$$\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = (\vec{d} - \vec{a}) + (\vec{e} - \vec{b}) + (\vec{f} - \vec{c})$$

But $\vec{d} = \frac{\vec{b} + \vec{c}}{2}$, $\vec{e} = \frac{\vec{c} + \vec{a}}{2}$ and $\vec{f} = \frac{\vec{a} + b}{2}$

$$\Rightarrow \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \left(\frac{\vec{b} + \vec{c}}{2} - \vec{a}\right) + \left(\frac{\vec{c} + \vec{a}}{2} - \vec{b}\right) + \left(\frac{\vec{a} + b}{2} - \vec{c}\right)$$

$$\Rightarrow \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \left(\frac{\vec{b} + \vec{c} - 2\vec{a}}{2}\right) + \left(\frac{\vec{c} + \vec{a} - 2\vec{b}}{2}\right) + \left(\frac{\vec{a} + b - 2\vec{c}}{2}\right)$$

$$\Rightarrow \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{\vec{b} + \vec{c} - 2\vec{a} + \vec{c} + \vec{a} - 2\vec{b} + \vec{a} + b - 2\vec{c}}{2}$$

$$\Rightarrow \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \frac{2\vec{a} + 2\vec{b} + 2\vec{c} - 2\vec{a} - 2\vec{b} - 2\vec{c}}{2} = \frac{\vec{0}}{2}$$

$$\therefore \overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \vec{0}$$

Thus, the sum of the three vectors determined by the medians of a triangle is zero.

3. Question

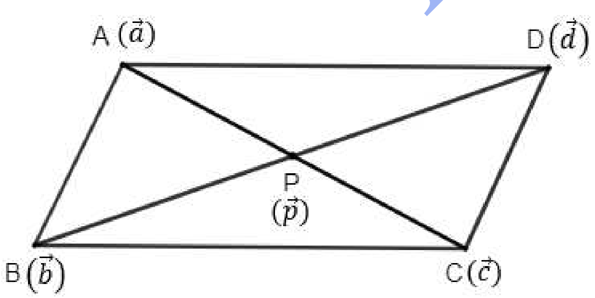
ABCD is a parallelogram and P is the point of intersection of its diagonals. If O is the origin of reference, show that $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} = 4\,\overline{OP}$.

Answer

Let position vectors of the vertices A, B, C and D of the parallelogram ABCD with respect to O be $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

$\Rightarrow \overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$ and $\overrightarrow{OD} = \vec{d}$

Also, let us assume position vector of P is $\vec{p}$.



Given ABCD is a parallelogram.

We know that the two diagonals of a parallelogram bisect each other. So, P is the midpoint of AC and BD.

As P is the midpoint of AC, using midpoint formula, we have

$$\vec{p} = \frac{\vec{a} + \vec{c}}{2}$$

$$\Rightarrow 2\vec{p} = \vec{a} + \vec{c}$$

$\Rightarrow \vec{a} + \vec{c} = 2\vec{p}$

P is also the midpoint of BC.

So, $\vec{p} = \frac{\vec{b} + \vec{d}}{2} \Rightarrow \vec{b} + \vec{d} = 2\vec{p}$

Now we have $\vec{a} + \vec{c} = 2\vec{p}$ and $\vec{b} + \vec{d} = 2\vec{p}$.

Adding these two equations, we get

$(\vec{a} + \vec{c}) + (\vec{b} + \vec{d}) = 2\vec{p} + 2\vec{p}$

$\therefore \vec{a} + \vec{b} + \vec{c} + \vec{d} = 4\vec{p}$

Thus $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$.

4. Question

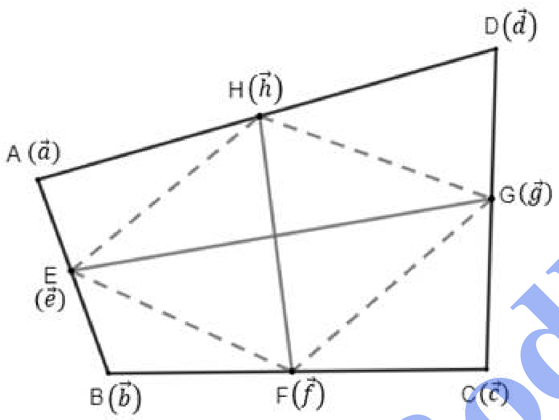
Show that the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other.

Answer

Let ABCD be a quadrilateral. E, F, G and H are the midpoints of sides AB, BC, CD and DA respectively.

We need to prove EG and HF bisect each other. It is sufficient to show EFGH is a parallelogram, as the diagonals in a parallelogram bisect each other.

Let the position vectors of these vertices and midpoints be as shown in the figure.



As E is the midpoint of AB, using midpoint formula, we have

$$\vec{e} = \frac{\vec{a} + \vec{b}}{2}$$

Similarly, $\vec{f} = \frac{\vec{b} + \vec{c}}{2}$, $\vec{g} = \frac{\vec{c} + \vec{d}}{2}$ and $\vec{h} = \frac{\vec{d} + \vec{a}}{2}$.

Recall the vector $\overrightarrow{EF}$ is given by

$\overrightarrow{EF} = \text{position vector of F} - \text{position vector of E}$

$\Rightarrow \overrightarrow{EF} = \vec{f} - \vec{e}$

$$\Rightarrow \overrightarrow{EF} = \frac{\vec{b} + \vec{c}}{2} - \frac{\vec{a} + \vec{b}}{2}$$

$$\Rightarrow \overrightarrow{EF} = \frac{\vec{b} + \vec{c} - \vec{a} - \vec{b}}{2}$$

$$\therefore \overrightarrow{EF} = \frac{\vec{c} - \vec{a}}{2}$$

Similarly $\overrightarrow{HG} = \text{position vector of G} - \text{position vector of H}$

$\Rightarrow \overrightarrow{HG} = \vec{g} - \vec{h}$

$\Rightarrow \overrightarrow{HG} = \dfrac{\vec{c} + \vec{d}}{2} - \dfrac{\vec{d} + \vec{a}}{2}$

$\Rightarrow \overrightarrow{HG} = \dfrac{\vec{c} + \vec{d} - \vec{d} - \vec{a}}{2}$

$\therefore \overrightarrow{HG} = \dfrac{\vec{c} - \vec{a}}{2}$

So, we have $\overrightarrow{EF} = \overrightarrow{HG}$.

Two vectors are equal only when both their magnitudes and directions are equal.

$\Rightarrow \overrightarrow{EF} \parallel \overrightarrow{HG}$ and $|\overrightarrow{EF}| = |\overrightarrow{HG}|$.

This means that the opposite sides in quadrilateral EFGH are parallel and equal, making EFGH a parallelogram.

EG and HF are diagonals of parallelogram EFGH. So, EG and HF bisect each other.

Thus, the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other.

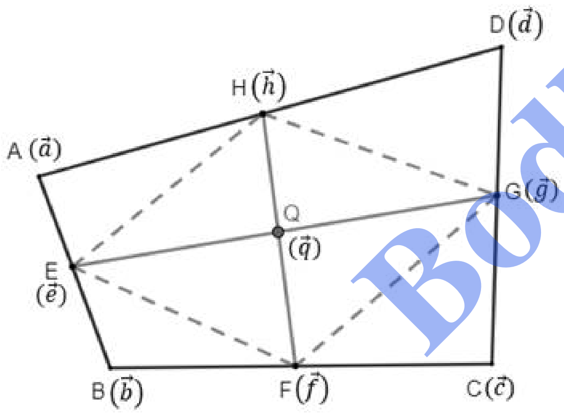
5. Question

ABCD are four points in a plane and Q is the point of intersection of the lines joining the mid-points of AB and CD ; BC and AD. Show that $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} + \overrightarrow{PD} = 4\overrightarrow{PQ}$, where P is any point.

Answer

Let E, F, G and H be the midpoints of sides AB, BC, CD and DA respectively of quadrilateral ABCD.

Let the position vectors of these vertices and midpoints be as shown in the figure.



As E is the midpoint of AB, using midpoint formula, we have

$\vec{e} = \dfrac{\vec{a} + \vec{b}}{2}$

Similarly, $\vec{f} = \frac{\vec{b} + \vec{c}}{2}$, $\vec{g} = \frac{\vec{c} + \vec{d}}{2}$ and $\vec{h} = \frac{\vec{d} + \vec{a}}{2}$.

We know that the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other.

⇒ Q is the midpoint of EG and HF.

Once again using midpoint formula, we get $\vec{q} = \frac{\vec{e} + \vec{g}}{2}$

But, we found $\vec{e} = \frac{\vec{a} + \vec{b}}{2}$ and $\vec{g} = \frac{\vec{c} + \vec{d}}{2}$.

$$\Rightarrow \vec{q} = \frac{\left(\frac{\vec{a}+\vec{b}}{2}\right) + \left(\frac{\vec{c}+\vec{d}}{2}\right)}{2}$$

$$\Rightarrow \vec{q} = \frac{\left(\frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{2}\right)}{2} = \frac{\vec{a}+\vec{b}+\vec{c}+\vec{d}}{4}$$

$\therefore \vec{a}+\vec{b}+\vec{c}+\vec{d} = 4\vec{q}$

Now, consider the vector $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD}$.

Let the position vector of point P be $\vec{p}$.

Recall the vector $\overrightarrow{PA}$ is given by

$\overrightarrow{PA} = \text{position vector of A} - \text{position vector of P}$

$\Rightarrow \overrightarrow{PA} = \vec{a} - \vec{p}$

Similarly, $\overrightarrow{PB} = \vec{b} - \vec{p}$, $\overrightarrow{PC} = \vec{c} - \vec{p}$ and $\overrightarrow{PD} = \vec{d} - \vec{p}$.

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = (\vec{a}-\vec{p}) + (\vec{b}-\vec{p}) + (\vec{c}-\vec{p}) + (\vec{d}-\vec{p})$

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = (\vec{a}+\vec{b}+\vec{c}+\vec{d}) - \vec{p} - \vec{p} - \vec{p} - \vec{p}$

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = (\vec{a}+\vec{b}+\vec{c}+\vec{d}) - 4\vec{p}$

But, we found $\vec{a}+\vec{b}+\vec{c}+\vec{d} = 4\vec{q}$

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = 4\vec{q} - 4\vec{p}$

$\Rightarrow \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = 4(\vec{q} - \vec{p})$

Observe, $\vec{q} - \vec{p} = \text{position vector of Q} - \text{position vector of P}$

$\Rightarrow \vec{q} - \vec{p} = \overrightarrow{PQ}$

$\therefore \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = 4\overrightarrow{PQ}$

Thus, $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}+\overrightarrow{PD} = 4\overrightarrow{PQ}$

6. Question

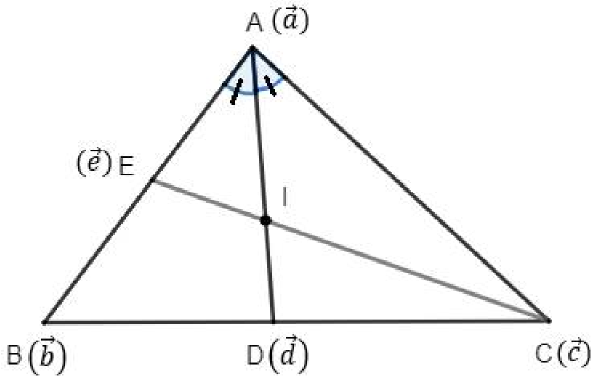
Prove by vector method that the internal bisectors of the angles of a triangle are concurrent.

Answer

Consider ΔABC with vertices A, B, C and sides BC = α, AC = β and AB = γ.

Let the position vectors of A, B and C be $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

Let D and E (with position vectors $\vec{d}$ and $\vec{e}$) be points on BC and AB such that AD and CE are the bisectors of ∠A and ∠C. Let, AB and CE meet at point I.



Observe from the figure that D divides BC in the ratio BD:DC.

Using the angular bisector theorem, we know that the angle bisector of an angle in a triangle bisects the opposite side in the ratio equal to the ratio of the other two sides.

$\Rightarrow \frac{BD}{DC} = \frac{AB}{AC} = \frac{\gamma}{\beta}$ (from our initial assumption)

Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

Here, we have D dividing BC internally in the ratio m:n where m = γ and n = β.

$$\Rightarrow \vec{d} = \frac{\gamma\vec{c} + \beta\vec{b}}{\gamma + \beta}$$

From angular bisector theorem above, we had $\frac{BD}{DC} = \frac{AB}{AC}$.

Adding 1 to both sides,

$$\frac{BD}{DC} + 1 = \frac{AB}{AC} + 1$$

$$\Rightarrow \frac{BD + DC}{DC} = \frac{AB + AC}{AC}$$

$$\Rightarrow \frac{BC}{DC} = \frac{AB + AC}{AC}$$

$$\Rightarrow \frac{AC}{DC} = \frac{AB + AC}{BC}$$

In addition, as CI is the angular bisector of ∠C in ΔACD, using the angular bisector theorem, we have

$$\frac{ID}{AI} = \frac{CD}{AC} \Rightarrow \frac{AI}{ID} = \frac{AC}{DC}$$

So, we get $\frac{AI}{ID} = \frac{AB + AC}{BC} = \frac{\beta + \gamma}{\alpha}$

So, I divides AD in the ratio (β + γ):α.

Let the position vector of I be $\vec{x}$.

Using the aforementioned section formula, we can write

$$\vec{x} = \frac{(\beta + \gamma)\vec{d} + \alpha\vec{a}}{\beta + \gamma + \alpha}$$

But, we already found $\vec{d} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$.

$$\Rightarrow \vec{x} = \frac{(\beta + \gamma)\left[\frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}\right] + \alpha\vec{a}}{\beta + \gamma + \alpha}$$

$$\Rightarrow \vec{x} = \frac{\beta\vec{b} + \gamma\vec{c} + \alpha\vec{a}}{\beta + \gamma + \alpha}$$

$$\therefore \vec{x} = \frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

Now, observe E divides AB in the ratio AE:EB.

$\Rightarrow \frac{AE}{EB} = \frac{AC}{BC} = \frac{\beta}{\alpha}$ (from angular bisector theorem)

So, $\vec{e} = \frac{\beta\vec{b} + \alpha\vec{a}}{\beta + \alpha}$ (using section formula)

By doing similar calculations as above for ∠C, we get

$$\frac{CI}{IE} = \frac{BC + AC}{AB} = \frac{\alpha + \beta}{\gamma}$$

So, I divides CE in the ratio (α + β):γ.

Let the position vector of I now be $\vec{y}$.

Using the aforementioned section formula, we can write

$$\vec{y} = \frac{(\alpha + \beta)\vec{e} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

But, we already found $\vec{e} = \frac{\beta\vec{b} + \alpha\vec{a}}{\beta + \alpha}$.

$$\Rightarrow \vec{y} = \frac{(\beta + \alpha)\left[\frac{\beta\vec{b} + \alpha\vec{a}}{\beta + \alpha}\right] + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

$$\Rightarrow \vec{y} = \frac{\beta\vec{b} + \alpha\vec{a} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

$$\therefore \vec{y} = \frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$

Observe that $\vec{x} = \vec{y}$ meaning the point I with position vector $\frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$ lies on both AB and CE.

Similarly, it can be shown that this point I also lies on the third angular bisector.

Thus, the internal bisectors of the angles of a triangle are concurrent with the point of concurrency given by the position vector $\frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$ where α,β and γ are sides of the ΔABC opposite to the vertices A, B and C respectively.

Exercise 23.5

1. Question

If the position vector of a point (–4, –3) be $\vec{a}$, find $|\vec{a}|$.

Answer

Given $\vec{a}$ is the position vector of point (–4, –3).

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

$\Rightarrow \vec{a} = (-4)\hat{i} + (-3)\hat{j}$

Now, we need to find magnitude of $\vec{a}$ i.e. $|\vec{a}|$.

Recall the magnitude of the vector $x\hat{i} + y\hat{j}$ is given as

$|x\hat{i} + y\hat{j}| = \sqrt{x^2 + y^2}$

Here, x = –4 and y = –3

$\Rightarrow |\vec{a}| = \sqrt{(-4)^2 + (-3)^2}$

$\Rightarrow |\vec{a}| = \sqrt{16 + 9}$

$\Rightarrow |\vec{a}| = \sqrt{25}$

$\therefore |\vec{a}| = 5$

Thus, $|\vec{a}| = 5$.

2. Question

If the position vector $\vec{a}$ of a point (12, n) is such that $\left|\vec{a}\right| = 13$, find the value(s) of n.

Answer

Given $\vec{a}$ is the position vector of point (12, n).

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

$\Rightarrow \vec{a} = 12\hat{i} + n\hat{j}$

Now, we need to find n such that $|\vec{a}| = 13$.

Recall the magnitude of the vector $x\hat{i} + y\hat{j}$ is given as

$|x\hat{i} + y\hat{j}| = \sqrt{x^2 + y^2}$

Here, x = 12 and y = n

$\Rightarrow |\vec{a}| = \sqrt{12^2 + n^2}$

$\Rightarrow 13 = \sqrt{144 + n^2}$

Squaring both the sides, we have

$13^2 = 144 + n^2$

$\Rightarrow n^2 + 144 = 169$

$\Rightarrow n^2 = 25$

$\Rightarrow n = \pm\sqrt{25}$

$\therefore n = \pm 5$

Thus, n = 5 or –5.

3. Question

Find a vector of magnitude 4 units which is parallel to the vector $\sqrt{3}\,\hat{i} + \hat{j}$.

Answer

Let $\vec{a}$ be the required vector that is parallel to $\sqrt{3}\hat{i}+\hat{j}$.

We know any vector parallel to a given vector $x\hat{i}+y\hat{j}$ is of the form $\lambda(x\hat{i}+y\hat{j})$, where λ is a real number.

$\Rightarrow \vec{a}=\lambda(\sqrt{3}\hat{i}+\hat{j})$

Now, we need to find λ such that $|\vec{a}|=4$.

Recall the magnitude of the vector $x\hat{i}+y\hat{j}$ is given as

$|x\hat{i}+y\hat{j}|=\sqrt{x^2+y^2}$

Here, x = $\lambda\sqrt{3}$ and y = λ

$\Rightarrow |\vec{a}|=\sqrt{(\lambda\sqrt{3})^2+\lambda^2}$

$\Rightarrow 4=\sqrt{3\lambda^2+\lambda^2}$

$\Rightarrow 4=\sqrt{4\lambda^2}$

Squaring both the sides, we have

$4^2=4\lambda^2$

$\Rightarrow 4\lambda^2=16$

$\Rightarrow \lambda^2=4$

$\therefore \lambda=2$

Thus, the required vector is $2\sqrt{3}\hat{i}+2\hat{j}$.

4. Question

Express $\overrightarrow{AB}$ in terms of unit vectors $\hat{i}$ and $\hat{j}$, when the points are :

(i) A (4, -1), B(1, 3)

(ii) A(-6, 3), B(-2, -5)

Find $\left|\overrightarrow{AB}\right|$ in each case.

Answer

(i) Given A = (4, –1) and B = (1, 3)

We know position vector of a point (x, y) is given by $x\hat{i}+y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vectors of points A and B be $\vec{a}$ and $\vec{b}$ respectively.

$\Rightarrow \vec{a}=4\hat{i}+(-1)\hat{j}=4\hat{i}-\hat{j}$

We also have $\vec{b}=\hat{i}+3\hat{j}$.

Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}=\text{position vector of B}-\text{position vector of A}$

$\Rightarrow \overrightarrow{AB}=\vec{b}-\vec{a}$

$\Rightarrow \overrightarrow{AB}=(\hat{i}+3\hat{j})-(4\hat{i}-\hat{j})$

$\Rightarrow \overrightarrow{AB}=(1-4)\hat{i}+(3+1)\hat{j}$

$\therefore \overrightarrow{AB} = -3\hat{i} + 4\hat{j}$

Recall the magnitude of the vector $x\hat{i} + y\hat{j}$ is given as

$|x\hat{i} + y\hat{j}| = \sqrt{x^2 + y^2}$

Here, x = –3 and y = 4

$\Rightarrow |\overrightarrow{AB}| = \sqrt{(-3)^2 + 4^2}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{9 + 16}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{25}$

$\therefore |\overrightarrow{AB}| = 5$

Thus, $\overrightarrow{AB} = -3\hat{i} + 4\hat{j}$ and $|\overrightarrow{AB}| = 5$.

(ii) Given A = (–6, 3) and B = (–2, –5)

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vectors of points A and B be $\vec{a}$ and $\vec{b}$ respectively.

$\Rightarrow \vec{a} = (-6)\hat{i} + 3\hat{j} = -6\hat{i} + 3\hat{j}$

We also have $\vec{b} = (-2)\hat{i} + (-5)\hat{j} = -2\hat{i} - 5\hat{j}$.

Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{position vector of B} - \text{position vector of A}$

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

$\Rightarrow \overrightarrow{AB} = (-2\hat{i} - 5\hat{j}) - (-6\hat{i} + 3\hat{j})$

$\Rightarrow \overrightarrow{AB} = (-2 + 6)\hat{i} + (-5 - 3)\hat{j}$

$\therefore \overrightarrow{AB} = 4\hat{i} - 8\hat{j}$

Recall the magnitude of the vector $x\hat{i} + y\hat{j}$ is given as

$|x\hat{i} + y\hat{j}| = \sqrt{x^2 + y^2}$

Here, x = 4 and y = –8

$\Rightarrow |\overrightarrow{AB}| = \sqrt{4^2 + (-8)^2}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{16 + 64}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{80}$

$\therefore |\overrightarrow{AB}| = 4\sqrt{5}$

Thus, $\overrightarrow{AB} = 4\hat{i} - 8\hat{j}$ and $|\overrightarrow{AB}| = 4\sqrt{5}$.

5. Question

Find the coordinates of the tip of the position vector which is equivalent to $\overline{AB}$, where the coordinates of A and B are (-1, 3) and (-2, 1) respectively.

Answer

Given A = (–1, 3) and B = (–2, 1)

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vectors of points A and B be $\vec{a}$ and $\vec{b}$ respectively.

$\Rightarrow \vec{a} = (-1)\hat{i} + 3\hat{j} = -\hat{i} + 3\hat{j}$

We also have $\vec{b} = (-2)\hat{i} + \hat{j} = -2\hat{i} + \hat{j}$.

Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{position vector of B} - \text{position vector of A}$

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

$\Rightarrow \overrightarrow{AB} = (-2\hat{i} + \hat{j}) - (-\hat{i} + 3\hat{j})$

$\Rightarrow \overrightarrow{AB} = (-2 + 1)\hat{i} + (1 - 3)\hat{j}$

$\therefore \overrightarrow{AB} = -\hat{i} - 2\hat{j}$

Now, it is given that there exists a point say (x, y) whose position vector is same as $\overrightarrow{AB}$.

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$.

$\Rightarrow x\hat{i} + y\hat{j} = \overrightarrow{AB}$

$\Rightarrow x\hat{i} + y\hat{j} = -\hat{i} - 2\hat{j}$

By comparing both the sides, we get x = –1 and y = –2

Thus, (–1, –2) is the tip of position vector that is same as $\overrightarrow{AB}$.

6. Question

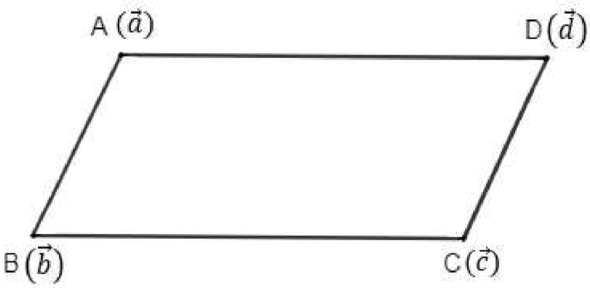
ABCD is a parallelogram. If the coordinates of A, B and C are (–2, 1), (3, 0) and (1, –2), find the coordinates of D.

Answer

Given A = (–2, –1), B = (3, 0) and C = (1, –2)

Let the other vertex D = (x, y)

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.



Let position vectors of points A, B, C and D be $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

$\Rightarrow \vec{a} = (-2)\hat{i} + (-1)\hat{j} = -2\hat{i} - \hat{j}$

We also have $\vec{b} = 3\hat{i} + 0\hat{j} = 3\hat{i}$.

Similarly $\vec{c} = \hat{i} - 2\hat{j}$ and $\vec{d} = x\hat{i} + y\hat{j}$.

Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = position vector of B − position vector of A

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

$\Rightarrow \overrightarrow{AB} = (3\hat{\imath}) - (-2\hat{\imath} - \hat{\jmath})$

$\Rightarrow \overrightarrow{AB} = (3 + 2)\hat{\imath} - (-1)\hat{\jmath}$

$\therefore \overrightarrow{AB} = 5\hat{\imath} + \hat{\jmath}$

Similarly, the vector $\overrightarrow{DC}$ is given by

$\overrightarrow{DC}$ = position vector of C − position vector of D

$\Rightarrow \overrightarrow{DC} = \vec{c} - \vec{d}$

$\Rightarrow \overrightarrow{DC} = (\hat{\imath} - 2\hat{\jmath}) - (x\hat{\imath} + y\hat{\jmath})$

$\Rightarrow \overrightarrow{DC} = (1 - x)\hat{\imath} + (-2 - y)\hat{\jmath}$

$\therefore \overrightarrow{DC} = (1 - x)\hat{\imath} - (2 + y)\hat{\jmath}$

But, it is given that ABCD is a parallelogram.

$\Rightarrow \overrightarrow{AB} = \overrightarrow{DC}$ (as the opposite sides are parallel and equal)

$\Rightarrow 5\hat{\imath} + \hat{\jmath} = (1 - x)\hat{\imath} - (2 + y)\hat{\jmath}$

By comparing both sides, we get 1 – x = 5 and 2 + y = –1

⇒ x = 1 – 5 = –4

and y = –1 – 2 = –3

So, x = –4 and y = –3

Thus, vertex D of parallelogram ABCD = (–4, –3).

7. Question

If the position vectors of the points A (3, 4), B (5, -6) and C (4, -1) are $\vec{a}, \vec{b}, \vec{c}$ respectively, compute $\vec{a} + 2\vec{b} - 3\vec{c}$.

Answer

Given A = (3, 4), B = (5, –6) and C = (4, –1)

We know position vector of a point (x, y) is given by $x\hat{\imath} + y\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in X and Y directions.

$\Rightarrow \vec{a} = 3\hat{\imath} + 4\hat{\jmath}$ (position vector of point A)

We also have $\vec{b} = 5\hat{\imath} + (-6)\hat{\jmath} = 5\hat{\imath} - 6\hat{\jmath}$

Similarly $\vec{c} = 4\hat{\imath} - \hat{\jmath}$.

We need to compute $\vec{a} + 2\vec{b} - 3\vec{c}$.

$\vec{a} + 2\vec{b} - 3\vec{c} = (3\hat{\imath} + 4\hat{\jmath}) + 2(5\hat{\imath} - 6\hat{\jmath}) - 3(4\hat{\imath} - \hat{\jmath})$

$\Rightarrow \vec{a} + 2\vec{b} - 3\vec{c} = (3 + 2 \times 5 - 3 \times 4)\hat{\imath} + (4 - 2 \times 6 + 3)\hat{\jmath}$

$\Rightarrow \vec{a} + 2\vec{b} - 3\vec{c} = (3 + 10 - 12)\hat{\imath} + (4 - 12 + 3)\hat{\jmath}$

$\therefore \vec{a} + 2\vec{b} - 3\vec{c} = \hat{\imath} - 5\hat{\jmath}$

Thus, $\vec{a} + 2\vec{b} - 3\vec{c} = \hat{i} - 5\hat{j}$.

8. Question

If $\vec{a}$ be the position vector whose tip is (5, -3), find the coordinates of a point B such that $\overrightarrow{AB} = \vec{a}$, the coordinates of A being (4, -1).

Answer

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

So, position vector of (5, –3) is $\vec{a} = 5\hat{i} + (-3)\hat{j}$

$\therefore \vec{a} = 5\hat{i} - 3\hat{j}$

Given A = (4, –1) and let the coordinates of B = (x, y)

Let position vectors of points A and B be $\vec{p}$ and $\vec{q}$ respectively.

$\Rightarrow \vec{p} = 4\hat{i} + (-1)\hat{j} = 4\hat{i} - \hat{j}$

We also have $\vec{q} = x\hat{i} + y\hat{j}$.

Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{position vector of B} - \text{position vector of A}$

$\Rightarrow \overrightarrow{AB} = \vec{q} - \vec{p}$

$\Rightarrow \overrightarrow{AB} = (x\hat{i} + y\hat{j}) - (4\hat{i} - \hat{j})$

$\therefore \overrightarrow{AB} = (x - 4)\hat{i} + (y + 1)\hat{j}$

But, it is given that $\overrightarrow{AB} = \vec{a}$

$\Rightarrow (x - 4)\hat{i} + (y + 1)\hat{j} = 5\hat{i} - 3\hat{j}$

By comparing both sides, we get x – 4 = 5 and y + 1 = –3

⇒ x = 5 + 4 = 9

and y = –3 – 1 = –4

So, x = 9 and y = –4

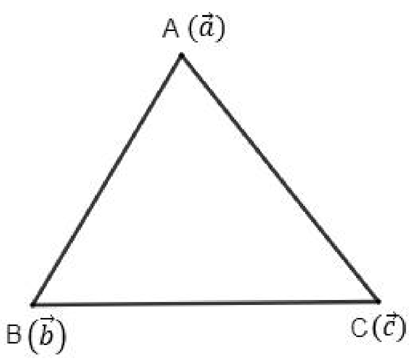
Thus, coordinates of point B are (9, –4).

9. Question

Show that the points $2\hat{i}, -\hat{i} - 4\hat{j}$ and $\hat{i} + 4\hat{j}$ form an isosceles triangle.

Answer

Let $\vec{a} = 2\hat{i}$, $\vec{b} = -\hat{i} - 4\hat{j}$ and $\vec{c} = -\hat{i} + 4\hat{j}$ be the position vectors corresponding to the vertices A, B and C of ΔABC.



Recall the vector $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{position vector of B} - \text{position vector of A}$

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

$\Rightarrow \overrightarrow{AB} = (-\hat{i} - 4\hat{j}) - (2\hat{i})$

$\Rightarrow \overrightarrow{AB} = (-1 - 2)\hat{i} + (-4)\hat{j}$

$\therefore \overrightarrow{AB} = -3\hat{i} - 4\hat{j}$

Recall the magnitude of the vector $x\hat{i} + y\hat{j}$ is given as

$|x\hat{i} + y\hat{j}| = \sqrt{x^2 + y^2}$

Now, we find the magnitude of $\overrightarrow{AB}$.

$|\overrightarrow{AB}| = \sqrt{(-3)^2 + (-4)^2}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{9 + 16}$

$\Rightarrow |\overrightarrow{AB}| = \sqrt{25}$

$\therefore |\overrightarrow{AB}| = 5$

Similarly, the vector $\overrightarrow{BC}$ is given by

$\overrightarrow{BC} = \text{position vector of C} - \text{position vector of B}$

$\Rightarrow \overrightarrow{BC} = \vec{c} - \vec{b}$

$\Rightarrow \overrightarrow{BC} = (-\hat{i} + 4\hat{j}) - (-\hat{i} - 4\hat{j})$

$\Rightarrow \overrightarrow{BC} = (-1 + 1)\hat{i} + (4 + 4)\hat{j}$

$\therefore \overrightarrow{BC} = 8\hat{j}$

Now, we find the magnitude of $\overrightarrow{BC}$.

$|\overrightarrow{BC}| = \sqrt{0^2 + 8^2}$

$\Rightarrow |\overrightarrow{BC}| = \sqrt{8^2}$

$\therefore |\overrightarrow{BC}| = 8$

Similarly, the vector $\overrightarrow{AC}$ is given by

$\overrightarrow{AC} = \text{position vector of C} - \text{position vector of A}$

$\Rightarrow \overrightarrow{AC} = \vec{c} - \vec{a}$

$\Rightarrow \overrightarrow{AC} = (-\hat{i} + 4\hat{j}) - (2\hat{i})$

$\Rightarrow \overrightarrow{AC} = (-1 - 2)\hat{i} + (4)\hat{j}$

$\therefore \overrightarrow{AC} = -3\hat{i} + 4\hat{j}$

Now, we find the magnitude of $\overrightarrow{AC}$.

$|\overrightarrow{AC}| = \sqrt{(-3)^2 + 4^2}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{9 + 16}$

$\Rightarrow |\overrightarrow{AC}| = \sqrt{25}$

$\therefore |\overrightarrow{AC}| = 5$

Observe that $|\overrightarrow{AB}| = |\overrightarrow{AC}|$ which means the sides AB and AC of ΔABC are equal in length, making it an isosceles triangle.

Thus, the triangle formed by the given points is isosceles.

10. Question

Find a unit vector parallel to the vector $\hat{i} + \sqrt{3}\,\hat{j}$.

Answer

Let $\vec{a}$ be the required vector that is parallel to $\hat{i} + \sqrt{3}\hat{j}$.

We know any vector parallel to a given vector $x\hat{i} + y\hat{j}$ is of the form $\lambda(x\hat{i} + y\hat{j})$, where λ is a real number.

$\Rightarrow \vec{a} = \lambda(\hat{i} + \sqrt{3}\hat{j})$

Now, we need to find λ such that $|\vec{a}| = 1$.

Recall the magnitude of the vector $x\hat{i} + y\hat{j}$ is given as

$|x\hat{i} + y\hat{j}| = \sqrt{x^2 + y^2}$

Here, x = λ and y = $\lambda\sqrt{3}$

$\Rightarrow |\vec{a}| = \sqrt{\lambda^2 + (\lambda\sqrt{3})^2}$

$\Rightarrow 1 = \sqrt{\lambda^2 + 3\lambda^2}$

$\Rightarrow 1 = \sqrt{4\lambda^2}$

Squaring both the sides, we have

$1^2 = 4\lambda^2$

$\Rightarrow 4\lambda^2 = 1$

$\Rightarrow \lambda^2 = \frac{1}{4}$

$\therefore \lambda = \frac{1}{2}$

Thus, the required vector is $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$.

11. Question

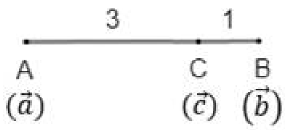
The position vectors of points A, B and C are $\lambda\hat{i} + 3\hat{j}, 12\hat{i} + \mu\hat{j}$ and respectively. If C divides the lien segment joining A and B in the ratio 3 : 1, find the values of λ and μ.

Answer

Let the position vectors of points A, B and C be $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

Given: $\vec{a} = \lambda\hat{i} + 3\hat{j}$, $\vec{b} = 12\hat{i} + \mu\hat{j}$ and $\vec{c} = -11\hat{i} - 3\hat{j}$

C divides AB internally in the ratio 3:1.



Recall the position vector of point P which divides AB, the line joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ respectively, internally in the ratio m : n is

$$\vec{p}=\frac{m\vec{b}\ +\ n\vec{a}}{m\ +\ n}$$

Here, m = 3 and n = 1.

$$\Rightarrow \vec{c}=\frac{(3)\vec{b}\ +\ (1)\vec{a}}{3\ +\ 1}$$

$$\Rightarrow \vec{c}=\frac{3\vec{b}\ +\ \vec{a}}{4}$$

We have $\vec{a}=\lambda\hat{i}+3\hat{j}$, $\vec{b}=12\hat{i}+\mu\hat{j}$ and $\vec{c}=-11\hat{i}-3\hat{j}$

$$\Rightarrow -11\hat{i}-3\hat{j}=\frac{3(12\hat{i}\ +\ \mu\hat{j})\ +\ (\lambda\hat{i}\ +\ 3\hat{j})}{4}$$

$$\Rightarrow -44\hat{i}-12\hat{j}=3(12\hat{i}+\mu\hat{j})+(\lambda\hat{i}+3\hat{j})$$

$$\Rightarrow -44\hat{i}-12\hat{j}=(36+\lambda)\hat{i}+(3\mu+3)\hat{j}$$

$$\Rightarrow (36+\lambda)\hat{i}+(3\mu+3)\hat{j}=-44\hat{i}-12\hat{j}$$

By comparing both sides, we get 36 + λ = –44

⇒ λ = –44 – 36

∴ λ = –80

We also have 3μ + 3 = –12

⇒ 3μ = –15

∴ μ = –5

Thus, λ = –80 and μ = –5

12. Question

Find the components along the coordinate axes of the position vector of each of the following points –

i. P (3, 2)

ii. Q (5, 1)

iii. R (–11, –9)

iv. S (4, –3)

Answer

(i) Given P = (3, 2)

We know position vector of a point (x, y) is given by $x\hat{i}+y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vector of point P be $\vec{p}$.

$$\Rightarrow \vec{p}=3\hat{i}+2\hat{j}$$

So, component of $\vec{p}$ along the X-axis is $3\hat{i}$, that is a vector of magnitude 3 along the positive direction of the X-axis.

Also, component of $\vec{p}$ along the Y-axis is $2\hat{j}$, that is a vector of magnitude 2 along the positive direction of the Y-axis.

(ii) Given Q = (5, 1)

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vector of point Q be $\vec{q}$.

$\Rightarrow \vec{q} = 5\hat{i} + \hat{j}$

So, component of $\vec{q}$ along the X-axis is $5\hat{i}$, that is a vector of magnitude 5 along the positive direction of the X-axis.

Also, component of $\vec{q}$ along the Y-axis is $\hat{j}$, that is a vector of magnitude 1 along the positive direction of the Y-axis.

(iii) Given R = (–11, –9)

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vector of point R be $\vec{r}$.

$\Rightarrow \vec{r} = (-11)\hat{i} + (-9)\hat{j} = -11\hat{i} - 9\hat{j}$

So, component of $\vec{r}$ along the X-axis is $-11\hat{i}$, that is a vector of magnitude 11 along the negative direction of the X-axis.

Also, component of $\vec{r}$ along the Y-axis is $-9\hat{j}$, that is a vector of magnitude 9 along the negative direction of the Y-axis.

(iv) Given S = (4, –3)

We know position vector of a point (x, y) is given by $x\hat{i} + y\hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors in X and Y directions.

Let position vector of point S be $\vec{s}$.

$\Rightarrow \vec{s} = 4\hat{i} + (-3)\hat{j} = 4\hat{i} - 3\hat{j}$

So, component of $\vec{s}$ along the X-axis is $4\hat{i}$, that is a vector of magnitude 4 along the positive direction of the X-axis.

Also, component of $\vec{s}$ along the Y-axis is $-3\hat{j}$, that is a vector of magnitude 3 along the negative direction of the Y-axis.

Exercise 23.6

1. Question

Find the magnitude of the vector $\vec{a} = 2\hat{i} + 3\hat{j} - 6\hat{k}$.

Answer

If a vector is given by $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$ then the magnitude of vector is generally denoted by $|\vec{a}|$ which is equal to $\sqrt{a^2 + b^2 + c^2}$

So the magnitude

$|\vec{a}| = \sqrt{2^2 + 3^2 + (-6)^2}$

$|\vec{a}| = \sqrt{4 + 9 + 36}$

$|\vec{a}| = \sqrt{49}$

$|\vec{a}| = 7$

So the magnitude of the vector is 7.

2. Question

Find the unit vector in the direction of $3\hat{i}+4\hat{j}-12\hat{k}$.

Answer

Let the unit vector in the direction of $\vec{A}=3\hat{i}+4\hat{j}-12\hat{k}$

So any unit vector in the direction of $\vec{A}=3\hat{i}+4\hat{j}-12\hat{k}$

$$\hat{B}=\frac{\vec{A}}{|\vec{A}|}$$

So the magnitude of the vector $\overrightarrow{|A|}=\sqrt{3^2+4^2+(-12)^2}$

$|\vec{A}|=\sqrt{9+16+144}$

$|\vec{A}|=\sqrt{169}$

$|\vec{A}|=13$

So, the unit vector $\hat{B}=\frac{3\hat{i}+4\hat{j}-12\hat{k}}{13}$

3. Question

Find a unit vector in the direction of the resultant of the vectors $\hat{i}-\hat{j}+3\hat{k}$, $2\hat{i}+\hat{j}-2\hat{k}$ and $\hat{i}+2\hat{j}-2\hat{k}$.

Answer

To find the resultant vector we add all the vector by vector addition.

$\vec{a}=\hat{i}-\hat{j}+3\hat{k}$

$\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$

$\vec{c}=\hat{i}+2\hat{j}-2\hat{k}$

So, resultant vector is

$\vec{P}=\vec{a}+\vec{b}+\vec{c}$

$\vec{P}=(\hat{i}-\hat{j}+3\hat{k})+(2\hat{i}+\hat{j}-2\hat{k})+(\hat{i}+2\hat{j}-2\hat{k})$

$\vec{P}=4\hat{i}+2\hat{j}-\hat{k}$

So, the unit vector $\hat{P}=\frac{\vec{P}}{|\vec{P}|}$

Magnitude of $|\vec{P}|=\sqrt{4^2+2^2+(-1)^2}$

$|\vec{P}|=\sqrt{16+4+1}$

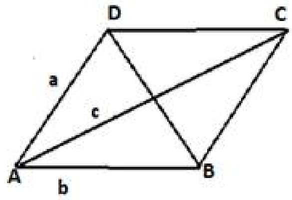
$|\vec{P}|=\sqrt{21}$

$$\hat{P}=\frac{4\hat{i}+2\hat{j}-\hat{k}}{\sqrt{21}}$$

4. Question

The adjacent sides of a parallelogram are represented by the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\hat{j}+2\hat{k}$. Find unit vectors parallel to the diagonals of the parallelogram.

Answer



Side BC parallel to

$\overrightarrow{AD} = \vec{a}, \overrightarrow{AB} = \vec{b}$

So resultant vector c = b + a

So, vector $\vec{C} = \vec{a} + \vec{b} = (\hat{\imath} + \hat{\jmath} - \hat{k}) + (-2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

$\vec{C} = \hat{\imath} + \hat{\jmath} - \hat{k} - 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$\vec{C} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

So unit vector along the diagonal of

Parallelogram is

$$\hat{C} = \frac{\vec{C}}{|\vec{C}|}$$

$$\hat{C} = \frac{-\hat{\imath} + 2\hat{\jmath} + \hat{k}}{\sqrt{(-1)^2 + 2^2 + 1^2}}$$

$$\hat{C} = \frac{-\hat{\imath} + 2\hat{\jmath} + \hat{k}}{\sqrt{6}}$$

5. Question

If $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, find $\left|3\vec{a} - 2\vec{b} + 4\vec{c}\right|$.

Answer

We want to find the magnitude of vector $3\vec{a} - 2\vec{b} + 4\vec{c}$

So,

$3\vec{a} - 2\vec{b} + 4\vec{c} = 3(3\hat{\imath} - \hat{\jmath} - 4\hat{k}) - 2(-2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}) + 4(\hat{\imath} + 2\hat{\jmath} - \hat{k})$

$3\vec{a} - 2\vec{b} + 4\vec{c} = 9\hat{\imath} - 3\hat{\jmath} - 12\hat{k} + 4\hat{\imath} - 8\hat{\jmath} + 6\hat{k} + 4\hat{\imath} + 8\hat{\jmath} - 4\hat{k}$

$3\vec{a} - 2\vec{b} + 4\vec{c} = 17\hat{\imath} - 3\hat{\jmath} - 10\hat{k}$

If a vector is given by $\vec{A} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$ then the magnitude of vector is generally denoted by $|\vec{a}|$ which is equal to $\sqrt{a^2 + b^2 + c^2}$

$|3\vec{a} - 2\vec{b} + 4\vec{c}| = \sqrt{17^2 + (-3)^2 + (-10)^2}$

$|3\vec{a} - 2\vec{b} + 4\vec{c}| = \sqrt{289 + 9 + 100}$

$|3\vec{a} - 2\vec{b} + 4\vec{c}| = \sqrt{398}$

6. Question

If $\overrightarrow{PQ} = 3\hat{i} + 2\hat{j} - \hat{k}$ and the coordinates of P are (1, -1, 2), find the coordinates of Q.

Answer

Position vector of 'P' is $\hat{\imath} - \hat{\jmath} + 2\hat{k}$

Let the position vector of point 'Q' is 'a.'

So we need to find the value of 'a.'

$\overrightarrow{PQ} =$ Position vector of 'Q' - Position vector of 'P'

$3\hat{\imath} + 2\hat{\jmath} - \hat{k} = a - (\hat{\imath} - \hat{\jmath} + 2\hat{k})$

$3\hat{\imath} + 2\hat{\jmath} - \hat{k} = a - \hat{\imath} + \hat{\jmath} - 2\hat{k}$

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k} + \hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\vec{a} = 4\hat{\imath} + \hat{\jmath} + \hat{k}$

So the position vector of 'Q" is (4, 1, 1)

7. Question

Prove that the points $\hat{i} - \hat{j}, 4\hat{j} - 3\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + 5\hat{k}$ are the vertices of a right-angled triangle.

Answer

In a right angle triangle $CA^2 = AB^2 + BC^2$

Where CA is the hypotenuse

BC is the perpendicular and AB is the base

Vertices of the triangle are given below

A = (1, -1, 0), B = (4, -3, 1), C = (2, -4, 5)

So,

$\overrightarrow{AB} = \vec{B} - \vec{A} = (4\hat{\imath} - 3\hat{\jmath} + \hat{k}) - (\hat{\imath} - \hat{\jmath})$

$\overrightarrow{AB} = 4\hat{\imath} - 3\hat{\jmath} + \hat{k} - \hat{\imath} + \hat{\jmath}$

$\overrightarrow{AB} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$|\overrightarrow{AB}| = \sqrt{3^2 + (-2)^2 + 1^2}$

$|\overrightarrow{AB}| = \sqrt{9 + 4 + 1}$

$|\overrightarrow{AB}| = \sqrt{14}$ ....(1)

Similarly,

$\overrightarrow{BC} = \vec{C} - \vec{B} = (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) - (4\hat{\imath} - 3\hat{\jmath} + \hat{k})$

$\overrightarrow{BC} = -2\hat{\imath} - \hat{\jmath} + 4\hat{k}$

$|\overrightarrow{BC}| = \sqrt{(-2)^2 + (-1)^2 + 4^2}$

$|\overrightarrow{BC}| = \sqrt{4 + 1 + 16}$

$|\overrightarrow{BC}| = \sqrt{21}$ ....(2)

$\overrightarrow{CA} = \vec{A} - \vec{C} = (\hat{\imath} - \hat{\jmath}) - (2\hat{\imath} - 4\hat{\jmath} + 5\hat{k})$

$\overrightarrow{CA} = -\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$

$|\overrightarrow{CA}| = \sqrt{(-1)^2 + 3^2 + (-5)^2}$

$|\overrightarrow{CA}| = \sqrt{1 + 9 + 25}$

$|\overrightarrow{CA}| = \sqrt{35}$ ....(3)

$CA^2 = AB^2 + BC^2$

$(\sqrt{35})^2 = (\sqrt{14})^2 + (\sqrt{21})^2$

35 = 14 + 21

35 = 35

LHS = RHS

So, these point form a right angle triangle

8. Question

If the vertices A, B, C of a triangle ABC are the points with position vectors $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $b_1\hat{i} + b_2\hat{j} + b_3\hat{k}, c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ respectively, what are the vectors determined by its sides? Find the length of these vectors.

Answer

Let the position vector of the vertex 'A' is $a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$,

And similarly B$= b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and C$= c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Side AB is

$\overrightarrow{AB} = \vec{B} - \vec{A} = (b_1\hat{i} + b_2\hat{j} + b_3\hat{k}) - (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$

$\overrightarrow{AB} = (b_1 - a_1)\hat{i} + (b_2 - a_2)\hat{j} + (b_3 - a_3)\hat{k}$ ....(1)

Equation (1) vector representation of the side AB

Magnitude of side AB,

$|\overrightarrow{AB}| = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + (b_3 - a_3)^2}$

And similarly for side BC and CA

$\overrightarrow{BC} = \vec{C} - \vec{B} = (c_1\hat{i} + c_2\hat{j} + c_3\hat{k}) - (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$

$\overrightarrow{BC} = (c_1 - b_1)\hat{i} + (c_2 - b_2)\hat{j} + (c_3 - b_3)\hat{k}$ ....(2)

$\overrightarrow{CA} = \vec{A} - \vec{C} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k}) - (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$

$\overrightarrow{CA} = (a_1 - c_1)\hat{i} + (a_2 - c_2)\hat{j} + (a_3 - c_3)\hat{k}$

Length of side BC and CA

$|\overrightarrow{BC}| = \sqrt{(c_1 - b_1)^2 + (c_2 - b_2)^2 + (c_3 - b_3)^2}$

$|\overrightarrow{CA}| = \sqrt{(a_1 - c_1)^2 + (a_2 - c_2)^2 + (a_3 - c_3)^2}$

9. Question

Find the vector from the origin O to the centroid of the triangle whose vertices are (1, -1, 2), (2, 1, 3) and (-1, 2, -1).

Answer

Centeroid of the triangle with Vertices (x_1, y_1, z_1), (x_2, y_2, z_2), and (x_3, y_3, z_3) is given by,

$$(x, y, z) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$$

In vector algebra, 'x' consider as a coefficient of $\hat{i}$ and 'y' as a coefficient of $\hat{j}$ and 'z' as a coefficient of $\hat{k}$

So the position vector of the centroid,

$$(\hat{i}, \hat{j}, \hat{k}) = \frac{1 + 2 - 1}{3}, \frac{-1 + 1 + 2}{3}, \frac{2 + 3 - 1}{3}$$

$$(\hat{i}, \hat{j}, \hat{k}) = \frac{2}{3}, \frac{2}{3}, \frac{4}{3}$$

So the location of the centroid is $\left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}\right)$

And the vector is,

$$\overrightarrow{OC} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{4}{3}\hat{k}$$

10. Question

Find the position vector of a point R which divides the line segment joining points $p\left(\hat{i} + 2\hat{j} + \hat{k}\right)$ and $Q\left(-\hat{i} + \hat{j} + \hat{k}\right)$ in the ratio 2 : 1.

(i) Internally

(ii) Externally

Answer

By using section formula,

(1) Internally $= \frac{m\vec{Q} + n\vec{P}}{m+n}$

Position vectors of P and Q are given as

$\overrightarrow{OP} = (\hat{i} + 2\hat{j} + \hat{k})$ and $\overrightarrow{OQ} = (-i + j + k)$

$$\frac{m}{n} = \frac{2}{1}$$

The position vector of point R which divides the line joining two points P and Q internally in the ratio 2 : 1 is given by,

$$\overrightarrow{OR} = \frac{(\hat{i} + 2\hat{j} + \hat{k}) + 2(-i + j + k)}{2 + 1}$$

$$\overrightarrow{OR} = \frac{-\hat{i} + 4\hat{j} + 3\hat{k}}{3}$$

The position vector of point R which divides the line joining two points P and Q externally in the ratio 2 : 1 is given by,

(2)Externally$= \frac{m\vec{Q} - n\vec{P}}{m-n}$

$$\overrightarrow{OR} = \frac{(\hat{i} + 2\hat{j} + \hat{k}) - 2(-i + j + k)}{1 - 2}$$

$$\overrightarrow{OR} = \frac{3\hat{i} - \hat{k}}{-1}$$

$$\overrightarrow{OR} = -3\hat{i} + \hat{k}$$

11. Question

Find the position vector of the mid-point of the vector joining the points $P\left(2\hat{i}-3\hat{j}+4\hat{k}\right)$ and $Q\left(4\hat{i}+\hat{j}-2\hat{k}\right)$.

Answer

If P and Q are two points with position vector $P(2\hat{i}-3\hat{j}+4\hat{k})$ and $Q(4\hat{i}+\hat{j}-2\hat{k})$ then the position vector of mid point A is given by

$$=\frac{\vec{P}+\vec{Q}}{2}$$

Let A is the mid point of PQ.

So, position vector of A $=\frac{\vec{P}+\vec{Q}}{2}$

$$\vec{A}=\frac{2\hat{i}-3\hat{j}+4\hat{k}+4\hat{i}+\hat{j}-2\hat{k}}{2}$$

$$\vec{A}=\frac{6i-2j+2k}{2}$$

$$\vec{A}=\frac{2(3i-j+k)}{2}$$

$$\vec{A}=3\hat{i}-\hat{j}+\hat{k}$$

12. Question

Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points (1, 2, 3) and (4, 5, 6).

Answer

First we need to create vector PQ

Position vector of P = OP = (1, 2, 3) and position vector of Q = OQ = (4, 5, 6)

$$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=\left(4\hat{i}+5\hat{j}+6\hat{k}\right)-\left(\hat{i}+2\hat{j}+3\hat{k}\right)$$

$$\overrightarrow{PQ}=3\hat{i}+3\hat{j}+3\hat{k}$$

$$\overrightarrow{PQ}=3\left(\hat{i}+\hat{j}+\hat{k}\right)$$

So unit vector in the direction PQ,

$$\widehat{PQ}=\frac{\overrightarrow{PQ}}{\left|\overrightarrow{PQ}\right|}$$

$$\widehat{PQ}=\frac{3\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3^2+3^2+3^2}}$$

$$\widehat{PQ}=\frac{3\left(\hat{i}+\hat{j}+\hat{k}\right)}{3\sqrt{1+1+1}}$$

$$\widehat{PQ}=\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}$$

13. Question

Show that the points $A\left(2\hat{i}-\hat{j}+\hat{k}\right)$, $B\left(\hat{i}-3\hat{j}-5\hat{k}\right)$, $C\left(3\hat{i}-4\hat{j}-4\hat{k}\right)$ are the vertices of a right angled triangle.

Answer

If A, B and C are the vertices of the right angle triangle

So,

In a right angle triangle $AB^2 = CA^2 + BC^2$

Where AB is the hypotenuse

BC is the perpendicular and CA is the base

Vertices of the triangle are given bellow

$A(2\hat{\imath} - \hat{\jmath} + \hat{k})$, $B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and $C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$

So,

$\overrightarrow{AB} = \vec{B} - \vec{A} = (\hat{\imath} - 3\hat{\jmath} - 5\hat{k}) - (2\hat{\imath} - \hat{\jmath} + \hat{k})$

$\overrightarrow{AB} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2}$

$|\overrightarrow{AB}| = \sqrt{1 + 4 + 36}$

$|\overrightarrow{AB}| = \sqrt{41}$ ……(1)

Similarly,

$\overrightarrow{BC} = \vec{C} - \vec{B} = (3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}) - (\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$

$\overrightarrow{BC} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$|\overrightarrow{BC}| = \sqrt{(2)^2 + (-1)^2 + 1^2}$

$|\overrightarrow{BC}| = \sqrt{4 + 1 + 1}$

$|\overrightarrow{BC}| = \sqrt{6}$ ……(2)

$\overrightarrow{CA} = \vec{A} - \vec{C} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) - (3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$

$\overrightarrow{CA} = -\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

$|\overrightarrow{CA}| = \sqrt{(-1)^2 + 3^2 + (5)^2}$

$|\overrightarrow{CA}| = \sqrt{1 + 9 + 25}$

$|\overrightarrow{CA}| = \sqrt{35}$ ……(3)

$AB^2 = CA^2 + BC^2$

$(\sqrt{41})^2 = (\sqrt{35})^2 + (\sqrt{6})^2$

41 = 35 + 6

41 = 41

LHS = RHS

14. Question

Find the position vector of the mid-point of the vector joining the points P (2, 3, 4) and Q (4, 1, -2).

Answer

If P and Q are two points with position vector $P(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ and $Q(4\hat{\imath} + \hat{\jmath} - 2\hat{k})$ then the position vector of mid point A is given by

$$= \frac{\vec{P}+\vec{Q}}{2}$$

Let A is the mid point of PQ.

So, position vector of A $= \frac{\vec{P}+\vec{Q}}{2}$

$$\vec{A} = \frac{(2\hat{i}+3\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}-2\hat{k})}{2}$$

$$\vec{A} = \frac{6i+4j+2k}{2}$$

$$\vec{A} = \frac{2(3i+2j+k)}{2}$$

$$\vec{A} = 3\hat{i}+2\hat{j}+\hat{k}$$

15. Question

Find the value of x for which $x\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector.

Answer

We need to find the value of 'x' for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector

If any vector is a unit vector, then its magnitude should be one.

So, the magnitude of the vector is,

$$|x(\hat{i}+\hat{j}+\hat{k})| = 1$$

$$\sqrt{x^2(1^2+1^2+1^2)} = 1$$

$$x\sqrt{3} = 1$$

$$x = \pm\frac{1}{\sqrt{3}}$$

For this value of 'x' the above vector is a unit vector

16. Question

If $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = 2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c} = \hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to $2\vec{a}-\vec{b}+3\vec{c}$.

Answer

First, we need to create a vector in the direction of $2\vec{a}-\vec{b}+3\vec{c}$

So,

$$2\vec{a}-\vec{b}+3\vec{c} = 2(\hat{i}+\hat{j}+\hat{k})-(2\hat{i}-\hat{j}+3\hat{k})+3(\hat{i}-2\hat{j}+\hat{k})$$

$$2\vec{a}-\vec{b}+3\vec{c} = 3\hat{i}-3\hat{j}+2\hat{k}$$

So the unit vector in the direction of $2\vec{a}-\vec{b}+3\vec{c}$ is,

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|} = \frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{3^2+(-3)^2+2^2}}$$

$$\frac{2\vec{a}-\vec{b}+3\vec{c}}{|2\vec{a}-\vec{b}+3\vec{c}|} = \frac{3\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{22}}$$

This is the unit vector in the direction of $2\vec{a}-\vec{b}+3\vec{c}$

17. Question

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a vector of magnitude 6 units which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

Answer

Vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$ is,

If a vector parallel $\vec{d}$ to other vector, so we can write a scalar multiple of the other so,

$\vec{d} = \lambda(2\vec{a} - \vec{b} + 3\vec{c})$

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$2\vec{a} - \vec{b} + 3\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$

So,

$\vec{d} = \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$

$|\vec{d}| = 6$ this is given in the question

$|\lambda(\hat{i} - 2\hat{j} + 2\hat{k})| = 6$

$\lambda\sqrt{1^2 + (-2)^2 + 2^2} = 6$

$\lambda\sqrt{1 + 4 + 4} = 6$

$\lambda\sqrt{9} = 6$

$\pm 3\lambda = 6$

$\lambda = \pm 2$

So the vector parallel to the $2\vec{a} - \vec{b} + 3\vec{c}$ is $\vec{d} = \pm 2(\hat{i} - 2\hat{j} + 2\hat{k})$

18. Question

Find a vector of magnitude of 5 units parallel to the resultant of the vector $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$.

Answer

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Let resultant vector is 'R' so the resultant vector by using the vector triangle law

$\vec{a} + \vec{b} = \vec{R} = (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k})$

$\vec{R} = 3\hat{i} + \hat{j}$

If a vector parallel $\vec{d}$ to other vector so we can write scalar multiple of the other so,

$\vec{d} = \lambda(\vec{R})$

$\vec{d} = \lambda(3\hat{i} + \hat{j})$ has a magnitude of 5 unit so

$|\vec{d}| = 5$

$|\lambda(3\hat{i} + \hat{j})| = 5$

$\lambda\sqrt{3^2 + 1^2} = 5$

$\lambda\sqrt{10} = 5$

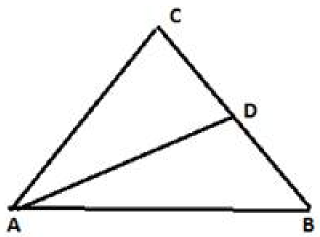
$\lambda = \dfrac{5}{\sqrt{10}}$

So the vector is $\vec{d} = \frac{5}{\sqrt{10}}(3\hat{i} + \hat{j})$

19. Question

The two vectors $\hat{j} + \hat{i}$ and $3\hat{i} + \hat{j} + 4\hat{k}$ represent the sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of triangle ABC. Find the length of the median through A.

Answer



Let D be the point at which median drawn from A touches side BC.

Let $\vec{a}, \vec{b}, \text{and } \vec{c}$ be the position vectors of the vertices A, B and C.

So position vector of D = $\frac{\vec{b}+\vec{c}}{2}$

So we creating a vector in the direction of AD

$\overrightarrow{AD} =$ Position vector of D – position vector of A

$$\overrightarrow{AD} = \frac{\vec{b} + \vec{c}}{2} - \vec{a}$$

$$\overrightarrow{AD} = \frac{\vec{b} + \vec{c} - 2\vec{a}}{2}$$

$$\overrightarrow{AD} = \frac{\vec{b} - \vec{a} + \vec{c} - \vec{a}}{2} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AD} = \frac{(\hat{j} + \hat{i}) + (3\hat{i} + \hat{j} + 4\hat{k})}{2} = \frac{4\hat{i} + 2\hat{j} + 4\hat{k}}{2}$$

$$\overrightarrow{AD} = (2\hat{i} + \hat{j} + 2\hat{k})$$

So length of AD

$|\overrightarrow{AD}| = \sqrt{2^2 + 1^2 + 2^2}$

$|\overrightarrow{AD}| = \sqrt{9}$

$|\overrightarrow{AD}| = 3$

Exercise 23.7

1. Question

Show that the points A, B, C with position vectors $\vec{a} - 2\vec{b} + 3\vec{c}$, $2\vec{a} + 3\vec{b} - 4\vec{c}$ and $-7\vec{b} + 10\vec{c}$ are collinear.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

We have been given that,

$$\text{Position vector of A} = \vec{a} - 2\vec{b} + 3\vec{c}$$

$$\text{Position vector of B} = 2\vec{a} + 3\vec{b} - 4\vec{c}$$

$$\text{Position vector of C} = -7\vec{b} + 10\vec{c}$$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$$

$$\Rightarrow \overrightarrow{AB} = (2\vec{a} + 3\vec{b} - 4\vec{c}) - (\vec{a} - 2\vec{b} + 3\vec{c})$$

$$\Rightarrow \overrightarrow{AB} = 2\vec{a} - \vec{a} + 3\vec{b} + 2\vec{b} - 4\vec{c} - 3\vec{c}$$

$$\Rightarrow \overrightarrow{AB} = \vec{a} + 5\vec{b} - 7\vec{c}$$

And $\overrightarrow{BC}$ is given by

$$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$$

$$\Rightarrow \overrightarrow{BC} = (-7\vec{b} + 10\vec{c}) - (2\vec{a} + 3\vec{b} - 4\vec{c})$$

$$\Rightarrow \overrightarrow{BC} = -2\vec{a} - 7\vec{b} - 3\vec{b} + 10\vec{c} + 4\vec{c}$$

$$\Rightarrow \overrightarrow{BC} = -2\vec{a} - 10\vec{b} + 14\vec{c}$$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = -2\vec{a} - 10\vec{b} + 14\vec{c}$

Or $\overrightarrow{BC} = -2(\vec{a} + 5\vec{b} - 7\vec{c})$

Or $\overrightarrow{BC} = -2 \times \overrightarrow{AB}$ $[\because \overrightarrow{AB} = \vec{a} + 5\vec{b} - 7\vec{c}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, A, B and C are collinear.

2 A. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, prove that the points having the following position vectors are collinear:

$\vec{a}, \vec{b}, \ 3\vec{a} - 2\vec{b}$

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Given that, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar vectors.

And we know that, vectors that do not lie on the same plane or line are called non-coplanar vectors.

To Prove: $\vec{a}$, $\vec{b}$ and $3\vec{a}-2\vec{b}$ are collinear.

Proof: Let the points be A, B and C.

Then,

Position vector of A $= \vec{a}$

Position vector of B $= \vec{b}$

Position vector of C $= 3\vec{a}-2\vec{b}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (\vec{b}) - (\vec{a})$

$\Rightarrow \overrightarrow{AB} = \vec{b} - \vec{a}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$\Rightarrow \overrightarrow{BC} = (3\vec{a}-2\vec{b}) - (\vec{b})$

$\Rightarrow \overrightarrow{BC} = 3\vec{a}-2\vec{b}-\vec{b}$

$\Rightarrow \overrightarrow{BC} = 3\vec{a}-3\vec{b}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = 3\vec{a}-3\vec{b}$

Or $\overrightarrow{BC} = 3(\vec{a}-\vec{b})$

Or $\overrightarrow{BC} = -3(\vec{b}-\vec{a})$

Or $\overrightarrow{BC} = -3 \times \overrightarrow{AB}$ $[\because \overrightarrow{AB} = \vec{b}-\vec{a}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, A, B and C are collinear.

2 B. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors, prove that the points having the following position vectors are collinear:

$$\vec{a}+\vec{b}+\vec{c},\ 4\vec{a}+3\vec{b},\ 10\vec{a}+7\vec{b}-2\vec{c}$$

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Given that, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar vectors.

And we know that, vectors that do not lie on the same plane or line are called non-coplanar vectors.

To Prove: $\vec{a}+\vec{b}+\vec{c}$, $4\vec{a}+3\vec{b}$ and $10\vec{a}+7\vec{b}-2\vec{c}$ are collinear.

Proof: Let the points be A, B and C.

Then,

$$\text{Position vector of A } = \vec{a}+\vec{b}+\vec{c}$$

$$\text{Position vector of B } = 4\vec{a}+3\vec{b}$$

$$\text{Position vector of C } = 10\vec{a}+7\vec{b}-2\vec{c}$$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$$

$$\Rightarrow \overrightarrow{AB} = \left(4\vec{a}+3\vec{b}\right) - \left(\vec{a}+\vec{b}+\vec{c}\right)$$

$$\Rightarrow \overrightarrow{AB} = 4\vec{a}+3\vec{b}-\vec{a}-\vec{b}-\vec{c}$$

$$\Rightarrow \overrightarrow{AB} = 3\vec{a}+2\vec{b}-\vec{c}$$

And $\overrightarrow{BC}$ is given by

$$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$$

$$\Rightarrow \overrightarrow{BC} = \left(10\vec{a}+7\vec{b}-2\vec{c}\right) - \left(4\vec{a}+3\vec{b}\right)$$

$$\Rightarrow \overrightarrow{BC} = 10\vec{a}-4\vec{a}+7\vec{b}-3\vec{b}-2\vec{c}$$

$$\Rightarrow \overrightarrow{BC} = 6\vec{a}+4\vec{b}-2\vec{c}$$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = 6\vec{a}+4\vec{b}-2\vec{c}$

Or $\overrightarrow{BC} = 2(3\vec{a}+2\vec{b}-\vec{c})$

Or $\overrightarrow{BC} = 2 \times \overrightarrow{AB}$ $[\because \overrightarrow{AB} = 3\vec{a}+2\vec{b}-\vec{c}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, A, B and C are collinear.

3. Question

Prove that the points having position vectors $\hat{i}+2\hat{j}+3\hat{k},\ 3\hat{i}+4\hat{j}+7\hat{k},\ -3\hat{i}-2\hat{j}-5\hat{k}$ are collinear.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Let the points be A, B and C having position vectors such that,

$$\text{Position vector of A} = \hat{i}+2\hat{j}+3\hat{k}$$

Position vector of B $= 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$

Position vector of C $= -3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}) - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$

$\Rightarrow \overrightarrow{AB} = 3\hat{\imath} - \hat{\imath} + 4\hat{\jmath} - 2\hat{\jmath} + 7\hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{AB} = 2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$\Rightarrow \overrightarrow{BC} = (-3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}) - (3\hat{\imath} + 4\hat{\jmath} + 7\hat{k})$

$\Rightarrow \overrightarrow{BC} = -3\hat{\imath} - 3\hat{\imath} - 2\hat{\jmath} - 4\hat{\jmath} - 5\hat{k} - 7\hat{k}$

$\Rightarrow \overrightarrow{BC} = -6\hat{\imath} - 6\hat{\jmath} - 12\hat{k}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = -6\hat{\imath} - 6\hat{\jmath} - 12\hat{k}$

Or $\overrightarrow{BC} = -3(2\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$

Or $\overrightarrow{BC} = -3 \times \overrightarrow{AB}$ $[\because \overrightarrow{AB} = 2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\Rightarrow$ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, A, B and C are collinear.

4. Question

If the points with position vectors $10\hat{i} + 3\hat{j}, 12\hat{i} - 5\hat{j}$ and $a\hat{i} + 11\hat{j}$ are collinear, find the value of a.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Let the points be A, B and C having position vectors such that,

Position vector of A $= 10\hat{\imath} + 3\hat{\jmath}$

Position vector of B $= 12\hat{\imath} - 5\hat{\jmath}$

Position vector of C $= a\hat{\imath} + 11\hat{\jmath}$

So, let us find $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (12\hat{\imath} - 5\hat{\jmath}) - (10\hat{\imath} + 3\hat{\jmath})$

$\Rightarrow \overrightarrow{AB} = 12\hat{\imath} - 10\hat{\imath} - 5\hat{\jmath} - 3\hat{\jmath}$

$\Rightarrow \overrightarrow{AB} = 2\hat{\imath} - 8\hat{\jmath}$ ...(i)

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$\Rightarrow \overrightarrow{BC} = (a\hat{\imath} + 11\hat{\jmath}) - (12\hat{\imath} - 5\hat{\jmath})$

$\Rightarrow \overrightarrow{BC} = a\hat{\imath} - 12\hat{\imath} + 11\hat{\jmath} + 5\hat{\jmath}$

$\Rightarrow \overrightarrow{BC} = a\hat{\imath} - 12\hat{\imath} + 16\hat{\jmath}$ ...(ii)

Since, it has been given that points A, B and C are collinear.

So, we can write as

$\overrightarrow{BC} = \lambda\,\overrightarrow{AB}$

Where λ = a scalar quantity

Put the values of $\overrightarrow{BC}$ and $\overrightarrow{AB}$ from (i) and (ii), we get

$a\hat{\imath} - 12\hat{\imath} + 16\hat{\jmath} = \lambda\,(2\hat{\imath} - 8\hat{\jmath})$

$\Rightarrow (a\hat{\imath} - 12\hat{\imath}) + 16\hat{\jmath} = 2\lambda\hat{\imath} - 8\lambda\hat{\jmath}$

Comparing the vectors $\hat{\imath}$ and $\hat{\jmath}$ respectively, we get

a – 12 = 2λ ...(iii)

and, 16 = –8λ

From –8λ = 16, we can find the value of λ.

–8λ = 16

$\Rightarrow \lambda = -\frac{16}{8}$

⇒ λ = –2

Put λ = –2 in equation (iii), we get

a – 12 = 2λ

⇒ a – 12 = 2(–2)

⇒ a – 12 = –4

⇒ a = –4 + 12

⇒ a = 8

Thus, we have got a = 8.

5. Question

If $\vec{a}, \vec{b}$ are two non-collinear vectors, prove that the points with position vectors $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$ and $\vec{a} + \lambda\vec{b}$ are collinear for all real values of λ.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Given that, $\vec{a}$ and $\vec{b}$ are two non-collinear vectors.

Let the points be A, B and C having position vectors such that, Position vector of A $= \vec{a} + \vec{b}$

Position vector of B $= \vec{a} - \vec{b}$

Position vector of C $= \vec{a} + \lambda\vec{b}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (\vec{a} - \vec{b}) - (\vec{a} + \vec{b})$

$\Rightarrow \overrightarrow{AB} = \vec{a} - \vec{a} - \vec{b} - \vec{b}$

$\Rightarrow \overrightarrow{AB} = -2\vec{b}$

$\Rightarrow \vec{b} = -\frac{\overrightarrow{AB}}{2}$ ...(i)

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$\Rightarrow \overrightarrow{BC} = (\vec{a} + \lambda\vec{b}) - (\vec{a} - \vec{b})$

$\Rightarrow \overrightarrow{BC} = \vec{a} - \vec{a} + \lambda\vec{b} + \vec{b}$

$\Rightarrow \overrightarrow{BC} = \lambda\vec{b} + \vec{b}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = \lambda\vec{b} + \vec{b}$

Or $\overrightarrow{BC} = (\lambda + 1)\vec{b}$

Or $\overrightarrow{BC} = (\lambda + 1) \times -\frac{\overrightarrow{AB}}{2}$ [∵, from (i)]

Or $\overrightarrow{BC} = -\left(\frac{\lambda+1}{2}\right) \times \overrightarrow{AB}$ ...(ii)

If λ is any real value, then $\left(\frac{\lambda+1}{2}\right)$ is also a real value.

Then, for any real value $\left(\frac{\lambda+1}{2}\right)$, we can write

$$\left(\frac{\lambda + 1}{2}\right) = \mu$$

From (ii) equation, we can write

$\overrightarrow{BC} = -\mu \times \overrightarrow{AB}$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, A, B and C are collinear.

6. Question

If $\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$, prove that A, B, C are collinear points.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Given: $\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$

To Prove: A, B and C are collinear points.

Proof: We have been given that,

$\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$

Rearrange it so that we get a relationship between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\Rightarrow \overrightarrow{AO}-\overrightarrow{BO}=\overrightarrow{OC}-\overrightarrow{OB}$

$\Rightarrow (-\overrightarrow{OA})-(-\overrightarrow{OB})=\overrightarrow{OC}-\overrightarrow{OB}$

$\Rightarrow \overrightarrow{OB}-\overrightarrow{OA}=\overrightarrow{OC}-\overrightarrow{OB}$ …(i)

Now, we know that

$\overrightarrow{AB}=\vec{B}-\vec{A}$

But actually we are doing $\overrightarrow{OB}-\overrightarrow{OA}$, such that O is the point of origin so that the difference between the two vectors is a displacement.

So, $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}$ …(ii)

Similarly, $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}$ …(iii)

Substituting equation (ii) & (iii) in equation (i), we get

$\overrightarrow{AB}=\overrightarrow{BC}$

Thus, this relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Hence, A, B and C are collinear.

7. Question

Show that the vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$ are collinear.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

We have been given position vectors $2\hat{i}-3\hat{j}+4\hat{k}$ and $-4\hat{i}+6\hat{j}-8\hat{k}$.

Let

$\vec{A}=2\hat{i}-3\hat{j}+4\hat{k}$

$\vec{B}=-4\hat{i}+6\hat{j}-8\hat{k}$

Also, let O be the initial point having position vector as

$\vec{O}=0\hat{i}+0\hat{j}+0\hat{k}$

Now, let us find $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is given by

$\overrightarrow{OA}$ = Position vector of $\vec{A}$ − Position vector of $\vec{O}$

$\Rightarrow \overrightarrow{OA} = (2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) - (0\hat{\imath} + 0\hat{\jmath} + 0\hat{k})$

$\Rightarrow \overrightarrow{OA} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\overrightarrow{OB}$ is given by

$\overrightarrow{OB}$ = Position vector of $\vec{B}$ − Position vector of $\vec{O}$

$\Rightarrow \overrightarrow{OB} = (-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}) - (0\hat{\imath} + 0\hat{\jmath} + 0\hat{k})$

$\Rightarrow \overrightarrow{OB} = -4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$

We have $\overrightarrow{OB}$ as

$\overrightarrow{OB} = -4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$

$\Rightarrow \overrightarrow{OB} = 2(-2\hat{\imath} + 3\hat{\jmath} - 4\hat{k})$

$\Rightarrow \overrightarrow{OB} = -2(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$

$\Rightarrow \overrightarrow{OB} = -2 \times \overrightarrow{OA}$

$[\because \overrightarrow{OA} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}]$

Thus, this relation shows that $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are parallel to each other.

But also, $\vec{O}$ is the common vector in $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

⇒ $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are not parallel but lies on a straight line.

⇒ A and B are collinear.

Hence, $2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $-4\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$ are collinear.

8. Question

If the points A (m, –1), B (2, 1) and C(4, 5) are collinear, find the value of m.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

We have been given points:

A (m, –1), B (2, 1) and C (4, 5).

These points are collinear.

Let us define the position vectors as,

Position vector of A $= m\hat{\imath} - \hat{\jmath}$

Position vector of B $= 2\hat{\imath} + \hat{\jmath}$

Position vector of C $= 4\hat{\imath} + 5\hat{\jmath}$

Now, we need to find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of $\vec{B}$ − Position vector of $\vec{A}$

$\Rightarrow \overrightarrow{AB} = (2\hat{\imath} + \hat{\jmath}) - (m\hat{\imath} - \hat{\jmath})$

$\Rightarrow \overrightarrow{AB} = 2\hat{i} + \hat{j} - m\hat{i} + \hat{j}$

$\Rightarrow \overrightarrow{AB} = (2 - m)\hat{i} + 2\hat{j}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC} = \text{Position vector of } \vec{C} - \text{Position vector of } \vec{B}$

$\Rightarrow \overrightarrow{BC} = (4\hat{i} + 5\hat{j}) - (2\hat{i} + \hat{j})$

$\Rightarrow \overrightarrow{BC} = 4\hat{i} + 5\hat{j} - 2\hat{i} - \hat{j}$

$\Rightarrow \overrightarrow{BC} = 4\hat{i} - 2\hat{i} + 5\hat{j} - \hat{j}$

$\Rightarrow \overrightarrow{BC} = 2\hat{i} + 4\hat{j}$

Since, A, B, C and D are collinear. We can draw a relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\overrightarrow{AB} = \lambda\overrightarrow{BC}$

Putting the values of $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$\Rightarrow (2 - m)\hat{i} + 2\hat{j} = \lambda \times (2\hat{i} + 4\hat{j})$

$\Rightarrow (2 - m)\hat{i} + 2\hat{j} = 2\lambda\hat{i} + 4\lambda\hat{j}$

Comparing L.H.S and R.H.S, we get

2 – m = 2λ

And 2 = 4λ

We need to find the value of λ in order to find m.

We have

2 = 4λ

$\Rightarrow \lambda = \frac{2}{4}$

$\Rightarrow \lambda = \frac{1}{2}$

Putting the value of λ in equation (2 – m) = 2λ

$\Rightarrow 2 - m = 2 \times \frac{1}{2}$

⇒ 2 – m = 1

⇒ m = 2 – 1

⇒ m = 1

Thus, the value of m = 1.

9. Question

Show that the points (3, 4), (–5, 16), (5, 1) are collinear.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Let the points be A (3, 4), B (–5, 16) and C (5, 1).

Let

$\text{Position vector of A} = 3\hat{i} + 4\hat{j}$

$\text{Position vector of B} = -5\hat{i} + 16\hat{j}$

$\text{Position vector of C} = 5\hat{i} + \hat{j}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AB} = (-5\hat{i} + 16\hat{j}) - (3\hat{i} + 4\hat{j})$

$\Rightarrow \overrightarrow{AB} = -5\hat{i} + 16\hat{j} - 3\hat{i} - 4\hat{j}$

$\Rightarrow \overrightarrow{AB} = -5\hat{i} - 3\hat{i} + 16\hat{j} - 4\hat{j}$

$\Rightarrow \overrightarrow{AB} = -8\hat{i} + 12\hat{j}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$

$\Rightarrow \overrightarrow{BC} = (5\hat{i} + \hat{j}) - (-5\hat{i} + 16\hat{j})$

$\Rightarrow \overrightarrow{BC} = 5\hat{i} + \hat{j} + 5\hat{i} - 16\hat{j}$

$\Rightarrow \overrightarrow{BC} = 5\hat{i} + 5\hat{i} + \hat{j} - 16\hat{j}$

$\Rightarrow \overrightarrow{BC} = 10\hat{i} - 15\hat{j}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = 10\hat{i} - 15\hat{j}$

Or $\overrightarrow{BC} = 5(2\hat{i} - 3\hat{j})$ …(i)

And we know, $\overrightarrow{AB} = -8\hat{i} + 12\hat{j}$

Or $\overrightarrow{AB} = -4(2\hat{i} - 3\hat{j})$

Or $(2\hat{i} - 3\hat{j}) = -\frac{\overrightarrow{AB}}{4}$ …(ii)

Substituting the value of $2\hat{i} - 3\hat{j}$ in equation (i), we get

$$\overrightarrow{BC} = 5 \times -\left(\frac{\overrightarrow{AB}}{4}\right)$$

$$\Rightarrow \overrightarrow{BC} = -\frac{5}{4} \times \overrightarrow{AB}$$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, A, B and C are collinear.

10. Question

If the vectors $\vec{a} = 2\hat{i} - 3\hat{j}$ and $\vec{b} = -6\hat{i} + m\hat{j}$ are collinear, find the value of m.

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

We have the position vectors as,

$\text{Position vector of a} = 2\hat{\imath} - 3\hat{\jmath}$

$\text{Position vector of b} = -6\hat{\imath} + m\hat{\jmath}$

Since, a and b are collinear. We can draw a relation between $\vec{a}$ and $\vec{b}$.

$\vec{a} = \lambda\vec{b}$

Putting the values of $\vec{a}$ and $\vec{b}$, we get

$\Rightarrow 2\hat{\imath} - 3\hat{\jmath} = \lambda \times (-6\hat{\imath} + m\hat{\jmath})$

$\Rightarrow 2\hat{\imath} - 3\hat{\jmath} = -6\lambda\hat{\imath} + m\lambda\hat{\jmath}$

Comparing L.H.S and R.H.S, we get

2 = –6λ

And –3 = mλ

We need to find the value of λ in order to find m.

We have

2 = –6λ

$\Rightarrow \lambda = -\frac{2}{6}$

$\Rightarrow \lambda = -\frac{1}{3}$

Putting the value of λ in equation –3 = mλ

$\Rightarrow -3 = -m \times \frac{1}{3}$

⇒ m = 3 × 3

⇒ m = 9

Thus, the value of m = 9.

11. Question

Show that the points A (1, –2, –8), B (5, 0, –2) and C(11, 3, 7) are collinear, and find the ratio in which B divides AC.

Answer

We have been given the points A (1, –2, –8), B (5, 0, –2) and C (11, 3, 7).

We need to show that A, B and C are collinear.

Let us define the position vector.

$\text{Position vector of A} = \hat{\imath} - 2\hat{\jmath} - 8\hat{k}$

$\text{Position vector of B} = 5\hat{\imath} - 2\hat{k}$

$\text{Position vector of C} = 11\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

So, in this case if we find a relation between $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{AC}$, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (5\hat{i} - 2\hat{k}) - (\hat{i} - 2\hat{j} - 8\hat{k})$

$\Rightarrow \overrightarrow{AB} = 5\hat{i} - 2\hat{k} - \hat{i} + 2\hat{j} + 8\hat{k}$

$\Rightarrow \overrightarrow{AB} = 5\hat{i} - \hat{i} + 2\hat{j} - 2\hat{k} + 8\hat{k}$

$\Rightarrow \overrightarrow{AB} = 4\hat{i} + 2\hat{j} + 6\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$\Rightarrow \overrightarrow{BC} = (11\hat{i} + 3\hat{j} + 7\hat{k}) - (5\hat{i} - 2\hat{k})$

$\Rightarrow \overrightarrow{BC} = 11\hat{i} + 3\hat{j} + 7\hat{k} - 5\hat{i} + 2\hat{k}$

$\Rightarrow \overrightarrow{BC} = 11\hat{i} - 5\hat{i} + 3\hat{j} + 7\hat{k} + 2\hat{k}$

$\Rightarrow \overrightarrow{BC} = 6\hat{i} + 3\hat{j} + 9\hat{k}$

And $\overrightarrow{AC}$ is given by

$\overrightarrow{AC}$ = Position vector of C − Position vector of A

$\Rightarrow \overrightarrow{AC} = (11\hat{i} + 3\hat{j} + 7\hat{k}) - (\hat{i} - 2\hat{j} - 8\hat{k})$

$\Rightarrow \overrightarrow{AC} = 11\hat{i} - \hat{i} + 3\hat{j} + 2\hat{j} + 7\hat{k} + 8\hat{k}$

$\Rightarrow \overrightarrow{AC} = 10\hat{i} + 5\hat{j} + 15\hat{k}$

Let us add $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$\overrightarrow{AB} + \overrightarrow{BC} = (4\hat{i} + 2\hat{j} + 6\hat{k}) + (6\hat{i} + 3\hat{j} + 9\hat{k})$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = 4\hat{i} + 6\hat{i} + 2\hat{j} + 3\hat{j} + 6\hat{k} + 9\hat{k}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = 10\hat{i} + 5\hat{j} + 15\hat{k}$

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

Thus, clearly A, B and C are collinear.

We need to find the ratio in which B divides AC.

Let the ratio at which B divides AC be λ : 1. Then, position vector of B is:

$\left(\frac{11\lambda + 1}{\lambda + 1}\right)\hat{i} + \left(\frac{3\lambda - 2}{\lambda + 1}\right)\hat{j} + \left(\frac{7\lambda - 8}{\lambda + 1}\right)\hat{k}$

But the position vector of B is $5\hat{i} - 2\hat{k}$.

So, by comparing the position vectors of B, we can write

$\left(\frac{11\lambda + 1}{\lambda + 1}\right) = 5$

$\left(\frac{3\lambda - 2}{\lambda + 1}\right) = 0$

$\left(\frac{7\lambda - 8}{\lambda + 1}\right) = -2$

Solving these equations separately, we get

$\left(\frac{11\lambda + 1}{\lambda + 1}\right) = 5$

⇒ $11\lambda + 1 = 5(\lambda + 1)$

⇒ $11\lambda + 1 = 5\lambda + 5$

⇒ $11\lambda - 5\lambda = 5 - 1$

⇒ $6\lambda = 4$

$\Rightarrow \lambda = \frac{4}{6}$

$\Rightarrow \lambda = \frac{2}{3}$

The ratio at which B divides AC is λ : 1.

Since, $\lambda = \frac{2}{3}$

We can say

$\lambda : 1 = \frac{2}{3} : 1$

Solving it further, multiply the ratio by 3.

$\lambda : 1 = \frac{2}{3} \times 3 : 1 \times 3$

⇒ λ : 1 = 2 : 3

Thus, the ratio in which B divides AC is 2 : 3.

12. Question

Using vectors show that the pointsA(–2, 3, 5), B(7, 0, 1) C(–3, –2, –5) and D(3, 4, 7) are such that AB and CD intersect at the point P(1, 2, 3).

Answer

We have been given the points A(–2, 3, 5), B(7, 0, 1), C(–3, –2, –5), D(3, 4, 7) and P(1, 2, 3).

Let us define it position vectors.

So,

$\text{Position vector of A} = -2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$

$\text{Position vector of B} = 7\hat{\imath} + \hat{k}$

$\text{Position vector of C} = -3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

$\text{Position vector of D} = 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$

$\text{Position vector of P} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Now, we need to show that AB and CD intersect at the point P.

For this, if we prove that A, B and P are collinear & C, D and P are collinear so that P is the common point between them and we can show that AB and CD intersect at P.

Let us find position vector of AP and PB.

$\overrightarrow{AP} = \text{Position vector of P} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AP} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$

$\Rightarrow \overrightarrow{AP} = \hat{\imath} + 2\hat{\imath} + 2\hat{\jmath} - 3\hat{\jmath} + 3\hat{k} - 5\hat{k}$

$\Rightarrow \overrightarrow{AP} = 3\hat{i} - \hat{j} - 2\hat{k}$

And

$\overrightarrow{PB} = \text{Position vector of B} - \text{Position vector of P}$

$\Rightarrow \overrightarrow{PB} = (7\hat{i} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{PB} = 7\hat{i} - \hat{i} - 2\hat{j} + \hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{PB} = 6\hat{i} - 2\hat{j} - 2\hat{k}$

Now, we can draw out a relation between $\overrightarrow{AP}$ and $\overrightarrow{PB}$.

We know, $\overrightarrow{PB} = 6\hat{i} - 2\hat{j} - 2\hat{k}$

$\Rightarrow \overrightarrow{PB} = 2(3\hat{i} - \hat{j} - \hat{k})$

$\Rightarrow \overrightarrow{PB} = 2 \times \overrightarrow{AP}$

This relation clearly shows that $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are parallel.

And since, P is the common point between them, we can say that these vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are actually not parallel but lie on a straight line.

⇒ Points A, P, B are collinear

[∵, Two more points are said to be collinear if they all lie on a single straight line.]

Now let us find the position vector of CP and PD.

$\overrightarrow{CP} = \text{Position vector of P} - \text{Position vector of C}$

$\Rightarrow \overrightarrow{CP} = (\hat{i} + 2\hat{j} + 3\hat{k}) - (-3\hat{i} - 2\hat{j} - 5\hat{k})$

$\Rightarrow \overrightarrow{CP} = \hat{i} + 3\hat{i} + 2\hat{j} + 2\hat{j} + 3\hat{k} + 5\hat{k}$

$\Rightarrow \overrightarrow{CP} = 4\hat{i} + 4\hat{j} + 8\hat{k}$

And

$\overrightarrow{PD} = \text{Position vector of D} - \text{Position vector of P}$

$\Rightarrow \overrightarrow{PD} = (3\hat{i} + 4\hat{j} + 7\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{PD} = 3\hat{i} - \hat{i} + 4\hat{j} - 2\hat{j} + 7\hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{PD} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

Now, we can draw out a relation between $\overrightarrow{CP}$ and $\overrightarrow{PD}$.

We know, $\overrightarrow{CP} = 4\hat{i} + 4\hat{j} + 8\hat{k}$

$\Rightarrow \overrightarrow{CP} = 2(2\hat{i} + 2\hat{i} + 4\hat{i})$

$\Rightarrow \overrightarrow{CP} = 2 \times \overrightarrow{PD}$

This relation clearly shows that $\overrightarrow{CP}$ and $\overrightarrow{PD}$ are parallel.

And since, P is the common point between them, we can say that these vectors $\overrightarrow{CP}$ and $\overrightarrow{PD}$ are actually not parallel but lie on a straight line.

⇒ Points C, P and D are collinear.

[∵, Two more points are said to be collinear if they all lie on a single straight line.]

Since, we know that A, P, B and C, P, D are collinear separately.

Note that, P is the common point between the two pairs of collinear points.

Thus, AB and CD intersect each other at a point P.

13. Question

Using vectors, find the value of λ such that the points $(\lambda, -10, 3)$, (1, –1, 3) and (3, 5, 3) are collinear.

Answer

Let the points be A (λ, –10, 3), B (1, –1, 3), C (3, 5, 3).

Let us define the position vectors of A, B and C.

$$\text{Position vector of A} = \lambda\hat{i} - 10\hat{j} + 3\hat{k}$$

$$\text{Position vector of B} = \hat{i} - \hat{j} + 3\hat{k}$$

$$\text{Position vector of C} = 3\hat{i} + 5\hat{j} + 3\hat{k}$$

Then,

$$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$$

$$\Rightarrow \overrightarrow{AB} = (\hat{i} - \hat{j} + 3\hat{k}) - (\lambda\hat{i} - 10\hat{j} + 3\hat{k})$$

$$\Rightarrow \overrightarrow{AB} = \hat{i} - \lambda\hat{i} - \hat{j} + 10\hat{j} + 3\hat{k} - 3\hat{k}$$

$$\Rightarrow \overrightarrow{AB} = (1 - \lambda)\hat{i} + 9\hat{j}$$

And

$$\overrightarrow{AC} = \text{Position vector of C} - \text{Position vector of A}$$

$$\Rightarrow \overrightarrow{AC} = (3\hat{i} + 5\hat{j} + 3\hat{k}) - (\lambda\hat{i} - 10\hat{j} + 3\hat{k})$$

$$\Rightarrow \overrightarrow{AC} = 3\hat{i} - \lambda\hat{i} + 5\hat{j} + 10\hat{j} + 3\hat{k} - 3\hat{k}$$

$$\Rightarrow \overrightarrow{AC} = (3 - \lambda)\hat{i} + 15\hat{j}$$

And since, A, B and C are collinear.

Then, it has a relation as such

$\overrightarrow{AB} = k\overrightarrow{AC}$, where k is scalar quantity.

$$\Rightarrow (1 - \lambda)\hat{i} + 9\hat{j} = k[(3 - \lambda)\hat{i} + 15\hat{j}]$$

$$\Rightarrow (1 - \lambda)\hat{i} + 9\hat{j} = k(3 - \lambda)\hat{i} + 15k\hat{j}$$

Comparing the coefficients of $\hat{i}$ and $\hat{j}$. We get

1 – λ = k (3 – λ)

And 9 = 15k

First, we need to find the value of k.

So take 9 = 15k

$$\Rightarrow k = \frac{9}{15}$$

$$\Rightarrow k = \frac{3}{5}$$

Substitute the value of k in (1 – λ) = k (3 – λ)

$\Rightarrow 1 - \lambda = \frac{3}{5}(3 - \lambda)$

⇒ 5(1 – λ) = 3(3 – λ)

⇒ 5 – 5λ = 9 – 3λ

⇒ 5λ – 3λ = 5 – 9

⇒ 2λ = –4

$\Rightarrow \lambda = -\frac{4}{2}$

⇒ λ = –2

Hence, the value of λ is –2.

Exercise 23.8

1 A. Question

Show that the points whose position vectors are as given below are collinear :

$2\hat{i} + \hat{j} - \hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + 4\hat{j} - 3\hat{k}$

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Let us assume points to be A, B and C such that

$\text{Position vector of A} = 2\hat{i} + \hat{j} - \hat{k}$

$\text{Position vector of B} = 3\hat{i} - 2\hat{j} + \hat{k}$

$\text{Position vector of C} = \hat{i} + 4\hat{j} - 3\hat{k}$

Then, we need to find $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AB} = (3\hat{i} - 2\hat{j} + \hat{k}) - (2\hat{i} + \hat{j} - \hat{k})$

$\Rightarrow \overrightarrow{AB} = 3\hat{i} - 2\hat{i} - 2\hat{j} - \hat{j} + \hat{k} + \hat{k}$

$\Rightarrow \overrightarrow{AB} = \hat{i} - 3\hat{j} + 2\hat{k}$

And

$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$

$\Rightarrow \overrightarrow{BC} = (\hat{i} + 4\hat{j} - 3\hat{k}) - (3\hat{i} - 2\hat{j} + \hat{k})$

$\Rightarrow \overrightarrow{BC} = \hat{i} - 3\hat{i} + 4\hat{j} + 2\hat{j} - 3\hat{k} - \hat{k}$

$\Rightarrow \overrightarrow{BC} = -2\hat{i} + 6\hat{j} - 4\hat{k}$

Now, we need to draw a relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know that,

$\overrightarrow{BC} = -2\hat{i} + 6\hat{j} - 4\hat{k}$

Or $\overrightarrow{BC} = -2(\hat{i} - 3\hat{j} + 2\hat{k})$

Or $\overrightarrow{BC} = -2 \times \overrightarrow{AB}$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But since, B is the common point in AB and BC.

⇒ AB and BC actually lies on a straight line.

Thus, A, B and C are collinear.

1 B. Question

Show that the points whose position vectors are as given below are collinear :

$3\hat{i}+2\hat{j}+4\hat{k}, \hat{i}+\hat{j}+\hat{k}$ and $-\hat{i}+4\hat{j}-2\hat{k}$

Answer

Let us assume points to be A, B and C such that

$\text{Position vector of A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

$\text{Position vector of B} = \hat{i} + \hat{j} + \hat{k}$

$\text{Position vector of C} = -\hat{i} + 4\hat{j} - 2\hat{k}$

Then, we need to find $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AB} = (\hat{i} + \hat{j} + \hat{k}) - (3\hat{i} - 2\hat{j} + 4\hat{k})$

$\Rightarrow \overrightarrow{AB} = \hat{i} + \hat{j} + \hat{k} - 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\Rightarrow \overrightarrow{AB} = \hat{i} - 3\hat{i} + \hat{j} + 2\hat{j} + \hat{k} - 4\hat{k}$

$\Rightarrow \overrightarrow{AB} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

And

$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$

$\Rightarrow \overrightarrow{BC} = (-\hat{i} + 4\hat{j} - 2\hat{k}) - (\hat{i} + \hat{j} + \hat{k})$

$\Rightarrow \overrightarrow{BC} = -\hat{i} - \hat{i} + 4\hat{j} - \hat{j} - 2\hat{k} - \hat{k}$

$\Rightarrow \overrightarrow{BC} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

Now, we need to draw a relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know that,

$\overrightarrow{BC} = -2\hat{i} + 3\hat{j} - 3\hat{k}$

Or $\overrightarrow{BC} = \overrightarrow{AB}$

Or $\overrightarrow{BC} = 1 \times \overrightarrow{AB}$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But since, B is the common point in AB and BC.

⇒ AB and BC actually lies on a straight line.

Thus, A, B and C are collinear.

2 A. Question

Using vector method, prove that the following points are collinear.

A(6, –7, –1), B(2 –3, 1) and C(4, –5, 0)

Answer

Let us understand that, two more points are said to be collinear if they all lie on a single straight line.

Given: A (6, –7, –1), B (2, –3, 1) and C (4, –5, 0).

To Prove: A, B and C are collinear.

Proof:

Let us define position vectors. So,

$\text{Position vector of A} = 6\hat{\imath} - 7\hat{\jmath} - \hat{k}$

$\text{Position vector of B} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$

$\text{Position vector of C} = 4\hat{\imath} - 5\hat{\jmath}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AB} = \left(2\hat{\imath} - 3\hat{\jmath} + \hat{k}\right) - \left(6\hat{\imath} - 7\hat{\jmath} - \hat{k}\right)$

$\Rightarrow \overrightarrow{AB} = 2\hat{\imath} - 6\hat{\imath} - 3\hat{\jmath} + 7\hat{\jmath} + \hat{k} + \hat{k}$

$\Rightarrow \overrightarrow{AB} = -4\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$

$\Rightarrow \overrightarrow{BC} = (4\hat{\imath} - 5\hat{\jmath}) - \left(2\hat{\imath} - 3\hat{\jmath} + \hat{k}\right)$

$\Rightarrow \overrightarrow{BC} = 4\hat{\imath} - 2\hat{\imath} - 5\hat{\jmath} + 3\hat{\jmath} - \hat{k}$

$\Rightarrow \overrightarrow{BC} = 2\hat{\imath} - 2\hat{\jmath} - \hat{k}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{AB} = -4\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$

Or $\overrightarrow{AB} = -2(2\hat{\imath} - 2\hat{\jmath} - \hat{k})$

Or $\overrightarrow{AB} = -2 \times \overrightarrow{BC}$ $[\because \overrightarrow{BC} = 2\hat{\imath} - 2\hat{\jmath} - \hat{k}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, proved that A, B and C are collinear.

2 B. Question

Using vector method, prove that the following points are collinear.

A(2, –1, 3), B(4, 3, 1) and C(3, 1, 2)

Answer

Given: A (2, –1, 3), B (4, 3, 1) and C (3, 1, 2).

To Prove: A, B and C are collinear.

Proof:

Let us define position vectors. So,

Position vector of $A = 2\hat{i} - \hat{j} + 3\hat{k}$

Position vector of $B = 4\hat{i} + 3\hat{j} + \hat{k}$

Position vector of $C = 3\hat{i} + \hat{j} + 2\hat{k}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AB} = (4\hat{i} + 3\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{AB} = 4\hat{i} - 2\hat{i} + 3\hat{j} + \hat{j} + \hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$

$\Rightarrow \overrightarrow{BC} = (3\hat{i} + \hat{j} + 2\hat{k}) - (4\hat{i} + 3\hat{j} + \hat{k})$

$\Rightarrow \overrightarrow{BC} = 3\hat{i} - 4\hat{i} + \hat{j} - 3\hat{j} + 2\hat{k} - \hat{k}$

$\Rightarrow \overrightarrow{BC} = -\hat{i} - 2\hat{j} + \hat{k}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 2\hat{k}$

Or $\overrightarrow{AB} = -2(-\hat{i} - 2\hat{j} + \hat{k})$

Or $\overrightarrow{AB} = -2 \times \overrightarrow{BC}$ $[\because, \overrightarrow{BC} = -\hat{i} - 2\hat{j} + \hat{k}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$\Rightarrow$ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, proved that A, B and C are collinear.

2 C. Question

Using vector method, prove that the following points are collinear.

A(1, 2, 7), B(2, 6, 3) and C(3, 10 –1)

Answer

Given: A (1, 2, 7), B (2, 6, 3) and C (3, 10, –1).

To Prove: A, B and C are collinear.

Proof:

Let us define position vectors. So,

Position vector of $A = \hat{i} + 2\hat{j} + 7\hat{k}$

$\text{Position vector of B} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

$\text{Position vector of C} = 3\hat{\imath} + 10\hat{\jmath} - \hat{k}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB} = \text{Position vector of B} - \text{Position vector of A}$

$\Rightarrow \overrightarrow{AB} = (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}) - (\hat{\imath} + 2\hat{\jmath} + 7\hat{k})$

$\Rightarrow \overrightarrow{AB} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k} - \hat{\imath} - 2\hat{\jmath} - 7\hat{k}$

$\Rightarrow \overrightarrow{AB} = 2\hat{\imath} - \hat{\imath} + 6\hat{\jmath} - 2\hat{\jmath} + 3\hat{k} - 7\hat{k}$

$\Rightarrow \overrightarrow{AB} = \hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC} = \text{Position vector of C} - \text{Position vector of B}$

$\Rightarrow \overrightarrow{BC} = (3\hat{\imath} + 10\hat{\jmath} - \hat{k}) - (2\hat{\imath} + 6\hat{\jmath} + 3\hat{k})$

$\Rightarrow \overrightarrow{BC} = 3\hat{\imath} + 10\hat{\jmath} - \hat{k} - 2\hat{\imath} - 6\hat{\jmath} - 3\hat{k}$

$\Rightarrow \overrightarrow{BC} = 3\hat{\imath} - 2\hat{\imath} + 10\hat{\jmath} - 6\hat{\jmath} - \hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{BC} = \hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = \hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

Or $\overrightarrow{BC} = \overrightarrow{AB}$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, proved that A, B and C are collinear.

2 D. Question

Using vector method, prove that the following points are collinear.

A(–3, –2, –5), B(1, 2, 3) and C(3, 4, 7)

Answer

Given: A (–3, –2, –5), B (1, 2, 3) and C (3, 4, 7).

To Prove: A, B and C are collinear.

Proof:

Let us define position vectors. So,

$\text{Position vector of A} = -3\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

$\text{Position vector of B} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\text{Position vector of C} = 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - (-3\hat{\imath} - 2\hat{\jmath} - 5\hat{k})$

$\Rightarrow \overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + 3\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$

$\Rightarrow \overrightarrow{AB} = \hat{\imath} + 3\hat{\imath} + 2\hat{\jmath} + 2\hat{\jmath} + 3\hat{k} + 5\hat{k}$

$\Rightarrow \overrightarrow{AB} = 4\hat{\imath} + 4\hat{\jmath} + 8\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$\Rightarrow \overrightarrow{BC} = (3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}) - (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$

$\Rightarrow \overrightarrow{BC} = 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k} - \hat{\imath} - 2\hat{\jmath} - 3\hat{k}$

$\Rightarrow \overrightarrow{BC} = 3\hat{\imath} - \hat{\imath} + 4\hat{\jmath} - 2\hat{\jmath} + 7\hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{BC} = 2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{AB} = 4\hat{\imath} + 4\hat{\jmath} + 8\hat{k}$

Or $\overrightarrow{AB} = 2(2\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$

Or $\overrightarrow{AB} = 2 \times \overrightarrow{BC}$ $[\because \overrightarrow{BC} = 2\hat{\imath} + 2\hat{\jmath} + 4\hat{k}]$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, proved that A, B and C are collinear.

2 E. Question

Using vector method, prove that the following points are collinear.

A (2, –1, 3), B (3, –5, 1) and C (–1, 11, 9).

Answer

A (2, –1, 3), B (3, –5, 1) and C (–1, 11, 9).

To Prove: A, B and C are collinear.

Proof:

Let us define position vectors. So,

Position vector of A $= 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$

Position vector of B $= 3\hat{\imath} - 5\hat{\jmath} + \hat{k}$

Position vector of C $= -\hat{\imath} + 11\hat{\jmath} + 9\hat{k}$

So, in this case if we prove that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other, then we can easily show that A, B and C are collinear.

Therefore, $\overrightarrow{AB}$ is given by

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$\Rightarrow \overrightarrow{AB} = (3\hat{i} - 5\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$

$\Rightarrow \overrightarrow{AB} = 3\hat{i} - 5\hat{j} + \hat{k} - 2\hat{i} + \hat{j} - 3\hat{k}$

$\Rightarrow \overrightarrow{AB} = 3\hat{i} - 2\hat{i} - 5\hat{j} + \hat{j} + \hat{k} - 3\hat{k}$

$\Rightarrow \overrightarrow{AB} = \hat{i} - 4\hat{j} - 2\hat{k}$

And $\overrightarrow{BC}$ is given by

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$\Rightarrow \overrightarrow{BC} = (-\hat{i} + 11\hat{j} + 9\hat{k}) - (3\hat{i} - 5\hat{j} + \hat{k})$

$\Rightarrow \overrightarrow{BC} = -\hat{i} + 11\hat{j} + 9\hat{k} - 3\hat{i} + 5\hat{j} - \hat{k}$

$\Rightarrow \overrightarrow{BC} = -\hat{i} - 3\hat{i} + 11\hat{j} + 5\hat{j} + 9\hat{k} - \hat{k}$

$\Rightarrow \overrightarrow{BC} = -4\hat{i} + 16\hat{j} + 8\hat{k}$

Let us note the relation between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

We know, $\overrightarrow{BC} = -4\hat{i} + 16\hat{j} + 8\hat{k}$

Or $\overrightarrow{BC} = -4(\hat{i} - 4\hat{j} - 2\hat{k})$

Or $\overrightarrow{BC} = -4 \times \overrightarrow{AB}$ [$\because$, $\overrightarrow{AB} = \hat{i} - 4\hat{j} - 2\hat{k}$

This relation shows that $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel to each other.

But also, $\vec{B}$ is the common vector in $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

⇒ $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are not parallel but lies on a straight line.

Thus, proved that A, B and C are collinear.

3 A. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non–zero, non-coplanar vectors, prove that the following vectors are coplanar :

$5\vec{a} + 6\vec{b} + 7\vec{c}, 7\vec{a} - 8\vec{b} + 9\vec{c}$ and $3\vec{a} + 20\vec{b} + 5\vec{c}$

Answer

Vectors parallel to the same plane, or lie on the same plane are called coplanar vectors

The three vectors are coplanar if one of them is expressible as a linear combination of the other two.

We have been given that, $5\vec{a} + 6\vec{b} + 7\vec{c}$, $7\vec{a} - 8\vec{b} + 9\vec{c}$ and $3\vec{a} + 20\vec{b} + 5\vec{c}$.

We can form a relation using these three vectors. Say,

$5\vec{a} + 6\vec{b} + 7\vec{c} = x(7\vec{a} - 8\vec{b} + 9\vec{c}) + y(3\vec{a} + 20\vec{b} + 5\vec{c})$

$\Rightarrow 5\vec{a} + 6\vec{b} + 7\vec{c} = 7x\vec{a} - 8x\vec{b} + 9x\vec{c} + 3y\vec{a} + 20y\vec{b} + 5y\vec{c}$

$\Rightarrow 5\vec{a} + 6\vec{b} + 7\vec{c} = (7x + 3y)\vec{a} + (-8x + 20y)\vec{b} + (9x + 5y)\vec{c}$

Compare the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. We get

5 = 7x + 3y …(1)

6 = –8x + 20y …(2)

7 = 9x + 5y …(3)

Solving equations (1) and (2) for x and y.

Equation (1), 7x + 3y = 5

Equation (2), –8x + 20y = 6

Multiply equation (1) by 8 and equation (2) by 7, we get

7x + 3y = 5 [× 8

–8x + 20y = 6 [× 7

We get

$$\begin{array}{r} 56x + 24y = 40 \\ -56x + 140y = 42 \\ \hline 0 + 164y = 82 \\ \hline \end{array}$$

⇒ 164y = 82

$\Rightarrow y = \frac{82}{164}$

$\Rightarrow y = \frac{41}{82}$

$\Rightarrow y = \frac{1}{2}$

Put $y = \frac{1}{2}$ in equation (2), we get

$-8x + 20\left(\frac{1}{2}\right) = 6$

⇒ –8x + 10 = 6

⇒ –8x = 6 – 10

⇒ –8x = –4

⇒ 8x = 4

$\Rightarrow x = \frac{4}{8}$

$\Rightarrow x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ and $y = \frac{1}{2}$ in equation (3), we get

7 = 9x + 5y

Or 9x + 5y = 7

$\Rightarrow 9\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right) = 7$

$\Rightarrow \frac{9}{2} + \frac{5}{2} = 7$

$\Rightarrow \frac{9+5}{2} = 7$

⇒ 14 = 7 × 2

⇒ 14 = 14

∵, L.H.S = R.H.S

⇒ The value of x and y satisfy equation (3).

Thus, $5\vec{a}+6\vec{b}+7\vec{c}$, $7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are coplanar.

3 B. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non–zero, non-coplanar vectors, prove that the following vectors are coplanar :

$\vec{a}-2\vec{b}+3\vec{c}, \vec{a}-3\vec{b}+5\vec{c}$ and $-2\vec{a}+3\vec{b}-4\vec{c}$

Answer

We have been given that, $\vec{a}-2\vec{b}+3\vec{c}$, $-3\vec{b}+5\vec{c}$ and $-2\vec{a}+3\vec{b}-4\vec{c}$.

We can form a relation using these three vectors. Say,

$$\vec{a}-2\vec{b}+3\vec{c}=x(-3\vec{b}+5\vec{c})+y(-2\vec{a}+3\vec{b}-4\vec{c})$$

$$\Rightarrow \vec{a}-2\vec{b}+3\vec{c}=-3x\vec{b}+5x\vec{c}-2y\vec{a}+3y\vec{b}-4y\vec{c}$$

$$\Rightarrow \vec{a}-2\vec{b}+3\vec{c}=-2y\vec{a}+(-3x+3y)\vec{b}+(5x-4y)\vec{c}$$

Compare the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. We get

1 = – 2y ...(1)

-2 = -3x + 3y ...(2)

3 = 5x – 4y ...(3)

Solving equation (1) for y,

Equation (1), -2y = 1

$$\Rightarrow y=-\frac{1}{2}$$

Put $y=-\frac{1}{2}$ in equation (2), we get

$$-3x+3\left(-\frac{1}{2}\right)=-2$$

$$\Rightarrow -3x-\frac{3}{2}=-2$$

$$\Rightarrow \frac{-6x-3}{2}=-2$$

⇒ -6x – 3 = -2 × 2

⇒ -6x – 3 = -4

⇒ -6x = -4 + 3

⇒ -6x = -1

$$\Rightarrow x=\frac{1}{6}$$

Substituting $x=\frac{1}{6}$ and $y=-\frac{1}{2}$ in equation (3), we get

3 = 5x – 4y

Or 5x – 4y = 3

$\Rightarrow 5\left(\frac{1}{6}\right) - 4\left(-\frac{1}{2}\right) = 3$

$\Rightarrow \frac{5}{6} + \frac{4}{2} = 3$

$\Rightarrow \frac{5}{6} + 2 = 3$

$\Rightarrow \frac{5+12}{6} = 3$

$\Rightarrow \frac{17}{6} = 3$

But $\frac{17}{6} \neq 3$

∵, L.H.S ≠ R.H.S

⇒ The value of x and y doesn't satisfy equation (3).

Thus, $\vec{a} - 2\vec{b} + 3\vec{c}$, $\vec{a} - 3\vec{b} + 5\vec{c}$ and $-2\vec{a} + 3\vec{b} - 4\vec{c}$ are not coplanar.

4. Question

Show that the four points having position vectors $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$, $2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar.

Answer

Vectors parallel to the same plane, or lie on the same plane are called coplanar vectors

The three vectors are coplanar if one of them is expressible as a linear combination of the other two.

Let the four points be denoted be P, Q, R and S for $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ respectively such that we can say,

Position vector of $P = 6\hat{i} - 7\hat{j}$

Position vector of $Q = 16\hat{i} - 19\hat{j} - 4\hat{k}$

Position vector of $R = 3\hat{j} - 6\hat{k}$

Position vector of $S = 2\hat{i} - 5\hat{j} + 10\hat{k}$

Let us find $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$.

So,

$\overrightarrow{PQ}$ = Position vector of Q − Position vector of P

$\Rightarrow \overrightarrow{PQ} = (16\hat{i} - 19\hat{j} - 4\hat{k}) - (6\hat{i} - 7\hat{j})$

$\Rightarrow \overrightarrow{PQ} = 16\hat{i} - 19\hat{j} - 4\hat{k} - 6\hat{i} + 7\hat{j}$

$\Rightarrow \overrightarrow{PQ} = 16\hat{i} - 6\hat{i} - 19\hat{j} + 7\hat{j} - 4\hat{k}$

$\Rightarrow \overrightarrow{PQ} = 10\hat{i} - 12\hat{j} - 4\hat{k}$

Also,

$\overrightarrow{PR}$ = Position vector of R − Position vector of P

$\Rightarrow \overrightarrow{PR} = (3\hat{j} - 6\hat{k}) - (6\hat{i} - 7\hat{j})$

$\Rightarrow \overrightarrow{PR} = 3\hat{j} - 6\hat{k} - 6\hat{i} + 7\hat{j}$

$\Rightarrow \overrightarrow{PR} = -6\hat{i} + 3\hat{j} + 7\hat{j} - 6\hat{k}$

$\Rightarrow \overrightarrow{PR} = -6\hat{i} + 10\hat{j} - 6\hat{k}$

And,

$\overrightarrow{PS} = \text{Position vector of S} - \text{Position vector of P}$

$\Rightarrow \overrightarrow{PS} = (2\hat{i} - 5\hat{j} + 10\hat{k}) - (6\hat{i} - 7\hat{j})$

$\Rightarrow \overrightarrow{PS} = 2\hat{i} - 5\hat{j} + 10\hat{k} - 6\hat{i} + 7\hat{j}$

$\Rightarrow \overrightarrow{PS} = 2\hat{i} - 6\hat{i} - 5\hat{j} + 7\hat{j} + 10\hat{k}$

$\Rightarrow \overrightarrow{PS} = -4\hat{i} + 2\hat{j} + 10\hat{k}$

Now, we need to show a relation between $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and $\overrightarrow{PS}$.

So, $\overrightarrow{PQ} = x\overrightarrow{PR} + y\overrightarrow{PS}$

$\Rightarrow 10\hat{i} - 12\hat{j} - 4\hat{k} = x(-6\hat{i} + 10\hat{j} - 6\hat{k}) + y(-4\hat{i} + 2\hat{j} + 10\hat{k})$

$\Rightarrow 10\hat{i} - 12\hat{j} - 4\hat{k} = -6x\hat{i} + 10x\hat{j} - 6x\hat{k} - 4y\hat{i} + 2y\hat{j} + 10y\hat{k}$

$\Rightarrow 10\hat{i} - 12\hat{j} - 4\hat{k} = (-6x - 4y)\hat{i} + (10x + 2y)\hat{j} + (-6x + 10y)\hat{k}$

Comparing coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

–6x – 4y = 10 …(i)

10x + 2y = –12 …(ii)

–6x + 10y = –4 …(iii)

For solving equation (i) and (ii) for x and y, multiply equation (ii) by 2.

10x + 2y = –12 [× 2

⇒ 20x + 4y = –24 …(iv)

Solving equations (iv) and (i), we get

$$\begin{array}{l} 20x + 4y = -24 \\ \underline{-6x - 4y = 10} \\ \underline{14x + 0 = -14} \end{array}$$

⇒ 14x = –14

$\Rightarrow x = -\frac{14}{14}$

⇒ x = –1

Put x = –1 in equation (i), we get

–6(–1) – 4y = 10

⇒ 6 – 4y = 10

⇒ –4y = 10 – 6

⇒ –4y = 4

$\Rightarrow y = -\frac{4}{4}$

⇒ y = –1

Substitute x = –1 and y = –1 in equation (iii), we get

–6x + 10y = –4

⇒ –6(–1) + 10(–1) = –4

⇒ 6 – 10 = –4

⇒ –4 = –4

∵, L.H.S = R.H.S

⇒ The value of x and y satisfy equation (iii).

Thus, $6\hat{i} - 7\hat{j}$, $16\hat{i} - 19\hat{j} - 4\hat{k}$, $3\hat{j} - 6\hat{k}$ and $2\hat{i} - 5\hat{j} + 10\hat{k}$ are coplanar.

5 A. Question

Prove that the following vectors are coplanar :

$2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$

Answer

Vectors parallel to the same plane, or lie on the same plane are called coplanar vectors

The three vectors are coplanar if one of them is expressible as a linear combination of the other two.

We have been given that, $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$.

We can form a relation using these three vectors. Say,

$2\hat{i} - \hat{j} + \hat{k} = x(\hat{i} - 3\hat{j} - 5\hat{k}) + y(3\hat{i} - 4\hat{j} - 4\hat{k})$

$\Rightarrow 2\hat{i} - \hat{j} + \hat{k} = x\hat{i} - 3x\hat{j} - 5x\hat{k} + 3y\hat{i} - 4y\hat{j} - 4y\hat{k}$

$\Rightarrow 2\hat{i} - \hat{j} + \hat{k} = (x + 3y)\hat{i} + (-3x - 4y)\hat{j} + (-5x - 4y)\hat{k}$

Comparing coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

2 = x + 3y …(1)

–1 = –3x – 4y …(2)

1 = –5x – 4y …(3)

Solving equations (1) and (2) for x and y.

Equation (1), x + 3y = 2

Equation (2), –3x – 4y = –1

Multiply equation (1) by 3.

x + 3y = 2 [× 3

⇒ 3x + 9y = 6 …(4)

Solving equations (4) and (2), we get

$$\begin{array}{l} 3x + 9y = 6 \\ \underline{-3x - 4y = -1} \\ \underline{0 + 5y = 5} \end{array}$$

⇒ 5y = 5

$\Rightarrow y = \frac{5}{5}$

⇒ y = 1

Put $y = 1$ in equation (1), we get

2 = x + 3y

⇒ x + 3(1) = 2

⇒ x = 2 – 3

⇒ x = –1

Substituting x = –1 and y = 1 in equation (3), we get

–5x – 4y = 1

⇒ –5(–1) – 4(1) = 1

⇒ 5 – 4 = 1

⇒ 1 = 1

∵, L.H.S = R.H.S

⇒ The value of x and y satisfy equation (3).

Thus, $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ are coplanar.

5 B. Question

Prove that the following vectors are coplanar :

$\hat{i} + \hat{j} + \hat{k}, 2\hat{i} + 3\hat{j} - \hat{k}$ and $-\hat{i} - 2\hat{j} + 2\hat{k}$

Answer

We have been given that, $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j} - \hat{k}$ and $-\hat{i} - 2\hat{j} + 2\hat{k}$.

We can form a relation using these three vectors. Say,

$\hat{i} + \hat{j} + \hat{k} = x(2\hat{i} + 3\hat{j} - \hat{k}) + y(-\hat{i} - 2\hat{j} + 2\hat{k})$

$\Rightarrow \hat{i} + \hat{j} + \hat{k} = 2x\hat{i} + 3x\hat{j} - x\hat{k} - y\hat{i} - 2y\hat{j} + 2y\hat{k}$

$\Rightarrow \hat{i} + \hat{j} + \hat{k} = (2x - y)\hat{i} + (3x - 2y)\hat{j} + (-x + 2y)\hat{k}$

Comparing coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

1 = 2x – y …(1)

1 = 3x – 2y …(2)

1 = –x + 2y …(3)

Solving equations (1) and (2) for x and y.

Equation (1), 2x – y = 1

Equation (2), 3x – 2y = 1

Multiply equation (1) by 2.

2x – y = 1 [× 2

⇒ 4x – 2y = 2 …(4)

Solving equations (4) and (2), we get

4x – 2y = 2
3x – 2y = 1
(–) (+) (–)
x + 0 = 1

⇒ x = 1

Put x = 1 in equation (1), we get

1 = 2x – y

⇒ 1 = 2(1) – y

⇒ 1 = 2 – y

⇒ y = 2 – 1

⇒ y = 1

Substituting x = 1 and y = 1 in equation (3), we get

1 = –x + 2y

Or –x + 2y = 1

⇒ –(1) + 2(1) = 1

⇒ –1 + 2 = 1

⇒ 1 = 1

∵, L.H.S = R.H.S

⇒ The value of x and y satisfy equation (3).

Thus, $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j} - \hat{k}$ and $-\hat{i} - 2\hat{j} + 2\hat{k}$ are coplanar.

6 A. Question

Prove that the following vectors are non-coplanar :

$3\hat{i} + \hat{j} - \hat{k}, 2\hat{i} - \hat{j} + 7\hat{k}$ and $7\hat{i} + \hat{j} + 23\hat{k}$

Answer

Vectors parallel to the same plane, or lie on the same plane are called coplanar vectors

The three vectors are coplanar if one of them is expressible as a linear combination of the other two.

We have been given that, $3\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 7\hat{k}$ and $7\hat{i} - \hat{j} + 23\hat{k}$.

We can form a relation using these three vectors. Say,

$3\hat{i} + \hat{j} - \hat{k} = x(2\hat{i} - \hat{j} + 7\hat{k}) + y(7\hat{i} - \hat{j} + 23\hat{k})$

$\Rightarrow 3\hat{i} + \hat{j} - \hat{k} = 2x\hat{i} - x\hat{j} + 7x\hat{k} + 7y\hat{i} - y\hat{j} + 23y\hat{k}$

$\Rightarrow 3\hat{i} + \hat{j} - \hat{k} = (2x + 7y)\hat{i} + (-x - y)\hat{j} + (7x + 23y)\hat{k}$

Comparing coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

3 = 2x + 7y …(1)

1 = –x – y …(2)

–1 = 7x + 23y …(3)

Solving equations (1) and (2) for x and y.

Equation (1), 2x + 7y = 3

Equation (2), –x – y = 1

Multiply equation (2) by 2.

–x – y = 1 [× 2

⇒ –2x – 2y = 2 …(4)

Solving equations (4) and (1), we get

$$\begin{array}{l} 2x + 7y = 3 \\ -2x - 2y = 2 \\ \hline 0 + 5y = 5 \\ \hline \end{array}$$

⇒ 5y = 5

$\Rightarrow y = \frac{5}{5}$

⇒ y = 1

Put y = 1 in equation (2), we get

1 = –x – y

⇒ 1 = –x – (1)

⇒ 1 = –x – 1

⇒ x = –1 – 1

⇒ x = –2

Substituting x = –2 and y = 1 in equation (3), we get

–1 = 7x + 23y

Or 7x + 23y = –1

⇒ 7(–2) + 23(1) = –1

⇒ –14 + 23 = –1

⇒ 9 ≠ –1

∵, L.H.S ≠ R.H.S

⇒ The value of x and y doesn't satisfy equation (3).

Thus, $3\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 7\hat{k}$ and $7\hat{i} - \hat{j} + 23\hat{k}$ are not coplanar.

6 B. Question

Prove that the following vectors are non-coplanar :

$\hat{i} + 2\hat{j} + 3\hat{k}, 2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$

Answer

We have been given that, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$.

We can form a relation using these three vectors. Say,

$\hat{i} + 2\hat{j} + 3\hat{k} = x(2\hat{i} + \hat{j} + 3\hat{k}) + y(\hat{i} + \hat{j} + \hat{k})$

$\Rightarrow \hat{i} + 2\hat{j} + 3\hat{k} = 2x\hat{i} + x\hat{j} + 3x\hat{k} + y\hat{i} + y\hat{j} + y\hat{k}$

$\Rightarrow \hat{i} + 2\hat{j} + 3\hat{k} = (2x + y)\hat{i} + (x + y)\hat{j} + (3x + y)\hat{k}$

Comparing coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

1 = 2x + y ...(1)

2 = x + y ...(2)

3 = 3x + y ...(3)

Solving equations (1) and (2) for x and y.

Equation (1), 2x + y = 1

Equation (2), x + y = 2

$$\begin{array}{l} 2x + y = 1 \\ x + y = 2 \\ (-)\ (-)\ (-) \\ \hline x + 0 = -1 \\ \hline \end{array}$$

⇒ x = –1

Put x = –1 in equation (2), we get

2 = x + y

⇒ 2 = (–1) + y

⇒ y = 2 + 1

⇒ y = 3

Substituting x = –1 and y = 3 in equation (3), we get

3 = 3x + y

Or 3x + y = 3

⇒ 3(–1) + (3) = 3

⇒ –3 + 3 = 3

⇒ 0 ≠ 3

∵, L.H.S ≠ R.H.S

⇒ The value of x and y doesn't satisfy equation (3).

Thus, $\hat{i} + 2\hat{j} + 3\hat{k}$, $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ are not coplanar.

7 A. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar vectors, prove that the following vectors are non–coplanar :

$2\vec{a} - \vec{b} + 3\vec{c}, \vec{a} + \vec{b} - 2\vec{c}$ and $\vec{a} + \vec{b} - 3\vec{c}$

Answer

Vectors parallel to the same plane, or lie on the same plane are called coplanar vectors

The three vectors are coplanar if one of them is expressible as a linear combination of the other two.

We have been given that, $2\vec{a} - \vec{b} + 3\vec{c}$, $\vec{a} + \vec{b} - 2\vec{c}$ and $\vec{a} + \vec{b} - 3\vec{c}$.

We can form a relation using these three vectors. Say,

$2\vec{a} - \vec{b} + 3\vec{c} = x(\vec{a} + \vec{b} - 2\vec{c}) + y(\vec{a} + \vec{b} - 3\vec{c})$

$\Rightarrow 2\vec{a} - \vec{b} + 3\vec{c} = x\vec{a} + x\vec{b} - 2x\vec{c} + y\vec{a} + y\vec{b} - 3y\vec{c}$

$\Rightarrow 2\vec{a} - \vec{b} + 3\vec{c} = (x + y)\vec{a} + (x + y)\vec{b} + (-2x - 3y)\vec{c}$

Compare the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. We get

2 = x + y ...(1)

–1 = x + y ...(2)

3 = –2x – 3y ...(3)

Solving equations (1) and (2) for x and y.

Equation (1), x + y = 2

Equation (2), x + y = –1

We get

$$\begin{array}{l} x + y = 2 \\ x + y = -1 \\ \underline{(-1)\ (-1)\ (+)} \\ \underline{0 + 0 = 3} \end{array}$$

The value of x and y cannot be found so it won't satisfy equation (3).

Thus, $2\vec{a} - \vec{b} + 3\vec{c}$, $\vec{a} + \vec{b} - 2\vec{c}$ and $\vec{a} + \vec{b} - 3\vec{c}$ are not coplanar.

7 B. Question

If $\vec{a}, \vec{b}, \vec{c}$ are non–coplanar vectors, prove that the following vectors are non-coplanar :

$\vec{a} + 2\vec{b} + 3\vec{c}, 2\vec{a} + \vec{b} + 3\vec{c}$ and $\vec{a} + \vec{b} + \vec{c}$

Answer

We have been given that, $\vec{a} + 2\vec{b} + 3\vec{c}$, $2\vec{a} + \vec{b} + 3\vec{c}$ and $\vec{a} + \vec{b} + \vec{c}$.

We can form a relation using these three vectors. Say,

$$\vec{a} + 2\vec{b} + 3\vec{c} = x\left(2\vec{a} + \vec{b} + 3\vec{c}\right) + y\left(\vec{a} + \vec{b} + \vec{c}\right)$$

$$\Rightarrow \vec{a} + 2\vec{b} + 3\vec{c} = 2x\vec{a} + x\vec{b} + 3x\vec{c} + y\vec{a} + y\vec{b} + y\vec{c}$$

$$\Rightarrow \vec{a} + 2\vec{b} + 3\vec{c} = (2x + y)\vec{a} + (x + y)\vec{b} + (3x + y)\vec{c}$$

Compare the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. We get

1 = 2x + y ...(1)

2 = x + y ...(2)

3 = 3x + y ...(3)

Solving equation (1) and (2) for x and y,

$$\begin{array}{l} 2x + y = 1 \\ x + y = 2 \\ \underline{(-)\ (-)\ (-)} \\ \underline{x + 0 = -1} \end{array}$$

⇒ x = –1

Put x = –1 in equation (2), we get

⇒ 2 = x + y

⇒ 2 = –1 + y

⇒ y = 2 + 1

⇒ y = 3

Substituting x = –1 and y = 3 in equation (3), we get

3 = 3x + y

Or 3x + y = 3

$\Rightarrow 3(-1) + 3 = 3$

$\Rightarrow -3 + 3 = 3$

$\Rightarrow 0 \neq 3$

$\because$, L.H.S ≠ R.H.S

⇒ The value of x and y doesn't satisfy equation (3).

Thus, $\vec{a} + 2\vec{b} + 3\vec{c}$, $2\vec{a} + \vec{b} + 3\vec{c}$ and $\vec{a} + \vec{b} + \vec{c}$ are not coplanar.

8. Question

Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ givenby $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ are non-coplanar. Express vector $\vec{d} = 2\hat{i} - \hat{j} - 3\hat{k}$ as a linear combination of the vectors $\vec{a}, \vec{b}$ and $\vec{c}$.

Answer

Vectors parallel to the same plane, or lie on the same plane are called coplanar vectors

The three vectors are coplanar if one of them is expressible as a linear combination of the other two.

Given that

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} + \hat{j} + \hat{k}$

Let

$\vec{a} = x\vec{b} + y\vec{c}$

$\Rightarrow \hat{i} + 2\hat{j} + 3\hat{k} = x(2\hat{i} + \hat{j} + 3\hat{k}) + y(\hat{i} + \hat{j} + \hat{k})$

$\Rightarrow \hat{i} + 2\hat{j} + 3\hat{k} = 2x\hat{i} + x\hat{j} + 3x\hat{k} + y\hat{i} + y\hat{j} + y\hat{k}$

$\Rightarrow \hat{i} + 2\hat{j} + 3\hat{k} = (2x + y)\hat{i} + (x + y)\hat{j} + (3x + y)\hat{k}$

Comparing the coefficients of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

1 = 2x + y ...(1)

2 = x + y ...(2)

3 = 3x + y ...(3)

Solving equation (1) and (2), we get

$$\begin{array}{l} 2x + y = 1 \\ x + y = 2 \\ \underline{(-)\ (-)\ (-)} \\ \underline{x + 0 = -1} \end{array}$$

$\Rightarrow x = -1$

Substitute x = –1 in equation (2), we get

2 = x + y

$\Rightarrow 2 = -1 + y$

$\Rightarrow y = 2 + 1$

$\Rightarrow y = 3$

Put x = –1 and y = 3 in equation (3), we get

3 = 3x + y

⇒ 3 = 3(–1) + 3

⇒ 3 = –3 + 3

⇒ 3 ≠ 0

∴ L.H.S ≠ R.H.S

⇒ The value of x and y doesn't satisfy equation (3).

Thus, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} + \hat{j} + \hat{k}$ are not coplanar.

Let $\vec{d}$ be depicted as,

$\vec{d} = x\vec{a} + y\vec{b} + z\vec{c}$ ...(*)

Substitute the value of $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$.

$$2\hat{i} - \hat{j} - 3\hat{k} = x(\hat{i} + 2\hat{j} + 3\hat{k}) + y(2\hat{i} + \hat{j} + 3\hat{k}) + z(\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow 2\hat{i} - \hat{j} - 3\hat{k} = x\hat{i} + 2x\hat{j} + 3x\hat{k} + 2y\hat{i} + y\hat{j} + 3y\hat{k} + z\hat{i} + z\hat{j} + z\hat{k}$$

$$\Rightarrow 2\hat{i} - \hat{j} - 3\hat{k} = (x + 2y + z)\hat{i} + (2x + y + z)\hat{j} + (3x + 3y + z)\hat{k}$$

Comparing the coefficients in $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

2 = x + 2y + z ...(1)

–1 = 2x + y + z ...(2)

–3 = 3x + 3y + z ...(3)

From equation (1),

2 = x + 2y + z

⇒ z = 2 – x – 2y ...(4)

Putting the value of z from equation (4) in equations (2) & (3), we get

From equation (2),

–1 = 2x + y + z

⇒ –1 = 2x + y + (2 – x – 2y)

⇒ –1 = 2x + y + 2 – x – 2y

⇒ 2x – x + y – 2y = –1 – 2

⇒ x – y = –3 ...(5)

From equation (3),

–3 = 3x + 3y + z

⇒ –3 = 3x + 3y + (2 – x – 2y)

⇒ –3 = 3x + 3y + 2 – x – 2y

⇒ 3x – x + 3y – 2y = –3 – 2

⇒ 2x + y = –5 ...(6)

Solving equation (5) and (6), we have

$$\begin{array}{l} 2x + y = -5 \\ \underline{x - y = -3} \\ \underline{3x + 0 = -8} \end{array}$$

⇒ 3x = –8

$$\Rightarrow x = -\frac{8}{3}$$

Substituting $x = -\frac{8}{3}$ in equation (5), we get

x – y = –3

$$\Rightarrow \left(-\frac{8}{3}\right) - y = -3$$

$$\Rightarrow \frac{-8 - 3y}{3} = -3$$

⇒ –8 – 3y = –3 × 3

⇒ –8 – 3y = –9

⇒ 3y = 9 – 8

⇒ 3y = 1

$$\Rightarrow y = \frac{1}{3}$$

Now, substitute $x = -\frac{8}{3}$ and $y = \frac{1}{3}$ in z = 2 – x – 2y, we get

$$\Rightarrow z = 2 - \left(-\frac{8}{3}\right) - 2\left(\frac{1}{3}\right)$$

$$\Rightarrow z = 2 + \frac{8}{3} - \frac{2}{3}$$

$$\Rightarrow z = 2 + \frac{8 - 2}{3}$$

$$\Rightarrow z = \frac{6 + 8 - 2}{3}$$

$$\Rightarrow z = \frac{6 + 6}{3}$$

$$\Rightarrow z = \frac{12}{3}$$

⇒ z = 4

We have got $x = -\frac{8}{3}, y = \frac{1}{3}$ and z = 4.

Put these values in equation (*), we get

$$\vec{d} = \left(-\frac{8}{3}\right)\vec{a} + \left(\frac{1}{3}\right)\vec{b} + 4\vec{c}$$

Thus, we have found the relation.

9. Question

Prove that a necessary and sufficient condition for three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ to be coplanar is that there exist scalars l, m, n not all zero simultaneously such that $l\vec{a} + m\vec{b} + n\vec{c} = \vec{0}$.

Answer

Given: The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

To Prove: (a). Necessary condition: The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ will be coplanar if there exist scalar l, m, n not all zero simultaneously such that $l\vec{a} + m\vec{b} + n\vec{c} = 0$.

(b). Sufficient condition: For vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, there exist scalar l, m, n not all zero simultaneously such that $l\vec{a} + m\vec{b} + n\vec{c} = 0$

Proof:

(a). Necessary condition: Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three coplanar vectors.

Then, one of them can be expressed as a linear combination of the other two.

Then, let $\vec{c} = x\vec{a} + y\vec{b}$

Rearranging them we get,

$x\vec{a} + y\vec{b} - \vec{c} = 0$

Here, let

x = l

y = m

–1 = n

We have,

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

Thus, if $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanars, there exists scalar l, m and n (not all zero simultaneously zero) such that $l\vec{a} + m\vec{b} + n\vec{c} = 0$.

∴ necessary condition is proved.

(b). Sufficient condition: Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that there exists scalars l, m and n not all simultaneously zero such that $l\vec{a} + m\vec{b} + n\vec{c} = 0$.

$l\vec{a} + m\vec{b} + n\vec{c} = 0$

$\Rightarrow n\vec{c} = -l\vec{a} - m\vec{b}$

Now, divide by n on both sides, we get

$\Rightarrow \frac{n\vec{c}}{n} = \frac{-l\vec{a} - m\vec{b}}{n}$

$\Rightarrow \vec{c} = \left(-\frac{l}{n}\right)\vec{a} - \left(\frac{m}{n}\right)\vec{b}$

Here, we can see that

$\vec{c}$ is the linear combination of $\vec{a}$ and $\vec{b}$.

⇒ Clearly, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

∴ sufficient condition is also proved.

Hence, proved.

10. Question

Show that the four points A, B, C and D with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively are coplanar if and only if $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = \vec{0}$.

Answer

Given that,

Position vector of A $= \vec{a}$

Position vector of B $= \vec{b}$

Position vector of C $= \vec{c}$

Position vector of D $= \vec{d}$

⇒

Let A, B, C and D be coplanar.

As we know that, the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ will be coplanar if there exist scalar x, y, z, u not all zero simultaneously such that $x\vec{a} + y\vec{b} + z\vec{c} + u\vec{d} = 0$.

Then, we can write

$x\vec{a} + y\vec{b} + z\vec{c} + u\vec{d} = 0$

Where, (x + y + z + u) = 0

Provided x, y, z, u are scalars not all simultaneously zero.

Let x = 3, y = –2, z = 1 and u = –2

So, we get

$\Rightarrow 3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$

Thus, A, B, C and D are coplanar if $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$.

⇐

If $3\vec{a} - 2\vec{b} + \vec{c} - 2\vec{d} = 0$ is true.

Rearranging it, we get

$\Rightarrow 3\vec{a} + \vec{c} = 2\vec{b} + 2\vec{d}$

Dividing this from the sum of its coefficient (that is, 4) on both sides,

$$\frac{3\vec{a} + \vec{c}}{4} = \frac{2\vec{b} + 2\vec{d}}{4}$$

Or $\frac{3\vec{a}+\vec{c}}{3+1} = \frac{2\vec{b}+2\vec{d}}{2+2}$

⇒ There is a point say P, which divides the line AC in ratio 1:3 and BD in ratio 2:2 internally.

Thus, P is the point of interaction of AC and BD.

As, vectors parallel to the same plane, or lie on the same plane are called coplanar vectors.

Hence, A, B, C and D are coplanar.

Exercise 23.9

1. Question

Can a vector have direction angles 45°, 60°, 120°.

Answer

We know that, If l, m, n are the direction cosine of a vector and α, β, γ are the direction angle, then –

l = cosα, m = cosβ, n = cosγ

And, $l^2 + m^2 + n^2 = 1$ ...... (i)

∴ $l = \cos 45°$, $m = \cos 60°$, $n = \cos 120°$

$$l = \frac{1}{\sqrt{2}}, m = \frac{1}{2}, n = -\frac{1}{2}$$

Now, substituting l, m, n in equation (i), we get -

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$$

$$\Rightarrow \frac{2 + 1 + 1}{4} = 1$$

$$\Rightarrow \frac{4}{4} = 1$$

⇒ 1 = 1

⇒ L.H.S = R.H.S

∴ A vector can have direction angles 45^o, 60^o, 120^o.

2. Question

Prove that 1, 1, and 1 cannot be direction cosines of a straight line.

Answer

Here, l = 1, m = 1, n = 1

And, we know that –

$l^2 + m^2 + n^2 = 1$

Taking LHS,

$l^2 + m^2 + n^2 = (1)^2 + (1)^2 + (1)^2$

= 3

≠1

⇒ LHS≠RHS

∴ 1, 1, and 1 cannot be direction cosines of a straight line.

3. Question

A vector makes an angle of π/4 with each of x - axis and y - axis. Find the angle made by it with the z - axis.

Answer

Given, $\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{4}, \gamma = \gamma$

l = cosα

$$= \cos\frac{\pi}{4}$$

$$\Rightarrow l = \frac{1}{\sqrt{2}}$$

m = cosα

$$= \cos\frac{\pi}{4}$$

$\Rightarrow \ m = \frac{1}{\sqrt{2}}$

And, n = cosγ

Also,

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\gamma = 1$

$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1$

$\Rightarrow 1 + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 0$

$\Rightarrow \cos\gamma = 0$

$\Rightarrow \gamma = \cos^{-1}0$

$\Rightarrow \gamma = \frac{\pi}{2}$

The angle made by the vector with the z - axis = $\frac{\pi}{2}$

4. Question

A vector $\vec{r}$ is inclined at equal acute angles to x – axis, y – axis, and z - axis. If $|\vec{r}| = 6$ units, find $\vec{r}$.

Answer

Here, α = β = γ

⇒ cosα = cosβ = cosγ

⇒ l = m = n = p(say)

Now, we know that -

$l^2 + m^2 + n^2 = 1$

$\Rightarrow p^2 + p^2 + p^2 = 1$

$\Rightarrow 3p^2 = 1$

$\Rightarrow p = \pm\frac{1}{\sqrt{3}}$

∴ the direction cosines of $\vec{r}$ are -

$\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

$\Rightarrow \vec{r} = |\vec{r}|(l\,\hat{i} + m\,\hat{j} + n\,\hat{k})$

$= 6(\pm\frac{1}{\sqrt{3}}\hat{i} + \pm\frac{1}{\sqrt{3}}\hat{j} + \pm\frac{1}{\sqrt{3}}\hat{k})$

$= \pm\frac{6}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

Now, multiplying and dividing it by √3

$$\Rightarrow \vec{r} = \pm\frac{6 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

$\Rightarrow \vec{r} = \pm 2\sqrt{3}(\hat{i} + \hat{j} + \hat{k})$

5. Question

A vector $\vec{r}$ is inclined to the x - axis at 45^0 and y - axis at 60^0. If $|\vec{r}| = 8$ units, find $\vec{r}$.

Answer

Here, $\alpha = 45°, \beta = 60°, \gamma = \theta$ (say)

$l = \cos\alpha$

$= \cos 45°$

$$l = \frac{1}{\sqrt{2}}$$

$m = \cos\alpha$

$= \cos 45°$

$$m = \frac{1}{2}$$

$n = \cos\theta$

Now, substituting l, m,n in

$l^2 + m^2 + n^2 = 1$,

$$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4}$$

$$\Rightarrow \cos^2\theta = \frac{1}{4}$$

$$\Rightarrow \cos\theta = \pm\frac{1}{2}$$

$$\Rightarrow n = \pm\frac{1}{2}$$

$\therefore \vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k})$

$$= 8\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{2}\hat{j} + \pm\frac{1}{2}\hat{k}\right)$$

$$= \frac{8}{2}(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$$

$\Rightarrow \vec{r} = 4(\sqrt{2}\hat{i} + \hat{j} \pm \hat{k})$

6. Question

Find the direction cosines of the following vectors :

i. $2\hat{i} + 2\hat{j} - \hat{k}$

ii. $6\hat{i} - 2\hat{j} - 3\hat{k}$

iii. $3\hat{i}-4\hat{k}$

Answer

(i) $2\hat{i}+2\hat{j}-\hat{k}$

Here,

The direction ratios of the vector 2 î + 2 j - k̂ are 2, 2, - 1

The direction cosines of the vector $= \frac{2}{|\vec{r}|}, \frac{2}{|\vec{r}|}, \frac{-1}{|\vec{r}|}$

$$= \frac{2}{\sqrt{(2^2 + 2^2 + (-1)^2}}, \frac{2}{\sqrt{(2^2 + 2^2 + (-1)^2}}, \frac{-1}{\sqrt{(2^2 + 2^2 + (-1)^2}}$$

$$= \frac{2}{\sqrt{9}}, \frac{2}{\sqrt{9}}, -\frac{1}{\sqrt{9}}$$

$$= \frac{2}{3}, \frac{2}{3}, -\frac{1}{3}$$

∴ The direction cosines of $\vec{r}$ are given by $\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}$

(ii) $6\hat{i}-2\hat{j}-3\hat{k}$

Here,

The direction ratios of the vector 6 î - 2 j - 3 k̂ are 6, - 2, - 3

The direction cosines of the vector $= \frac{6}{|\vec{r}|}, \frac{-2}{|\vec{r}|}, \frac{-3}{|\vec{r}|}$

$$= \frac{6}{\sqrt{(6^2 + (-2)^2 + (-3)^2}}, \frac{-2}{\sqrt{(6^2 + (-2)^2 + (-3)^2}}, \frac{-3}{\sqrt{(6^2 + (-2)^2 + (-3)^2}}$$

$$= \frac{6}{\sqrt{49}}, \frac{-2}{\sqrt{49}}, -\frac{3}{\sqrt{49}}$$

$$= \frac{6}{7}, \frac{-2}{7}, -\frac{3}{7}$$

∴ The direction cosines of $\vec{r}$ are given by $\frac{6}{7}, -\frac{2}{7}, -\frac{3}{7}$

(iii) $3\hat{i}-4\hat{k}$

Here,

The direction ratios of the vector 3 î - 4k̂ are 3, 0, - 4

The direction cosines of the vector $= \frac{3}{|\vec{r}|}, \frac{0}{|\vec{r}|}, \frac{-4}{|\vec{r}|}$

$$= \frac{3}{\sqrt{(3^2 + 0^2 + (-4)^2}}, \frac{0}{\sqrt{(3^2 + 0^2 + (-4)^2}}, \frac{-4}{\sqrt{(3^2 + 0^2 + (-4)^2}}$$

$$= \frac{3}{\sqrt{25}}, 0, -\frac{4}{\sqrt{25}}$$

$$= \frac{3}{5}, 0, -\frac{4}{5}$$

∴ The direction cosines of $\vec{r}$ are given by $\frac{3}{5}, 0, -\frac{4}{5}$

7. Question

Find the angles at which the following vectors are inclined to each of the coordinate axes :

i. $\hat{i}-\hat{j}+\hat{k}$

ii. $\hat{j}-\hat{k}$

iii. $4\hat{i}+8\hat{j}+\hat{k}$

Answer

(i) $\hat{i}-\hat{j}+\hat{k}$

Let, $\vec{r}$ = î - j + k̂

The direction ratios of the vector $\vec{r}$ = 1, - 1, 1

And, $|\vec{r}\,|$ = √ ((1)^2 + (- 1)^2 + (1)^2)

= √3

The direction cosines of the vector $\vec{r} = \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$$= \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

So,

$$l = \cos\alpha = \frac{1}{\sqrt{3}}$$

$$\alpha = \cos^{-1}\frac{1}{\sqrt{3}}$$

$$m = \cos\beta = -\frac{1}{\sqrt{3}}$$

$$\beta = \cos^{-1}(-\frac{1}{\sqrt{3}})$$

$$n = \cos\gamma = \frac{1}{\sqrt{3}}$$

$$\gamma = \cos^{-1}\frac{1}{\sqrt{3}}$$

Thus, angles made by with the coordinate axes are given by $\cos^{-1}\frac{1}{\sqrt{3}}, \cos^{-1}(-\frac{1}{\sqrt{3}}), \cos^{-1}\frac{1}{\sqrt{3}}$

(ii) $\hat{j}-\hat{k}$

Let, $\vec{r}$ = 0 î + j - k̂

The direction ratios of the vector $\vec{r}$ = 0, 1, - 1

And, $|\vec{r}\,|$ = √ ((0)^2 + (1)^2 + (- 1)^2)

= √2

The direction cosines of the vector $\vec{r} = \frac{0}{|\vec{r}|}, \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}$

$$= \frac{0}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

So,

$l = \cos\alpha = 0$

$\alpha = \cos^{-1}0$

$\alpha = \frac{\pi}{2}$

$m = \cos\beta = \frac{1}{\sqrt{2}}$

$\beta = \cos^{-1}(\frac{1}{\sqrt{2}})$

$\beta = \frac{\pi}{4}$

$n = \cos\gamma = -\frac{1}{\sqrt{2}}$

$\gamma = \cos^{-1}(-\frac{1}{\sqrt{2}})$

$\gamma = \pi - \frac{\pi}{4}$

$\gamma = \frac{3\pi}{4}$

Thus, angles made by with the coordinate axes are given by $\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}$

(iii) $4\hat{i} + 8\hat{j} + \hat{k}$

Let, $\vec{r} = 4\hat{i} + 8\ j + \hat{k}$

The direction ratios of the vector $\vec{r}$ = 4, 8, 1

And, $|\vec{r}\,| = \sqrt{((4)^2 + (8)^2 + (1)^2)}$

$= \sqrt{81}$

$= 9$

The direction cosines of the vector $\vec{r} = \frac{4}{|\vec{r}|}, \frac{8}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$= \frac{4}{9}, \frac{-81}{9}, \frac{1}{9}$

So,

$l = \cos\alpha = \frac{4}{9}$

$\alpha = \cos^{-1}\frac{4}{9}$

$m = \cos\beta = \frac{8}{9}$

$\beta = \cos^{-1}\frac{8}{9}$

$n = \cos\gamma = \frac{1}{9}$

$\gamma = \cos^{-1}\frac{1}{9}$

Thus, angles made by with the coordinate axes are given by $\cos^{-1}\frac{4}{9}, \cos^{-1}\frac{8}{9}, \cos^{-1}\frac{1}{9}$

8. Question

Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined with the axes OX, OY and OZ.

Answer

Let $\vec{r} = \hat{\imath} + j + \hat{k}$

And, $|\vec{r}\,| = \sqrt{((1)^2 + (1)^2 + (1)^2)}$

$= \sqrt{3}$

Therefore, The direction cosines of the vector $\vec{r} = \frac{1}{|\vec{r}|}, \frac{-1}{|\vec{r}|}, \frac{1}{|\vec{r}|}$

$$= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

Now, let α, β and γ be the angles formed by $\vec{r}$ with the positive directions of x, y and z axes.

Then,

We have,

$$\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}$$

Hence, the given vector is equally inclined to axes OX, OY and OZ.

9. Question

Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Answer

Let a vector be equally inclined to axes OX, OY, and OZ at angle α.

Then, the direction cosines of the vector are $l = \cos\alpha$, $m = \cos\alpha$ and $n = \cos\alpha$

And, we know that –

$l^2 + m^2 + n^2 = 1$

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

10. Question

If a unit vector $\vec{a}$ makes an angle $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find θ and hence, the components of $\vec{a}$.

Answer

Let unit vector $\vec{r}$ have (r_1, r_2, r_3) components.

$\Rightarrow \vec{r} = r_1\hat{\imath} + r_2\, j + r_3\,\hat{k}$

Since, $\vec{r}$ is a unit vector.

$\Rightarrow |\vec{r}\,| = 1$

Also, given that $\vec{r}$ makes angles $\frac{\pi}{3}$ with $\hat{\imath}$, $\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle θ with $\hat{k}$

Then, we have:

$$\cos\frac{\pi}{3} = \frac{r_1}{|\vec{r}|}$$

$\Rightarrow \frac{1}{2} = r_1$ ($\because |\vec{r}\,| = 1$)

And, $\cos\frac{\pi}{4} = \frac{r_2}{|\vec{r}|}$

$\Rightarrow \frac{1}{\sqrt{2}} = r_2$ ($\because |\vec{r}\,| = 1$)

Also, $\cos\theta = \frac{r_3}{|\vec{r}|}$

$\Rightarrow \cos\theta = r_3$ ($\because |\vec{r}\,| = 1$)

Now, $|\vec{r}\,| = 1$

$\Rightarrow \sqrt{(r_1^2 + r_2^2 + r_3^2)} = 1$

$$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \frac{3}{4} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = 1 - \frac{3}{4}$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

$$\therefore r_3 = \cos\frac{\pi}{3}$$

$$= \frac{1}{2}$$

$\therefore \theta = \frac{\pi}{3}$ and the components of $\vec{r}$ are - $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

11. Question

Find a vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes an angle of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with y and z - axes respectively.

Answer

Let l, m, n be the direction cosines of the vector $\vec{r}$

l = cosα,

m = cosβ

$$= \cos\frac{\pi}{4}$$

$= \frac{1}{\sqrt{2}}$,

And, n = cosγ

$= \cos\frac{\pi}{2}$

= 0

Also, $l^2 + m^2 + n^2 = 1$

$\Rightarrow\ l^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + 0 = 1$

$\Rightarrow\ l = \pm\frac{1}{\sqrt{2}}$

$\therefore \vec{r} = |\vec{r}|(l\,\hat{i} + m\,\hat{j} + n\,\hat{k})$

$= 3\sqrt{2}\left(\pm\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + 0\,\hat{k}\right)$

(Given, $|\vec{r}| = 3\sqrt{2}$)

$\Rightarrow \vec{r} = \pm 3\,\hat{i} + 3\,\hat{j}$

12. Question

A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$, find $\vec{r}$.

Answer

Let l, m, n be the direction cosines of the vector $\vec{r}$

Vector $\vec{r}$ is inclined at equal angles to the three axes.

l = cosα, m = cos α, n = cosα

⇒ l = m = n.

Also, we know that -

$l^2 + m^2 + n^2 = 1$

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

$\Rightarrow\ \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Hence, the direction cosines of the vector which are equally inclined to the axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

$\therefore \vec{r} = |\vec{r}|(l\,\hat{i} + m\,\hat{j} + n\,\hat{k})$

$= 2\sqrt{3}\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm 1/\sqrt{3}\,\hat{k}\right)$

(Given, $|\vec{r}| = 2\sqrt{3}$)

$\Rightarrow \vec{r} = \pm(2\,\hat{i} + 2\hat{j} + 2\,\hat{k})$

Very short answer

1. Question

Define “zero vector”.

Answer

Zero vector is a vector which has magnitude is 0. It is denoted by $\vec{0}$.

2. Question

Define unit vector.

Answer

A unit vector is a vector whose magnitude is 1. It is denoted by capping the vector whose unit vector is required. For instance, the unit vector of $\vec{a}$ will be $\hat{a}$.

3. Question

Define position vector of a point.

Answer

A position vector is a vector which tells the relative position of any point in space with respect to origin. This vector starts from origin and its head lies on the point itself. If The x, y, z coordinates of the point is x_1, y_1, z_1, the position vector will be equal to $x_1\hat{\imath} + y_1\hat{\imath} + z_1\hat{\imath}$.

4. Question

Write $\overrightarrow{PQ} + \overrightarrow{RP} + \overrightarrow{QR}$ in the simplified form.

Answer

Any vector $\overrightarrow{PQ}$, if the position vectors of point P ($\vec{p}$) and Q ($\vec{q}$) are known, can be written as $\vec{q} - \vec{p}$.

Let the position vectors of points P, Q, R be $\vec{p}, \vec{q}, \vec{r}$.

Then $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = (\vec{q} - \vec{p}) + (\vec{r} - \vec{q}) + (\vec{p} - \vec{r}) = \vec{0}$

5. Question

If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors such that $x\vec{a} + y\vec{b} = \vec{0}$, then write the values of x and y.

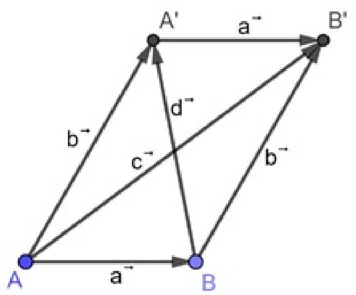
Answer

Since $\vec{a}$ and $\vec{b}$ are non-colinear vectors, the only way the equality $x\vec{a} + y\vec{b} = \vec{0}$ will hold is if x=y=0.

6. Question

If $\vec{a}$ and $\vec{b}$ represent two adjacent sides of a parallelogram, then write vectors representing its diagonals.

Answer



In the above figure, $\overrightarrow{AB} = \overrightarrow{A'B'} = \vec{a}$, $\overrightarrow{AA'} = \overrightarrow{BB'} = \vec{b}$, $\overrightarrow{AB'} = \vec{c}$ and

$\overrightarrow{BA'} = \vec{d}$

Using parallelogram law of vector addition, we can say that

$\vec{a} + \vec{b} = \vec{c}$ and $\vec{a} + \vec{d} = \vec{b}$ or $\vec{d} = \vec{b} - \vec{a}$

Also, $-\vec{c}\ and\ -\vec{d}$ are the diagonals of the parallelogram.

Hence the diagonal vectors of a parallelogram formed by vectors $\vec{a}$ and $\vec{b}$ will be $\pm(\vec{a} + \vec{b})$ and $\pm(\vec{b} - \vec{a})$.

7. Question

If $\vec{a}, \vec{b}, \vec{c}$ represent the sides of a triangle taken in order, then write the value of $\vec{a} + \vec{b} + \vec{c}$.

Answer

Let ΔABC be the required triangle with $\overrightarrow{AB} = \vec{c}, \overrightarrow{BC} = \vec{a}$ and $\overrightarrow{CA} = \vec{b}$.

Any vector $\overrightarrow{AB}$, if the position vectors of point A ($\vec{a}$) and B ($\vec{b}$) are known, can be written as $\vec{b} - \vec{a}$.

Let the position vectors of points A, B, C be $\vec{A}, \vec{B}, \vec{C}$.

Then $\vec{a} + \vec{b} + \vec{c} = \overrightarrow{BC} + \overrightarrow{CA} + \overrightarrow{AB} = (\vec{C} - \vec{B}) + (\vec{A} - \vec{C}) + (\vec{B} - \vec{A}) = \vec{0}$

8. Question

If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices A, B and C respectively, of a triangle ABC, write the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$.

Answer

Any vector $\overrightarrow{AB}$, if the position vectors of point A ($\vec{a}$) and B ($\vec{b}$) are known, can be written as $\vec{b} - \vec{a}$.

Since the position vectors of points A, B, C are $\vec{a}, \vec{b}, \vec{c}$, we get

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = (\vec{b} - \vec{a}) + (\vec{c} - \vec{b}) + (\vec{a} - \vec{c}) = \vec{0}$

19. Question

If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the points A, B and C respectively, write the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AC}$.

Answer

Any vector $\overrightarrow{AB}$, if the position vectors of point A ($\vec{a}$) and B ($\vec{b}$) are known, can be written as $\vec{b} - \vec{a}$.

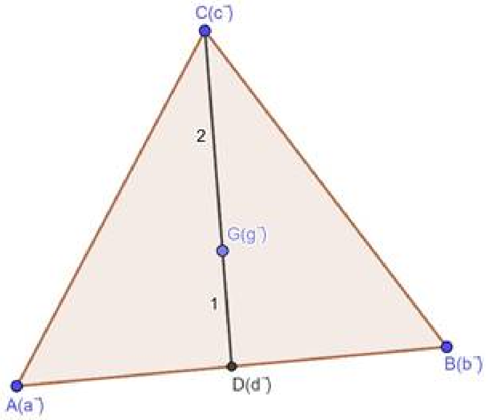
Since the position vectors of points A, B, C are $\vec{a}, \vec{b}, \vec{c}$, we get

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = (\vec{b} - \vec{a}) + (\vec{c} - \vec{b}) + (\vec{a} - \vec{c}) = \vec{0}$

10. Question

If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of a triangle, then write the position vector of its centroid.

Answer



In the figure, D is the mid-point of AB, so it divides AB in 1:1 ratio. CD is a median of ΔABC. G is the centroid of the triangle and by the property of triangle, G divides CD in 2:1 ratio.

The position vector of point D can be calculated using the section formula for vector, which states that the position vector of a point($\vec{c}$) dividing two position vectors($\vec{a}$ *and* $\vec{b}$) in ration m:n, internally is

$$\vec{c} = \frac{m\vec{b} + n\vec{a}}{m + n}$$

So, $\vec{d} = \frac{\vec{b}+\vec{a}}{1+1} = \frac{\vec{a}+\vec{b}}{2}$

Similarly, using section formula for G between points C and D, we get

$$\vec{g} = \frac{2\vec{d} + \vec{c}}{2 + 1} = \frac{2 \times \frac{\vec{a} + \vec{b}}{2} + \vec{c}}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

11. Question

If G denotes the centroid of Δ ABC, then write the value of $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$.

Answer

Let the position vector points A, B, C be $\vec{a}, \vec{b}$ and $\vec{c}$. Then the position vector of G will be $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$.

Any vector $\overrightarrow{AB}$, if the position vectors of point A ($\vec{a}$) and B ($\vec{b}$) are known, can be written as $\vec{b} - \vec{a}$.

Then, $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \left(\vec{a} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) + \left(\vec{b} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right) + \left(\vec{c} - \frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$

$= \vec{a} + \vec{b} + \vec{c} - \left(\vec{a} + \vec{b} + \vec{c}\right) = \vec{0}$

12. Question

If $\vec{a}$ and $\vec{b}$ denote the position vectors of points A and B respectively and C is a point on AB such that 3AC = 2AB, then write the position vector of C.

Answer

Given that the position vector points A, B are $\vec{a}, \vec{b}$.

Let us assume that C lies between A and B.

Then AB=AC+BC

Given that 3AC=2AB=2(AC+BC)

⇒ AC=2BC

Therefore AC: BC=1:2

Also, since the ratio is positive, our assumption was correct.

Using section formula, $\vec{c}=\frac{1\times\vec{b}+2\times\vec{a}}{1+2}=\frac{\vec{b}+2\vec{a}}{3}$

13. Question

If D is the mid-point of side BC of a triangle ABC such that $\overline{AB}+\overline{AC}=\lambda\,\overline{AD},$ write the value of λ.

Answer

Let the position vectors of A, B and C be $\vec{a},\vec{b}$ and $\vec{c}$

Then the position vector of $\vec{d}$ will be $\frac{\vec{b}+\vec{c}}{2}$

Any vector $\overrightarrow{AB}$, if the position vectors of point A ($\vec{a}$) and B ($\vec{b}$) are known, can be written as $\vec{b}-\vec{a}$.

Then, $\overrightarrow{AB}+\overrightarrow{AC}=\lambda\overrightarrow{AD}$

$\Rightarrow(\vec{b}-\vec{a})+(\vec{c}-\vec{a})=\lambda(\vec{d}-\vec{a})$

Substituting value of $\vec{d}$, we get

$$\vec{b}+\vec{c}-2\vec{a}=\lambda\left(\frac{\vec{b}+\vec{c}}{2}-\vec{a}\right)=\lambda\left(\frac{\vec{b}+\vec{c}-2\vec{a}}{2}\right)$$

$\Rightarrow\frac{\lambda}{2}=1$ or λ=2

14. Question

If D, E, F are the mid-points of the sides BC, CA and AB respectively of a triangle ABC, write the value of $\overline{AD}+\overline{BE}+\overline{CF}.$

Answer

Let the position vectors of A, B and C be $\vec{a},\vec{b}$ and $\vec{c}$

Then the position vector of $\vec{d},\vec{e}$ and $\vec{f}$ will be $\frac{\vec{b}+\vec{c}}{2}$, $\frac{\vec{a}+\vec{c}}{2}$ and $\frac{\vec{b}+\vec{a}}{2}$ respectively.

Any vector $\overrightarrow{AB}$, if the position vectors of point A ($\vec{a}$) and B ($\vec{b}$) are known, can be written as $\vec{b}-\vec{a}$.

Then, $\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}=(\vec{d}-\vec{a})+(\vec{e}-\vec{b})+(\vec{f}-\vec{c})$

$$=\left(\frac{\vec{b}+\vec{c}}{2}-\vec{a}\right)+\left(\frac{\vec{a}+\vec{c}}{2}-\vec{b}\right)+\left(\frac{\vec{b}+\vec{a}}{2}-\vec{c}\right)$$

$$=(\vec{a}+\vec{b}+\vec{c})-(\vec{a}+\vec{b}+\vec{c})=\vec{0}$$

15. Question

If $\vec{a}$ is non-zero vector of modulus a and m is a non-zero scalar such that m $\vec{a}$ is a unit vector, write the value of m.

Answer

The modulus of $\vec{a}$ is a, therefore $\vec{a}$ can be written as modulus×unit-direction=$a\times\hat{a}$

Given that m$\vec{a}$ has the magnitude of 1, therefore $ma\times\hat{a}$ has magnitude of 1 or ma=1.Hence $m=\frac{1}{a}$

16. Question

If $\vec{a},\vec{b},\vec{c}$ are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then write the value of $\vec{a}+\vec{b}+\vec{c}$.

Answer

Since in an equilateral triangle, orthocenter and centroid coincide, therefore the position vector of centroid is $\vec{0}$.

Also, the position vector of centroid G ($\vec{g}$) can be defined as $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

Therefore, $\frac{\vec{a}+\vec{b}+\vec{c}}{3}=\vec{0}$ hence $\vec{a}+\vec{b}+\vec{c}=\vec{0}$

17. Question

Write a unit vector making equal acute angles with a coordinates axes.

Answer

Let the angle made be α. We know that the sum of squares of direction cosines of a vector is 1. SO, we get

$\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$

$cos^2\alpha=\frac{1}{3}\Rightarrow cos\alpha=\pm\frac{1}{\sqrt{3}}$

Since, α is acute therefore $cos\alpha=\frac{1}{\sqrt{3}}$

Any vector, if it's magnitude and direction cosines are given can be written as $m(cos\alpha\hat{\imath}+cos\beta\hat{\jmath}+cos\gamma\hat{k})$

So the required vector is $1(\frac{1}{\sqrt{3}}\hat{\imath}+\frac{1}{\sqrt{3}}\hat{\jmath}+\frac{1}{\sqrt{3}}\hat{k})$

Rationalizing, we get $\frac{\sqrt{3}}{3}(\hat{\imath}+\hat{\jmath}+\hat{k})$

18. Question

If a vector makes angles α, β, γ with OX, OY and OZ respectively, then write the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.

Answer

The sum of squares of direction cosines of a vector is 1.

Let the angles made by vector be α, β, γ. Then, we get

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

using $\cos^2\theta=1-\sin^2\theta$, we get

$(1-\sin^2\alpha)+(1-\sin^2\beta)+(1-\sin^2\gamma)=1$

Or, $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=2$

19. Question

Write a vector of magnitude 12 units which makes 45° angle with X-axis, 60° angle with y-axis and an obtuse angle with Z-axis.

Answer

Let the angles made by vector be α, β, γ and the magnitude be m.

Given that α=45°, β=60° and m= 12. We have to figure out the vector.

Since $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, we get

$\cos^2 45°+\cos^2 60°+\cos^2\gamma=1$

$= \frac{1}{2} + \frac{1}{4} + cos^2\gamma = 1 \Rightarrow cos^2\gamma = \frac{1}{4}\ or\ cos\gamma = \pm\frac{1}{2}$

Since γ is obtuse, $cos\gamma = -\frac{1}{2}$.

Any vector, if it's magnitude and direction cosines are given can be written as $m(cos\alpha\hat{\imath} + cos\beta\hat{\jmath} + cos\gamma\hat{k})$

So the required vector is $12(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{2}\hat{\jmath} - \frac{1}{2}\hat{k})$

Rationalizing, we get $6\sqrt{2}\hat{\imath} + 6\hat{\jmath} - 6\hat{k}$

20. Question

Write the length (magnitude) of a vector whose projections on the coordinate axes are 12, 3 and 4 units.

Answer

Since $L_x=12$, $L_y=3$ and $L_z=4$ are given, we can find out L by

$L^2= L_x^2+L_y^2+L_z^2$

$=12^2+3^2+4^2$

$=144+9+16=169$

Hence L=13 units.

21. Question

Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1 : 4, where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + \hat{k}$.

Answer

If a point R($\vec{r}$) divides the vector joining point P($\vec{p}$) and Q($\vec{q}$) externally in the ratio m:n, then

$$\vec{r} = \frac{m\vec{p} - n\vec{q}}{m - n}$$

Here, $\vec{p} = \vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{q} = \vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$, m=1 and n=4

Then $\vec{r} = \frac{1(2\hat{\imath}+3\hat{\jmath}+4\hat{k})-4(-\hat{\imath}+\hat{\jmath}+\hat{k})}{1-4} = \frac{6\hat{\imath}-\hat{\jmath}}{-3} = \frac{1}{3}(-6\hat{\imath} + \hat{\jmath})$

22. Question

Write the direction cosines of the vector $\vec{r} = 6\hat{i} - 2\hat{j} + 3\hat{k}$.

Answer

The direction cosines of a vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ are

$$cos\alpha = \frac{a_1}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}, cos\beta = \frac{a_2}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}\ and$$

$$cos\gamma = \frac{a_3}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$$

In this question, $a_1=6$, $a_2=-2$ and $a_3=3$, Substituting in formulas we get

$$cos\alpha = \frac{6}{\sqrt{49}} = \frac{6}{7}, cos\beta = \frac{-2}{\sqrt{49}} = \frac{-2}{7}\ and\ cos\gamma = \frac{3}{\sqrt{49}} = \frac{3}{7}$$

23. Question

If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \hat{k} + \hat{i}$, write unit vectors parallel to $\vec{a} + \vec{b} - 2\vec{c}$.

Answer

Given that $\vec{a} = \hat{\imath} + \hat{\jmath}, \vec{b} = \hat{\jmath} + \hat{k}$ *and* $\vec{c} = \hat{\imath} + \hat{k}$, we get

$\vec{a} + \vec{b} - 2\vec{c} = \hat{\imath} + \hat{\jmath} + \hat{\jmath} + \hat{k} - 2 \times (\hat{\imath} + \hat{k}) = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

Let $\vec{d} = \vec{a} + \vec{b} - 2\vec{c}$

Then, $\hat{d} = \frac{-\hat{\imath} + 2\hat{\jmath} - \hat{k}}{\sqrt{(-1)^2 + 2^2 + (-1)^2}} = \frac{\sqrt{6}}{6} \times (-\hat{\imath} + 2\hat{\jmath} - \hat{k})$

Both $\hat{d}$ *and* $-\hat{d}$ will be parallel to $\vec{d}$, therefore the answer is $\pm\frac{\sqrt{6}}{6} \times (2\hat{\jmath} - \hat{\imath} - \hat{k})$

24. Question

If $\vec{a} = \hat{i} + 2\hat{j}, \vec{b} = \hat{j} + 2\hat{k}$, write a unit vector along the vector $3\vec{a} - 2\vec{b}$.

Answer

Given that $\vec{a} = \hat{\imath} + 2\hat{\jmath}, \vec{b} = \hat{\jmath} + 2\hat{k}$, we get

$3\vec{a} - 2\vec{b} = 3 \times (\hat{\imath} + 2\hat{\jmath}) - 2 \times (\hat{\jmath} + 2\hat{k}) = 3\hat{\imath} + 4\hat{\jmath} - 4\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

Let $\vec{d} = 3\vec{a} - 2\vec{b}$

Then, $\hat{d} = \frac{3\hat{\imath} + 4\hat{\jmath} - 4\hat{k}}{\sqrt{3^2 + 4^2 + (-4)^2}} = \frac{\sqrt{41}}{41} \times (3\hat{\imath} + 4\hat{\jmath} - 4\hat{k})$

The unit vector in direction of $\vec{d}$ is $\hat{d}$

25. Question

Write the position vector of a point dividing the line segment joining points having position vectors $\hat{i} + \hat{j} - 2\hat{k}$ and $2\hat{i} - \hat{j} + 3\hat{k}$ externally in the ratio 2 : 3.

Answer

. If a point R($\vec{r}$) divides the vector joining point P($\vec{p}$) and Q($\vec{q}$) externally in the ratio m:n, then

$$\vec{r} = \frac{m\vec{p} - n\vec{q}}{m - n}$$

Here, $\vec{p} = \hat{\imath} + \hat{\jmath} - 2\hat{k}$, $\vec{q} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, m=2 and n=3

Then $\vec{r} = \frac{2(\hat{\imath} + \hat{\jmath} - 2\hat{k}) - 3(2\hat{\imath} - \hat{\jmath} + 3\hat{k})}{2 - 3} = \frac{-4\hat{\imath} + 5\hat{\jmath} - 13\hat{k}}{-1} = 4\hat{\imath} - 5\hat{\jmath} + 13\hat{k}$

26. Question

If $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{j} + \hat{k}, \vec{c} = \hat{k} + \hat{i}$, fine the unit vector in the direction of $\vec{a} + \vec{b} + \vec{c}$.

Answer

Given that $\vec{a} = \hat{\imath} + \hat{\jmath}, \vec{b} = \hat{\jmath} + \hat{k}$ *and* $\vec{c} = \hat{\imath} + \hat{k}$, we get

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} + \hat{\jmath} + \hat{\jmath} + \hat{k} + \hat{\imath} + \hat{k} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2+a_2{}^2+a_3{}^2}}$

Let $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

Then, $\hat{d} = \frac{2\hat{\imath}+2\hat{\jmath}+2\hat{k}}{\sqrt{2^2+2^2+2^2}} = \frac{\sqrt{3}}{3} \times (\hat{\imath} + \hat{\jmath} + \hat{k})$

27. Question

If $\vec{a} = 3\hat{i} - \hat{j} - 4\hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, find $|3\vec{a} - 2\vec{b} + 4\vec{c}|$.

Answer

Given that $\vec{a} = 3\hat{\imath} - \hat{\jmath} - 4\hat{k}, \vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}\ and\ \vec{c} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$, we get

$$3\vec{a} - \vec{b} + 4\vec{c} = 3 \times (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) - (2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}) + 4 \times (\hat{\imath} + 2\hat{\jmath} - \hat{k})$$
$$= 11\hat{\imath} + \hat{\jmath} - 13\hat{k}$$

The magnitude of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $|\vec{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$

$$|3\vec{a} - \vec{b} + 4\vec{c}| = \sqrt{11^2 + 1^2 + (-13)^2} = \sqrt{291}$$

28. Question

A unit vector $\vec{r}$ makes angles $\frac{\pi}{3}$ and $\frac{\pi}{2}$ with $\hat{j}$ and $\hat{k}$ respectively and a acute angle θ with $\hat{i}$. Find θ.

Answer

The sum of squares of direction cosines of a vector is 1.

Let the angles made by vector be α, β, θ. Then, we get

$\cos^2\alpha+\cos^2\beta+\cos^2\theta=1$

Given that $\alpha = \frac{\pi}{3}\ and\ \beta = \frac{\pi}{2}$, we have to calculate θ

$$cos^2\frac{\pi}{3} + cos^2\frac{\pi}{2} + cos^2\theta = 1$$

$\Rightarrow cos^2\theta = \frac{3}{4} = cos^2\frac{\pi}{6}$ (Since θ is acute)

Hence, $\theta = \frac{\pi}{6}$

29. Question

Write a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

Answer

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2+a_2{}^2+a_3{}^2}}$

Given that $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$

We get $\hat{a} = \frac{3\hat{\imath}-2\hat{\jmath}+6\hat{k}}{\sqrt{3^2+(-2)^2+6^2}} = \frac{1}{7}(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$

30. Question

If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 9\hat{k}$, find a unit vector parallel to $\vec{a} + \vec{b}$.

Answer

Given that $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} + 9\hat{k}$, we get

$\vec{a} + \vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k} + 2\hat{\imath} + 4\hat{\jmath} + 9\hat{k} = 3\hat{\imath} + 6\hat{\jmath} + 6\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

Let $\vec{d} = \vec{a} + \vec{b}$

Then, $\hat{d} = \frac{3\hat{\imath} + 6\hat{\jmath} + 6\hat{k}}{\sqrt{3^2 + 6^2 + 6^2}} = \frac{1}{3} \times (\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

Both $\hat{d}$ and $-\hat{d}$ will be parallel to $\vec{d}$, therefore the answer is $\pm\frac{1}{3} \times (\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

31. Question

Write a unit vector in the direction of $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$.

Answer

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

Given that $\vec{b} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

We get $\hat{b} = \frac{2\hat{\imath} + \hat{\jmath} + 2\hat{k}}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{1}{3}(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

32. Question

Find the position vector of the mid-point of the line segment AB, where A is the point (3, 4, –2) and B is the point (1, 2, 4).

Answer

If the co-ordinates of a point A≡ (x_1, y_1, z_1), then the position vector of A($\vec{a}$) is

$\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$

Given that A≡(3,4,2) and B≡(1,2,4), we get position vector of A($\vec{a}$) and B($\vec{b}$). Let the midpoint be C($\vec{c}$)

$\vec{a} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

The position vector of midpoint of two vectors is defined by $\frac{\vec{a} + \vec{b}}{2}$

$$\vec{c} = \frac{3\hat{\imath} + 4\hat{\jmath} + 2\hat{k} + \hat{\imath} + 2\hat{\jmath} + 4\hat{k}}{2} = 2\hat{\imath} + 3\hat{\jmath} + 3\hat{k}$$

33. Question

Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude of 6 units.

Answer

Given that $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

$$\hat{a} = \frac{2\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{1}{3}(2\hat{\imath} - \hat{\jmath} + 2\hat{k})$$

Let the required vector be $\vec{d}$.

Any vector ($\vec{p}$) with magnitude m and unit vector $\hat{a}$ can be written as $\vec{p} = m \times \hat{a}$.

Since the magnitude of $\vec{d}$ is 6 and it's unit vector is $\hat{a}$, we get

$$\vec{d} = 6 \times \frac{1}{3}\left(2\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) = 4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$$

34. Question

What is the cosine of the angle which the vector $\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}$ makes with y-axis?

Answer

The angle that a vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ makes with y-axis is

$$cos\beta = \frac{a_2}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}$$

In this question, $a_1=\sqrt{2}$, $a_2=1$ and $a_3=1$, Substituting in formulas we get

$$cos\beta = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

35. Question

Write two different vectors having same magnitude.

Answer

$\hat{\imath}$ and $\hat{\jmath}$ both have magnitude 1 but different directions. $\hat{\imath}$ is along x-axis and $\hat{\jmath}$ is along y-axis.

36. Question

Write two different vectors having same direction.

Answer

$\hat{\imath}$ and $2\hat{\imath}$ both have the same direction but different magnitudes, 1 and 2.

37. Question

Write a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude of 8 unit.

Answer

Given that $\vec{a} = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{{a_1}^2+{a_2}^2+{a_3}^2}}$

$$\hat{a} = \frac{5\hat{\imath} - \hat{\jmath} + 2\hat{k}}{\sqrt{5^2 + (-1)^2 + 2^2}} = \frac{\sqrt{30}}{30}(5\hat{\imath} - \hat{\jmath} + 2\hat{k})$$

Let the required vector be $\vec{d}$.

Any vector ($\vec{p}$) with magnitude m and unit vector $\hat{a}$ can be written as $\vec{p} = m \times \hat{a}$.

Since the magnitude of $\vec{d}$ is 8 and it's unit vector is $\hat{a}$, we get

$$\vec{d} = 8 \times \frac{\sqrt{30}}{30}\left(5\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) = \frac{4\sqrt{30}}{15}\left(5\hat{\imath} - \hat{\jmath} + 2\hat{k}\right)$$

38. Question

Write the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.

Answer

The direction cosines of a vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ are

$$cos\alpha = \frac{a_1}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}, cos\beta = \frac{a_2}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}\ and$$

$$cos\gamma = \frac{a_3}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}$$

In this question, a_1=1, a_2=2 and a_3=3, Substituting in formulas we get

$$cos\alpha = \frac{1}{\sqrt{14}} = \frac{\sqrt{14}}{14}, cos\beta = \frac{2}{\sqrt{14}} = \frac{2\sqrt{14}}{14}\ and\ cos\gamma = \frac{3}{\sqrt{14}} = \frac{3\sqrt{14}}{14}$$

39. Question

Find a unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

Answer

Given that $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}$

$$\hat{a} = \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{1}{7}(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$$

40. Question

For what value of 'a' the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

Answer

Two vectors $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ are collinear , if and only if, $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$.

Here $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = a\hat{\imath} + 6\hat{\jmath} - 8\hat{k}$

Hence, $\frac{2}{a} = \frac{-3}{6} = \frac{4}{-8}$

Solving this equality, we get

a=-4

41. Question

Write the direction cosines of the vectors $-2\hat{i} + \hat{j} - 5\hat{k}$.

Answer

The direction cosines of a vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ are

$$cos\alpha = \frac{a_1}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}, cos\beta = \frac{a_2}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}\ and$$

$$cos\gamma = \frac{a_3}{\sqrt{{a_1}^2 + {a_2}^2 + {a_3}^2}}$$

In this question, a_1=-2, a_2=1 and a_3=-5, Substituting in formulas we get

$$cos\alpha = \frac{-2}{\sqrt{30}} = \frac{-\sqrt{30}}{15}, cos\ \beta = \frac{1}{\sqrt{30}} = \frac{\sqrt{30}}{30}\ and\ cos\gamma = \frac{-5}{\sqrt{30}} = \frac{-\sqrt{30}}{6}$$

42. Question

Find the sum of the following vectors: $\vec{a} = \hat{i} - 2\hat{j}, \vec{b} = 2\hat{i} - 3\hat{j}, \vec{c} = 2\hat{i} + 3\hat{k}$.

Answer

$\vec{a} = \hat{\imath} - 2\hat{\jmath}, \vec{b} = 2\hat{\imath} - 3\hat{\jmath}$ *and* $\vec{c} = 2\hat{\imath} + 3\hat{k}$

Then $\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 2\hat{\jmath} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{\imath} + 3\hat{k} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

43. Question

Find a unit vector in the direction of the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

Answer

Given that $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

$$\hat{a} = \frac{3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}}{\sqrt{3^2 + (-2)^2 + 6^2}} = \frac{1}{7}(3\hat{\imath} - 2\hat{\jmath} + 6\hat{k})$$

44. Question

If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$. are two equal vectors, then write the value of x + y + z.

Answer

Two vectors $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ are equal , if and only if, $a_1=b_1$, $a_2=b_2$, $a_3=b_3$

Here $\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$ and $\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$

Hence, we get x=3, -y=2⇒y=-2 and -z=1⇒ z=-1

45. Question

Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$.

Answer

$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k}$ and $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 7\hat{k}$

$\vec{a} + \vec{b} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k} + 2\hat{\imath} + \hat{\jmath} - 7\hat{k} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$

Let $\vec{d} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}}$

Hence, $\hat{d} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{\sqrt{4^2 + 3^2 + (-12)^2}} = \frac{1}{13}(4\hat{\imath} + 3\hat{\jmath} - 12\hat{k})$

46. Question

Find the value of ‘p’ for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

Answer

Two vectors $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$ are parallel , if and only if, $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3}$.

Here $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$

Hence, $\frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

Solving this equality, we get

$p = -\frac{1}{3}$

47. Question

Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of π/4 with x-axis π/2 with y-axis and an acute angle θ with z-axis.

Answer

Let the angles made by vector be α, β, θ and the magnitude be m.

Given that $\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{2}$ and $m = 5\sqrt{2}$. We have to figure out the vector.

Since $\cos^2\alpha+\cos^2\beta+\cos^2\theta=1$, we get

$cos^2\frac{\pi}{4} + cos^2\frac{\pi}{2} + cos^2\theta = 1$

$\Rightarrow \frac{1}{2} + cos^2\theta = 1 \Rightarrow cos^2\theta = \frac{1}{2}$ or $cos\theta = \pm\frac{1}{\sqrt{2}}$

Since θ is acute, $cos\theta = \frac{1}{\sqrt{2}}$.

Any vector, if it's magnitude and direction cosines are given can be written as $m(cos\alpha\hat{\imath} + cos\beta\hat{\jmath} + cos\theta\hat{k})$

So, the required vector is $5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{k}\right) = 5\hat{\imath} + 5\hat{k}$

48. Question

Write a unit vector in the direction of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 0) and (4, 5, 6) respectively.

Answer

If the co-ordinates of points A≡ (x_1, y_1, z_1) and B≡ (x_2, y_2, z_2), then the vector $\overrightarrow{AB}$ is

$\overrightarrow{AB} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$

Given that P≡(1,3,0) and Q≡(4,5,6), we get

$\overrightarrow{PQ} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2+a_2{}^2+a_3{}^2}}$

Hence, $\widehat{PQ} = \frac{3\hat{\imath}+2\hat{\jmath}+6\hat{k}}{\sqrt{3^2+2^2+6^2}} = \frac{1}{7}(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$

49. Question

Find a vector in the direction of vector $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ which has magnitude 21 units.

Answer

Given that $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

The unit vector of any vector $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ can be written as $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\vec{a}}{\sqrt{a_1{}^2+a_2{}^2+a_3{}^2}}$

$$\hat{a} = \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{1}{7}(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$$

Let the required vector be $\vec{d}$.

Any vector ($\vec{p}$) with magnitude m and unit vector $\hat{a}$ can be written as $\vec{p} = m \times \hat{a}$.

Since the magnitude of $\vec{d}$ is 21 and it's unit vector is $\hat{a}$, we get

$$\vec{d} = 21 \times \frac{1}{7}(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}) = 6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$$

50. Question

If $|\vec{a}| = 4$ and –3 ≤ λ ≤ 2, then write the range of $|\lambda\vec{a}|$.

Answer

Given that $|\vec{a}| = 4$ and -3≤λ≤2

We have to figure out range of $|\lambda\vec{a}|$

In calculating the modulus of a vector multiplied by a scalar quantity, the sign of the scalar quantity does not matter, only it's absolute value does.

Hence the minimum value of $|\lambda\vec{a}| = 0$ when λ=0 and maximum value of $|\lambda\vec{a}| = 12$ when λ=-3.

51. Question

In a triangle OAC, if B is the mid-point of side AC and $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$, then what is $\overrightarrow{OC}$?

Answer

Given that the position vectors of point A and B are $\vec{a}$ and $\vec{b}$. Let the position vector of point C be $\vec{c}$.

The position vector of B will be defined as

$$\vec{b} = \frac{\vec{a} + \vec{c}}{2}$$

$$\Rightarrow \vec{c} = 2\vec{b} - \vec{a}$$

52. Question

Write the position vector of the point which divides the join of points with position vectors $3\vec{a} - 2\vec{b}$ and $2\vec{a} + 3\vec{b}$ in the ratio 2:1.

Answer

If a point R($\vec{r}$) divides the vector joining point P($\vec{p}$) and Q($\vec{q}$) externally in the ratio m:n, then

$$\vec{r} = \frac{m\vec{p} - n\vec{q}}{m - n}$$

Here, $\vec{p} = 3\vec{a} - 2\vec{b}$, $\vec{q} = 2\vec{a} + 3\vec{b}$, m=2 and n=1

We get, $\vec{r} = \frac{2(3\vec{a}-2\vec{b})-(2\vec{a}+3\vec{b})}{2-1} = 4\vec{a} - 7\vec{b}$

MCQ

1. Question

Mark the correct alternative in each of the following:

If in a Δ ABC, A ≡ (0, 0), B ≡ (3, 3, √3), C ≡ (–3, √3, 3), then the vector of magnitude $2\sqrt{2}$ units directed along AO, where O is the circumcentre of Δ ABC is

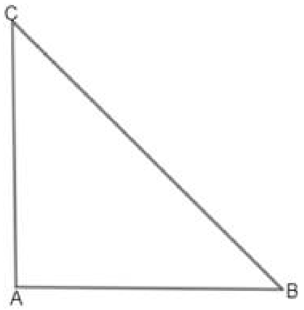
A. $(1-\sqrt{3})\hat{i} + (1+\sqrt{3})\hat{j}$

B. $(1+\sqrt{3})\hat{i} + (1-\sqrt{3})\hat{j}$

C. $\left(1+\sqrt{3}\right)\hat{\imath}+\left(\sqrt{3}-1\right)\hat{\jmath}$

D. none of these

Answer



Slope of a line joining two points $= \frac{y_2-y_1}{x_2-x_1}$

Slope of AC $= \frac{0-3}{0+3\sqrt{3}}$

$= \frac{-3}{3\sqrt{3}}$

$= -\frac{1}{\sqrt{3}}$

Slope of AB $= \frac{0-3\sqrt{3}}{0-3}$

$= \frac{-3\sqrt{3}}{-3}$

$= \sqrt{3}$

Product of Slopes (AC $\times$ AB) = $\left(= -\frac{1}{\sqrt{3}} \times \sqrt{3}\right)$

= -1

As the Product of Slopes (AC $\times$ AB) = -1, so AC ▯ AB, ie.., $\angle$ CAB = 90°.

Circumcentre (O) of Triangle ABC = Mid-Point of BC

Mid-Point of BC $= \frac{3-\sqrt{3}}{2}, \frac{3+3\sqrt{3}}{2}$

$$\overrightarrow{OA} = \left(\frac{3-\sqrt{3}}{2} - 0\right)\hat{\imath} + \left(\frac{3+3\sqrt{3}}{2}\right)\hat{\jmath}\overrightarrow{OA} = \left(\frac{3-\sqrt{3}}{2}\right)\hat{\imath} + \left(\frac{3+3\sqrt{3}}{2}\right)\hat{\jmath}$$

Now, $\left|\overrightarrow{OA}\right| = \sqrt{\left(\frac{3-\sqrt{3}}{2}\right)^2 + \left(\frac{3+3\sqrt{3}}{2}\right)^2}$

$$\left|\overrightarrow{OA}\right| = \frac{\sqrt{(9+27-18\sqrt{3}+9+27+18\sqrt{3})}}{\sqrt{4}}$$

$\left|\overrightarrow{OA}\right| = \sqrt{18}$

$\left|\overrightarrow{OA}\right| = 3\sqrt{2}$

Unit Vector $\overrightarrow{OA} = \frac{\overrightarrow{OA}}{\left|\overrightarrow{OA}\right|}$

$$= \frac{1}{3\sqrt{2}}\left(\left(\frac{3-\sqrt{3}}{2}\right)\hat{\imath} + \left(\frac{3+3\sqrt{3}}{2}\right)\hat{\jmath}\right)$$

Vector along $\overrightarrow{OA}$, whose magnitude is $2\sqrt{2} \times \left\{\frac{1}{3\sqrt{2}}\left(\left(\frac{3-\sqrt{3}}{2}\right)\hat{\imath} + \left(\frac{3+3\sqrt{3}}{2}\right)\hat{\jmath}\right)\right\}$

$$= \frac{2}{3} \times \frac{3}{2}\left((1-\sqrt{3})\hat{\imath} + (1+\sqrt{3})\hat{\jmath}\right)$$

$$= \left((1-\sqrt{3})\hat{\imath} + (1+\sqrt{3})\hat{\jmath}\right)$$

Option (A) is the answer.

2. Question

Mark the correct alternative in each of the following:

If $\bar{a}, \bar{b}$ are the vectors forming consecutive sides of a regular hexagon ABCDEF, then the vector representing side CD is

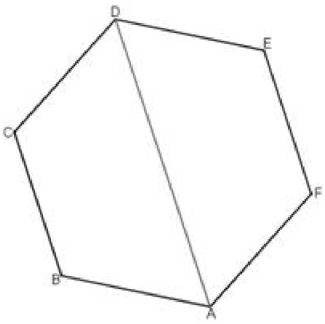
A. $\bar{a} + \bar{b}$

B. $\bar{a} - \bar{b}$

C. $\bar{b} - \bar{a}$

D. $-(\bar{a} + \bar{b})$

Answer



$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ --------- (i)

As, AD = 2BC {Properties of a regular hexagon, also AD || BC (Parallel)}

$\overrightarrow{AD} = 2\overrightarrow{BC}$Putting $\overrightarrow{AD} = 2\overrightarrow{BC}$ in equation (i),

$2\overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

$\overrightarrow{BC} = \overrightarrow{AB} + \overrightarrow{CD}$

$\overrightarrow{CD} = \overrightarrow{BC} - \overrightarrow{AB}$

$\overrightarrow{CD} = \vec{b} - \vec{a}$

$\therefore \overrightarrow{CD} = \vec{b} - \vec{a}$

Option(C)is the answer.

3. Question

Mark the correct alternative in each of the following:

Forces $3\bar{OA}, 5\bar{OB}$ act along OA and OB. If their resultant passes through C on AB, then

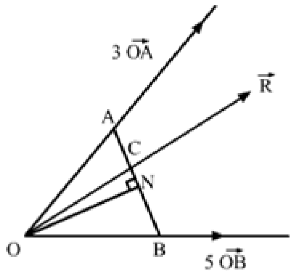
A. C is a mid-point of AB

B. C divides AB in the ratio 2 : 1

C. 3 AC = 5 CB

D. 2AC = 3 CB

Answer



Draw ON perpendicular to the line AB.

Let $\hat{\imath}$ be the unit vector along ON,

The resultant force $\vec{R} = 3\overrightarrow{OA} + 5\overrightarrow{OB}$ --------- (i)

The angles between $\hat{\imath}$ and the forces $\vec{R}$, $3\overrightarrow{OA}$, $5\overrightarrow{OB}$ are $\angle$CON, $\angle$AON & $\angle$BON respectively,

$\vec{R}.\vec{\imath} = 3\overrightarrow{OA}.\vec{\imath} + 5\overrightarrow{OB}.\vec{\imath}$

$R.1.Cos\angle CON = 3OA.1.Cos\angle AON + 5OB.1.Cos\angle BON$

$R.\frac{ON}{OC} = 3OA \times \frac{ON}{OA} + \frac{5OBON}{OB}$

$\frac{R}{OC} = (3+5)$

$\frac{R}{OC} = 8$

R = $8\overrightarrow{OC}$

$\because \overrightarrow{OA} = \overrightarrow{OC} + \overrightarrow{CA}$ ---------- (i)

Multiplying the equation (i) by 3,

$3\overrightarrow{OA} = 3\overrightarrow{OC} + 3\overrightarrow{CA}$ ----------(ii)

Also, $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$ ----------(iv)

Multiplying the equation (iv) by 5,

$5\overrightarrow{OB} = 5\overrightarrow{OC} + 5\overrightarrow{CB}$ ----------(v)

Adding equation (iv) & (v) respectively,

$3\overrightarrow{OA} + 5\overrightarrow{OB} = 3\overrightarrow{OC} + 3\overrightarrow{CA} + 5\overrightarrow{OC} + 5\overrightarrow{CB}$

$\vec{R} = 8\overrightarrow{OC} + 3\overrightarrow{CA} + 5\overrightarrow{CB}$

$8\overrightarrow{OC} = 8\overrightarrow{OC} + 3\overrightarrow{CA} + 5\overrightarrow{CB}$

$|3\overrightarrow{CA}| = |5\overrightarrow{CB}|$

$\therefore$ 3AC = 5CB

Option(C)is the answer.

4. Question

Mark the correct alternative in each of the following:

If $\bar{a}, \bar{b}, \bar{c}$ are three non-zero vectors, no two of which are collinear and the vector $\bar{a}+\bar{b}$ is collinear with $\bar{c}$, $\bar{b}+\bar{c}$ is collinear with $\bar{a}$, then $\bar{a}+\bar{b}+\bar{c}$

A. $\bar{a}$

B. $\bar{b}$

C. $\bar{c}$

D. none of these

Answer

As $\vec{a}+\vec{b}$ is collinear with $\vec{c}$,

$\therefore \vec{a}+\vec{b}=\lambda\vec{c}$ -------- (i)

As $\vec{b}+\vec{c}$ is collinear with $\vec{a}$,

$\therefore \vec{b}+\vec{c}=\mu\vec{a}$ -------- (ii)

Adding $\vec{c}$ both sides of the equation (i),

$\vec{a}+\vec{b}+\vec{c}=\lambda\vec{c}+\vec{c}$

$\vec{a}+\vec{b}+\vec{c}=\vec{c}(\lambda+1)$-------- (iii)

Adding $\vec{a}$ both sides of the equation (iii),

$\vec{a}+\vec{b}+\vec{c}=\mu\vec{a}+\vec{a}$

$\vec{a}+\vec{b}+\vec{c}=\vec{a}(\mu+1)$ --------- (iv)

Equating the RHS of equation (iii)& (iv), being their LHS equal,

$\vec{c}(\lambda+1)=\vec{a}(\mu+1)$

As, a is not collinear with c,

$\therefore \lambda+1=\mu+1=0$

$\vec{a}+\vec{b}+\vec{c}=0$ {From equation (iv)}

$\therefore \vec{a}+\vec{b}+\vec{c}=0$

Option (D)is the answer.

5. Question

Mark the correct alternative in each of the following:

If $A(60\hat{i}+3\hat{j})$, $B(40\hat{i}-8\hat{j})$ and $C(a\hat{i}-52\hat{j})$ points are collinear, then a is equal to

A. 40

B. –40

C. 20

D. –20

Answer

A $(60\hat{\imath}+3\hat{\jmath})$, B $(40\hat{\imath}-8\hat{\jmath})$ & C $(a\hat{\imath}-52\hat{\jmath})$ are collinear, Then $\overrightarrow{AB} = \lambda\overrightarrow{BC}$

$\overrightarrow{AB} = (40\hat{\imath} - 8\hat{\jmath}) - (60\hat{\imath} + 3\hat{\jmath})$

$\overrightarrow{AB} = 40\hat{\imath} - 8\hat{\jmath} - 60\hat{\imath} - 3\hat{\jmath}$

$\overrightarrow{AB} = -20\hat{\imath} - 11\hat{\jmath}$

$\overrightarrow{BC} = (a\hat{\imath} - 52\hat{\jmath}) - (40\hat{\imath} - 8\hat{\jmath})$

$\overrightarrow{BC} = a\hat{\imath} - 52\hat{\jmath} - 40\hat{\imath} + 8\hat{\jmath}$

$\overrightarrow{BC} = (a - 40)\hat{\imath} - 44\hat{\jmath}$

$\therefore\ \overrightarrow{AB} = \lambda\overrightarrow{BC}$

$-20\hat{\imath} - 11\hat{\jmath} = \lambda\{(a - 40)\hat{\imath} - 44\hat{\jmath}\}$

$-20\hat{\imath} - 11\hat{\jmath} = \lambda(a - 40)\hat{\imath} - \lambda 44\hat{\jmath}$

Comparing the LHS & RHS of the above mentioned equation,

$\therefore -20 = \lambda(a - 40)\ \& - 44\lambda = -11$

$\lambda = \frac{-11}{-44}$

$\lambda = \frac{1}{4}$

$-20 = \frac{1}{4}(a - 40)$

-80 = a - 40

a = 40 - 80

a = -40

$\therefore$ a = -40

Option (B) is the answer.

6. Question

Mark the correct alternative in each of the following:

If G is the intersection of diagonals of a parallelogram ABCD and O is any point, then $\overline{OA} + \overline{OB} + \overline{OC} + \overline{OD} =$

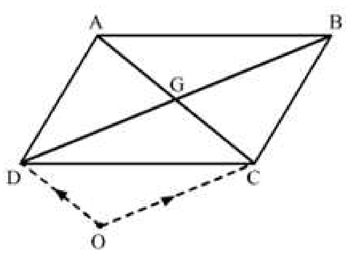
A. $2\,\overline{OG}$

B. $4\,\overline{OG}$

C. $5\,\overline{OG}$

D. $3\,\overline{OG}$

Answer



Let us consider the point O as origin.

G is the mid – point of AC.

$$\therefore \overrightarrow{OG} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$$

$2\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OC}$ ------- (i)

Also, G is the mid– point of BD,

$$\therefore \overrightarrow{OG} = \frac{\overrightarrow{OB} + \overrightarrow{OD}}{2}$$

$2\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{OD}$ ------- (ii)

Adding eq. (i) & eq. (ii),

$2\overrightarrow{OG} + 2\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OC} + \overrightarrow{OB} + \overrightarrow{OD}$

$4\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{OC} + \overrightarrow{OB} + \overrightarrow{OD}$

$\therefore \overrightarrow{OA} + \overrightarrow{OC} + \overrightarrow{OB} + \overrightarrow{OD} = 4\overrightarrow{OG}$

Option (B) is the answer

7. Question

Mark the correct alternative in each of the following:

The vector $\cos\alpha \cos\beta \hat{i} + \cos\alpha \sin B \hat{j} + \sin\alpha \hat{k}$ is a

A. null vector

B. unit vector

C. constant vector

D. none of these

Answer

$Cos\alpha Cos\beta \hat{i} + Cos\alpha Sin\beta \hat{j} + Sin\alpha \hat{k}$

$|Cos\alpha Cos\beta \hat{i} + Cos\alpha Sin\beta \hat{j} + Sin\alpha \hat{k}|$

$= \sqrt{Cos^2\alpha Cos^2\beta + Cos\alpha^2 Sin^2\beta + Sin^2\alpha}$

$= \sqrt{Cos^2\alpha(Cos^2\beta + Sin^2\beta) + Sin^2\alpha}$

$= \sqrt{Cos^2\alpha(1) + Sin^2\alpha}$

$= \sqrt{Cos^2\alpha + Sin^2\alpha}$

$= \sqrt{1}$

=1

Hence, the given vector is a unit vector.

Option (B) is the answer

8. Question

Mark the correct alternative in each of the following:

In a regular hexagon ABCDEF, $\bar{AB} = a, \bar{BC} = \bar{b}$ and $\bar{CD} = \bar{c}$. Then, $\bar{AE} =$

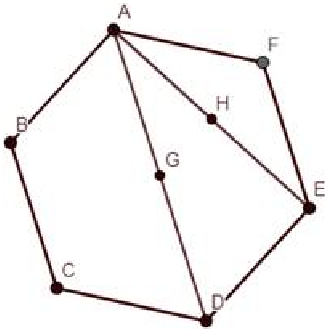
A. $\bar{a}+\bar{b}+\bar{c}$

B. $2\bar{a}+\bar{b}+\bar{c}$

C. $\bar{b}+\bar{c}$

D. $\bar{a}+2\bar{b}+2\bar{c}$

Answer



$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$

$\overrightarrow{AD} = \vec{a} + \vec{b} + \vec{c}$ ------- (i)

In the triangle ADE,

$\overrightarrow{AD} + \overrightarrow{DE} = \overrightarrow{AE}$

$\vec{a} + \vec{b} + \vec{c} - \vec{a} = \overrightarrow{AE}$ $\{\because \overrightarrow{DE} = -\overrightarrow{AB}\}$

$\therefore \overrightarrow{AE} = \vec{b} + \vec{c}$

Option (C) is the answer

9. Question

Mark the correct alternative in each of the following:

The vector equation of the plane passing through $\bar{a}, \bar{b}, \bar{c}$ is $\bar{r} = \alpha\bar{a} + \beta\bar{b} + \gamma\bar{c}$, provided that

A. $\alpha + \beta + \gamma = 0$

B. $\alpha + \beta + \gamma = 1$

C. $\alpha + \beta = \gamma$

D. $\alpha^2 + \beta^2 + \gamma^2 = 1$

Answer

As a plane passing through $\vec{a}, \vec{b}, \vec{c},$

Lines $\vec{a} - \vec{b}$ and $\vec{c} - \vec{a}$ lie on the plane.

The parametric equation of the plane can be expressed as,

$\vec{r} = \vec{a} + \lambda_1(\vec{a} - \vec{b}) + \lambda_2(\vec{c} - \vec{a})$

$\vec{r} = \vec{a}(1 + \lambda_1 - \lambda_2) - \lambda_1\vec{b} + \lambda_2\vec{c}$

As, $\vec{r} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$

$\therefore\ \alpha + \beta + \gamma$

$= 1 + \lambda_1 - \lambda_2 - \lambda_1 + \lambda_2$

$\therefore \alpha + \beta + \gamma = 1$

Option (B) is the answer.

10. Question

Mark the correct alternative in each of the following:

If O and O’ are circumcentre and orthocentre of Δ ABC, then $O\bar{A} + O\bar{B} + O\bar{C}$ equals

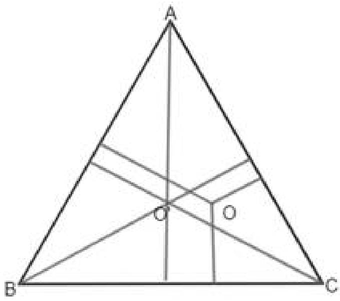
A. $2\bar{O}O'$

B. $O\bar{O}'$

C. $\bar{O}'O$

D. $2\bar{O}'O$

Answer



Let the vertices of the triangle ABC be A ($\vec{a}$), B ($\vec{b}$) & C ($\vec{c}$), with respect to the origin.

O (x,y) is the circumcentre & O’ (0,0) is the orthocenter.

$$\therefore G = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

As the centroid ‘G’ divides the orthocentre ‘C’ (x,y) and circumcentre (0,0) in the ratio 2 : 1.

By using Section Formula,

$$\left(\frac{\vec{a}+\vec{b}+\vec{c}}{3}\right)$$

$$= \frac{2(0)+OO'}{3}$$

$$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = 3\overrightarrow{OG}$$

$$\because 3\overrightarrow{OG} = O\vec{O}$$

$$\therefore \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OO'}$$

Option (B)is the answer.

11. Question

Mark the correct alternative in each of the following:

If $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ are the position vectors of points A, B, C, D such that no three of them are collinear and $\bar{a}+\bar{b}+\bar{c}=\bar{b}+\bar{d}$, then ABCD is a

A. rhombus

B. rectangle

C. square

D. parallelogram

Answer

$\vec{a} + \vec{c} = \vec{b} + \vec{d}$

$\vec{c} - \vec{d} = \vec{b} - \vec{a}$

$\overrightarrow{AB} = \overrightarrow{DC}$

And, $\vec{a} + \vec{c} = \vec{b} + \vec{d}$

$\vec{c} - \vec{b} = \vec{d} - \vec{a}$

$\overrightarrow{AD} = \overrightarrow{BC}$

$\vec{a} + \vec{c} = \vec{b} + \vec{d}$ (Given)

Multiplying the aqbove mentioned equation by $\frac{1}{2}$,

$$\frac{1}{2}(\vec{a} + \vec{c}) = \frac{1}{2}(\vec{b} + \vec{d})$$

So, the position vector of mid – point of BD = Position Vector of mid - point of AC.

Hence the diagonals bisect each other.

Therefore the given figure ABCD is a parallelogram.

Option (D) is the answer

12. Question

Mark the correct alternative in each of the following:

Let G be the centroid of Δ ABC. If $\overline{AB} + \bar{a}$, $\overline{AC} = \bar{b}$, then the bisector $\overline{AG}$, in terms of $\bar{a}$ and $\bar{b}$ is

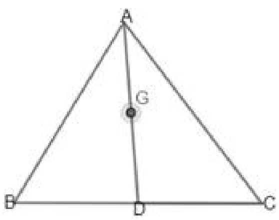
A. $\frac{2}{3}(\bar{a} + \bar{b})$

B. $\frac{1}{6}(\bar{a} + \bar{b})$

C. $\frac{1}{3}(\bar{a} + \bar{b})$

D. $\frac{1}{2}(\bar{a} + \bar{b})$

Answer



Let A be the origin, then $\overrightarrow{AB} = \vec{a}, \overrightarrow{AC} = \vec{b}$, implies that the position vectors of B and C are $\vec{b}$ & $\vec{c}$ respectively.

Let AD be the median and G be the centroid.

Then,

Position Vector of $D = \frac{\vec{a}+\vec{b}}{2}$

Position Vector of $G = \frac{\vec{a}+\vec{b}}{3}$

$\therefore \overrightarrow{AG} = \frac{2}{3}(\vec{a} + \vec{b})$

$\therefore \overrightarrow{AG} = \frac{2}{3}(\vec{a} + \vec{b})$

Option (A) is the answer.

13. Question

Mark the correct alternative in each of the following:

If ABCDEF is a regular hexagon, then $\overline{AD} + \overline{EB} + \overline{FC}$ equals.

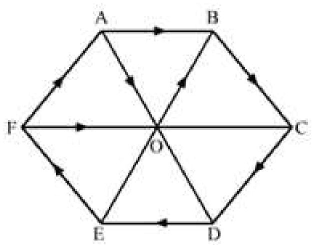
A. $2\bar{A}B$

B. $\bar{0}$

C. $3\bar{A}B$

D. $4\bar{A}B$

Answer



$\because \overrightarrow{AD} = 2\overrightarrow{BC}$

$\overrightarrow{EB} = 2\overrightarrow{FA}$

$\overrightarrow{FC} = 2\overrightarrow{AB}$

$\overrightarrow{AD} + \overrightarrow{EB} = 2\overrightarrow{BC} + 2\overrightarrow{FA}$

$\overrightarrow{AD} + \overrightarrow{EB} = 2(\overrightarrow{BC} + \overrightarrow{FA})$

$\overrightarrow{AD} + \overrightarrow{EB} = 2(\overrightarrow{AO} + \overrightarrow{FA})$

As, $\overrightarrow{BC} = \overrightarrow{AO}$

In triangle AOF,

$\overrightarrow{FA} + \overrightarrow{AO} + \overrightarrow{FO} = 0$

$\therefore \overrightarrow{FA} + \overrightarrow{AO} = -\overrightarrow{FO}$

$\therefore \overrightarrow{AD} + \overrightarrow{EB} = -2\overrightarrow{FO}$

And, $\overrightarrow{AB} = -\overrightarrow{FO}$

$\therefore \overrightarrow{AD} + \overrightarrow{EB} = 2\overrightarrow{AB}$

$\therefore \overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} = 2\overrightarrow{AB} + 2\overrightarrow{AB}$

$\therefore \overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} = 4\overrightarrow{AB}$

$= 4\overrightarrow{AB}$

Option (D) is the answer

14. Question

Mark the correct alternative in each of the following:

The position vectors of the points A, B, C are $2\hat{i}+\hat{j}-\hat{k}$, $3\hat{i}-2\hat{j}+\hat{k}$ and $\hat{i}+4\hat{j}-3\hat{k}$ respectively. These points

A. form an isosceles triangle

B. form a right triangle

C. are collinear

D. form a scalene triangle

Answer

$2\hat{\imath}+\hat{\jmath}-\hat{k}, 3\hat{\imath}-2\hat{\jmath}+\hat{k}$ & $\hat{\imath}+4\hat{\jmath}-3\hat{k}$

$\overrightarrow{AB}=\left(3\hat{\imath}-2\hat{\jmath}+\hat{k}\right)-\left(2\hat{\imath}+\hat{\jmath}-\hat{k}\right)$

$\overrightarrow{AB}=3\hat{\imath}-2\hat{\jmath}+\hat{k}-2\hat{\imath}-\hat{\jmath}+\hat{k}$

$\overrightarrow{AB}=\hat{\imath}-3\hat{\jmath}+2\hat{k}$

$\overrightarrow{BC}=\left(\hat{\imath}+4\hat{\jmath}-3\hat{k}\right)-\left(3\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$

$\overrightarrow{BC}=\hat{\imath}+4\hat{\jmath}-3\hat{k}-3\hat{\imath}+2\hat{\jmath}-\hat{k}$

$\overrightarrow{BC}=-2\hat{\imath}+6\hat{\jmath}-4\hat{k}$

$\overrightarrow{CA}=\left(2\hat{\imath}+\hat{\jmath}-\hat{k}\right)-\left(\hat{\imath}+4\hat{\jmath}-3\hat{k}\right)$

$\overrightarrow{CA}=2\hat{\imath}+\hat{\jmath}-\hat{k}-\hat{\imath}-4\hat{\jmath}+3\hat{k}$

$\overrightarrow{CA}=\hat{\imath}-3\hat{\jmath}+2\hat{k}$

$\overrightarrow{AB}=\sqrt{(1)^2+(-3)^2+(2)^2}$

$\overrightarrow{AB}=\sqrt{1+9+4}$

$\overrightarrow{AB}=\sqrt{14}$

$\overrightarrow{CA}=\sqrt{(1)^2+(-3)^2+(2)^2}$

$\overrightarrow{CA}=\sqrt{1+9+4}$

$\overrightarrow{CA}=\sqrt{14}$

$\overrightarrow{BC}=\sqrt{(-2)^2+(6)^2+(-4)^2}$

$\overrightarrow{BC}=\sqrt{4+36+16}$

$\overrightarrow{BC}=\sqrt{56}$

$\therefore \left|\overrightarrow{AB}\right|=\left|\overrightarrow{CA}\right|$

Hence the triangle is isosceles with two sides equal.

Option (A) is the answer

15. Question

Mark the correct alternative in each of the following:

If three points A, B and C have position vectors $\hat{i}+x\hat{j}+3\hat{k}$, $3\hat{i}+4\hat{j}+7\hat{k}$ and $y\hat{i}-2\hat{j}-5\hat{k}$ respectively are collinear, then (x, y) =

A. (2, –3)

B. (–2, 3)

C. (–2, –3)

D. (2, 3)

Answer

$\hat{\imath} + x\hat{\jmath} + 3\hat{k}, 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$ & $y\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$

$\overrightarrow{AB} = 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k} - (\hat{\imath} + x\hat{\jmath} + 3\hat{k})$

$\overrightarrow{AB} = 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k} - \hat{\imath} - x\hat{\jmath} - 3\hat{k}$

$\overrightarrow{AB} = 2\hat{\imath} + (4 - x)\hat{\jmath} + 4\hat{k}$

$\overrightarrow{BC} = y\hat{\imath} - 2\hat{\jmath} - 5\hat{k} - (3\hat{\imath} + 4\hat{\jmath} + 7\hat{k})$

$\overrightarrow{BC} = y\hat{\imath} - 2\hat{\jmath} - 5\hat{k} - 3\hat{\imath} - 4\hat{\jmath} - 7\hat{k}$

$\overrightarrow{BC} = (y - 3)\hat{\imath} - 6\hat{\jmath} - 12\hat{k}$

$\because$ the given vectors are collinear.

$\therefore\ \overrightarrow{AB} = \lambda\overrightarrow{BC}$

$2\hat{\imath} + (4 - x)\hat{\jmath} + 4\hat{k} = \lambda(y - 3)\hat{\imath} - 6\lambda\hat{\jmath} - 12\lambda\hat{k}$

After comparing the equations,

$\lambda(y - 3) = 2, 4 - x = -6\lambda$ & $-12\lambda = 4$

$\lambda = \frac{4}{-12} = -\frac{1}{3}$

$\therefore \left(-\frac{1}{3}\right)(y - 3) = 2$

y – 3 = -6

y = -6 + 3

y = -3

$\therefore 4 - x = -6\left(-\frac{1}{3}\right)$

4 – x = 2

x = 4 – 2

x = 2

(x, y) = (2, -3)

Option (A) is the answer.

16. Question

Mark the correct alternative in each of the following:

ABCD is a parallelogram with AC and BD as diagonals. Then, $\bar{AC} - \bar{BD} =$

A. $4\,\overline{AB}$

B. $3\,\overline{AB}$

C. $2\,\overline{AB}$

D. $\overline{AB}$

Answer

$\because$ ABCD is a parallelogram with diagonals AC and BD.

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$ &

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} \therefore \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

$\therefore \overrightarrow{AC} - \overrightarrow{BD}$

$= \overrightarrow{AB} + \overrightarrow{BC} - (\overrightarrow{AD} - \overrightarrow{AB})$ $\{\because \overrightarrow{AD} = \overrightarrow{BC}\}$

$= \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AD} + \overrightarrow{AB}$

$= 2\overrightarrow{AB}$

Option (C) is the answer.

17. Question

Mark the correct alternative in each of the following:

If OACB is a parallelogram with $\vec{OC} = \bar{a}$ and $\vec{AB} = \bar{b}$, then $\vec{OA} =$

A. $(\bar{a} + \bar{b})$

B. $(\bar{a} - \bar{b})$

C. $\frac{1}{2}(\bar{b} - \bar{a})$

D. $\frac{1}{2}(\bar{a} - \bar{b})$

Answer

$\overrightarrow{OC} = \vec{a}$, $\overrightarrow{AB} = \vec{b}$

$\therefore$ $\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$

$\overrightarrow{OB} = \overrightarrow{OC} - \overrightarrow{BC}$

$\overrightarrow{OB} = \overrightarrow{OC} - \overrightarrow{OA}$ $\{\because \overrightarrow{BC} = \overrightarrow{OA}\}$

$\overrightarrow{OB} = \vec{a} - \overrightarrow{OA}$ --------(i)

$\therefore$ $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$2\overrightarrow{OA} = \vec{a} - \vec{b}$

$$\overrightarrow{OA} = \frac{(\vec{a} - \vec{b})}{2}$$

$$\overrightarrow{OA} = \frac{1}{2}(\vec{a} - \vec{b})$$

Option (D) is the answer.

18. Question

Mark the correct alternative in each of the following:

If $\bar{a}$ and $\bar{b}$ are two collinear vectors, then which of the following are incorrect?

A. $\bar{b} = \lambda \bar{a}$ for some scalar λ

B. $\bar{a} = \pm\bar{b}$

C. the respective components of $\bar{a}$ and $\bar{b}$ are proportional

D. both the vectors $\bar{a}$ and $\bar{b}$ have the same direction but different magnitudes

Answer

If $\vec{a}$ & $\vec{b}$ are collinear vectors, then they are parallel,

Then, $\vec{b} = \lambda\vec{a}$

For some scalar λ

If $\lambda = \pm 1$, then,

$\vec{a} = \pm\vec{b}$

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ & $\vec{b} = b\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\vec{b} = \lambda\vec{a}$

$b\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = \lambda(a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k})$

$b\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k} = \lambda a_1\hat{\imath} + \lambda a_2\hat{\jmath} + \lambda a_3\hat{k}$

$\therefore b_1 = \lambda a_1$

$\therefore b_2 = \lambda a_2$

$\therefore b_3 = \lambda a_3$

$\therefore \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$

Thus, respective components of $\vec{a}$ & $\vec{b}$ are proportional.

However the vectors $\vec{a}$ & $\vec{b}$ can have different directions.

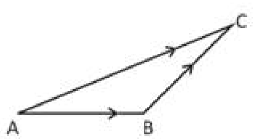
Statement given in D is incorrect.

Option (D) is the answer.

19. Question

Mark the correct alternative in each of the following:

If figure which of the following is not true?



A. $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$

B. $\vec{AB} + \vec{BC} - \vec{AC} = \vec{0}$

C. $\vec{AB} + \vec{BC} - \vec{CA} = \vec{0}$

D. $\vec{AB} - \vec{CB} + \vec{CA} = \vec{0}$

Answer

$\because \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

Subtracting $\overrightarrow{CA}$, from both the sides of the above mentioned equation,

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \overrightarrow{AC} - \overrightarrow{CA}$

Solving RHS,

$= -\overrightarrow{CA} - \overrightarrow{CA}$

$= -2\overrightarrow{CA}$

∵LHS ≠ RHS

Hence, it is not true.

$\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA}$

Option (C) is the answer.