20. Mensuration-I (Area of a Trapezium and a Polygon)

Exercise 20.1

1. Question

A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2?

Answer

Given: Base of parallelogram = 24cm and height of parallelogram = 10cm

Now, we know that area of parallelogram = base × heightTherefore,Area of 1 tile = 24 × 10 = 240cm^2

And, Area of floor 1080m^2

We know that 1m = 100cmSo, 1080m^2 = 1080 × 100 × 100 cm^2

Now,Number of tiles = $\frac{area\ of\ floor}{area\ of\ one\ tile}$

Number of tiles = (1080 × 100 × 100)/(24 × 10) = 45000

Therefore number of tiles = 45000

2. Question

A plot is in the form of a rectangle *ABCD* having semi-circle on *BC* as shown in Fig. 20.23. If *AB* = 60 m and *BC* = 28 m, Find the area of the plot.

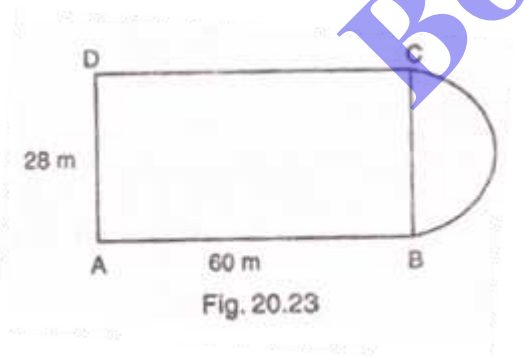
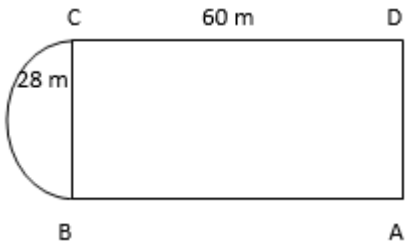


Fig. 20.23

Answer



Area of the plot = area of the rectangle + area of semi-circle

Radius of semi-circle = $\frac{BC}{2} = \frac{28}{2} = 14m$

Area of the rectagular plot = length × breadth = 60 × 28 = 1680 m^2

Area of the semi-circular portion = $\frac{\pi r^2}{2}$

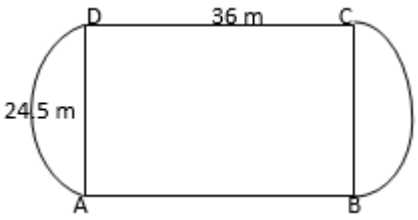
= $\frac{1}{2} \times \frac{22}{7} \times 14 \times 14$ = 308 m^2

Therefore, the total area of the plot = 1680 + 308 = 1988 m^2

3. Question

A playground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36 m and 24.5 m, find the area of the playground. (Take π= 22/7.)

Answer



Area of the plot = area of the rectangle + 2 × area of one semi-circle

Radius of semi-circle = $\frac{BC}{2} = \frac{24.5}{2} = 12.25m$

Area of the plot = length × breadth + $\frac{2 \times \pi r^2}{2}$

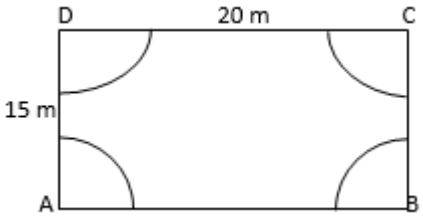
Area of the plot = 36 × 24.5 + $\frac{22}{7} \times 12.25 \times 12.25$

Area of the plot = 882 + 471.625 = 1353.625 m^2

4. Question

A rectangular piece is 20 m long and 15 m wide. From its four corners, quadrants of radii 3.5 m have been cut. Find the area of the remaining part.

Answer



Area of the plot = area of the rectangle - 4 × area of one quadrant

Radius of semi-circle = 3.5 m

Area of four quadrants = area of one circle

Area of the plot = length × breadth + πr^2

Area of the plot = $20 \times 15 - \frac{22}{7} \times 3.5 \times 3.5$

Area of the plot = 300 – 38.5 = 261.5 m^2

5. Question

The inside perimeter of a running track (shown in Fig. 20.24) is 400 m. The length of each of the straight portion is 90 m and the ends are semi-circles. If track is everywhere 14 m wide, find the area of the track. Also, find the length of the outer running track.

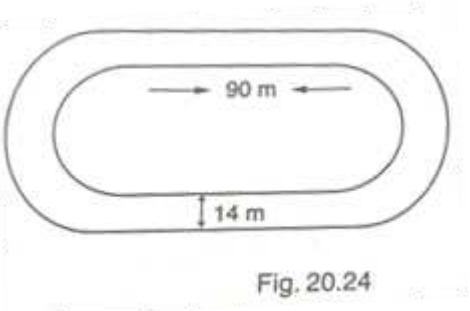


Fig. 20.24

Answer

Perimeter of the inner track = 2× Length of rectangle + perimeter of two semi-circular ends

Perimeter of the inner track = length + length + 2πr

$$400 = 90 + 90 + \frac{2 \times 22 \times r}{7}$$

$$\frac{2 \times 22 \times r}{7} = 400 - 180$$

$$r = 35\ m$$

Hence, the radius of inner circle = 35 m

Radius of outer track = radius of inner track + width of the track

Radius of outer track = 35 + 14 = 49m

Length of outer track = 2× Length of rectangle + perimeter of two outer semi-circular ends

Length of outer track = 2× 90 + 2πr

Length of outer track = $180 + \frac{2 \times 22 \times 49}{7}$

Length of outer track = $488\ m$

Area of inner track = area of inner rectangle + area of two inner semi-circles

Area of inner track = length× breadth + πr^2

Area of inner track = $90\times70+\frac{22\times35\times35}{7}$

Area of inner track = 6300 + 3850

Area of inner track = 10150 m^2

Area of outer track = area of outer rectangle + area of two outer semi-circles

Breadth of outer track = 35 + 35 +14 + 14 = 98 m

Area of outer track = length× breadth + πr^2

Area of outer track = $90\times98+\frac{22\times49\times49}{7}$

Area of outer track = 8820 + 7546

Area of outer track = 16366 m^2

Area of path = area of outer track – area of inner track

Area of path = 16366 – 10150 = 6216 m^2

6. Question

Find the area of Fig. 20.25, in square cm, correct to one place of decimal. (Take π =22/7)

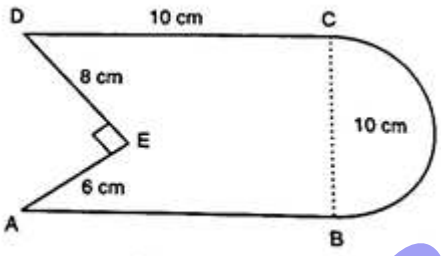


Fig. 20.25

Answer

Area of the figure = area of square + area of semi circle – area of right angled triangle

Area of the figure = side × side + $\frac{1}{2}\times\pi r^2-\frac{1}{2}base\times altitude$

Area of the figure = 10 × 10 + $\frac{1}{2}\times\frac{22}{7}\times5\times5-\frac{1}{2}\times8\times6$

Area of the figure = 100 + 78.57 - 24

Area of the figure = 100 + 78.57 – 24 = 154.57 cm^2

Area of the figure = 154.57 cm^2

7. Question

The diamerer of a wheel of a bus is 90 cm which makes 315 revolutions per minute. Determine its speed in kilometres per hour. (Take π=22/7)

Answer

Diameter of wheel = 90 cm

Perimeter of wheel = 2πd

Perimeter of wheel = $\frac{22}{7} \times 90$

Perimeter of wheel = 282.857 cm

Distance covered in 315 revolutions = 282.857× 315 = 89099.955cm

One km = 100000 cm

Distance covered = $\frac{89099.955}{100000} = 0.89\ km$

Speed in km per hour = 0.89 × 60 = 53.4 km per hour

8. Question

The area of a rhombus is 240 cm^2 and one of the diagonal is 16 cm. Find another diagonal.

Answer

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

240 = $\frac{1}{2} \times 16 \times d_2$

$d_2 = \frac{240}{8} = 30$

Hence, other diameter is 30 cm

9. Question

The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

Answer

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

Area of rhombus = $\frac{1}{2} \times 7.5 \times 12$

Area of rhombus = $6 \times 7.5 = 45.0$

Hence, area of rhombus is 45 cm^2

10. Question

The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.

Answer

Area of quadrilateral = $\frac{1}{2} \times d_1 \times (p_1 + p_2)$

Area of rhombus = $\frac{1}{2} \times 24 \times (8 + 13)$

Area of rhombus = $12 \times 21 = 252$

Hence, area of quadrilateral is 252 cm^2

11. Question

Find the area of a rhombus whose side is 6 cm and whose altitude is 4 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

Answer

Side of rhombus = 6 cm

Altitude of rhombus = 4 cm

Since rhombus is a parallelogram, thereore area of parallelogram = base× altitude

Area of parallelogram = 6 × 4 = 24 cm^2

Area of parallelogram = area of rhombus

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

24 = $\frac{1}{2} \times 8 \times d_2$

$d_2 = \frac{24}{4} = 6$

Hence, length of other diagonal of rhombus is 6 cm

12. Question

The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is Rs. 4.

Answer

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

Area of rhombus = $\frac{1}{2} \times 45 \times 30$

Area of rhombus = 675 cm^2

Area of one tile = 675 cm^2

Area of 3000 tiles = 675× 3000 = 2025000 cm^2

Area of toles in m^2 = $\frac{2025000}{10000} = 202.5\ m^2$

Cost of tiles = 202.5× 4 = Rs 810.0

13. Question

A rectangular grassy plot is 112 m long and 78 m broad. It has gravel path 2.5 m wide all around it on the side. Find the area of the path and the cost of constructing it at Rs. 4.50 per square metre.

Answer

Inner area of rectangle = length× breadth

Inner area of rectangle = 112× 78 = 8736 m^2

Width of path = 2.5 m

Length of outer rectangle = 112 +2.5 + 2.5 = 117 m

Breadth of outer rectangle = 78 +2.5 + 2.5 = 83 m

Outer area of rectangle = length× breadth

= 117 × 83 = 9711 m^2

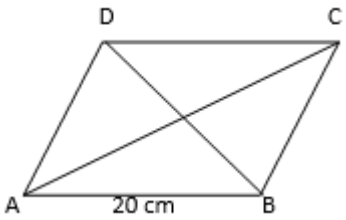
Area of path = outer area of rectangle – inner area of rectangle

Area of path = 9711 – 8736 = 975 m^2Cost of construction for 1 m^2 = 4.50 RsCost of construction for 975 m^2 = 975(4.50) = 4387.5 Rs

14. Question

Find the area of a rhombus, each side of which measures 20 cm and one of whose diagonals is 24 cm.

Answer



Length of side of rhombus = 20 cm

Length of one diagonal = 24 cm

In ΔAOB

Using pythagorous theorem:

$AB^2 = OA^2 + OB^2$

$20^2 - 12^2 = OB^2$

$OB^2 = 400 – 144$

$OB^2 = 256$

OB = 16

Hence, length of other diameter = 16 × 2 = 32 cm

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

Area of rhombus = $\frac{1}{2} \times 24 \times 32$

Area of rhombus = 384 cm^2

15. Question

The length of a side of a square field is 4 m. What will be the altitude of the rhombus, if the area of the rhombus is equal to the square field and one of its diagonal is 2 m?

Answer

Length of square = 4 m

Area of square = $side^2$

Area of square = 4 × 4 = 16 m^2

Area of square = area of rhombus

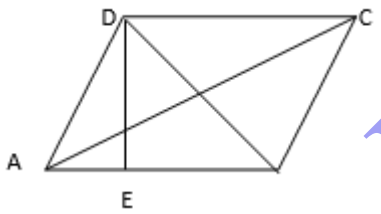
Area of rhombus = 16 m^2

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

16 = $\frac{1}{2} \times 2 \times d_2$

Other diagonal of rhombus = 16 m

In ΔAOB



Using pythagorous theorem:

$AB^2 = OA^2 + OB^2$

$AB^2 = 8^2 + 1^2$

$AB^2 = 65$

AB = $\sqrt{65}$

Rhombus is a parallelogram, area of parallelogram = base× altitude

Area of parallelogram = AB × DE

Area of parallelogram = $\sqrt{65}$ × DE

DE = $\frac{16}{\sqrt{65}}$

Altitude of Rhombus = $\frac{16}{\sqrt{65}}$ cm

16. Question

Find the area of the field in the form of a rhombus, if the length of each side be 14 cm and the altitude be 16 cm.

Answer

Side of rhombus = 14 cm

Altitude of rhombus = 16 cm

Rhombus is a type of parallelogram

Area of parallelogram = base × altitude

Area of parallelogram = 14 × 16 = 224 cm^2

17. Question

The cost of fencing a square field at 60 paise per metre is Rs. 1200. Find the cost of reaping the field at the rate of 50 paise per 100 sq. metres.

Answer

Perimeter of square field = $\frac{cost\ of\ fencing}{rate\ of\ fencing}$

Perimeter of square field = $\frac{1200}{0.6} = 2000\ m$

Perimeter of square = 4 × side

Side of square = $\frac{perimeter}{4} = \frac{2000}{4} = 500\ m$

Area of square = $side^2$

Area of square = 500 × 500 = 250000 m^2

Cost of reaping = $\frac{250000 \times 0.5}{100} = 1250$

Cost of reaping is Rs 1250

18. Question

In exchange of a square plot one of whose sides is 84 m, a man wants to buy a rectangular plot 144 m long and of the same area as of the square plot. Find the width of the rectangular plot.

Answer

Area of square = $side^2$

Area of square = 84 × 84

Area of square = area of rectangle

84× 84 = 144 × width

Width = $\frac{84\times84}{144} = 49\ m$

Therefore width of rectangle = 49 m

19. Question

The area of a rhombus is 84 m^2. If its perimeter is 40 m, then find its altitude.

Answer

Perimeter of rhombus = 4 × side

Side of rhombus = $\frac{perimeter}{4} = \frac{40}{4} = 10\ m$

Rhombus is a type of parallelogram, Hence area oa rhombus = area of parallelogram

Therefore area of parallelogram = base× altitude

Base× altitude = area of parallelogram

10 × altitude = 84

Altitude of rhombus = 8.4 m

20. Question

A garden is in the form of a rhombus whose side is 30 metres and the corresponding altitudes is 16 m. Find the cost of levelling the garden at the rate of Rs. 2 per m^2.

Answer

Side of rhombus = 30 m

Altitude of rhombus = 16 m

Rhombus is a type of parallelogram

Area of parallelogram = base × altitude

Area of parallelogram = 30 × 16 = 480 m^2

Cost of levelling the garden = area × rate

Cost of levelling the garden = 480 × 2 = 960

Cost of levelling the garden is Rs 960

21. Question

A field in the form of a rhombus has each side of length 64 m and altitude 16 m. What is the side of a square field which has the same area as that of a rhombus?

Answer

Side of rhombus = 30 m

Altitude of rhombus = 16 m

Rhombus is a type of parallelogram

Area of parallelogram = base × altitude

Area of parallelogram = 64 × 16 = 1024 m^2

Since area of rhombus = area of square

Area of square = $side^2$

Side of a square = $\sqrt{area}$

Side of square = $\sqrt{1024} = 32$

Therefore side of square = 32 m

22. Question

The area of a rhombus is equal to the area of a triangle whose base and the corresponding altitude are 24.8 cm and 16.5 cm respectively. If one of the diagonals of the rhombus is 22 cm, find the length of the other diagonal.

Answer

Length of base of triangle = 24.8 cm

Length of altitude of triangle= 16.5 cm

Area of triangle = $\frac{1}{2} \times base \times altitude$

Area of triangle = $\frac{1}{2} \times 24.8 \times 16.5 = 204.6$

Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$

204.6 = $\frac{1}{2} \times 22 \times d_2$

$d_2 = \frac{204.6}{11} = 18.6$

Hence, the length of other diagonal is 18.6 cm

Exercise 20.2

1. Question

Find the area, in square metres, of the trapezium whose bases and altitudes are as under:

(i) bases = 12 dm and 20 dm, altitude = 10 dm

(ii) bases = 28 cm and 3 dm, altitude = 25 cm

(iii) bases = 8 m and 60 dm, altitude = 40 dm

(iv) bases = 150 cm and 30 dm, altitude = 9 dm

Answer

(i) Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 12 dm and 20 dm

10 dm = 1 m

Therefore length of bases in m = 1.2 m and 2 m

Similarly length of altitude in m = 1 m

Area of trapezium = $\frac{1}{2}(1.2+2.0)\times 1$

Area of trapezium = $\frac{1}{2}\times 3.2 = 1.6\ m^2$

(ii) Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 28 cm and 3 dm

10 dm = 1 m

Therefore length of bases in m = 0.28 m and 0.3 m

Similarly length of altitude in m = 0.25 m

Area of trapezium = $\frac{1}{2}(0.28+0.3)\times 0.25$

Area of trapezium = $\frac{1}{2}\times 0.58\times 0.25 = 0.0725\ m^2$

(iii) Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 8 m and 60 dm

10 dm = 1 m

Therefore length of bases in m = 8 m and 6 m

Similarly length of altitude in m = 4 m

Area of trapezium = $\frac{1}{2}(8+6)\times 4$

Area of trapezium = $\frac{1}{2}\times 14\times 4 = 28\ m^2$

(iv) Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 150 cm and 30 dm

10 dm = 1 m

Therefore length of bases in m = 1.5 m and 3 m

Similarly length of altitude in m = 0.9 m

Area of trapezium = $\frac{1}{2}(1.5+3)\times 0.9$

Area of trapezium = $\frac{1}{2}\times 4.5\times 0.9 = 2.025\ m^2$

2. Question

Find the area of trapezium with base 15 cm and height 8 cm, if the side parallel to the given base is 9 cm long.

Answer

Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 15 cm and 9 cm

Similarly length of altitude in m = 8 cm

Area of trapezium = $\frac{1}{2}(15+9)\times 8$

Area of trapezium = $\frac{1}{2}\times 24\times 8 = 96\ m^2$

3. Question

Find the area of a trapezium whose parallel sides are of length 16 dm and 22 dm and whose height is 12 dm.

Answer

Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 15 cm and 9 cm

Similarly length of altitude in m = 8 cm

Area of trapezium = $\frac{1}{2}(15+9)\times 8$

Area of trapezium = $\frac{1}{2}\times 24\times 8 = 96\ m^2$

4. Question

Find the height of a trapezium, the sum of the lengths of whose bases (parallel sides) is 60 cm and whose area is 600 cm^2.

Answer

Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 60 cm and 60 cm

Area of trapezium = $\frac{1}{2}(60+60)\times altitude$

Length of altitude = $\frac{2\times area}{120} = \frac{2\times 600}{120} = 10$

Therefore length of altitude = 10 cm

5. Question

Find the altitude of a trapezium whose area is 65 cm^2 and whose base are 13 cm and 26 cm.

Answer

Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Length of bases of trapezium = 13 cm and 26 cm

Area of trapezium = $\frac{1}{2}(13+26)\times altitude$

Length of altitude = $\frac{2\times area}{39} = \frac{2\times 65}{39} = 3.33$

Therefore length of altitude = 3.33 cm

6. Question

Find the sum of the lengths of the bases of a trapezium whose area is 4.2 m^2 and whose height is 280 cm.

Answer

Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides)\times altitude$

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times altitude$

Sum of parallel sides = $\frac{2\times area}{2.8} = \frac{2\times 4.2}{2.8} = 3$

Therefore sum of length of parallel sides = 3 m

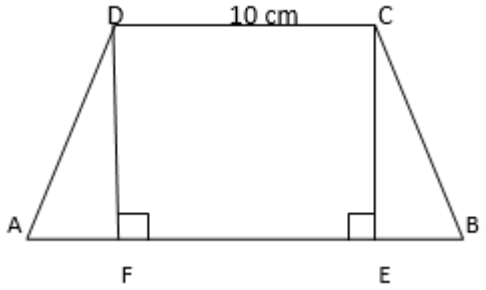
7. Question

Find the area of a trapezium whose parallel sides of lengths 10 cm and 15 cm are at a distance of 6 cm from each other. Calculate this area as

(i) the sum of the areas of two triangles and one rectangle.

(ii) the difference of the area of a rectangle and the sum of the areas of two triangles.

Answer



Area of a trapezium ABCD= area (ΔDFA) + area (rectangle DFEC) + area (ΔCEB)= (1/2 × AF × DF) + (FE × DF) + (1/2 × EB × CE)= (1/2 × AF × h) + (FE × h) + (1/2 × EB × h)

= 1/2 × h × (AF + 2FE + EB)= 1/2 × h × (AF + FE + EB + FE)= 1/2 × h × (AB + FE)

= 1/2 × h × (AB + CD) [Opposite sides of rectangle are equal]

= 1/2 × 6 × (15 + 10)

= 1/2 × 6 × 25 = 75

Area of trapezium = 75 cm^2

8. Question

The area of a trapezium is 960 cm^2. If the parallel sides are 34 cm and 46 cm, find the distance between them.

Answer

Area of trapezium = $\frac{1}{2}(sum\ of\ lengths\ of\ parallel\ sides) \times distance\ between\ parallel\ sides$

Area of trapezium = $\frac{1}{2}(sum\ of\ sides) \times distance\ between\ parallel\ sides$

$\text{Distance between parallel sides} = \frac{2 \times area}{sum\ of\ sides} = \frac{2 \times 960}{80} = 24$

Therefore $\text{Distance between parallel sides}$ = 24 cm

9. Question

Find the area of Fig. 20.35 as the sum of the areas of two trapezium and a rectangle.

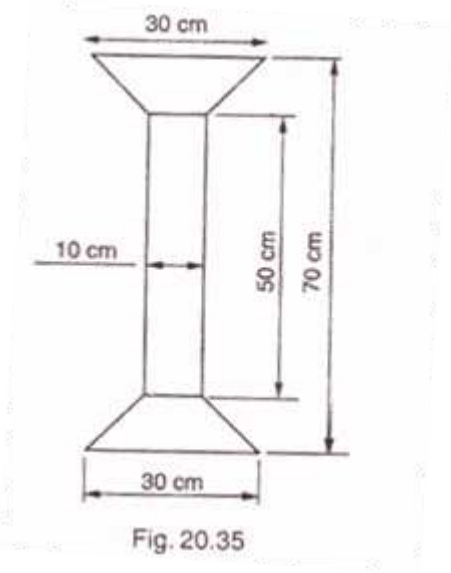


Fig. 20.35

Answer

Area of figure = Area of two trapeziums + area of rectangle

Length of rectangle = 50 cm

Breadth of rectangle = 10 cm

Length of parallel sides of trapezium = 30 cm and 10 cm

Distance between parallel sides of trapezium = $\frac{70-50}{2} = \frac{20}{2} = 10\ cm$

Area of figure = $2 \times \frac{1}{2}(sum\ of\ sides) \times distance\ between\ parallel\ sides + length \times breadth$

Area of figure = $2 \times \frac{1}{2}(30+10) \times 10 + 50 \times 10$

Area of figure = $400 + 500$ = 900 cm^2

10. Question

Top surface of a table is trapezium in shape. Find its area if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

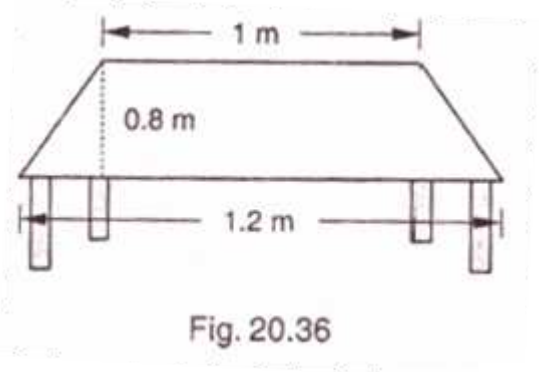


Fig. 20.36

Answer

Length of parallel sides of trapezium = 1.2m and 1m

Distance between parallel sides of trapezium = $0.8m$

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times distance\ between\ parallel\ sides$

Area of trapezium = $\frac{1}{2}(1.2+1)\times 0.8$

Area of trapezium = 0.88 m^2

11. Question

The cross-section of a canal is a trapqzium in shape. If the canal is 10 m wide at the top 6 m wide at the bottom and the area of cross-section is 72 m^2 determine its depth.

Answer

Length of parallel sides of trapezium = 10m and 6m

Distance between parallel sides of trapezium = x meter

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times distance\ between\ parallel\ sides$

$72 = \frac{1}{2}(10+6)\times x$

$x = \frac{72}{8} = 9$

Therefore depth of river is 9m

12. Question

The area of a trapezium is 91 cm^2 and its height is 7 cm. If one of the parallel sides is longer than the other by 8 cm, find the two parallel sides.

Answer

Let the length of one parallel side of trapezium = $x\ meter$

Length of other parallel side of trapezium = $(x+8)m$

Area of trapezium = 91 cm^2

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times distance\ between\ parallel\ sides$

$91 = \frac{1}{2}(x+x+8)\times 7$

$x+4 = \frac{91}{7}$

$x+4 = 13$

$x = 13-4 = 9$

Therefore length of one parallel side of trapezium = 9 cm

Length of other parallel side of trapezium = 9+8 = 17 cm

13. Question

The area of a trapqzium is 384 cm^2. Its parallel sides are in the ratio 3 : 5 and the perpendicular distance between them is 12 cm. Find the length of each one of the parallel sides.

Answer

Let the length of one parallel side of trapezium = $3x\ meter$

Length of other parallel side of trapezium = $5x\ meter$

Area of trapezium = 384 cm^2

Distance between parallel sides = 12 cm

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times distance\ between\ parallel\ sides$

384 = $\frac{1}{2}(3x+5x)\times 12$

$$4x = \frac{384}{12}$$

$$4x = 32$$

$$x = \frac{32}{4} = 8$$

Therefore length of one parallel side of trapezium = 3x = 3× 8 = 24 cm

Length of other parallel side of trapezium = 5x = 5× 8 = 40 cm

14. Question

Mohan wants to buy a trapezium shaped field. Its side along the river is parallel and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.

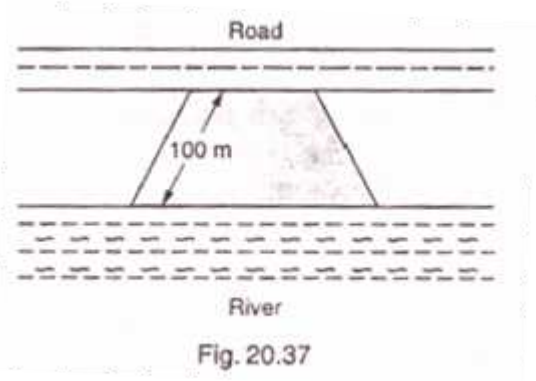


Fig. 20.37

Answer

Let the length of side of trapezium shaped field along road = $x\ meter$

Length of other side of trapezium shaped field along road = $2x\ meter$

Area of trapezium = 10500 cm^2

Distance between parallel sides = 100 m

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times distance\ between\ parallel\ sides$

10500 = $\frac{1}{2}(x+2x)\times 100$

$$3x = \frac{10500}{50}$$

$$x = \frac{210}{3}$$

$$x = 70$$

Therefore length of side along road = 70 m

Length of side along river = 70× 2 = 140 m

15. Question

The area of a trapezium is 1586 cm^2 and the distance between the parallel sides is 26 cm. If one of the parallel sides is 38 cm, find the other.

Answer

Let the length of other parallel side of trapezium = $x\ cm$

Length of one parallel side of trapezium = $38\ cm$

Area of trapezium = 1586 cm^2

Distance between parallel sides = 26 cm

Area of trapezium = $\frac{1}{2}(sum\ of\ sides)\times distance\ between\ parallel\ sides$

1586 = $\frac{1}{2}(38+x)\times 26$

$$38 + x = \frac{1586}{13}$$

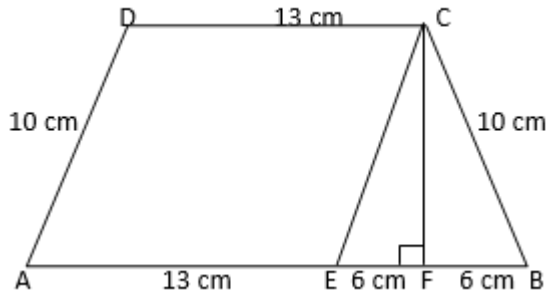
$$x = 122 - 38 = 84$$

Therefore the length of other parallel side of trapezium = $84\ cm$

16. Question

The parallel sides of a trapezium are 25 cm and 13 cm; its nonparallel sides are equal, each being 10 cm, find the area of the trapezium.

Answer



In ΔCEF

CE = 10 cm and EF = 6cm

Using pythagorous theorem:

$CE^2 = CF^2 + EF^2$

$CF^2 = CE^2 - EF^2$

$CF^2 = 10^2 - 6^2$

$CF^2 = 100-36$

$CF^2 = 64$

CF = 8 cm

Area of trapezium = area of parallelogram AECD + area of area of triangle CEF

Area of trapezium = $base \times height + \frac{1}{2}(base \times hight)$

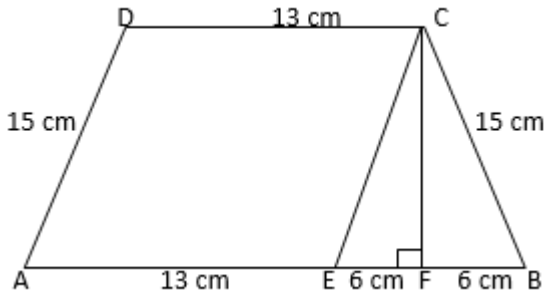
Area of trapezium = $13 \times 8 + \frac{1}{2}(12 \times 8)$

Area of trapezium = 104 + 48 = 152 cm^2

17. Question

Find the area of a trapezium whose parallel sides are 25 cm, 13 cm and the other sides are 15 cm each.

Answer



In ΔCEF

CE = 10 cm and EF = 6cm

Using pythagorous theorem:

$CE^2 = CF^2 + EF^2$

$CF^2 = CE^2 - EF^2$

$CF^2 = 15^2 - 6^2$

$CF^2 = 225-36$

$CF^2 = 189$

CF = 17 cm

Area of trapezium = area of parallelogram AECD + area of area of triangle CEF

Area of trapezium = $base \times height + \frac{1}{2}(base \times hight)$

Area of trapezium = $13 \times 17 + \frac{1}{2}(12 \times 17)$

Area of trapezium = 221 + 102 = 323 cm^2

18. Question

If the area of a trapezium is 28 cm^2 and one of its parallel sides is 6 cm, find the other parallel side if its altitude is 4 cm.

Answer

Let the length of other parallel side of trapezium = $x\ cm$

Length of one parallel side of trapezium = $6\ cm$

Area of trapezium = 28 cm^2

Length of altitude of trapezium = 4 cm

Area of trapezium = $\frac{1}{2}(sum\ of\ sides) \times distance\ between\ parallel\ sides$

$28 = \frac{1}{2}(6 + x) \times 4$

$6 + x = \frac{28}{2}$

$x = 14 - 6 = 8$

Therefore the length of other parallel side of trapezium = $8\ cm$

19. Question

In Fig. 20.38, a parallelogram is drawn in a trapezium, the area of the parallelogram is 80 cm^2, find the area of the trapezium.

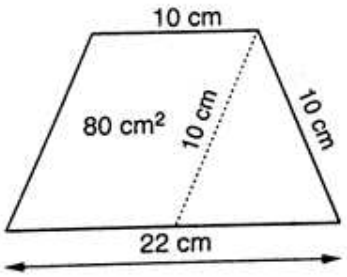
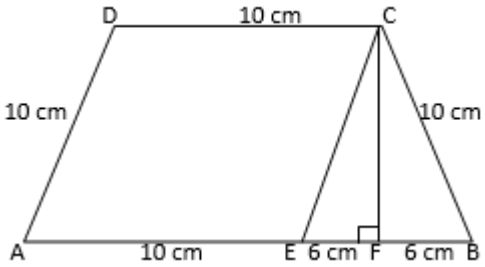


Fig. 20.38

Answer



In ΔCEF

CE = 10 cm and EF = 6cm

Using pythagorous theorem:

$CE^2 = CF^2 + EF^2$

$CF^2 = CE^2 - EF^2$

$CF^2 = 10^2 - 6^2$

$CF^2 = 100-36$

$CF^2 = 64$

CF = 8 cm

Area of parallelogram = 80 cm^2

Area of trapezium = area of parallelogram AECD + area of area of triangle CEF

Area of trapezium = $base \times height + \frac{1}{2}(base \times hight)$

Area of trapezium = $80 + \frac{1}{2}(12 \times 8)$

Area of trapezium = 80 + 48 = 128 cm^2

20. Question

Find the area of the field shown in Fig. 20.39 by dividing it into a square, a rectangle and a trapezium.

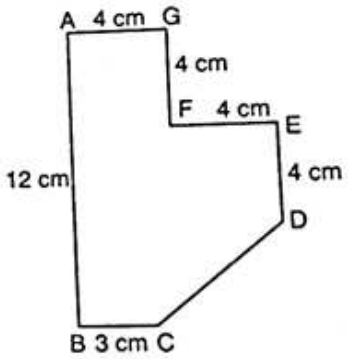
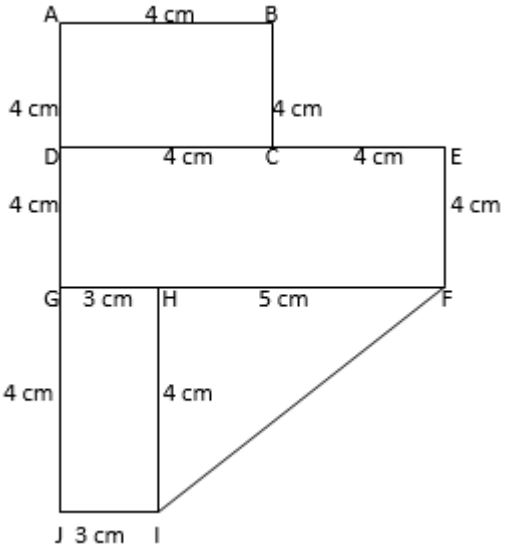


Fig. 20.39

Answer



Area of given figure = area of square ABCD + area of rectangle DEFG + area of rectangle GHIJ + area of triangle FHI

Area of given figure = $Side \times side + length \times breadth + length \times breadth + \frac{1}{2} \times base \times altitude$

Area of given figure = $4 \times 4 + 8 \times 4 + 3 \times 4 + \frac{1}{2} \times 5 \times 4$

Area of given figure = $16 + 32 + 12 + 10$ = 70 cm^2

Exercise 20.3

1. Question

Find the area of the pentagon shown in fig. 20.48, if *AD* = 10 cm, *AG* = 8 cm, *AH* = 6 cm, *AF* = 5 cm, *BF* = 5 cm, *CG* = 7 cm and *EH* = 3 cm.

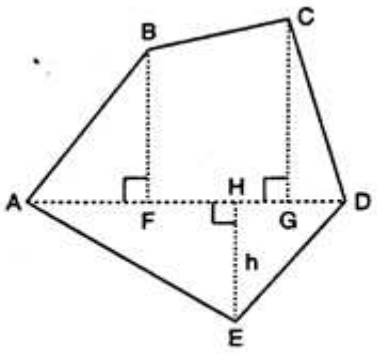


Fig. 20.48

Answer

GH = AG – AH = 8 – 6 = 2 cm

HF = AH – AF = 6 – 5 = 1 cm

GD = AD – AG = 10 – 8 = 2 cm

Area of given figure = area of triangle AFB + area of trapezium BCGF + area of triangle CGD + area of triangle AHE + area of triangle EGD

Area of right angled triangle = $\frac{1}{2}\times base\times altitude$

Area of trapezium = $\frac{1}{2}\times(sum\ of\ parallel\ sides)\times altitude$

Area of given pentagon = $\frac{1}{2}\times AF\times BF+\frac{1}{2}\times(CG+BF)\times FG+\frac{1}{2}\times GD\times CG+\frac{1}{2}\times AH\times EH+\frac{1}{2}\times HD\times EH$

Area of given pentagon = $\frac{1}{2}\times5\times5+\frac{1}{2}\times(7+5)\times3+\frac{1}{2}\times2\times7+\frac{1}{2}\times6\times3+\frac{1}{2}\times4\times3$

Area of given pentagon = $12.5+18+7+9+6$ = 52.5 cm^2

Therefore area of given pentagon is 70 cm^2

2. Question

Find the area enclosed by each of the following figures [fig. 20.49 (i)-(ii)] as the sum of the areas of a rectangle and a trapezium.

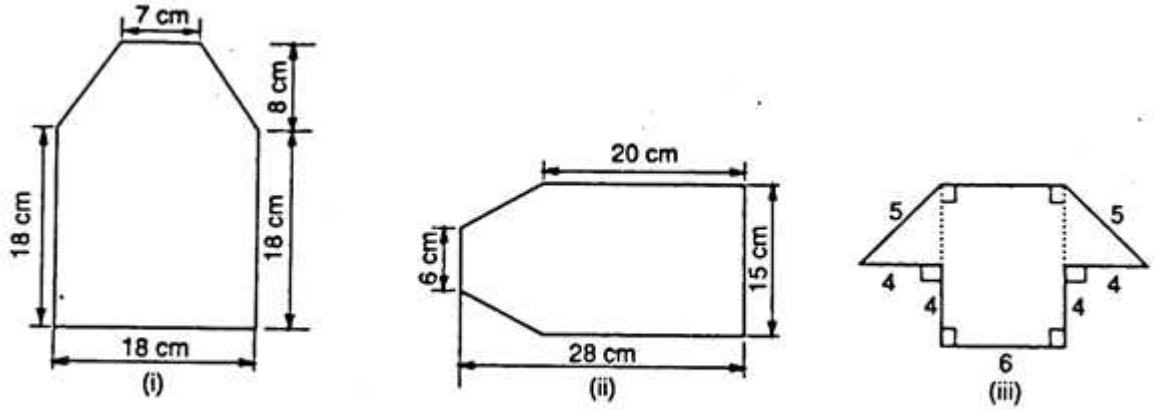


Fig. 20.49

Answer

Figure (i)

Area of given figure = Area of trapezium + area of rectangle

Area of given figure = $\frac{1}{2}\times(sum\ of\ parallel\ sides)\times altitude + length\times breadth$

Area of given figure = $\frac{1}{2}\times(18+7)\times 8 + 18\times 18$

Area of given figure = $100 + 324 = 424$

Therefore area of given figure is 424 cm^2

Figure (ii)

Area of given figure = Area of trapezium + area of rectangle

Area of given figure = $\frac{1}{2}\times(sum\ of\ parallel\ sides)\times altitude + length\times breadth$

Area of given figure = $\frac{1}{2}\times(15+6)\times 8 + 15\times 20$

Area of given figure = $84 + 300 = 384$

Therefore area of given figure is 384 cm^2

Figure (iii)

Using pythagorous theorem in the right angled triangle

$5^2 = 4^2 + x^2$

$x^2 = 25 - 16$

$x^2 = 9$

$x = 3$ cm

Area of given figure = Area of trapezium + area of rectangle

Area of given figure = $\frac{1}{2}\times(sum\ of\ parallel\ sides)\times altitude + length\times breadth$

Area of given figure = $\frac{1}{2}\times(14+6)\times 3 + 4\times 6$

Area of given figure = $30 + 24 = 54$

Therefore area of given figure is 54 cm^2

3. Question

There is a pentagonal shaped park as shown in Fig. 20.50. Jyoti and Kavita divided it in two different ways.

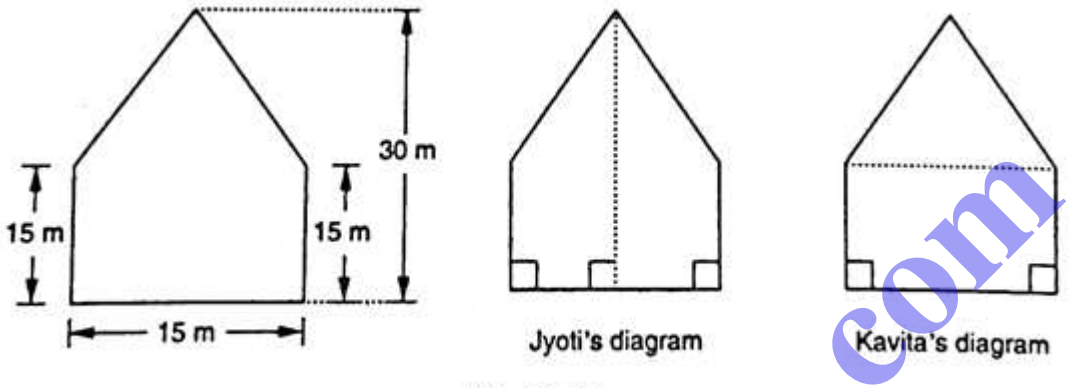


Fig. 20.50

Find the area of this park using both ways. Can you suggest some another way of finding its area?

Answer

Area of given figure = Area of trapezium + area of rectangle

Area of Jyoti's diagram = $2\times\frac{1}{2}\times(sum\ of\ parallel\ sides)\times altitude$

Area of given figure = $2\times\frac{1}{2}\times(15+30)\times 7.5$

Area of given figure = 337.5 cm^2

Therefore area of given figure is 54 cm^2

Area of given pentagon = Area of triangle + area of rectangle

Area of given pentagon = $\frac{1}{2}\times base\times altitude + length\times breadth$

Area of given pentagon = $\frac{1}{2}\times 15\times 15 + 15\times 15$

Area of given pentagon = $112.5 + 225 = 337.5$

Therefore area of given pentagon is 337.5 cm^2

4. Question

Find the area of the following polygon, if *AL* = 10 cm, *AM* = 20 cm, *AN* = 50 cm. *AO* = 60 cm and *AD* = 90 cm.

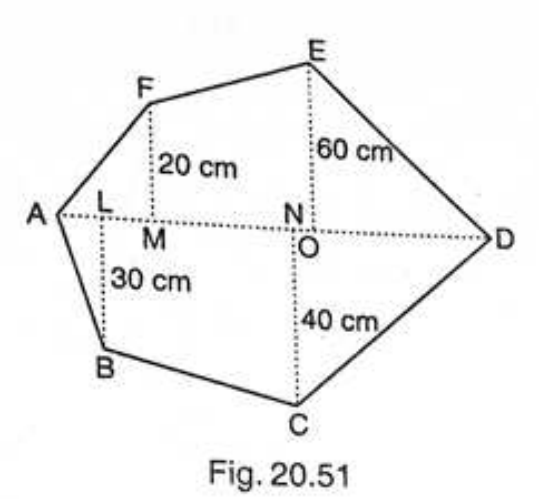


Fig. 20.51

Answer

AL = 10 cm; AM = 20 cm; AN = 50 cm; AO = 60 cm; AD = 90 cm given

LM = AM – AL = 20 – 10 = 10 cm

MN = AN – AM = 50 – 20 = 30 cm

OD = AD – AO = 90 – 60 = 30 cm

ON = AO – AN = 60 – 50 = 10 cm

DN = OD + ON = 30 + 10 = 40 cm

OM = MN + ON = 30 + 10 = 40 cm

LN = LM + MN = 10 + 30 = 40 cm

Area of given figure = area of triangle AMF + area of trapezium FMNE + area of triangle END + area of triangle ALB + area of trapezium LBCN + area of triangle DNC

Area of right angled triangle = $\frac{1}{2}\times base\times altitude$

Area of trapezium = $\frac{1}{2}\times(sum\ of\ parallel\ sides)\times altitude$

Area of given hexagon =
$\frac{1}{2}\times AM\times FM+\frac{1}{2}\times(MF+OE)\times OM+\frac{1}{2}\times OD\times OE+\frac{1}{2}\times AL\times BL+\frac{1}{2}\times(BL+CN)\times LN+\frac{1}{2}\times DN\times CN$

Area of given hexagon =
$\frac{1}{2}\times 20\times 20+\frac{1}{2}\times(20+60)\times 40+\frac{1}{2}\times 30\times 60+\frac{1}{2}\times 10\times 30+\frac{1}{2}\times(30+40)\times 40+\frac{1}{2}\times 40\times 40$

Area of given hexagon = $200+1600+900+150+1400+800$ = 5050 cm^2

Therefore area of given hexagon is 5050 cm^2

5. Question

Find the area of the following regular hexagon.

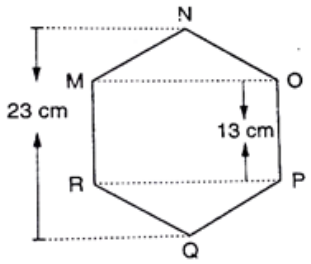


Fig. 20.52

Answer

NQ = 23 cm given

ND + QC = 23 -13 = 10

ND = QC = 5 cm

OM = 24 cm

Area of given hexagon = 2 times area of triangle MNO + area of rectangle MOPR

Area of right angled triangle = $\frac{1}{2}\times base\times altitude$

Area of rectangle = length × breadth

Area of regular hexagon = $\frac{1}{2}\times OM\times ND + MO\times OP + \frac{1}{2}\times RP\times QC$

Area of given hexagon = $\frac{1}{2}\times 24\times 5 + 24\times 13 + \frac{1}{2}\times 24\times 5$ = 432 cm^2

Therefore area of given hexagon is 432 cm^2