2. Relations

Exercise 2.1

1 A. Question

If $\left(\frac{a}{3}+1, b-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of a and b.

Answer

Given: $\left(\frac{a}{3}+1, b-\frac{2}{3}\right)=\left(\frac{5}{3}, \frac{1}{3}\right)$

To find: values of a and b

By the definition of equality of ordered pairs, we have and simultaneously solving for a and b

$\frac{a}{3}+1=\frac{5}{3}$

and $b-\frac{2}{3}=\frac{1}{3}$

$\Rightarrow \frac{a}{3}=\frac{5}{3}-1$

$\Rightarrow b=\frac{1}{3}+\frac{2}{3}$

$\Rightarrow \frac{a}{3}=\frac{2}{3}$

⇒ b = 1

⇒ a = 2

1 B. Question

If (x + 1, 1) = (3y, y - 1), find the values of x and y.

Answer

given: (x + 1, 1) = (3y, y - 1)

To find: values of a and b

By the definition of equality of ordered pairs, we have

x + 1 = 3y and 1 = y - 1

⇒ x = 3y - 1 and y = 2So,x = 3(2) - 1 = 6 - 1 = 5

⇒ x = 5 and y = 2

2. Question

If the ordered pairs (x, - 1) and (5, y) belong to the set {(a, b): b = 2a - 3}, find the values of x and y.

Answer

given the ordered pairs (x, - 1) and (5, y) belong to the set {(a, b): b = 2a - 3}

To find: values of x and y

solving for first order pair

⇒ (x, - 1) = {(a, b): b = 2a - 3}

⇒ x = a and b = - 1

If b = - 1 then 2a = - 1 + 3 = 2

So, a = 1

$\Rightarrow$ x = 1

Similarly, solving for second order pair

$\Rightarrow$ (5, y) = {(a, b): b = 2a - 3}

$\Rightarrow$ a = 5 and y = b

If a = 5 then b = 2×5 - 3

So, b = 7

$\Rightarrow$ y = 7

3. Question

If a ∈ { - 1, 2, 3, 4, 5} and b ∈ {0, 3, 6}, write the set of all ordered pairs (a, b) such that a + b = 5.

Answer

given a ∈ { - 1, 2, 3, 4, 5} and b ∈ {0, 3, 6},

To find: the ordered pair (a, b) such that a + b = 5

then the ordered pair (a, b) such that a + b = 5 are as follows

(a, b)∈ {(- 1, 6), (2, 3), (5, 0)}

4. Question

If a ∈ {2, 4, 6, 9} and b ∈{4, 6, 18, 27}, then form the set of all ordered pairs (a, b) such that a divides b and a<b.

Answer

given that a ∈ {2, 4, 6, 9} and b ∈{4, 6, 18, 27}.

To find: ordered pairs (a, b) such that a divides b and a<b

Here,

2 divides 4, 6, 18 and is also less than all of them

4 divides 4 and is also less than none of them

6 divides 6, 18 and is less than 18 only

9 divides 18, 27 and is less than all of them

Therefore, ordered pairs (a, b) are (2, 4), (2, 6), (2, 18),

(6, 18), (9, 18) and (9, 27)

5. Question

If A = {1, 2} and B = {1, 3}, find A x B and B x A.

Answer

Given A = {1, 2} and B = {1, 3}

To find: A × B, B × A

A × B = {(1, 1), (1, 3), (2, 1), (2, 3)}

B × A = {(1, 1), (1, 2), (3, 1), (3, 2)}

6. Question

Let A = {1, 2, 3} and B = {3, 4}. Find A x B and show it graphically

Answer

given: A = {1, 2, 3} and B = {3, 4}

To find: graphical representation of A × B

A x B = {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

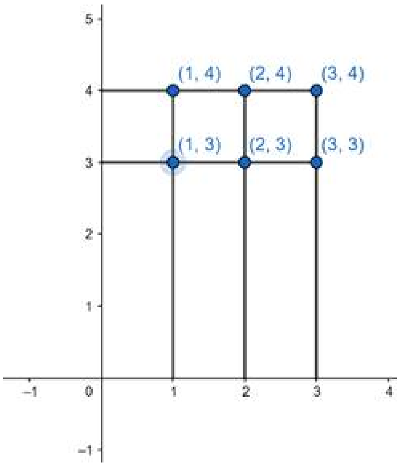
To represent A × B graphically, given steps must be followed:

a. One horizontal and one vertical axis should be drawn

b. Element of set A should be represented in horizontal axis and on vertical axis elements of set B should be represented

c. Draw dotted lines perpendicular to horizontal and vertical axes through the elements of set A and B

d. Point of intersection of these perpendicular represents A × B.



7. Question

If A = {1, 2, 3} and B = {2, 4}, what are A x B, B x A, A x A, B x B, and (A x B)∩ (B x A)?

Answer

given: A = {1, 2, 3} and B = {2, 4}

To find: A × B, B × A, A × A, (A × B) ∩ (B × A)

Now,

A × B = {(1, 2), (1, 4), (2, 2), (2, 4), (3, 2), (3, 4)}

B × A = {(2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}

A × A = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

B × B = {(2, 2), (2, 4), (4, 2), (4, 4)}

Intersection of two sets represents common elements of both the sets

So,

(A × B) ∩ (B × A) = {(2, 2)}

8. Question

If A and B are two sets having 3 elements in common. If n(A) = 5, n(B) = 4, find n(A x B) and n[(A x B)∩ (B x A)].

Answer

given: (A) = 5 and n(B) = 4

To find: [(A × B) ∩ (B×A)]

n (A × B) = n(A) × n(B) = 5 x 4 = 20

n (A ∩ B) = 3 (given: A and B has 3 elements in common)

In order to calculate n [(A × B) ∩ (B × A)], we will assume

A = (x, x, x, y, z) and B = (x, x, x, p)

So, we have

(A × B) = {(x, x), (x, x), (x, x), (x, p), (x, x), (x, x), (x, x), (x, p), (x, x), (x, x), (x, x), (x, p), (y, x), (y, x), (y, x), (y, p), (z, x), (z, x), (z, x), (z, p)}

(B × A) = {(x, x), (x, x), (x, x), (x, y), (x, z), (x, x), (x, x), (x, x), (x, y), (x, z), (x, x), (x, x), (x, x), (x, y), (x, z), (p, x), (p, x), (p, x), (p, y), (p, z)}

[(A × B) ∩ (B × A)] = {(x, x), (x, x), (x, x), (x, x), (x, x), (x, x), (x, x), (x, x), (x, x)}

∴ We can say that n [(A × B) ∩ (B × A)] = 9

9. Question

Let A and B be two sets. Show that the sets A x B and B x A have an element in common if the sets A and B be two sets such that n (A) = 3 and n (B) = 2.

Answer

given: n (A) = 3 n (B) = 2

To prove: The sets A x B and B x A have an element in common if the sets A and B be two sets such that n (A) = 3 and n (B) = 2

Proof:

Case 1: No elements are common

Assuming:

A = (a, b, c) and B = (e, f)

So, we have:

A × B = {(a, e), (a, f), (b, e), (b, f), (c, e), (c, f)}

B × A = {(e, a), (e, b), (e, c), (f, a), (f, b), (f, c)}

There are no common ordered pair in A × B and B × A.

Case 2: One element is common

Assuming:

A = (a, b, c) and B = (a, f)

So, we have:

A × B = {(a, a), (a, f), (b, a), (b, f), (c, a), (c, f)}

B × A = {(a, a), (a, b), (a, c), (f, a), (f, b), (f, c)}

Here, A × B and B × A have one ordered pair in common.

Therefore, we can say that A × B and B × A will have elements in common if and only if sets A and B have an element in common.

10. Question

Let A and B be two sets such that n(A) x B, find A and B, where x, y, z are distinct elements

Answer

given: n(A) = 3 and n(B) = 2

To find: distinct elements of set A and B

Also, it is given that {(x, 1), (y, 2), (z, 1)} $\subset$ A × B

Set A has 3 elements whereas Set B has 2 elements.

Also, A × B = {(a, b): a ∈ A and b ∈ B}

Therefore, A ∈ {x, y, z} and B ∈ {1, 2}

11. Question

Let A = {1, 2, 3, 4} and R = {(a, b): a∈ A, b ∈ A, a divides b}. Write R explicitly.

Answer

given: A = {1, 2, 3, 4} and R = {(a, b): a∈ A, b ∈ A, a divides b}

To find: set R

Both elements of R, a and b, belongs to set A and relation between and elements a and b is that a divides b

So,

1 divides 1, 2, 3 and 4.

2 divides 2 and 4.

3 divides 3.

4 divides 4.

∴ R = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)}

12. Question

If A = { - 1, 1}, find A x A x A.

Answer

given: A = {−1, 1}

To find: A × A × A

So, A × A = {(−1, −1), (−1, 1), (1, −1), (1, 1)}

And, A × A × A = {(−1, −1, −1), (−1, −1, 1), (−1, 1, −1), (−1, 1, 1), (1, −1, −1), (1, −1, 1), (1, 1, −1), (1, 1, 1)}

13. Question

State whether each of the following statements are true of false. If the statement is false, re - write the given statement correctly:

i. If P = [m, n} and Q = {n, m} then P x Q = {(m, n), (n, m)}

ii. If A and B are non - empty sets, then A x B is a non - empty set of ordered pairs (x, y) such that x∈ B and y ∈ A.

iii. If A = {1, 2}, B]{3, 4}, then A x (B∩ φ) = φ.

Answer

(i) False

given: P = {m, n} and Q = {n, m}, then

P × Q = {(m, n), (m, m), (n, n), (n, m)}.

(ii) False

given: A and B are non - empty sets

Then A × B is a non - empty set of an ordered pair (x, y) such that x∈ A and y ∈ B

(iii) True

given: A = {1, 2} and B = {3, 4}

∅ is represents null setand intersection of any set with null set gives null set as null set has no elements

⇒ (B ∩ ∅) = ∅

The Cartesian product of any set and an empty set will be an empty set.

∴ A × (B ∩ ∅) = ∅

14. Question

If A = {1, 2}, form the set A x A x A.

Answer

given A = {1, 2}

To find A × A × A

Firstly, we will findCartesian product of A with A

A × A = {(1, 1), (1, 2), (2, 1), (2, 2)}

Now, Cartesian product of A x A with A

∴ A × A × A = {(1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2)}

15. Question

If A = {1, 2, 4} and B = {1, 2, 3}, represent following sets graphically:

i. A x B ii. B x A

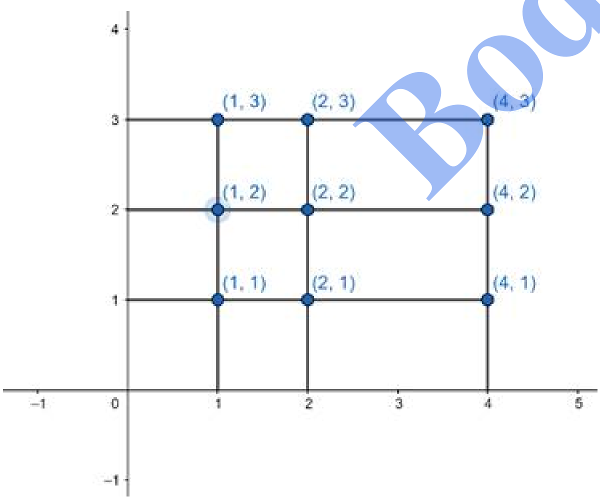
iii A x A iv. B x B

Answer

given: A = {1, 2, 4} B = {1, 2, 3}

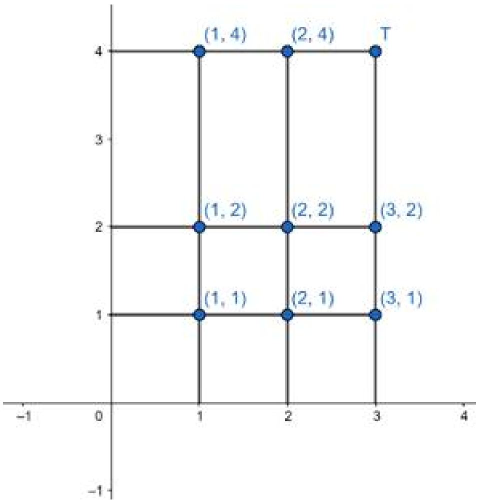
(i) To find: A × B

A × B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (4, 1), (4, 2), (4, 3)}



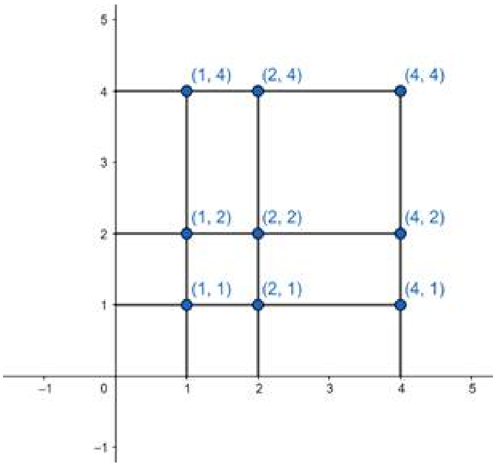
(ii) To find: B × A

B × A = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (3, 1), (3, 2), (3, 4)}



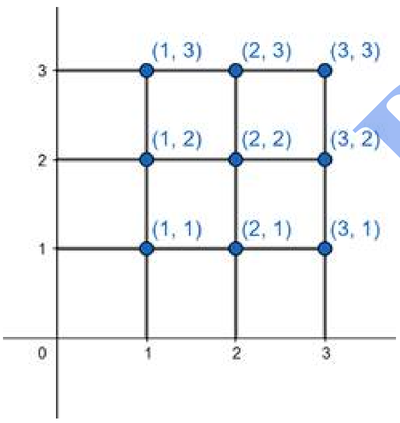
(iii) To find: A × A

A × A = {(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (2, 4), (4, 1), (4, 2), (4, 4)}



(iv) To find: B × B

B × B = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}



Exercise 2.2

1. Question

Given A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, find (A x B)∩ (B x C).

Answer

given: A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

To find: (A × B) ∩ (B × C)

(A × B) = {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

$(B \times C) = \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$

$\therefore (A \times B) \cap (B \times C) = \{(3, 4)\}$

2. Question

If $A = \{2, 3\}$, $B = \{4, 5\}$, $C = \{5, 6\}$ find $A \times (B \cup C)$, $(A \times B) \cup (A \times C)$.

Answer

given: $A = \{2, 3\}$, $B = \{4, 5\}$ and $C = \{5, 6\}$

To find: $(A \times B) \cup (A \times C)$

Since, $(B \cup C) = \{4, 5, 6\}$

$\therefore A \times (B \cup C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

$(A \times B) = \{(2, 4), (2, 5), (3, 4), (3, 5)\}$

$(A \times C) = \{(2, 5), (2, 6), (3, 5), (3, 6)\}$

$\therefore (A \times B) \cup (A \times C) = \{(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

3. Question

If $A = \{1, 2, 3\}$, $B = \{4\}$, $c = \{5\}$, then verify that:

i. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii. $A \times (B \cap C) = (A \times B) \cap (A \times C)$

iii. $A \times (B - C) = (A \times B) - (A \times C)$.

Answer

given $A = \{1, 2, 3\}$, $B = \{4\}$ and $C = \{5\}$

(i) To prove: $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS: $(B \cup C) = \{4, 5\}$

therefore $A \times (B \cup C) = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

RHS:

$(A \times B) = \{(1, 4), (2, 4), (3, 4)\}$

$(A \times C) = \{(1, 5), (2, 5), (3, 5)\}$

$(A \times B) \cup (A \times C) = \{(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)\}$

$\therefore$ LHS = RHS

(ii) To prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

LHS: $(B \cap C) = \varnothing$ (No common element)

$A \times (B \cap C) = \varnothing$

RHS: $(A \times B) = \{(1, 4), (2, 4), (3, 4)\}$

$(A \times C) = \{(1, 5), (2, 5), (3, 5)\}$

$(A \times B) \cap (A \times C) = \varnothing$

$\therefore$ LHS = RHS

(iii) To prove: $A \times (B - C) = (A \times B) - (A \times C)$

LHS: $(B - C) = \varnothing$

$A \times (B - C) = \varnothing$

RHS: (A × B) = {(1, 4), (2, 4), (3, 4)}

(A × C) = {(1, 5), (2, 5), (3, 5)}

(A × B) − (A × C) = ∅

∴ LHS = RHS

4. Question

Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:

i. A x C B x D

ii. A x (B ∩ C) = (A x B) ∩ (A x C)

Answer

given: A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}

(i) To prove: A × C ⊂ B × D

LHS: A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

RHS: B × D = {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

Since, all elements of A × C is in B × D.

∴We can A × C ⊂ B × D

(ii) To prove: A × (B ∩ C) = (A × B) ∩ (A × C)

LHS: (B ∩ C) = ∅

A × (B ∩ C) = ∅

RHS: (A × B) = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

(A × C) = {(1, 5), (1, 6), (2, 5), (2, 6)}

Since, there is no common element between A × B and A × C

⇒(A × B) ∩ (A × C) = ∅

∴ LHS = RHS

5. Question

If A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, find

i. A x (B ∩ C)

ii. (A x B) ∩ (A xC)

iii. A x (B ∪ C)

iv. (A x B) ∪ (A x C)

Answer

given: A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

(i) To find: A × (B ∩ C)

(B ∩ C) = {4}

∴ A × (B ∩ C) = {(1, 4), (2, 4), (3, 4)}

(ii) To find: (A × B) ∩ (A × C)

(A × B) = {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(A × C) = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

$\therefore (A \times B) \cap (A \times C) = \{(1, 4), (2, 4), (3, 4)\}$

(iii) To find: $A \times (B \cup C)$

$(B \cup C) = \{3, 4, 5, 6\}$

$\therefore A \times (B \cup C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

(iv) To find: $(A \times B) \cup (A \times C)$

$(A \times B) = \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$

$(A \times C) = \{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$

$\therefore (A \times B) \cup (A \times C) = \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$

6 A. Question

Prove that:

$(A \cup B) x C = (A x C) = (A x C) \cup (B x C)$

Answer

To prove: $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Proof:

Let (x, y) be an arbitrary element of $(A \cup B) \times C$.

$\Rightarrow (x, y) \in (A \cup B) C$

Since, (x, y) are elements of Cartesian product of $(A \cup B) \times C$

$\Rightarrow x \in (A \cup B)$ and $y \in C$

$\Rightarrow (x \in A$ or $x \in B)$ and $y \in C$

$\Rightarrow (x \in A$ and $y \in C)$ or $(x \in B$and $y \in C)$

$\Rightarrow (x, y) \in A \times C$ or $(x, y) \in B \times C$

$\Rightarrow (x, y) \in (A \times C) \cup (B \times C)$ ...1

Let (x, y) be an arbitrary element of $(A \times C) \cup (B \times C)$.

$\Rightarrow (x, y) \in (A \times C) \cup (B \times C)$

$\Rightarrow (x, y) \in (A \times C)$ or $(x, y) \in (B \times C)$

$\Rightarrow (x \in A$ and $y \in C)$ or $(x \in B$and $y \in C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $y \in C$

$\Rightarrow x \in (A \cup B)$ and $y \in C$

$\Rightarrow (x, y) \in (A \cup B) \times C$ ...2

From 1 and 2, we get: $(A \cup B) \times C = (A \times C) \cup (B \times C)$

6 B. Question

Prove that:

$(A \cap B) x C = (A x C) \cap (B x C)$

Answer

To prove: $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Proof:

Let (x, y) be an arbitrary element of $(A \cap B) \times C$.

$\Rightarrow (x, y) \in (A \cap B) \times C$

Since, (x, y) are elements of Cartesian product of $(A \cap B) \times C$

$\Rightarrow x \in (A \cap B)$ and $y \in C$

$\Rightarrow (x \in A$ and $x \in B)$ and $y \in C$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \in B$ and $y \in C)$

$\Rightarrow (x, y) \in A \times C$ and $(x, y) \in B \times C$

$\Rightarrow (x, y) \in (A \times C) \cap (B \times C)$ ...1

Let (x, y) be an arbitrary element of $(A \times C) \cap (B \times C)$.

$\Rightarrow (x, y) \in (A \times C) \cap (B \times C)$

$\Rightarrow (x, y) \in (A \times C)$ and $(x, y) \in (B \times C)$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \in B$ and $y \in C)$

$\Rightarrow (x \in A$ and $x \in B)$ and $y \in C$

$\Rightarrow x \in (A \cap B)$ and $y \in C$

$\Rightarrow (x, y) \in (A \cap B) \times C$ ...2

From 1 and 2, we get: $(A \cap B) \times C = (A \times C) \cap (B \times C)$

7. Question

If $A \times B \subseteq C \times D$ and $A \cap B \in \varnothing$, Prove that $A \subseteq C$ and $B \subseteq D$.

Answer

given $A \times B \subseteq C \times D$ and $A \cap B \in \varnothing$

To prove: $A \subseteq C$ and $B \subseteq D$

$A \times B \subseteq C \times D$ denotes $A \times B$ is subset of $C \times D$ that is every element $A \times B$ is in $C \times D$

And $A \cap B \in \varnothing$ denotes A and B does not have any common element between them.

$A \times B = \{(a, b): a \in A \text{ and } b \in B\}$

Since,

$A \times B \subseteq C \times D$ (Given)

$\therefore$We can say $(a, b) \subseteq C \times D$

$\Rightarrow a \in C$ and $b \in D$

$\Rightarrow A \in C$ and $B \in D$ (A and B does not have common elements)

Exercise 2.3

1. Question

If A = {1, 2, 3}, B = {4, 5, 6}, which of the following are relations from A to B?

Give reasons in support of your answer.

Answer

Given,

A = {1, 2, 3}, B = {4, 5, 6}

A relation from A to B can be defined as:

A × B = {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

i. {(1, 6), (3, 4),(5, 2)}

No, it is not a relation from A to B. The given set is not a subset of A × B as (5, 2) is not a part of the relation from A to B.

ii. {(1, 5), (2, 6), (3, 4),(3, 6)}

Yes, it is a relation from A to B. The given set is a subset of A × B.

iii. {(4, 2), (4, 3), (5, 1)}

No, it is not a relation from A to B. The given set is not a subset of A × B as (4, 2), (4, 3), (5, 1) are not a part of the relation from A to B.

iv. A × B

A × B is a relation from A to B can be defined as:

{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6),(3, 4),(3, 5),(3, 6)}

2. Question

A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows: (x, y)R x is relatively prime to y. Express R as a set of ordered pairs and determine its domain and range.

Answer

Relatively prime numbers are also known as co-prime numbers. If there is no integer greater than one that divides both (that is, their greatest common divisor is one). For example, 12 and 13 are relatively prime, but 12 and14 are not as their greatest common divisor is two.

Given, (x, y) R x is relatively prime to y

Here,

2 is co-prime to 3 and 7.

3 is co-prime to 7 and 10.

4 is co-prime to 3 and 7.

5 is co-prime to 3, 6 and 7.

∴ R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}

Domain of relation R = {2, 3, 4, 5}

Range of relation R = {3, 6, 7, 10}

3. Question

Let A be the set of first five natural and let R be a relation on A defined as follows: (x, y) R x ≤ y

Express R and R^{-1} as sets of ordered pairs. Determine also

i. the domain of R^{-1}

ii. The Range of R.

Answer

A is set of first five natural numbers.

Therefore, A= {1, 2, 3, 4, 5}

Given, (x, y) R x ≤ y

1 is less than 2, 3, 4 and 5.

2 is less than 3, 4 and 5.

3 is less than 4 and 5.

4 is less than 5.

5 is not less than any number A

∴ R = {(1, 2), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)}

An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation. If the graph of a function contains a point (a, b), then the graph of the inverse relation of this function contains the point (b, a).

∴ R^{-1} = {(2, 1), (3, 1), (4, 1), (5, 1), (3, 2), (4, 2), (5, 2), (4, 3), (5, 3), (5, 4)}

⇒ R^{-1} = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)}

i. Domain of R^{-1} = {2, 3, 4, 5}

ii. Range of R = {2, 3, 4, 5}

NOTE: You can see that Domain of R^{-1} is same as Range of R. Similarly, Domain of R is same as Range of R^{-1}

4. Question

Find the inverse relation R^{-1} in each of the following cases:

i. R= {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}

ii. R= {(x, y) : x, y N; x + 2y = 8}

iii. R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x – 3

Answer

An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation. If the graph of a function contains a point (a, b), then the graph of the inverse relation of this function contains the point (b, a).

i. Given, R= {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}

∴ R^{-1} = {(2, 1), (3, 1), (3, 2), (2, 3), (6, 5)}

⇒ R^{-1} = {(2, 1), (2, 3), (3, 1), (3, 2), (6, 5)}

ii. Given, R= {(x, y) : x, y N; x + 2y = 8}

Here, x + 2y = 8

⇒ x = 8 – 2y

As y N, Put the values of y = 1, 2, 3,...... till x N

On putting y=1, x = 8 – 2(1) = 8 – 2 = 6

On putting y=2, x = 8 – 2(2) = 8 – 4 = 4

On putting y=3, x = 8 – 2(3) = 8 – 6 = 2

On putting y=4, x = 8 – 2(4) = 8 – 8 = 0

Now, y cannot hold value 4 because x = 0 for y = 4 which is not a natural number.

∴ R = {(2, 3), (4, 2), (6, 1)}

R^{-1} = {(3, 2), (2, 4), (1, 6)}

⇒ R^{-1} = {(1, 6), (2, 4), (3, 2)}

iii. Given, R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x – 3

Here,

x {11, 12, 13} and y (8, 10, 12}

y = x – 3

On putting x = 11, y = 11 – 3 = 8(8, 10, 12}

On putting x = 12, y = 12 – 3 = 9∉ (8, 10, 12}

On putting x = 13, y = 13 – 3 = 10(8, 10, 12}

∴ R = {(11, 8), (13, 10)}

R^{-1} = {(8, 11), (10, 13)}

5 A. Question

Write the following relations as the sets of ordered pairs:

A relation R from the set {2, 3, 4, 5, 6} to the set {1, 2, 3} defined by x = 2y.

Answer

Let A = {2, 3, 4, 5, 6} and B = {1, 2, 3}

Given, x = 2y where x {2, 3, 4, 5, 6} and y {1, 2, 3}

On putting y = 1, x = 2(1) = 2 A

On putting y = 2, x = 2(2) = 4 A

On putting y = 3, x = 2(3) = 6 A

∴ R = {(2, 1), (4, 2), (6, 3)}

5 B. Question

Write the following relations as the sets of ordered pairs:

A relation R on the set {1,2,3,4,5,6,7} defined by

(x, y) R x is relatively prime to y.

Answer

Relatively prime numbers are also known as co-prime numbers. If there is no integer greater than one that divides both (that is, their greatest common divisor is one). For example, 12 and 13 are relatively prime, but 12 and14 are not as their greatest common divisor is two.

Given, (x, y) R x is relatively prime to y

Here,

2 is co-prime to 3, 5 and 7.

3 is co-prime to 2, 4, 5 and 7.

4 is co-prime to 3, 5 and 7.

5 is co-prime to 2, 3, 4, 6 and 7.

6 is co-prime to 5 and 7.

7 is co-prime to 2, 3, 4, 5 and 6.

∴ R ={(2,3), (2,5), (2,7), (3,2), (3,4), (3,5), (3,7), (4,3), (4.5), (4,7), (5,2), (5,3), (5,4), (5,6), (5,7), (6,5), (6,7), (7,2), (7,3), (7,4), (7,5), (7,6)}

5 C. Question

Write the following relations as the sets of ordered pairs:

A relation R on the set {0,1,2,...,10} defined by

2x + 3y = 12.

Answer

Given, (x, y) R 2x + 3y = 12

where x and y {0,1,2,...,10}

2x + 3y = 12

⇒ 2x = 12 – 3y

$$\Rightarrow x = \frac{12 - 3y}{2}$$

On putting y=0,

$$\Rightarrow x = \frac{12 - 3(0)}{2} = \frac{12}{2} = 6$$

On putting y=2,

$$\Rightarrow x = \frac{12 - 3(2)}{2} = \frac{12 - 6}{2} = \frac{6}{2} = 3$$

On putting y=4,

$$\Rightarrow x = \frac{12 - 3(4)}{2} = \frac{12 - 12}{2} = \frac{0}{2} = 0$$

∴ R = {(0, 4), (3, 2), (6, 0)}

5 D. Question

Write the following relations as the sets of ordered pairs:

A relation R form a set A = {5, 6, 7, 8} to the set B = {10, 12, 15, 16, 18} defined by (x, y)R x divides y.

Answer

Given, (x, y) R x divides y

where x {5, 6, 7, 8} and y {10, 12, 15, 16, 18}

Here,

5 divides 10 and 15.

6 divides 12 and 18.

7 divides none of the value of set B.

8 divides 16.

∴ R = {(5, 10), (5, 15), (6, 12), (6, 18), (8, 16)}

6. Question

Let R be a relation in N defined by (x, y) R x + 2y = 8. Express R and R^{-1} as sets of ordered pairs.

Answer

Given, (x, y) R x + 2y = 8 where x N and y N

x = 8 – 2y

As y N, Put the values of y = 1, 2, 3,...... till x N

On putting y=1, x = 8 – 2(1) = 8 – 2 = 6

On putting y=2, x = 8 – 2(2) = 8 – 4 = 4

On putting y=3, x = 8 – 2(3) = 8 – 6 = 2

On putting y=4, x = 8 – 2(4) = 8 – 8 = 0

Now, y cannot hold value 4 because x = 0 for y = 4 which is not a natural number.

∴ R = {(2, 3), (4, 2), (6, 1)}

An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation. If the graph of a function contains a point (a, b), then the graph of the inverse relation of this function contains the point (b, a).

R^{-1} = {(3, 2), (2, 4), (1, 6)}

⇒R^{-1} = {(1, 6), (2, 4), (3, 2)}

7. Question

Let A = {3, 5} and B = {7, 11}. Let R = {(a, b): a A, b B, a-b is odd}. Show that R is an empty relation from into B.

Answer

Given, A = {3, 5} and B = {7, 11}

R = {(a, b): a A, b B, a-b is odd}

On putting a = 3 and b = 7:

⇒ a – b = 3 – 7 = -4 which is not odd

On putting a = 3 and b = 11:

⇒ a – b = 3 – 11 = -8 which is not odd

On putting a = 5 and b = 7:

⇒ a – b = 5 – 7 = -2 which is not odd

On putting a = 5 and b = 11:

⇒ a – b = 5 – 11 = -6 which is not odd

∴ R = { } = Φ

⇒ R is an empty relation from into B

8. Question

Let A = {1, 2} and B={3, 4}. Find the total number of relations from A into B.

Answer

Given,

A= {1, 2}, B= {3, 4}

n(A) = 2 (Number of elements in set A).

n(B) = 2 (Number of elements in set B).

We know,

n(A × B) = n(A) × n(B) = 2 × 2 = 4

∴ Number of relations from A to B are 4.

NOTE:

Given,

A= {1, 2}, B= {3, 4}

A relation from A to B can be defined as:

A × B = {(1, 3), (1, 4), (2, 3), (2, 4)}

∴ Number of relations from A to B are 4

9 A. Question

Determine the domain and range of the relation R defined by

R = {(x, x+5): x {0, 1, 2, 3, 4, 5}

Answer

Given,

R = {(x, x+5): x {0, 1, 2, 3, 4, 5}

∴ R = {(0, 0+5), (1, 1+5), (2, 2+5), (3, 3+5), (4, 4+5), (5, 5+5)}

⇒ R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}

So,

Domain of relation R = {0, 1, 2, 3, 4, 5}

Range of relation R = {5, 6, 7, 8, 9, 10}

9 B. Question

Determine the domain and range of the relation R defined by

R= {(x, x^3): x is a prime number less than 10}

Answer

Given,

R = {(x, x^3): x is a prime number less than 10}

Prime numbers less than 10 are 2, 3, 5 and 7

∴ R = {(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)}

⇒ R = {(2, 8), (3, 27), (5, 125), (7, 343)}

So,

Domain of relation R = {2, 3, 5, 7}

Range of relation R = {8, 27, 125, 343}

10 A. Question

Determine the domain and range of the following relations:

R= {a, b): a N, a < 5, b = 4}

Answer

Given,

R= {a, b): a N, a < 5, b = 4}

Natural numbers less than 5 are 1, 2, 3 and 4

Therefore, a {1, 2, 3, 4} and b {4}

⇒ R = {(1, 4), (2, 4), (3, 4), (4, 4)}

So,

Domain of relation R = {1, 2, 3, 4}

Range of relation R = {4}

10 B. Question

Determine the domain and range of the following relations:

S= {a, b): b = |a-1|, a Z and |a| ≤ 3}

Answer

Given,

S= {a, b): b = |a-1|, a Z and |a| ≤ 3}

Z denotes integer which can be positive as well as negative

Now, |a| ≤ 3 and b = |a-1|

∴ a {-3, -2, -1, 0, 1, 2, 3}

S = {a, b): b = |a-1|, a Z and |a| ≤ 3}

⇒ S = {a, |a-1|): b = |a-1|, a Z and |a| ≤ 3}

⇒ S = {(-3, |-3 – 1|), (-2, |-2 – 1|), (-1, |-1 – 1|), (0, |0 – 1|), (1, |1 – 1|), (2, |2 – 1|), (3, |3 – 1|)}

⇒S = {(-3, |-4|), (-2, |-3|), (-1, |-2|), (0, |-1|), (1, |0|), (2, |1|), (3, |2|)}

⇒S = {(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}

So,

Domain of relation S = {-3, -2, -1, 0, 1, 2, 3}

Range of relation S = {0, 1, 2, 3, 4}

11. Question

Let A = {a, b}. List all relations on A and find their number.

Answer

The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A)=p and n(B)=q, then n(A × B)= p q. So, the total number of relations is 2^{pq}.

Now,

A × A = {(a, a), (a, b), (b, a), (b, b)}

Total number of relations are all possible subsets of A × A:

{ Φ, {(a, a)}, {(a, b)}, {(b, a)}, {(b, b)}, {(a, a), (a, b)}, {(a, a), (b, a)}, {(a, a), (b, b)}, {(a, b), (b, a)}, {(a, b), (b, b)}, {(b, a), (b, b)}, {(a, a), (a, b), (b, a)}, {(a, b), (b, a), (b, b)}, {(a, a), (b, a), (b, b)}, {(a, a), (a, b), (b, b)}, {(a, a), (a, b), (b, a), (b, b)}}

n(A) = 2 ⇒ n(A × A) = 2 × 2 = 4

∴ Total number of relations = 2^4 = 16

12. Question

Let A= {x, y, z} and B= {a, b}. Find the total number of relations from A into B.

Answer

The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n(A)=p and n(B)=q, then n(A × B)= pq. So, the total number of relations is 2^{pq}.

n(A) = 3 and n(B) = 2 ⇒ n(A × B) = 3 × 2 = 6

∴ Total number of relations = 2^6 = 64

13. Question

Let R be a relation from N to N defined by R= {(a, b): a, b N and a = b^2}.

Are the following statements true?

i. (a, a) R for all a N

ii. (a, b) R (b, a) R

iii. (a, b) R and (b, c) R (a, c) R

Answer

Given, R= {(a, b): a, b N and a = b^2}

i. (a, a) R for all a N

Here, take b = 2

⇒ a = b^2 = 2^2 = 4

∴ (4, 2) R but (2, 2) ∉ R

As, $2^2 \neq 2$

So,

No, the statement is false.

ii. (a, b) R (b, a) R

Here, take b = 2

⇒ a = b^2 = 2^2 = 4

∴ (4, 2) R but (2, 4) ∉ R

As, $4^2 \neq 2$

So,

No, the statement is false.

iii. (a, b) R and (b, c) R (a, c) R

Here, take b = 4

⇒ a = b^2 = 4^2 = 16

⇒ (16, 4) R

Now, b = c^2

⇒ 4 = c^2

⇒ c = -2 ∉ N or 2 N

⇒ (4, 2) R

But (16, 2) ∉ R

As, $2^2 \neq 16$

So,

No, the statement is false.

14. Question

Let A= {1, 2, 3,....,14}. Define a relation on a set A by R= {(x, y): 3x – y = 0, where x, yA}. Depict this relationship using an arrow diagram. Write down its domain, co-domain and range.

Answer

Given, R= {(x, y): 3x – y = 0, where x, y A}

A= {1, 2, 3,...,14}

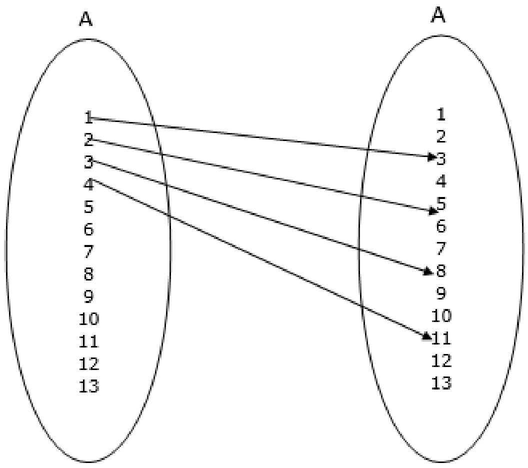
As, y = 3x

∴ R= {(x, 3x): where x, 3x A}

⇒ R= {(1, 3×1), (2, 3×2), (3, 3×3), (4, 3×4)}

<u>NOTE: We cannot include (5, 3×5) as 15 ∉ A</u>

⇒ R = {(1, 3), (2, 6), (3, 9), (4, 12)}



Co-Domain

So,

Domain of relation R= {1, 2, 3, 4}

Co-Domain of relation R= {1, 2, 3,...,14}= A

Range of relation R= {3, 6, 9, 12}

15. Question

Define a relation R on the set N of natural numbers by R = {(x, y): y = x + 5, x is a natural less than 4, x, y N}.Depict this relationship using (i) roster form an arrow diagram. Write down the domain and range of R.

Answer

Given,

R = {(x, y): y = x + 5, x is a natural less than 4, x, yN}

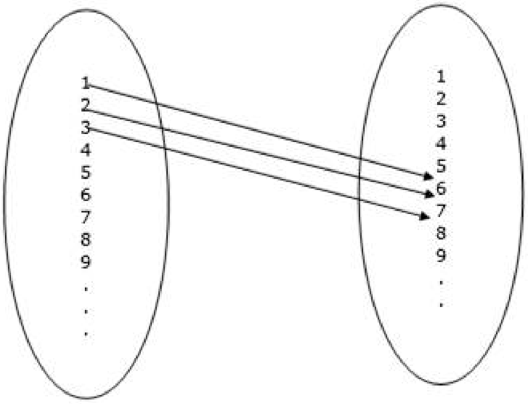
Natural numbers less than 4 are 1, 2 and 3.

On putting x= 1, y = 1 + 5 = 6

On putting x= 2, y = 2 + 5 = 7

On putting x= 3, y = 3 + 5 = 8

i. R = {(1, 6), (2, 7), (3, 8)}



So,

Domain of relation R= {1, 2, 3}

Range of relation R= {6, 7, 8}

16. Question

A = {1, 2, 3, 5} and B = {4, 6, 9}. Define a relation R from A to B by R = {(x, y): the difference between x and y is odd, x A, y B}. Write R in Roster form.

Answer

Given,

Relation R from A to B by R = {(x, y): the difference between x and y is odd, x A, y B}

A = {1, 2, 3, 5} and B = {4, 6, 9}

For x = 1:

y – x = 4 – 1 = 3 which is odd ⇒ 4 y

y – x = 6 – 1 = 5 which is odd ⇒ 6 y

y – x = 9 – 1 = 8 which is even ⇒ 8 ∉ y

For x = 2:

y – x = 4 – 2 = 2 which is even ⇒ 4 ∉ y

y – x = 6 – 2 = 4 which is even ⇒ 6 ∉ y

y – x = 9 – 2 = 7 which is odd ⇒ 8 y

For x = 3:

y – x = 4 – 3 = 1 which is odd ⇒ 4 y

y – x = 6 – 3 = 3 which is odd ⇒ 6 y

y – x = 9 – 3 = 6 which is even ⇒ 8 ∉ y

For x = 5:

x – y = 5 – 4 = 1 which is odd ⇒ 4 y

y – x = 6 – 5 = 4 which is even ⇒ 6 ∉ y

y – x = 9 – 4 = 4 which is even ⇒ 8 ∉ y

∴ R = {(1, 4), (1, 6), (2, 8), (3, 4), (3, 6), (5, 4)}

NOTE:

Domain of relation R= {1, 2, 3, 5}

Range of relation R= {4, 6, 8}

17. Question

Write the relation R = {(x, x^3): x is a prime number less than 10} in roster form.

Answer

Given,

R = {(x, x^3): x is a prime number less than 10}

Prime numbers less than 10 are 2, 3, 5 and 7

∴ R = {(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)}

⇒ R = {(2, 8), (3, 27), (5, 125), (7, 343)}

So,

Domain of relation R = {2, 3, 5, 7}

Range of relation R = {8, 27, 125, 343}

18. Question

Let A= {1, 2, 3, 4, 5, 6}. Let R be a relation on A defined by

R= {(a, b): a, b A, b is exactly divisible by a}

i. Write R in roster form

ii. Find the domain of R

iii. Find the range of R.

Answer

Given,

R= {(a, b): a, b A, b is exactly divisible by a}

A= {1, 2, 3, 4, 5, 6}

Here,

6 is exactly divisible by 1, 2, 3 and 6

5 is exactly divisible by 1 and 5

4 is exactly divisible by 1, 2 and 4

3 is exactly divisible by 1 and 3

2 is exactly divisible by 1 and 2

1 is exactly divisible by 1

i. R = {(1, 1), (2, 1), (2, 2), (3, 1), (3, 3), (4, 1), (4, 2), (4,4), (5, 1), (5, 5), (6, 1), (6, 2), (6, 3), (6, 6)}

ii. Domain of relation R = {1, 2, 3, 4, 5, 6}

iii. Range of relation R = {1, 2 , 3, 4, 5, 6}

19. Question

Figure 2.15 shows a relationship between the sets P and Q. Write this relation in

a. Set builder form

b. Roster form

c. What is its domain and range?

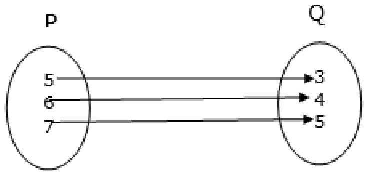


Fig. 2.15

Answer

i. Since 5 – 3 = 6 – 4 = 7 – 5 = 2

∴ x – y = 2 where x P and y Q

So,

R = {(x, y): x – y = 2, x P, y Q}

ii. Now, R = {(5, 3), (6, 4), (7, 5)}

iii. Domain of relation R = {5, 6, 7}

Range of relation R = {3, 4, 5}

20. Question

Let R be the relation on Z defined by R = {(a, b) Z, a – b is an integer}. Find the domain and range of R.

Answer

Given, R = {(a, b) Z, a – b is an integer}

Z denotes integer, and here a and b both are integers

We know that difference of two integers is always an integer

∴ a and b can be any integer in relation R

⇒ The domain of relation R = Z (as a Z)

The range of relation R = Z (as b Z)

21. Question

For the relation R_1 defined on R by the rule (a, b) R_1 1 + ab > 0. Prove that: (a, b) R_1 and (b,c) R_1 (a, c) R_1 is not true for all a, b, c R.

Answer

To prove: (a, b) R_1 and (b,c) R_1 (a, c) R_1 is not true for all a, b, c R.

Given R_1 = {(a, b): 1 + ab > 0}

Let a = 1, b = -0.5, c = -4

Here, (1, -0.5) R_1 [∵ 1+(1×-0.5) = 0.5 > 0]

And, (-0.5, -4) R_1 [∵ 1+(-0.5×-4) = 3 > 0]

But, (1, -4) ∉ R_1 [∵ 1+(1×-4) = -3 < 0]

∴ (a, b) R_1 and (b,c) R_1 (a, c) R_1 is not true for all a, b, c R

Hence Proved.

NOTE:

Here R_1 is a relation whereas R denotes a real number.

22. Question

Let R be a relation on N x N defined by (a, b) R (c, d) a + d = b + c for all (a, b), (c, d) N x N.

Show that:

i. (a, b) R (a, b) for all (a, b) N x N

ii. (a, b) R (c, d) (c, d) R (a, b) for all (a, b), (c, d) N x N

iii. (a, b) R (c, d) and (c, d) R (e, f) (a, b) R (e, f) for all (a, b), (c, d), (e, f) N × N

Answer

Given,

(a, b) R (c, d) a + d = b + c for all (a, b), (c, d) N x N

i. (a, b) R (a, b)

⇒ a + b = b + a for all (a, b) N x N

∴ (a, b) R (a, b) for all (a, b) N x N

ii. (a, b) R (c, d)

⇒ a + d = b + c ⇒ c + b = d + a

⇒ (c, d) R (a, b) for all (c, d), (a, b) N x N

iii. (a, b) R (c, d) and (c, d) R (e, f)

a + d = b + c and c + f = d + e

⇒ a + d + c + f = b + c + d + e

⇒ a + f = b + c + d + e – c – d

⇒ a + f = b + e

⇒ (a, b) R (e, f) for all (a, b), (c, d), (e, f) N × N

Very Short Answer

1. Question

If A = {1, 2, 4}, B = {2, 4, 5} and C = {2, 5}, write (A − C) × (B − C).

Answer

Here, A={1,2,4} ,B={2,4,5} ,C={2,5}

So, A \ C={1,2,4} \ {2,5}

={1,4}

Also,B \ C={2,4,5} \ {2,5}

={4}

Now, (A \ B)×(B \ C)={(x,y):x϶(A \ C) and y϶(B \ C) }

={(1,4), (4,4)}

2. Question

If n(A) = 3, n(B) = 4, ten write n (A × A × B).

Answer

We know ,n(A×B)=n(A)×n(B)

Similarly, n(A×B×C)=n(A)×n(B)×n(C)

Here, n(A)=3 and n(B)=4

n(A×A×C)=n(A)×n(A)×n(B)

$=3\times3\times4$

$=36$

3. Question

If R is a relation defined on the set Z of integers by the rule $(x, y) \in R \Leftrightarrow x^2 + y^2 = 9$, then write domain of R.

Answer

Here, relation R is defined on the set Z.

And by the given definition of relation, $R=\{(x,y) : x,y\in Z ,x^2 +y^2=9\}$

$=\{(-3,0),(0,-3),(3,0),(0,3)\}$

Now we know,Domain is the set which consist all first elements of ordered pairs in relation R.

So,Domain(R)={-3,0,3}

4. Question

If $R = \{(x, y) : x, y\in Z, x^2 + y^2\le 4\}$ is a relation defined on the set Z of integers, then write domain of R.

Answer

Here, relation R is defined on the set Z.

And $R=\{(x,y) : x,y\in Z,x^2+y^2\le4\}$

$=\{(-2,0),(-1,0),(0,0),(1,0),(2,0),(0,-2),(0,-1),(0,1),(0,2),(1,1),(-1,-1),(1,-1),(-1,1)\}$

Now we know,Domain is the set which consist all first elements of ordered pairs in relation R.

So,Domain(R)={-2,-1,0,1,2}

5. Question

If R is a relation from set $A = \{11, 12, 13\}$ to set $B = \{8, 10, 12\}$ defined by $y = x - 3$, the write R^{-1}.

Answer

Here,A={11,12,13} and B={8,10,12}

Also,$R=\{(x,y) : y=x-3 ,x\in A ,y\in B\}$

$=\{(11,8),(13,10)\}$

Now,$R^{-1} =\{(x,y) : x=y+3 ,x\in B ,y\in A\}$

$=\{(8,11),(10,13)\}$

6. Question

Let $A = \{1, 2, 3\}$ and $R = \{(a, b) : | a^2- b^2 | \le5, a, b \in A\}$. Then write R as set of ordered pairs.

Answer

Here,A={1,2,3}

$R=\{(a,b) : |a^2-b^2|\le5,a,b \in A\}$

$=\{(1,1),(1,2),(2,1),(2,2),(2,3),(3,2),(3,3)\}$

7. Question

Let $R = \{(x, y) : x, y \in Z, y = 2x - 4\}$. If $(a, -2)$ and $(4, b^2)\in R$, then write the values of a and b.

Answer

Here,$R=\{(x,y) : x, y\in Z, y=2x-4\}$

Now,(a,-2)ϵ R ⇒ -2=2a-4

∴ 2a=-2+4

∴ 2a=2

∴ a=1

Also,$(4,b^2)$ϵ R ⇒ b^2=2×4-4

∴ b^2=8-4

∴ b^2=4

∴ b=±2

8. Question

If $R = \{(2, 1), (4, 7), (1, -2), ...\}$, then write the linear relation between the components of the ordered pairs of the relation R.

Answer

Here,R={(2,1),(4,7),(1,-2),...}

It is seen that all the elements of R have a rule that (x,y)ϵ R ⇒ y+5=3x

i.e y=3x-5.

9. Question

If $A = \{1, 3, 5\}$ and $B = \{2, 4\}$, list the elements of R, if $R = \{(x, y) : x, y \in A \times B$ and $x > y\}$.

Answer

Here, A={1,3,5} and B={2,4}

Also, R={ (x,y) : x,yϵ A×B and x>y}

={(3,2) ,(5,2) ,(5,4)}

10. Question

If $R = \{(x, y) : x, y \in W, 2x + y = 8\}$, then write the domain and range of R.

Answer

Here, R={ (x, y) :x ,yϵ W,2x+y=8}

={(0,8) ,(1,6) ,(2,4) ,(3,2) ,(4,0)}

Now we know, Domain is the set which consist all first elements of ordered pairs in relation R.

So, Domain(R)= {0,1,2,3,4}

Also we know, Range is the set which consist all second elements of ordered pairs in relation R.

So, Range(R)={0,2,4,6,8}

11. Question

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, write A and B.

Answer

Here, A×B ={(x,1) ,(y,2) ,(z,1)} and n(A)=3 , n(B)=2

We know ,A×B = {(x,y) : xϵA and yϵ B}

∴ A={x,y,z} and B={1,2}

12. Question

Let $A = \{1, 2, 3, 5\}$, $B = \{4, 6, 9\}$ and R be a relation from A to B defined by $R = \{(x, y) : x - y$ is odd$\}$. Write

R in roster form.

Answer

Here, A= {1, 2, 3, 5} and B={4,6,9}

Also, R= {(x, y) : x-y is odd}

={(1,4),(1,6),(2,9),(3,4),(3,6),(5,4),(5,6)}

MCQ

1. Question

Mark the correct alternative in the following:

If $A = \{1, 2, 4\}$, $B = \{2, 4, 5\}$, $C = \{2, 5\}$, then $(A-B) \times (B - C)$ is

A. {(1, 2), (1, 5), (2, 5)}

B. {(1, 4)}

C. (1, 4)

D. none of these.

Answer

When we subtract two sets, say (A – B), the result will be a set obtained on removing those elements from A which also exist in B.

Note : We do not consider the elements of the subtracted set(here B) if it is not present in A.

So, we have (A – B) = {1, 2, 4} – {2, 4, 5}

= {1}

Similarly, we have (B - C) = {2, 4, 5} – {2, 5}

= {4}

[When we multiply two sets, each element of first set is paired with every element of other in an ordered pair of form (x, y)

where x belongs to first set and y to the other.]

Now, $(A-B) \times (B - C) = \{1\} \times \{4\}$

= {(1, 4)}

Since, it is a set so it is written in curly braces.

Therefore, option B is correct.

2. Question

Mark the correct alternative in the following:

If R is a relation on the set $A = \{1, 2, 3, 4, 5, 6, 7, 9\}$ given by $xRyy = 3x$, then $R =$

A. {(3, 1), (6, 2), (8, 2), (9, 3)}

B. {(3, 1), (6, 2), (9, 3)}

C. {(3, 1), (2, 6), (3, 9)}

D. none of these.

Answer

Here, y = 3x;

If x = 1; then y = 3.

If x = 2; then y = 6.

If x = 3; then y = 9.

Therefore the required relation will be R = {(1, 3), (2, 6), (3, 9)}.

So, correct answer is D.

3. Question

Mark the correct alternative in the following:

Let $A = \{1, 2, 3\}$, $B = \{1, 3, 5\}$. If relation R and A to B is given by $R = \{(1, 3), (2, 5), (3, 3)\}$. Then R^{-1} is

A. {(3, 3), (3, 1), (5, 2)}

B. {(1, 3), (2, 5), (3, 3)}

C. {(1, 3), (5, 2)}

D. none of these.

Answer

Inverse of a relation is given by interchanging the element's position in each pair.

Ex: Inverse of relation P = {(x, y)} is given by P^{-1} = {(y, x)}.

Therefore, R^{-1} = {(3, 1), (5, 2), (3, 3)}.

So, option A is correct.

4. Question

Mark the correct alternative in the following:

If $A = \{1, 2, 3\}$ $B = \{1, 4, 6, 9\}$ and R is a relation from A to B defined by 'x is greater than y. The range of R is

A. {1, 4, 6, 9}

B. {4, 6, 9}

C. {1}

D. none of these.

Answer

As per condition of the relation, x > y.

So, required relation will be : {(2, 1), (3, 1)}

Since we know that Range is the set of elements written after comma in each ordered pair.

Therefore, Range = {1}

So, option C is correct.

5. Question

Mark the correct alternative in the following:

If $R = \{(x, y) : x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation on Z, *then domain of* R is

A. {0, 1, 2}

B. {0, −1, −2}

C. {−2, −1, 0, 1, 2}

D. none of these.

Answer

Domain of R is a set constituting all values of x.

Here, possible values for x by equation $x^2 + y^2 \leq 4$ will be 0, 1, -1, 2, -2.

So, Domain of R is : {-2, -1, 0, 1, 2}.

Therefore, option C is correct.

6. Question

Mark the correct alternative in the following:

A relation *R* is defined from {2, 3, 4, 5} to {3, 6, 7, 10} by :$xRyx$ is relatively prime to is relatively prime to y. Then, domain of *R* is

A. {2, 3, 5}

B. {3, 5}

C. {2, 3, 4}

D. {2, 3, 4, 5}.

Answer

Relatively prime numbers are those numbers which have only 1 as the common factor.

So, according to this definition we get to know that (2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6) ,(5, 7) are relatively prime.

So, R ={(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6) ,(5, 7)}.

Therefore, Domain of R is the values of x or the first elemenyt of the ordered pair.

So, Domain = {2, 3, 4, 5}.

So, option D is correct.

7. Question

Mark the correct alternative in the following:

A relation φ from *C* to *R* is defined by $x\varphi y\ |x| = y$. Which one is correct?

A. $(2 + 3i)\ \varphi\ 13$

B. $3\ \varphi\ (-3)$

C. $(1 + i)\ \varphi\ 2$

D. $i\varphi\ 1$.

Answer

We have $x\varphi y\ |x| = y$

By checking the options,

A. $(2 + 3i)\ \varphi\ 13$

X = 2 + 3i;

$|x| = \sqrt{2^2 + 3^2}$

= √13

Therefore, |x|≠ y.

So, option A is incorrect.

B. $3\ \varphi\ (-3)$

X = 3;

$|x| = \sqrt{3^2}$

= 3

3 ≠(-3)

Therefore, option B is incorrect.

C. (1 + *i*) φ 2

X = 1+ i;

$|x| = \sqrt{1^2 + 1^2}$

= √2

√2 ≠ 2

Therefore, option C is also incorrect.

D. *i*φ 1

x = i;

$|x| = \sqrt{1^2}$

= 1

1 = 1

|x| = y.

Therefore, option D is correct.

8. Question

Mark the correct alternative in the following:

Let *R* be a relation on *N* defined by *x* + 2 *y* = 8. The domain of *R* is

A. {2, 4, 8}

B. {2, 4, 6, 8}

C. {2, 4, 6}

D. {1, 2, 3, 4}.

Answer

We have , x + 2y = 8

$y = \frac{8-x}{2}$

since, x and y are Natural numbers, So .. x must be an even number..

if x = 2, y = 3;

if x = 4, y = 2;

if x = 6, y = 1.

So, relation R = {(2, 3), (4, 2), (6, 1)}

Now, domain of R is {2, 4, 6}.

So, option C is correct.

9. Question

Mark the correct alternative in the following:

R is a relation from {11, 12, 13} to {8, 10, 12} defined by $y = x - 3$. Then, R^{-1} is

A. {(8, 11), (10, 13)}

B. {(11, 8), (13, 10)}

C. {(10, 13), (8, 11), (12, 10)}

D. none of these.

Answer

Since, y = x – 3;

Therefore, for x = 11, y = 8.

For x = 12, y = 9.[But the value y = 9 does not exist in the given set.]

For x = 13, y =10.

So, we have R = {(11, 8), (13, 10)}

Now, R^{-1} = {(8, 11), (10, 13)}.

Therefore, option A is correct.

10. Question

Mark the correct alternative in the following:

If the set A has p elements, B has q elements, then the number of elements in $A \times B$ is

A. $p + q$

B. $p + q + 1$

C. pq

D p^2

Answer

Since A has p elements, so each element of A will make a pair with each element of set B , and so other elements of A will do the same..

This is all forms q + q + q+ q +.......{p-times} ordered pairs...

Therefore, we will have a total of pq elements in $A \times B$.

11. Question

Mark the correct alternative in the following:

Let R be a relation from a set A to a set B, then

A. $\mathrm{R} = \mathrm{A} \cup \mathrm{B}$

B. $\mathrm{R} = \mathrm{A} \cap \mathrm{B}$

C. $\mathrm{R} \subseteq \mathrm{A} \times \mathrm{B}$

D. $\mathrm{R} \subseteq \mathrm{B} \times \mathrm{A}$

Answer

Since, R is a relation from set A to set B, therefore it will always be a subset of A x B.

So, option C is correct.

12. Question

Mark the correct alternative in the following:

If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is

A. 2^{mn}

B. $2^{mn} - 1$

C. $2mn$

D. m^n

Answer

Since we know that a relation from A to B consists of mn ordered pairs if they contain m and n elements respectively..

Each subset of those mn pairs will be a relation..so, each pair has two choices, either to be in that particular relation or not.

So, we have a tptal of 2^{mn} relations.

Therefore, option A is correct.

13. Question

Mark the correct alternative in the following:

If R is a relation on a finite set having n elements, then the number of relations on A is

A. 2^n

B. 2^{n^2}

C. n^2

D. n^n

Answer

A is a set of n elements.

A x A will have a total of n^2 elements.

Then, the number of relations on A will be 2^{n^2}.

Therefore, option B is correct.