2. Powers

Exercise 2.1

1. Question

Express each of the following as a rational number of the form $\frac{q}{p}$, where p and q are integers and $q \neq 0$:

(i) 2^{-3}

(ii) $(-4)^{-2}$

(iii) $\frac{1}{3^{-2}}$

(iv) $\left(\frac{1}{2}\right)^{-5}$

(v) $\left(\frac{2}{3}\right)^{-2}$

Answer

(i) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ [Using $a^{-n} = \frac{1}{a^n}$]

(ii) $(-4)^{-2} = \frac{1}{-4^2} = \frac{1}{(-4)\times(-4)} = \frac{1}{16}$ [Using $a^{-n} = \frac{1}{a^n}$]

(iii) $\frac{1}{3^{-2}} = \frac{3\times3}{1} = \frac{9}{1}$ [Using $\frac{1}{a^{-n}} = a^n$]

(iv) $\left(\frac{1}{2}\right)^{-5} = \frac{2^5}{1^5} = \frac{2^5}{1} = \frac{32}{1}$ [Using $a^{-n} = \frac{1}{a^n}$]

(v) $\left(\frac{2}{3}\right)^{-2} = \frac{3^2}{2^2} = \frac{9}{4}$ [Using $a^{-n} = \frac{1}{a^n}$; $a^2 = a\times a$]

2. Question

Find the values of each of the following:

(i) $3^{-1} + 4^{-1}$

(ii) $\left(3^0 + 4^{-1}\right) \times 2^2$

(iii) $\left(3^{-1} + 4^{-1} + 5^{-1}\right)^0$

(iv) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} + \left(\frac{1}{4}\right)^{-2}\right\}$

Answer

(i) $3^{-1} + 4^{-1}$

$\Rightarrow \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}$ (LCM of 3 and 4 is 12) [Using $a^{-n} = \frac{1}{a^n}$]

(ii) $\left(3^0 + 4^{-1}\right) \times 2^2$

$\Rightarrow \left(1 + \frac{1}{4}\right)\times4 = \left(\frac{4+1}{4}\right)\times4 = \frac{5\times4}{4} = 5$ (LCM of 1 and 4 is 4) [Using $a^{-n} = \frac{1}{a^n}$; $a^o = 1$; $a^2 = a\times a$]

(iii) $\left(3^{-1} + 4^{-1} + 5^{-1}\right)^0$

$\Rightarrow \left(3^{-1} + 4^{-1} + 5^{-1}\right)^0 = 1$ [Using $a^o = 1$]

We know that any number to power zero is always equal to 1.

(iv) $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1} + \left(\frac{1}{4}\right)^{-2}\right\}$

$\Rightarrow (3) - (4) + (4^2)$ [Using $a^{-n} = \frac{1}{a^n}$; $a^2 = a \times a$]

$3 - 4 + 16 = 15$

3. Question

Find the values of each of the following:

(i) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{4}\right)^{-1}$

(ii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iii) $\left(2^{-1} \times 4^{-1}\right) \div 2^{-2}$

(iv) $\left(5^{-1} \times 2^{-1}\right) \div 6^{-1}$

Answer

(i) $\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{3}\right)^{-1} + \left(\frac{1}{4}\right)^{-1}$

$\Rightarrow (2) + (3) + (4)$ [Using $a^{-n} = \frac{1}{a^n}$]

$\Rightarrow 2 + 3 + 4 = 9$

(ii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

$\Rightarrow (2^2) + (3^2) + (4^2)$ [Using $a^{-n} = \frac{1}{a^n}$; $a^2 = a \times a$]

$\Rightarrow 4 + 9 + 16 = 29$

(iii) $\left(2^{-1} \times 4^{-1}\right) \div 2^{-2}$

$\Rightarrow \left(\frac{1}{2} \times \frac{1}{4}\right) \div \left(\frac{1}{2^2}\right)$ [Using $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

$\Rightarrow \left(\frac{1 \times 1}{2 \times 4}\right) \times 2^2$ [Using $\frac{1}{a} \times \frac{1}{b} = \frac{1}{ab}$; $a^2 = a \times a$]

$\Rightarrow \frac{1}{8} \times 4 = \frac{1}{2}$

(iv) $\left(5^{-1} \times 2^{-1}\right) \div 6^{-1}$

$\Rightarrow \left(\frac{1}{5} \times \frac{1}{2}\right) \div \left(\frac{1}{6}\right)$ [Using $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

$\Rightarrow \left(\frac{1 \times 1}{5 \times 2}\right) \times \left(\frac{6}{1}\right)$ [Using $\frac{1}{a} \times \frac{1}{b} = \frac{1}{ab}$]

$\Rightarrow \frac{1}{10} \times 6 = \frac{6}{10} = \frac{3}{5}$

4. Question

Simplify:

(i) $\left(4^{-1} \times 3^{-1}\right)^2$

(ii) $\left(5^{-1} \div 6^{-1}\right)^{3}$

(iii) $\left(2^{-1} \times 4^{-1}\right) \div 2^{-2}$

(iv) $\left(3^{-1} \times 4^{-1}\right)^{-1} \times 5^{-1}$

Answer

(i) $\left(4^{-1} \times 3^{-1}\right)^{2}$

$\Rightarrow \left(\frac{1}{4} \times \frac{1}{3}\right)^{2}$ [Using $a^{-n} = \frac{1}{a^n}; a^2 = a \times a$]

$\Rightarrow \frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$

(ii) $\left(5^{-1} \div 6^{-1}\right)^{3}$

$\Rightarrow \left(\frac{1}{5} \div \frac{1}{6}\right)^{3}$ [Using $a^{-n} = \frac{1}{a^n}; a^n = a \times a \ldots \ldots n\ times$]

$\Rightarrow \left(\frac{1}{5} \times \frac{6}{1}\right)^{3} = \left(\frac{6}{5}\right)^{3} = \frac{216}{125}$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

(iii) $\left(2^{-1} \times 4^{-1}\right) \div 2^{-2}$

$\Rightarrow \left(\frac{1}{2} \times \frac{1}{4}\right) \div \frac{1}{2^2}$ [Using $a^{-n} = \frac{1}{a^n}; a^2 = a \times a$]

$\Rightarrow \frac{1}{8} \times \frac{4}{1} = \frac{1}{2}$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

(iv) $\left(3^{-1} \times 4^{-1}\right)^{-1} \times 5^{-1}$

$\Rightarrow \left(\frac{1}{3} \times \frac{1}{4}\right) \times \frac{1}{5}$ [Using $a^{-n} = \frac{1}{a^n}$]

$\Rightarrow \frac{1}{12} \times \frac{1}{5} = \frac{1}{60}$

5. Question

Simplify:

(i) $\left(3^2 + 2^2\right) \times \left(\frac{1}{2}\right)^{3}$

(ii) $\left(3^2 + 2^2\right) \times \left(\frac{2}{3}\right)^{-3}$

(iii) $\left[\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$

(iv) $\left(2^2 + 3^2 - 4^2\right) \div \left(\frac{3}{2}\right)^{2}$

Answer

(i) $\left(3^2 + 2^2\right) \times \left(\frac{1}{2}\right)^{3}$

$\Rightarrow (9 + 4) \times \frac{1}{2^3}$ [Using $a^n = a \times a \ldots \ldots n\ times$]

$\Rightarrow 13 \times \frac{1}{8} = \frac{13}{8}$

(ii) $\left(3^2+2^2\right)\times\left(\frac{2}{3}\right)^{-3}$

$\Rightarrow (9+4)\times\left(\frac{3}{2}\right)^3$ [Using $a^{-n}=\frac{1}{a^n}$; $a^n = a\times a\ldots\ldots n\ times$]

$\Rightarrow\ 13\times\frac{27}{8}=\frac{351}{8}$

(iii) $\left[\left(\frac{1}{3}\right)^{-3}-\left(\frac{1}{2}\right)^{-3}\right]\div\left(\frac{1}{4}\right)^{-3}$

$\Rightarrow [(3^3)-(2)^3]\div(4^3)$ [Using $a^{-n}=\frac{1}{a^n}$]

$\Rightarrow\ [27-8]\div 4^3$

$\Rightarrow 19\times\frac{1}{4^3}$ [Using $and\ \frac{1}{a}\div\frac{1}{b}=\frac{1}{a}\times\frac{b}{1}$]

$\Rightarrow 19\times\frac{1}{64}=\frac{19}{64}$ [Using $a^n = a\times a\ldots\ldots n\ times$]

(iv) $\left(2^2+3^2-4^2\right)\div\left(\frac{3}{2}\right)^2$

$\Rightarrow (4+9-16)\div\left(\frac{3}{2}\right)^2$ [Using $a^n = a\times a\ldots\ldots n\ times$]

$\Rightarrow\ (-3)\div\frac{9}{4}$

$\Rightarrow -3\times\frac{4}{9}=\ -\frac{4}{3}$ [Using $and\ \frac{1}{a}\div\frac{1}{b}=\frac{1}{a}\times\frac{b}{1}$]

6. Question

By what number should 5^{-1} be multiplied so that the product many be equal to $(-7)^{-1}$?

Answer

Let the number 5^{-1} shlould be multiplied by x

According to the question:

$x\times 5^{-1} = (-7)^{-1}$

$x\times\frac{1}{5}=\frac{1}{-7}$ [Using $a^{-n}=\frac{1}{a^n}$]

$x=\frac{1}{-7}\times\frac{5}{1}=\frac{5}{-7}$

Therefore 5^{-1} should be multiplied by $\frac{-5}{7}$

7. Question

By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product may be equal to $\left(-\frac{4}{7}\right)^{-1}$?

Answer

Let the number $\left(\frac{1}{2}\right)^{-1}$ shlould be multiplied by x

According to the question:

$x\times\left(\frac{1}{2}\right)^{-1}=\left(-\frac{4}{7}\right)^{-1}$

$x \times 2 = -\frac{7}{4}$ [Using $a^{-n} = \frac{1}{a^n}$]

$x = -\frac{7}{4} \times \frac{1}{2} = -\frac{7}{8}$

Therefore $\left(\frac{1}{2}\right)^{-1}$ should be multiplied by $-\frac{7}{8}$

8. Question

By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Answer

Let the number $(-15)^{-1}$ shlould be devided by x

According to the question:

$(-15)^{-1} \div x = (-5)^{-1}$

$\frac{1}{-15} \div x = -\frac{1}{5}$ [Using $a^{-n} = \frac{1}{a^n}$]

$-\frac{1}{15} \times \frac{1}{x} = -\frac{1}{5}$ [using $p \div q = p \times \frac{1}{q}$]

$\frac{1}{x} = \frac{(-1)\times(-15)}{5}$

$\frac{1}{x} = 3$

$x = \frac{1}{3}$

Therefore $(-15)^{-1}$ should be devided by $\frac{1}{3}$

Exercise 2.2

1. Question

Write each of the following in exponential form:

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1}$

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$

Answer

(i) $\left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1} \times \left(\frac{3}{2}\right)^{-1}$

$\Rightarrow \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^{4}$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots\ldots n\ times$]

(ii) $\left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2} \times \left(\frac{2}{5}\right)^{-2}$

$\Rightarrow \left(\frac{2}{5}\right)^{-6} = \left(\frac{5}{2}\right)^{6}$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots\ldots n\ times$]

2. Question

Evaluate:

(i) 5^{-2}

(ii) $(-3)^{-2}$

(iii) $\left(\frac{1}{3}\right)^{-4}$

(iv) $\left(\frac{-1}{2}\right)^{-1}$

Answer

(i) 5^{-2}

$\Rightarrow \frac{1}{5^2} = \frac{1}{25}$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots \ldots n\, times$]

(ii) $(-3)^{-2}$

$\Rightarrow \left(\frac{1}{-3}\right)^2 = \frac{1}{-3} \times \frac{1}{-3} = \frac{1}{9}$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots \ldots n\, times$]

(iii) $\left(\frac{1}{3}\right)^{-4}$

$\Rightarrow \left(\frac{3}{1}\right)^4 = 3^4 = 81$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots \ldots n\, times$]

(iv) $\left(\frac{-1}{2}\right)^{-1}$

$\Rightarrow \frac{-2}{1} = -2$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots \ldots n\, times$]

3. Question

Express each of the following as a rational number in the form $\frac{p}{q}$:

(i) 6^{-1}

(ii) $(-7)^{-1}$

(iii) $\left(\frac{1}{4}\right)^{-1}$

(iv) $(-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1}$

(v) $\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1}$

Answer

(i) 6^{-1}

$\Rightarrow 6^{-1} = \frac{1}{6}$ [Using $a^{-n} = \frac{1}{a^n}$]

(ii) $(-7)^{-1}$

$\Rightarrow (-7)^{-1} = \frac{1}{-7} = -\frac{1}{7}$ [Using $a^{-n} = \frac{1}{a^n}$]

(iii) $\left(\frac{1}{4}\right)^{-1}$

$\Rightarrow \left(\frac{1}{4}\right)^{-1} = 4$ [Using $a^{-n} = \frac{1}{a^n}$]

(iv) $(-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1}$

$\Rightarrow (-4)^{-1} \times \left(\frac{-3}{2}\right)^{-1} = \frac{1}{-4} \times \frac{2}{-3} = \frac{2}{12} = \frac{1}{6}$ [Using $a^{-n} = \frac{1}{a^n}$]

(v) $\left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1}$

$\Rightarrow \left(\frac{3}{5}\right)^{-1} \times \left(\frac{5}{2}\right)^{-1} = \frac{5}{3} \times \frac{2}{5} = \frac{10}{15} = \frac{2}{3}$ [Using $a^{-n} = \frac{1}{a^n}$]

4. Question

Simplify:

(i) $\{4^{-1} \times 3^{-1}\}^2$

(ii) $\{5^{-1} \div 6^{-1}\}^3$

(iii) $(2^{-1} + 3^{-1})^{-1}$

(iv) $\{3^{-1} \times 4^{-1}\}^{-1} \times 5^{-1}$

(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$

Answer

(i) $\{4^{-1} \times 3^{-1}\}^2$

$\Rightarrow \left(\frac{1}{4} \times \frac{1}{3}\right)^2 = \left(\frac{1}{12}\right)^2 = \frac{1}{144}$ [Using $a^{-n} = \frac{1}{a^n}$; $a^n = a \times a \ldots \ldots n\ times$]

(ii) $\{5^{-1} \div 6^{-1}\}^3$

$\Rightarrow \left(\frac{1}{5} \div \frac{1}{6}\right)^3$ [Using $a^{-n} = \frac{1}{a^n}$]

$\Rightarrow (\frac{1}{5} \times 6)^3 = \frac{6^3}{5^3} = \frac{216}{125}$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

$\Rightarrow \frac{6^3}{5^3} = \frac{216}{125}$ [Using $a^n = a \times a \ldots \ldots n\ times$]

(iii) $(2^{-1} + 3^{-1})^{-1}$

$\Rightarrow \left(\frac{1}{2} + \frac{1}{3}\right)^{-1}$ [Using $a^{-n} = \frac{1}{a^n}$]

$\Rightarrow \left\{\frac{3+2}{6}\right\}^{-1} = \left(\frac{5}{6}\right)^{-1} = \frac{6}{5}$ [Using $a^{-n} = \frac{1}{a^n}$]

(iv) $\{3^{-1} \times 4^{-1}\}^{-1} \times 5^{-1}$

$\Rightarrow \left(\frac{1}{3} \times \frac{1}{4}\right)^{-1} \times \frac{1}{5}$ [Using $a^{-n} = \frac{1}{a^n}$]

$\Rightarrow \left\{\frac{1}{12}\right\}^{-1} \times \frac{1}{5} = \frac{12}{5}$ [Using $a^{-n} = \frac{1}{a^n}$]

(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$

$\Rightarrow \left(\frac{1}{4} - \frac{1}{5}\right)^{-1} \div \frac{1}{3}$ [Using $a^{-n} = \frac{1}{a^n}$]

$\Rightarrow \left\{\frac{5-4}{20}\right\}^{-1} \times 3$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

$\Rightarrow \frac{1}{20} \times 3 = \frac{3}{20}$

5. Question

Express each of the following rational numbers with a negative exponent:

(i) $\left(\frac{1}{4}\right)^{3}$

(ii) 3^5

(iii) $\left(\frac{3}{5}\right)^{4}$

(iv) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-3}$

(v) $\left\{\left(\frac{7}{3}\right)^{4}\right\}^{-3}$

Answer

(i) $\left(\frac{1}{4}\right)^{3}$

$\Rightarrow \left(\frac{1}{4}\right)^{3} = (4)^{-3}$ [Using $\frac{1}{a^n} = a^{-n}$]

(ii) 3^5

$\Rightarrow 3^5 = \left(\frac{1}{3}\right)^{-5} = (4)^{-3}$ [Using $\frac{1}{a^n} = a^{-n}$]

(iii) $\left(\frac{3}{5}\right)^{4}$

$\Rightarrow \left(\frac{3}{5}\right)^{4} = \left(\frac{5}{3}\right)^{-4}$ [Using $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$]

(iv) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-3}$

$\Rightarrow \left\{\left(\frac{3}{2}\right)^{4}\right\}^{-3} = \left\{\left(\frac{3}{2}\right)^{4}\right\}^{-3} = \left(\frac{3}{2}\right)^{-12}$ [Using $(a^n)^m = a^{mn}$]

(v) $\left\{\left(\frac{7}{3}\right)^{4}\right\}^{-3}$

$\Rightarrow \left\{\left(\frac{7}{3}\right)^{4}\right\}^{-3} = \left\{\left(\frac{7}{3}\right)^{4}\right\}^{-3} = \left(\frac{7}{3}\right)^{-12}$ [Using $(a^n)^m = a^{mn}$]

6. Question

Express each of the following rational numbers with a positive exponent:

(i) $\left(\frac{3}{4}\right)^{-2}$

(ii) $\left(\frac{5}{4}\right)^{-3}$

(iii) $4^3 \times 4^{-9}$

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

(v) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-2}$

Answer

(i) $\left(\frac{3}{4}\right)^{-2}$

$\Rightarrow \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^{2}$ [Using $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$]

(ii) $\left(\frac{5}{4}\right)^{-3}$

$\Rightarrow \left(\frac{5}{4}\right)^{-3} = \left(\frac{4}{5}\right)^{3}$ [Using $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$]

(iii) $4^{3} \times 4^{-9}$

$\Rightarrow 4^{3} \times 4^{-9} = 4^{3-9} = 4^{-6}$ [Using $(a^n \times a^m = a^{m+n}$]

$\Rightarrow 4^{-6} = \left(\frac{1}{4}\right)^{6}$ [Using $\frac{1}{a^n} = a^{-n}$]

(iv) $\left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4}$

$\Rightarrow \left\{\left(\frac{4}{3}\right)^{-3}\right\}^{-4} = \left(\frac{4}{3}\right)^{12}$ [Using $(a^n)^m = a^{mn}$]

(v) $\left\{\left(\frac{3}{2}\right)^{4}\right\}^{-2}$

$\Rightarrow \left\{\left(\frac{3}{2}\right)^{4}\right\}^{-2} = \left(\frac{3}{2}\right)^{-8} = \left(\frac{2}{3}\right)^{8}$ [Using $(a^n)^m = a^{mn}$ and $\frac{1}{a^n} = a^{-n}$]

7. Question

Simplify:

(i) $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

(ii) $(3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3}$

(iii) $\left\{\left(\frac{1}{2}\right)^{-1} \times (-4)^{-1}\right\}^{-1}$

(iv) $\left[\left\{\left(\frac{-1}{4}\right)^{2}\right\}^{-2}\right]^{-1}$

(v) $\left\{\left(\frac{2}{3}\right)^{2}\right\}^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

Answer

(i) $\left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

$\Rightarrow \left\{\left(\frac{1}{3}\right)^{-3} - \left(\frac{1}{2}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-3}$

$\Rightarrow (3^3 - 2^3) \div 4^3$ [Using $\frac{1}{a^n} = a^{-n}$]

$\Rightarrow (27-8) \div 4^3$

$\Rightarrow 19 \div 4^3$

$\Rightarrow 19 \times \frac{1}{64} = \frac{19}{64}$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

(ii) $(3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3}$

$\Rightarrow (3^2 - 2^2) \times \left(\frac{2}{3}\right)^{-3}$

$\Rightarrow (9 - 4) \times \left(\frac{3}{2}\right)^{-3}$ [Using $\frac{1}{a^n} = a^{-n}$]

$\Rightarrow (5) \times \left(\frac{3}{2}\right)^{3}$

$\Rightarrow (5) \times \left(\frac{27}{8}\right)$ [Using $a^n = a \times a \ldots \ldots n\ times$]

$\Rightarrow \frac{135}{8}$

(iii) $\left\{\left(\frac{1}{2}\right)^{-1} \times (-4)^{-1}\right\}^{-1}$

$\Rightarrow \left\{\left(\frac{1}{2}\right)^{-1} \times (-4)^{-1}\right\}^{-1}$

$\Rightarrow \left\{(2) \times \left(\frac{1}{-4}\right)\right\}^{-1}$ [Using $\frac{1}{a^n} = a^{-n}$]

$\Rightarrow \left\{-\frac{1}{2}\right\}^{-1}$

$\Rightarrow -2$

(iv) $\left[\left\{\left(\frac{-1}{4}\right)^{2}\right\}^{-2}\right]^{-1}$

$\Rightarrow \left[\left\{\left(\frac{-1}{4}\right)^{2}\right\}^{-2}\right]^{-1}$

$\Rightarrow \left\{-\frac{1}{4}\right\}^{4}$ [Using $(a^n)^m = a^{mn}$]

$\Rightarrow \frac{1}{256}$ [Using $a^n = a \times a \ldots \ldots n\ times$]

(v) $\left\{\left(\frac{2}{3}\right)^{2}\right\}^{3} \times \left(\frac{1}{3}\right)^{-4} \times 3^{-1} \times 6^{-1}$

$$\left(\frac{2}{3}\right)^{6} \times \left(\frac{1}{3}\right)^{-4} \times \frac{1}{3} \times \frac{1}{2 \times 3}$$

$$= \left(\frac{2}{3}\right)^{6} \times \left(\frac{1}{3}\right)^{-4} \times \frac{1}{3} \times \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{2^{6-1}}{3^{6-4+1+1}} \qquad a^m \times a^n = a^{m+n}$$

$$= \frac{2^5}{3^9} = \frac{32}{81}$$

8. Question

By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

Answer

Let the number 5^{-1} shlould be multiplied by x

According to the question:

$x \times 5^{-1} = (-7)^{-1}$

$x \times \frac{1}{5} = \frac{1}{-7}$ [Using $a^{-n} = \frac{1}{a^n}$]

$$x = \frac{1}{-7} \div \frac{5}{1} = \frac{5}{-7}$$

$x = \frac{1}{-7} \times \frac{5}{1} = \frac{5}{-7} = -\frac{5}{7}$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

Therefore 5^{-1} should be multiplied by $\frac{-5}{7}$

9. Question

By what number should $\left(\frac{1}{2}\right)^{-1}$ be multiplied so that the product may be equal to $\left(\frac{-4}{7}\right)^{-1}$?

Answer

Let the number $\left(\frac{1}{2}\right)^{-1}$ shlould be multiplied by x

According to the question:

$x \times \left(\frac{1}{2}\right)^{-1} = \left(-\frac{4}{7}\right)^{-1}$

$x \times 2 = -\frac{7}{4}$ [Using $a^{-n} = \frac{1}{a^n}$]

$$x = -\frac{7}{4} \div 2$$

$x = -\frac{7}{4} \times \frac{1}{2} = -\frac{7}{8}$ [Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

Therefore $\left(\frac{1}{2}\right)^{-1}$ should be multiplied by $-\frac{7}{8}$

10. Question

By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Answer

Let the number $(-15)^{-1}$ shlould be devided by x

According to the question:

$(-15)^{-1} \div x = (-5)^{-1}$

$\frac{1}{-15} \div x = -\frac{1}{5}$ [Using $a^{-n} = \frac{1}{a^n}$]

$-\frac{1}{15} \times \frac{1}{x} = -\frac{1}{5}$ [Using and $\frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

$$\frac{1}{x} = \frac{(-1)\times(-15)}{5}$$

$$\frac{1}{x} = 3$$

$$x = \frac{1}{3}$$

Therefore $(-15)^{-1}$ should be devided by $\frac{1}{3}$

11. Question

By what number should $\left(\frac{5}{3}\right)^{-2}$ be multiplied so that the product may be $\left(\frac{7}{3}\right)^{-1}$?

Answer

Let the number $\left(\frac{5}{3}\right)^{-2}$ shlould be multiplied by x

According to the question:

$$x\times \left(\frac{5}{3}\right)^{-2} = \left(\frac{7}{3}\right)^{-1}$$

$x\times \left(\frac{3}{5}\right)^{2} = \frac{3}{7}$ [Using $a^{-n} = \frac{1}{a^n}$]

$$x = \frac{3}{7} \div \left(\frac{3}{5}\right)^{2}$$

$x = \frac{3}{7} \times \frac{25}{9} = \frac{25}{21}$ [Using and $\frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

Therefore $\left(\frac{1}{2}\right)^{-1}$ should be multiplied by $\frac{25}{21}$

12. Question

Find x, if

(i) $\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} \times \left(\frac{1}{4}\right)^{-4x}$

(ii) $\left(\frac{-1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

(iii) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

(iv) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

(v) $\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$

(vi) $\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

Answer

(i) $\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} \times \left(\frac{1}{4}\right)^{-4x}$

$\left(\frac{1}{4}\right)^{-4} \times \left(\frac{1}{4}\right)^{-8} = \left(\frac{1}{4}\right)^{-4x}$

$\Rightarrow \left(\frac{1}{4}\right)^{-4-8} = \left(\frac{1}{4}\right)^{-4x}$ [Using $a^{n} \times a^{m} = a^{m+n}$]

Equating coefficients when bases are equal.

$-4 - 8 = -4x$

$-12 = -4x$

$-12 = -4x$

$x = 3$

(ii) $\left(\frac{-1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

$\left(\frac{-1}{2}\right)^{-19} \div \left(\frac{-1}{2}\right)^{8} = \left(\frac{-1}{2}\right)^{-2x+1}$

$\Rightarrow \left(\frac{1}{2}\right)^{-19-8} = \left(\frac{1}{2}\right)^{-2x+1}$ [Using $a^{n} \div a^{m} = a^{m-n}$]

Equating coefficients when bases are equal.

$-19 - 8 = -2x + 1$

$-27 = -2x + 1$

$-27 - 1 = -2x$

$-28 = -2x$

$x = 14$

(iii) $\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

$\left(\frac{3}{2}\right)^{-3} \times \left(\frac{3}{2}\right)^{5} = \left(\frac{3}{2}\right)^{2x+1}$

$\Rightarrow \left(\frac{3}{2}\right)^{-3+5} = \left(\frac{3}{2}\right)^{2x+1}$ [Using $a^{n} \times a^{m} = a^{m+n}$]

Equating coefficients when bases are equal.

$-3 + 5 = 2x + 1$

$2 - 1 = 2x$

$1 = 2x$

$x = \frac{1}{2}$

(iv) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

$\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{15} = \left(\frac{2}{5}\right)^{2+3x}$

$\Rightarrow \left(\frac{2}{5}\right)^{-3+15} = \left(\frac{2}{5}\right)^{2+3x}$ [Using $a^n \times a^m = a^{m+n}$]

Equating coefficients when bases are equal.

$-3 + 15 = 2 + 3x$

$12 - 2 = 3x$

$10 = 3x$

$x = \frac{10}{3}$

(v) $\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$

$\left(\frac{5}{4}\right)^{-x} \div \left(\frac{5}{4}\right)^{-4} = \left(\frac{5}{4}\right)^{5}$

$\Rightarrow \left(\frac{5}{4}\right)^{-x+4} = \left(\frac{5}{4}\right)^{5}$ [Using $a^n \div a^m = a^{m-n}$]

Equating coefficients when bases are equal.

$-x + 4 = 5$

$-x = 5 - 4$

$-x = 1$

$x = -1$

(vi) $\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

$\left(\frac{8}{3}\right)^{2x+1} \times \left(\frac{8}{3}\right)^{5} = \left(\frac{8}{3}\right)^{x+2}$

$\Rightarrow \left(\frac{8}{3}\right)^{2x+1+5} = \left(\frac{8}{3}\right)^{x+2}$ [Using $a^n \times a^m = a^{m+n}$]

Equating coefficients when bases are equal.

$2x + 1 + 5 = x + 2$

$2x + 6 = x + 2$

$2x - x = 2 - 6$

$x = -4$

13 A. Question

If x=$\left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$, find the value of x^{-2}.

Answer

x =$\left(\frac{3}{2}\right)^{2} \times \left(\frac{2}{3}\right)^{-4}$

$x = \left(\frac{3}{2}\right)^{2} \times \left(\frac{3}{2}\right)^{4}$ [Using $a^m \times a^n = a^{m+n}$]

$x = \left(\frac{3}{2}\right)^{6}$

$x^{-2} = \left(\frac{2}{3}\right)^{12}$

13 B. Question

If x = $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$, find the value of x^{-1}.

Answer

X = $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{1}{4}\right)^{2}$

On using, [Using $a^{-n} = \frac{1}{a^n}$], we get,

$x = \left(\frac{5}{4}\right)^{2} \div \left(\frac{1}{4}\right)^{2}$

[Using $and\ \frac{1}{a} \div \frac{1}{b} = \frac{1}{a} \times \frac{b}{1}$]

$x = \left(\frac{5}{4}\right)^{2} \times \left(\frac{4}{1}\right)^{2}$

$x = \frac{5}{4} \times \frac{5}{4} \times 4 \times 4$

x = 25

$x^{-1} = \frac{1}{25}$

14. Question

Find the value of x for which $5^{2x} \div 5^{-3} = 5^5$.

Answer

$5^{2x} \div 5^{-3} = 5^5$

$\Rightarrow (5)^{2\,x-3} = 5^5$ [Using $a^n \div a^m = a^{m-n}$]

Equating coefficients when bases are equal.

$2x + 3 = 5$

$2x = 5 - 3$

$2x = 2$

$x = 1$

Exercise 2.3

1. Question

Express the following numbers in standard form:

(i) 6020000000000000

(ii) 0.00000000000942

(iii) 0.00000000085

(iv) 846×10^7

(v) 3759×10^{-4}

(vi) 0.00072984

(vii) 0.000437 $\times 10^4$

(viii) $4 \div 100000$

Answer

(i) 6020000000000000

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

In this question total number of digits leaving one digit from left are 15.

Therefore the standard form is: 6.02×10^{15}

(ii) 0.00000000000942

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

If the number has all the digits to the right of the decimal then powers will be negative. In this question total number of digits after decimal are 12.

Therefore the standard form is: 9.42×10^{-12}

(iii) 0.00000000085

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

If the number has all the digits to the right of the decimal then powers will be negative. In this question total number of digits after decimal are 12.

Therefore the standard form is: 8.5×10^{-10}

(iv) 846×10^7

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

In this question total number of digits are 2.

Therefore the standard form is: 8.46×10^{9}

(v) 3759×10^{-4}

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

In this question total number of digits are 3.

Therefore the standard form is: 8.46×10^{-1}

(vi) 0.00072984

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

If the number has all the digits to the right of the decimal then powers will be negative. In this question total number of digits after decimal are 4.

Therefore the standard form is: 7.2984×10^{-4}

(vii) 0.000437 $\times 10^4$

To write in the standard form, count the number of digits leaving one digit from the left. The total number of digits so obtained becomes power of 10. Decimal comes after first left digit.

If the number has all the digits to the right of the decimal then powers will be negative. In this question total number of digits after decimal are 4.

Therefore the standard form is: 4.37

(viii) $4 \div 100000$

To write in the standard form, Count the number of zeros of the divisor. This number of zeros becomes negative power of 10.

Therefore the standard form is: 4×10^{-5}

2. Question

Write the following numbers in the usual form:

(i) 4.83×10^{7}

(ii) 3.02×10^{-6}

(iii) 4.5×10^{4}

(iv) 3×10^{-8}

(v) 1.0001×10^{9}

(vi) 5.8×10^{2}

(vii) 3.61492×10^{6}

(viii) 3.25×10^{-7}

Answer

(i) 4.83×10^{7}

In cae of positive power of 10. The usual form of the number is written after multiplying the given numbers and then counts the numbers from the right and put decimal.

Step 1: Multipy the given numbers: 4.83× 10000000 = 4830000000

Step 2: Put decimal after two places from the right: 48300000.00

Step 3: Write the number after in the usual form: 48300000

(ii) 3.02×10^{-6}

In case of negative powers, decimal sifts to left equal to the power of 10.

Step 1: Here the power of 10 is negative 6.

Step 2: Therefore decimal will sift six places to the left. i.e: 0.00000302

(iii) 4.5×10^{4}

In cae of positive power of 10. The usual form of the number is written after multiplying the given numbers and then counts the numbers from the right and put decimal.

Step 1: Multipy the given numbers: 4.5× 10000 = 450000

Step 2: Put decimal after one place from the right: 45000.0

Step 3: Write the number after in the usual form: 45000

(iv) 3×10^{-8}

In case of negative powers, decimal sifts to left equal to the power of 10.

Step 1: Here the power of 10 is negative 8.

Step 2: Therefore decimal will sift eight places to the left, and we write zeros before the number to make eight places . i.e: 0.00000003

(v) 1.0001×10^{9}

In cae of positive power of 10. The usual form of the number is written after multiplying the given numbers and then counts the numbers from the right and put decimal.

Step 1: Multipy the given numbers: 1.0001× 1000000000 = 10001000000000

Step 2: Put decimal after four places from the right: 1000100000.0000

Step 3: Write the number after in the usual form: 1000100000

(vi) 5.8×10^{2}

In cae of positive power of 10. The usual form of the number is written after multiplying the given numbers and then counts the numbers from the right and put decimal.

Step 1: Multipy the given numbers: 5.8× 100 = 5800

Step 2: Put decimal after one place from the right: 580.0

Step 3: Write the number after in the usual form: 580

(vii) 3.61492×10^{6}

In cae of positive power of 10. The usual form of the number is written after multiplying the given numbers and then counts the numbers from the right and put decimal.

Step 1: Multipy the given numbers: 3.61492 × 1000000 = 5800

Step 2: Put decimal after five places from the right: 3614920.00000

Step 3: Write the number after in the usual form: 3614920

(viii) 3.25×10^{-7}

In case of negative powers, decimal sifts to left equal to the power of 10.

Step 1: Here the power of 10 is negative 8.

Step 2: Therefore decimal will sift seven places to the left, and we write zeros before the number to make seven places . i.e: 0.000000325