2. Functions

Exercise 2.1

1 A. Question

Give an example of a function

Which is one – one but not onto.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, Let, $f: N \rightarrow N$ given by f(x) = x^2

Check for Injectivity:

Let x,y be elements belongs to N i.e $x, y \in N$ such that

So, from definition

⇒ f(x) = f(y)

⇒ $x^2 = y^2$

⇒ $x^2 - y^2 = 0$

⇒ (x – y)(x + y) = 0

As $x, y \in N$ therefore x + y>0

⇒ x – y = 0

⇒ x = y

Hence f is One – One function

Check for Surjectivity:

Let y be element belongs to N i.e $y \in N$ be arbitrary, then

⇒ f(x) = y

⇒ x^2 = y

⇒ $x = \sqrt{y}$

⇒ $\sqrt{y}$ not belongs to N for non–perfect square value of y.

Therefore no non – perfect square value of y has a pre image in domain N.

Hence, $f: N \rightarrow N$ given by f(x) = x^2 is One – One but not onto.

1 B. Question

Give an example of a function

Which is not one – one but onto.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, Let, $f: R \rightarrow R$ given by $f(x) = x^3 - x$

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

So, from definition

⇒ f(x) = f(y)

⇒ $x^3 - x = y^3 - y$

⇒ $x^3 - y^3 - (x - y) = 0$

⇒ $(x - y)(x^2 + xy + y^2 - 1) = 0$

As $x^2 + xy + y^2 \geq 0$

⇒ therefore $x^2 + xy + y^2 - 1 \geq - 1$

⇒ x – y≠0

⇒ x ≠ y for some $x, y \in R$

Hence f is not One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ $x^3 - x = y$

⇒ $x^3 - x - y = 0$

Now, we know that for 3 degree equation has a real root

So, let $x = \alpha$ be that root

⇒ $\alpha^3 - \alpha = y$

⇒ $f(\alpha) = y$

Thus for clearly $y \in R$, there exist $\alpha \in R$ such that f(x) = y

Therefore f is onto

⇒ Hence, $f: R \to R$ given by f(x) = x^3 – x is not One – One but onto

1 C. Question

Give an example of a function

Which is neither one – one nor onto.

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, Let, $f: R \to R$ given by f(x) = 5

As we know

A constant function is neither one – one nor onto.

So, here f(x) = 5 is constant function

Therefore

$f: R \to R$ given by f(x) = 5 is neither one – one nor onto function.

2 A. Question

Which of the following functions from A to B are one – one and onto?

f_1 = {(1, 3), (2, 5), (3, 7)}; A = {1, 2, 3}, B = {3, 5, 7}

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, As given,

$f_1 = \{(1, 3), (2, 5), (3, 7)\}$

$A = \{1, 2, 3\}$, $B = \{3, 5, 7\}$

Thus we can see that,

Check for Injectivity:

Every element of A has a different image from B

Hence f is a One – One function

Check for Surjectivity:

Also, each element of B is an image of some element of A

Hence f is Onto.

2 B. Question

Which of the following functions from A to B are one – one and onto?

$f_2 = \{(2, a), (3, b), (4, c)\}$; $A = \{2, 3, 4\}$, $B = \{a, b, c\}$

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

$\Leftrightarrow a \neq b$

$\Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow f(a) = f(b)$

$\Rightarrow a = b$ for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, As given,

$f_2 = \{(2, a), (3, b), (4, c)\}$

$A = \{2, 3, 4\}$, $B = \{a, b, c\}$

Thus we can see that

Check for Injectivity:

Every element of A has a different image from B

Hence f is a One – One function

Check for Surjectivity:

Also, each element of B is an image of some element of A

Hence f is Onto.

2 C. Question

Which of the following functions from A to B are one – one and onto?

$f_3 = \{(a, x), (b, x), (c, z), (d, z)\}$; $A = \{a, b, c, d\}$, $B = \{x, y, z\}$

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, As given,

f_3 = {(a, x), (b, x), (c, z), (d, z)}

A = {a, b, c, d}, B = {x, y, z}

Thus we can clearly see that

Check for Injectivity:

Every element of A does not have different image from B

Since,

$f_3(a) = x = f_3(b)$ and $f_3(c) = z = f_3(d)$

Therefore f is not One – One function

Check for Surjectivity:

Also each element of B is not image of any element of A

Hence f is not Onto.

3. Question

Prove that the function f : N → N, defined by f(x) = x^2 + x + 1 is one – one but not onto.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, $f: N \rightarrow N$ given by f(x) = x^2 + x + 1

Check for Injectivity:

Let x,y be elements belongs to N i.e $x, y \in N$ such that

So, from definition

⇒ f(x) = f(y)

⇒ $x^2 + x + 1 = y^2 + y + 1$

⇒ $x^2 - y^2 + x - y = 0$

⇒ (x – y)(x + y + 1) = 0

As $x, y \in N$ therefore x + y + 1>0

⇒ x – y = 0

⇒ x = y

Hence f is One – One function

Check for Surjectivity:

y be element belongs to N i.e $y \in N$ be arbitrary

Since for y > 1, we do not have any pre image in domain N.

Hence, f is not Onto function.

4. Question

Let A = {–1, 0, 1} and f = {(x, x^2) : x ∈ A}. Show that f : A → A is neither one – one nor onto.

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, We have, A = {–1, 0, 1} and f = {(x, x^2) : x ∈ A}.

To Prove: – f : A → A is neither One – One nor onto function

Check for Injectivity:

We can clearly see that

f(1) = 1

and f(– 1) = 1

Therefore

f(1) = f(– 1)

⇒ Every element of A does not have different image from A

Hence f is not One – One function

Check for Surjectivity:

Since, y = – 1 be element belongs to A

i.e $-1 \in A$ in co – domain does not have any pre image in domain A.

Hence, f is not Onto function.

5 A. Question

Classify the following functions as injection, surjection or bijection:

f : N → N given by f(x) = x^2

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, $f: N \rightarrow N$ given by f(x) = x^2

Check for Injectivity:

Let x,y be elements belongs to N i.e $x, y \in N$ such that

So, from definition

⇒ f(x) = f(y)

⇒ $x^2 = y^2$

⇒ $x^2 - y^2 = 0$

⇒ (x – y)(x + y) = 0

As $x, y \in N$ therefore x + y>0

⇒ x – y = 0

⇒ x = y

Hence f is One – One function

Check for Surjectivity:

Let y be element belongs to N i.e $y \in N$ be arbitrary, then

⇒ f(x) = y

⇒ $x^2 = y$

$\Rightarrow x = \sqrt{y}$

$\Rightarrow \sqrt{y}$ not belongs to N for non–perfect square value of y.

Therefore no non – perfect square value of y has a pre–image in domain N.

Hence, f is not Onto function.

Thus, Not Bijective also.

5 B. Question

Classify the following functions as injection, surjection or bijection:

f : Z → Z given by $f(x) = x^2$

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : Z → Z given by $f(x) = x^2$

Check for Injectivity:

Let x_1, – x_1 be elements belongs to Z i.e $x_1, -x_1 \in Z$ such that

So, from definition

$\Rightarrow x_1 \neq - x_1$

$\Rightarrow (x_1)^2 = (- x_1)^2$

$\Rightarrow f(x_1)^2 = f(- x_1)^2$

Hence f is not One – One function

Check for Surjectivity:

Let y be element belongs to Z i.e $y \in Z$ be arbitrary, then

⇒ f(x) = y

$\Rightarrow x^2 = y$

$\Rightarrow x = \pm\sqrt{y}$

$\Rightarrow \sqrt{y}$ not belongs to Z for non–perfect square value of y.

Therefore no non – perfect square value of y has a pre–image in domain Z.

Hence, f is not Onto function.

Thus, Not Bijective also

5 C. Question

Classify the following functions as injection, surjection or bijection:

f : N → N given by f(x) = x^3

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : N → N given by f(x) = x^3

Check for Injectivity:

Let x,y be elements belongs to N i.e $x, y \in N$ such that

⇒ f(x) = f(y)

⇒ $x^3 = y^3$

⇒ $x^3 - y^3 = 0$

⇒ $(x - y)(x^2 + y^2 + xy) = 0$

As $x, y \in N$ therefore $x^2 + y^2 + xy > 0$

⇒ x – y = 0

⇒ x = y

Hence f is One – One function

Check for Surjectivity:

Let y be element belongs to N i.e $y \in N$ be arbitrary, then

⇒ f(x) = y

⇒ $x^3 = y$

⇒ $x = \sqrt[3]{y}$

⇒ $\sqrt[3]{y}$ not belongs to N for non–perfect cube value of y.

Since f attain only cubic number like 1,8,27…., Therefore no non – perfect cubic values of y in N (co – domain) has a pre–image in domain N.

Hence, f is not onto function

Thus, Not Bijective also

5 D. Question

Classify the following functions as injection, surjection or bijection:

f : Z → Z given by f(x) = x^3

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : Z → Z given by f(x) = x^3

<u>Check for Injectivity</u>:

Let x,y be elements belongs to Z i.e $x, y \in Z$ such that

⇒ f(x) = f(y)

⇒ $x^3 = y^3$

⇒ $x^3 - y^3 = 0$

⇒ x = y

Hence f is One – One function

<u>Check for Surjectivity</u>:

Let y be element belongs to Z i.e $y \in Z$ be arbitrary, then

⇒ f(x) = y

⇒ $x^3 = y$

⇒ $x = \sqrt[3]{y}$

⇒ $\sqrt[3]{y}$ not belongs to Z for non – perfect cube value of y.

Since f attain only cubic number like 1,8,27….

Therefore no non – perfect cubic values of y in Z (co – domain) have a pre–image in domain Z.

Hence, f is not onto function

Thus, Not Bijective also

5 E. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = |x|

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R, defined by f(x) = |x|

<u>Check for Injectivity</u>:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

Case i

⇒ x = y

⇒ |x| = |y|

Case ii

⇒ – x = y

⇒ | – x| = |y|

⇒ x = |y|

Hence from case i and case ii f is not One – One function

<u>Check for Surjectivity</u>:

Since f attain only positive values, for negative real numbers in R

(co – domain) there is no pre–image in domain R.

Hence, f is not onto function

Thus, Not Bijective also

5 F. Question

Classify the following functions as injection, surjection or bijection:

f : Z → Z, defined by f(x) = x^2 + x

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : Z → Z given by $f(x) = x^2 + x$

Check for Injectivity:

Let x,y be elements belongs to Z i.e $x, y \in Z$ such that

⇒ f(x) = f(y)

⇒ $x^2 + x = y^2 + y$

⇒ $x^2 – y^2 + x – y = 0$

⇒ (x – y)(x + y + 1) = 0

Either (x – y) = 0 or (x + y + 1) = 0

Case i :

If x – y = 0

⇒ x = y

Hence f is One – One function

Case ii :

If x + y + 1 = 0

⇒ x + y = – 1

⇒ x ≠ y

Hence f is not One – One function

Thus from case i and case ii f is not One – One function

Check for Surjectivity:

As $1 \in Z$

Let x be element belongs to Z i.e $y \in Z$ be arbitrary, then

⇒ f(x) = 1

⇒ $x^2 + x = 1$

⇒ $x^2 + x – 1 = 0$

⇒ $x = \frac{-1 \pm \sqrt{1+4}}{2}$

Above value of x does not belong to Z

Therefore no values of x in Z (co – domain) have a pre–image in domain Z.

Hence, f is not onto function

Thus, Not Bijective also

5 G. Question

Classify the following functions as injection, surjection or bijection:

f : Z → Z, defined by f(x) = x – 5

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : Z → Z given by f(x) = x – 5

<u>Check for Injectivity</u>:

Let x,y be elements belongs to Z i.e $x, y \in Z$ such that

⇒ f(x) = f(y)

⇒ x – 5 = y – 5

⇒ x = y

Hence, f is One – One function

<u>Check for Surjectivity</u>:

Let y be element belongs to Z i.e $y \in Z$ be arbitrary, then

⇒ f(x) = y

⇒ x – 5 = y

⇒ x = y + 5

Above value of x belongs to Z

Therefore for each element in Z (co – domain) there exists an element in domain Z.

Hence, f is onto function

Thus, Bijective function

5 H. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = sin x

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R, defined by f(x) = sin x

<u>Check for Injectivity</u>:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

⇒ f(x) = f(y)

⇒ sin x = sin y

⇒ $x = n\pi + (-1)^n y$

⇒ x ≠ y

Hence, f is not One – One function

<u>Check for Surjectivity</u>:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ sin x = y

⇒ $x = \sin^{-1} y$

Now, for y>1 x not belongs to R (Domain)

Hence, f is not onto function

Thus, It is also not Bijective function

5 I. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = x^3 + 1

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, Let, $f: R \rightarrow R$ given by f(x) = x^3 + 1

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

So, from definition

⇒ f(x) = f(y)

⇒ $x^3 + 1 = y^3 + 1$

⇒ $x^3 = y^3$

⇒ x = y

Hence f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ $x^3 + 1 = y$

Now, we know that for 3 degree equation has a real root

So, let $x = \alpha$ be that root

$\Rightarrow \alpha^3 + 1 = y$

$\Rightarrow f(\alpha) = y$

Thus for clearly $y \in R$, there exist $\alpha \in R$ such that f(x) = y

Therefore f is onto

Thus, It is also Bijective function

5 J. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = $x^3 - x$

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, Let, $f: R \rightarrow R$ given by f(x) = x^3 + x

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

So, from definition

⇒ f(x) = f(y)

⇒ x^3 – x = y^3 – y

⇒ x^3 – y^3 – (x – y) = 0

⇒ (x – y)(x^2 + xy + y^2 – 1) = 0

Hence f is not One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ x^3 – x = y

⇒ x^3 – x – y = 0

Now, we know that for 3 degree equation has a real root

So, let $x = \alpha$ be that root

⇒ $\alpha^3 - \alpha = y$

$f(\alpha) = y$

Thus for clearly $y \in R$, there exist $\alpha \in R$ such that f(x) = y

Therefore f is onto

Thus, It is not Bijective function

5 K. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = $\sin^2$x + $\cos^2$x

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R, defined by $f(x) = \sin^2 x + \cos^2 x$

Check for Injectivity and Check for Surjectivity

Let x be element belongs to R i.e $x \in R$ such that

So, from definition

$\Rightarrow f(x) = \sin^2 x + \cos^2 x$

$\Rightarrow f(x) = \sin^2 x + \cos^2 x$

⇒ f(x) = 1

⇒ f(x) = constant

We know that a constant function is neither One – One function nor onto function.

Thus, It is not Bijective function

5 L. Question

Classify the following functions as injection, surjection or bijection:

f: Q – {3} → Q, defined by $f(x) = \dfrac{2x+3}{x-3}$

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R given by $f(x) = \frac{2x+3}{x-3}$

Check for Injectivity:

Let x,y be elements belongs to Q i.e $x, y \in Q$ such that

⇒ f(x) = f(y)

$\Rightarrow \frac{2x+3}{x-3} = \frac{2y+3}{y-3}$

⇒ (2x + 3)(y – 3) = (2y + 3)(x – 3)

⇒ 2xy – 6x + 3y – 9 = 2xy – 6y + 3x – 9

⇒ – 6x + 3y = – 6y + 3x

⇒ – 6x + 3y + 6y – 3x = 0

⇒ – 9x + 9y = 0

⇒ x = y

Thus, f is One – One function

Check for Surjectivity:

Let y be element belongs to Q i.e $y \in Q$ be arbitrary, then

⇒ f(x) = y

$\Rightarrow \frac{2x+3}{x-3} = y$

⇒ 2x + 3 = y (x – 3)

⇒ 2x + 3 = xy – 3y

⇒ 2x – xy = – 3(y + 1)

$\Rightarrow x = \frac{-3(y+1)}{2-y}$

Above value of x belongs to Q – [3] for y = 2

Therefore for each element in Q – [3] (co – domain), there does not exist an element in domain Q.

Hence, f is not onto function

Thus, Not Bijective function

5 M. Question

Classify the following functions as injection, surjection or bijection:

f : Q → Q, defined by f(x) = x^3 + 1

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : Q → Q, defined by $f(x) = x^3 + 1$

Check for Injectivity:

Let x,y be elements belongs to Q i.e $x, y \in Q$ such that

$\Rightarrow f(x) = f(y)$

$\Rightarrow x^3 + 1 = y^3 + 1$

$\Rightarrow x^3 = y^3$

$\Rightarrow x = y$

Hence, f is One – One function

Check for Surjectivity:

Let y be element belongs to Q i.e $y \in Q$ be arbitrary, then

$\Rightarrow x^3 + 1 = y$

$\Rightarrow x^3 + 1 - y = 0$

Now, we know that for 3 degree equation has a real root

So, let $x = \alpha$ be that root

$\Rightarrow \alpha^3 + 1 = y$

$\Rightarrow f(\alpha) = y$

Thus for clearly $y \in Q$, there exist $\alpha \in Q$ such that f(x) = y

Therefore f is onto

Thus, It is a Bijective function

5 N. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by $f(x) = 5x^3 + 4$

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

$\Leftrightarrow a \neq b$

$\Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow f(a) = f(b)$

$\Rightarrow a = b$ for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R, defined by $f(x) = 5x^3 + 4$

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

⇒ f(x) = f(y)

⇒ $5x^3 + 4 = 5y^3 + 4$

⇒ $x^3 = y^3$

⇒ x = y

Hence, f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ $5x^3 + 4 = y$

⇒ $5x^3 + 4 - y = 0$

Now, we know that for 3 degree equation has a real root

So, let $x = \alpha$ be that root

⇒ $5\alpha^3 + 4 = y$

$f(\alpha) = y$

Thus for clearly $y \in R$, there exist $\alpha \in R$ such that f(x) = y

Therefore f is onto

Thus, It is a Bijective function

5 O. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = 3 – 4x

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R given by f(x) = 3 – 4x

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

⇒ f(x) = f(y)

⇒ 3 – 4x = 3 – 4y

⇒ x = y

Hence, f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ 3 – 4x = y

⇒ $x = \frac{3-y}{4}$

Above value of x belongs to R

Therefore for each element in R (co – domain), there exists an element in domain R.

Hence, f is onto function

Thus, Bijective function

5 P. Question

Classify the following functions as injection, surjection or bijection:

f : R → R, defined by f(x) = 1 + x^2

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, $f: R \rightarrow R$ given by f(x) = 1 + x^2

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

So, from definition

⇒ f(x) = f(y)

⇒ $x^2 + 1 = y^2 + 1$

⇒ $x^2 = y^2$

⇒ ±x = ±y

Therefore, either x = y or x = – y or x ≠ y

Hence f is not One – One function

Check for Surjectivity:

1 be element belongs to R i.e $1 \in R$ be arbitrary, then

⇒ f(x) = 1

⇒ $x^2 + x = 1$

⇒ $x^2 + x - 1 = 0$

⇒ $x = \pm\sqrt{y-1}$

Above value of x not belongs to R for y < 1

Therefore f is not onto

Thus, It is also not Bijective function

5 Q. Question

Classify the following functions as injection, surjection or bijection:

f: R → R, defined by $f(x) = \frac{x}{x^2+1}$

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f: R → R given by $f(x) = \frac{x}{x^2+1}$

Check for Injectivity:

Let x,y be elements belongs to R i.e. $x, y \in R$ such that

⇒ f(x) = f(y)

⇒ $\frac{x}{x^2+1} = \frac{y}{y^2+1}$

⇒ $xy^2 + x = yx^2 + y$

⇒ $xy^2 + x - yx^2 - y = 0$

⇒ xy (y – x) + (x – y) = 0

⇒ (x – y)(1 – xy) = 0

Case i :

⇒ x – y = 0

⇒ x = y

f is One – One function

Case ii :

⇒ 1 – xy = 0

⇒ xy = 1

Thus from case i and case ii f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ $\frac{x}{x^2+1} = y$

⇒ x = x^2y + y

⇒ x – x^2y = y

Above value of x belongs to R

Therefore for each element in R (co – domain) there exists an element in domain R.

Hence, f is onto function

Thus, Bijective function

6. Question

If f: A → B is an injection such that range of f = {a}. Determine the number of elements in A.

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Here, Range {f} = {a}

Since it is injective map, different elements have different images.

Thus A has only one element

7. Question

Show that the function f : R – {3} → R – {1} given by $f(x) = \frac{x-2}{x-3}$ is a bijection.

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f: R → R given by $f(x) = \frac{x-2}{x-3}$

To Prove: – $f(x) = \frac{x-2}{x-3}$ is a bijection

Check for Injectivity:

Let x,y be elements belongs to R i.e. $x, y \in R$ such that

⇒ f(x) = f(y)

⇒ $\frac{x-2}{x-3} = \frac{y-2}{y-3}$

⇒ (x – 2)(y – 3) = (x – 3)(y – 2)

⇒ xy – 3x – 2y + 6 = xy – 2x – 3y + 6

⇒ – 3x – 2y + 2x + 3y = 0

⇒ – x + y = 0

⇒ x = y

Hence, f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ f(x) = y

⇒ $\frac{x-2}{x-3} = y$

⇒ x – 2 = xy – 3y

⇒ x – xy = 2 – 3y

⇒ $x = \frac{2-3y}{1-y}$

$x = \frac{2-3y}{1-y}$ is a real number for all y ≠ 1.

Also, $\frac{2-3y}{1-y} \neq 2$ for any y

Therefore for each element in R (co – domain), there exists an element in domain R.

Hence, f is onto function

Thus, Bijective function

8 A. Question

Let A = [–1, 1], Then, discuss whether the following functions from A to itself are one – one, onto or bijective:

$$f(x) = \frac{x}{2}$$

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, here f: A → A: A = [–1, 1] given by function is $f(x) = \frac{x}{2}$

<u>Check for Injectivity</u>:

Let x, y be elements belongs to A i.e. $x, y \in A$ such that

⇒ f(x) = f(y)

⇒ $\frac{x}{2} = \frac{y}{2}$

⇒ 2x = 2y

⇒ x = y

1 belongs to A then

$f(1) = \frac{1}{2}$

Not element of A co – domain

Hence, f is not One – One function

<u>Check for Surjectivity</u>:

Let y be element belongs to A i.e $y \in A$ be arbitrary, then

⇒ f(x) = y

⇒ $\frac{x}{2} = y$

⇒ x = 2y

Now,

1 belongs to A

⇒ x = 2, which not belong to A co – domain

Hence, f is not onto function

Thus, It is not Bijective function

8 B. Question

Let A = [–1, 1], Then, discuss whether the following functions from A to itself are one – one, onto or bijective:

g(x) = |x|

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

Now, here f : A → A : A = [–1, 1] given by function is g(x) = |x|

Check for Injectivity:

Let x, y be elements belongs to A i.e $x, y \in A$ such that

⇒ g(x) = g(y)

⇒ |x| = |y|

⇒ x = y

1 belongs to A then

⇒ g(1) = 1 = g(– 1)

Since, it has many element of A co – domain

Hence, g is not One – One function

Check for Surjectivity:

Let y be element belongs to A i.e $y \in A$ be arbitrary, then

⇒ f(x) = y

⇒ $\frac{x}{2} = y$

⇒ x = 2y

Now,

1 belongs to A

⇒ x = 2, which not belong to A co – domain

Since g attain only positive values, for negative – 1 in A (co – domain) there is no pre–image in domain A.

Hence, g is not onto function

Thus, It is not Bijective function

8 C. Question

Let A = [–1, 1], Then, discuss whether the following functions from A to itself are one – one, onto or bijective:

$h(x) = x^2$

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, here f : A → A : A = [–1, 1] given by function is $h(x) = x^2$

<u>Check for Injectivity</u>:

Let x, y be elements belongs to A i.e. $x, y \in A$ such that

⇒ h(x) = h(y)

⇒ $x^2 = y^2$

⇒ ±x = ±y

Since it has many elements of A co – domain

Hence, h is not One – One function

<u>Check for Surjectivity</u>:

Let y be element belongs to A i.e. $y \in A$ be arbitrary, then

⇒ h(x) = y

⇒ $x^2 = y$

⇒ $x = \pm\sqrt{y}$

Since h have no pre–image in domain A.

Hence, h is not onto function

Thus, It is not Bijective function

9 A. Question

Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective:

{(x, y): x is a person, y is the mother of x}

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Here, It is given (x, y): x is a person, y is the mother of x

As we know each person "x" has only one biological mother

Thus,

Given relation is a function

Since more than one person may have the same mother

Function, not One – One (injective) but Onto (Surjective)

9 B. Question

Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective:

{(a, b) : a is a person, b is an ancestor of a}

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Here, It is given (a, b): a is a person, b is an ancestor of a

As we know any person "a" has more than one ancestor

Thus,

Given relation is not a function

10. Question

Let A = {1, 2, 3}. Write all one – one from A to itself.

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

We have A = {1, 2, 3}

So all one – one functions from A = {1, 2, 3} to itself are obtained by re – arranging elements of A.

Thus all possible one – one functions are:

f(1) = 1, f(2) = 2, f(3) = 3

f(1) = 2, f(2) = 3, f(3) = 1

f(1) = 3, f(2) = 1, f(3) = 2

f(1) = 1, f(2) = 3, f(3) = 2

f(1) = 3, f(2) = 2, f(3) = 1

f(1) = 2, f(2) = 1, f(3) = 3

11. Question

If f : R → R be the function defined by f(x) = $4x^3 + 7$, show that f is a bijection.

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

<u>Onto Function</u>: – A function $f: A \to B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \to B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

<u>Bijection Function</u>: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, f : R → R, defined by f(x) = $4x^3 + 7$

To Prove : – f : R → R is bijective defined by f(x) = $4x^3 + 7$

<u>Check for Injectivity</u>:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

⇒ f(x) = f(y)

⇒ $4x^3 + 7 = 4y^3 + 7$

⇒ $x^3 = y^3$

⇒ $x = y$

Hence, f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e $y \in R$ be arbitrary, then

⇒ $f(x) = y$

⇒ $4x^3 + 7 = y$

⇒ $4x^3 + 7 - y = 0$

Now, we know that for 3 degree equation has a real root

So, let $x = \alpha$ be that root

⇒ $4\alpha^3 + 7 = y$

⇒ $f(\alpha) = y$

Thus for clearly $y \in R$, there exist $\alpha \in R$ such that $f(x) = y$

Therefore f is onto

Thus, It is Bijective function

Hence Proved

12. Question

Show that the exponential function f: R → R, given by $f(x) = e^x$, is one – one but not onto. What happens if the co – domain is replaced by R_0^+ (set of all positive real numbers).

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ $a \neq b$

⇒ $f(a) \neq f(b)$ for all $a, b \in A$

⇔ $f(a) = f(b)$

⇒ $a = b$ for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that $f(a) = b$

Now, $f: R \rightarrow R$ given by $f(x) = e^x$

Check for Injectivity:

Let x,y be elements belongs to R i.e $x, y \in R$ such that

So, from definition

⇒ $f(x) = f(y)$

⇒ $e^x = e^y$

$\Rightarrow \frac{e^x}{e^y} = 1$

$\Rightarrow e^{x - y} = 1$

$\Rightarrow e^{x - y} = e^0$

$\Rightarrow x - y = 0$

$\Rightarrow x = y$

Hence f is One – One function

Check for Surjectivity:

Here range of f = $(0,\infty) \neq R$

Therefore f is not onto

Now if co – domain is replaced by $R_0{}^+$ (set of all positive real numbers) i.e $(0,\infty)$ then f becomes an onto function.

13. Question

Show that the logarithmic function f : $R_+{}^0 \rightarrow R$ given by f(x) = $\log_a x$, a > 0 is a bijection.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

$\Leftrightarrow a \neq b$

$\Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow f(a) = f(b)$

$\Rightarrow a = b$ for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \rightarrow B$ is said to be a bijection function if it is one – one as well as onto function.

To Prove : – Logarithmic function f : $R_{++} \rightarrow R$ given by f(x) = $\log_a x$, a > 0 is a bijection.

Now, f : $R_0{}^+ \rightarrow R$ given by f(x) = $\log_a x$, a > 0

Check for Injectivity:

Let x,y be elements belongs to $R_0{}^+$ i.e $x, y \in R_0^+$ such that

So, from definition

$\Rightarrow f(x) = f(y)$

$\Rightarrow \log_a x = \log_a y$

$\Rightarrow \log_a x - \log_a y = 0$

$\Rightarrow \log_a\left(\frac{x}{y}\right) = 0$

$\Rightarrow \frac{x}{y} = 1$

$\Rightarrow x = y$

Hence f is One – One function

Check for Surjectivity:

Let y be element belongs to R i.e. $y \in R$ be arbitrary, then

$\Rightarrow f(x) = y$

$\Rightarrow \log_a x = y$

$\Rightarrow x = a^y$

Above value of x belongs to $R_0{}^+$

Therefore, for all $y \in R$ there exist $x = a^y$ such that $f(x) = y$.

Hence, f is Onto function.

Thus, it is Bijective also

14. Question

If A = {1, 2, 3}, show that a one – one function f : A→ A must be onto.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

$\Leftrightarrow a \neq b$

$\Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow f(a) = f(b)$

$\Rightarrow a = b$ for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, f: A → A where A = {1, 2, 3} and its a One – One function

To Prove: – A is Onto function

Since it is given that f is a One – One function,

Three elements of A = {1, 2, 3} must be taken to 3 different elements of co – domain A = {1, 2, 3} under f.

Thus by definition of Onto Function

f has to be Onto function.

Hence Proved

15. Question

If A = {1, 2, 3}, show that an onto function f : A→ A must be one – one.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, f : A → A where A = {1, 2, 3} and its an Onto function

To Prove: – A is a One – One function

Let's assume f is not Onto function,

Then,

There must be two elements let it be 1 and 2 in Domain A = {1, 2, 3} whose images in co–domain A = {1, 2, 3} is same.

Also, Image of 3 under f can be only one element.

Therefore,

Range set can have at most two elements in co – domain A = {1, 2, 3}

⇒ f is not an onto function

Hence it contradicts

⇒ f must be One – One function

Hence Proved

16. Question

Find the number of all onto functions from the set A = {1, 2, 3, …., n} to itself.

Answer

TIP: –

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, f : A → A where A = {1, 2, 3,….,n}

All onto function

It’s a permutation of n symbols 1,2,3,….n

Thus,

Total number of Onto maps from A = {1, 2, 3, …., n} to itself =

Total number of permutations of n symbols 1,2,3,….n.

17. Question

Give examples of two one – one functions f_1 and f_2 from R to R such that $f_1 + f_2$: R → R, defined by $(f_1 + f_2)(x) = f_1(x) + f_2(x)$ is not one – one.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ $a \neq b$

⇒ $f(a) \neq f(b)$ for all $a, b \in A$

⇔ $f(a) = f(b)$

$a = b$ for all $a, b \in A$

Let, f_1: R → R and f_2: R → R be two functions given by (Examples)

$f_1(x) = x$

$f_1(x) = – x$

From above function it is clear that both are One – One functions

Now,

⇒ $(f_1 + f_2)(x) = f_1(x) + f_2(x)$

⇒ $(f_1 + f_2)(x) = x – x$

⇒ $(f_1 + f_2)(x) = 0$

Therefore,

$f_1 + f_2$: R → R is a function given by

$(f_1 + f_2)(x) = 0$

Since $f_1 + f_2$ is a constant function,

Hence it is not an One – One function.

18. Question

Give examples of two surjective function f_1 and f_2 from Z to Z such that $f_1 + f_2$ is not surjective.

Answer

TIP: –

<u>Onto Function</u>: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that $f(a) = b$

Let, f_1: Z → Z and f_2: Z → Z be two functions given by (Examples)

$f_1(x) = x$

$f_1(x) = – x$

From above function it is clear that both are Onto or Surjective functions

Now,

$f_1 + f_2$: Z → Z

⇒ $(f_1 + f_2)(x) = f_1(x) + f_2(x)$

⇒ $(f_1 + f_2)(x) = x – x$

⇒ $(f_1 + f_2)(x) = 0$

Therefore,

$f_1 + f_2$: Z → Z is a function given by

$(f_1 + f_2)(x) = 0$

Since $f_1 + f_2$ is a constant function,

Hence it is not an Onto/Surjective function.

19. Question

Show that if f_1 and f_2 are one – one maps from R to R, then the product $f_1 \times f_2 : R \to R$ defined by $(f_1 \times f_2)(x) = f_1(x)f_2(x)$ need not be one – one.

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

$\Leftrightarrow a \neq b$

$\Rightarrow f(a) \neq f(b)$ for all $a, b \in A$

$\Leftrightarrow f(a) = f(b)$

$a = b$ for all $a, b \in A$

Let, $f_1: R \to R$ and $f_2: R \to R$ are two functions given by

$f_1(x) = x$

$f_2(x) = x$

From above function it is clear that both are One – One functions

Now, $f_1 \times f_2 : R \to R$ given by

$\Rightarrow (f_1 \times f_2)(x) = f_1(x) \times f_2(x) = x^2$

$\Rightarrow (f_1 \times f_2)(x) = x^2$

Also,

$f(1) = 1 = f(-1)$

Therefore,

f is not One – One

$\Rightarrow f_1 \times f_2 : R \to R$ is not One – One function.

Hence Proved

20. Question

Suppose f_1 and f_2 are non – zero one – one functions from R to R. Is $\frac{f_1}{f_2}$ necessarily one – one? Justify your answer. Here, $\frac{f_1}{f_2} : R \to R$ is given by $\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)}$ for all $x \in R$.

Answer

TIP: – One – One Function: – A function $f: A \to B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \to B$ is One – One function

$\Leftrightarrow a \neq b$

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

a = b for all $a, b \in A$

Let, f_1: R → R and f_2: R → R are two non – zero functions given by

$f_1(x) = x^3$

$f_1(x) = x$

From above function it is clear that both are One – One functions

Now, $\frac{f_1}{f_2}: R \rightarrow R$ given by

$\Rightarrow \frac{f_1}{f_2}(x) = \frac{f_1(x)}{f_2(x)}$

$\Rightarrow \frac{f_1}{f_2}(x) = x^2$ for all $x \in R$

Again,

$\frac{f_1}{f_2} = f(\text{let}): R \rightarrow R$ defined by

$f(x) = x^2$

Now,

⇒ f(1) = 1 = f(– 1)

Therefore,

f is not One – One

$\Rightarrow \frac{f_1}{f_2}: R \rightarrow R$ is not One – One function.

Hence it is not necessarily to $\frac{f_1}{f_2}$ be one – one function.

21 A. Question

Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:

an injective map from A to B

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Now, f : A → B, denotes a mapping such that

⇒ f = {(x,y):y = x + 3}

It can be written as follows in roster form

f = {(2,5),(3,6),(4,7)}

Hence this is injective mapping

21 B. Question

Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:

a mapping from A to B which is not injective

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Now, f : A → B, denotes a mapping such that

f = {(2,2),(3,5),(4,5)}

Hence this is not injective mapping

21 C. Question

Given A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following:

a mapping from A to B.

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Now, f : A → B, denotes a mapping such that

f = {(2,2),(5,3),(6,4),(7,4)}

Here it is clear that every first component is from B and second component is from A

Hence this is mapping from B to A

22. Question

Show that f : R → R, given by f(x) = x – [x], is neither one – one nor onto.

Answer

TIP: – <u>One – One Function</u>: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Now, f : A → A given by f(x) = x – [x]

To Prove: – f(x) = x – [x], is neither one – one nor onto

Check for Injectivity:

Let x be element belongs to Z i.e $x \in Z$ such that

So, from definition

⇒ f(x) = x – [x]

⇒ f(x) = 0 for $x \in Z$

Therefore,

Range of f = [0,1] ≠ R

Hence f is not One – One function

Check for Surjectivity:

Since Range of f = [0,1] ≠ R

Hence, f is not Onto function.

Thus, it is neither One – One nor Onto function

Hence Proved

23. Question

Let f : N → N be defined by

$$f(n)=\begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$$

Show that f is a bijection.

Answer

TIP: – One – One Function: – A function $f: A \rightarrow B$ is said to be a one – one functions or an injection if different elements of A have different images in B.

So, $f: A \rightarrow B$ is One – One function

⇔ a≠b

⇒ f(a)≠f(b) for all $a, b \in A$

⇔ f(a) = f(b)

⇒ a = b for all $a, b \in A$

Onto Function: – A function $f: A \rightarrow B$ is said to be a onto function or surjection if every element of A i.e, if f(A) = B or range of f is the co – domain of f.

So, $f: A \rightarrow B$ is Surjection iff for each $b \in B$, there exists $a \in B$ such that f(a) = b

Bijection Function: – A function $f: A \to B$ is said to be a bijection function if it is one – one as well as onto function.

Now, suppose

$f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$\Rightarrow n_1 + 1 = n_2 - 2$

$\Rightarrow n_2 - n_1 = 2$

Not possible

Suppose both n_1 even and n_2 is odd.

Then, $f(n_1) = f(n_2)$

$\Rightarrow n_1 - 1 = n_2 + 1$

$\Rightarrow n_1 - n_2 = 2$

Not possible

Therefore, both n_1 and n_2 must be either odd or even

Suppose both n_1 and n_2 are odd.

Then, $f(n_1) = f(n_2)$

$\Rightarrow n_1 + 1 = n_2 + 1$

$\Rightarrow n_1 = n_2$

Suppose both n_1 and n_2 are even.

Then, $f(n_1) = f(n_2)$

$\Rightarrow n_1 - 1 = n_2 - 1$

$\Rightarrow n_1 = n_2$

Then, f is One – One

Also, any odd number 2r + 1 in the co – domain N will have an even number as image in domain N which is

$\Rightarrow f(n) = 2r + 1$

$\Rightarrow n - 1 = 2r + 1$

$\Rightarrow n = 2r + 2$

Any even number 2r in the co – domain N will have an odd number as image in domain N which is

$\Rightarrow f(n) = 2r$

$\Rightarrow n + 1 = 2r$

$\Rightarrow n = 2r - 1$

Thus f is Onto function.

Exercise 2.2

1 A. Question

Find gof and fog when f: R → R and g: R → R is defined by

$f(x) = 2x + 3$ and $g(x) = x^2 + 5$

Answer

Since, f:R → R and g:R → R

fog:R → R and gof:R → R

Now, $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

$gof(x) = g(2x + 3) = (2x + 3)^2 + 5$

$\Rightarrow gof(x) = 4x^2 + 12x + 9 + 5 = 4x^2 + 12x + 14$

$fog(x) = f(g(x)) = f(x^2 + 5) = 2(x^2 + 5) + 3$

$\Rightarrow fog(x) = 2x^2 + 10 + 3 = 2x^2 + 13$

Hence, $gof(x) = 4x^2 + 12x + 14$ and $fog(x) = 2x^2 + 13$

1 B. Question

Find gof and fog when f: R → R and g: R → R is defined by

$f(x) = 2x + x^2$ and $g(x) = x^3$

Answer

Since, f:R → R and g:R → R

fog:R → R and gof:R → R

$f(x) = 2x + x^2$ and $g(x)=x^3$

Now, $gof(x) = g(f(x)) = g(2x + x^2)$

$gof(x)=(2x + x^2)^3 = x^6 + 8x^3 + 6x^5 + 12x^4$

and $fog(x)=f(g(x))= f(x^3)$

$\Rightarrow fog(x) = 2x^3 + x^6$

So, $gof(x) = x^6 + 6x^5 + 12x^4 + 8x^3$ and $fog(x) = 2x^3 + x^6$

1 C. Question

Find gof and fog when f: R → R and g: R → R is defined by

$f(x) = x^2 + 8$ and $g(x) = 3x^3 + 1$

Answer

Since, f:R → R and g:R → R

fog:R → R and gof:R → R

$f(x)=x^2 + 8$ and $g(x)=3x^3 + 1$

So, $gof(x)= g(f(x))$

$gof(x)= g(x^2 + 8)$

$gof(x)= 3(x^2 + 8)^3 + 1$

$\Rightarrow gof(x)= 3(x^6 + 512 + 24x^4 + 192x^2) + 1$

$\Rightarrow gof(x)= 3x^6 + 72x^4 + 576x^2 + 1537$

Similarly, $fog(x)=f(g(x))$

$\Rightarrow fog(x)= f(3x^3 + 1)$

$\Rightarrow fog(x)=(3x^3 + 1)^2 + 8$

$\Rightarrow$ fog(x)=$(9x^6 + 1 + 6x^3) + 8$

$\Rightarrow$ fog(x)=$9x^6 + 6x^3 + 9$

So, gof(x) = $3x^6 + 72x^4 + 576x^2 + 1537$ and fog(x) = $9x^6 + 6x^3 + 9$

1 D. Question

Find gof and fog when f: R → R and g: R → R is defined by

f(x) = x and g(x) = |x|

Answer

Since, f:R → R and g:R → R

fog:R → R and gof:R → R

f (x) = x and g (x) = |x|

Now, gof(x)=g(f(x) =g(x)

$\Rightarrow$ gof(x) =|x|

and, fog(x) = f(g(x)) = f (|x|) $\Rightarrow$ fog(x)=|x|

Hence, gof(x) = fog(x) = |x|

1 E. Question

Find gof and fog when f: R → R and g: R → R is defined by

f(x) = $x^2 + 2x - 3$ and g(x) = $3x - 4$

Answer

Since, f:R → R and g:R → R

fog:R → R and gof:R → R

f(x) = $x^2 + 2x - 3$ and g(x) = $3x - 4$

Now, gof(x)=g(f(x))= g($x^2 + 2x - 3$)

gof(x) = $3(x^2 + 2x-3) - 4$

$\Rightarrow$ gof(x)= $3x^2 + 6x - 9 - 4$

$\Rightarrow$ gof(x) = $3x^2 + 6x - 13$

and, fog= f(g(x)) = f(3x – 4)

fog(x) = $(3x - 4)^2 + 2(3x - 4) - 3$

= $9x^2 + 16 - 24x + 6x - 8 - 3$

$\therefore$ fog(x) = $9x^2 - 18x + 5$

Thus, gof(x) = $3x^2 + 6x - 13$ and fog(x) = $9x^2 - 18x + 5$

1 F. Question

Find gof and fog when f: R → R and g: R → R is defined by

f(x) = $8x^3$ and g(x) = $x^{1/3}$

Answer

Since, f:R → R and g:R → R

fog:R → R and gof:R → R

$f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

Now, $gof(x) = g(f(x)) = g(8x^3)$

$\Rightarrow gof(x) = (8x^3)^{\frac{1}{3}}$

$gof(x) = 2x$

and, $fog(x) = f(g(x)) = f(x^{\frac{1}{3}})$

$= 8\left(x^{\frac{1}{3}}\right)^3$

$fog(x) = 8x$

Thus, $gof(x) = 2x$ and $fog(x) = 8x$

2. Question

Let f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3), (4, 9), (5, 9)}. Show that gof and fog are both defined, Also, find fog and gof.

Answer

Let f = {(3,1), (9,3), (12,4)} and

g = {(1,3), (3,3), (4,9), (5,9)}

Now,

range of f = (1, 3, 4}

domain of f = {3, 9, 12}

range of g = {3,9}

domain of g = (1, 3, 4, 5}

since, range of f ⊂ domain of g

∴ gof is well defined.

Again, the range of g ⊆ domain of f

∴ fog in well defined.

Finally, gof = {(3,3), (9 ,3), (12,9)}

fog = {(1,1) , (3,1), (4,3), (5,3)}

3. Question

Let f = {(1, – 1), (4, – 2), (9, – 3), (16, 4)} and g = {(– 1, – 2), (– 2, – 4), (– 3, – 6), (4, 8)}. Show that gof is defined while fog is not defined. Also, find gof.

Answer

We have,

f = {(1, – 1), (4, – 2) , (9, – 3), (16,4)} and

g = {(– 1, – 2), (– 2, – 4), (– 3, – 6), (4,8)}

Now,

Domain of f = {1,4,9,16}

Range of f = {– 1, – 2, – 3, 4}

Domain of g = (– 1, – 2, – 3,4}

Range of g = (– 2, – 4, – 6, 8}

Clearly range of f = domain of g

∴ gof is defined.

but, range of g ≠ domain of fSo, fog is not defined.

Now,

gof(1) = g(– 1)= – 2

gof(4) = g(– 2) = – 4

gof(9) = g (– 3) = – 6

gof(16) = g(4)= 8

So, gof = {(1, – 2), (4, – 4), (9 , – 6), (16,8)}

4. Question

Let A = {a, b c}, B = {u, v, w} and let f and g be two functions from A to B and from B to A respectively defined as: f = {(a, v), (b, u), (c, w)}, g = {(u, b), (v, a), (w, c)}.

Show that f and g both are bijections and find fog and gof.

Answer

Given, A = {a, b, c}, B = {u, v, w} and

f = A → B and g: B → A defined by

f = {(a, v), (b, u), (c, w)} and

g = {(u, b), (v, a), (w, c)}

For both f and g, different elements of domain have different

images

∴ f and g are one – one

Again, for each element in co – domain of f and g, there is a pre – image in the domain

∴ f and g are onto

Thus, f and g are bijective.

Now,

gof = {(a, a), (b, b), (c, c)} and

fog = {(u, u), (v, v), (w, w)}

5. Question

Find fog (2) and gof (1) when: f: R → R; f(x) = x^2 + 8 and g: R → R; g(x) = $3x^3$ + 1.

Answer

We have, f: R → R given by f(x) = x^2 + 8 and

g : R → R given by g (x) = $3x^3$ + 1

fog(x) = f (g(x)) = f ($3x^3$ + 1)

= $(3x^3 + 1)^2 + 8$

fog(2) = $(3 \times 8 + 1)^{2\,+}$ 8 = 625 + 8 = 633

Again,

$gof(x) = g(f(x)) = g(x^2 + 8)$

$= 3(x^2 + 8)^3 + 1$

$gof(1) = 3(1 + 8)^{3} + 1 = 2188$

6. Question

Let R^+ be the set of all non – negative real numbers. If f: $R^+ \rightarrow R^+$ and g: $R^+ \rightarrow R^+$ are defined as $f(x) = x^2$ and $g(x) = +\sqrt{x}$. Find fog and gof. Are they equal functions.

Answer

We have, f : $R^+ \rightarrow R^+$ given by

$f(x) = x^2$

g: $R^+ \rightarrow R^+$ given by

$g(x) = \sqrt{x}$

$fog(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

Also,

$gof(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$

Thus,

$fog(x) = gof(x)$

They are equal functions as their domain and range are also equal.

7. Question

Let f: R → R and g: R → R be defined by $f(x) = x^2$ and $g(x) = x + 1$. Show that fog ≠ gof.

Answer

We have, f: R → R and g: R → R are two functions defined by

$f(x) = x^2$ and $g(x) = x + 1$

Now,

$fog(x) = f(g(x)) = f(x + 1) = (x + 1)^2$

$\Rightarrow fog(x) = x^2 + 2x + 1$ ……(i)

$gof(x) = g(f(x)) = g(x^2) = x^2 + 1$ ……(ii)

from (i) & (ii)

fog ≠ gof

8. Question

Let f: R → R an g: R → R be defined by $f(x) = x + 1$ and $g(x) = x - 1$. Show that fog = gof = I_R.

Answer

Let f: R → R and g: R → R are defined as

$f(x) = x + 1$ and $g(x) = x - 1$

Now,

$fog(x) = f(g(x)) = f(x - 1) = x - 1 + 1$

$= x = I_R$ ……(i)

Again,

$fog(x) = f(g(x)) = g(x + 1) = x + 1 – 1$

$= x = I_R$ ......(ii)

from (i)& (ii)

$fog = gof = I_R$

9. Question

Verify associativity for the following three mappings: f: N → Z_0 (the set of non – zero integers), g: Z_0→ Q and h: Q → R given by f(x) = 2x, g(x) = 1/x and h(x) = e^x.

Answer

We have, f: N → Z_0, g: Z_0 → Q and h: Q → R

Also, f(x) = 2x, $g(x) = \frac{1}{x}$ and h(x) = e^x

Now, f: N → Z_0 and hog: Z_0 → R

∴ (hog)of: N → R

Also, gof: N → Q and h: Q → R

∴ ho(gof): N → R

Thus, (hog)of and ho(gof) exist and are function from N to set R.

Finally. (hog)of(x) = (hog)(f(x)) = (hog)(2x)

$$= h\left(\frac{1}{2}\right) = e^{\frac{1}{2x}}$$

Now, ho(gof)(x) = ho(g(2x)) = $h\left(\frac{1}{2x}\right)$

$$= e^{\frac{1}{2x}}$$

Hence, associativity verified.

10. Question

Consider f: N → N, g: N → N and h: N → R defined as f(x) = 2x, g(y) = 3y + 4 and h(z) = sin z for all x, y, z∈ N. Show that ho (gof) = (hog) of.

Answer

We have,

ho(gof)(x)=h(gof(x))=h(g(f(x)))

= h(g(2x)) = h(3(2x) + 4)

= h(6x + 4) = sin(6x + 4) ∀x ∈N

((hog)of)(x) = (hog)(f(x))= (hog)(2x)

=h(g(2x))=h(3(2x) + 4)

=h(6x + 4) = sin(6x + 4) ∀x ∈N

This shows, ho(gof) = (hog)of

11. Question

Give examples of two functions f: N → N and g: N → N such that gof is onto, but f is not onto.

Answer

Define f:N → N by, f(x) = x + 1 And, g: N → N by,

$$g(x) = \begin{cases} x - 1 \text{ if } x > 1 \\ 1 \text{ if } x = 1 \end{cases}$$

We first show that f is not onto.

For this, consider element 1 in co – domain N. It is clear that this element is not an image of any of the elements in domain N.

Therefore, f is not onto.

12. Question

Give examples of two functions f: N → Z and g: Z → Z such that gof is injective, but g is not injective.

Answer

Define f: N → Z as f(x) = x and g: N → N as g(x)=|x|.

We first show that g is not injective.

It can be observed that:

g(– 1)=| – 1| = 1

g(1) =|1| = 1

Therefore, g(– 1) = g(1), but —1 ≠ 1.

Therefore, g is not injective.

Now, gof: N → Z is defined as gof(x) = g(f(x)) =g(x)=|x|.

Let x, y ∈N such that gof(x) = gof(y).

⇒ |x|=|y|

Since x and y ∈N both are positive.

∴ |x|=|y| ⇒ x=y

Hence, gof is injective

13. Question

If f: A → B and g: B → C are one – one functions show that gof is a one – one function.

Answer

We have, f : A → B and g : B → C are one – one functions.

Now we have to prove : gof: A → C in one – one

let x, y ∈ A such that

gof(x) = gof(y)

g(f(x))=g(f(y))

f (x) = f (y) [As, g in one – one]

x = y [As, f in one – one]

gof is one – one function

14. Question

If f: A → B and g: B → C are onto functions show that gof is an onto function.

Answer

We have, f: A → B and g: B → C are onto functions.

Now, we need to prove: gof: A → C is onto.

let $y \in C$, then

gof (x) = y

g(f(x)) = y ……(i)

Since g is onto, for each element in C, there exists a preimage in B.

g(x)=y ……(ii)

From (i) & (ii)

f(x)=x

Since f is onto, for each element in B there exists a preim age in el

f(x)=x ……(iii)

From (ii)and(iii) we can conclude that for each $y \in C$, there exists a preimage in A such that gof(x) = y

∴ gof is onto.

Exercise 2.3

1 A. Question

Find fog and gof, if

$f(x) = e^x$, $g(x) = \log_e x$

Answer

$f(x) = e^x$ and $g(x) = \log_e x$

Now, $fog(x) = f(g(x)) = f(\log_e x) = e^{\log_e x} = x$

⇒ fog(x) = x

$gof(x) = g(f(x)) = g(e^x) = \log_e e^x = x$

⇒ gof(x)=x

Hence, fog(x) = x and gof(x) = x

1 B. Question

Find fog and gof, if

$f(x) = x^2$, g(x) = cos x

Answer

f(x)= x, g (x) = cos x

Domain of f and Domain of g = R

Range of f = (0, ∞)

Range of g = (– 1,1)

∴ Range of f ⊂ domain of g ⇒ gof exist

Also, Range of g ⊂ domain of f ⇒ fog exist

Now,

$gof (x) = g (f (x)) = g (x^2) = \cos x^2$

And

fog (x) = f(g(x)) = f (cos x) = $\cos^2 x$

Hence, fog(x) = $\cos x^2$ and gof(x) = $\cos^2 x$

1 C. Question

Find fog and gof, if

f(x) = |x|, g(x) = sin x

Answer

f(x)= |x| and g (x)= sin x

Range of f = (0, ∞) ⊂ Domain g (R) ⇒ gof exist

Range of g= [– 1,1] ⊂ Domain f (R) ⇒ fog exist

Now, fog (x)= f(g(x)) = f(sin x) = |sin x| and

gof(x) = g(f(x)) = g(|x|) =sin |x|

Hence, fog(x) = |sin x| and gof(x) = sin |x|

1 D. Question

Find fog and gof, if

f(x) = x + 1, g(x) = e^x

Answer

f(x) = x + 1 and g(x) = e^x

Range of f = R ⊂ Domain of g= R ⇒ gof exist

Range of g = (0, ∞) = Domain of f = R ⇒ fog exist

Now,

gof(x) = g(f(x)) = g(x + 1) = e^{x+1}

And

fog (x) = f(g(x))= f(e^x) = $e^x + 1$

Hence, fog(x) = e^{x+} 1 and gof(x) = $e^x + 1$

1 E. Question

Find fog and gof, if

f(x) = $\sin^{-1} x$, g(x) = x^2

Answer

f(x) = $\sin^{-1} x$ and g (x) = x^2

Range of f=$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ⊂ Domain of g = R ⇒ gof exist

Range of g= (0,∞) ⊂ Domain of f = R ⇒ fog exist

Now,

fog (x) = f(g(x)) = f(x^2) = $\sin^{-1} x^2$ and

gof(x) = g(f(x)) = g ($\sin^{-1} x$) = $(\sin^{-1} x)^2$

Hence, fog(x) = $\sin^{-1} x^2$ and gof(x) = $(\sin^{-1} x)^2$

1 F. Question

Find fog and gof, if

$f(x) = x + 1$, $g(x) = \sin x$

Answer

$f(x) = x + 1$ and $g(x) = \sin x$

Range of $f = R \subset$ Domain of $g = R \Rightarrow$ gof exists

Range of $g = [-1,1] \subset$ Domain of $f \Rightarrow$ fog exists

Now,

$fog(x) = f(g(x)) = f(\sin x) = \sin x + 1$

And

$gof(x) = g(f(x)) = g(x + 1) = \sin(x + 1)$

Hence, $fog(x) = \sin x + 1$ and $gof(x) = \sin(x + 1)$

1 G. Question

Find fog and gof, if

$f(x) = x + 1$, $g(x) = 2x + 3$

Answer

$f(x) = x + 1$ and $g(x) = 2x + 3$

Range of $f = R \subset$ Domain of $g = R \Rightarrow$ gof exists

Range of $g = R \subset$ Domain of $f \Rightarrow$ fog exists

Now,

$fog(x) = f(g(x) = f(2x + 3) = (2x + 3) + 1 = 2x + 4$ and

$gof(x) = g(f(x)) = g(x + 1) = 2(x + 1) + 3 = 2x + 5$

So, $fog(x) = 2x + 4$ and $gof(x) = 2x + 5$

1 H. Question

Find fog and gof, if

$f(x) = c$, $c \in R$, $g(x) = \sin x^2$

Answer

$f(x) = c$, $c \in R$ and

$g(x) = \sin x^2$

Range of $f = R \subset$ Domain of $g = R \Rightarrow$ gof exists

Range of $g = [-1,1] \subset$ Domain of $f = R \Rightarrow$ fog exists

Now,

$gof(x) = g(f(x)) = g(c) = \sin c^2$ and

$fog(x) = f(g(x)) = f(\sin x^2) = c$

Thus, $gof(x) = \sin c^2$ and $fog(x) = c$

1 I. Question

Find fog and gof, if

f(x) = x^2 + 2, $g(x) = 1 - \frac{1}{1-x}$

Answer

f(x) = x^2 + 1 and $g(x) = 1 - \frac{1}{1-x}$

Range of f = (2,∞) ⊂ Domain of g = R ⇒ gof exists

Range of g= R – [– 1] ⊂ Domain of f =R ⇒ fog exists

Now,

$fog(x) = f(g(x)) = f\left(-\frac{x}{1-x}\right) = \frac{x^2}{(1-x)^2} + 2$ and

$gof(x) = g(f(x)) = g(x2 + 2) = -\frac{x^2 + 2}{1-(x^2 + 2)}$

$gof(x) = \frac{x^2 + 2}{(x^2 + 1)}$

Hence, $fog(x) = \frac{x^2}{(1-x)^2} + 2$ and $gof(x) = -\frac{x^2+2}{1-(x^2+2)}$

2. Question

Let f(x) = x^2 + x + 1 and g(x) = sin x. Show that fog ≠ gof.

Answer

We have, f (x) = x^2 + x + 1 and g(x) = sin x

Now,

fog(x) = f(g(x)) = f(sin x)

⇒ fog(x) = $\sin^2$ x + sin x + 1

Again, gof(x) = g(f(x)) = g (x^2 + x + 1)

⇒ gof(x) = sin(x^2 + x + 1)

Clearly,

fog ≠ gof

3. Question

If f(x) = |x|, prove that fof = f.

Answer

We have, f(x) = |x|

We assume the domain of f = R and range of f = (0,∞)

Range of f ⊂ domain of f

∴ fof exists,

Now,

fof(x) = f(f(x)) = f(|x|) = ||x|| = f(x)

∴ fof = f

Hence proved.

4. Question

If $f(x) = 2x + 5$ and $g(x) = x^2 + 1$ be two real functions, then describe each of the following functions:

(i) fog

(ii) gof

(iii) fof

(iv) f^2

Also, show that fof $\neq f^2$.

Answer

$f(x)= 2x + 5$ and $g(x)= x^{2\,+}\ 1$

The range of f = R and range of g = $[1,\infty]$

The range of f $\subset$ Domain of g (R) and range of g $\subset$ domain of f (R)

$\therefore$ both fog and gof exist.

(i) $fog(x) = f(g(x)) = f\ (x^2 + 1)$

$= 2(x^2 + 1) + 5$

$\Rightarrow fog(x)=2x^2 + 7$

Hence $fog(x) = 2x^2 + 7$

(ii) $gof(x) = g(f(x))\ ^{-} = g\ (2x + 5)$

$= (2x + 5)^2 + 1$

$gof(x)= 4x^2 + 20x + 26$

Hence $gof(x) = 4x^2 + 20x + 26$

(iii) $fof(x) = f(f(x)) = f(2x + 5)$

$= 2\ (2x + 5) + 5$

$fof(x) = 4x + 15$

Hence $fof(x) = 4x + 15$

(iv) $f^2(x) = [f(x)]^2= (2x + 5)^2$

$= 4x^2 + 20x + 25$

$\therefore$ from (iii) and (iv)

fof $\neq f^2$

5. Question

If $f(x) = \sin x$ and $g(x) = 2x$ be two real functions, then describe gof and fog. Are these equal functions?

Answer

We have, $f\ (x) = \sin x$ and $g\ (x) = 2x$.

Domain of f and g is R

Range of f = [– 1,1], Range of g = R

$\therefore$ Range of f $\subset$ Domain g and Range of g $\subset$ Domain f

fog and gof both exist.

$gof(x) = g(f(x)) = g(\sin x)$

⇒ gof(x) = 2sin x

fog(x) = f(g(x)) = f(2x) = sin 2x

∴ gof ≠ fog

6. Question

Let f, g, h be real functions given by f(x) = sin x, g(x) = 2x and h(x) = cos x. Prove that fog = go(fh).

Answer

f, g and h are real functions given by f(x) = sin x, g(x) = 2x and

h(x) = cos x

To prove: fog=go(fh)

L.H.S

fog(x) = f(g(x))

= f(2x) = sin 2x

⇒ fog(x)=2sin x cos x ……(A)

R.H .S

go(fh)(x) = go(f(x).h(x))

= g(sin x cos x) = 2sin x cos x

go(fh)(x) = 2 sin x cos x ……(B)

from A and B

fog(x) = go(fh)(x)

Hence proved

7. Question

Let f be any real function and let g be a function given by g(x) = 2x. Prove that gof = f + f.

Answer

We are given that f is a real function and g is a function given by

g(x) = 2x

To prove; gof=f + f.

L.H.S

gof(x) = g(f(x)) = 2f(x)

=f + f = R.H.S

gof=f + f

Hence proved

8. Question

If $f(x) = \sqrt{1-x}$ and g(x) = $\log_e$x are two real functions, then describe functions fog and gof.

Answer

f(x) = $\sqrt{1-x}$, g(x) = $\log_e x$

Domain of f and g are R.

Range of f= (– ∞, 1) Range of g = (0, e)

Range of f ⊂ Domain of g ⇒ gof exists

Range of g ⊂ Domain f ⇒ fog exists

∴ gof (x) = g(f(x)) = $g\left(\sqrt{1-x}\right)$

∴ gof (x) = $\log_e \sqrt{1-x}$

Again

fog (x) = f(g(x)) = f($\log_e x$)

fog (x) = $\sqrt{1-\log_e x}$

9. Question

If f: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ → R and g: [–1, 1] → R be defined as f(x) = tan x and $g(x)=\sqrt{1-x^2}$ respectively. Describe fog and gof.

Answer

f: $\left(-\frac{\pi}{2},\frac{\pi}{2}\right) \to \mathrm{R}$ and g: [– 1,1] → R defined as f(x) = tan x and g(x) = $\sqrt{1-x^2}$

Range of f: let y = f(x)

⇒ y = tan x

⇒ x = $\tan^{-1} y$

Since, x ∈ $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, y ∈ (– ∞, ∞)

As Range of f ⊂ Domain of g

∴ gof exists.

Similarly, let y = g(x)

⇒ y = $\sqrt{1-x^2}$

⇒ x = $\sqrt{1-y^2}$

∴ Range of g is [– 1,1]

As, Range of g ⊂ Domain of f

Hence, fog also exists

Now,

fog(x) = f(g(x)) = $f\left(\sqrt{1-x^2}\right)$

⇒ fog(x) = tan $\sqrt{1-x^2}$

Again,

gof(x) = g(f(x)) = g(tan x)

⇒ gof(x) = $\sqrt{1-\tan^2 x}$

10. Question

If $f(x)=\sqrt{x+3}$ and g(x) = x^2 + 1 be two real functions, then find fog and gof.

Answer

$f(x) = \sqrt{x + 3}$, $g(x) = x^2 + 1$

Now,

Domain of f = [– 3, ∞], domain of g = (– ∞, ∞)

Range of f = [0, ∞), range of g = [1, ∞)

Then, range of f ⊂ Domain of g and range of g ⊂ Domain of f

Hence, fog and gof exists

Now,

$fog(x) = f(g(x)) = f(x^2 + 1)$

$\Rightarrow fog(x) = \sqrt{x^2 + 4}$

Again,

$gof(x) = g(f(x)) = g(\sqrt{x + 3})$

$\Rightarrow gof(x) = \left(\sqrt{x + 3}\right)^2 + 1$

$\Rightarrow gof(x) = x + 4$

11 A. Question

Let f be a real function given by $f(x) = \sqrt{x - 2}$. Find each of the following:

fof

Answer

We have, $f(x) = \sqrt{x - 2}$

Clearly, domain of f = [2, ∞] and range of f = [0, ∞)

We observe that range of f is not a subset of domain of f

∴ Domain of (fof) = {x: x ϵ Domain of f and f(x) ϵ Domain of f}

= {x: x ϵ [2, ∞) and $\sqrt{x - 2}$ ϵ [2, ∞)}

= {x: x ϵ [2, ∞) and $\sqrt{x - 2} \geq 2$}

= {x: x ϵ [2, ∞) and x – 2 ≥ 4}

= {x: x ϵ [2, ∞) and x ≥ 6}

= [6, ∞)

Now,

$fof(x) = f(f(x)) = f\left(\sqrt{x - 2}\right) = \sqrt{\sqrt{x - 2} - 2}$

11 B. Question

Let f be a real function given by $f(x) = \sqrt{x - 2}$. Find each of the following:

fofof

Answer

We have, $f(x) = \sqrt{x - 2}$

Clearly, domain of f = [2, ∞] and range of f = [0, ∞)

We observe that range of f is not a subset of domain of f

∴ Domain of (fof) = {x: x ϵ Domain of f and f(x) ϵ Domain of f}

= {x: x ϵ [2, ∞) and $\sqrt{x-2}$ ϵ [2, ∞)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2} \geq 2$}

= {x: x ϵ [2, ∞) and x – 2 ≥ 4}

= {x: x ϵ [2, ∞) and x ≥ 6}

= [6, ∞)

Clearly, range of f = [0, ∞) ⊄ Domain of (fof)

∴ Domain of ((fof)of) = {x: x ϵ Domain of f and f(x) ϵ Domain of (fof)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2}$ ϵ [6, ∞)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2} \geq 6$}

= {x: x ϵ [2, ∞) and x – 2 ≥ 36}

= {x: x ϵ [2, ∞) and x ≥ 38}

= [38, ∞)

Now,

(fof)(x) = f(f(x)) = $f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$

(fofof)(x) = (fof)(f(x)) = (fof)$\left(\sqrt{x-2}\right) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$

∴ fofof : [38, ∞) → R defined as

(fofof)(x) = $\sqrt{\sqrt{\sqrt{x-2}-2}-2}$

11 C. Question

Let f be a real function given by $f(x) = \sqrt{x-2}$. Find each of the following:

(fofof)(38)

Answer

We have, f(x) = $\sqrt{x-2}$

Clearly, domain of f = [2, ∞] and range of f = [0, ∞)

We observe that range of f is not a subset of domain of f

∴ Domain of (fof) = {x: x ϵ Domain of f and f(x) ϵ Domain of f}

= {x: x ϵ [2, ∞) and $\sqrt{x-2}$ ϵ [2, ∞)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2} \geq 2$}

= {x: x ϵ [2, ∞) and x – 2 ≥ 4}

= {x: x ϵ [2, ∞) and x ≥ 6}

= [6, ∞)

Clearly, range of f = [0, ∞) ⊄ Domain of (fof)

∴ Domain of ((fof)of) = {x: x ϵ Domain of f and f(x) ϵ Domain of (fof)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2}$ ϵ [6, ∞)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2} \geq 6$}

= {x: x ϵ [2, ∞) and x – 2 ≥ 36}

= {x: x ϵ [2, ∞) and x ≥ 38}

= [38, ∞)

Now,

(fof)(x) = f(f(x)) = $f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$

(fofof)(x) = (fof)(f(x)) = (fof)$\left(\sqrt{x-2}\right) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$

∴ fofof : [38, ∞) → R defined as

(fof)(x) = f(f(x)) = $f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$

(fofof)(x) = (fof)(f(x)) = (fof)$\left(\sqrt{x-2}\right) = \sqrt{\sqrt{\sqrt{x-2}-2}-2}$

∴ fofof : [38, ∞) → R defined as

(fofof)(x) = $\sqrt{\sqrt{\sqrt{x-2}-2}-2}$

(fofof)(38) = $\sqrt{\sqrt{\sqrt{38-2}-2}-2} = \sqrt{\sqrt{\sqrt{36}-2}-2}$

$= \sqrt{\sqrt{6-2}-2} = \sqrt{\sqrt{4}-2} = \sqrt{2-2} = 0$

11 D. Question

Let f be a real function given by $f(x) = \sqrt{x-2}$. Find each of the following:

f^2

Also, show that fof ≠ f^2.

Answer

We have, f(x) = $\sqrt{x-2}$

Clearly, domain of f = [2, ∞] and range of f = [0, ∞)

We observe that range of f is not a subset of domain of f

∴ Domain of (fof) = {x: x ϵ Domain of f and f(x) ϵ Domain of f}

= {x: x ϵ [2, ∞) and $\sqrt{x-2}$ ϵ [2, ∞)}

= {x: x ϵ [2, ∞) and $\sqrt{x-2} \geq 2$}

= {x: x ϵ [2, ∞) and x – 2 ≥ 4}

= {x: x ϵ [2, ∞) and x ≥ 6}

= [6, ∞)

Now,

(fof)(x) = f(f(x)) = $f\left(\sqrt{x-2}\right) = \sqrt{\sqrt{x-2}-2}$

∴ fof: [6, ∞) → R defined as

$(fof)(x) = \sqrt{\sqrt{x-2}-2}$

$f^2(x) = [f(x)]^2 = \left[\sqrt{x-2}\right]^2 = x - 2$

$\therefore f^2: [2, \infty) \to R$ defined as

$f^2(x) = x - 2$

$\therefore fof \neq f^2$

12. Question

Let $f(x) = \begin{cases} 1+x, & 0 \le x \le 2 \\ 3-x, & 2 < x \le 3 \end{cases}$. Find fof.

Answer

$f(x) = \begin{cases} 1 + x & 0 \le x \le 2 \\ 3 - x & 2 < x \le 3 \end{cases}$

Range of f = [0, 3] ⊂ Domain of f

$\therefore fof(x) = f(f(x)) = f\left(\begin{cases} 1 + x & 0 \le x \le 2 \\ 3 - x & 2 < x \le 3 \end{cases}\right) = f\left(\begin{cases} 1 + (1 + x) & 0 \le x \le 1 \\ 3 - (1 + x) & 1 < x \le 2 \\ 1 + (3 - x) & 2 < x \le 3 \end{cases}\right)$

So, $fof(x) = \begin{cases} 2 + x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 4 - x & 2 < x \le 3 \end{cases}$

13. Question

If f, g: R → R be two functions defined as f(x) = |x| + x and

g(x) = |x|–x for all x ∈ R. Then, find fog and gof. Hence, find fog (–3), fog (5) and gof(–2).

Answer

Domain of f(x) and g(x) is R.

Range of f(x) = [0, ∞) and range of g(x) = [0, ∞)

As, range of f ⊂ Domain of g and range of g ⊂ Domain of f

So, gof and fog exists

Now,

fog(x) = f(g(x)) = f(|x|–x)

⇒ fog(x) = ||x|–x| + |x|–x

As, range of g(x) ≥ 0 so, ||x|–x| = |x|–x

So, fog(x) = ||x|–x| + |x|–x = |x|–x + |x|–x

⇒ fog(x) = 2(|x|–x)

Also,

gof(x) = g(f(x)) = g(|x| + x) = ||x| + x| – (|x| + x)

As, range of f(x) ≥ 0 so, ||x| + x| = |x| + x

So, gof(x) = ||x| + x| – (|x| + x) = |x| + x – (|x| + x) = 0

Thus, gof(x) = 0

Now, fog(– 3) = 2(| – 3|–(– 3)) = 2(3 + 3) = 6,

fog(5) = 2(|5| – 5) = 0, gof(– 2) =0

Exercise 2.4

1. Question

State with reasons whether the following functions have inverse:

(i) f : [1, 2, 3, 4] → {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

(ii) g : {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

(iii) h : {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Answer

(i) f : [1, 2, 3, 4] → {10} with f = {(1, 10), (2, 10), (3, 10), (4, 10)}

Recall that a function is invertible only when it is both one-one and onto.

Here, we have f(1) = 10 = f(2) = f(3) = f(4)

Hence, f is not one-one.

Thus, the function f does not have an inverse.

(ii) g : {5, 6, 7, 8} → {1, 2, 3, 4} with g = {(5, 4), (6, 3), (7, 4), (8, 2)}

Recall that a function is invertible only when it is both one-one and onto.

Here, we have g(5) = 4 = g(7)

Hence, g is not one-one.

Thus, the function g does not have an inverse.

(iii) h : {2, 3, 4, 5} → {7, 9, 11, 13} with h = {(2, 7), (3, 9), (4, 11), (5, 13)}

Recall that a function is invertible only when it is both one-one and onto.

Here, observe that distinct elements of the domain {2, 3, 4, 5} are mapped to distinct elements of the co-domain {7, 9, 11, 13}.

Hence, h is one-one.

Also, each element of the range {7, 9, 11, 13} is the image of some element of {2, 3, 4, 5}.

Hence, h is also onto.

Thus, the function h has an inverse.

2. Question

Find f^{-1} if it exists for f: A → B where

(i) A = {0, –1, –3, 2}; B = {–9, –3, 0, 6} & f(x) = 3x

(ii) A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} & f(x) = x^2

Answer

(i) A = {0, –1, –3, 2}; B = {–9, –3, 0, 6} & f(x) = 3x

We have f : A → B and f(x) = 3x.

⇒ f = {(0, 3×0), (–1, 3×(-1)), (–3, 3×(-3)), (2, 3×2)}

∴ f = {(0, 0), (–1, –3), (–3, –9), (2, 6)}

Recall that a function is invertible only when it is both one-one and onto.

Here, observe that distinct elements of the domain {0, –1, –3, 2} are mapped to distinct elements of the co-domain {0, –3, –9, 6}.

Hence, f is one-one.

Also, each element of the range {–9, –3, 0, 6} is the image of some element of {0, –1, –3, 2}.

Hence, f is also onto.

Thus, the function f has an inverse.

We have f^{-1} = {(0, 0), (–3, –1), (–9, –3), (6, 2)}

(ii) A = {1, 3, 5, 7, 9}; B = {0, 1, 9, 25, 49, 81} & f(x) = x^2

We have f : A → B and f(x) = x^2.

⇒ f = {(1, 1^2), (3, 3^2), (5, 5^2), (7, 7^2), (9, 9^2)}

∴ f = {(1, 1), (3, 9), (5, 25), (7, 49), (9, 81)}

Recall that a function is invertible only when it is both one-one and onto.

Here, observe that distinct elements of the domain {1, 3, 5, 7, 9} are mapped to distinct elements of the co-domain {1, 9, 25, 49, 81}.

Hence, f is one-one.

However, the element 0 of the range {0, 1, 9, 25, 49, 81} is not the image of any element of {1, 3, 5, 7, 9}.

Hence, f is not onto.

Thus, the function f does not have an inverse.

3. Question

Consider f : {1, 2, 3} → {a, b, c} and g : {a, b, c} → {apple, ball, cat} defined as f(1) = a, f(2) = b, f(3) = c, g(a) = apple, g(b) = ball and g(c) = cat. Show that f, g and gof are invertible. Find f^{-1}, g^{-1}, $(gof)^{-1}$ and show that $(gof)^{-1} = f^{-1}og^{-1}$.

Answer

f : {1, 2, 3} → {a, b, c} and f(1) = a, f(2) = b, f(3) = c

⇒ f = {(1, a), (2, b), (3, c)}

Recall that a function is invertible only when it is both one-one and onto.

Here, observe that distinct elements of the domain {1, 2, 3} are mapped to distinct elements of the co-domain {a, b, c}.

Hence, f is one-one.

Also, each element of the range {a, b, c} is the image of some element of {1, 2, 3}.

Hence, f is also onto.

Thus, the function f has an inverse.

We have f^{-1} = {(a, 1), (b, 2), (c, 3)}

g : {a, b, c} → {apple, ball, cat} and g(a) = apple, g(b) = ball, g(c) = cat

⇒ g = {(a, apple), (b, ball), (c, cat)}

Similar to the function f, g is also one-one and onto.

Thus, the function g has an inverse.

We have g^{-1} = {(apple, a), (ball, b), (cat, c)}

We know (gof)(x) = g(f(x))

Thus, gof : {1, 2, 3} → {apple, ball, cat} and

(gof)(1) = g(f(1)) = g(a) = apple

(gof)(2) = g(f(2)) = g(b) = ball

(gof)(3) = g(f(3)) = g(c) = cat

⇒ gof = {(1, apple), (2, ball), (3, cat)}

As the functions f and g, gof is also both one-one and onto.

Thus, the function gof has an inverse.

We have $(gof)^{-1}$ = {(apple, 1), (ball, 2), (cat, 3)}

Now, let us consider $f^{-1}og^{-1}$.

We know $(f^{-1}og^{-1})(x) = f^{-1}(g^{-1}(x))$

Thus, $f^{-1}og^{-1}$: {apple, ball, cat} → {1, 2, 3} and

$(f^{-1}og^{-1})$(apple) = $f^{-1}(g^{-1}$(apple)) = $f^{-1}(a) = 1$

$(f^{-1}og^{-1})$(ball) = $f^{-1}(g^{-1}$(ball)) = $f^{-1}(b) = 2$

$(f^{-1}og^{-1})$(cat) = $f^{-1}(g^{-1}$(cat)) = $f^{-1}(c) = 3$

⇒ $f^{-1}og^{-1}$ = {(apple, 1), (ball, 2), (cat, 3)}

Therefore, we have $(gof)^{-1} = f^{-1}og^{-1}$.

4. Question

Let A = {1, 2, 3, 4}; B = {3, 5, 7, 9}; C = {7, 23, 47, 79} and f : A→ B, g : B → C be defined as f(x) = 2x + 1 and $g(x) = x^2 – 2$. Express $(gof)^{-1}$ and $f^{-1}og^{-1}$ as the sets of ordered pairs and verify $(gof)^{-1} = f^{-1}og^{-1}$.

Answer

We have f : A → B & f(x) = 2x + 1

⇒ f = {(1, 2×1 + 1), (2, 2×2 + 1), (3, 2×3 + 1), (4, 2×4 + 1)}

∴ f = {(1, 3), (2, 5), (3, 7), (4, 9)}

Function f is clearly one-one and onto.

Thus, f^{-1} exists and f^{-1} = {(3, 1), (5, 2), (7, 3), (9, 4)}

We have g : B → C & $g(x) = x^2 – 2$

⇒ g = {(3, 3^2 – 2), (5, 5^2 – 2), (7, 7^2 – 2), (9, 9^2 – 2)}

∴ g = {(3, 7), (5, 23), (7, 47), (9, 79)}

Function g is clearly one-one and onto.

Thus, g^{-1} exists and g^{-1} = {(7, 3), (23, 5), (47, 5), (79, 9)}

We know (gof)(x) = g(f(x))

Thus, gof : A → C and

(gof)(1) = g(f(1)) = g(3) = 7

(gof)(2) = g(f(2)) = g(5) = 23

(gof)(3) = g(f(3)) = g(7) = 47

(gof)(4) = g(f(4)) = g(9) = 79

⇒ gof = {(1, 7), (2, 23), (3, 47), (4, 79)}

Clearly, gof is also both one-one and onto.

Thus, the function gof has an inverse.

We have $(gof)^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\}$

Now, let us consider $f^{-1}og^{-1}$.

We know $(f^{-1}og^{-1})(x) = f^{-1}(g^{-1}(x))$

Thus, $f^{-1}og^{-1} : C \rightarrow A$ and

$(f^{-1}og^{-1})(7) = f^{-1}(g^{-1}(7)) = f^{-1}(3) = 1$

$(f^{-1}og^{-1})(23) = f^{-1}(g^{-1}(23)) = f^{-1}(5) = 2$

$(f^{-1}og^{-1})(47) = f^{-1}(g^{-1}(47)) = f^{-1}(7) = 3$

$(f^{-1}og^{-1})(79) = f^{-1}(g^{-1}(79)) = f^{-1}(9) = 4$

$\Rightarrow f^{-1}og^{-1} = \{(7, 1), (23, 2), (47, 3), (79, 4)\}$

Therefore, we have $(gof)^{-1} = f^{-1}og^{-1}$.

5. Question

Show that the function $f : Q \rightarrow Q$ defined by $f(x) = 3x + 5$ is invertible. Also, find f^{-1}.

Answer

We have $f : Q \rightarrow Q$ and $f(x) = 3x + 5$.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in Q$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 + 5 = 3x_2 + 5$

$\Rightarrow 3x_1 = 3x_2$

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let $y \in Q$ (co-domain) such that $f(x) = y$

$\Rightarrow 3x + 5 = y$

$\Rightarrow 3x = y - 5$

$\therefore x = \frac{y-5}{3}$

Clearly, for every $y \in Q$, there exists $x \in Q$ (domain) such that $f(x) = y$ and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \frac{y-5}{3}$

Hence, $f^{-1}(y) = \frac{y-5}{3}$

Thus, f(x) is invertible and $f^{-1}(x) = \frac{x-5}{3}$

6. Question

Show that the function f : R → R defined by f(x) = 4x + 3 is invertible. Find the inverse of f.

Answer

We have f : R → R and f(x) = 4x + 3.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in$ R (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow 4x_1 + 3 = 4x_2 + 3$

$\Rightarrow 4x_1 = 4x_2$

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ∈ R (co-domain) such that f(x) = y

⇒ 4x + 3 = y

⇒ 4x = y – 3

$\therefore x = \frac{y-3}{4}$

Clearly, for every y ∈ R, there exists x ∈ R (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have f(x) = y ⇒ x = $f^{-1}(y)$

But, we found f(x) = y ⇒ $x = \frac{y-3}{4}$

Hence, $f^{-1}(y) = \frac{y-3}{4}$

Thus, f(x) is invertible and $f^{-1}(x) = \frac{x-3}{4}$

7. Question

Consider f : $R^+ \to [4, \infty)$ given by f(x) = $x^2 + 4$. Show that f is invertible with f^{-1} of f given by $f^{-1}(x) = \sqrt{x-4}$, where R^+ is the set of all non-negative real numbers.

Answer

We have f : $R^+ \to [4, \infty)$ and f(x) = $x^2 + 4$.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in R^+$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + 4 = x_2^2 + 4$

$\Rightarrow x_1^2 = x_2^2$

$\therefore x_1 = x_2$ ($x_1 \neq -x_2$ as $x_1, x_2 \in R^+$)

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ∈ [4, ∞) (co-domain) such that f(x) = y

$\Rightarrow x^2 + 4 = y$

$\Rightarrow x^2 = y - 4$

$\therefore x = \sqrt{y-4}$

Clearly, for every y ∈ [4, ∞), there exists x ∈ R^+ (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \sqrt{y-4}$

Hence, $f^{-1}(y) = \sqrt{y-4}$

Thus, f(x) is invertible and $f^{-1}(x) = \sqrt{x-4}$

8. Question

If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that (fof)(x) = x for all $x \neq \frac{2}{3}$. What is the inverse of f?

Answer

We have $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$

We know (fof)(x) = f(f(x))

$$\Rightarrow (fof)(x) = f\left(\frac{4x+3}{6x-4}\right)$$

$$\Rightarrow (fof)(x) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$$

$$\Rightarrow (fof)(x) = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)}$$

$$\Rightarrow (fof)(x) = \frac{16x+12+18x-12}{24x+18-24x+16}$$

$$\Rightarrow (fof)(x) = \frac{34x}{34}$$

$$\therefore (fof)(x) = x$$

As (fof)(x) = x = I_x (the identity function), $f(x) = f^{-1}(x)$.

Thus, $f^{-1}(x) = \frac{4x+3}{6x-4}$

9. Question

Consider f : $R^+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$.

Answer

We have f : $R^+ \rightarrow [-5, \infty)$ and $f(x) = 9x^2 + 6x - 5$.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in R^+$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9x_1^2 + 6x_1 = 9x_2^2 + 6x_2$

$\Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 = 0$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow 9(x_1 - x_2)(x_1 + x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[9(x_1 + x_2) + 6] = 0$

$\Rightarrow x_1 - x_2 = 0$ (as $x_1, x_2 \in R^+$)

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let $y \in [-5, \infty)$ (co-domain) such that $f(x) = y$

$\Rightarrow 9x^2 + 6x - 5 = y$

Adding 6 to both sides, we get

$9x^2 + 6x - 5 + 6 = y + 6$

$\Rightarrow 9x^2 + 6x + 1 = y + 6$

$\Rightarrow (3x + 1)^2 = y + 6$

$\Rightarrow 3x + 1 = \sqrt{y + 6}$

$\Rightarrow 3x = \sqrt{y + 6} - 1$

$\therefore x = \dfrac{\sqrt{y + 6} - 1}{3}$

Clearly, for every $y \in [4, \infty)$, there exists $x \in R^+$ (domain) such that $f(x) = y$ and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$

Hence, $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

Thus, f(x) is invertible and $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$

10. Question

If f : R → R be defined by $f(x) = x^3 - 3$, then prove that f^{-1} exists and find a formula for f^{-1}. Hence, find $f^{-1}(24)$

and $f^{-1}(5)$.

Answer

We have f : R → R and $f(x) = x^3 - 3$.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in$ R (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 - 3 = x_2^3 - 3$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

$\Rightarrow x_1 - x_2 = 0$ (as $x_1, x_2 \in R^+$)

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ∈ R (co-domain) such that f(x) = y

$\Rightarrow x^3 - 3 = y$

$\Rightarrow x^3 = y + 3$

$\therefore x = \sqrt[3]{y+3}$

Clearly, for every y ∈ R, there exists x ∈ R (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \sqrt[3]{y+3}$

Hence, $f^{-1}(y) = \sqrt[3]{y+3}$

Thus, f(x) is invertible and $f^{-1}(x) = \sqrt[3]{x+3}$

Hence, we have

$f^{-1}(24) = \sqrt[3]{24+3} = \sqrt[3]{27} = 3$

$f^{-1}(5) = \sqrt[3]{5+3} = \sqrt[3]{8} = 2$

Thus, $f^{-1}(24) = 3$ and $f^{-1}(5) = 2$.

11. Question

A function f : R → R is defined as $f(x) = x^3 + 4$. Is it a bijection or not? In case it is a bijection, find $f^{-1}(3)$.

Answer

We have f : R → R and $f(x) = x^3 + 4$.

Recall that a function is a bijection only if it is both one-one and onto.

First, we will check if f is one-one.

Let $x_1, x_2 \in$ R (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 + 4 = x_2^3 + 4$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

As $x_1, x_2 \in R$ and the second factor has no real roots,

$x_1 - x_2 = 0$

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will check if f is onto.

Let y ϵ R (co-domain) such that f(x) = y

$\Rightarrow x^3 + 4 = y$

$\Rightarrow x^3 = y - 4$

$\therefore x = \sqrt[3]{y-4}$

Clearly, for every y ϵ R, there exists x ϵ R (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f is a bijection and has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \sqrt[3]{y-4}$

Hence, $f^{-1}(y) = \sqrt[3]{y-4}$

Thus, f(x) is invertible and $f^{-1}(x) = \sqrt[3]{y-4}$

Hence, we have

$f^{-1}(3) = \sqrt[3]{3-4} = \sqrt[3]{-1} = -1$

Thus, $f^{-1}(3) = -1$.

12. Question

If f : Q → Q, g : Q → Q are two functions defined by f(x) = 2x and g(x) = x + 2, show that f and g are bijective maps. Verify that $(gof)^{-1} = f^{-1}og^{-1}$.

Answer

We have f : Q → Q and f(x) = 2x.

Recall that a function is a bijection only if it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in Q$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ϵ Q (co-domain) such that f(x) = y

⇒ 2x = y

$\therefore x = \frac{y}{2}$

Clearly, for every y ϵ Q, there exists x ϵ Q (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f is a bijection and has an inverse.

We have f(x) = y ⇒ x = $f^{-1}(y)$

But, we found f(x) = y ⇒ $x = \frac{y}{2}$

Hence, $f^{-1}(y) = \frac{y}{2}$

Thus, $f^{-1}(x) = \frac{x}{2}$

Now, we have g : Q → Q and g(x) = x + 2.

First, we will prove that g is one-one.

Let x_1, x_2ϵ Q (domain) such that $g(x_1) = g(x_2)$

⇒ $x_1 + 2 = x_2 + 2$

∴ $x_1 = x_2$

So, we have $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$.

Thus, function g is one-one.

Now, we will prove that g is onto.

Let y ϵ Q (co-domain) such that g(x) = y

⇒ x + 2 = y

∴ x = y – 2

Clearly, for every y ϵ Q, there exists x ϵ Q (domain) such that g(x) = y and hence, function g is onto.

Thus, the function g is a bijection and has an inverse.

We have g(x) = y ⇒ x = $g^{-1}(y)$

But, we found g(x) = y ⇒ x = y – 2

Hence, $g^{-1}(y) = y - 2$

Thus, $g^{-1}(x) = x - 2$

We have $(f^{-1}og^{-1})(x) = f^{-1}(g^{-1}(x))$

We found $f^{-1}(x) = \frac{x}{2}$ and $g^{-1}(x) = x - 2$

⇒ $(f^{-1}og^{-1})(x) = f^{-1}(x - 2)$

$\therefore (f^{-1}og^{-1})(x) = \frac{x-2}{2}$

We know (gof)(x) = g(f(x)) and gof : Q → Q

⇒ (gof)(x) = g(2x)

∴ (gof)(x) = 2x + 2

Clearly, gof is a bijection and has an inverse.

Let y ∈ Q (co-domain) such that (gof)(x) = y

⇒ 2x + 2 = y

⇒ 2x = y – 2

$$\therefore x = \frac{y-2}{2}$$

We have (gof)(x) = y ⇒ x = $(gof)^{-1}(y)$

But, we found (gof)(x) = y ⇒ $x = \frac{y-2}{2}$

Hence, $(gof)^{-1}(y) = \frac{y-2}{2}$

Thus, $(gof)^{-1}(x) = \frac{x-2}{2}$

So, it is verified that $(gof)^{-1} = f^{-1}og^{-1}$.

13. Question

Let A = R – {3} and B = R – {1}. Consider the function f : A→ B defined by $f(x) = \frac{x-2}{x-3}$. Show that f is one-one and onto and hence find f^{-1}.

Answer

We have f : A → B where A = R – {3} and B = R – {1}

$$f(x) = \frac{x-2}{x-3}$$

First, we will prove that f is one-one.

Let $x_1, x_2 \in A$ (domain) such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow -3x_1 + 2x_1 = 2x_2 - 3x_2$

$\Rightarrow -x_1 = -x_2$

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ∈ B (co-domain) such that f(x) = y

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow \frac{(x-3)+1}{x-3} = y$$

$$\Rightarrow 1 + \frac{1}{x-3} = y$$

$\Rightarrow \frac{1}{x-3} = y - 1$

$\Rightarrow \frac{1}{y-1} = x - 3$

$\Rightarrow x = 3 + \frac{1}{y-1}$

$\therefore x = \frac{3y-2}{y-1}$

Clearly, for every y ϵ B, there exists x ϵ A (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have f(x) = y ⇒ x = $f^{-1}(y)$

But, we found f(x) = y ⇒ $x = \frac{3y-2}{y-1}$

Hence, $f^{-1}(y) = \frac{3y-2}{y-1}$

Thus, f(x) is invertible and $f^{-1}(x) = \frac{3x-2}{x-1}$

14. Question

Consider the function f : $R^+ \rightarrow [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$. Prove that f is invertible with $f^{-1}(y) = \frac{\sqrt{54+5y} - 3}{4}$.

Answer

We have f : $R^+ \rightarrow [-9, \infty)$ and $f(x) = 5x^2 + 6x - 9$.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in R^+$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow 5x_1^2 + 6x_1 - 9 = 5x_2^2 + 6x_2 - 9$

$\Rightarrow 5x_1^2 + 6x_1 = 5x_2^2 + 6x_2$

$\Rightarrow 5x_1^2 - 5x_2^2 + 6x_1 - 6x_2 = 0$

$\Rightarrow 5(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow 5(x_1 - x_2)(x_1 + x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[5(x_1 + x_2) + 6] = 0$

$\Rightarrow x_1 - x_2 = 0$ (as $x_1, x_2 \in R^+$)

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ϵ $[-9, \infty)$ (co-domain) such that f(x) = y

$\Rightarrow 5x^2 + 6x - 9 = y$

$\Rightarrow 5\left(x^2 + \frac{6}{5}x - \frac{9}{5}\right) = y$

$\Rightarrow x^2 + \frac{6}{5}x - \frac{9}{5} = \frac{y}{5}$

$\Rightarrow x^2 + \frac{6}{5}x = \frac{y+9}{5}$

Adding $\frac{9}{25}$ to both sides, we get

$\Rightarrow x^2 + \frac{6}{5}x + \frac{9}{25} = \frac{y+9}{5} + \frac{9}{25}$

$\Rightarrow \left(x + \frac{3}{5}\right)^2 = \frac{(5y+45)+9}{25}$

$\Rightarrow \left(x + \frac{3}{5}\right)^2 = \frac{5y+54}{25}$

$\Rightarrow x + \frac{3}{5} = \sqrt{\frac{5y+54}{25}}$

$\Rightarrow x + \frac{3}{5} = \frac{\sqrt{5y+54}}{5}$

$\Rightarrow x = \frac{\sqrt{5y+54}}{5} - \frac{3}{5}$

$\therefore x = \frac{\sqrt{5y+54} - 3}{5}$

Clearly, for every y ϵ [–9, ∞), there exists x ϵ R^+ (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have f(x) = y ⇒ x = $f^{-1}(y)$

But, we found f(x) = y ⇒ $x = \frac{\sqrt{5y+54}-3}{5}$

Hence, $f^{-1}(y) = \frac{\sqrt{5y+54}-3}{5}$

15. Question

Let f : N → N be a function defined as f(x) = $9x^2$ + 6x – 5. Show that f : N → S, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and $f^{-1}(163)$.

Answer

We have f : N → N and f(x) = $9x^2$ + 6x – 5.

We need to prove f : N → S is invertible.

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let x_1, x_2ϵ N (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow 9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9x_1^2 + 6x_1 = 9x_2^2 + 6x_2$

$\Rightarrow 9x_1^2 - 9x_2^2 + 6x_1 - 6x_2 = 0$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow 9(x_1 - x_2)(x_1 + x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[9(x_1 + x_2) + 6] = 0$

$\Rightarrow x_1 - x_2 = 0$ (as $x_1, x_2 \in R^+$)

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ϵ S (co-domain) such that f(x) = y

$\Rightarrow 9x^2 + 6x - 5 = y$

Adding 6 to both sides, we get

$9x^2 + 6x - 5 + 6 = y + 6$

$\Rightarrow 9x^2 + 6x + 1 = y + 6$

$\Rightarrow (3x + 1)^2 = y + 6$

$\Rightarrow 3x + 1 = \sqrt{y+6}$

$\Rightarrow 3x = \sqrt{y+6} - 1$

$\therefore x = \dfrac{\sqrt{y+6} - 1}{3}$

Clearly, for every y ϵ S, there exists x ϵ N (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \frac{\sqrt{y+6}-1}{3}$

Hence, $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$

Thus, f(x) is invertible and $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$

Hence, we have

$$f^{-1}(43) = \frac{\sqrt{43+6} - 1}{3} = \frac{\sqrt{49} - 1}{3} = \frac{7-1}{3} = 2$$

$$f^{-1}(163) = \frac{\sqrt{163+6} - 1}{3} = \frac{\sqrt{169} - 1}{3} = \frac{13-1}{3} = 4$$

Thus, $f^{-1}(43) = 2$ and $f^{-1}(163) = 4$.

16. Question

Let $f : R - \left\{-\frac{4}{3}\right\} \rightarrow R$ be a function defined as $f(x) = \dfrac{4x}{3x+4}$. Show that $f : R - \left\{-\frac{4}{3}\right\} \rightarrow$ range(f) is one-one and onto. Hence, find f^{-1}.

Answer

We have f : R – $\left\{-\frac{4}{3}\right\}$ → R and $f(x) = \frac{4x}{3x+4}$

We need to prove f : R – $\left\{-\frac{4}{3}\right\}$ → range(f) is invertible.

First, we will prove that f is one-one.

Let x_1, x_2ϵ A (domain) such that $f(x_1) = f(x_2)$

$$\Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$$

$\Rightarrow (4x_1)(3x_2 + 4) = (3x_1 + 4)(4x_2)$

$\Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow 16x_1 = 16x_2$

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let y ϵ range(f) (co-domain) such that f(x) = y

$$\Rightarrow \frac{4x}{3x+4} = y$$

$\Rightarrow 4x = 3xy + 4y$

$\Rightarrow 4x - 3xy = 4y$

$\Rightarrow x(4 - 3y) = 4y$

$$\therefore x = \frac{4y}{4-3y}$$

Clearly, for every y ϵ range(f), there exists x ϵ A (domain) such that f(x) = y and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \frac{4y}{4-3y}$

Hence, $f^{-1}(y) = \frac{4y}{4-3y}$

Thus, f(x) is invertible and $f^{-1}(x) = \frac{4x}{4-3x}$

17. Question

If f: R → (–1, 1) defined by $f(x) = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is invertible, find f^{-1}.

Answer

We have f: R → (–1, 1) and $f(x) = \frac{10^x - 10^{-x}}{10^x + 10^{-x}}$

Given that f^{-1} exists.

Let y ϵ (–1, 1) such that f(x) = y

$$\Rightarrow \frac{10^x - 10^{-x}}{10^x + 10^{-x}} = y$$

$$\Rightarrow \frac{10^x - \frac{1}{10^x}}{10^x + \frac{1}{10^x}} = y$$

$$\Rightarrow \frac{10^{2x} - 1}{10^{2x} + 1} = y$$

$\Rightarrow 10^{2x} - 1 = y\,(10^{2x} + 1)$

$\Rightarrow 10^{2x} - 1 = 10^{2x}y + y$

$\Rightarrow 10^{2x} - 10^{2x}y = 1 + y$

$\Rightarrow 10^{2x}\,(1 - y) = 1 + y$

$$\Rightarrow 10^{2x} = \frac{1 + y}{1 - y}$$

Taking $\log_{10}$ on both sides, we get

$$\log_{10} 10^{2x} = \log_{10}\left(\frac{1 + y}{1 - y}\right)$$

$$\Rightarrow 2x\log_{10} 10 = \log_{10}\left(\frac{1 + y}{1 - y}\right)$$

$$\Rightarrow 2x = \log_{10}\left(\frac{1 + y}{1 - y}\right)$$

$$\therefore x = \frac{1}{2}\log_{10}\left(\frac{1 + y}{1 - y}\right)$$

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

Hence, $f^{-1}(y) = \frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

Thus, $f^{-1}(x) = \frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$

18. Question

If f: R → (0, 2) defined by $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} + 1$ is invertible, find f^{-1}.

Answer

We have f: R → (0, 2) and $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1$

Given that f^{-1} exists.

Let y ϵ (0, 2) such that f(x) = y

$$\Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} + 1 = y$$

$$\Rightarrow \frac{e^x - e^{-x} + (e^x + e^{-x})}{e^x + e^{-x}} = y$$

$$\Rightarrow \frac{2e^x}{e^x + e^{-x}} = y$$

$\Rightarrow \frac{2e^x}{e^x + \frac{1}{e^x}} = y$

$\Rightarrow \frac{2e^{2x}}{e^{2x} + 1} = y$

$\Rightarrow 2e^{2x} = y(e^{2x} + 1)$

$\Rightarrow 2e^{2x} = e^{2x}y + y$

$\Rightarrow 2e^{2x} - e^{2x}y = y$

$\Rightarrow e^{2x}(2 - y) = y$

$\Rightarrow e^{2x} = \frac{y}{2 - y}$

Taking ln on both sides, we get

$\ln e^{2x} = \ln\left(\frac{y}{2 - y}\right)$

$\Rightarrow 2x \ln e = \ln\left(\frac{y}{2 - y}\right)$

$\Rightarrow 2x = \ln\left(\frac{y}{2 - y}\right)$

$\therefore x = \frac{1}{2}\ln\left(\frac{y}{2 - y}\right)$

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \frac{1}{2}\ln\left(\frac{y}{2-y}\right)$

Hence, $f^{-1}(y) = \frac{1}{2}\ln\left(\frac{y}{2-y}\right)$

Thus, $f^{-1}(x) = \frac{1}{2}\ln\left(\frac{x}{2-x}\right)$

19. Question

Let $f : [-1, \infty) \to [-1, \infty)$ is given by $f(x) = (x + 1)^2 - 1$. Show that f is invertible. Also, find the set $S = \{x: f(x) = f^{-1}(x)\}$

Answer

We have $f : [-1, \infty) \to [-1, \infty)$ and $f(x) = (x + 1)^2 - 1$

Recall that a function is invertible only when it is both one-one and onto.

First, we will prove that f is one-one.

Let $x_1, x_2 \in [-1, \infty)$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow (x_1 + 1)^2 - 1 = (x_2 + 1)^2 - 1$

$\Rightarrow (x_1 + 1)^2 = (x_2 + 1)^2$

$\Rightarrow x_1^2 + 2x_1 + 1 = x_2^2 + 2x_2 + 1$

$\Rightarrow x_1^2 + 2x_1 = x_2^2 + 2x_2$

$\Rightarrow x_1^2 - x_2^2 + 2x_1 - 2x_2 = 0$

$\Rightarrow (x_1^2 - x_2^2) + 2(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) + 2(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)[x_1 + x_2 + 2] = 0$

$\Rightarrow x_1 - x_2 = 0$ (as $x_1, x_2 \in R^+$)

$\therefore x_1 = x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Thus, function f is one-one.

Now, we will prove that f is onto.

Let $y \in [-1, \infty)$ (co-domain) such that $f(x) = y$

$\Rightarrow (x + 1)^2 - 1 = y$

$\Rightarrow (x + 1)^2 = y + 1$

$\Rightarrow x + 1 = \sqrt{y + 1}$

$\therefore x = \sqrt{y + 1} - 1$

Clearly, for every $y \in [-1, \infty)$, there exists $x \in [-1, \infty)$ (domain) such that $f(x) = y$ and hence, function f is onto.

Thus, the function f has an inverse.

We have $f(x) = y \Rightarrow x = f^{-1}(y)$

But, we found $f(x) = y \Rightarrow x = \sqrt{y + 1} - 1$

Hence, $f^{-1}(y) = \sqrt{y + 1} - 1$

Thus, f(x) is invertible and $f^{-1}(x) = \sqrt{x + 1} - 1$

Now, we need to find the values of x for which $f(x) = f^{-1}(x)$.

We have $f(x) = f^{-1}(x)$

$\Rightarrow (x + 1)^2 - 1 = \sqrt{x + 1} - 1$

$\Rightarrow (x + 1)^2 = \sqrt{x + 1}$

We can write $(x + 1)^2 = \left(\sqrt{x + 1}\right)^4$

$\Rightarrow \left(\sqrt{x + 1}\right)^4 = \sqrt{x + 1}$

On substituting $t = \sqrt{x + 1}$, we get

$t^4 = t$

$\Rightarrow t^4 - t = 0$

$\Rightarrow t\,(t^3 - 1) = 0$

$\Rightarrow t\,(t - 1)(t^2 + t + 1) = 0$

$t^2 + t + 1 \neq 0$ because this equation has no real root t.

$\Rightarrow t = 0$ or $t - 1 = 0$

$\Rightarrow t = 0$ or $t = 1$

<u>Case – I</u>: $t = 0$

$\Rightarrow \sqrt{x+1} = 0$

$\Rightarrow x + 1 = 0$

$\therefore x = -1$

Case – II: $t = 1$

$\Rightarrow \sqrt{x+1} = 1$

$\Rightarrow x + 1 = 1$

$\therefore x = 0$

Thus, $S = \{0, -1\}$

20. Question

Let $A = \{x \in R \mid -1 \le x \le 1\}$ and let $f : A \to A$, $g : A \to A$ be two functions defined by $f(x) = x^2$ and $g(x) = \sin \pi x/2$. Show that g^{-1} exists but f^{-1} does not exist. Also, find g^{-1}.

Answer

We have $f : A \to A$ where $A = \{x \in R \mid -1 \le x \le 1\}$ defined by $f(x) = x^2$.

Recall that a function is invertible only when it is both one-one and onto.

First, we will check if f is one-one.

Let $x_1, x_2 \in A$ (domain) such that $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$

$\Rightarrow x_1 - x_2 = 0$ or $x_1 + x_2 = 0$

$\therefore x_1 = \pm x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = \pm x_2$.

This means that two different elements of the domain are mapped to the same element by the function f.

For example, consider $f(-1)$ and $f(1)$.

We have $f(-1) = (-1)^2 = 1$ and $f(1) = 1^2 = 1 = f(-1)$

Thus, f is not one-one and hence f^{-1} doesn't exist.

Now, let us consider $g : A \to A$ defined by $g(x) = \sin \frac{\pi x}{2}$

First, we will prove that g is one-one.

Let $x_1, x_2 \in A$ (domain) such that $g(x_1) = g(x_2)$

$\Rightarrow \sin\frac{\pi x_1}{2} = \sin\frac{\pi x_2}{2}$

$\Rightarrow \frac{\pi x_1}{2} = \frac{\pi x_2}{2}$ (in the given range)

$\therefore x_1 = x_2$

So, we have $g(x_1) = g(x_2) \Rightarrow x_1 = x_2$.

Thus, function g is one-one.

Let $y \in A$ (co-domain) such that $g(x) = y$

$\Rightarrow \sin\frac{\pi x}{2} = y$

$\Rightarrow \frac{\pi x}{2} = \sin^{-1} y$

$\Rightarrow \pi x = 2\sin^{-1} y$

$\therefore x = \frac{2}{\pi}\sin^{-1} y$

Clearly, for every y ϵ A, there exists x ϵ A (domain) such that g(x) = y and hence, function g is onto.

Thus, the function g has an inverse.

We have g(x) = y ⇒ x = $g^{-1}(y)$

But, we found g(x) = y ⇒ $x = \frac{2}{\pi}\sin^{-1} y$

Hence, $g^{-1}(y) = \frac{2}{\pi}\sin^{-1} y$

Thus, g(x) is invertible and $g^{-1}(x) = \frac{2}{\pi}\sin^{-1} x$

21. Question

Let f be a function from R to R such that f(x) = cos (x + 2). Is f invertible? Justify your answer.

Answer

We have f : R → R and f(x) = cos (x + 2).

Recall that a function is invertible only when it is both one-one and onto.

First, we will check if f is one-one.

Let x_1, x_2ϵ R (domain) such that $f(x_1) = f(x_2)$

⇒ $\cos(x_1 + 2) = \cos(x_2 + 2)$

As the cosine function repeats itself with a period 2π, we have

$x_1 + 2 = x_2 + 2$ or $x_1 + 2 = 2\pi + (x_2 + 2)$

$\therefore x_1 = x_2$ or $x_1 = 2\pi + x_2$

So, we have $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ or $2\pi + x_2$

This means that two different elements of the domain are mapped to the same element by the function f.

For example, consider f(0) and f(2π).

We have f(0) = cos (0 + 2) = cos 2 and

f(2π) = cos (2π + 2) = cos 2 = f(0)

Thus, f is not one-one.

Hence, f is not invertible and f^{-1} does not exist.

22. Question

If A = {1, 2, 3, 4} and B = {a, b, c, d}, define any four bijections from A to B. Also, give their inverse function.

Answer

Given A = {1, 2, 3, 4} and B = {a, b, c, d}.

We need to define bijections f_1, f_2, f_3 and f_4 from A to B.

Consider $f_1 = \{(1, a), (2, b), (3, c), (4, d)\}$

(1) f_1 is one-one because no two elements of the domain are mapped to the same element.

f_1 is also onto because each element in the co-domain has a pre-image in the domain.

Thus, f_1 is a bijection from A to B.

We have $f_1^{-1} = \{(a, 1), (b, 2), (c, 3), (d, 4)\}$

Using similar explanation, we also have the following bijections defined from A to B -

(2) $f_2 = \{(1, b), (2, c), (3, d), (4, a)\}$

We have $f_2^{-1} = \{(b, 1), (c, 2), (d, 3), (a, 4)\}$

(3) $f_3 = \{(1, c), (2, d), (3, a), (4, b)\}$

We have $f_3^{-1} = \{(c, 1), (d, 2), (a, 3), (b, 4)\}$

(4) $f_4 = \{(1, d), (2, a), (3, b), (4, c)\}$

We have $f_4^{-1} = \{(d, 1), (a, 2), (b, 3), (c, 4)\}$

23. Question

Let A and B be two sets each with finite number of elements. Assume that there is an injective map from A to B and that there is an injective map from B to A. Prove that there is a bijection from A to B.

Answer

Given A and B are two finite sets. There are injective maps from both A to B and B to A.

Let f be the injective map defined from A to B.

Thus, we have f is one-one.

We also know that there is a one-one mapping from B to A.

This means that each element of B is mapped to a distinct element of A.

But, B is the co-domain of f and A is the domain of f.

So, every element of the co-domain of the function f has a pre-image in the domain of the function f.

Thus, f is also onto.

Therefore, f is a bijection as it is both one-one and onto.

Hence, there exists a bijection defined from A to B.

24. Question

If $f : A \rightarrow A$ and $g : A \rightarrow A$ are two bijections, then prove that

(i) fog is an injection

(ii) fog is a surjection

Answer

Given $f : A \rightarrow A$ and $g : A \rightarrow A$ are two bijections. So, both f and g are one-one and onto functions.

We know $(fog)(x) = f(g(x))$

Thus, fog is also defined from A to A.

(i) First, we will prove that fog is an interjection.

Let $x_1, x_2 \epsilon$ A (domain) such that $(fog)(x_1) = (fog)(x_2)$

$\Rightarrow f(g(x_1)) = f(g(x_2))$

$\Rightarrow g(x_1) = g(x_2)$ [since f is one-one]

$\therefore x_1 = x_2$ [since g is one-one]

So, we have $(fog)(x_1) = (fog)(x_2) \Rightarrow x_1 = x_2$.

Thus, function fog is an interjection.

(ii) Now, we will prove that fog is a surjection.

Let $z \in A$, the co-domain of fog.

As f is onto, we have $y \in A$ (domain of f) such that $f(y) = z$.

However, as g is also onto and y belongs to the co-domain of g, we have $x \in A$ (domain of g) such that $g(x) = y$.

Hence, $(fog)(x) = f(g(x)) = f(y) = z$.

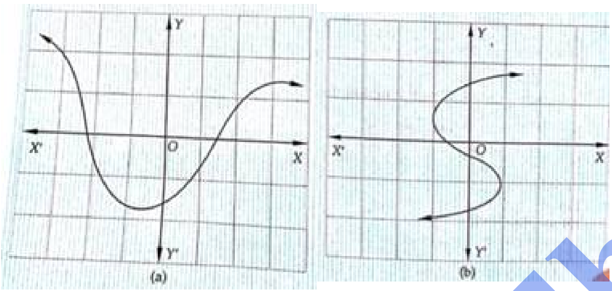
Here, x belongs to the domain of fog (A) and z belongs to the co-domain of fog (A).

Thus, function fog is a surjection.

Very short answer

1. Question

Which one of the following graphs represent a function?



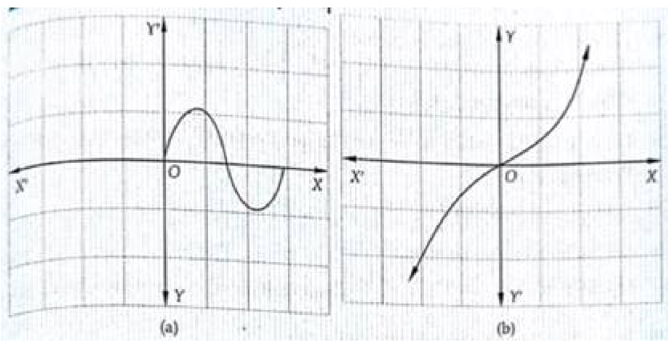
Answer

(a) It have unique image therefore a function

(b) It have more than one image

2. Question

Which one of the following graphs represent a one-one function?



Answer

Formula:-

(i) A function $f: A \rightarrow B$ is one-one function or an injection if

f(x)=f(y)

⇒ x=y for all x, y ∈ A

or f(x)$\neq$f(y)

⇒ x$\neq$y for all x, y ∈ A

(a) It is not one-one function as it has same image on x axis

(b) It is one-one function s it have unique image

3. Question

If A = {1, 2, 3} and B = {a, b}, write total number of functions from A to B.

Answer

Formula:-if A and B are two non-empty finite sets containing m and n

(i) Number of function from A to B = n^m

(ii) Number of one–one function from A to B= $\begin{cases} C^n_m . m!, \text{if } n \geq m \\ 0, \text{if } n < m \end{cases}$

(iii) Number of one-one and onto function from A to B= $\begin{cases} n!, \text{if } m = n \\ 0, \text{if } m \neq n \end{cases}$

(iv) Number of onto function from A to B=$\sum_{r=1}^{n} (-1)^{n-r}\, C^n_r\, r^m$,if $m \geq n$

given: -

A = {1, 2, 3} and B = {a, b}

n(A)=3, and n(B)=2

total number of functions=2^3=8

4. Question

If A = {a, b, c} and B = {–2, –1, 0, 1, 2}, write total number of one-one functions from A to B.

Answer

Formula:-

(I)A function $f: A \to B$ is one-one function or an injection if

f(x)=f(y)

⇒ x=y for all x, y ∈ A

or f(x)$\neq$f(y)

⇒ x$\neq$y for all x, y ∈ A

(II)if A and B are two non-empty finite sets containing m and n

(i) Number of function from A to B = n^m

(ii) Number of one–one function from A to B= $\begin{cases} C^n_m . m!, \text{if } n \geq m \\ 0, \text{if } n < m \end{cases}$

(iii) Number of one-one and onto function from A to B= $\begin{cases} n!, \text{if } m = n \\ 0, \text{if } m \neq n \end{cases}$

(iv) Number of onto function from A to B=$\sum_{r=1}^{n} (-1)^{n-r}\, C^n_r\, r^m$,if $m \geq n$

Let f: A→B be one-one function

F(a)=3 and f(B)=5

Using formula

Number of one–one function from A to B$= \begin{cases} C^n_m . m!, \text{if } n \geq m \\ 0, \text{if } n < m \end{cases}$

$\Rightarrow {}^3C_5.5!=60$

5. Question

Write total number of one-one functions from set A = {1, 2, 3, 4} to set B = {a, b c}.

Answer

Formula:-

(I) A function $f: A \rightarrow B$ is one-one function or an injection if

f(x)=f(y)

⇒ x=y for all x, y ∈ A

or f(x)$\neq$f(y)

⇒ x$\neq$y for all x, y ∈ A

(II)if A and B are two non-empty finite sets containing m and n

(i) Number of function from A to B = n^m

(ii) Number of one–one function from A to B$= \begin{cases} C^n_m . m!, \text{if } n \geq m \\ 0, \text{if } n < m \end{cases}$

(iii) Number of one-one and onto function from A to B$= \begin{cases} n!, \text{if } m = n \\ 0, \text{if } m \neq n \end{cases}$

(iv) Number of onto function from A to B=$\sum_{r=1}^{n} (-1)^{n-r}\, C^n_r\, r^m$, if $m \geq n$

F(A)=4 and f(B)=3

Using formula

Number of one–one function from A to B$= \begin{cases} C^n_m . m!, \text{if } n \geq m \\ 0, \text{if } n < m \end{cases}$

Number of one-one function from A to B=0

6. Question

If f: R→ R is defined by f(x) = x^2, write $f^{-1}(25)$.

Answer

Formula:-

(i)A function f: X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof =I_x and fog = I_y. The function g is called the inverse of f and is denoted by f^{-1}

f(x)=y

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow x^2=25$

$\Rightarrow$ x=-5,5

$\Rightarrow f^{-1}(25) =\{-5,5\}$

7. Question

If f : C → C is defined by f(x) = x^2, write $f^{-1}(-4)$. Here, C denotes the set of all complex numbers.

Answer

Formula:-

(i)A function f: X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$ .The function g is called the inverse of f and is denoted by f^{-1}

f(x)=y

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow f(x)=-4$

$\Rightarrow x^2=-4$

$\Rightarrow x=2i,-2i$

8. Question

If f: R → R is given by $f(x) = x^3$, write $f^{-1}(1)$.

Answer

Formula:-

(i)A function f: X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$. The function g is called the inverse of f and is denoted by f^{-1}

f(x)=y

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow f^{-1}(1) =x$

$\Rightarrow f(x)=1$

$\Rightarrow x^3=1$

$\Rightarrow x^3-1=0$

$\Rightarrow (x-1)(x^2+x+1)=0$

$\Rightarrow X=1$

9. Question

Let C denote the set of all complex numbers. A function f : C → C is defined by $f(x) = x^3$.

Write $f^{-1}(1)$.

Answer

Formula:-

(i)A function f: X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$. The function g is called the inverse of f and is denoted by f^{-1}

f(x)=y

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow f^{-1}(1) =x$

$\Rightarrow f(x)=1$

$\Rightarrow x^3=1$

$\Rightarrow x^3-1=0$

$\Rightarrow (x-1)(x^2+x+1)=0$

$\Rightarrow x=1, w, w^2$

10. Question

Let f be a function from C (set of all complex numbers) to itself given by $f(x) = x^3$. Write $f^{-1}(-1)$.

Answer

Formula:-

(i)A function $f: X \rightarrow Y$ is defined to be invertible, if there exists a function $g : Y \rightarrow X$

such that gof $=I_X$ and fog $= I_Y$ .The function g is called the inverse of f and is denoted by f^{-1}

$f(x)=y$

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow f(x)=-1$

$\Rightarrow f^{-1}(-1)=x$

$\Rightarrow x^3=-1$

$\Rightarrow x^3+1=0$

$\Rightarrow (x+1)(x^2-x+1)=0$

$\Rightarrow x=-1,-w,-w^2$

11. Question

Let $f : R \rightarrow R$ be defined by $f(x) = x^4$, write $f^{-1}(1)$.

Answer

Formula:-

(i)A function $f : X \rightarrow Y$ is defined to be invertible, if there exists a function $g : Y \rightarrow X$

such that gof $=I_X$ and fog $= I_Y$ .The function g is called the inverse of f and is denoted by f^{-1}

$f(x)=y$

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow f(x)=1$

$\Rightarrow f^{-1}(1)=x$

$\Rightarrow x^4=1$

$\Rightarrow x^4-1=0$

$\Rightarrow (x-1)(x^2+1)=0$

$\Rightarrow x=-1,1$

$\Rightarrow f^{-1}(1)=\{-1,1\}$

12. Question

If f : C → C is defined by $f(x) = x^4$, $f^{-1}(1)$.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$ .The function g is called the inverse of f and is denoted by f^{-1}

$f(x)=y$

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow f(x)=1$

$\Rightarrow f^{-1}(1)=x$

$\Rightarrow x^4=1$

$\Rightarrow x^4-1=0$

$\Rightarrow$ $(x-1)(x^2+1)=0$

$\Rightarrow x=-1,1,i,-i$

$\Rightarrow f^{-1}(1)=\{-1,-i,1,i\}$

13. Question

If f :R→R is defined by $f(x) = x^2$, $f^{-1}(–25)$.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$ .The function g is called the inverse of f and is denoted by f^{-1}

$f(x)=y$

$\Rightarrow f^{-1}(y)=x$

$\Rightarrow x^2=-25$

but x should be Real number

$f^{-1}(-25) = \emptyset$

14. Question

If f : C → C is defined by $f(x) = (x – 2)^3$, write $f^{-1}(–1)$.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$ .The function g is called the inverse of f and is denoted by f^{-1}

$f(x)=y$

$f^{-1}(y)=x$

$\Rightarrow$ $(x – 2)^3=-1$

$\Rightarrow x-2=-1, x-2=w$ and $x-2=-w^2$

$\Rightarrow$ x=1,-w+2,2-w^2

$\Rightarrow$ $f^{-1}(25)=\{1,2-w,2-w^2\}$

15. Question

If f :R→R is defined by f(x) = 10x –7, then write $f^{-1}(x)$.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof =I_x and fog = I_y .The function g is called the inverse of f and is denoted by f^{-1}

$f^{-1}(x)=y$

$\Rightarrow$ f(y)=x

$\Rightarrow$ 10y-7=x

$$\Rightarrow\ y=\frac{x+7}{10}$$

$$\Rightarrow\ f^{-1}(x)=\frac{x+7}{10}$$

16. Question

Let $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\rightarrow R$ be a function defined by f(x) = cos[x]. Write range (f).

Answer

Given:-

(i) $f\colon(-\frac{\pi}{2},\frac{\pi}{2})$

(ii)f(x)=cos[x]

Domain==$-\frac{\pi}{2},\frac{\pi}{2}$

For f(x)= cos [x]

Range ={1,cos1,cos2}

17. Question

If f : R → R defined by f(x) = 3x – 4 is invertible then write $f^{-1}(x)$.

Answer

Given:- (i) f : R → R

(ii) f(x)=3x-4

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof =I_x and fog = I_y .The function g is called the inverse of f and is denoted by f^{-1}

For $f^{-1}(x)=y$

$\Rightarrow$ f(y)=x

$\Rightarrow$ 3y – 4 =x

$\Rightarrow\ y = \frac{x+4}{3}$

$\Rightarrow\ f^{-1}(x) = \frac{x+4}{3}$

18. Question

If f : R → R, g : R → R are given by f(x) = $(x + 1)^2$ and g(x) = $x^2 + 1$, then write the value of fog (–3).

Answer

Formula:-

(I)Let f : A→B and g : B→C be two functions.

Then, the composition of f andg, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i) f : R → R

(ii) g : R → R

(iii) f(x) = $(x + 1)^2$

(iv) g(x) = $x^2 + 1$

fog(-3)=f(g(-3))

$\Rightarrow$ fog(-3)=f($(-3)^2$+1)

$\Rightarrow$ fog(-3)=f(10)

$\Rightarrow$ fog(-3)=$(10+1)^2$

$\Rightarrow$ fog(-3)=121

19. Question

Let A = {x ϵR : –4 ≤ x ≤ x ≤ 4 and x ≠ 0} and f : A→ R be defined by $f(x) = \frac{|x|}{x}$. Write the range of f.

Answer

Given:-

(i)$A = \{x \in R : -4 \le x \le x \le 4 \text{ and } x \ne 0\}$

(ii) f : A → R

(iii)$f(x) = \frac{|x|}{x}$

For $f(x) = \frac{|x|}{x}$

Range ={-1,1}

20. Question

Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to A$ be defined by f(x) = sinx. If f is a bijection, write set A.

Answer

Formula:-

(i)A function $f: A \rightarrow B$ is a bijection if it is one-one as well as onto

(ii)A function $f: A \rightarrow B$ is onto function or surjection if

Range (f)=co-domain(f)

Given:-

(i)f: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(ii)f(x)=sinx

(ii) f is bijection

For f(x)=sinx

Codomain =range

Set A=[-1,1]

21. Question

Let f : R → R^+ be defined by f(x) = a^x, a > 0 and a ≠ 1. Write $f^{-1}(x)$.

Answer

Given:-

(i)f : R → R^+

(ii)f(x) = a^x, a > 0 and a ≠ 1

Let

f(y)=x

a^y=x

$\Rightarrow\ y = \log_a x$

$\Rightarrow\ f^{-1}(x) = \log_a x$

22. Question

Let f : R – {-1} → R – {1} be given by $f(x) = \frac{x}{x+1}$. Write $f^{-1}(x)$.

Answer

Given:-

(i)f : R – {-1} → R – {1}

(ii)$f(x) = \frac{x}{x+1}$

F(y)=x

$\Rightarrow\ \frac{y}{y+1} = x$

$\Rightarrow$ Y=xy+x

$\Rightarrow\ y = \frac{x}{1-x}$

$\Rightarrow\ f^{-1} = \frac{x}{1-x}$

23. Question

Let $f : R - \left\{-\frac{3}{5}\right\} \to R$ be a function defined as $f(x) = \frac{2x}{5x+3}$.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog = I_y .The function g is called the inverse of f and is denoted by f^{-1}

Given:-

(i)$f: R - \left\{-\frac{3}{5}\right\} \to R$

(ii)$f(x) = \frac{2x}{5x+3}$

F(y)=x

$\Rightarrow \frac{2y}{5x+3} = x$

$\Rightarrow$ 2y-3x-5xy=0

$\Rightarrow y = \frac{3x}{2-5x}$

$\Rightarrow f^{-1}(x) = \frac{3x}{2-5x}$

24. Question

Let f : R → R, g : R → R be two functions defined by $f(x) = x^2 + x + 1$ and $g(x) = 1 – x^2$. Write fog(–2).

Answer

Formula :- (I)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i)f : R → R

(ii)g : R → R

(iii) $f(x) = x^2 + x + 1$

(iv)$g(x) = 1 – x^2$

Fog(-2)=f(g(-2))

$\Rightarrow$ Fog(-2)=f(1-(-2)^2)

$\Rightarrow$ Fog(-2)=f(-3)

$\Rightarrow$ Fog(-2)=(-3)^2-3+1=7

25. Question

Let f : R →R be defined as $f(x) = \frac{2x-3}{4}$. Write fof^{-1}(1).

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$ .The function g is called the inverse of f and is denoted by f^{-1}

(II)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i)f : R →R

$$(ii) f(x) = \frac{2x-3}{4}$$

F(y)=x

$$\Rightarrow \frac{2y-3}{4} = x$$

⇒ 2y-3-4x=0

$$\Rightarrow y = \frac{4x+3}{2}$$

Now

$$\Rightarrow fof^{-1}(1) = f(\frac{7}{2})$$

$$\Rightarrow fof^{-1}(1) = \frac{7-3}{4} = 1$$

26. Question

Let f be an invertible real function. Write (f^{-1} of) (1) + (f^{-1} of) (2) + … +(f^{-1} of) (100).

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog $= I_y$ .The function g is called the inverse of f and is denoted by f^{-1}

(II)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i)f be an invertible real function

(f^{-1} of) (1) + (f^{-1} of) (2) + … +(f^{-1} of) (100)

=1+2+3+………+100

$$= \frac{100(100+1)}{2} = 5050$$

27. Question

Let A = {1, 2, 3, 4} and B = {a, b} be two sets. Write total number of onto functions from A to B.

Answer

Formula:-

(I)A function $f: A \to B$ is onto function or surjection if

Range (f)=co-domain(f)

(II)if A and B are two non-empty finite sets containing m and n

(i) Number of function from A to B = n^m

(ii) Number of one–one function from A to B= $\begin{cases} C^n_m . m!, \text{if } n \geq m \\ 0, \text{if } n < m \end{cases}$

(iii) Number of one-one and onto function from A to B= $\begin{cases} n!, \text{if } m = n \\ 0, \text{if } m \neq n \end{cases}$

(iv) Number of onto function from A to B=$\sum_{r=1}^{n}(-1)^{n-r}\, C^n_r\, r^m$,$\text{if } m \geq n$

Given:-

(i)A = {1, 2, 3, 4}=4

(ii)B = {a, b}=2

Using formula (iv)

Number of onto function from A to B=$\sum_{r=1}^{n}(-1)^{n-r}\, C^n_r\, r^m$,$\text{if } m \geq n$

Where m=4,n=2

$$\sum_{r=1}^{n}(-1)^{n-r} C^n_r\, r^m = (-1)^2\, C^2_1(1)^4 + (-1)^0 C^2_2(2)^4$$

=-2+16=14

28. Question

Write the domain of the real function $f(x) = \sqrt{x - [x]}$.

Answer

$f(x) = \sqrt{x - [x]}$ where x is for all real number

Then,

domain=R

129. Question

Write the domain of the real function $f(x) = \sqrt{[x] - x}$.

Answer

$f(x) = \sqrt{[x] - x}$ where x is not for real number

Domain=$\emptyset$

30. Question

Write the domain of the real function $f(x) = \dfrac{1}{\sqrt{|X| - x}}$

Answer

$$f(x) = \frac{1}{\sqrt{|x| - x}}$$

When x<0 negative

$$\frac{1}{\sqrt{|x|-x}}=\frac{1}{\sqrt{-x-x}}$$

$$=\frac{1}{\sqrt{-2x}}$$

When x>0

$$\frac{1}{\sqrt{|x|-x}}=\frac{1}{\sqrt{x-x}}=\infty$$

Domain=$(-\infty,0)$

31. Question

Write whether f : R → R given by $f(x)=x+\sqrt{x^2}$ is one-one, many-one, onto or into.

Answer

(I)A function $f: A \rightarrow B$ is one-one function or an injection if

f(x)=f(y)$\Rightarrow$ x=y for all x,y$\in$A

or f(x)$\neq$f(y)$\Rightarrow$ x$\neq$y for all x,y$\in$A

(II) A function $f: A \rightarrow B$ is onto function or surjection if

Range (f)=co-domain(f)

(III) A function $f: A \rightarrow B$ is not onto function, then

$f: A \rightarrow A$ is always an onto function

Given:-

(i)f : R → R

(ii)$f(x)=x+\sqrt{x^2}$

$f(x)=x+\sqrt{x^2}$

=x ± x

=0, 2x

Now putting x=0

$F(0)=0+\sqrt{0^2}=0$

Again putting x=-1

$F(-1)=-1+\sqrt{-1^2}=0$

Hence f is many one

32. Question

If f(x) = x + 7 and g(x) = x – 7, x∈ R, write fog(7).

Answer

Formula:-

(i)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C given by g o f (x) = g (f (x))

Given:-

(i)f(x) = x + 7

(ii)g(x) = x – 7, x ∈ R

Fog(7)=f(g(7))

⇒ Fog(7)=f(7-7)

⇒ Fog(7)=f(0)

⇒ Fog(7)=0+7

⇒ Fog(7)=7

33. Question

What is the range of the function $f(x) = \frac{|x-1|}{x-1}$?

Answer

$$f(x) = \frac{|x-1|}{x-1}$$

=±1

Range of f={-1,1}

34. Question

If f : R → R be defined by f(x) = $(3 - x^3)^{1/3}$, then find fof(x).

Answer

Formula:-

(i)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i) f : R → R

$(ii) f(x) = (3 - x^3)^{\frac{1}{3}}$

Fof(x)=f(f(x))

$\Rightarrow \ fof(x) = f((3 - x^3)^{\frac{1}{3}})$

$\Rightarrow \ fof(x) = \left(3 - (3 - x^3)\right)^{\frac{1}{3}}$

$\Rightarrow \ fof(x) = (x^3)^{\frac{1}{3}} = x$

35. Question

If f : R → R is defined by f(x) = 3x + 2, find f(f (x)).

Answer

Given:-

(i)f : R → R

F(f(x))=f(3x+2)

$\Rightarrow$ F(f(x))=3(3x+2)+2

$\Rightarrow$ F(f(x))=9x+8

36. Question

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. State whether f is one-one or not.

Answer

Given:-

(i) A = {1, 2, 3}

(ii) B = {4, 5, 6, 7}

(iii) f = {(1, 4), (2, 5), (3, 6)}

each element has a unique image

hence ,f is one-one

37. Question

If f : {5, 6} → {2, 3} and g : {2, 3} → {5, 6} are given by f = {(5, 2), (6, 3)} and g = {(2, 5), (3, 6)}, find fog.

Answer

Formula:-

(i)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i) f : {5, 6} → {2, 3}

(ii) g : {2, 3} → {5, 6}

(iv) f = {(5, 2), (6, 3)}

(v) g = {(2, 5), (3, 6)}

for fog(2)=f(g(2))

$\Rightarrow$ fog(2)=f(5)

$\Rightarrow$ fog(2)=2

38. Question

Let f : R → R be the function defined by f(x) = 4x – 3 for all x ϵ R. Then write f^{-1}.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof =I_X and fog = I_Y .The function g is called the inverse of f and is denoted by f^{-1}

Given:-

(i) f : R → R

(ii) f(x) = 4x – 3 for all x ϵ R.

f(x)=y

$\Rightarrow$ 4x-3=y

$\Rightarrow\ x = \frac{y+3}{4}$

$f^{-1}(y) = x = \frac{y+3}{4}$

$f^{-1}(x) = \frac{x+3}{4}$

39. Question

Which one the following relations on A = {1, 2, 3} is a function?

f = {(1, 3), (2, 3), (3, 2)}, g = {(1, 2), (1, 3), (3, 1)}.

Answer

Given:-

(i) A = {1, 2, 3}

(ii)f = {(1, 3), (2, 3), (3, 2)}

(iii)g = {(1, 2), (1, 3), (3, 1)}.

In case of set A and f

Every element in A has a unique image in f

So, f is a function

In case of set A and g

Only one element has image in g

So, g is not a function

40. Question

Write the domain of the real function f defined by $f(x) = \sqrt{25 - x^2}$.

Answer

$f(x) = \sqrt{25 - x^2}$

$\Rightarrow 25-x^2 \geq 0$

$\Rightarrow -(x+5)(x-5)\geq 0$

$\Rightarrow (x+5)(x-5)\leq 0$

$\Rightarrow x\leq -5$ or 5

Domain = [-5,5]

41. Question

Let A = {a, b, c, d} and f : A → A be given by f = {(a, b), (b, d), (c, a), (d, c)}, write f^{-1}.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof $=I_x$ and fog = I_y .The function g is called the inverse of f and is denoted by f^{-1}

(ii)A function $f: A \to B$ is onto function or surjection if

Range (f)=co-domain(f)

Given:-

(i)A = {a, b, c, d}

(ii)f : A → A

(iii)f = {(a, b), (b, d), (c, a), (d, c)}

f is one-one since each element of A is assigned to distinct element of the set A. Also, f is onto since f (A) = A.

f^{-1}= {(b, a), (d, b), (a, c), (c, d)}.

42. Question

Let f, g : R → R be defined by f(x) = 2x + 1 and g(x) = x^2 – 2 for all x ϵ R, respectively. Then, find gof.

Answer

Formula:-

(i)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i)f, g : R → R

(ii)f(x) = 2x + 1

(ii)g(x) = x^2 – 2 for all x ϵ R

gof(x)=g(f(x))

⇒ gof(x)=g(2x+1)

⇒ gof(x)=$(2x+1)^2$-2

⇒ gof(x)=$4x^2$+4x-1

43. Question

If the mapping f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3}, given by

f = {(1, 2), (3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3), write fog.

Answer

Formula:-

(i)Let f : A→B and g : B→C be two functions.

Then, the composition of f and g, denoted by g o f, is defined as the function g o f : A→C

given by g o f (x) = g (f (x))

Given:-

(i)f : {1, 3, 4} → {1, 2, 5}

(ii)g : {1, 2, 5} → {1, 3}

(iii)f = {(1, 2), (3, 5), (4, 1)}

(iv)g = {(2, 3), (5, 1), (1, 3)

fog(1)=f(g(1))=f(3)=5

fog(2)=f(g(2))=f(3)=5

fog(5)=f(g(5))=f(1)=2

⇒ fog={(1,5)(2,5)(5,2)}

44. Question

If a function g = {(1, 1), (2, 3), (3, 5), (4, 7)} is described by g(x) = αx + β, find the values of α and β.

Answer

Given:-

(i)g={(1, 1), (2, 3), (3, 5), (4, 7)}

(ii)g(x) = αx + β

For x=1 and αx + β

g(1)=α(1)+ β=1

⇒ α+ β=1

For x=2

g(2)=α(2)- β=3

⇒ 2α- β=3

Similarly with g(3) and g(4)

Using above value

α=2

β=1

45. Question

If f(x) = 4 – (x – 7)^3, write $f^{-1}(x)$.

Answer

Formula:-

(i)A function f : X → Y is defined to be invertible, if there exists a function g : Y → X

such that gof =I_x and fog = I_y .The function g is called the inverse of f and is denoted by f^{-1}

Given:-

(i) f(x) = 4 – (x – 7)^3

Let f(x)=y

y= 4 – (x – 7)^{3*}

$x = 7 + \sqrt[3]{4 - y}$

$f^{-1}(x) = 7 + \sqrt[3]{4 - x}$

MCQ

1. Question

Mark the correct alternative in each of the following:

Let A = {x ϵ R : –1 ≤ x ≤ 1} = B and C = {xϵ R : X ≥ 0} and let S = {(x, y) ϵ A × B : $x^2 + y^2 = 1$} and S_0 =

$\{(x, y) \in A \times C : x^2 + y^2 = 1\}$ Then

A. S defines a function from A to B

B. S_0 defines a function from A to C

C. S_0 defines a function from A to B

D. S defines a function from A to C

Answer

Given that

$A = \{x \in R: –1 \leq x \leq 1\} = B$

$C = \{x \in R: X \geq 0\}$

$S = \{(x, y) \in A \times B: x^2 + y^2 = 1\}$

$S_0 = \{(x, y) \in A \times C: x^2 + y^2 = 1\}$

$x^2 + y^2 = 1$

$\Rightarrow y^2 =1 - x^2$

$\Rightarrow y = \sqrt{1-x^2}$

$\therefore y \in B$

Hence, S defines a function from A to B.

2. Question

Mark the correct alternative in each of the following:

f: R → R given by $f(x) = x+\sqrt{x^2}$ is

A. injective B. surjective

C. bijective D. none of these

Answer

Given function is f: R → R given

$f(x) = x + \sqrt{x^2}$

For this function if we take x = 2,

$f(x) = 2 + \sqrt{4}$

$\Rightarrow f(x) =2$

For this function if we take x = -2,

$f(x) = -2 + \sqrt{4}$

$\Rightarrow f(x) = 0$

So, in general for every negative x, f(x) will be always 0. There is no $x \in R$ for which $f(x) \in (-\infty, 0)$.

Hence, it is neither injective nor surjective and so it is not bijective either.

3. Question

Mark the correct alternative in each of the following:

If f : A → B given by $3^{f(x)} + 2^{-x} = 4$ is a bijection, then

A. $A = \{x \in R : –1 < x < \infty\}$, $B = \{x \in R : 2 < x < 4\}$

B. A = {x ∈ R : –3 < x < ∞}, B = {x∈ R : 0 < x < 4}

C. A = {x ∈R : –2 < x < ∞}, B = {x∈ R : 0 < x < 4}

D. none of these

Answer

Given that f: A → B given by $3^{f(x)} + 2^{-x} = 4$ is a bijection.

$3^{f(x)} + 2^{-x} = 4$

$\Rightarrow 3^{f(x)} = 4 - 2^{-x}$

$\Rightarrow 4 - 2^{-x} \geq 0$

$\Rightarrow 4 \geq 2^{-x}$

$\Rightarrow 2 \geq -x$

$\Rightarrow x \geq -2$

So, x ∈ (-2, ∞)

But, for x =0, f(x) = 1.

Hence, the correct option is none of these.

4. Question

Mark the correct alternative in each of the following:

The function f : R → R defined by $f(x) = 2^x + 2^{|x|}$ is

A. one-one and onto

B. many-one and onto

C. one-one and into

D. many-one and into

Answer

Given that f: R → R where $f(x) = 2^x + 2^{|x|}$

Here, for each value of x we will get different value of f(x).

So, it is one-one.

Also, f(x) is always positive for x ∈ R.

There is no x ∈ R for which f(x) ∈ (-∞, 0).

So, it is into.

Hence, the given function is one-one and into.

5. Question

Mark the correct alternative in each of the following:

Let the function f : R – {–b} → R – {1} be defined by $f(x) = \frac{x+a}{x+b}, a \neq b,$ then

A. f is one-one but not onto

B. f is onto but not one-one

C. f is both one-one and onto

D. none of these

Answer

Given that f: R – {–b} → R – {1} where

$$f(x) = \frac{x + a}{x + b}, a \neq b.$$

Here, f(x) = f(y) only when x=y.

Hence, it is one-one.

Now, f(x) = y

$$\Rightarrow \frac{x + a}{x + b} = y$$

⇒ x + a = y(x + b)

⇒ x – yx = yb – a

$$\Rightarrow x = \frac{yb - a}{1 - y}, y \neq 1$$

So, x ϵ R – {1}

Hence, it is onto.

6. Question

Mark the correct alternative in each of the following:

The function f : A → B defined by f(x) = $-x^2 + 6x - 8$ is a bijection, if

A. A = (–∞, 5] and B = (–∞, 1]

B. A = [–3, ∞] and B = (–∞, 1]

C. A = (–∞, 3] and B = [1, ∞)

D. A = [3, ∞) and B = [1, ∞)

Answer

Given that f: A → B defined by f(x) = $-x^2 + 6x - 8$ is a bijection.

f(x) = $-x^2 + 6x - 8$

⇒ f(x) = $-(x^2 - 6x + 8)$

⇒ f(x) = $-(x^2 - 6x + 8 + 1 - 1)$

⇒ f(x) = $-(x^2 - 6x + 9 - 1)$

⇒ f(x) = $-[(x - 3)^2 - 1]$

Hence, x ϵ (–∞, 5] and f(x) ϵ (–∞, 1]

7. Question

Mark the correct alternative in each of the following:

Let A = {x ϵR : –1 ≤ x ≤ 1} = B. Then, the mapping f : A→ B given by f(x) = x |x| is

A. injective but not surjective

B. surjective but not injective

C. bijective

D. none of these

Answer

Given that A = {x ϵ R: –1 ≤ x ≤ 1} = B. Then, the mapping f: A→ B given by f(x) = x |x|.

For x <0, f(x) <0

⇒ $y = -x^2$

⇒ x = √-y, which is not possible for x > 0.

Hence, f is one-one and onto.

∴ the given function is bijective.

8. Question

Mark the correct alternative in each of the following:

Let f: R → R be given by $f(x) = [x]^2 + [x + 1]–3$, where [x] denotes the greatest integer less than or equal to x. Then, f(x) is

A. many-one and onto

B. many-one and into

C. one-one and into

D. one-one and onto

Answer

Given that f: R → R be given by $f(x) = [x]^2 + [x + 1] – 3$

As [x] is the greatest integer so for different values of x, we will get same value of f(x).

$[x]^2 + [x + 1]$ will always be an integer.

So, f is many-one.

Similarly, in this function co domain is mapped with at most one element of domain because for every x ϵ R, f(x) ϵ Z.

So, f is into.

9. Question

Mark the correct alternative in each of the following:

Let M be the set of all 2 × 2 matrices with entries from the set R of real numbers. Then the function f : M→ R defined by f(A) = |A| for every A ϵ M, is

A. one-one and onto

B. neither one-one nor onto

C. one-one not one-one

D. onto but not one-one

Answer

Given that M is the set of all 2 × 2 matrices with entries from the set R of real numbers. Then the function f: M → R defined by f(A) = |A| for every A ϵ M.

If f(a) =f(b)

⇒ |a| = |b|

But this does not mean that a=b.

So, f is not one-one.

As a ≠ b but |a|=|b|

So, f is onto.

10. Question

Mark the correct alternative in each of the following:

The function f : [0, ∞) → R given by f(x) $f(x) = \frac{x}{x+1}$ is

A. one-one and onto

B. one-one but not onto

C. onto but not one-one

D. neither one-one nor onto

Answer

Given that f: [0, ∞) → R where $f(x) = \frac{x}{x+1}$

Let f(x) = f(y)

$\Rightarrow \frac{x}{x\ +\ 1} = \frac{y}{y\ +\ 1}$

⇒ xy + x = xy + y

⇒ x = y

So, f is one-one.

Now, y = f(x)

$\Rightarrow y = \frac{x}{x\ +\ 1}$

⇒ xy + y = x

⇒ y = x – xy

$\Rightarrow \frac{y}{1-y} = x$

Here, y≠1 i.e. y ϵ R.

So, f is not onto.

11. Question

Mark the correct alternative in each of the following:

The range of the function f(x) = ${}^{7-x}P_{x-3}$ is

A. {1, 2, 3, 4, 5}

B. {1, 2, 3, 4, 5, 6}

C. {1, 2, 3, 4}

D. {1, 2, 3}

Answer

Given that f(x) = ${}^{7-x}P_{x-3}$

Here, 7-x ≥ x-3

⇒ 10 ≥ 2x

⇒ 5 ≥ x

So, domain = {3, 4, 5}

Range = $\{{}^4P_0, {}^3P_1, {}^2P_2\} = \{1, 3, 2\}$

12. Question

Mark the correct alternative in each of the following:

A function f from the set on natural numbers to integers defined by

$$.f(n) = \begin{cases} \frac{n-1}{2}, & \text{when n is odd} \\ -\frac{n}{2}, & \text{when n is even} \end{cases} .\text{is}$$

A. neither one-one nor onto

B. one-one but not onto

C. onto but not one-one

D. one-one and onto both

Answer

Given that a function f from the set on natural numbers to integers where

$$f(n) = \begin{Bmatrix} \frac{n-1}{2}, \text{when n is odd} \\ \frac{-n}{2}, \text{when n is even} \end{Bmatrix}$$

For n is odd

Let f(n) = f(m)

$$\Rightarrow \frac{n-1}{2} = \frac{m-1}{2}$$

⇒ n = m

For n is even

Let f(n) = f(m)

$$\Rightarrow \frac{-n}{2} = \frac{-m}{2}$$

⇒ n = m

So, f is one-one.

Also, each element of y is associated with at least one element of x, so f is onto.

Hence, f is one-one and onto.

13. Question

Mark the correct alternative in each of the following:

Let f be an injective map with domain {x, y, z} and range {1, 2, 3} such that exactly one of the following statements is correct and the remaining are false.

f(x) = 1, f(y) ≠ 1, f(z) ≠ 2.

The value of $f^{-1}(1)$ is

A. x

B. y

C. z

D. none of these

Answer

Given that f is an injective map with domain {x, y, z} and range {1, 2, 3}.

Case-1

Let us assume that f(x) =1 is true and f(y) ≠ 1, f(z) ≠ 2 is false.

Then f(x) = 1, f(y) = 1 and f(z) = 2.

This violates the injectivity of f because it is one-one.

Case-2

Let us assume that f(y) ≠1 is true and f(x)= 1, f(z) ≠ 2 is false.

Then f(x) ≠ 1, f(y) ≠ 1 and f(z) = 2.

This means there is no pre image of 1 which contradicts the fact that the range of f is {1, 2, 3}.

Case-3

Let us assume that f(z) ≠2 is true and f(x)= 1, f(y) ≠ 1 is false.

Then f(z) ≠ 2, f(y) = 1 and f(x) ≠1.

⇒ $f^{-1}(1) = y$

14. Question

Mark the correct alternative in each of the following:

Which of the following functions from Z to itself are bijections?

A. $f(x) = x^3$

B. f(x) = x + 2

C. f(x) = 2x + 1

D. $f(x) = x^2 + x$

Answer

a. $f(x) = x^3$

⇒ For no value of x ϵ Z, f(x) = 2.

Hence, it is not bijection.

b. f(x) = x + 2

If f(x) = f(y)

⇒ x + 2 = y + 2

⇒ x = y

So, f is one-one.

Also, y = x + 2

⇒ x = y – 2 ϵ Z

So, f is onto.

Hence, this function is bijection.

c. f(x) = 2x + 1

If f(x) = f(y)

$\Rightarrow 2x + 1 = 2y + 1$

$\Rightarrow x = y$

So, f is one-one.

Also, $y = 2x + 1$

$\Rightarrow 2x = y - 1$

$\Rightarrow x = \frac{y-1}{2}$

So, f is into because x can never be odd for any value of y.

d. $f(x) = x^2 + x$

For this function if we take $x = 2$,

$f(x) = 4 + 2$

$\Rightarrow f(x) = 6$

For this function if we take $x = -2$,

$f(x) = 4 - 2$

$\Rightarrow f(x) = 2$

So, in general for every negative x, f(x) will be always 0. There is no $x \in R$ for which $f(x) \in (-\infty, 0)$.

It is not bijection.

15. Question

Mark the correct alternative in each of the following:

Which of the following functions from $A = \{x : -1 \le x \le 1\}$ to itself are bijections?

A. $f(x) = \frac{x}{2}$

B. $g(x) = \sin(\frac{\pi x}{2})$

C. $h(x) = |x|$

D. $k(x) = x^2$

Answer

Given that $A = \{x : -1 \le x \le 1\}$

a. $f(x) = \frac{x}{2}$

It is one-one but not onto.

b. $g(x) = \sin(\frac{\pi x}{2})$

It is bijective as it is one-one and onto with range [-1, 1].

c. $h(x) = |x|$

It is not one-one because h(-1)=1 and h(1)=1.

d. $k(x) = x^2$

It is not one-one because k(-1)=1 and k(1)=1.

16. Question

Mark the correct alternative in each of the following:

Let A = {x : –1 ≤ x ≤ 1} and f : A→ A such that f(x) = x |x|, then f is

A. a bijection

B. injective but not surjective

C. surjective but not injective

D. neither injective nor surjective

Answer

Given that A = {x: –1 ≤ x ≤ 1} and f: A→ A such that f(x) = x |x|.

For x <0, f(x) <0

⇒ $y = -x^2$

⇒ $x = \sqrt{-y}$, which is not possible for x > 0.

Hence, f is one-one and onto.

∴ the given function is bijective.

17. Question

Mark the correct alternative in each of the following:

If the function f: R → A given by $f(x) = \frac{x^2}{x^2+1}$ is a surjection, then A =

A. R

B. [0, 1]

C. (0, 1]

D. [0, 1)

Answer

Given that f: R → A such that $f(x) = \frac{x^2}{x^2+1}$ is a surjection.

f(x) = y

$\Rightarrow y = \frac{x^2}{x^2 + 1}$

⇒ $y(x^2 + 1) = x^2$

⇒ $yx^2 + y = x^2$

⇒ $yx^2 - x^2 = -y$

$\Rightarrow x^2 = \frac{y}{1-y}$

$\Rightarrow x = \sqrt{\frac{y}{1-y}}$

Here, $\frac{y}{1-y} \geq 0$

So, y ϵ [0, 1)

18. Question

Mark the correct alternative in each of the following:

If a function f : [2, ∞) → B defined by $f(x) = x^2 - 4x + 5$ is a bijection, then B =

A. R

B. [1, ∞)

C. [4, ∞)

D. [5, ∞)

Answer

Given that a function f : [2, ∞) → B defined by $f(x) = x^2 - 4x + 5$ is a bijection.

Put x = 2 in f(x),

$f(x) = 2^2 - 4\times 2 + 5$

⇒ f(x=2) = 4 – 8 + 5

⇒ f(x=2) = 1

So, B ϵ [1, ∞)

19. Question

Mark the correct alternative in each of the following:

The function f : R → R defined by f(x) = (x – 1) (x – 2) (x – 3) is

A. one-one but not onto

B. onto but not one-one

C. both one and onto

D. neither one-one nor onto

Answer

Given that function f : R → R where f(x) = (x – 1) (x – 2) (x – 3)

If f(x) = f(y)

Then

(x – 1) (x – 2) (x – 3) = (y – 1) (y – 2) (y – 3)

⇒ f(1) = f(2) = f(3) = 0

So, f is not one-one.

y = f(x)

∵ x ϵ R also y ϵ R so f is onto.

20. Question

Mark the correct alternative in each of the following:

The function f : [–1/2, 1/2] → [π/2, π/2] defined by $f(x) = \sin^{-1}(3x - 4x^3)$ is

A. bijection

B. injection but not a surjection

C. surjection but not an injection

D. neither an injection nor a surjection

Answer

Given that f : [–1/2, 1/2] → [π/2, π/2] where $f(x) = \sin^{-1}(3x - 4x^3)$

Put x = sinθ in $f(x) = \sin^{-1}(3x - 4x^3)$

$\Rightarrow f(x=\sin\theta) = \sin^{-1}(3\sin\theta - 4\sin\theta^3)$

$\Rightarrow f(x) = \sin^{-1}(\sin 3\theta)$

$\Rightarrow f(x) = 3\theta$

$\Rightarrow f(x) = 3\sin^{-1}x$

If f(x) = f(y)

Then

$3\sin^{-1}x = 3\sin^{-1}y$

$\Rightarrow x = y$

So, f is one-one.

$y = 3\sin^{-1}x$

$\Rightarrow\ x = \sin\frac{y}{3}$

∵ x ϵ R also y ϵ R so f is onto.

Hence, f is bijection.

21. Question

Mark the correct alternative in each of the following:

Let f : R → R be a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$. Then,

A. f is a bijection

B. f is an injection only

C. f is surjection on only

D. f is neither an injection nor a surjection

Answer

Given that f: R → R is a function defined as

$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$

Here, $e^{|x|}$ is always positive whether x is negative or positive. So, we will get same values of f(x) for different values of x.

Hence, it is not one-one and onto.

∴ f is neither an injection nor a surjection

22. Question

Mark the correct alternative in each of the following:

Let f : R –{n} → R be a function defined by $f(x) = \frac{x - m}{x - n}$, where m ≠ n. Then,

A. f is one-one onto

B. f is one-one into

C. f is many one onto

D. f is many one into

Answer

Given that f : R –{n} → R where

$f(x) = \frac{x-m}{x-n}$, such that $m \neq n$

Let f(x) = f(y)

$\Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n}$

⇒ (x-m)(y-n)=(x-n)(y-m)

⇒ xy – xn – my + mn = xy – xm – ny + mn

⇒ x = y

So, f is one-one.

$f(x) = \frac{x-m}{x-n}$

$\Rightarrow y = \frac{x-m}{x-n}$

⇒ y(x-n)=(x-m)

⇒ xy – ny = x – m

⇒ x(y-1) = ny – m

$\Rightarrow x = \frac{ny-m}{y-1}, y \neq 1$

For y = 1 , no x is defined.

So, f is into.

23. Question

Mark the correct alternative in each of the following:

Let f : R → R be a function defined by $f(x) = \frac{x^2-8}{x^2+2}$. Then, f is

A. one-one but not onto

B. one-one and onto

C. onto but not one-one

D. neither one-one nor onto

Answer

Given that f : R → R be a function where

$f(x) = \frac{x^2-8}{x^2 + 2}$

Here, we can see that for negative as well as positive x we will get same value.

So, it is not one-one.

y = f(x)

$\Rightarrow y = \frac{x^2-8}{x^2 + 2}$

$\Rightarrow y(x^2 + 2) = (x^2 - 8)$

$\Rightarrow x^2(y-1) = -2y - 8$

$\Rightarrow x = \sqrt{\frac{2y + 8}{1 - y}}$

For y = 1 , no x is defined.

So, f is not onto.

24. Question

Mark the correct alternative in each of the following:

f : R → R is defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ is

A. one-one but not onto

B. one-one and onto

C. onto but not one-one

D. neither one-one nor onto

Answer

Given that f : R → R where

$f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$

Here, we can see that for negative as well as positive x we will get same value.

So, it is not one-one.

f(x)=y

By definition of onto, each element of y is not mapped to at least one element of x.

So, it is not onto.

25. Question

Mark the correct alternative in each of the following:

The function f : R → R, $f(x) = x^2$ is

A. injective but not surjective

B. surjective but not injective

C. injective as well as surjective

D. neither injective nor surjective

Answer

Given that f : R → R, $f(x) = x^2$

Let f(x)=y(x)

$\Rightarrow x^2 = y^2$

$\Rightarrow x = \pm y$

So, it is not one-one.

f(x) = y

⇒ $x^2 = y$

⇒ x = ±√y

But co domain is R.

Hence, f is neither injective nor surjective.

26. Question

Mark the correct alternative in each of the following:

A function f from the set of natural, numbers to the set of integers defined by

$$f(n)=\begin{cases}\frac{n-1}{2}, & \text{when n is odd}\\ -\frac{n}{2}, & \text{when n is even}\end{cases}$$

A. neither one-one nor onto

B. one-one but not onto

C. onto but not one-one

D. one-one and onto both

Answer

Given that a function f from the set on natural numbers to integers where

$$f(n) = \begin{Bmatrix}\frac{n-1}{2}, \text{when n is odd}\\ \frac{-n}{2}, \text{when n is even}\end{Bmatrix}$$

For n is odd

Let f(n) = f(m)

$$\Rightarrow \frac{n-1}{2} = \frac{m-1}{2}$$

⇒ n = m

For n is even

Let f(n) = f(m)

$$\Rightarrow \frac{-n}{2} = \frac{-m}{2}$$

⇒ n = m

So, f is one-one.

Also, each element of y is associated with at least one element of x, so f is onto.

Hence, f is one-one and onto.

27. Question

Mark the correct alternative in each of the following:

Which of the following functions from A = {x ϵ R : –1 ≤ x ≤ 1} to itself are bijections?

A. f(x) = |x|

B. $f(x)=\sin\frac{\pi x}{2}$

C. $f(x)=\sin\frac{\pi x}{4}$

D. none of these

Answer

Given that A = {x : –1 ≤ x ≤ 1}

a. f(x)=|x|

It is not one-one because f(-1)=1 and f(1)=1.

b. $f(x) = \sin(\frac{\pi x}{2})$

It is bijective as it is one-one and onto with range [-1, 1].

28. Question

Mark the correct alternative in each of the following:

Let f: Z → Z be given by $f(x)=\begin{cases}\frac{x}{2}, & \text{if x is even}\\ 0, & \text{if x is odd}\end{cases}$. Then, f is

A. onto but not one-one

B. one-one but not onto

C. one-one and onto

D. neither one-one nor onto

Answer

Given function f: Z → Z defined as

$f(x) = \begin{Bmatrix}\frac{x}{2}, \text{when x is even}\\ 0, \text{when x is odd}\end{Bmatrix}$

For x = 3, f(x) = 0

For x = 5, f(x) = 0

But 3 ≠ 5

So, f is not one-one.

Y=f(x)

∵ x ϵ R ⇒ y ϵ R

∴ Domain = Range

Hence, f is not one-one but onto.

29. Question

Mark the correct alternative in each of the following:

The function f : R → R defined by $f(x) = 6^x + 6^{|x|}$ is

A. one-one and onto

B. many one and onto

C. one-one and into

D. many one and into

Answer

Given that function f : R → R defined by $f(x) = 6^x + 6^{|x|}$

Let f(x) = f(y)

$\Rightarrow 6^x + 6^{|x|} = 6^y + 6^{|y|}$

Only when x = y

So, f is one-one.

Now for y=f(x)

y can never be negative which means for no x ϵ R y is negative.

So, f is not onto but into.

30. Question

Mark the correct alternative in each of the following:

Let $f(x) = x^2$ and $g(x) = 2^x$. Then the solution set of the equation fog(x) = gof (x) is

A. R

B. {0}

C. {0, 2}

D. none of these

Answer

Given that $f(x) = x^2$ and $g(x) = 2^x$.

Also, fog(x) = gof (x)

$\Rightarrow f(2^x)=g(x^2)$

$\Rightarrow 2^{2x} = 2^{x^2}$

$\Rightarrow 2x = x^2$

$\Rightarrow x^2 - 2x=0$

$\Rightarrow x(x-2)=0$

$\Rightarrow x = 0$ or $x = 2$

31. Question

Mark the correct alternative in each of the following:

If f: R → R is given by f(x) = 3x – 5, then $f^{-1}(x)$

A. is given by $\frac{1}{3x-5}$

B. is given by $\frac{x+5}{3}$

C. does not exist because f is not one-one

D. does not exist because f is not onto

Answer

Given that f: R → R is given by f(x) = 3x – 5

To find $f^{-1}(x)$:

y =f(x)

⇒ y = 3x – 5

⇒ y + 5 = 3x

$\Rightarrow\ x = \frac{y + 5}{3}$

$\Rightarrow f^{-1}(x) = \frac{x + 5}{3}$

32. Question

Mark the correct alternative in each of the following:

If g(f(x)) = |sin x| and f (g(x)) = $\left(\sin\sqrt{x}\right)^2$, then

A. f(x) = $\sin^2 x$, g(x) = √x

B. f(x) = sin x, g(x) = |x|

C. f(x) = x^2, g(x) = sin √x

D. f and g cannot be determined

Answer

Given that g(f(x)) = |sin x| and $f(g(x)) = (\sin\sqrt{x})^2$

a. For f(x) = $\sin^2 x$, g(x) = √x

f(g(x))=f(√x)= $(\sin\sqrt{x})^2$

g(f(x))=g($\sin^2 x$)= √ $\sin^2 x$= |sinx|

Correct

b. For f(x) = sin x, g(x) = |x|

f(g(x)) = f(|x|) = sin |x|

g(f(x) = g(sin x) = |sin x|

Incorrect

c. f(x) = x^2, g(x) = sin √x

f(g(x)) = f(sin √x) = $(\sin\sqrt{x})^2$

g(f(x)) = g(x^2) = sin |x|

Incorrect

33. Question

Mark the correct alternative in each of the following:

The inverse of the function f : R → [x ϵ R : x < 1] given by $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, is

A. $\frac{1}{2}\log\frac{1+x}{1-x}$

B. $\frac{1}{2}\log\frac{2+x}{2-x}$

C. $\frac{1}{2}\log\frac{1-x}{1+x}$

D. none of these

Answer

Given that f: R → [x ϵ R : x < 1] defined by

$$f(x)=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

Put y = f(x)

$$\Rightarrow y=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$$

$$\Rightarrow y=\frac{e^{2x}-1}{e^{2x}+1}$$

⇒ y(e^{2x} + 1)= e^{2x} - 1

⇒ e^{2x} (y -1) = -y -1

$$\Rightarrow e^{2x}=\frac{y+1}{1-y}$$

$$\Rightarrow 2x=\log(\tfrac{y+1}{1-y})$$

$$\Rightarrow x=\tfrac{1}{2}\log(\tfrac{y+1}{1-y})$$

So, $f^{-1}(x)=\frac{1}{2}\log(\frac{x+1}{1-x})$

34. Question

Mark the correct alternative in each of the following:

Let A = {x ϵR : x ≥ 1}. The inverse of the function f : A→ A given by f(x) = $2^{x(x-1)}$, is

A. $\left(\frac{1}{2}\right)^{x(x-1)}$

B. $\frac{1}{2}\left\{1+\sqrt{1+4\log_2 x}\right\}$

C. $\frac{1}{2}\left\{1-\sqrt{1+4\log_2 x}\right\}$

D. not defined

Answer

Given that A = {x ϵ R : x ≥ 1}. The function f: A→ A given by f(x) = $2^{x(x-1)}$

Put y = f(x)

⇒ y = $2^{x(x-1)}$

$$\Rightarrow \log_2 y=x(x-1)$$

$$\Rightarrow \log_2 y=x^2-x$$

$\Rightarrow \log_2 y + \frac{1}{4} = x^2 - x + \frac{1}{4}$

$\Rightarrow \log_2 y + \frac{1}{4} = (x - \frac{1}{2})^2$

$\Rightarrow \sqrt{\frac{4\log_2 y + 1}{4}} + \frac{1}{2} = x$

$\Rightarrow \frac{1 + \sqrt{4\log_2 y + 1}}{2} = x$

$f^{-1}(x) = \frac{1 + \sqrt{4\log_2 x + 1}}{2}$

35. Question

Mark the correct alternative in each of the following:

Let A = {x ϵ R: x ≤ 1} and f : A → A given by f(x) = x(2 – x).Then, $f^{-1}(x)$ is

A. $1+\sqrt{1-x}$

B. $1-\sqrt{1-x}$

C. $\sqrt{1-x}$

D. $1\pm\sqrt{1-x}$

Answer

Given that A = {x ϵ R: x ≤ 1} and f : A → A given by f(x) = x(2 – x).

y = f(x)

⇒ y = x(2 – x)

⇒ y = 2x – x^2

⇒ y - 1 = 2x – x^2 -1

⇒ y – 1 = - (x^2 + 1 – 2x)

⇒ $(x – 1)^2$ = 1 – y

$\Rightarrow \ x = 1 - \sqrt{1 - y}$

$f^{-1}(x) = 1 - \sqrt{1 - x}$

36. Question

Mark the correct alternative in each of the following:

Let $f(x) = \frac{1}{1-x}$. Then, {fo(fof)} (x)

A. x for all x ϵ R

B. x for all x ϵ R – {1}

C. x for all x ϵ R –{0, 1}

D. none of these

Answer

Given that $f(x) = \frac{1}{1-x}$

$fof(x) = f(\frac{1}{1-x})$, for x≠1

$\Rightarrow fof = \dfrac{1}{1 - \frac{1}{1-x}}$

$\Rightarrow fof = \dfrac{1-x}{1-x-1}$

$\Rightarrow fof = \dfrac{x-1}{x}$

$fofof(x) = f(\frac{x-1}{x})$, for x≠0

$\Rightarrow fofof = \dfrac{1}{1 - \frac{x-1}{x}}$

$\Rightarrow fofof = \dfrac{x}{x - x + 1}$

⇒ fofof = x for all x ϵ R –{0, 1}

37. Question

Mark the correct alternative in each of the following:

If the function f: R → R be such that f(x) = x – [x], where [x] denotes the greatest integer less than or equal to x, then f^{-1} (x) is

A. $\dfrac{1}{x - [x]}$

B. [x] – x

C. not defined

D. none of these

Answer

Given that f: R → R be such that f(x) = x – [x], where [x] denotes the greatest integer less than or equal to x

We will have same value of f for different values of x.

So, the function is not one-one.

∵ f is not bijective

∴ f does not have inverse.

38. Question

Mark the correct alternative in each of the following:

If F: [1, ∞) → [2, ∞) is given by $f(x) = x + \dfrac{1}{x}$, then $f^{-1}(x)$ equals.

A. $\dfrac{x + \sqrt{x^2 - 4}}{2}$

B. . $\dfrac{x}{1 + x^2}$.

C. $\dfrac{x-\sqrt{x^2-4}}{2}$

D. $1+\sqrt{x^2-4}$.

Answer

Given that F: [1, ∞) → [2, ∞) defined as

$$f(x) = x + \frac{1}{x}$$

y = f(x)

$$\Rightarrow y = x + \frac{1}{x}$$

$$\Rightarrow y = \frac{x^2 + 1}{x}$$

⇒ $xy = x^2 + 1$

$$\Rightarrow x^2 - xy + \frac{y^2}{4} = \frac{y^2}{4} - 1$$

$$\Rightarrow (x - \frac{y}{2})^2 = \frac{y^2}{4} - 1$$

$$\Rightarrow x = \frac{y}{2} + \sqrt{\frac{y^2-4}{4}}$$

$$\Rightarrow x = \frac{y}{2} + \frac{1}{2}\sqrt{y^2-4}$$

$$f^{-1}(x) = \frac{x + \sqrt{x^2-4}}{2}$$

39. Question

Mark the correct alternative in each of the following:

Let g(x) = 1 + x – [x] and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$, where [x] denotes the greatest integer less than or equal to x.

Then for all x, f (g(x)) is equal to

A. x

B. 1

C. f(x)

D. g(x)

Answer

Given that g(x) = 1 + x – [x] and

$$f(x) = \begin{Bmatrix} -1, x < 0 \\ 0, x = 0 \\ 1, x > 0 \end{Bmatrix}$$

where [x] denotes the greatest integer less than or equal to x.

(i) -1 < x <0

g(x) = 1 + x – [x]

⇒ g(x) = 1 + x + 1 {∵ [x] = -1}

⇒ g(x) = 2 + x

f(g(x))= f(2 + x)

⇒ f(g(x))=1 + 2 + x-[2 + x]

⇒ f(g(x)) = 3 + x -2 – x

⇒ f(g(x)) = 1

(ii) x = 0

f(g(x)) = f(1 + x-[x])

⇒ f(g(x)) = 1 + 1 + x –[x] – [1 + x + [x]]

⇒ f(g(x)) = 2 + 0 -1

⇒ f(g(x)) = 1

(iii) x > 1

f(g(x)) = f(1 + x-[x])

⇒ f(g(x)) = f(x>0) = 1

Hence, f(g(x)) = 1 for all cases.

40. Question

Mark the correct alternative in each of the following:

Let $f(x) = \frac{\alpha x}{x+1}, x \neq -1$. Then, for what value of α is f(f(x)) = x?

A. √2

B. –√2

C. 1

D. –1

Answer

Given that $(x) = \frac{\alpha x}{x+1}$, x≠-1 and f(f(x)) = x

$\Rightarrow\ f\left(\frac{\alpha x}{x\ +\ 1}\right) = x$

$\Rightarrow \frac{\alpha\frac{\alpha x}{x+1}}{\frac{\alpha x}{x+1}\ +\ 1} = x$

$\Rightarrow \frac{\alpha^2 x}{x\ +\ 1} = x\left(\frac{\alpha x}{x\ +\ 1}\ +\ 1\right)$

⇒ $\alpha^2 x = x(\alpha x + x + 1)$

⇒ $\alpha^2 = \alpha x + x + 1$

⇒ $\alpha^2 - \alpha x = x + 1$

On comparing - αx with x,

We get α = -1

41. Question

Mark the correct alternative in each of the following:

The distinct linear functions which map [–1, 1] onto [0, 2] are

A. $f(x) = x + 1, g(x) = -x + 1$

B. $f(x) = x - 1, g(x) = x + 1$

C. $f(x) = -x - 1\ g(x) = x - 1$

D. none of these

Answer

a. $f(x) = x + 1, g(x) = -x + 1$

$f(-1) = -1 + 1= 0$

$f(1) = 1 + 1 = 2$

Also, $g(-1) = 1 + 1 = 2$

$g(1) = -1 + 1= 0$

These functions map [–1, 1] onto [0, 2].

b. $f(x) = x - 1, g(x) = x + 1$

$f(-1) = -1 - 1= -2$

$f(1) = 1-1 = 0$

Also, $g(-1) = -1 + 1 = 0$

$g(1) = 1 + 1= 2$

These functions do not map [–1, 1] onto [0, 2].

c. $f(x) = -x - 1\ g(x) = x - 1$

$f(-1) = 1 - 1= 0$

$f(1) = -1 - 1 = -2$

Also, $g(-1) = -1 - 1 = -2$

$g(1) = 1 - 1= 0$

These functions do not map [–1, 1] onto [0, 2].

42. Question

Mark the correct alternative in each of the following:

Let $f : [2, \infty) \rightarrow X$ be defined by $f(x) = 4x - x^2$. Then, f is invertible, if X =

A. $[2, \infty)$

B. $(-\infty, 2]$

C. $(-\infty, 4]$

D. $[4, \infty)$

Answer

Given that $f: [2, \infty) \rightarrow X$ be defined by

$f(x) = 4x - x^2$

Let $y = f(x)$

$\Rightarrow y = 4x - x^2$

$\Rightarrow -y + 4 = 4 - 4x + x^2$

$\Rightarrow 4 - y = (x - 2)^2$

$\Rightarrow x - 2 = \sqrt{4 - y}$

$\Rightarrow x = 2 + \sqrt{4 - y}$

So, $f^{-1}(x) = 2 + \sqrt{4 - x}$

where $x < 4$

So, $x \in (-\infty, 4]$

43. Question

Mark the correct alternative in each of the following:

If $f : R \rightarrow (-1, 1)$ is defined by $f(x) = \frac{-x\,|\,x\,|}{1 + x^2}$, then $f^{-1}(x)$ equals

A. $\sqrt{\frac{|\,x\,|}{1 - |\,x\,|}}$

B. $-\mathrm{Sgn}\,(x)\sqrt{\frac{|\,x\,|}{1 - |\,x\,|}}$

C. $-\sqrt{\frac{x}{1 - x}}$

D. none of these

Answer

Given that $f : R \rightarrow (-1, 1)$ is defined by

$$f(x) = \frac{-x|x|}{1 + x^2}$$

Here for mod function we will consider three cases, x = 0, x<0 and x>0.

For x< 0

$$f(x) = \frac{-x(-x)}{1 + x^2}$$

$$y = \frac{x^2}{1 + x^2}$$

$\Rightarrow y(1 + x^2) = x^2$

$\Rightarrow x^2 (1 - y) = y$

$$\Rightarrow x = -\sqrt{\frac{y}{1 - y}}$$

$$\Rightarrow x = -\sqrt{\frac{|y|}{1 - |y|}},\ x < 0$$

Also, checking on x>0 and x = 0 we find that

$$f^{-1}(x) = -\text{sgn}(x)\sqrt{\frac{|y|}{1-|y|}},$$

44. Question

Mark the correct alternative in each of the following:

Let [x] denote the greatest integer less than or equal to x. If f(x) = $\sin^{-1} x$, g(x) = $[x^2]$ and $h(x) = 2x, \frac{1}{2} \le x \le \frac{1}{\sqrt{2}}$, then

A. fogoh (x) = π/2

B. fogoh (x) = π

C. hofog = hogof

D. hofog ≠ hogof

Answer

Given that f(x) = $\sin^{-1} x$, g(x) = $[x^2]$ and $h(x) = 2x, \frac{1}{2} \le x \le \frac{1}{\sqrt{2}}$

a. goh(x) = g(2x)

⇒ goh(x) = $[4x^2]$

fogoh(x) = f($[4x^2]$)

⇒ fogoh(x) = $\sin^{-1} [4x^2]$

Hence, given option is incorrect.

b. Similarly, this option is also incorrect.

c. fog(x) = f($[x^2]$)

⇒ fog(x) = $\sin^{-1} [x^2]$

hofog(x) = h($\sin^{-1} [x^2]$)

⇒ hofog(x) = 2($\sin^{-1} [x^2]$)

gof(x) = g($\sin^{-1} x$)

⇒ gof(x) = $[(\sin^{-1} x)^2]$

hogof(x) = h($[(\sin^{-1} x)^2]$)

⇒ hogof(x) = 2$[(\sin^{-1} x)^2]$

Hence, hogof(x) ≠ hofog(x)

45. Question

Mark the correct alternative in each of the following:

If g(x) = $x^2 + x - 2$ and $\frac{1}{2}$gof (x) = $2x^2 - 5x + 2$, then f(x) is equal to

A. 2x – 3

B. 2x + 3

C. $2x^2 + 3x + 1$

D. $2x^2 - 3x - 1$

Answer

Given that g(x) = x^2 + x – 2 and

$$\frac{1}{2}\text{gof}(x) = 2x^2 - 5x + 2$$

a. Let f(x) = 2x – 3

gof(x) = g(2x – 3)

⇒ gof(x) = $(2x - 3)^2$ + 2x - 3 – 2

⇒ gof(x) = $4x^2$ - 12x + 9 + 2x – 5

⇒ gof(x) = $4x^2$ - 10x + 4

$$\frac{1}{2}\text{gof}(x) = \frac{4x^2 - 10x + 4}{2}$$

$$\Rightarrow \frac{1}{2}\text{gof}(x) = 2x^2 - 5x + 2$$

Hence, this option is the required value of f(x).

b. Let f(x) = 2x + 3

gof(x) = g(2x + 3)

⇒ gof(x) = $(2x + 3)^2$ + 2x + 3 – 2

⇒ gof(x) = $4x^2$ + 12x + 9 + 2x + 1

⇒ gof(x) = $4x^2$ + 14x + 10

$$\frac{1}{2}\text{gof}(x) = \frac{4x^2 + 14x + 10}{2}$$

$$\Rightarrow \frac{1}{2}\text{gof}(x) = 2x^2 + 7x + 10$$

Hence, this option is not the required value of f(x).

c and d option are incorrect because their degree is more than 1. So, the degree of gof will be more than 2.

46. Question

Mark the correct alternative in each of the following:

If f(x) = $\sin^2$ x and the composite function g(f(x)) = |sin x|, then g(x) is equal to

A. $\sqrt{x-1}$

B. $\sqrt{x}$

C. $\sqrt{x+1}$

D. $-\sqrt{x}$

Answer

Given that f(x) = $\sin^2$ x and the composite function g(f(x)) = |sin x|.

g(f(x)) = g($\sin^2$ x)

a. If $g(x) = \sqrt{x-1}$

$$g(f(x)) = \sqrt{\sin^2 x - 1}$$

Hence, given option is incorrect.

b. If g(x) =√x

$g(f(x)) = \sqrt{\sin^2 x}$

⇒ g(f(x)) = |sin x|

Hence, given option is correct.

c. If $g(x) = \sqrt{x + 1}$

$g(f(x)) = \sqrt{\sin^2 x + 1}$

Hence, given option is incorrect.

d. If g(x) =-√x

$g(f(x)) = -\sqrt{\sin^2 x}$

⇒ g(f(x)) = - sin x

Hence, given option is incorrect.

47. Question

Mark the correct alternative in each of the following:

If f : R → R is given by f(x) = x^3 + 3, then $f^{-1}(x)$ is equal to

A. $x^{1/3} - 3$

B. $x^{1/3} + 3$

C.$(x - 3)^{1/3}$

D. $x + 3^{1/3}$

Answer

Given that f : R → R is given by f(x) = $x^3 + 3$

Then $f^{-1}(x)$:

y = f(x)

⇒ y = x^3 + 3

⇒ y - 3 = x^3

$\Rightarrow x = \sqrt[3]{y - 3}$

So, $f^{-1}(x) = \sqrt[3]{x - 3}$

48. Question

Mark the correct alternative in each of the following:

Let f(x) = x^3 be a function with domain {0, 1, 2, 3}. Then domain of f^{-1} is

A. {3, 2, 1, 0}

B. {0, –1, –2, –3}

C. {0, 1, 8, 27}

D. {0, –1, –8, –27}

Answer

Given that $f(x) = x^3$ be a function with domain {0, 1, 2, 3}.

Then range = {0, 1, 8, 27}

f can be written as {(0, 0), (1, 1), (2, 8), (3, 27)}

f^{-1} can be written as {(0, 0), (1, 1), (8, 2), (27, 3)}

So, the domain of f^{-1} is {0, 1, 8, 27}

49. Question

Mark the correct alternative in each of the following:

Let $f : R \to R$ be given by $f(x) = x^2 - 3$. Then, f^{-1} is given by

A. $\sqrt{x+3}$

B. $\sqrt{x}+3$

C. $x+\sqrt{3}$

D. none of these

Answer

Given that $f : R \to R$ defined by $f(x) = x^2 - 3$

For f^{-1}:

$y = f(x)$

$\Rightarrow y = x^2 - 3$

$\Rightarrow x = \pm\sqrt{y + 3}$

$f^{-1}(x) = \pm\sqrt{x + 3}$

50. Question

Mark the correct alternative in each of the following:

Let $f : R \to R$ be given by $f(x) = \tan x$. Then, $f^{-1}(1)$ is

A. $\frac{\pi}{4}$

B. $\left\{n\pi+\frac{\pi}{4} : n \in Z\right\}$

C. does not exist

D. none of these

Answer

Given that $f: R \to R$ be given by $f(x) = \tan x$

For f^{-1}:

$y = f(x)$

$\Rightarrow y = \tan x$

$\Rightarrow x = \tan^{-1} y$

$f^{-1} = \tan^{-1} x$

$\Rightarrow f^{-1}(x) = n\pi + \frac{\pi}{4}; n \in Z$

51. Question

Mark the correct alternative in each of the following:

Let f: R → R be defined as $f(x) = \begin{cases} 2x, \text{ if } x > 3 \\ x^2, \text{ if } 1 < x \le 3. \\ 3x, \text{ if } x \le 1 \end{cases}$

Then, find f(–1) + f(2) + f(4)

A. 9

B. 14

C. 5

D. none of these

Answer

Given that f: R → R be defined as

$$f(x) = \begin{Bmatrix} 2x, \text{if } x > 3 \\ x^2, \text{if } 1 < x \le 3 \\ 3x, \text{if } x \le 1 \end{Bmatrix}$$

For f(-1):

f(x) = 3x

⇒ f(-1) = -3

For f(2):

f(x) = x^2

⇒ f(2) = 4

For f(4):

f(x) = 2x

⇒ f(4) = 8

f(–1) + f(2) + f(4) = -3 + 4 + 8

⇒ f(–1) + f(2) + f(4) = 9

52. Question

Mark the correct alternative in each of the following:

Let A = {1, 2, ..., n} and B = {a, b}. Then the number of subjections from A into B is

A. $^nP_2$

B. 2^n – 2

C. 0

D. none of these

Answer

Given that A = {1, 2, ..., n} and B = {a, b}

The number of functions from a set with n number of elements into a set of 2 number of elements = 2^n

But two functions can be many-one into functions.

Hence, answer is $2^n - 2$.

53. Question

Mark the correct alternative in each of the following:

If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

A. 720

B. 120

C. 0

D. none of these

Answer

Given that set A contains 5 elements and set B contains 6 elements.

Number of one-one and onto mappings from A to B means bijections from A to B.

Number of bijections are possible only when n(B) < n(A).

But here, n(A) < n(B)

So, the number of one-one and onto mappings from A to B is 0.

54. Question

Mark the correct alternative in each of the following:

If the set A contains 7 elements and the set B contains 10 elements, then the number one-one functions from A to B is

A. ${}^{10}C_7$

B. ${}^{10}C_7 \times 7!$

C. 7^{10}

D. 10^7

Answer

Given that set A contains 7 elements and set B contains 10 elements.

The number one-one functions from A to B is ${}^{10}C_7 \times 7!$.

55. Question

Mark the correct alternative in each of the following:

Let $f : R - \left\{\frac{3}{5}\right\} \to R$ be defined by $f(x) = \frac{3x+2}{5x-3}$. Then,

A. $f^{-1}(x) = x$

B. $f^{-1}(x) = -f(x)$

C. fof (x) = x

D. $f^{-1}(x) = \frac{1}{19} f(x)$

Answer

Given that $f: R - \left\{\frac{3}{5}\right\} \to R$ defined as $f(x) = \frac{3x + 2}{5x - 3}$

For f^{-1}:

$$y = \frac{3x + 2}{5x - 3}$$

$\Rightarrow y(5x - 3) = 3x + 2$

$\Rightarrow x(5y - 3) = 2 + 3y$

$$\Rightarrow x = \frac{2 + 3y}{5y - 3}$$

So, $f^{-1}(x) = \frac{2 + 3x}{5x - 3}$

$$fof(x) = f\left(\frac{3x + 2}{5x - 3}\right)$$

$$\Rightarrow fof(x) = \frac{3\frac{3x+2}{5x-3} + 2}{5\frac{3x+2}{5x-3} - 3}$$

$$\Rightarrow fof(x) = \frac{3(3x + 2) + 2(5x - 3)}{5(3x + 2) - 3(5x - 3)}$$

$$\Rightarrow fof(x) = \frac{19x}{19}$$

$\Rightarrow fof(x) = x$

Hence, option C is correct.