19. Arithmetic Progressions

Exercise 19.1

1. Question

If the n^{th} term of a sequence is given by $a_n = n^2 - n+1$, write down its first five terms.

Answer

Given,

$a_n = n^2 - n+1$

We can find first five terms of this sequence by putting values of n from 1 to 5.

When n = 1:

$a_1 = (1)^2 - 1 + 1$

$\Rightarrow a_1 = 1 - 1 + 1$

$\Rightarrow a_1 = 1$

When n = 2:

$a_2 = (2)^2 - 2 + 1$

$\Rightarrow a_2 = 4 - 2 + 1$

$\Rightarrow a_2 = 3$

When n = 3:

$a_3 = (3)^2 - 3 + 1$

$\Rightarrow a_3 = 9 - 3 + 1$

$\Rightarrow a_3 = 7$

When n = 4:

$a_4 = (4)^2 - 4 + 1$

$\Rightarrow a_4 = 16 - 4 + 1$

$\Rightarrow a_4 = 13$

When n = 5:

$a_5 = (5)^2 - 5 + 1$

$\Rightarrow a_5 = 25 - 5 + 1$

$\Rightarrow a_5 = 21$

$\therefore$ First five terms of the sequence are 1, 3, 7, 13, 21.

2. Question

A sequence is defined by $a_n = n^3 - 6n^2 + 11n - 6, n \in N$. Show that the first three terms of the sequence are zero and all other terms are positive.

Answer

Given,

$a_n = n^3 - 6n^2 + 11n - 6, n \in N$

We can find first three terms of sequence by putting the values of n form 1 to 3.

When n = 1:

$a_1 = (1)^3 – 6(1)^2 + 11(1) – 6$

$\Rightarrow a_1 = 1 – 6 + 11 – 6$

$\Rightarrow a_1 = 12 – 12$

$\Rightarrow a_1 = 0$

When n = 2:

$a_2 = (2)^3 – 6(2)^2 + 11(2) – 6$

$\Rightarrow a_2 = 8 – 6(4) + 22 – 6$

$\Rightarrow a_2 = 8 – 24 + 22 – 6$

$\Rightarrow a_2 = 30 – 30$

$\Rightarrow a_2 = 0$

When n = 3:

$a_3 = (3)^3 – 6(3)^2 + 11(3) – 6$

$\Rightarrow a_3 = 27 – 6(9) + 33 – 6$

$\Rightarrow a_3 = 27 – 54 + 33 – 6$

$\Rightarrow a_3 = 60 – 60$

$\Rightarrow a_3 = 0$

This shows that the first three terms of the sequence is zero.

When n = n:

$a_n = n^3 – 6n^2 + 11n – 6$

$\Rightarrow a_n = n^3 – 6n^2 + 11n – 6 – n + n – 2 + 2$

$\Rightarrow a_n = n^3 – 6n^2 + 12n – 8 – n + 2$

$\Rightarrow a_n = (n)^3 – 3\times 2n(n – 2) – (2)^3 – n + 2$

$\{(a – b)^3 = (a)^3 – (b)^3 – 3ab(a – b)\}$

$\Rightarrow a_n = (n – 2)^3 – (n – 2)$

Here, n – 2 will always be positive for n > 3

$\therefore a_n$ is always positive for n > 3

3. Question

Find the first four terms of the sequence defined by $a_1 = 3$ and $a_n = 3a_{n–1} + 2$, for all n > 1.

Answer

Given,

$a_1 = 3$ and $a_n = 3a_{n–1} + 2$, for all n > 1

We can find the first four terms of a sequence by putting values of n

from 1 to 4

When n = 1:

$a_1 = 3$

When n = 2:

$a_2 = 3a_{2-1} + 2$

$\Rightarrow a_2 = 3a_1 + 2$

$\Rightarrow a_2 = 3(3) + 2$

$\Rightarrow a_2 = 9 + 2$

$\Rightarrow a_2 = 11$

When n = 3:

$a_3 = 3a_{3-1} + 2$

$\Rightarrow a_3 = 3a_2 + 2$

$\Rightarrow a_3 = 3(11) + 2$

$\Rightarrow a_3 = 33 + 2$

$\Rightarrow a_3 = 35$

When n = 4:

$a_4 = 3a_{4-1} + 2$

$\Rightarrow a_4 = 3a_3 + 2$

$\Rightarrow a_4 = 3(35) + 2$

$\Rightarrow a_4 = 105 + 2$

$\Rightarrow a_4 = 107$

∴ First four terms of sequence are 3, 11, 35, 107.

4 A. Question

Write the first five terms in each of the following sequences:

$a_1 = 1, a_n = a_{n-1} + 2, n > 1$

Answer

Given,

$a_1 = 1, a_n = a_{n-1} + 2, n > 1$

We can find the first five terms of a sequence by putting values of n

from 1 to 5

When n = 1:

$a_1 = 1$

When n = 2:

$a_2 = a_{2-1} + 2$

$\Rightarrow a_2 = a_1 + 2$

$\Rightarrow a_2 = 1 + 2$

$\Rightarrow a_2 = 3$

When n = 3:

$a_3 = a_{3-1} + 2$

$\Rightarrow a_3 = a_2 + 2$

$\Rightarrow a_3 = 3 + 2$

$\Rightarrow a_3 = 5$

When n = 4:

$a_4 = a_{4-1} + 2$

$\Rightarrow a_4 = a_3 + 2$

$\Rightarrow a_4 = 5 + 2$

$\Rightarrow a_4 = 7$

When n = 5:

$a_5 = a_{5-1} + 2$

$\Rightarrow a_5 = a_4 + 2$

$\Rightarrow a_5 = 7 + 2$

$\Rightarrow a_5 = 9$

∴ First five terms of the sequence are 1, 3, 5, 7, 9.

4 B. Question

Write the first five terms in each of the following sequences:

$a_1 = 1 = a_2$, $a_n = a_{n-1} + a_{n-2}$, n>2

Answer

Given,

$a_1 = 1 = a_2$, $a_n = a_{n-1} + a_{n-2}$, n>2

We can find the first five terms of a sequence by putting values of n

from 1 to 5

When n = 1:

$a_1 = 1$

When n = 2:

$a_1 = 1$

When n = 3:

$a_3 = a_{3-1} + a_{3-2}$

$\Rightarrow a_3 = a_2 + a_1$

$\Rightarrow a_3 = 1 + 1$

$\Rightarrow a_3 = 2$

When n = 4:

$a_4 = a_{4-1} + a_{4-2}$

⇒ $a_4 = a_3 + a_2$

⇒ $a_4 = 2 + 1$

⇒ $a_4 = 3$

When n = 5:

$a_5 = a_{5-1} + a_{5-2}$

⇒ $a_5 = a_4 + a_3$

⇒ $a_5 = 3 + 2$

⇒ $a_5 = 5$

∴ First five terms of the sequence are 1, 1, 2, 3, 5.

4 C. Question

Write the first five terms in each of the following sequences:

$a_1 = a_2 = 2$, $a_n = a_{n-1} - 1$, n>2

Answer

Given,

$a_1 = 2 = a_2$, $a_n = a_{n-1} - 1$, n>2

We can find the first five terms of a sequence by putting values of n

from 1 to 5

When n = 1:

$a_1 = 2$

When n = 2:

$a_1 = 2$

When n = 3:

$a_3 = a_{3-1} - 1$

⇒ $a_3 = a_2 - 1$

⇒ $a_3 = 2 - 1$

⇒ $a_3 = 1$

When n = 4:

$a_4 = a_{4-1} - 1$

⇒ $a_4 = a_3 - 1$

⇒ $a_4 = 1 - 1$

⇒ $a_4 = 0$

When n = 5:

$a_5 = a_{5-1} - 1$

⇒ $a_5 = a_4 - 1$

$\Rightarrow a_5 = 0 - 1$

$\Rightarrow a_5 = -1$

∴ First five terms of the sequence are 2, 2, 1, 0, -1.

5. Question

The Fibonacci sequence is defined by $a_1 = 1 = a_2$, $a_n = a_{n-1} + a_{n-2}$ for n > 2. Find $\frac{a_{n+1}}{a_n}$ for n = 1, 2, 3, 4, 5.

Answer

Given: $a_1 = 1 = a_2$, $a_n = a_{n-1} + a_{n-2}$, n>2

When n = 1:

$$\frac{a_{n+1}}{a_n} = \frac{a_{1+1}}{a_1} = \frac{a_2}{a_1} = \frac{1}{1} = 1$$

$a_3 = a_{3-1} + a_{3-2}$

$\Rightarrow a_3 = a_2 + a_1$

$\Rightarrow a_3 = 1 + 1$

$\Rightarrow a_3 = 2$

When n = 2:

$$\frac{a_{n+1}}{a_n} = \frac{a_{2+1}}{a_2} = \frac{a_3}{a_2} = \frac{2}{1} = 2$$

$a_4 = a_{4-1} + a_{4-2}$

$\Rightarrow a_4 = a_3 + a_2$

$\Rightarrow a_4 = 2 + 1$

$\Rightarrow a_4 = 3$

When n = 3:

$$\frac{a_{n+1}}{a_n} = \frac{a_{3+1}}{a_3} = \frac{a_4}{a_3} = \frac{3}{2}$$

$a_5 = a_{5-1} + a_{5-2}$

$\Rightarrow a_5 = a_4 + a_3$

$\Rightarrow a_5 = 3 + 2$

$\Rightarrow a_5 = 5$

When n = 4:

$$\frac{a_{n+1}}{a_n} = \frac{a_{4+1}}{a_5} = \frac{a_5}{a_4} = \frac{5}{3}$$

$a_6 = a_{6-1} + a_{6-2}$

$\Rightarrow a_6 = a_5 + a_4$

$\Rightarrow a_6 = 5 + 3$

$\Rightarrow a_6 = 8$

When n = 5:

$$\frac{a_{n+1}}{a_n} = \frac{a_{5+1}}{a_5} = \frac{a_6}{a_5} = \frac{8}{5}$$

$\therefore$ Value of $\frac{a_{n+1}}{a_n}$ when $n = 1, 2, 3, 4, 5$ are $1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$

6 A. Question

Show that each of the following sequences is an A.P. Also, find the common difference and write 3 more terms in each case.

3, -1, -5, -9…

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

$a_1 = 3$, $a_2 = -1$, $a_3 = -5$, $a_4 = -9$

Now, $a_2 - a_1 = -1 - 3 = -4$

$a_3 - a_2 = -5 - (-1) = -5 + 1 = -4$

$a_4 - a_3 = -9 - (-5) = -9 + 5 = -4$

As, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

The given sequence is A.P

Common difference, $d = a_2 - a_1 = -4$

To find next three more terms of A.P, firstly find a_n

We know, $a_n = a + (n-1) d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 3 + (n-1) -4$

$\Rightarrow a_n = 3 - 4n + 4$

$\Rightarrow a_n = 7 - 4n$

When n = 5:

$a_5 = 7 - 4(5)$

$\Rightarrow a_5 = 7 - 20$

$\Rightarrow a_5 = -13$

When n = 6:

$a_6 = 7 - 4(6)$

$\Rightarrow a_6 = 7 - 24$

$\Rightarrow a_6 = -17$

When n = 7:

$a_7 = 7 - 4(7)$

$\Rightarrow a_7 = 7 - 28$

$\Rightarrow a_7 = -21$

Hence, A.P is 3, -1, -5, -9, -13, -17, -21,…

6 B. Question

Show that each of the following sequences is an A.P. Also, find the common difference and write 3 more terms in each case.

$-1, \frac{1}{4}, \frac{3}{2}, \frac{11}{4}, \ldots$

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

$a_1 = -1, a_2 = \frac{1}{4}, a_3 = \frac{3}{2}, a_4 = \frac{11}{4}$

$\text{Now}, a_2 - a_1 = \frac{1}{4} - (-1) = \frac{1}{4} + 1 = \frac{5}{4}$

$a_3 - a_2 = \frac{3}{2} - \frac{1}{4} = \frac{6-1}{4} = \frac{5}{4}$

$a_4 - a_3 = \frac{11}{4} - \frac{3}{2} = \frac{11-6}{4} = \frac{5}{4}$

As, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

The given sequence is A.P

Common difference, d = $a_2 - a_1 = \frac{5}{4}$

To find next three more terms of A.P, firstly find a_n

We know, $a_n = a + (n-1)\,d$ where a is first term or a_1 and d is common difference

$\therefore a_n = -1 + (n-1)\frac{5}{4}$

$\Rightarrow a_n = -1 + \frac{5n}{4} - \frac{5}{4}$

$\Rightarrow a_n = \frac{5n}{4} + \frac{(-5-4)}{4}$

$\Rightarrow a_n = \frac{5n}{4} + \frac{(-9)}{4}$

$\Rightarrow a_n = \frac{5n}{4} - \frac{9}{4}$

When n = 5:

$a_n = \frac{5(5)}{4} - \frac{9}{4}$

$\Rightarrow a_5 = \frac{25}{4} - \frac{9}{4}$

$\Rightarrow a_5 = \frac{16}{4}$

$\Rightarrow a_5 = 4$

When n = 6:

$a_n = \frac{5(6)}{4} - \frac{9}{4}$

$$\Rightarrow a_6 = \frac{30}{4} - \frac{9}{4}$$

$$\Rightarrow a_6 = \frac{21}{4}$$

When n = 7:

$$a_n = \frac{5(7)}{4} - \frac{9}{4}$$

$$\Rightarrow a_7 = \frac{35}{4} - \frac{9}{4}$$

$$\Rightarrow a_7 = \frac{26}{4}$$

$$\Rightarrow a_7 = \frac{13}{2}$$

$$\text{Hence, A.P. is } -1, \frac{1}{4}, \frac{3}{2}, \frac{11}{4}, 4, \frac{21}{4}, \frac{13}{2}, \ldots..$$

6 C. Question

Show that each of the following sequences is an A.P. Also, find the common difference and write 3 more terms in each case.

$$\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, \ldots\ldots$$

Answer

A.P is known for Arithmetic Progression whose common difference = a_n - a_{n-1} where n > 0

$$a_1 = \sqrt{2}, a_2 = 3\sqrt{2}, a_3 = 5\sqrt{2}, a_4 = 7\sqrt{2}$$

$$\text{Now, } a_2 - a_1 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$$

$$a_3 - a_2 = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$$

$$a_4 - a_3 = 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$$

As, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

The given sequence is A.P

Common difference,d $= a_2 - a_1 = 2\sqrt{2}$

To find the next three more terms of A.P, firstly find a_n

We know, a_n = a + (n-1) d where a is a first term or a_1 and d is common difference

$$\therefore a_n = \sqrt{2} + (n-1)2\sqrt{2}$$

$$\Rightarrow a_n = \sqrt{2} + 2\sqrt{2}n - 2\sqrt{2}$$

$$\Rightarrow a_n = 2\sqrt{2}n - \sqrt{2}$$

When n = 5:

$$a_n = 2\sqrt{2}(5) - \sqrt{2}$$

$$\Rightarrow a_5 = 10\sqrt{2} - \sqrt{2}$$

$$\Rightarrow a_5 = 9\sqrt{2}$$

When n = 6:

$a_n = 2\sqrt{2}(6) - \sqrt{2}$

$\Rightarrow a_6 = 12\sqrt{2} - \sqrt{2}$

$\Rightarrow a_6 = 11\sqrt{2}$

When n = 7:

$a_n = 2\sqrt{2}(7) - \sqrt{2}$

$\Rightarrow a_7 = 14\sqrt{2} - \sqrt{2}$

$\Rightarrow a_7 = 13\sqrt{2}$

Hence, A.P. is $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, 11\sqrt{2}, 13\sqrt{2} \ldots$

6 D. Question

Show that each of the following sequences is an A.P. Also, find the common difference and write 3 more terms in each case.

9, 7, 5, 3 …

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

$a_1 = 9$, $a_2 = 7$, $a_3 = 5$, $a_4 = 3$

Now, $a_2 - a_1 = 7 - 9 = -2$

$a_3 - a_2 = 5 - 7 = -2$

$a_4 - a_3 = 3 - 5 = -2$

As, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

The given sequence is A.P

Common difference, $d = a_2 - a_1 = -2$

To find the next three more terms of A.P, firstly find a_n

We know, $a_n = a + (n-1) d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 9 + (n-1) -2$

$\Rightarrow a_n = 9 - 2n + 2$

$\Rightarrow a_n = 11 - 2n$

When n = 5:

$a_5 = 11 - 2(5)$

$\Rightarrow a_5 = 11 - 10$

$\Rightarrow a_5 = 1$

When n = 6:

$a_6 = 11 - 2(6)$

$\Rightarrow a_6 = 11 - 12$

$\Rightarrow a_6 = -1$

When n = 7:

$a_7 = 11 – 2(7)$

$\Rightarrow a_7 = 11 – 14$

$\Rightarrow a_7 = -3$

Hence, A.P is 9, 7, 5, 3, 1, -1, -3,....

7. Question

The n^{th} term of a sequence is given by $a_n = 2n + 7$. Show that it is an A.P. Also, find its 7^{th} term.

Answer

Given,

$a_n = 2n + 7$

We can find first five terms of this sequence by putting values of n from 1 to 5.

When n = 1:

$a_1 = 2(1) + 7$

$\Rightarrow a_1 = 2 + 7$

$\Rightarrow a_1 = 9$

When n = 2:

$a_2 = 2(2) + 7$

$\Rightarrow a_2 = 4 + 7$

$\Rightarrow a_2 = 11$

When n = 3:

$a_3 = 2(3) + 7$

$\Rightarrow a_3 = 6 + 7$

$\Rightarrow a_3 = 13$

When n = 4:

$a_4 = 2(4) + 7$

$\Rightarrow a_4 = 8 + 7$

$\Rightarrow a_4 = 15$

When n = 5:

$a_5 = 2(5) + 7$

$\Rightarrow a_5 = 10 + 7$

$\Rightarrow a_5 = 17$

∴ First five terms of the sequence are 9, 11, 13, 15, 17.

A.P is known for Arithmetic Progression whose common difference = $a_n – a_{n-1}$ where n > 0

$a_1 = 9$, $a_2 = 11$, $a_3 = 13$, $a_4 = 15$, $a_5 = 17$

Now, $a_2 – a_1 = 11 – 9 = 2$

$a_3 - a_2 = 13 - 11 = 2$

$a_4 - a_3 = 15 - 13 = 2$

$a_5 - a_4 = 17 - 15 = 2$

As, $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = a_5 - a_4$

The given sequence is A.P

Common difference, $d = a_2 - a_1 = 2$

To find the seventh term of A.P, firstly find a_n

We know, $a_n = a + (n-1)\ d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 3 + (n-1)\ 2$

$\Rightarrow a_n = 3 + 2n - 2$

$\Rightarrow a_n = 2n + 1$

When n = 7:

$a_7 = 2(7) + 1$

$\Rightarrow a_7 = 14 + 1$

$\Rightarrow a_7 = 15$

Hence, the 7^{th} term of A.P. is 15

8. Question

The n^{th} term of a sequence is given by $a_n = 2n^2 + n + 1$. Show that it is not an A.P.

Answer

Given,

$a_n = 2n^2 + n + 1$

We can find first three terms of this sequence by putting values of n from 1 to 3.

When n = 1:

$a_1 = 2(1)^2 + 1 + 1$

$\Rightarrow a_1 = 2(1) + 2$

$\Rightarrow a_1 = 2 + 2$

$\Rightarrow a_1 = 4$

When n = 2:

$a_2 = 2(2)^2 + 2 + 1$

$\Rightarrow a_2 = 2(4) + 3$

$\Rightarrow a_2 = 8 + 3$

$\Rightarrow a_2 = 11$

When n = 3:

$a_3 = 2(3)^2 + 3 + 1$

$\Rightarrow a_3 = 2(9) + 4$

$\Rightarrow a_3 = 18 + 4$

$\Rightarrow a_3 = 22$

$\therefore$ First three terms of the sequence are 4, 11, 22.

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

$a_1 = 4$, $a_2 = 11$, $a_3 = 22$

Now, $a_2 - a_1 = 11 - 4 = 7$

$a_3 - a_2 = 22 - 11 = 11$

As $a_2 - a_1$ is not equal to $a_3 - a_2$

The given sequence is not an A.P.

Exercise 19.2

1 A. Question

Find:

10^{th} term of the A.P. 1, 4, 7,10,.....

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

$a = a_1 = 1$, $a_2 = 4$

Common difference, $d = a_2 - a_1 = 4 - 1 = 3$

To find the tenth term of A.P, firstly find a_n

We know, $a_n = a + (n-1)\,d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 1 + (n-1)\,3$

$\Rightarrow a_n = 1 + 3n - 3$

$\Rightarrow a_n = 3n - 2$

When n = 10:

$a_{10} = 3(10) - 2$

$\Rightarrow a_{10} = 30 - 2$

$\Rightarrow a_{10} = 28$

Hence, 10^{th} term of A.P. is 28

1 B. Question

Find:

18^{th} term of the A.P. $\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}$........

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

$a = a_1 = \sqrt{2},\ a_2 = 3\sqrt{2}$

Common difference, $d = a_2 - a_1 = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$

To find 18^{th} term of A.P, firstly find a_n

We know, a_n = a + (n-1) d where a is a first term or a_1 and d is common difference

$\therefore a_n = \sqrt{2} + (n-1)\ 2\sqrt{2}$

$\Rightarrow a_n = \sqrt{2} + 2\sqrt{2}n - 2\sqrt{2}$

$\Rightarrow a_n = 2\sqrt{2}n - \sqrt{2}$

When n = 18:

$a_{18} = 2\sqrt{2}(18) - \sqrt{2}$

$\Rightarrow a_{18} = 36\sqrt{2} - \sqrt{2}$

$\Rightarrow a_{18} = 35\sqrt{2}$

Hence, 18th term of A.P. is 34√2

1 C. Question

Find:

nth term of the A.P. 13, 8, 3, -2,……

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n - a_{n-1}$ where n > 0

a = a_1 = 13, a_2 = 8

Common difference, d = $a_2 - a_1$ = 8 – 13 = -5

We know, a_n = a + (n-1) d where a is first term or a_1 and d is common difference

∴ a_n = 13 + (n-1) -5

⇒ a_n = 13 – 5n + 5

⇒ a_n = 18 – 5n

Hence, n^{th} term of A.P. is 18 – 5n

2. Question

In an A.P., show that $a_{m+n} + a_{m-n} = 2a_m$.

Answer

Let common difference of an A.P is d and first term is a

We know, a_n = a + (n – 1)d where a is first term or a_1 and d is common difference

Now, take L.H.S.:

$a_{m+n} + a_{m-n}$ = a + (m + n – 1)d + a + (m - n – 1)d

⇒ $a_{m+n} + a_{m-n}$ = a + md + nd – d + a + md - nd – d

⇒ $a_{m+n} + a_{m-n}$ = 2a + 2md – 2d

⇒ $a_{m+n} + a_{m-n}$ = 2(a + md – d)

⇒ $a_{m+n} + a_{m-n}$ = 2[a + d(m – 1)]

{∵ a_n = a + (n – 1)d}

$\Rightarrow a_{m+n} + a_{m-n} = 2a_m$

Hence Proved

3 A. Question

Which term of the A.P. 3, 8, 13,... is 248 ?

Answer

Given A.P is 3, 8, 13,...

Here, $a_1 = a = 3$, $a_2 = 8$

Common difference, $d = a_2 - a_1 = 8 - 3 = 5$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 3 + (n - 1)5$

$\Rightarrow a_n = 3 + 5n - 5$

$\Rightarrow a_n = 5n - 2$

Now, to find which term of A.P is 248

Put $a_n = 248$

$\therefore 5n - 2 = 248$

$\Rightarrow 5n = 248 + 2$

$\Rightarrow 5n = 250$

$\Rightarrow n = \frac{250}{5}$

$\Rightarrow n = 50$

Hence, 50^{th} term of given A.P is 248

3 B. Question

Which term of the A.P. 84, 80, 76,... is 0 ?

Answer

Given A.P is 84, 80, 76,...

Here, $a_1 = a = 84$, $a_2 = 88$

Common difference, $d = a_2 - a_1 = 80 - 84 = -4$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 84 + (n - 1)-4$

$\Rightarrow a_n = 84 - 4n + 4$

$\Rightarrow a_n = 88 - 4n$

Now, to find which term of A.P is 0

Put $a_n = 0$

$\therefore 88 - 4n = 0$

$\Rightarrow -4n = -88$

$\Rightarrow n = \frac{-88}{-4}$

$\Rightarrow$ n = 22

Hence, 22^{th} term of given A.P is 0

3 C. Question

Which term of the A.P. 4, 9, 14,... is 254 ?

Answer

Given A.P is 4, 9, 14,...

Here, $a_1 = a = 4$, $a_2 = 9$

Common difference, $d = a_2 - a_1 = 9 - 4 = 5$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference

$\therefore a_n = 4 + (n - 1)5$

$\Rightarrow a_n = 4 + 5n - 5$

$\Rightarrow a_n = 5n - 1$

Now, to find which term of A.P is 254

Put $a_n = 254$

$\therefore 5n - 1 = 254$

$\Rightarrow 5n = 254 + 1$

$\Rightarrow 5n = 255$

$\Rightarrow n = \frac{255}{5}$

$\Rightarrow$ n = 51

Hence, 51^{st} term of given A.P is 254

4 A. Question

Is 68 a term of the A.P. 7, 10, 13,...?

Answer

Given A.P is 7, 10, 13,...

Here, $a_1 = a = 7$, $a_2 = 10$

Common difference, $d = a_2 - a_1 = 10 - 7 = 3$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$\therefore a_n = 7 + (n - 1)3$

$\Rightarrow a_n = 7 + 3n - 3$

$\Rightarrow a_n = 3n + 4$

Now, to find whether 68 is a term of this A.P. or not

Put $a_n = 68$

$\therefore 3n + 4 = 68$

$\Rightarrow 3n = 68 - 4$

$\Rightarrow 3n = 64$

$\Rightarrow n = \frac{64}{3}$

$\frac{64}{3}$ is not a natural number

Hence, 68 is not a term of given A.P.

4 B. Question

Is 302 a term of the A.P. 3, 8, 13,...?

Answer

Given A.P is 3, 8, 13,...

Here, $a_1 = a = 3$, $a_2 = 8$

Common difference, $d = a_2 - a_1 = 8 - 3 = 5$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$\therefore a_n = 3 + (n - 1)5$

$\Rightarrow a_n = 3 + 5n - 5$

$\Rightarrow a_n = 5n - 2$

To find: whether 302 is a term of this A.P. or not

Put $a_n = 302$

$\therefore 5n - 2 = 302$

$\Rightarrow 5n = 302 + 2$

$\Rightarrow 5n = 304$

$\Rightarrow n = \frac{304}{5}$

$\frac{304}{5}$ is not a natural number

Hence,304 is not a term of given A.P.

5 A. Question

Which term of the sequence $24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}...$ is the first negative term?

Answer

$\text{Given, A.P. is } 24, 23\frac{1}{4}, 22\frac{1}{2}, 21\frac{3}{4}, ... = 24, \frac{93}{4}, \frac{45}{2}, \frac{87}{4}, ...$

$\text{Here, } a_1 = a = 24, a_2 = \frac{93}{4}$

$\text{Common difference, } d = a_2 - a_1 = \frac{93}{4} - 24 = \frac{93-96}{4} = -\frac{3}{4}$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$\therefore a_n = 24 + (n - 1)\left(-\frac{3}{4}\right)$

$\Rightarrow a_n = 24 - \frac{3}{4}n + \frac{3}{4}$

$\Rightarrow a_n = \frac{96+3}{4} - \frac{3}{4}n$

$\Rightarrow a_n = \frac{99}{4} - \frac{3}{4}n$

Now, to find first negative term

Put $a_n < 0$

$\therefore a_n = \frac{99}{4} - \frac{3}{4}n < 0$

$\Rightarrow \frac{99}{4} < \frac{3}{4}n$

$\Rightarrow 3n > 99$

$\Rightarrow n > \frac{99}{3}$

$\Rightarrow n > 33$

Hence, 34^{th} term is first negative term of given A.P

5 B. Question

Which term of the sequence 12 + 8i, 11 + 6i, 10 + 4i, … is (a) purely real (b) purely imaginary ?

Answer

Given A.P is 12 + 8i, 11 + 6i, 10 + 4i, …

Here, $a_1 = a = 12 + 8i$, $a_2 = 11 + 6i$

Common difference, $d = a_2 - a_1$

$= 11 + 6i - (12 + 8i) = 11 - 12 + 6i - 8i = -1 - 2i$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$\therefore a_n = 12 + 8i + (n - 1) -1 - 2i$

$\Rightarrow a_n = 12 + 8i - n - 2ni + 1 + 2i$

$\Rightarrow a_n = 13 + 10i - n - 2ni$

$\Rightarrow a_n = (13 - n) + (10 - 2n)i$

To find purely real term of this A.P., imaginary part have to be zero

$\therefore 10 - 2n = 0$

$\Rightarrow 2n = 10$

$\Rightarrow n = \frac{10}{2}$

$\Rightarrow n = 5$

Hence, 5^{th} term is purely real

To find purely imaginary term of this A.P., real part have to be zero

$\therefore 13 - n = 0$

$\Rightarrow n = 13$

Hence, 13th term is purely imaginary

6 A. Question

How many terms are in A.P. 7, 10, 13,...43?

Answer

Given A.P is 7, 10, 13,...

Here, $a_1 = a = 7$, $a_2 = 10$

Common difference, $d = a_2 - a_1 = 10 - 7 = 3$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$\therefore a_n = 7 + (n - 1)3$

$\Rightarrow a_n = 7 + 3n - 3$

$\Rightarrow a_n = 3n + 4$

To find total terms of the A.P., put $a_n = 43$ as 43 is last term of A.P.

$\therefore 3n + 4 = 43$

$\Rightarrow 3n = 43 - 4$

$\Rightarrow 3n = 39$

$$\Rightarrow n = \frac{39}{3}$$

$\Rightarrow n = 13$

Hence, there are total 13 terms in the given A.P.

6 B. Question

How many terms are there in the A.P. $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}$?

Answer

Given, A.P. is $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots.$

Here, $a_1 = a = -1, a_2 = -\frac{5}{6}$

$$\text{Common difference, } d = a_2 - a_1 = -\frac{5}{6} - (-1) = -\frac{5}{6} + 1 = \frac{-5+6}{6} = \frac{1}{6}$$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$$\therefore a_n = -1 + (n-1)\frac{1}{6}$$

$$\Rightarrow a_n = -1 + \frac{1}{6}n - \frac{1}{6}$$

$$\Rightarrow a_n = \frac{-6-1}{6} + \frac{1}{6}n$$

$\Rightarrow a_n = \frac{-7}{6} + \frac{1}{6}n$

To find total terms of the A.P., put $a_n = \frac{10}{3}$ as $\frac{10}{3}$ is last term of A.P.

$\therefore a_n = \frac{-7}{6} + \frac{1}{6}n = \frac{10}{3}$

$\Rightarrow \frac{1}{6}n = \frac{10}{3} + \frac{7}{6}$

$\Rightarrow \frac{1}{6}n = \frac{20+7}{6}$

$\Rightarrow \frac{1}{6}n = \frac{27}{6}$

⇒ n = 27

Hence, there are total 27 terms in the given A.P.

7. Question

The first term of an A.P. is 5, the common difference is 3, and the last term is 80; find the number of terms.

Answer

Given,

a = 5, last term, l = a_n = 80

Common difference, d = 3

We know, $a_n = a + (n – 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

$\therefore a_n = 5 + (n – 1)3$

$\Rightarrow a_n = 5 + 3n – 3$

$\Rightarrow a_n = 3n + 2$

To find total terms of the A.P., put a_n = 80 as 80 is last term of A.P.

∴ 3n + 2 = 80

⇒ 3n = 80 – 2

⇒ 3n = 78

$\Rightarrow n = \frac{78}{3}$

⇒ n = 26

Hence, there are total 26 terms in the given A.P.

8. Question

The 6^{th} and 17^{th} terms of an A.P. are 19 and 41 respectively. Find the 40^{th} term.

Answer

Given,

6^{th} term of an A.P is 19 and 17^{th} terms of an A.P. is 41

$\Rightarrow a_6 = 19$ and $a_{17} = 41$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = 6:

$\therefore a_6 = a + (6 - 1)d$

$\Rightarrow a_6 = a + 5d$

Similarly, When n = 17:

$\therefore a_{17} = a + (17 - 1)d$

$\Rightarrow a_{17} = a + 16d$

According to question:

$a_6 = 19$ and $a_{17} = 41$

$\Rightarrow a + 5d = 19$ ....(i)

And $a + 16d = 41$..............(ii)

Subtracting equation (i) from (ii):

$a + 16d - (a + 5d) = 41 - 19$

$\Rightarrow a + 16d - a - 5d = 22$

$\Rightarrow 11d = 22$

$\Rightarrow d = \frac{22}{11}$

$\Rightarrow d = 2$

Put the value of d in equation (i):

$a + 5(2) = 19$

$\Rightarrow a + 10 = 19$

$\Rightarrow a = 19 - 10$

$\Rightarrow a = 9$

As, $a_n = a + (n - 1)d$

$a_{40} = a + (40 - 1)d$

$\Rightarrow a_{40} = a + 39d$

Now put the value of a = 9 and d = 2 in a_{40}

$\Rightarrow a_{40} = 9 + 39(2)$

$\Rightarrow a_{40} = 9 + 78$

$\Rightarrow a_{40} = 87$

Hence, 40^{th} term of the given A.P. is 87

9. Question

If 9^{th} term of an A.P. is Zero, prove that its 29^{th} term is double the 19^{th} term.

Answer

Given,

9^{th} term of an A.P is 0

$\Rightarrow a_9 = 0$

To prove: $a_{29} = 2a_{19}$

We know, $a_n = a + (n – 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = 9:

$\therefore a_9 = a + (9 – 1)d$

$\Rightarrow a_9 = a + 8d$

According to question:

$a_9 = 0$

$\Rightarrow a + 8d = 0$

$\Rightarrow a = -8d$

When n = 19:

$\therefore a_{19} = a + (19 – 1)d$

$\Rightarrow a_{19} = a + 18d$

$\Rightarrow a_{19} = -8d + 18d$

$\Rightarrow a_{19} = 10d$

When n = 29:

$\therefore a_{29} = a + (29 – 1)d$

$\Rightarrow a_{29} = a + 28d$

$\{\because a = -8d\}$

$\Rightarrow a_{29} = -8d + 28d$

$\Rightarrow a_{29} = 20d$

$\Rightarrow a_{29} = 2\times10d$

$\{\because a_{19} = 10d\}$

$\Rightarrow a_{29} = 2a_{19}$

Hence Proved

10. Question

If 10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term, show that the 25^{th} term of the A.P. is Zero.

Answer

Given,

10 times the 10^{th} term of an A.P. is equal to 15 times the 15^{th} term

$\Rightarrow 10a_{10} = 15a_{15}$

To prove: $a_{25} = 0$

We know, $a_n = a + (n – 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = 10:

$\therefore a_{10} = a + (10 - 1)d$

$\Rightarrow a_{10} = a + 9d$

When n = 15:

$\therefore a_{15} = a + (15 - 1)d$

$\Rightarrow a_{15} = a + 14d$

When n = 25:

$\therefore a_{25} = a + (25 - 1)d$

$\Rightarrow a_{25} = a + 24d$ ....(i)

According to question:

$10a_{10} = 15a_{15}$

$\Rightarrow 10(a + 9d) = 15(a + 14d)$

$\Rightarrow 10a + 90d = 15a + 210d$

$\Rightarrow 10a - 15a + 90d - 210d = 0$

$\Rightarrow -5a - 120d = 0$

$\Rightarrow -5(a + 24d) = 0$

$\Rightarrow a + 24d = 0$

$\Rightarrow a_{25} = 0$ (From (i))

Hence Proved

11. Question

The 10^{th} and 18^{th} term of an A.P. are 41 and 73 respectively, find 26^{th} term.

Answer

Given: 10^{th} term of an A.P is 41, and 18^{th} terms of an A.P. is 73

$\Rightarrow a_{10} = 41$ and $a_{18} = 73$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is the common difference and n is any natural number

When n = 10:

$\therefore a_{10} = a + (10 - 1)d$

$\Rightarrow a_{10} = a + 9d$

Similarly, When n = 18:

$\therefore a_{18} = a + (18 - 1)d$

$\Rightarrow a_{18} = a + 17d$

According to question:

$a_{10} = 41$ and $a_{18} = 73$

$\Rightarrow a + 9d = 41$ ....(i)

And $a + 17d = 73$.............(ii)

Subtracting equation (i) from (ii):

$a + 17d - (a + 9d) = 73 - 41$

$\Rightarrow a + 17d - a - 9d = 32$

$\Rightarrow 8d = 32$

$\Rightarrow d = \frac{32}{8}$

$\Rightarrow d = 4$

Put the value of d in equation (i):

$a + 9(4) = 41$

$\Rightarrow a + 36 = 41$

$\Rightarrow a = 41 - 36$

$\Rightarrow a = 5$

As, $a_n = a + (n - 1)d$

$a_{26} = a + (26 - 1)d$

$\Rightarrow a_{26} = a + 25d$

Now put the value of a = 5 and d = 4 in a_{26}

$\Rightarrow a_{26} = 5 + 25(4)$

$\Rightarrow a_{26} = 5 + 100$

$\Rightarrow a_{26} = 105$

Hence, 26^{th} term of the given A.P. is 105

12. Question

In a certain A.P. the 24^{th} term is twice the 10^{th} term. Prove that the 72^{nd} term is twice the 34^{th} term.

Answer

Given: 24^{th} term is twice the 10^{th} term

$\Rightarrow a_{24} = 2a_{10}$

To prove: $a_{72} = 2a_{34}$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = 10:

$\therefore a_{10} = a + (10 - 1)d$

$\Rightarrow a_{10} = a + 9d$

When n = 24:

$\therefore a_{24} = a + (24 - 1)d$

$\Rightarrow a_{24} = a + 23d$

When n = 34:

$\therefore a_{34} = a + (34 - 1)d$

$\Rightarrow a_{34} = a + 33d$ ....(i)

When n = 72:

$\therefore a_{72} = a + (72 - 1)d$

$\Rightarrow a_{72} = a + 71d$

According to question:

$a_{24} = 2a_{10}$

$\Rightarrow a + 23d = 2(a + 9d)$

$\Rightarrow a + 23d = 2a + 18d$

$\Rightarrow a - 2a + 23d - 18d = 0$

$\Rightarrow -a + 5d = 0$

$\Rightarrow a = 5d$

Now, $a_{72} = a + 71d$

$\Rightarrow a_{72} = 5d + 71d$

$\Rightarrow a_{72} = 76d$

$\Rightarrow a_{72} = 10d + 66d$

$\Rightarrow a_{72} = 2(5d + 33d)$

$\{\because a = 5d\}$

$\Rightarrow a_{72} = 2(a + 33d)$

$\Rightarrow a_{72} = 2a_{34}$ (From (i))

Hence Proved

13. Question

If $(m + 1)^{th}$ term of an A.P. is twice the $(n + 1)^{th}$ term, prove that $(3m + 1)^{th}$ term is twice the $(m + n + 1)^{th}$ term.

Answer

Given: $(m + 1)^{th}$ term of an A.P. is twice the $(n + 1)^{th}$ term

$\Rightarrow a_{m+1} = 2a_{n+1}$

To prove: $a_{3m+1} = 2a_{m+n+1}$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = m + 1:

$\therefore a_{m+1} = a + (m + 1 - 1)d$

$\Rightarrow a_{m+1} = a + md$

When n = n + 1:

$\therefore a_{n+1} = a + (n + 1 - 1)d$

$\Rightarrow a_{n+1} = a + nd$

According to question:

$a_{m+1} = 2a_{n+1}$

$\Rightarrow a + md = 2(a + nd)$

$\Rightarrow a + md = 2a + 2nd$

$\Rightarrow a - 2a + md - 2nd = 0$

$\Rightarrow -a + (m - 2n)d = 0$

$\Rightarrow a = (m - 2n)d$.........(i)

$a_n = a + (n - 1)d$

When $n = m + n + 1$:

$\therefore a_{m+n+1} = a + (m + n + 1 - 1)d$

$\Rightarrow a_{m+n+1} = a + md + nd$

$\Rightarrow a_{m+n+1} = (m - 2n)d + md + nd$......... (From (i))

$\Rightarrow a_{m+n+1} = md - 2nd + md + nd$

$\Rightarrow a_{m+n+1} = 2md - nd$.........(ii)

When $n = 3m + 1$:

$\therefore a_{3m+1} = a + (3m + 1 - 1)d$

$\Rightarrow a_{3m+1} = a + 3md$

$\Rightarrow a_{3m+1} = (m - 2n)d + 3md$......... (From (i))

$\Rightarrow a_{3m+1} = md - 2nd + 3md$

$\Rightarrow a_{3m+1} = 4md - 2nd$

$\Rightarrow a_{3m+1} = 2(2md - nd)$

$\Rightarrow a_{3m+1} = 2a_{m+n+1}$............(From (ii))

Hence Proved

14. Question

If the n^{th} term of the A.P. 9, 7, 5,... is same as the nth term of the A.P. 15, 12, 9 ... Find n.

Answer

Given: n^{th} term of the A.P. 9, 7, 5,... is same as the nth term of the A.P. 15, 12, 9 ...

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

Let A.P. 9, 7, 5,... has first term a_1 and common difference d_1

$\Rightarrow a_1 = 9$ and $a_2 = 7$

Common difference, $d_1 = a_2 - a_1 = 7 - 9 = -2$

Now, $a_n = a_1 + (n - 1)d_1$

$\Rightarrow a_n = 9 + (n - 1)(-2)$

$\Rightarrow a_n = 9 - 2n + 2$

$\Rightarrow a_n = 11 - 2n$

Let A.P. 15, 12, 9 ... has first term a_1 and common difference d_1

$\Rightarrow b_1 = 15$ and $b_2 = 12$

Common difference, $d_2 = b_2 - b_1 = 12 - 15 = -3$

Now, $b_n = b_1 + (n – 1)d_2$

$\Rightarrow b_n = 15 + (n – 1)(-3)$

$\Rightarrow b_n = 15 – 3n + 3$

$\Rightarrow b_n = 12 – 3n$

According to question:

$a_n = b_n$

$\Rightarrow 11 – 2n = 12 – 3n$

$\Rightarrow 3n – 2n = 12 – 11$

$\Rightarrow n = 1$

Hence, the value of n is 1

15 A. Question

Find the 12th term from the end of the following arithmetic progression:

3, 5, 7, 9, ... 201

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n – a_{n-1}$ where n > 0

$a = a_1 = 3, a_2 = 5, l = 201$

Common difference, $d = a_2 – a_1 = 5 – 3 = 2$

We know, nth term from end, $b_n = l – (n – 1)d$ where l is last term or a_1 and d is common difference and n is any natural number

$\therefore b_{12} = 201 – (12 – 1)2$

$\Rightarrow b_{12} = 201 – 24 + 2$

$\Rightarrow b_{12} = 203 – 24$

$\Rightarrow b_{12} = 179$

Hence, 12thterm from end for the given A.P is 179

15 B. Question

Find the 12th term from the end of the following arithmetic progression:

3, 8, 13, ... 253

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n – a_{n-1}$ where n > 0

$a = a_1 = 3, a_2 = 8, l = 253$

Common difference, $d = a_2 – a_1 = 8 – 3 = 5$

We know, nth term from end, $b_n = l – (n – 1)d$ where l is last term or a_1 and d is common difference and n is any natural number

$\therefore b_{12} = 253 – (12 – 1)5$

$\Rightarrow b_{12} = 253 – 60 + 5$

$\Rightarrow b_{12} = 258 – 60$

⇒ $b_{12} = 198$

Hence, 12^{th}term from end for the given A.P is 198

15 C. Question

Find the 12^{th} term from the end of the following arithmetic progression:

1, 4, 7, 10, … 88

Answer

A.P is known for Arithmetic Progression whose common difference = $a_n – a_{n-1}$ where n > 0

$a = a_1 = 1, a_2 = 4, l = 88$

Common difference, $d = a_2 – a_1 = 4 – 1 = 3$

We know, n^{th} term from end, $b_n = l – (n – 1)d$ where l is last term or a_1 and d is common difference and n is any natural number

∴ $b_{12} = 88 – (12 – 1)3$

⇒ $b_{12} = 88 – 36 + 3$

⇒ $b_{12} = 91 – 36$

⇒ $b_{12} = 55$

Hence, 12^{th}term from end for the given A.P is 55

16. Question

The 4^{th} term of an A.P. is three times the first and the 7^{th} term exceeds twice the third term by 1. Find the first term and the common difference.

Answer

Given,

4^{th} term of an A.P. is three times the first and the 7^{th} term exceeds twice the third term by 1

⇒ $a_4 = 3a$ and $a_7 = 2a_3 + 1$

We know, $a_n = a + (n – 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = 4:

∴ $a_4 = a + (4 – 1)d$

⇒ $a_4 = a + 3d$

When n = 7:

∴ $a_7 = a + (7 – 1)d$

⇒ $a_7 = a + 6d$

When n = 3:

∴ $a_3 = a + (3 – 1)d$

⇒ $a_3 = a + 2d$

According to question:

$a_7 = 2a_3 + 1$

⇒ $a + 6d = 2(a + 2d) + 1$

⇒ a + 6d = 2a + 4d + 1

⇒ a – 2a + 6d – 4d = 1

⇒ -a + 2d = 1

⇒ a – 2d = -1……(i)

a_4 = 3a

⇒ a + 3d = 3a

⇒ 3d = 3a – a

⇒ 3d = 2a

$\Rightarrow d = \frac{2a}{3} \ldots\ldots\ldots (ii)$

Put this value of d in equation (i):

$a - 2\left(\frac{2a}{3}\right) = -1$

$\Rightarrow a - \frac{4a}{3} = -1$

$\Rightarrow \frac{3a - 4a}{3} = -1$

$\Rightarrow \frac{-a}{3} = -1$

⇒ a = 3

Now, put this value of a in equation (ii):

$d = \frac{2a}{3}$

$\Rightarrow d = \frac{2}{3}(3)$

⇒ d = 2

Hence, the value of a and d are 3 and 2 respectively

17. Question

Find the second term and nth term of an A.P. whose 6^{th} term is 12 and 8^{th} term is 22.

Answer

Given: 6^{th}term of an A.P is 12 and 8^{th} terms of an A.P. is 22

⇒ a_6 = 12 and a_8 = 22

We know, a_n = a + (n – 1)d where a is first term or a_1 and d is common difference and n is any natural number

When n = 6:

∴ a_6 = a + (6 – 1)d

⇒ a_6 = a + 5d

Similarly, When n = 8:

∴ a_8 = a + (8 – 1)d

$\Rightarrow a_8 = a + 7d$

According to question:

$a_6 = 12$ and $a_8 = 22$

$\Rightarrow a + 5d = 12$ ....(i)

And $a + 7d = 22$..............(ii)

Subtracting equation (i) from (ii):

$a + 7d - (a + 5d) = 22 - 12$

$\Rightarrow a + 7d - a - 5d = 10$

$\Rightarrow 2d = 10$

$\Rightarrow d = \frac{10}{2}$

$\Rightarrow d = 5$

Put the value of d in equation (i):

$a + 5(5) = 12$

$\Rightarrow a + 25 = 12$

$\Rightarrow a = 12 - 25$

$\Rightarrow a = -13$

As, $a_n = a + (n - 1)d$

$a_2 = a + (2 - 1)d$

$\Rightarrow a_2 = a + d$

Now put the value of a = 9 and d = 2 in a_n and a_2

$\Rightarrow a_n = a + (n - 1)d$

$\Rightarrow a_n = -13 + (n - 1)5$

$\Rightarrow a_n = -13 + 5n - 5$

$\Rightarrow a_n = -18 + 5n$

$a_2 = a + d$

$\Rightarrow a_2 = -13 + 5$

$\Rightarrow a_2 = -8$

Hence, 2^{th}term and n^{th} of the given A.P. are -8 and 5n – 18 respectively

18. Question

How many numbers of two digit are divisible by 3?

Answer

For finding total two-digit numbers which are divisible by 3, firstly we will make an A.P. of those two-digit numbers which are divisible by 3.

First two digit number which is divisible by 3 is 12

$\therefore a_1 = a = 12$

Next two digit number which is divisible by 3 is 15

$\therefore a_2 = 15$

Largest two digit number which is divisible by 3 is 99

$\therefore a_n = 99$

⇒ A.P. is 12, 15,………,99

We know, $a_n = a + (n - 1)d$ where a is first term or a and d is common difference and n is any natural number

⇒ $a_1 = 12$, $a_2 = 15$ and $a_n = 99$

Common difference, $d_1 = a_2 - a_1 = 15 - 12 = 3$

Now, $a_n = a_1 + (n - 1)d$

⇒ $a_n = 12 + (n - 1)3$

⇒ $99 = 12 + 3n - 3$

⇒ $99 = 3n + 9$

⇒ $99 - 9 = 3n$

⇒ $90 = 3n$

⇒ $n = 30$

Hence, there are total 30 two-digit numbers which are divisible by 3

19. Question

An A.P. consists of 60 terms. If the first and the last terms be 7 and 125 respectively, find 32^{nd} term.

Answer

Given: $a = 7$, $n = 60$ and $l = a_{60} = 125$

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is common difference and n is any natural number

When n = 60:

$\therefore a_{60} = a + (60 - 1)d$

⇒ $a_{60} = a + 59d$

When n = 32:

$\therefore a_{32} = a + (32 - 1)d$

⇒ $a_{32} = a + 31d$ …………(i)

According to question:

$a_{60} = 125$

⇒ $a + 59d = 125$

⇒ $7 + 59d = 125$

⇒ $59d = 125 - 7$

⇒ $59d = 118$

$$\Rightarrow d = \frac{118}{59}$$

⇒ $d = 2$

Now put the value of a = 7 and d = 2 in a_{32}

⇒ a_{32} = a + 31d.........(From (i))

⇒ a_{32} = 7 + 31(2)

⇒ a_{32} = 7 + 62

⇒ a_{32} = 69

Hence, 32^{th}term of the given A.P. is 69

20. Question

The sum of the 4^{th} and the 8^{th} terms of an A.P. is 24, and the sum of the 6^{th} and 10^{th} term is 34. Find the first term and the common difference of the A.P.

Answer

Given: the sum of the 4^{th} and the 8^{th} terms of an A.P. is 24, and the sum of the 6^{th} and 10^{th} term is 34

⇒ $a_4 + a_8 = 24$ and $a_6 + a_{10} = 34$

We know, $a_n = a + (n – 1)d$ where a is first term or a_1 and d is the common difference and n is any natural number

When n = 4:

∴ $a_4 = a + (4 – 1)d$

⇒ $a_4 = a + 3d$

When n = 6:

∴ $a_6 = a + (6 – 1)d$

⇒ $a_6 = a + 5d$

When n = 8:

∴ $a_8 = a + (8 – 1)d$

⇒ $a_8 = a + 7d$

When n = 10:

∴ $a_{10} = a + (10 – 1)d$

⇒ $a_{10} = a + 9d$

According to question:

$a_4 + a_8 = 24$

⇒ a + 3d + a + 7d = 24

⇒ 2a + 10d = 24

⇒ 2(a + 5d) = 24

⇒ $a + 5d = \frac{24}{2}$

⇒ a + 5d = 12.........(i)

$a_6 + a_{10} = 34$

⇒ a + 5d + a + 9d = 34

⇒ 2a + 14d = 34

⇒ $2(a + 7d) = 34$

$$\Rightarrow a + 7d = \frac{34}{2}$$

⇒ $a + 7d = 17$.........(ii)

Subtracting equation (i) from (ii):

$a + 7d - (a + 5d) = 17 - 12$

⇒ $a + 7d - a - 5d = 5$

⇒ $2d = 5$

$$\Rightarrow d = \frac{5}{2}$$

Put the value of d in equation (i):

$$a + 5\left(\frac{5}{2}\right) = 12$$

$$\Rightarrow a + \frac{25}{2} = 12$$

$$\Rightarrow a = 12 - \frac{25}{2}$$

$$\Rightarrow a = \frac{24 - 25}{2}$$

$$\Rightarrow a = \frac{-1}{2}$$

Hence, the value of a and d are $-\frac{1}{2}$ and $\frac{5}{2}$ respectively

21. Question

How many numbers are there between 1 and 1000 which when divided by 7 leave remainder 4?

Answer

For finding total numbers between 1 and 1000 which when divided by 7 leave remainder 4, firstly we will make an A.P. of those numbers which when divided by 7 leave remainder 4.

First number which when divided by 7 leave remainder 4 is 4

∴ $a_1 = a = 4$

Next number which when divided by 7 leave remainder 4 is 11

∴ $a_2 = 11$

Largest three-digit number which when divided by 7 leave remainder 4 is 998

∴ $a_n = 998$

⇒ A.P. is 4, 11,.........,998

We know, $a_n = a + (n - 1)d$ where a is first term or a and d is common difference and n is any natural number

⇒ $a_1 = 4$, $a_2 = 11$ and $a_n = 998$

Common difference, $d_1 = a_2 - a_1 = 11 - 4 = 7$

Now, $a_n = a_1 + (n - 1)d$

$\Rightarrow a_n = 4 + (n - 1)7$

$\Rightarrow 998 = 4 + 7n - 7$

$\Rightarrow 998 = 7n - 3$

$\Rightarrow 998 + 3 = 7n$

$\Rightarrow 1001 = 7n$

$\Rightarrow n = 143$

Hence, there are total 143 numbers between 1 and 1000 which when divided by 7 leave remainder 4.

22. Question

The first and the last term of an A.P. are a and l respectively. Show that the sum of the n^{th} term from the beginning and the nth term from the end is a + l.

Answer

To prove: sum of the n^{th} term from the beginning and the nth term from the end is a + l

We know, $a_n = a + (n - 1)d$ where a is first term or a_1 and d is the common difference, and n is any natural number, and n^{th} term from the end is $a_n' = l - (n - 1)d$

Now,

$a_n + a_n' = a + (n - 1)d + l - (n - 1)d$

$\Rightarrow a_n + a_n' = a + nd - d + l - nd + d$

$\Rightarrow a_n + a_n' = a + l$

Hence Proved

23. Question

If an A.P. is such that $\frac{a_4}{a_7} = \frac{2}{3}$, find $\frac{a_6}{a_8}$.

Answer

Given:

$$\frac{a_4}{a_7} = \frac{2}{3}$$

To find: $\frac{a_6}{a_8}$

We know, $a_n = a + (n - 1)d$ where a is a first term or a_1 and d is the common difference and n is any natural number

When n = 4:

$\therefore a_4 = a + (4 - 1)d$

$\Rightarrow a_4 = a + 3d$

When n = 6:

$\therefore a_6 = a + (6 - 1)d$

$\Rightarrow a_6 = a + 5d$

When n = 7:

$\therefore a_7 = a + (7 - 1)d$

$\Rightarrow a_7 = a + 6d$

When n = 8:

$\therefore a_8 = a + (8 - 1)d$

$\Rightarrow a_8 = a + 7d$

According to the question:

$$\frac{a_4}{a_7} = \frac{2}{3}$$

$$\Rightarrow \frac{a + 3d}{a + 6d} = \frac{2}{3}$$

$\Rightarrow 3(a + 3d) = 2(a + 6d)$

$\Rightarrow 3a + 9d = 2a + 12d$

$\Rightarrow 3a - 2a = 12d - 9d$

$\Rightarrow a = 3d$

Now,

$$\frac{a_6}{a_8} = \frac{a + 5d}{a + 7d}$$

$$\Rightarrow \frac{a_6}{a_8} = \frac{3d + 5d}{3d + 7d}$$

$$\Rightarrow \frac{a_6}{a_8} = \frac{8d}{10d}$$

$$\Rightarrow \frac{a_6}{a_8} = \frac{4}{5}$$

Hence, the value of $\frac{a_6}{a_8}$ is $\frac{4}{5}$

24. Question

If $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ are in AP, whose common difference is d, show that

$$\text{Sec}\ \theta_1.\text{Sec}\ \theta_2 + \text{Sec}\ \theta_2.\text{Sec}\ \theta_3 + \ldots + \text{Sec}\ \theta_{n-1}\text{Sec}\ \theta_n = \frac{\tan\theta_n - \tan\theta_1}{\text{Sin}\ d}$$

Answer

Given: $\theta_1, \theta_2, \theta_3, \ldots, \theta_n$ is A.P

$\therefore \theta_2 - \theta_1 = \theta_3 - \theta_2 = \theta_4 - \theta_3 = \ldots\ldots\ldots\ldots = \theta_n - \theta_{n-1} = d$ (Common Difference)

Now,

$$\sec\theta_1 . \sec\theta_2 = \frac{1}{\cos\theta_1 \cos\theta_2}$$

Multiplying and dividing by sin d:

$$\Rightarrow \sec\theta_1 . \sec\theta_2 = \frac{\sin d}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

$$\Rightarrow \sec\theta_1 . \sec\theta_2 = \frac{\sin(\theta_2 - \theta_1)}{\sin d\,(\cos\theta_1 \cos\theta_2)}$$

{$\because$ sin(A - B) = sin A cos B - cos A sin B}

$$\Rightarrow \sec\theta_1 . \sec\theta_2 = \frac{\sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1}{\sin d\,(\cos\theta_1\cos\theta_2)}$$

$$\Rightarrow \sec\theta_1 . \sec\theta_2 = \frac{1}{\sin d}\left\{\frac{\sin\theta_2\cos\theta_1}{\cos\theta_1\cos\theta_2} - \frac{\cos\theta_2\sin\theta_1}{\cos\theta_1\cos\theta_2}\right\}$$

$$\Rightarrow \sec\theta_1 . \sec\theta_2 = \frac{1}{\sin d}\left\{\frac{\sin\theta_2}{\cos\theta_2} - \frac{\sin\theta_1}{\cos\theta_1}\right\}$$

$$\Rightarrow \sec\theta_1 . \sec\theta_2 = \frac{1}{\sin d}\{\tan\theta_2 - \tan\theta_1\} \dots\dots (i)$$

Similarly,

$$\Rightarrow \sec\theta_2 . \sec\theta_3 = \frac{1}{\sin d}\{\tan\theta_3 - \tan\theta_2\} \dots \dots . (ii)$$

$$\Rightarrow \sec\theta_{n-1} . \sec\theta_n = \frac{1}{\sin d}\{\tan\theta_n - \tan\theta_{n-1}\} \dots \dots . (iii)$$

Take L.H.S.:

$\sec\theta_1 . \sec\theta_2 + \sec\theta_2 . \sec\theta_3 + \cdots + \sec\theta_{n-1} . \sec\theta_n$

Putting the value of (i), (ii) and (iii):

$$= \frac{1}{\sin d}\{\tan\theta_2 - \tan\theta_1\} + \frac{1}{\sin d}\{\tan\theta_3 - \tan\theta_2\} + .. + \frac{1}{\sin d}\{\tan\theta_n - \tan\theta_{n-1}\}$$

$$= \frac{1}{\sin d}\{\tan\theta_2 - \tan\theta_1 + \tan\theta_3 - \tan\theta_2 + \cdots + \tan\theta_n - \tan\theta_{n-1}\}$$

$$= \frac{1}{\sin d}\{\tan\theta_n - \tan\theta_1\}$$

= R.H.S.

Hence Proved

Exercise 19.3

1. Question

The Sum of the three terms of an A.P. is 21 and the product of the first, and the third terms exceed the second term by 6, find three terms

Answer

Given: the sum of first three terms is 21

To find: the first three terms of AP

Assume the first three terms are a - d, a, a + d where a is the first term and d is the common difference

So, sum of first three terms is a - d + a + a + d = 21

3a = 21

a = 7

it is also given that product of first and third term exceeds the second by 6

so (a - d)(a + d) - a = 6

a^2 - d^2 - a = 6

substituting a = 7

7^2 - d^2 - 7 = 6

$d^2 = 36$

d = 6 or d = - 6

Hence the terms of AP are a - d, a, a + d which is 1, 7, 13 or 13, 7, 1

2. Question

Three numbers are in A.P. If the sum of these numbers be 27 and the product 648, find the numbers

Answer

Given: sum of first three terms is 27

To find: the first three terms of AP

Assume the first three terms are a - d, a, a + d where a is the first term and d is the common difference

So, sum of first three terms is a - d + a + a + d = 27

3a = 27

a = 9

given that the product of three terms is 648

so $a^3 - ad^2 = 648$

substituting a = 9

$9^3 - 9d^2 = 648$

$729 - 9d^2 = 648$

$81 = 9d^2$

d = 3 or d = - 3

hence the given terms are a - d, a, a + d which is 6, 9, 12 or 12, 9, 6

3. Question

Find the four numbers in A.P., whose sum is 50 and in which the greatest number is 4 times the least.

Answer

Given: Sum of four terms is 50

To find: first four terms of AP

Assume these four terms are a - 2d, a - d, a + d, a + 2d

Sum of these terms is 4a = 50

$$a = \frac{25}{2}$$

It is also given that the greatest number is 4 time the least

a + 2d = 4(a - 2d)

3a = 10d

$$d = \frac{3}{10}a$$

Substituting $a = \frac{25}{2}$

$$d = \frac{15}{4}$$

Hence the terms of AP are a - d, a, a + d which is $5, \frac{35}{4}, \frac{65}{4}, 20$

4. Question

The sum of three numbers in A.P. is 12, and the sum of their cubes is 288. Find the numbers.

Answer

assume the numbers in AP are a - d, a, a + d

Given that the sum of three numbers is 12

To find: the first three terms of AP

So,

3a = 12

a = 4

It is also given that the sum of their cube is 288

$(a - d)^3 + a^3 + (a + d)^3 = 288$

$a^3 - d^3 - 3ad(a - d) + a^3 + a^3 + d^3 + 3ad(a + d) = 288$

substituting a = 4 we get

$64 - d^3 - 12d(4 - d) + 64 + 64 + d^3 + 12d(4 + d) = 288$

$192 + 24d^2 = 288$

d = 2 or d = - 2

hence the numbers are a - d, a, a + d which is 2, 4, 6 or 6, 4, 2

5. Question

If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.

Answer

Given: sum of first three terms is 24

To find: the first three terms of AP

Assume the first three terms are a - d, a, a + d where a is the first term and d is the common difference

So, sum of first three terms is a - d + a + a + d = 24

3a = 24

a = 8

given that the product of three terms is 440

so $a^3 - ad^2 = 440$

substituting a = 8

$8^3 - 8d^2 = 440$

$512 - 8d^2 = 440$

$72 = 8d^2$

d = 3 or d = - 3

hence the given terms are a - d, a, a + d which is 5, 8, 11 or 11, 8, 5

6. Question

The angles of a quadrilateral are in A.P. whose common difference is 10. Find the angles

Answer

assume the angles are a - 2d, a - d, a + d, a + 2d

We know that the sum of all angles in a quadrilateral is 360

Given: d = 10

To find: angles of the quadrilateral

So, a - 2d + a - d + a + d + a + 2d = 360

4a = 360

a = 90

hence the angles are a - 2d, a - d, a + d, a + 2d which is 70, 80, 100, 110

Exercise 19.4

1 A. Question

Find the sum of the following arithmetic progressions:

50, 46, 42, ... To 10 terms

Answer

for the **given** AP the first term a is 50, and common difference d is a difference of second term and first term, which is 46 - 50 = - 4

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{10}{2}\left(100 + (9)(-4)\right)$$

s = 5 × 64

s = 320

1 B. Question

Find the sum of the following arithmetic progressions:

1, 3, 5, 7, ... to 12 terms

Answer

for the **given** AP the first term a is 1, and common difference d is a difference of the second term and first term, which is 3 - 1 = 2

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{12}{2}\left(2 + (11)(2)\right)$$

s = 6 × 24

s = 144

1 C. Question

Find the sum of the following arithmetic progressions:

3, $\frac{9}{2}$, 6, $\frac{15}{2}$, ... to 25 terms

Answer

for the **given** AP the first term a is 3, and common difference d is a difference of the second term and first term, which is $\frac{9}{2} - 3 = \frac{3}{2}$

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{25}{2}\left(6 + (24)\left(\frac{3}{2}\right)\right)$$

s = 25 × 21

s = 525

1 D. Question

Find the sum of the following arithmetic progressions:

41, 36, 31, ... to 12 terms

Answer

for the **given** AP the first term a is 41, and common difference d is a difference of the second term and first term, which is 36 - 41 = - 5

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{12}{2}(82 + (11)(-5))$$

s = 6 × 27

s = 162

1 E. Question

Find the sum of the following arithmetic progressions:

a + b, a – b, a – 3b, ... to 22 terms

Answer

for the **given** AP the first term a is a + b, and common difference d is a difference of the second term and the first term, which is a - b - (a + b) = - 2b

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{22}{2}\big(2a + 2b + (21)(-2b)\big)$$

s = 11(2a - 40b)

1 F. Question

Find the sum of the following arithmetic progressions:

$(x-y)^2$, (x^2+y^2), $(x+y)^2$, ... to n terms

Answer

for the **given** AP the first term a is $(x - y)^2$ and common difference d is a difference of the second term and first term, which is $(x^2 + y^2) - (x-y)^2 = 2xy$

To find: the sum of given AP

Formula: for the sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{n}{2}(2(x-y)^2 + (n-1)(2xy)) = \frac{n}{2}\times 2\left[\left(x^2+y^2-2xy\right)+xyn-xy\right]$$

$$= n\left[x^2+y^2-3xy+xyn\right]$$

1 G. Question

Find the sum of the following arithmetic progressions:

$\frac{x-y}{x+y}, \frac{3x-2y}{x+y}, \frac{5x-3y}{x+y}$, ... to n terms

Answer

for the **given** AP the first term a is $(x - y)^2$ and common difference d is a difference of the second term and first term, which is $\frac{3x-2y}{x+y} - \frac{x-y}{x+y} = \frac{2x-y}{x+y}$

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{n}{2}\left(2\frac{x-y}{x+y} + (n-1)\left(\frac{2x-y}{x-y}\right)\right)$$

2 A. Question

Find the sum of the following series:

2 + 5 + 8 + ... + 182

Answer

for the **given** AP the first term a is 2, and common difference d is a difference of the second term and first term, which is 5 - 2 = 3

To find: the sum of given AP

The formula for the sum of AP is given by

$$s = \frac{n}{2}(2a + (n - 1)d)$$

Substituting the values in the above formula

$$s = \frac{n}{2}\left(4 + (n - 1)(3)\right)$$

$$n = \frac{182 - 2}{3} + 1 = 61$$

$$s = \frac{61}{2}\left(4 + (60)(3)\right)$$

s = 92 × 61

s = 5612

2 B. Question

Find the sum of the following series:

101 + 99 + 97 + … + 47

Answer

for the **given** AP the first term a is 101, and common difference d is a difference of second term and first term, which is 99 - 101 = - 2

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n - 1)d)$$

Substituting the values in the above formula

$$s = \frac{n}{2}\left(202 + (n - 1)(-2)\right)$$

$$n = \frac{47 - 101}{-2} + 1 = 28$$

$$s = \frac{28}{2}\left(202 + (27)(-2)\right)$$

s = 14 × 148

s = 2072

2 C. Question

Find the sum of the following series:

$(a - b)^2 + (a^2 + b^2) + (a + b)^2 + s.... + [(a + b)^2 + 6ab]$

Answer

for the **given** AP the first term a is $(a - b)^2$ and common difference d is a difference of second term and first term, which is $(a^2 + b^2) - (a - b)^2 = 2ab$

To find: the sum of given AP

The formula for sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{n}{2}\left(2(a-b)^2 + (n-1)(2ab)\right)$$

$$n = \frac{(a + b)^2 + 6ab - (a-b)^2}{2ab}$$

n = 5

$$s = \frac{5}{2}\left(2(a-b)^2 + 4(2ab)\right)$$

3. Question

Find the sum of first n natural numbers.

Answer

for the **given** AP the first term a is 1, and common difference d is a difference of the second term and first term, which is 2 - 1 = 1

To find: the sum of given AP

The formula for the sum of AP is given by

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above formula

$$s = \frac{n}{2}\left(2 + (n-1)(1)\right)$$

$$s = \frac{n(n + 1)}{2}$$

4. Question

Find the sum of all - natural numbers between 1 and 100, which are divisible by 2 or 5

Answer

According to the **given** question we have to find the sum of natural numbers between 1 and 100 which are divisible by 2 or 5, so the sequence is 2, 4, 6...98 which are multiples of 2 and another sequence 5, 15, 25....95

To find: the sum of all - natural numbers between 1 and 100, which are divisible by 2 or 5

For the first sequence, it is an AP with first term a as 2 and common difference d as 2

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

$$s_1 = \frac{49}{2}(4 + 48 \times 2)$$

$$s_1 = 49 \times 50$$

$$s_1 = 2450$$

For the second sequence, it is an AP with first term a as 5 and common difference d as 10

Hence the sum is given by the formula $s = \frac{n}{2}(2a + (n-1)d)$

$$s_2 = \frac{10}{2}(10 + 9 \times 10)$$

$s_2 = 500$

Hence total sum is $s_1 + s_2 = 2450 + 500 = 2950$

5. Question

Find the sum of first n odd natural numbers.

Answer

given an AP of first n odd natural numbers whose first term a is 1, and common difference d is 3

Given sequence is 1, 3, 5, 7……n

To find: the sum of first n natural numbers

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Substituting the values in the above equation we get

$$s = \frac{n}{2}(2 + (n-1)2)$$

$s = n^2$

6. Question

Find the sum of all odd numbers between 100 and 200

Answer

the **given** sequence is 101, 103, ….199

Given an AP whose first term a is 101, and common difference d is 2

To find: the sum of all odd numbers between 100 and 200

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Substituting the values in the above equation we get

$$s = \frac{n}{2}(202 + (n-1)2)$$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{199 - 101}{2} + 1 = 50$$

Substituting n is the sum formula we get

$$s = \frac{50}{2}(202 + (50-1)2)$$

$s = 25 \times 300$

$s = 7500$

7. Question

Show that the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667

Answer

according to **given** conditions the sequence is 3, 9, 15, 21….999

Given an AP whose first term is 3 and d is 6. Hence sum is given by

To prove: the sum of all odd integers between 1 and 1000 which are divisible by 3 is 83667

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{999 - 3}{6} + 1$$

n = 167

Substituting n is the sum formula we get

$$s = \frac{167}{2}(6 + (166) \times 6)$$

s = 83667

Hence, Proved.

8. Question

Find the sum of all integers between 84 and 719, which are multiples of 5

Answer

given an AP is required of all integers between 84 and 719, which are multiples of 5

To find: the sum of all integers between 84 and 719, which are multiples of 5

So, the sequence is 85, 90, 95….715

It is an AP whose first term is 85 and d is 5

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{715 - 85}{5} + 1$$

n = 127

Substituting n is the sum formula we get

$$s = \frac{127}{2}(2 \times 85 + (126) \times 5)$$

s = 50800

9. Question

Find the sum of all integers between 50 and 500 which are divisible by 7

Answer

given an AP is required of all integers between 50 and 500, which are multiples of 7

To find: the sum of all integers between 50 and 500 which are divisible by 7

So, the sequence is 56, 63, 70….497

It is an AP whose first term is 56 and d is 7

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{497 - 56}{7} + 1$$

n = 64

Substituting n is the sum formula we get

$$s = \frac{64}{2}(2 \times 56 + (63) \times 7)$$

s = 17696

10. Question

Find the sum of all even integers between 101 and 999

Answer

given an AP is required of all integers between 101 and 999 which are even

To find: the sum of all even integers between 101 and 999

So, the sequence is 102, 104, 106….998

It is an AP whose first term is 102 and d is 2

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{998 - 102}{2} + 1$$

n = 449

Substituting n is the sum formula we get

$$s = \frac{449}{2}(2 \times 102 + (448) \times 2)$$

s = 246950

11. Question

Find the sum of all integers between 100 and 550, which are divisible by 9

Answer

given an AP is required of all integers between 100 and 550, which are multiples of 9

To find: the sum of all integers between 100 and 550, which are divisible by 9

So, the sequence is 108, 117, 126….549

It is an AP whose first term is 108 and d is 9

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{549 - 108}{9} + 1$$

n = 50

substituting n in the sum formula we get

$$s = \frac{50}{2}(2 \times 108 + (49) \times 9)$$

s = 16425

12. Question

Find the sum of the series: 3 + 5 + 7 + 6 + 9 + 12 + 9 + 13 + 17 + ... to 3n terms

Answer

given an AP is whose first term is 3 and d is 2

So, the sum is given by **formula** $s = \frac{n}{2}(2a + (n-1)d)$

To find: sum of the given AP

Substituting the values in the **formula**

$$s = \frac{3n}{2}(2 \times 3 + (3n-1)2)$$

$$s = \frac{3n}{2}(6n + 4)$$

s = 3n(3n + 2)

13. Question

Find the sum of all those integers between 100 and 800 each of which on division by 16 leaves the remainder 7

Answer

given an AP is required of all integers between 100 and 800, which on division by 16 leaves remainder 7

To find: the sum of all those integers between 100 and 800 each of which on division by 16 leaves the remainder 7

So, the sequence is 103, 119, 135....791

It is an AP whose first term is 103 and d is 16

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n-1)d)$

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{791 - 103}{16} + 1$$

n = 44

substituting n in the sum formula, we get

$$s = \frac{44}{2}(2 \times 103 + (43) \times 16)$$

s = 19668

14 A. Question

Solve:

25 + 22 + 19 + 16 + ... + x = 115

Answer

given is an AP whose first term a is 25 and d is - 3(22 - 25) with a number of terms equal to..

To find: the value of x for the given AP

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{x - 25}{-3} + 1$$

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n - 1)d)$

Now substituting the values in the above **formula**

$$s = \frac{n}{2}(50 + (n - 1)(-3))$$

It is **given** that s = 115, so equating the above expression to 115

$$\frac{n}{2}(50 + (n - 1)(-3)) = 115$$

Solving we get n is 10 or $\frac{23}{3}$

Since n cannot be a fraction, so we choose n as 10

$$10 = \frac{x - 25}{-3} + 1$$

x = - 2

14 B. Question

Solve:

1 + 4 + 7 + 10 + ... + x = 590

Answer

given is an AP whose first term is 1 and d is 3(4 - 1) with a number of terms equal to

To find: the value of x for the given AP

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{x - 1}{3} + 1$$

Hence the sum is given by the **formula** $s = \frac{n}{2}(2a + (n - 1)d)$

Now substituting the values in the above formula

$$s = \frac{n}{2}(2 + (n - 1)(3))$$

It is **given** that s = 590, so equating the above expression to 590

$$\frac{n}{2}(2 + (n-1)(3)) = 590$$

Solving we get n is 20 or $-\frac{59}{3}$

Since n cannot be a fraction, so we choose n as 20

$$20 = \frac{x-1}{3} + 1$$

x = 58

15. Question

Find the r^{th}term of an A.P., the sum of whose first n terms is $3n^2 + 2n$

Answer

It is **given** that the sum of n terms is $3n^2 + 2n$

To find: rth term of an AP, the sum of whose first term is $3n^2 + 2n$

So, the sum of n - 1 terms is $3(n - 1)^2 + 2(n - 1)$

formula $T_N = S_N - S_{N-1}$

So, $T_r = 3r^2 + 2r - [3(r - 1)^2 + 2(r - 1)]$

$T_r = 3r^2 + 2r - [3(r^2 + 1 - 2r) + 2(r - 1)]$

$T_r = - 1 + 6r$

16. Question

How many terms are there in the A.P. whose first and fifth terms are - 14 and 2 respectively and the sum of the terms is 40?

Answer

given the first term is - 14, from which we can say that a = - 14

To find: the number of terms in an AP whose first and fifth terms is - 14 and 2 respectively, and the sum of the terms is 40

Given fifth term is 2, so a + 4d = 2

- 14 + 4d = 2

d = 4

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

now substituting the values in the above equation

$$\frac{n}{2}(2a + (n-1)d) = 40$$

$$\frac{n}{2}(-28 + (n-1)4) = 40$$

$4n^2 - 32n - 80 = 0$

$n^2 - 8n - 20 = 0$

solving we get n as 10 or - 2

Since the number of terms cannot be negative hence n is 10

17. Question

The sum of the first 7 terms of an A.P. is 10, and that of the next 7 terms is 17. Find the progression

Answer

Assuming the first term as a and common difference as d

To find: the progression

So, the sum of first 7 terms is **given** by

a + a + d + a + 2d + a + 3d.... a + 6d = 10

7a + 21d = 10...(i)

In the second part it is **given** that sum of next seven terms is 17

a + 7d + a + 8d + a + 9d.... a + 13d = 7

7a + 70d = 7...(ii)

Solving (i) and (ii) we get

10 - 21d = 7 - 70d

3 = - 49d

$$d = -\frac{3}{49}$$

$$a = \frac{79}{49}$$

Hence the sequence is given by a, a + d, a + 2d.... which is

$$\frac{79}{49}, \frac{76}{49}, \frac{73}{49}, \ldots..$$

18. Question

The third term of an A.P. is 7, and the seventh term exceeds 3 times the third term by 2. Find the first term, the common difference and the sum of the first 20 terms.

Answer

given third term of AP is 7

So, a + 2d = 7...(i)

It is also **given** that the seventh term exceeds 3 times the third term by 2

To find: first term, the common difference and the sum of the first 20 terms

a + 6d - 3(a + 2d) = 2

- 2a = 2

a = - 1

d = 4

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in above equation

$$s = \frac{20}{2}(-2 + (19)4)$$

$s = 740$

19. Question

The first term of an A.P. is 2, and the last term is 50. The sum of all these terms is 442. Find the common difference.

Answer

given $a_1 = 2$ and $a_n = 50$

To find: the common difference

we know that the sum of AP is given by the **formula:**

$s = \frac{n}{2}(a_1 + a_n)$

substituting the values in above equation we get

$442 = \frac{n}{2}(2 + 50)$

$n = 17$

Now for the finding number of terms, the **formula** is

$n = \frac{\text{last term} - \text{first term}}{d} + 1$

we know that $n = \frac{50-2}{d} + 1$

$d = 3$

20. Question

The number of terms of an A.P. is even; the sum of the odd term is 24, of the even terms is 30, and the last term exceeds the first by $20\frac{1}{2}$, find the number of terms and the series

Answer

let the number of terms be 2n with first term a and common difference d

To find: number of terms and the series

The last term is **given** as a + (2n - 1) d

It is given that the last term exceeds the first by $20\frac{1}{2}$

So $a + (2n-1)d - a = 10\frac{1}{2}$

$(2n-1)d = 10\frac{1}{2}$...(i)

Sum of odd terms is 24

$\frac{n}{2}(2a + (n-1)2d) = 24$...(ii)

Sum of even terms is 30

$\frac{n}{2}(2(a + d) + (n-1)2d) = 30$...(iii)

Solving (i), (ii) and (iii) we get

$d = 1.5$

substituting d in (i) we get $n = 4$

and substituting d and n in (ii) we get a = 1.5

so, the number of terms is 2n = 8, and the sequence is given by a, a + d, a + 2d….

1.5, 3, 4.5, 6, 7.5 , 9, 10.5 , 12

21. Question

If $S_n = n^2p$ and $S_m = m^2p, m \neq n,$ in an A.P., prove that $S_p = p^3$

Answer

given an AP whose sum of n terms is n^2p and same AP with m terms whose sum is m^2p

To prove: $S_p = p^3$

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in the above equation, we get

$\frac{n}{2}(2a + (n-1)d) = n^2p$.... (i)

Similarly, for series with m terms

$\frac{m}{2}(2a + (m-1)d) = m^2p$...(ii)

Subtracting (ii) from (i) we get

d = 2p

substituting d in (i) we get

a = p

Now using the sum **formula** for AP consisting of p terms we get

$$S_p = \frac{p}{2}(2a + (n-1)d)$$

Substituting the values in the above equation

$$S_p = \frac{p}{2}(2p + (p-1)2p)$$

$S_P = p^3$

22. Question

If the 12th term of an A.P. is - 13 and the sum of the first 4 terms is 24, what is the sum of the first 10 terms?

Answer

assuming an AP whose first term is a and the common difference is d

To find: the sum of first 10 terms

Given 12^{th} term is - 13

So, a + 11d = - 13...(i)

Given the sum of first four terms is 24

So 4a + 6d = 24...(ii)

Solving (i) and (ii) we get

a = 9 and d = - 2

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in the above equation

$$s = \frac{10}{2}(18 + (10-1)(-2))$$

s = 0

23. Question

If the 5th and 12th terms of an A.P. are 30 and 65 respectively, what is the sum of the first 20 terms?

Answer

assuming an AP whose first term is a and the common difference is d

To find: the sum of the first 20 terms

Given 5^{th} term is 30

So, a + 4d = 30...(i)

Given 12^{th} term is 65

So, a + 11d = 65...(ii)

Solving (i) and (ii) we get

a = 10 and d = 5

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in the above equation

$$s = \frac{20}{2}(20 + (20-1)5)$$

s = 1150

24. Question

Find the sum of n terms of the A.P. whose kth terms is 5k + 1

Answer

given an AP whose kth term is 5k + 1

To find: the sum of n terms of an AP whose kth term is 5k + 1

So substituting k = 1 to get the first term $a_1 = 6$

Substituting k = 2 to get the second term $a_2 = 11$

$d = a_2 - a_1 = 5$

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in the above equation

$$s = \frac{n}{2}(12 + (n-1)5)$$

$$s = \frac{5n^2 + 7n}{2}$$

25. Question

Find the sum of all two digit numbers which when divided by 4, yields 1 as the remainder.

Answer

the series which satisfies the above condition is

13, 17, 21....97

To find: the sum of all two - digit numbers which when divided by 4, yields 1 as the remainder

So, it is an AP whose first term is 13 and common difference d as 4

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

And $n = \frac{97-13}{4} + 1$

n = 22

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in the above equation

$$s = \frac{22}{2}(26 + (22-1)4)$$

s = 1210

26. Question

If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116. Find the last term

Answer

given an AP whose first term is 25 and the common difference is - 3

To find: the last term of a given AP

we know that the sum of AP is given by the **formula**

$$s = \frac{n}{2}(2a + (n-1)d)$$

$$116 = \frac{n}{2}(50 + (n-1)(-3))$$

Solving we get

$3n^2 - 53n + 232 = 0$

n = 8 or $n = \frac{29}{3}$

n cannot be a fraction hence we choose n as 8

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$8 = \frac{\text{last term} - 25}{-3} + 1$$

solving we get the last term as 4

27. Question

Find the sum of odd integers from 1 to 2001.

Answer

given is an AP whose first term is 1 and d is 2

To find: the sum of odd integers from 1 to 2001

Now for the finding number of terms, the **formula** is

$$n = \frac{\text{last term} - \text{first term}}{d} + 1$$

$$n = \frac{2001 - 1}{2} + 1$$

n = 1001

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n - 1)d)$$

substituting the values in the above equation

$$s = \frac{1001}{2}(2 + (1001 - 1)2)$$

s = 1002001

28. Question

How many terms of the A.P. - 6, - $\frac{11}{2}$, - 5, ..., are needed to give the sum - 25?

Answer

given is an AP whose first term is - 6 and d is $\frac{1}{2}$

To find: the number of terms required to make the sum of a given AP to - 25

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n - 1)d)$$

substituting the values in the above equation

$$-25 = \frac{n}{2}\left(-12 + \frac{(n - 1)}{2}\right)$$

$$-25 = \frac{n}{4}(-24 + n - 1)$$

$-100 = - 25n + n^2$

$n^2 - 25n + 100 = 0$

(n - 20) (n - 5) = 0

solving we get n = 20 or n = 5

29. Question

In an A.P. the first term is 2, and the sum of the first 5 terms is one-fourth of the next 5 terms. Show that 20th term is - 112

Answer

given an AP whose first term is 2

To find: 20^{th} term of an AP

It is **given** that the sum of the first five terms is one-fourth of the next 5 terms

So $5a + 10d = \frac{1}{4}(5a + 35d)$

20a + 40d = 5a + 35d

15a + 5d = 0

Substituting a as 2

We get d = - 6

Hence 20^{th} term is a + 19d = - 112

30. Question

If S_1 be the sum of (2n + 1) terms of an A.P. and S_2 sum of its odd terms, then prove that: S_1: S_2 = (2n + 1) : (n + 1).

Answer

To prove: S_1: S_2 = (2n + 1) : (n + 1)

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above equation

$$s_1 = \frac{2n + 1}{2}(2a + 2nd)$$

For the sum of odd terms, it is given by

$s_2 = a_1 + a_3 + a_5 + ... a_{2n+1}$

s_2 = a + a + 2d + a + 4d + ... + a + 2nd

s_2 = (n + 1)a + n(n + 1)d

s_2 = (n + 1) (a + nd)

Hence $s_1{:}s_2 = \frac{2n+1}{n+1}$

31. Question

Find an A.P. in which the sum of any number of terms is always three times the squared number of these terms

Answer

To find: AP with given conditions

Given that sum of n terms is $3n^2$

$S_n = 3n^2$

Similarly, sum of n - 1 terms is $3(n - 1)^2$

$S_{n-1} = 3(n - 1)^2$

formula $T_N = S_n - S_{n-1} = 3n^2 - 3(n - 1)^2$

Now substituting n = 1 to get the first term

$a_1 = 3$

Now substituting n = 2 to get the second term

$a_2 = 9$

$d = a_2 - a_1 = 6$

hence the series is given by a, a + d, a + 2d...which is 3, 9, 15, 21....

32. Question

If the sum of n terms of an A.P. is $nP + \frac{1}{2}n(n - 1)Q$ where P and Q are constants, find the common difference.

Answer

it is **given** that sum of n terms of AP is $nP + \frac{1}{2}n(n-1)Q$

To find: the common difference

Substituting n = 1 gives the sum of the first term, that is the first term itself

$a_1 = P$

Substituting n = 2 gives the sum of first two terms

$a_1 + a_2 = 2P + Q$

$a_2 = P + Q$

Now **common difference** $d = a_2 - a_1 = Q$

33. Question

The sums of n terms of two arithmetic progressions are in the ratio 5n + 4: 9n + 6. Find the ratio of their 18th terms

Answer

assuming first AP whose first term is a and common difference d

To find: the ratio of 18^{th} term of a given AP

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

Substituting the values in the above equation

$$s_1 = \frac{n}{2}(2a + (n-1)d)$$

assuming second AP whose first term is A and common difference D

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2A + (n-1)D)$$

substituting the values in the above equation

$$s_2 = \frac{n}{2}(2A + (n-1)D)$$

$$\frac{s_1}{s_2} = \frac{2a + (n-1)d}{2A + (n-1)D}$$

$$\frac{5n+4}{9n+6} = \frac{a + \frac{(n-1)d}{2}}{A + \frac{(n-1)D}{2}} \text{.... (i)}$$

Given the ratio of the 18th term is required of both AP's which is

$$\frac{a + 17d}{A + 17D}$$

Substituting n = 35 in (i) we get

$$\frac{a + 17d}{A + 17D} = \frac{179}{321}$$

34. Question

The sum of first n terms of two A.P.'s is in the ratio (7n + 2) : (n + 4). Find the ratio of their 5th terms.

Answer

assuming first AP whose first term is a and common difference d

To find: the ratio of 5th term of a given AP

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2a + (n-1)d)$$

substituting the values in the above equation

$$s_1 = \frac{n}{2}(2a + (n-1)d)$$

Assuming second AP whose first term is A and common difference D

we know that the sum of AP is given by the **formula:**

$$s = \frac{n}{2}(2A + (n-1)D)$$

substituting the values in the above equation

$$s_2 = \frac{n}{2}(2A + (n-1)D)$$

$$\frac{s_1}{s_2} = \frac{2a + (n-1)d}{2A + (n-1)D}$$

$$\frac{7n+2}{n+4} = \frac{a + \frac{(n-1)d}{2}}{A + \frac{(n-1)D}{2}} \text{.... (i)}$$

Given the ratio of the 5th term is required of both AP's which is

$$\frac{a + 4d}{A + 4D}$$

Substituting n = 9 in (i) we get

$$\frac{a + 4d}{A + 4D} = \frac{65}{13} = 5$$

Exercise 19.5

1 A. Question

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., prove that:

$\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

Answer

We know, if a, b, c are in AP,

Then b - a = c - b

It is said that, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP,

Hence, $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

If $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP, then

$$\frac{c\ +\ a}{b} - \frac{b\ +\ c}{a} = \frac{a\ +\ b}{c} - \frac{c\ +\ a}{b}$$

Taking LCM,

$$\frac{ca\ +\ a^2 - b^2 - cb}{ab} = \frac{ab\ +\ b^2 - c^2 - ac}{bc}$$

Now, LHS = $\frac{ca + a^2 - b^2 - cb}{ab}$

Multiply c in both numerator and denominator,

We get, $\frac{ca + a^2 - b^2 - cb}{ab} = \frac{c^2a + ca^2 - cb^2 - c^2b}{abc} = \frac{C(b-a)(a+b+c)}{abc}$

RHS $= \frac{ab + b^2 - c^2 - ac}{bc}$

Multiply a in both numerator and denominator,

We get, $\frac{ab + b^2 - c^2 - ac}{bc} = \frac{a^2b + ab^2 - ac^2 - a^2c}{abc} = \frac{a(b-c)(a+b+c)}{abc}$

Therefore, LHS = RHS

$\frac{C(b-a)(a+b+c)}{abc} = \frac{a(b-c)(a+b+c)}{abc}$

c(b - a) = a(b-c)

Also, $\frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$

c(b - a) = a(b - c)

Hence given terms are in AP

1 B. Question

If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., prove that:

a(b + c), b(c + a), c(a + b) are in A.P.

Answer

It is said that, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, Hence, $\frac{1}{b}-\frac{1}{a} = \frac{1}{c}-\frac{1}{b}$

Taking LCM,

$\frac{(b-a)}{ab} = \frac{(b-c)}{bc}$.

Multiply in both denominator and numerator with c in LHS and a in RHS

$$\frac{c(b-a)}{abc} = \frac{a(b-c)}{abc} = c(b-a) = a(b-c)$$

Since,

a(b + c), b(c + a), c(a + b) are to be proved in A.P.

b(c + a)-a(b + c) = c(a + b)-b(c + a)

bc + ba – ab - ca- = ca + ab – bc - ba

cb - ca = ca - ab

c(b - a) = a(c - b)

Hence, given terms are in AP.

2. Question

If a^2, b^2, c^2 are in A.P., prove that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P.

Answer

If a^2, b^2, c^2 are in A.P then, $b^2 - a^2 = c^2 - b^2$

If $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in A.P. then,

$$\frac{b}{c+a}-\frac{a}{b+c} = \frac{c}{a+b}-\frac{b}{c+a}$$

$$\frac{b^2+bc-a^2-ac}{(a+c)(b+c)} = \frac{ca+c^2-b^2-ab}{(a+b)(b+c)}$$

$$\frac{(b-a)(a+b+c)}{(a+c)(b+c)} = \frac{(c-b)(a+b+c)}{(a+b)(b+c)}$$

Since, $b^2 - a^2 = c^2 - b^2$

Put $b^2 - a^2 = c^2 - b^2$ in above,

We get LHS = RHS

Hence given terms are in AP

3. Question

If a, b, c are in A.P., then show that:

(i) $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are also in A.P.

(ii) b + c - a, c + a - b, a + b - c are in A.P.

(iii) $bc - a^2$, $ca - b^2$, $ab - c^2$ are in A.P.

Answer

(i) $b^2(c+a)-a^2(b+c) = c^2(a+b)-b^2(a+c)$

$b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2a - b^2c$

$c(b^2 - a^2) + ab(b - a) = a(c^2 - b^2) + bc(c - b)$

$(b - a)(ab + bc + ca) = (c - b)(ab + bc + ca)$

$b - a = c - b$

And since a, b, c are in AP,

$b - a = c - b$

Hence given terms are in AP

(ii) $b + c - a, c + a - b, a + b - c$ are in A.P.

Therefore, $(c + a - b) - (b + c - a) = (a + b - c) - (c + a - b)$

$2a - 2b = 2b - 2c$

$b - a = c - b$

And since a, b, c are in AP,

$b - a = c - b$

Hence given terms are in AP.

(iii) $bc - a^2, ca - b^2, ab - c^2$ are in A.P.

$(ca - b^2) - (bc - a^2) = (ab - c^2) - (ca - b^2)$

$(a - b)(a + b + c) = (b - c)(a + b + c)$

$a - b = b - c$

$b - a = c - b$

And since a, b, c are in AP,

$b - a = c - b$

Hence given terms are in AP

4 A. Question

If $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P., prove that:

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

Answer

Since, $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in AP,

$$\frac{c+a}{b} - \frac{b+c}{a} = \frac{a+b}{c} - \frac{c+a}{b}$$

$$\frac{b^2 + bc - a^2 - ac}{ab} = \frac{ca + c^2 - b^2 - ab}{bc}$$

$$\frac{(b-a)(a+b+c)}{ab} = \frac{(c-b)(a+b+c)}{bc}$$

$$\frac{a(b-c)}{abc} = \frac{c(a-b)}{abc}$$

a(b - c) = c(a - b)

Since, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$\frac{1}{b}-\frac{1}{a} = \frac{1}{c}-\frac{1}{b}$

$\frac{a-b}{ab} = \frac{b-c}{bc}$

Multiply in both denominator and numerator with,

C in LHS and a in RHS

$\frac{c(b-a)}{abc} = \frac{a(b-c)}{abc} = c(b-a) = a(b-c)$

Hence given terms are in AP

4 B. Question

If $\frac{b+c}{a},\frac{c+a}{b},\frac{a+b}{c}$ are in A.P., prove that:

bc, ca, ab are in A.P.

Answer

Since, bc, ca, ab are in A.P.

$= ca-bc = ab-ca$

$= c(a-b) = a(b-c)$

Since, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

$\frac{1}{b}-\frac{1}{a} = \frac{1}{c}-\frac{1}{b}$

$\frac{a-b}{ab} = \frac{b-c}{bc}$

Multiply in both denominator and numerator with,

C in LHS and a in RHS

$\frac{c(b-a)}{abc} = \frac{a(b-c)}{abc} = c(b-a) = a(b-c)$

Hence given terms are in AP

5 A. Question

If a, b, c are in A.P., prove that:

$(a - c)^2 = 4 (a - b) (b - c)$

Answer

$(a - c)^2 = 4 (a - b) (b - c)$

$a^2 + c^2 - 2ac = 4(ab – ac – b^2 + bc)$

$a^2 + 4c^2b^2 + 2ac - 4ab - 4bc = 0$

$(a + c - 2b)^2 = 0$

$a + c - 2b = 0$

Since a, b, c are in AP

$b - a = c - b$

$a + c - 2b = 0$

Hence,

$(a - c)^2 = 4 (a - b) (b - c)$

5 B. Question

If a, b, c are in A.P., prove that:

$a^2 + c^2 + 4ac = 2 (ab + bc + ca)$

Answer

$a^2 + c^2 + 4ac = 2 (ab + bc + ca)$

$a^2 + c^2 + 2ac - 2ab - 2bc = 0$

$(a + c - b)^2 – b^2 = 0$

$a + c - b = b$

$a + c - 2b = 0$

Since a, b, c are in AP

$b - a = c - b$

$a + c - 2b = 0$

Hence proved

5 C. Question

If a, b, c are in A.P., prove that:

$a^3 + c^3 + 6abc = 8b^3$

Answer

$a^3 + c^3 + 6abc = 8b^3$

$a^3 + c^3 - (2b)^3 + 6abc = 0$

$a^3 + (-2b)^3 + c^3 + 3a(-2b)c = 0$

Since, if $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$

$(a - 2b + c)^3 = 0$

$a - 2b + c = 0$

Since a, b, c are in AP

$b - a = c - b$

$= a + c - 2b = 0$

Hence proved

6. Question

If $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P., prove that a, b, c are in A.P.

Answer

Since $a\left(\frac{1}{b}+\frac{1}{c}\right), b\left(\frac{1}{c}+\frac{1}{a}\right), c\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P

Also $a\left(\frac{1}{b}+\frac{1}{c}\right)+1, b\left(\frac{1}{c}+\frac{1}{a}\right)+1, c\left(\frac{1}{a}+\frac{1}{b}\right)+1$ are also in AP

Therefore,

$= \frac{ac+ab+cb}{bc}, \frac{ac+ab+cb}{ac}, \frac{ac+ab+cb}{ba}$ are in AP

$= \frac{1}{ab}, \frac{1}{bc}, \frac{1}{ab}$ are in AP

Multiply by abc in numerator in all terms,

$= \frac{abc}{ac}, \frac{abc}{ab}, \frac{abc}{cb}$ are in AP

= a, b, c are in AP

Hence proved

7. Question

Show that $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are in consecutive terms of an A.P., if x, y and z are in A.P.

Answer

Since, $x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are in AP

$(z^2 + zx + x^2) - (x^2 + xy + y^2) = (y^2 + yz + z^2) - (z^2 + zx + x^2)$

Let d = common difference,

Sicex, y, z are in AP

Y = x + d and x = x + 2d

Therefore the above equation becomes,

$= (x + 2d)^2 + (x + 2d)x - x(x + d) - (x + d)^2 = (x + d)^2 + (x + d)(x + 2d) - (x + 2d)x - x^2$

$= x^2 + 4xd + 4d^2 + x^2 + 2xd - x^2 - xd - x^2 - 2xd - d^2 = x^2 + 2dx + d^2 + x^2 + 2dx + xd + 2d^2 - x^2 - 2dx - x^2$

$3xd + 3d^2 = 3xd + 3d^2$

Hence, proved.

$x^2 + xy + y^2$, $z^2 + zx + x^2$ and $y^2 + yz + z^2$ are in consecutive terms of an A.P.

Exercise 19.6

1. Question

Find the A.M. between:

(i) 7 and 13 (ii) 12 and - 8 (iii) (x - y) and (x + y)

Answer

(i) Let A be the Arithmetic mean

Then 7, A, 13 are in AP

A-7 = 13-A

2A = 13 + 7

A = 10

(ii) Let A be the Arithmetic mean

Then 12, A, - 8 are in AP

A - 12 = - 8 - A

2A = 12 + 8

A = 2

(iii) Let A be the Arithmetic mean

Then x - y, A, x + y are in AP

A - (x - y) = (x + y) – A

2A = x + y + x - y

A = x

2. Question

Insert 4 A.M.s between 4 and 19.

Answer

Let A_1, A_2, A_3, A_4 be the 4 AM Between 4 And 19

Then, 4, A_1, A_2, A_3, A_4, 19 are in AP.

We know,

$$d = \frac{b-a}{n+1} \quad d = \frac{19-4}{4+1} \quad d = \frac{15}{5}$$

d = 3

Hence,

A_1 = a + d = 4 + 3 = 7

A_2 = A1 + d = 7 + 3 = 10

A_3 = A2 + d = 10 + 3 = 13

A_4 = A3 + d = 13 + 3 = 16

3. Question

Insert 7 A.M.s between 2 and 17.

Answer

Let A_1, A_2, A_3, A_4, A_5, A_6, A_7 be the 7 AMs Between 2 And 17

Then, 2, A_1, A_2, A_3, A_4, A_5, A_6, A_7, 17 are in AP

We know,

A_n = a + (n - 1)d

A_9 = 17 = 2 + (9 - 1)d

$$d = \frac{15}{8}$$

Therefore,

$$A_1 = a + d = 2 + \frac{15}{8} = \frac{31}{8}$$

$A_2 = A_1 + d = \frac{31}{8} + \frac{15}{8} = \frac{46}{8}$

$A_3 = A_2 + d = \frac{46}{8} + \frac{15}{8} = \frac{61}{8}$

$A_4 = A_3 + d = \frac{61}{8} + \frac{15}{8} = \frac{76}{8}$

$A_5 = A_4 + d = \frac{76}{8} + \frac{15}{8} = \frac{91}{8}$

$A_6 = A_5 + d = \frac{91}{8} + \frac{15}{8} = \frac{106}{8}$

$A_7 = A_6 + d = \frac{106}{8} + \frac{15}{8} = \frac{121}{8}$So, 7 AMs between 2 and 7 are $\frac{31}{8}, \frac{46}{8}, \frac{61}{8}, \frac{76}{8}, \frac{91}{8}, \frac{106}{8}, \frac{121}{8}$

4. Question

Insert six A.M.s between 15 and - 13.

Answer

Let A_1, A_2, A_3, A_4, A_5, A_6 be the 7 AM Between 15 And - 13

Then, 15, A_1, A_2, A_3, A_4, A_5, A_6, - 13 are in AP

We know,

$A_n = a + (n - 1)d$

$A_{8\,=}$ - 13 = 15 + (8 - 1)d

d = -4

Hence,

A_1 = a + d = 15 - 4 = 11

A_2 = A1 + d = 11 - 4 = 7

A_3 = A2 + d = 7 - 4 = 3

A_4 = A3 + d = 3 - 4 = -1

A_5 = A4 + d = -1 - 4 = -5

A_6 = A5 + d = -5 - 4 = -9

5. Question

There are n A.M.s between 3 and 17. The ratio of the last mean to the first mean is 3 : 1. Find the value of n.

Answer

Let the series be 3, A_1, A_2, A_3, ..., A_n, 17

From the data, $\frac{A_n}{A_1} = \frac{3}{1}$

We know total terms in AP are n + 2

So 17 is the (n + 2)th term

We know,

$A_n = a + (n - 1)d$

So, 17 = 3 + (n + 2 - 1)d

$$d = \frac{14}{n + 1}$$

Therefore,

$$A_n = 3 + \frac{14n}{n + 1} = \frac{17n + 3}{n + 1}$$

And

$$A_1 = 3 + d = \frac{3n + 17}{n + 1}$$

Since,

$$\frac{A_n}{A_1} = \frac{3}{1}$$

$$\frac{17n + 3}{3n + 17} = \frac{3}{1}$$

9n + 51 = 17n + 3

8n = 48

$$n = \frac{48}{8}$$

n = 6

There are 6 terms in AP

6. Question

Insert A.M.s between 7 and 71 in such a way that the 5^{th} A.M. is 27. Find the number of A.M.s.

Answer

Let the series be 7, A_1, A_2, A_3, ..., A_n, 71

We know total terms in AP are n + 2

So 71 is the (n + 2)th term

We know,

$A_n = a + (n - 1)d$

So, $A_6 = a + (6 - 1)d$

a + 5d = 27 (5^{th} term)

d = 4

71 = (n + 2)th term

71 = a + (n + 2 - 1)d

71 = 7 + n(4)

n = 15

There are 15 terms in AP

7. Question

If n A.M.s are inserted between two numbers, prove that the sum of the means equidistant from the beginning and the end is constant.

Answer

Let a and b be the first and last terms and

The series be a, A_1, A_2, A_3, ..., A_n, b

So We know, Mean $= \frac{a+b}{2}$

Mean of A_1 and A_n $= \frac{(A_1 + A_n)}{2}$

$A_1 = a + d$

$A_n = a - d$

Therefore, AM $= \frac{(a+d+b-d)}{2} = \frac{a+b}{2}$

AM between A_2 and $A_{n-1} = \frac{(a+2d)+(b-2d)}{2} = \frac{a+b}{2}$

Similarly, it is (a + b)/2 for all such numbers, which is constant

Hence, AM = (a + b)/2

8. Question

If x, y, z are in A.P. and $A_1$is the A.M. of x and y, and A_2 is the A.M. of y and z, then prove that the A.M. of A_1 and A_2 is y.

Answer

Given that,

A_1 = AM of x and y

And A_2 = AM of y and z

So, $A_1 = \frac{x+y}{2}$

$A_2 = \frac{y + x}{2}$

AM of A_1 and $A_2 = \frac{A_1 + A_2}{2}$

$$\frac{\frac{x+y}{2} + \frac{y+z}{2}}{2}$$

$$\frac{x + y + y + z}{2} = \frac{(x + 2y + z)}{2}$$

Since x, y, z are in AP, $y = \frac{x+z}{2}$

Finally, AM $= \frac{\left(x+\frac{z}{2}\right)+\left(\frac{2y}{2}\right)}{2} = \frac{y+y}{2} = y$

Hence proved.

9. Question

Insert five numbers between 8 and 26 such that the resulting sequence is an A.P

Answer

Let A_1, A_2, A_3, A_4, A_5 be the 5 nos Between 8 And 26

Then, 8, A_1, A_2, A_3, A_4, A_5, 26 are in AP

We know,

$A_n = a + (n - 1)d$

$A_7 = 26 = 8 + (7 - 1)d$

$d = 3$

Hence,

$A_1 = a + d = 8 + 3 = 11$

$A_2 = A_1 + d = 11 + 3 = 14$

$A_3 = A_2 + d = 14 + 3 = 17$

$A_4 = A_3 + d = 17 + 3 = 20$

$A_5 = A_4 + d = 20 + 3 = 23$

Exercise 19.7

1. Question

A man saved ₹ 16500/- in ten years . in each year after the first he saved ₹ 100/- more then he did in the preceding year. How much did he saved in the first year?

Answer

Given: A man saved 16500/- in ten years

To find: His saving in the first year

Let he saved Rs. x in the first year

Since each year after the first he saved 100/- more then he did in the preceding year

So,

A.P will be x, 100 + x, 200 + x……………….

where x is first term and

common difference, d = 100 + x – x = 100

We know,

S_n is the sum of n terms of an A.P

Formula used:

$$S_n = \frac{n}{2}\{2a + (n - 1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

According to the question:

S_n = 16500 and n = 10

Therefore,

$$S_{10} = \frac{10}{2}\{2x + (10 - 1)100\}$$

⇒ 16500 = 5{2x + 9(100)}

⇒ 16500 = 5(2x + 900)

⇒ 16500 = 10x + 4500

⇒ -10x = 4500 – 16500

⇒ –10x = –12000

$$\Rightarrow x = \frac{-12000}{-10}$$

⇒ x = 1200

Hence, his saving in first year is 1200

2. Question

A man saves ₹ 32 during the first year, ₹ 36 in the second year and in this way he increases his savings by ₹4 every year. Find in what time his saving will be ₹200.

Answer

Given: A man saves Rs. 32 during first year and increases his savings by Rs. 4 every year. His total saving is Rs. 200

To find: Time taken in years by him to save Rs. 200.

A man saves in the first year is 32Rs

He saves in the second year is 36Rs.

In this process he increases his savings by Rs. 4 every year

Therefore,

A.P. will be 32, 36, 40,....................

where 32 is first term and

common difference, d = 36 – 32 = 4

We know,

S_n is the sum of n terms of an A.P

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

According to the question:

S_n = 200

Therefore,

$$S_n = \frac{n}{2}\{2 \times 32 + (n-1)4\}$$

$$\Rightarrow 200 = \frac{n}{2}\{64 + 4n - 4\}$$

$$\Rightarrow 400 = n\{60 + 4n\}$$

$$\Rightarrow 400 = 4n\{15 + n\}$$

$$\Rightarrow \frac{400}{4} = 15n + n^2$$

$$\Rightarrow n^2 + 15n - 100 = 0$$

$$\Rightarrow n^2 + 20n - 5n - 100 = 0$$

$$\Rightarrow n(n+20) - 5(n+20) = 0$$

$$\Rightarrow (n+20)(n-5) = 0$$

$$\Rightarrow n = -20 \text{ or } 5$$

n can not be a negative values as days can not hold negative values

⇒ n = 5

Hence, he has to do saving for 5 days to save Rs. 200

3. Question

A man arranges to pay off a debt of ₹ 3600 by 40 annual instalments which form an arithmetic series. When 30 of the instalments are paid, he dies leaving one-third of the debt unpaid, find the value of the instalment.

Answer

Given: Total debt is Rs. 3600 and total number of installments are 40 and in 30 installments, he has paid two-third of the debt and dies leaving one-third of amount

Let first instalment be “a”

Let the common difference of the instalments be “d”

Here, S_{40} = 3600 and n = 40

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$3600 = \frac{40}{2}\{2a + (40-1)d\}$$

$$\Rightarrow 3600 = 20\{2a + 39d\}$$

$$\Rightarrow \frac{3600}{20} = 2a + 39d$$

⇒ 2a+39d = 180 ....(1)

Since 30 installments are paid and one third of the debt unpaid

$$\therefore \text{Paid amount} = \frac{2}{3} \times 3600 = 2400$$

So,

S_{30} = 2400 and n = 30

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow 2400 = \frac{30}{2}\{2a + (30-1)d\}$$

$$\Rightarrow 2400 = 15\{2a + 29d\}$$

$$\Rightarrow \frac{2400}{15} = 2a + 29d$$

⇒ 2a+29d = 160 ....(2)

Solving (1) and(2) by substitution method,

2a + 39d = 180

⇒ 2a = 180 – 39d ......(3)

Put value of 2a from (3) in (2):

$2a + 29d = 160$

$\Rightarrow 180 - 39d + 29d = 160$

$\Rightarrow -10d = 160 - 180$

$\Rightarrow -10d = -20$

$$\Rightarrow d = \frac{-20}{-10}$$

$\Rightarrow d = 2$

Put this value of d in (3):

$2a = 180 - 39d$

$\Rightarrow 2a = 180 - 39(2)$

$\Rightarrow 2a = 180 - 78$

$\Rightarrow 2a = 102$

$$\Rightarrow a = \frac{102}{2}$$

$\Rightarrow a = 51$

Hence, value of first installment = a = Rs. 51

4. Question

A manufacturer of the radio sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the product increases uniformly by a fixed number every year, find

(i) the production in the first year

(ii) the total product in the 7 years and

(iii) the product in the 10^{th} year.

Solution |||

(i) the production in the first year

Answer:

Given: 600 and 700 radio sets units are produced in third and seventh year respectively

To find: the production in the first year i.e. a

$\Rightarrow a_3 = 600$ and $a_7 = 700$

Formula used:

For an A.P., a_n is n^{th} term which is given by,

$a_n = a + (n - 1)d$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$a_3 = a + (3 - 1)d$

$\Rightarrow 600 = a + 2d$……………………(1)

$a_7 = a + (7 - 1)d$

$\Rightarrow 700 = a + 6d$

$\Rightarrow a = 700 - 6d$………………………(2)

Now put this value of a in equation (1):

⇒ 600 = 700 – 6d + 2d

⇒ 600 – 700 = – 6d + 2d

⇒ –100 = –4d

$$\Rightarrow d = \frac{-100}{-4}$$

⇒ d = 25

Put d = 25 in equation (2):

⇒ a = 700 – 6(25)

⇒ a = 700 – 150

⇒ a = 550

Production in the first year = a = 550

(ii) the total product in the 7 years

Answer

To find: the sum of totals products in 7 years i.e. S_7

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_7 = \frac{7}{2}\{2 \times 550 + (7-1)25\}$$

$$\Rightarrow S_7 = \frac{7}{2}\{1100 + (6)25\}$$

$$\Rightarrow S_7 = \frac{7}{2}(1100 + 150)$$

$$\Rightarrow S_7 = \frac{7}{2}(1250)$$

⇒ S_7 = 7(625)

⇒ S_7 = 4375

Hence, the total product in the 7 years are 4375

(iii) the product in the 10^{th} year

Answer:

To find: the product in the 10^{th} year i.e. a_{10}

Formula used:

For an A.P., a_n is n^{th} term which is given by,

$a_n = a + (n – 1)d$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$a_{10} = 550 + (10 - 1)25$

$\Rightarrow a_{10} = 550 + (9)25$

$\Rightarrow a_{10} = 550 + 225$

$\Rightarrow a_{10} = 775$

Hence, the product in the 10^{th} year are 775 units

5. Question

There are 25 trees at equal distances of 5 meters in a line with a well, the distance of well from the nearest tree being 10 meters. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next. Find the total distance the gardener will cover in order to water all the trees.

Answer

Given: total trees are 25 and equal distance between two adjacent trees are 5 meters

To find: the total distance the gardener will cover

As gardener is coming back to well after watering every tree

Distance covered by him to water 1^{st} tree and return to the initial position is 10m + 10m = 20m

Now, distance between adjacent trees is 5m

Distance covered by him to water 2^{nd} tree and return to the initial position is 15m + 15m = 30m

Distance covered by the gardener to water 3^{rd} tree return to the initial postion is 20m + 20m = 40m

Hence distance covered by the gardener to water the trees are in A.P

A.P. is 20, 30, 40 ....upto 25 terms

Here first term,a = 20,common difference, d = 30 – 20 = 10

And n = 25

We need to find S_{25} which will be the total distance covered by gardener to water 25 trees

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_{25} = \frac{25}{2}\{2 \times 20 + (25-1)10\}$$

$$\Rightarrow S_{25} = \frac{25}{2}\{40 + (24)10\}$$

$$\Rightarrow S_{25} = \frac{25}{2}(40 + 240)$$

$$\Rightarrow S_{25} = \frac{25}{2}(280)$$

$\Rightarrow S_{25} = 25(140)$

$\Rightarrow S_{25} = 3500$

Hence, total distance covered by gardener to water trees is 3500 m

6. Question

A man is employed to count ₹ 10710. He counts at the rate of ₹ 180 per minute for half an hour. After this he counts at the rate of ₹ 3 less every minute then the preceding minute. Find the time taken by him to count the entire amount.

Answer

Given: Amount to be counted is Rs. 10710

To find: Time taken by man to count the entire amount

He counts at the rate of Rs. 180 per minute for half an hour or 30 minutes

Amount to be counted in an hour = 180 * 30 = Rs. 5400

Amount to be left = 10710 – 5400 = 5310

⇒ S_n = 5310

After an hour, rate of counting is decreasing at Rs. 3 per minute. This rate will form an A.P.

A.P. will be 177, 174, 171,......................

Here a = 177 and d = 174 – 177 = –3

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_n = \frac{n}{2}\{2 \times 177 + (n-1) - 3\}$$

$$\Rightarrow 5310 = \frac{n}{2}\{354 - 3n + 3\}$$

$$\Rightarrow 5310 \times 2 = n(357 - 3n)$$

$$\Rightarrow 10620 = 357n - 3n^2$$

⇒ 10620 = 3n(119 – n)

$$\Rightarrow \frac{10620}{3} = n(119 - n)$$

$$\Rightarrow n^2 - 119n + 3540 = 0$$

⇒ n^2 – 59n – 60n + 3540 = 0

⇒ n(n – 59) – 60(n – 59) = 0

⇒ (n – 59)(n – 60) = 0

⇒ n = 59 or 60

We will take value of n = 59 as at 60^{th} min he will count Rs.0

Therefore, total time taken by him to count the entire amount = 30 + 59 = **89 minutes**

7. Question

A piece of equipment cost a certain factory ₹ 600, 000. If it depreciates in value 15% the first, 13.5% the next year, 12% the third year, and so on. What will be its value at the end of 10 years, all percentages applying to the original cost?

Answer

Given: A piece of equipment cost a certain factory is 600,000

To find: Value of the equipment at the end of 10 years

It depreciates 15%, 13.5%, 12% in 1^{st}, 2^{nd}, 3^{rd} year and so on.

This means price of equipment is depreciating in an A.P.

A.P. will be 15, 13.5, 12,...........................upto 10 terms

Hence a = 15, d = 13.5 – 15 = –1.5

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

Total percentage of depreciation in 10 years,

$$S_{10} = \frac{10}{2}\{2 \times 15 + (10-1) \times -1.5\}$$

$$\Rightarrow S_{10} = 5(30 + 9 \times -1.5)$$

$$\Rightarrow S_{10} = 5(30 - 13.5) \Rightarrow S_{10} = 5(16.5)$$

$\Rightarrow S_{10} = 82.5$

Value of the equipment at the end of 10 years,

$$= \frac{100 - \text{Depreciation\%}}{100} \times \text{cost of equipment}$$

$$= \frac{100 - 82.5}{100} \times 600000$$

$$= \frac{175}{10} \times 6000$$

= 175 × 600

= 105000

Hence, value of equipment at the end of 10 years is Rs. 105000

8. Question

A farmer buys a used tractor for ₹ 12000. He pays ₹ 6000 cash and agrees to pay the balance in annual instalments of ₹ 500 plus 12% interest on the unpaid amount. How much the tractor cost him?

Answer

Given: Price of the tractor is Rs. 12000. He pays ₹ 6000 cash and agrees to pay the balance in annual instalments of ₹ 500 plus 12% interest on the unpaid amount.

To find: Total cost of the tractor if he buys it at installment

Total price = 12000

Paid amount = 6000

Unpaid amount = 12000 – 6000 = 6000

He pays remaining 6000 in n number of installments of 500 each

$$\Rightarrow n = \frac{6000}{500} = 12$$

Cost incurred by him to pay remaining 6000,

$= (500 + 12\% \text{ of } 6000) + (500 + 12\% \text{ of } 5500) + \cdots \text{ upto 12 terms}$

$= 500 \times 12 + 12\% \text{ of } (6000 + 5500 + \cdots \text{upto 12 terms})$

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$= 500 \times 12 + \frac{12}{100} \times \frac{12}{2}\{2 \times 6000 + (12-1) \times -500\}$$

$$= 6000 + \frac{72}{100}(12000 + 11 \times -500)$$

$$= 6000 + \frac{72}{100}(12000 - 5500)$$

$$= 6000 + \frac{72}{100}(6500)$$

$= 6000 + 4680$

= 10680

Total cost = 6000 + 10680 = 16680

Hence, total cost of the tractor if he buys it at installment is Rs. 16680

9. Question

Shamshad Ali buys a Scooter for ₹ 22000. He pays ₹ 4000 cash and agrees to pay the balance in annual instalments of ₹ 1000 plus 10% interest on the unpaid amount. How much the Scooter will cost him?

Answer

Given: Price of the scooter is Rs. 22000. He pays ₹ 4000 cash and agrees to pay the balance in annual instalments of Rs. 1000 plus 10% interest on the unpaid amount.

To find: Total cost of the scooter if he buys it at installment

Total price = 22000

Paid amount = 4000

Unpaid amount = 22000 – 4000 = 18000

He pays remaining 18000 in n number of installments of 1000 each

$$\Rightarrow n = \frac{18000}{1000} = 18$$

Cost incurred by him to pay remaining 18000,

$= (1000 + 10\%\text{of}18000) + (1000 + 10\% \text{ of}1700) + \cdots \text{ upto 18 terms}$

$= 1000 \times 18 + 10\% \text{ of } (18000 + 17000 + \cdots \text{ upto 18 terms})$

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$= 1000 \times 18 + \frac{10}{100} \times \frac{18}{2}\{2 \times 18000 + (18-1) \times -1000\}$

$= 18000 + \frac{90}{100}(36000 + 17 \times -1000)$

$= 18000 + \frac{90}{100}(36000 - 17000)$

$= 18000 + \frac{90}{100}(19000)$

$= 18000 + 17100$

= 35100

Total cost = 4000 + 35100 = 39100

Hence, total cost of the scooter if he buys it at installment is Rs. 39100

10. Question

The income of a person is ₹ 300, 000 in the first year and he receives an increase of ₹ 10, 000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.

Answer

Given: The income of a person is ₹ 300, 000 in the first year

and he receives an increase of ₹ 10, 000 to his income per year for the next 19 years.

To find: the total amount he received in 20 years i.e. S_{20}

According to question:

a = 300000, d = 10000 and n = 20

Formula used:

$S_n = \frac{n}{2}\{2a + (n-1)d\}$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$S_{20} = \frac{20}{2}\{2 \times 300000 + (20-1) \times 10000\}$

$\Rightarrow S_{20} = 10(600000 + 19 \times 10000)$

$\Rightarrow S_{20} = 10(600000 + 190000)$

$\Rightarrow S_{20} = 10(790000)$

$\Rightarrow S_{20} = 7900000$

Hence, the total amount he received at the end of 20 years is Rs. 790000

11. Question

A man starts repaying a loan as first instalment of Rs.100. If he increases the instalments by Rs .5 every month, what amount he will pay in 30th instalment?

Answer

Given: First installment is Rs. 100 and he increases it by Rs. 5 every month

To find: Total amount in 30th installment i.e. a_{30}

According to question:

A.P. will be 100, 105,.......................

a = 100 and d = 5

Formula used:

$a_n = a + (n - 1)d$

$\Rightarrow a_{30} = 100 + (30 - 1)5$

$\Rightarrow a_{30} = 100 + (29)5$

$\Rightarrow a_{30} = 100 + 145$

$\Rightarrow a_{30} = 245$

Hence, he will pay Rs. 245 in 30th installment

12. Question

A carpenter was hired to build 192 window frames. The first day he made five frames and each day there after he made two more frames than he made the day before. How many days did it take him to finish the job? [NCERT EXEMPLAR]

Answer

Given: Carpenter was hired to build 192 frames. He made 5 frames on first day and two more frames for every next day

To find: Number of days taken by him to build 192 frames

According to question:

A.P. will be 5, 7, 9,..................................

$\Rightarrow S_n = 192$, a = 5 and d = 7 – 5 = 2

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_n = \frac{n}{2}\{2 \times 5 + (n-1)2\}$$

$$\Rightarrow 192 = \frac{n}{2}\{10 + 2n - 2\}$$

$$\Rightarrow 192 \times 2 = n(8 + 2n)$$

$$\Rightarrow 192 \times 2 = 2(4n + n^2)$$

$$\Rightarrow \frac{192 \times 2}{2} = 4n + n^2$$

$$\Rightarrow n^2 + 4n - 192 = 0$$

$\Rightarrow n^2 + 16n - 12n + 192 = 0$

$\Rightarrow n(n + 16) - 12(n + 16) = 0$

$\Rightarrow (n + 16)(n - 12) = 0$

$\Rightarrow n = -16$ or 12

n is number of days and it cannot be negative

Therefore, we will take value of n = 12

Hence, he will take 12 days to build 192 frames

13. Question

We know that the sum of interior angles of a triangle is 180°. Show that the sums of interior angles of polygons with 3, 4, 5, 6 ... sides from an arithmetic progression. Find the sum of interior angles for a 21 sided polygon. [NCERT EXEMPLAR]

Answer

Given: the sum of interior angles of a triangle is 180°

To prove: Sum of interior angles of polygons with 3, 4, 5,...... forms an A.P.

To find: Sum of interior angles for a 21 sided polygon i.e. a_{21}

We know,

The sum of interior angles of a polygon with n sides,

$a_n = 180° \times (n – 2)$

Sum of interior angles of a polygon of 3 sides,

$a_3 = 180° \times (3 – 2) = 180° \times 1 = 180°$

Sum of interior angles of a polygon of 4 sides,

$a_4 = 180° \times (4 – 2) = 180° \times 2 = 360°$

Sum of interior angles of a polygon of 5 sides,

$a_5 = 180° \times (5 – 2) = 180° \times 3 = 540°$

A.P is known for Arithmetic Progression whose common difference, $d = a_n – a_{n-1}$ where $n > 0$

Here,

$d = a_4 – a_3 = 360° - 180° = 180°$

$d = a_5 – a_4 = 540° - 360° = 180°$

Common difference is same in both the cases

This shows that sum of interior angles of polygons with 3, 4, 5,...... forms an A.P.

Hence Proved

Sum of interior angles of a polygon of 21 sides,

$a_{21} = 180° \times (21 – 2) = 180° \times 19 = 3420°$

14. Question

In a potato race 20 potato are placed in a line at intervals of 4 meters with the first potato 24 meters from the starting point. A contestant is required to bring the potatoes back to the starting place one at a time. How far would he run in bringing back all the potatoes? [NCERT EXEMPLAR]

Answer

Given: total potatoes are 20. First potato is 24 meters away from starting point equal distance between two adjacent potatoes is 4 meters

To find: the total distance run by contestant

As contestant is coming back to starting point after bringing every potato

Distance covered by him to bring 1^{st} potato and return to the initial position is 24m + 24m = 48m

Now, distance between adjacent trees is 4m

Distance covered by him to bring 2nd potato and return to the initial position is 28m + 28m = 56m

Distance covered by him to bring 3rd potato return to the initial postion is 32m + 32m = 64m

Hence distance covered by him to bring the potatoes are in A.P

A.P. is 48, 56, 64……………………………………..upto 20 terms

Here first term, a = 48, common difference, d = 56 – 48 = 8

And n = 20

We need to find S_{20} which will be the total distance covered by contestant to bring 20 potatoes

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_{20} = \frac{20}{2}\{2 \times 48 + (20-1)8\}$$

$$\Rightarrow S_{20} = 10\{96 + (19)8\}$$

$$\Rightarrow S_{20} = 10(96 + 152)$$

$$\Rightarrow S_{20} = 10(248)$$

$\Rightarrow S_{20} = 2480$

Hence, total distance covered by contestant to bring potatoes is 2480 m

15. Question

A man accepts a position with an initial salary of ₹ 5200 per month. It is understood that he will receive an automatic increase of ₹ 320 in the very next month and each month thereafter.

(i) Find hi salary for the tenth month.

(ii) What is his total earnings during the first year?

Answer

Given: Initial salary is Rs. 5200 per month and will increase Rs. 320 every month i.e. a = 5200 and d = 320

(i) Find hi salary for the tenth month.

Answer:

To find: Salary for the tenth month i.e. a_{10}

Formula used:

$a_n = a + (n - 1)d$

$\Rightarrow a_{10} = 5200 + (10 - 1)320$

$\Rightarrow a_{10} = 5200 + (9)320$

$\Rightarrow a_{10} = 5200 + 2880$

$\Rightarrow a_{10} = 8080$

Hence, he will get Rs. 8080 in tenth salary

(ii) What is his total earnings during the first year?

Answer:

To find: His total earnings during the first year i.e. 12 months, S_{12}

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_{12} = \frac{12}{2}\{2 \times 5200 + (12-1)320\}$$

$$\Rightarrow S_{12} = 6\{10400 + (11)320\}$$

$$\Rightarrow S_{12} = 6(10400 + 3520)$$

$$\Rightarrow S_{12} = 6(13920)$$

⇒ S_{12} = 83520

Hence, his total earnings during an year is Rs. 83520

16. Question

A man saved ₹ 66000 in 20 years. In each succeeding year after the first year he saved ₹ 200 more than what he saved in the previous year. How much did he saved in the first year?

Answer

Given: A man saved ₹ 66000 in 20 years

To find: His saving in first year i.e. a

According to question:

n = 20, S_{20} = 66000 and d = 200

Formula used:

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

$$S_{20} = \frac{20}{2}\{2a + (20-1)200\}$$

$$\Rightarrow 66000 = 10\{2a + (19)200\}$$

$$\Rightarrow \frac{66000}{10} = 2a + 3800$$

$$\Rightarrow 6600 = 2a + 3800$$

⇒ 2a = 6600 – 3800

⇒ 2a = 2800

$$\Rightarrow a = \frac{2800}{2}$$

⇒ a = 1400

Hence, his saving in first year is Rs. 1400

17. Question

In a cricket team tournment 16 teams particpated a sum of 8000 is to to awarded among themselves a price money. It the last placed team is awarded 275 in price money and the award increased by the same amount for successive finishing place , how much amount will the first place team receive?

Answer

Given: Total teams in tournament are 16

Total prize money is Rs 8000

The last placed team is awarded Rs. 275

To find: Amount received by first place team

As the award money is increased by the same amount for successive finishing place, it forms an A.P.

Let a is amount receievd by first place team and d is successive increase in award

A.P. will be a, a + d, a + 2d,......................

According to question:

n = 16, S_{16} = 8000 and a_{16} = 275

Formula used:

a_n = a + (n – 1)d

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

where a is first term, d is common difference and n is number of terms in an A.P.

Therefore,

a_{16} = a + (16 – 1)d

⇒ 275 = a + 15d

⇒ a = 275 – 15d ....(1)

$$S_{16} = \frac{16}{2}\{2a + (16-1)d\}$$

$$\Rightarrow 8000 = 8(2a + 15d)$$

$$\Rightarrow \frac{8000}{8} = 2a + 15d$$

⇒ 1000 = 2a + 15d ....(2)

Now put the value of a from (1) in (2):

⇒ 1000 = 2(275 – 15d) + 15d

⇒ 1000 = 550 – 30d + 15d

⇒ 1000 = 550 – 15d

⇒ 15d = 550 – 1000

⇒ 15d = –450

$$\Rightarrow d = \frac{-450}{15}$$

⇒ d = -30

Put d = -30 in (1):

$a = 275 - 15(-30)$

$\Rightarrow a = 275 + 450$

$\Rightarrow a = 725$

Hence, amount received by first placed team is Rs. 725

Very Short Answer

1. Question

Write the common difference of an A.P. whose nth term is xn + y.

Answer

Here, $a_n = xn + y$

$\therefore a_1 = x + y$

And $a_2 = 2x + y$

So, common difference of an A.P. is given by

$d = a_2 - a_1$

$= (2x+y)-(x+y)$

$= x$

2. Question

Write the common difference of an A.P. the sum of whose first n terms is $\frac{P}{2}n^2 + Qn$.

Answer

Here, $S_n = \frac{P}{2}n^2 + Qn$

So, $S_1 = \frac{P}{2} + Q = t_1$

And $S_2 = \frac{P}{2}2^2 + 2Q$

$= 2P + 2Q$

So, $t_2 = S_2 - S_1$

$= (2P + 2Q) - (\frac{P}{2} + Q)$

$= \frac{3P}{2} + Q$

Now, common difference of A.P. is given by

$d = t_2 - t_1$

$= \frac{3P}{2} + Q - (\frac{P}{2} + Q)$

$= \frac{2P}{2}$

$= P$

3. Question

If the sum of n terms of an AP is $2n^2 + 3n$, then write its nth term.

Answer

Here, $S_n=2n^2+3n$

So, $S_1=2+3=5=t_1$

And $S_2=2\times 2^2+3\times 2$

$=8+6=14$

So, $t_2=S_2-S_1$

$=14-5$

$=9$

Now, common difference of A.P. is given by

$d=t_2-t_1$

$=9-5$

$=4$

So, n^{th} term of A.P. is

$a_n=t_1+(n-1)d$

$=5+(n-1)4$

$=5+4n-4$

$=4n+1$

4. Question

If log 2, $\log\left(2^x-1\right)$ and $\log\left(2^x+3\right)$ are in A.P., write the value of x.

Answer

Here, log 2, $\log(2^x-1)$ and $\log(2^x+3)$ are in A.P.

So, $\log(2^x-1)-\log 2=\log(2^x+3)-\log(2^x-1)$

$\therefore \log\frac{2^x-1}{2}=\log\frac{2^x+3}{2^x-1}$

$\therefore \frac{2^x-1}{2}=\frac{2^x+3}{2^x-1}$

$\therefore (2^x-1)^2=2(2^x+3)$

Let $2^x=y$

Then above equation is written as

$(y-1)^2=2(y+3)$

$\therefore y^2-2y+1=2y+6$

$\therefore y^2-4y-5=0$

$\therefore (y-5)(y+1)=0$

$\therefore$ y=5 or y=-1

$\therefore$ $2^x=5$ or $2^x=-1$

$\therefore$ $x=\log_2 5$ or $2^x=-1$ is not possible

$\therefore$ $x = \log_2 5$

5. Question

If the sum of n terms of two arithmetic progressions are in the ratio 2n + 5 : 3n + 4, then write the ratio of their mth terms.

Answer

Let n^{th} term of first A.P. be denoted by a_n

And n^{th} term of second A.P. be denoted by a_n'

$$\frac{a_m}{a'_m} = \frac{a + (m-1)d}{a' + (m-1)d'}$$

Now, multiplying both sides by 2 we get,

$$\frac{2a_m}{2a'_m} = \frac{2a + 2(m-1)d}{2a' + 2(m-1)d'}$$

$$\therefore \frac{a_m}{a'_m} = \frac{2a + (2m-1-1)d}{2a' + (2m-1-1)d'}$$

$$\therefore \frac{\frac{2m-1}{2}a_m}{\frac{2m-1}{2}a'_m} = \frac{\frac{2m-1}{2}\{2a + (2m-1-1)d\}}{\frac{2m-1}{2}\{2a' + (2m-1-1)d'\}}$$

$$\therefore \frac{a_m}{a'_m} = \frac{S_{2m-1}}{S_{2m-1}}$$

$$= \frac{\{2(2m-1)+5\}}{3(2m-1)+4}$$

$$= \frac{4m+3}{6m+1}$$

The ratio of m^{th} term is 4m+3:6m+1

6. Question

Write the sum of first n odd natural numbers.

Answer

Here, A.P is 1,3,5…

So, a=1 and d=3-1=2

Now, sum of n term is given by,

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(2 \times 1 + (n-1)2)$$

$$= \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{n}{2}(2n)$$

$$= n^2$$

7. Question

Write the sum of first n even natural numbers.

Answer

Here, A.P is 2,4,6…

So, a=2 and d=4-2=2

Now, sum of n term is given by,

$S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2 \times 2 + (n-1)2)$

$= \frac{n}{2}(4 + 2n - 2)$

$= \frac{n}{2}(2n + 2)$

=n(n+1)

8. Question

Write the value of n for which nth of the A.P.s 3, 10, 17, ... and 63, 65, 67, ... are equal.

Answer

Let n^{th} term of A.P. 3,10,17,… be a_n

And n^{th} term of A.P. 63,65,67,…be a'_n

So,a_1=3 and a_2=10

∴ d=a_2-a_1=10-3=7

And, a_1'=63 and a_2'=65

∴ d'=a_2'-a_1'=65-63=2

Now, a_n=a_1+(n-1)d

=3+(n-1)7

=3+7n-7

=7n-4

And, a_n'=a_1'+(n-1)d'

=63+(n-1)2

=63+2n-2

=2n+61

Now,it is given that a_n=a_n'

∴ 7n-4=2n+61

∴ 7n-2n=61+4

∴ 5n=65

∴ n=13

9. Question

If $\frac{3+5+7+\ldots+\text{upto n terms}}{5+8+11+\ldots\text{upto 10 terms}} = 7$, then find the value of n.

Answer

Here, $\frac{3+5+7+\cdots+\text{upto n terms}}{5+8+11+\cdots+\text{upto 10 terms}}=7$

i.e. $\frac{S_n}{S'_{10}}=7$

$\therefore \frac{\left\{\frac{n}{2}[2a+(n-1)d]\right\}}{\{\frac{10}{2}[2a'+(10-1)d'\}}=7$

$\therefore \frac{\left\{\frac{n}{2}[2\times3+(n-1)2]\right\}}{5[2\times5+9\times3]}=7$ (∵a=3;d=2;a'=5;d'=3)

$\therefore \frac{\left\{\frac{n}{2}[4+2n]\right\}}{5\times37}=7$

∴n(2+n) =7×5×37

∴n^2+2n-1295=0

∴(n-35) (n+37) =0

∴ n=35 or n=-37 which is not possible as n is non negative.

So,n=35

10. Question

If mth term of an A.P. is n and nth term is m, then write its pth term.

Answer

Here, a_m=n and a_n=m

∴ a+(m-1)d=n and a+(n-1)d=m_______(1)

Subtracting above two equation we get

a +(m-1)d-a-(n-1)d=n-m

∴ md-d-nd+d=n-m

∴ d(m-n)=n-m

∴ d=-1

Substituting d=-1 in a+(m-1)d=n we get

∴ a+(m-1)(-1)=n

∴ a-m+1=n

∴ a=m+n-1

Now, p^{th} term is given by a_p=a+(p-1)d

=m+n-1+(p-1)(-1)

=m+n-1-p+1

=m+n-p

11. Question

If the sum of n terms of two A.P.'s are in the ratio (3n + 2) : (2n + 3), find the ratio of their 12^{th} terms.

Answer

Here, $\frac{S_n}{S'_n}=\frac{3n+2}{2n+3}$

$$\therefore \frac{\left\{\frac{n}{2}[2a+(n-1)d]\right\}}{\frac{n}{2}[2a'+(n-1)d']} = \frac{3n+2}{2n+3}$$

$\therefore \frac{\{2a+(n-1)d\}}{2a'+(n-1)d'} = \frac{3n+2}{2n+3}$ _____(1)

Now,we want to find ratio of 12^{th} term,

i.e. $\frac{a_{12}}{a'_{12}} = \frac{a+11d}{a'+11d'}$

$\frac{a_{12}}{a'_{12}} = \frac{2a+22d}{2a'+22d'}$ ___(2)

From (1) and (2)we get n-1=22

i.e.n=23

$$\therefore \frac{a_{12}}{a'_{12}} = \frac{3(23)+2}{2(23)+3}$$

$$= \frac{71}{49}$$

MCQ

1. Question

Mark the correct alternative in the following:

If 7th and 13th terms of an A.P. be 34 and 64 respectively, then its 18th term is

A. 87

B. 88

C. 89

D. 90

Answer

Here,a_7=34 and a_{13}=64

i.e. a+6d=34____(i) and a+12d=64_____(ii)

Subtracting (i) from (ii) we get,

(12-6)d=64-34

∴6d=30

∴ d=5

Substituting d=5 in (i) we get,

a+6(5)=34

∴a+30=34

∴ a=4

Now,18^{th} term is given by a_{18}=a+17d

=4+17×5

=89

2. Question

Mark the correct alternative in the following:

If the sum of p terms of an A.P. is q and the sum of q terms is p, then the sum or p + q terms will be

A. 0

B. p − q

C. p + q

D. − (p + q)

Answer

Here, S_p=q and S_q=p

i.e. $\frac{p}{2}[2a+(p-1)d]=q$ and $\frac{q}{2}[2a+(q-1)d]=p$

i.e. $[2ap+p(p-1)d]=2q$ ____(i)

and $2aq+q(q-1)d=2p$ ____(ii)

Now,multiplying (i) with q and (ii) with p and then subtracting we get

$pq(p-1)d-pq(q-1)d=2q^2-2p^2$

$\therefore pqd(p-q)=2(q^2-p^2)$

$\therefore pqd(p-q)=2(p-q)(p+q)$

$$\therefore d=-\frac{2(p+q)}{pq}$$

Substituting the value of d in (i) we get,

$$2ap-\frac{p(p-1)2(p+q)}{pq}=2q$$

$\therefore 2ap^2q-2p(p-1)(p+q)=2pq^2$

$\therefore apq-(p-1)(p+q)=q^2$

$\therefore apq=q^2+(p-1)(p+q)$

$\therefore apq=q^2+p^2+pq-p-q$

$$\therefore a=\frac{q^2+p^2+pq-p-q}{pq}$$

Now,$S_{p+q}=\frac{p+q}{2}[2a+(p+q-1)d)]$

$$=\frac{p+q}{2}\left[2\frac{q^2+p^2+pq-p-q}{pq}-(p+q-1)\frac{2(p+q)}{pq}\right]$$

$$=\frac{p+q}{pq}[-pq]$$

=-(p + q)

3. Question

Mark the correct alternative in the following:

If the sum of n terms of an A.P. be $3n^2-n$ and its common difference is 6, then its first term is

A. 2

B. 3

C. 1

D. 4

Answer

Here, $S_n=3n^2-n$

Then, $S_1=3(1^2)-1$

=2

So,first term is $a_1=S_1=2$

4. Question

Mark the correct alternative in the following:

Sum of all two digit numbers which when divided by 4 yield unity as remainder is

A. 1200

B. 1210

C. 1250

D. none of these

Answer

Two digits numbers which when divided by 4 yield unity as remainder is A.P. as 13, 17, 21,...,97

In this A.P., first term is a=13

Last term is l=97

And d=17-13=4

Now, $l=a_n=a+(n-1)d$

∴ 97=13+(n-1)4

∴ 84=(n-1)4

∴ 21=n-1

∴ n=22

Now, $S_n=\frac{n}{2}[a+l]$

$=\frac{22}{2}[13+97]$

=11×110

=1210

5. Question

Mark the correct alternative in the following:

In A.M.'s are introduced between 3 and 17 such that the ratio of the last mean to the first mean is 3 : 1, then the value of n is

A. 6

B. 8

C. 4

D. none of these

Answer

Let a=4 ;b=17

Let $a_1, a_2, a_3, \ldots a_n$ be the n A.M's between 3 and 17

Then, common difference is given by

$$d=\frac{b-a}{n+1}$$

$$\therefore d=\frac{17-3}{n+1}$$

$$=\frac{14}{n+1}$$

Now, $a_1=a+d$

$$=3+\frac{14}{n+1}$$

$$=\frac{3n+3+14}{n+1}$$

$$=\frac{3n+17}{n+1}$$

And $a_n=a+nd$

$$=3+\frac{14n}{n+1}$$

$$=\frac{3n+3+14n}{n+1}$$

$$=\frac{17n+3}{n+1}$$

Now, given that $\frac{a_n}{a_1}=\frac{3}{1}$

$$\therefore \frac{\left\{\frac{17n+3}{n+1}\right\}}{\left\{\frac{3n+17}{n+1}\right\}}=\frac{3}{1}$$

$$\therefore \frac{17n+3}{3n+17}=\frac{3}{1}$$

$$\therefore 17n+3=3(3n+17)$$

$$\therefore 17n-9n=51-3$$

$$\therefore 8n=48$$

$$\therefore n=6$$

6. Question

Mark the correct alternative in the following:

If S_n denotes the sum of first n terms of an A.P. $<a_n>$ such that $\frac{S_m}{S_n}=\frac{m^2}{n^2}$, then $\frac{a_m}{a_n}=$

A. $\frac{2m+1}{2n+1}$

B. $\frac{2m-1}{2n-1}$

C. $\frac{m-1}{n-1}$

D. $\frac{m+1}{n+1}$

Answer

Here, $\frac{S_m}{S_n} = \frac{m^2}{n^2}$

$\therefore \frac{\left\{\frac{m}{2}[2a+(m-1)d]\right\}}{\left\{\frac{n}{2}[2a+(n-1)d]\right\}} = \frac{m^2}{n^2}$

$\therefore \frac{[2a+(m-1)d]}{[2a+(n-1)d]} = \frac{m}{n}$

$\therefore n[2a+(m-1)d] = m[2a+(n-1)d]$

$\therefore 2an + mnd - nd = 2am + mnd - md$

$\therefore 2a(n-m) = d(n-m)$

$\therefore 2a = d$

Now, $\frac{a_m}{a_n} = \frac{\{a+(m-1)d\}}{\{a+(n-1)d\}}$

$= \frac{\{a+(m-1)(2a)\}}{\{a+(n-1)(2a)\}}$

$= \frac{a\{1+2m-2\}}{a\{1+2n-2\}}$

$= \frac{2m-1}{2n-1}$

7. Question

Mark the correct alternative in the following:

The first and last terms of an A.P. are 1 and 11. If the sum of its terms is 36, then the number of terms will be

A. 5

B. 6

C. 7

D. 8

Answer

Here,a=1 and l=11 and S_n=36

Now,$S_n = \frac{n}{2}[a+l]$

$\therefore 36 = \frac{n}{2}[1+11]$

$\therefore 36 \times 2 = n(12)$

$\therefore \frac{72}{12} = n$

$\therefore n = 6$

8. Question

Mark the correct alternative in the following:

If the sum of n terms of an A.P., is $3n^2 + 5n$ which of its terms is 164?

A. 26th

B. 27th

C. 28th

D. none of these

Answer

Here, $S_n=3n^2+5n$

$\therefore S_1=3+5=8$

$\therefore a_1=S_1=8$

And $S_2=3\times2^2+5\times2=22$

Now, $a_2=S_2-S_1=22-8=14$

$\therefore d=a_2-a_1=14-8=6$

Now, $a_n=a_1+(n-1)d$

$\therefore 164=8+(n-1)6$

$\therefore 156=(n-1)6$

$\therefore 26=n-1$

$\therefore n=27$

9. Question

Mark the correct alternative in the following:

If the sum of n terms of an A.P. is $2n^2 + 5n$, then its nth term is

A. 4n − 3

B. 3n − 4

C. 4n + 3

D. 3n + 4

Answer

Here, $S_n=2n^2+5n$

$\therefore S_1=2+5=7$

$\therefore a_1=S_1=7$

And $S_2=2\times2^2+5\times2=18$

Now, $a_2=S_2-S_1=18-7=11$

$\therefore d=a_2-a_1=11-7=4$

Now, $a_n=a_1+(n-1)d$

$=7+(n-1)4$

$=7+4n-4$

$=4n+3$

10. Question

Mark the correct alternative in the following:

If $a_1, a_2, a_3, \ldots a_n$ are in A.P. with common difference d, then the sum of the series sin d $[\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \ldots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n]$ is

A. $\sec a_1 - \sec a_n$

B. $\operatorname{cosec} a_1 - \operatorname{cosec} a_n$

C. $\cot a_1 - \cot a_n$

D. $\tan a_1 - \tan a_n$

Answer

Here,a_1, a_2,...,a_n are in A.P.

Then $d=a_2-a_1=a_3-a_2=\ldots=a_n-a_{n-1}$

$\sin d[\operatorname{cosec} a_1 \operatorname{cosec} a_2 + \operatorname{cosec} a_2 \operatorname{cosec} a_3 + \cdots + \operatorname{cosec} a_{n-1} \operatorname{cosec} a_n$

$$= \frac{\sin d}{\sin a_1 \sin a_2} + \frac{\sin d}{\sin a_2 \sin a_3} + \cdots + \frac{\sin d}{\sin a_{(n-1)} \sin a_n}$$

$$= \frac{\sin(a_2 - a_1)}{\sin a_1 \sin a_2} + \frac{\sin(a_3 - a_2)}{\sin a_2 \sin a_3} + \cdots + \frac{\sin(a_n - a_{n-1})}{\sin a_{(n-1)} \sin a_n}$$

$$= \frac{(\sin a_2 \cos a_1 - \cos a_2 \sin a_1)}{\sin a_1 \sin a_2} + \frac{(\sin a_3 \cos a_2 - \cos a_3 \sin a_2)}{\sin a_2 \sin a_3} + \cdots + \frac{(\sin a_n \cos a_{(n-1)} - \cos a_n \sin a_{(n-1)})}{\sin a_{(n-1)} \sin a_n}$$

$= \cot a_1 - \cot a_2 + \cot a_2 - \cot a_3 + \cdots + \cot a_{n-1} \cot a_n$

$= \cot a_1 - \cot a_n$

11. Question

Mark the correct alternative in the following:

In the arithmetic progression whose common difference is non-zero, the sum of first 3 n terms is equal to the sum of next n terms. Then the ratio of the sum of the first 2 n terms to the next 2 n terms is

A. 1/5

B. 2/3

C. 3/4

D. none of these

Answer

Here, sum of first 3n terms=sum of next n terms i.e. $S_{3n}=S_{4n}-S_{3n}$

i.e. $2S_{3n}=S_{4n}$

$\therefore 2\frac{3n}{2}[2a+(3n-1)d]=\frac{4n}{2}[2a+(4n-1)d]$

$\therefore 6[2a+3nd-d]=4[2a+4nd-d]$

$\therefore 12a+18nd-6d=8a+16nd-4d$

$\therefore 4a+2nd-2d=0$

$\therefore 2a+nd-d=0$ ----------(1)

Sum of 2n terms is S_{2nand} sum of next 2n terms is given by $S_{4n}-S_{2n}$

Now, $\frac{S_{4n}-S_{2n}}{S_{2n}}=\frac{S_{4n}}{S_{2n}}-1$

$=\frac{\left\{\frac{4n}{2}[2a+(4n-1)d]\right\}}{\left\{\frac{2n}{2}[2a+(2n-1)d]\right\}}-1$

$=\frac{2\{[2a+4nd-d]\}}{\{2a+2nd-d\}}-1$

$=\frac{2\{(2a+nd-d)+3nd\}}{\{(2a+nd-d)+nd\}}-1$

$=\frac{2(0+3nd)}{(0+nd)}-1$ ($\because$ using (1))

$=\frac{6nd}{nd}-1$

$=\frac{5}{1}$

So, ratio of the sum of first 2n terms to the next 2n terms is 1:5

12. Question

Mark the correct alternative in the following:

If $a_1, a_2, a_3, \ldots a_n$ are in A.P. with common difference d, then the sum of the series $\sin d\,[\sec a_1 \sec a_2+\sec a_2 \sec a_3+\ldots+\sec a_{n-1} \sec a_n]$, is

A. $\sec a_1-\sec a_n$

B. $\operatorname{cosec} a_1-\operatorname{cosec} a_n$

C. $\cot a_1-\cot a_n$

D. $\tan a_n-\tan a_1$

Answer

Here,a_1, a_2,...,a_n are in A.P.

Then $d=a_2-a_1=a_3-a_2=\ldots=a_n-a_{n-1}$

$\sin d[\sec a_1 \sec a_2+\sec a_2 \sec a_3+\cdots+\sec a_{n-1} \sec a_n$

$=\frac{\sin d}{\cos a_1 \cos a_2}+\frac{\sin d}{\cos a_2 \cos a_3}+\cdots+\frac{\sin d}{\cos a_{(n-1)} \cos a_n}$

$$= \frac{\sin(a_2 - a_1)}{\cos a_1 \cos a_2} + \frac{\sin(a_3 - a_2)}{\cos a_2 \cos a_3} + \cdots + \frac{\sin(a_n - a_{n-1})}{\cos a_{(n-1)} \cos a_n}$$

$$= \frac{(\sin a_2 \cos a_1 - \cos a_2 \sin a_1)}{\cos a_1 \cos a_2} + \frac{(\sin a_3 \cos a_2 - \cos a_3 \sin a_2)}{\cos a_2 \cos a_3} + \cdots + \frac{(\sin a_n \cos a_{(n-1)} - \cos a_n \sin a_{(n-1)})}{\cos a_{(n-1)} \cos a_n}$$

$$= \tan a_2 - \tan a_1 + \tan a_3 - \tan a_2 + \cdots + \tan a_n \tan a_{n-1}$$

$$= \tan a_n - \tan a_1$$

13. Question

Mark the correct alternative in the following:

If four numbers in A.P. are such that their sum is 50 and the greatest number is 4 times the least, then the numbers are

A. 5, 10, 15, 20

B. 4, 10, 16, 22

C. 3, 7, 11, 15

D. none of these

Answer

Let 4 numbers of A.P. be the a, a+d, a+2d, a+3d.

Here, given that their sum is 50.

i.e. a+a+d+a+2d+a+3d=50

∴ 4a+6d=50

∴ 2a+3d=25_____(1)

Also, given that the greatest number is 4 times the least.

i.e.4a=a+3d

∴ 3a=3d

∴ a=d

Substituting a=d in (1) we get,

5a=25

∴ a=5

And, d=a=5

So, numbers are a=5;a+d=5+5;a+2d=5+2(5);a+3d=5+3(5)

i.e. 5,10,15,20

14. Question

Mark the correct alternative in the following:

In n arithmetic means are inserted between 1 and 31 such that the ratio of the first mean and nth mean is 3 : 29, then the value of n is

A. 10

B. 12

C. 13

D. 14

Answer

Let a=1;b=31

Let $a_1,a_2,a_3,...a_n$ be the n A.M's between 1 and 31

Then, common difference is given by

$$d = \frac{b-a}{n+1}$$

$$\therefore d = \frac{31-1}{n+1}$$

$$= \frac{30}{n+1}$$

Now, $a_1 = a + d$

$$= 1 + \frac{30}{n+1}$$

$$= \frac{n+1+30}{n+1}$$

$$= \frac{n+31}{n+1}$$

And $a_n = a + nd$

$$= 1 + \frac{30n}{n+1}$$

$$= \frac{n+1+30n}{n+1}$$

$$= \frac{31n+1}{n+1}$$

Now, given that $\frac{a_1}{a_n} = \frac{3}{29}$

$$\therefore \frac{\left\{\frac{n+31}{n+1}\right\}}{\left\{\frac{31n+1}{n+1}\right\}} = \frac{3}{29}$$

$$\therefore \frac{n+31}{31n+1} = \frac{3}{29}$$

$$\therefore 29(n+31) = 3(31n+1)$$

$$\therefore 29n - 93n = 3 - 899$$

$$\therefore -64n = -896$$

$$\therefore n = 14$$

15. Question

Mark the correct alternative in the following:

Let S_n denote the sum of n terms of an A.P. whose first term is a. If the common difference d is given by $d = S_n - k\,S_{n-1} + S_{n-2}$, then k =

A. 1

B. 2

C. 3

D. none of these

Answer

We know $S_n=S_{n-2}+a_{n-1}+a_n$

$=S_{n-2}+a+(n-2)d+a+(n-1)d$

$=S_{n-2}+2a+2nd-3d$

Also, $S_{n-1}=S_{n-2}+a_{n-1}$

$=S_{n-2}+a+(n-2)d$

$=S_{n-2}+a+nd-2d$

Now, $d=S_n-kS_{n-1}+S_{n-2}$

$= S_{n-2}+2a+2nd-3d-k(S_{n-2}+a+nd-2d)+S_{n-2}$

$=(2-k)[S_{n-2}+a+nd]+d(2k-3)$

Comparing coefficient of both the side we get,

2-k=0 and 2k-3=1

$\therefore$ k=2

16. Question

Mark the correct alternative in the following:

The first and last term of an A.P. are a and l respectively. If S is the sum of all terms of the A.P. and the common difference is given by $\frac{l^2-a^2}{k-(l+a)}$, then k =

A. S

B. 2S

C. 3S

D. none of these

Answer

Here, $a_n=l$

$\therefore$ a+(n-1)d=l

$\therefore$ (n-1)d=l-a

$\therefore n=\frac{l-a}{d}+1$

Now, $S_n=S$

$\therefore \frac{n}{2}[2a+(n-1)d]=S$

$\therefore (\frac{l-a}{d}+1)[2a+\left(\frac{l-a}{d}+1-1\right)d]=2S$

$\therefore \left(\frac{l-a}{d}+1\right)[2a+(l-a)]=2S$

$\therefore \left(\frac{l-a}{d}+1\right)[a+l]=2S$

$\therefore \left(\frac{l-a}{d}+1\right)=2S/(a+l)$

$\therefore \frac{l-a}{d}=\frac{2S}{a+l}-1$

$\therefore \frac{l-a}{d}=\frac{2S-a-l}{a+l}$

$\therefore d=\frac{(l-a)(a+l)}{2S-(a+l)}$

$\therefore d=\frac{(l^2-a^2)}{2S-(a+l)}$ ______(1)

Now, given that $d=\frac{l^2-a^2}{k-(a+l)}$ ______(2)

Comparing (1) and (2) we get

k=2S

17. Question

Mark the correct alternative in the following:

If the sum of first n even natural numbers is equal to k times the sum of first n odd natural numbers, then k =

A. $\frac{1}{n}$

B. $\frac{n-1}{n}$

C. $\frac{n+1}{2n}$

D. $\frac{n+1}{n}$

Answer

Let first A.P is 2,4,6...

So, a=2 and d=4-2=2

Let S_1 denotes the sum of n even numbers

Now, sum of n term is given by,

$S_n=\frac{n}{2}(2a+(n-1)d)$

$=\frac{n}{2}(2\times 2+(n-1)2)$

$=\frac{n}{2}(4+2n-2)$

$=\frac{n}{2}(2n+2)$

$=n(n+1)$

Let second A.P is 1,3,5...

So, a=1 and d=3-1=2

Let S_2 denotes the sum of n odd numbers

Now, sum of n term is given by,

$$S_2 = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(2 \times 1 + (n-1)2)$$

$$= \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{n}{2}(2n)$$

$$= n^2$$

Now, given that $S_1 = kS_2$

$\therefore n(n+1) = kn^2$

$$\therefore\ k = \frac{n+1}{n}$$

18. Question

Mark the correct alternative in the following:

If the first, second and last term of an A.P. are a, b and 2a respectively, then its sum is

A. $\frac{ab}{2(b-a)}$

B. $\frac{ab}{b-a}$

C. $\frac{3ab}{2(b-a)}$

D. none of these

Answer

Here, $a_1 = a; a_2 = b$ and $l = 2a$

Then $d = a_2 - a_1 = b - a$

So, $l = a_n = a_1 + (n-1)d$

$\therefore 2a = a + (n-1)(b-a)$

$\therefore a = n(b-a) - (b-a)$

$\therefore a = n(b-a) - b + a$

$\therefore n(b-a) = b$

$$\therefore n = \frac{b}{b-a}$$

Now, $S_n = \frac{n}{2}(a_1 + l)$

$$= \frac{\frac{b}{b-a}}{2}(a + 2a)$$

$= \frac{b}{2(b-a)}(3a)$

$= \frac{3ab}{2(b-a)}$

19. Question

Mark the correct alternative in the following:

If, S_1 is the sum of an arithmetic progression of 'n' odd number of terms and S_2 the sum of the terms of the series in odd places, then $\frac{S_1}{S_2} =$

A. $\frac{2n}{n+1}$

B. $\frac{n}{n+1}$

C. $\frac{n+1}{2n}$

D. $\frac{n+1}{n}$

Answer

Here, First A.P is 1,3,5...

So, a=1 and d=3-1=2

Now, sum of n term is given by,

$S_1 = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2 \times 1 + (n-1)2)$

$= \frac{n}{2}(2 + 2n - 2)$

$= \frac{n}{2}(2n)$

$= n^2$

Now, second A.P. is 1,5,9,...

In this A.P. a=1, d=5-1=4

Let us assume that total no. of term in first A.P. is even then total no. of term in second A.P. is $\frac{n}{2}$

Now, $S_2 = \frac{\frac{n}{2}}{2}\left(2a + \left(\frac{n}{2} - 1\right)d\right.$

$= \frac{n}{4}\left(2 + \left(\frac{n}{2} - 1\right)4\right)$

$= \frac{n}{4}(2 + 2n - 4)$

$= \frac{n}{4}(2n - 2)$

$= \frac{n}{2}(n-1)$

Then $\frac{S_1}{S_2} = \frac{n^2}{\left\{\frac{n(n-1)}{2}\right\}}$

$= \frac{2n}{n-1}$

Let us assume that total no. of term in first A.P. is odd then total no. of term in second A.P. is $\frac{n+1}{2}$

Now, $S_2 = \frac{\frac{n+1}{2}}{2}\left(2a + \left(\frac{n+1}{2} - 1\right)d\right.$

$= \frac{n+1}{4}\left(2 + \left(\frac{n+1}{2} - 1\right)4\right)$

$= \frac{n+1}{4}(2 + 2n + 2 - 4)$

$= \frac{n+1}{4}(2n)$

$= \frac{n}{2}(n+1)$

Then $\frac{S_1}{S_2} = \frac{n^2}{\left\{\frac{n(n+1)}{2}\right\}}$

$= \frac{2n}{n+1}$

20. Question

Mark the correct alternative in the following:

If in an A.P., $S_n = n^2p$ and $S_m = m^2p$, where S_r denotes the sum of r terms of the A.P., then S_p is equal to

A. $\frac{1}{2}p^3$

B. mn p

C. p^3

D. $(m+n)p^2$

Answer

Here, $S_n = n^2p$

Now, substituting n=p we get

$S_p = p^2p = p^3$

21. Question

Mark the correct alternative in the following:

If in an A.P., the pth term is q and $(p+q)^{th}$ term is zero, then the q^{th} term is

A. $-p$

B. p

C. p + q

D. p − q

Answer

Here, $a_p = q$ and $a_{(p+q)} = 0$

i.e. a+(p-1)d=q______(1)

and a+(p+q-1)d=0_____(2)

Subtracting (1) from (2) we get,

qd=-q

∴ d=-1

substituting this in (2) we get,

a+(p+q-1)(-1)=0

∴ a=p+q-1

So, $a_q = a+(q-1)d$

=p+q-1+(q-1)(-1)

22. Question

Mark the correct alternative in the following:

The 10th common term between the A.P.s 3, 7, 11, 15, ... and 1, 6, 11, 16, ... is

A. 191

B. 193

C. 211

D. none of these

Answer

Here, common difference of first A.P. is d=7-3=4

And common difference of second A.P. is d’=6-1=5

Now L.C.M of d and d’=(4,5)=20

Also first common term in both A.P.s is 11.

We have to find 10th common term i.e. n=10

Consider an A.P. in which a=11,d=20 and n=10

Then, $a_{10} = a+(10-1)d$

=11+9×20

=11+180

=191

23. Question

Mark the correct alternative in the following:

If in an A.P. $S_n = n^2 q$ and $S_m = m^2 q$, where S_r denotes the sum r terms of the A.P., then S_q equals

A. $\frac{q^3}{2}$

B. mnq

C. q^3

D. $\left(m^2+n^2\right)q$

Answer

Here, $S_n=n^2q$

Now, substituting n=q we get

$S_q=q^2p=q^3$

24. Question

Mark the correct alternative in the following:

Let S_n denote the sum of first n terms of an A.P. If $S_{2n}=3S_n$, then $S_{3n}:S_n$ is equal to

A. 4

B. 6

C. 8

D. 10

Answer

Here,$S_{2n}=3S_n$

$\therefore \frac{2n}{2}[2a+(2n-1)d]=\frac{3n}{2}[2a+(n-1)d]$

$\therefore 2[2a+2nd-d]=3[2a+nd-d]$

$\therefore 4a+4nd-2d=6a+3nd-3d$

$\therefore 2a-nd-d=0$

$\therefore 2a=nd+d$

Now,$\frac{S_{3n}}{S_n}=\frac{\left\{\frac{3n}{2}[2a+(3n-1)d]\right\}}{\left\{\frac{n}{2}[2a+(n-1)d]\right\}}$

$=\frac{\{3[2a+3nd-d]\}}{\{2a+nd-d\}}$

$=\frac{\{3[nd+d+3nd-d]\}}{\{nd+d+nd-d\}}$

$=\frac{3[4nd]}{2nd}$

$=\frac{12}{2}$

$=\frac{6}{1}$

$\therefore S_{3n}:S_n=6:1$