18. Surface Area and Volume of a Cuboid and a Cude

Exercise 18.1

1. Question

Find the lateral surface area and total surface area of a cuboid of length 80 cm, breadth 40 cm and height 20 cm.

Answer

Given,

Length of a cuboid =80 cm

Breadth of cuboid = 40 cm

Height of cuboid = 20 cm

So, lateral surface area of cuboid = $2(l+b)h = 2(80+40)\times20 = 4800\ cm^2$

Total surface area of cuboid = $2(lb+bh+hl) = 2(80\times40+40\times20+20\times80) = 11200\ cm^2$

2. Question

Find the lateral surface area and total surface area of a cube of edge 10 cm.

Answer

Given,

Edge of cube = 10 cm

So, total surface area of cube = $6a^2 = 6\times100 = 600cm^2$

Lateral surface area of cube = $4a^2 = 4\times100 = 400\ cm^2$

3. Question

Find the ratio of the total surface area and lateral surface area of a cube.

Answer

$$\frac{total\ surface\ area\ of\ cube}{lateral\ surface\ area\ of\ cube} = \frac{6a^2}{4a^2} = \frac{6}{4} = \frac{3}{2}$$

4. Question

Mary wants to decorate her Christmea tree. She wants to place the tree on a wooden block covered with coloured paper with picture of Sants Claus on it. She must know the exact quantity of paper to buy for this purpose. If the box has length, breadth and height as 80 cm, 40 cm and 20 cm respectively. How many square sheets of paper of side 40 cm would she require?

Answer

Given,

Dimensions of box = 80cm ×40cm×20cm

So,

quantity of paper required by Mary to cover the wooden block is total surface area = $2(80\times40+40\times20+20\times80)$

$= 11200\ cm^2$

Hence,

number of square sheet of side 40 cm required $= \frac{total\ surface\ area\ of\ box}{area\ of\ sheet}$

$= \frac{11200}{40 \times 40} = 7$

5. Question

The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of white washing the walls of the room and the ceiling at the rate of Rs. 7.50 m^2.

Answer

Given,

Dimension of room = 5m×4m×3m

Area of walls = 2(l+b)h = 2×9×3 = 54 m^2

Area of ceiling = 20m^2

So, total area to be whitewashed = 20+54 = 74m^2

Hence, cost of whitewashing at rate Rs 7.50 per m^2 = 7.50×74 = Rs.555

6. Question

Three equal cubes are placed adjacently in a row. Find the ratio to total surface area on the new cuboid to that of the sum of the surface areas of the three cubes.

Answer

Given,

Let length of an edge of three equal cubes = a

Total surface area of 3 cubes = 3× $6a^2$ = $18a^2$

(While putting these cubes in a row adjacently, a cuboid is formed)

∴ length of cuboid = 3a

Breadth of cuboid = a

Height of cuboid = a

So,

total surface area of cuboid = $2(3a^2+a^2+3a^2)$ = 14 a^2

Hence, $\frac{total\ surface\ area\ of\ new\ cuboid}{sum\ of\ area\ of\ 3\ cubes} = \frac{14a^2}{18a^2} = \frac{14}{18} = \frac{7}{9}$

7. Question

A 4 cm cube is cut into 1 cm cubes. Calculate the total surface area of all the small cubes.

Answer

Given,

Original edge of cube = 4 cm

Original volume of cube = 4^3 = 64 cm^3

After cutting ,

Edge of cube = 1 cm

Hence volume of new cube = 1^3 = 1 cm^3

Number of small cubes thus formed = $\frac{64}{1} = 64$

Hence , total surface area of 64 cubes = 64× $6a^2$ = 64×6 = 384 m^2

8. Question

The length of a hall is 18 m and the width 12 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall.

Answer

Given,

Length of a hall = 18 m

Width of hall = 12 m

Sum of area of floor and area of roof = sum of area of four walls

Let height of hall = h m

= l×b+l×b = 2(l+b)h

= 2lb = 2(l+b)h = h = $\frac{lb}{l+b} = \frac{18\times12}{30} = 7.2\ m$

9. Question

Hameed has built a cubical water tank with lid for his house, with each other edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if the cost of tiles is Rs. 360 per dpzen.

Answer

Given,

Each edge of cubical water tank = 1.5m

So,

area of 5 faces of tank(excluding base) = $5\times(1.5)^2 = 5\times150\times150\ cm^2$

Area of each tile = $25\times25 = 625\ cm^2$

Number of tiles required = $\frac{area\ of\ 5\ walls}{area\ of\ a\ tile} = \frac{5\times150\times150}{625} = 180.$

∵ cost of 1 dozen tiles = Rs.360

∴ cost of 180 tiles (15 dozen) = 15×360 = Rs. 5400

10. Question

Each edge of a cube is increases by 50%. Find the percentage increase in the surface of area of the cube.

Answer

Given,

Increment in each edge = 50%

Let edge of cube = a

So surface area of cube = a^2

(after 50% increase)

Length of edge of cube = a+ a×$\frac{50}{100} = a + \frac{a}{2} = \frac{3a}{2}$

Surface area of cube = $\left(\frac{3a}{2}\right)^2 = \frac{9a^2}{4}$.

Increment in area = $\frac{9a^2}{4} - a^2 = \frac{5a^2}{4}$

% increment = $\frac{(increased\ area)}{original\ area} = \frac{\frac{5a^2}{4}}{a^2}\times100 = 125\%$

11. Question

The dimensions of a rectangular box are in the ratio of 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs. 8 and Rs. 9.50 per m^2 is Rs. 1248. Find the dimensions of the box.

Answer

Given,

Ratio of dimensions of rectangular box = 2:3:4

Let dimensions are = 2x ,3x , 4x

So, total surface area of box = $2(6x^2+12x^2+8x^2) = 52\ x^2$

Let, c_1 = cost of covering at rate Rs. 8 per m^2

$C_1 = 8\times52x^2$

Let c_2 = cost of covering at rate Rs.9.50 per m^2

$C_2 = 9.50\times\ 52x^2$

$= C_2 - C_1 = 1248$ (given)

$= 52x^2\times9.50 - 52x^2\times8 = 1248$

$= 52x^2 = \frac{1248}{1.50}$

$= x^2 = \frac{1248}{1.50\times52} = 16$

= x = 4

Hence dimensions of box are = 2x = 2×4 = 8 cm

= 3x = 3×4 = 12 cm

=4x= 4×4 = 16cm

12. Question

A closed iron tank 12 m long, 9 m wide and 4 m deep is to be made. Determine the cost of iron sheet used at the rate of Rs. 5 per metre sheet, sheet being 2 m wide.

Answer

Given,

Length of closed iron tank = 12m

Breadth = 9m

Height = 4m

Total surface area of tank = $2(12\times9+9\times4+4\times12) = 2(192) = 384\ m^2$

Width of iron sheet = 2m (given)

So, length of iron sheet = $\frac{384}{2} = 192\ m$

Hence, cost of iron sheet at rate Rs.5 per meter = 5×192 = Rs.960

13. Question

Ravish wanted to make a temporary shelter for his car by making a box-like structure with tarpaulin that covers all the four sides and the top of the car (with the front face as a flap which can be rolled up). Assuming that the stitching margins are very small, and therefore negligible, how much tarpaulin would be required to make the shelter of height 2.5 m with base dimensions 4 m × 3 m?

Answer

Given,

Height of shelter = 2.5 m

Base dimensions = l×b = 4m×3m

So,

area of tarapaulin needed = lateral surface area of shelter +area of roof

= 2(l+b)h + l×b

= 2(4+3)2.5 +12 = 47 m^2

14. Question

An open box is made of wood 3 cm thick. Its external length, breadth and height are 1.48 m, 1.16 m and 8.3 dm. Find the cost of painting the inner surface of Rs. 50per sq. metre.

Answer

Given,

Thickness of wood = 3cm

External length of box = 1.48 m = 148 cm

External breadth of box = 1.16 m = 116 cm

External height of box = 8.3 dm = 83 cm

So, inner dimensions of box = external dimension – thickness of wood

Inner length = 148- (2×3) = 142 cm

Inner breadth = 116 –(2×3) = 110 cm

Inner height = 83-3 = 80 cm

Hence , inner surface area of box = lateral surface area +area of roof

=2(l+b)h + l×b

= 2 (142+110)83 + 142×110 = 55940 cm^2 = 5.5940 m^2

∴ cost of painting at rate Rs.50 per m^2 = 50×5.5940 = Rs. 279.70

15. Question

The cost of preparing the walls of a room 12 m long at the rate of Rs. 1.35 per square metre is Rs. 340.20 and the cost of matting the floor at 85 paise per square metre is Rs. 91.80. Find the height of the room.

Answer

Given,

Length of room = 12m

Let breadth of room = b m

Let height of room = h m

So, lateral surface area of room = 2(l+b)h m^2

Area of floor = l×b m^2

Cost of matting the floor = area of floor × rate of matting

= 91.80 = l×b ×.85

= b = $\frac{91.80\times100}{85\times12} = 9\ m$

Cost of preparing wall = area of four walls ×rate of preparing

= 340.20 = 2(l+b)h $\times\frac{135}{100}$

$= h = \frac{340.20\times100}{42\times135} = 6\,m$

16. Question

The dimensions of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room; each door measures 2.5 m by 1.2 m and each window 1.5 m by 1 m. Find the cost of painting the walls at Rs. 3.50 per square metre.

Answer

Given,

Dimension of room = 12.5m×9m×7m

Dimension of door = 2.5m×1.2m

Dimension of window = 1.5m×1m

So, area of room = 2(l+b)h = 2(12.5+9)×7 = 301 m^2

Area of 2 doors = 2(2.5×1.2) = 6m^2

Area of 4 windows = 4(1.5×1) = 6 m^2

So, area to be painted = (301-12) = 289 m^2

∴ cost of painting walls at rate Rs.3.50 per m^2 = 3.50×289 = Rs.1011.50

17. Question

The length and breadth of a hall are in the ratio 4 : 3 and its height is 5.5 metres. The cost of decorating its walls (including doors and windows) at Rs. 6.60 per square metre is Rs. 5082. Find the length and breadth of the room.

Answer

Given,

Ratio of length and breadth of a hall = 4:3

Let length of hall = 4x m

Let breadth of hall = 3x m

Height of hall = 5.5m (given)

So, lateral surface area of hall = 2(l+b)h =2(7x) ×5.5 = 77x m^2

Cost of decorating walls = lateral surface area +rate of decoration per m^2

= 5082 = $77x\times6.60$

$= x = \frac{5080}{77\times6.60} = 10$

So, length of hall = 4x = 4×10 = 40 m

Breadth of hall = 3x = 3×10 = 30 m

18. Question

A wooden bookshelf has external dimensions as follows : Height = 110 cm, Depth = 25 cm, Breadth = 85 cm (See Fig. 18.5). The thickness of the plank is 5 cm everywhere. The external faces are to be polished and the inner faces are to be painted. If the rate of polishing is 20 paise per cm^2 and the rate of painting is 10 paise per cm^2. Find the total expenses required for polishing and painting the surface of the bookshelf.

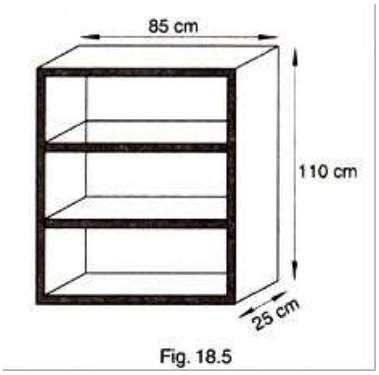

Fig. 18.5

Answer

Given,

External dimensions of wooden shelf = 85cm×25cm×110cm

Thickness of plank = 5 cm

So,

area to be polished = area of four walls +area of back+ area of front beading

= [2(110+85)×25 +110×85 +(75×5)×4] = 21700 cm^2

Cost of polishing at rate 20paise per cm^2 = $21700\times\frac{20}{100}=Rs.4340$

Internal surface area = area of five faces of 3 cuboids each of dimension (75cm×30cm×20cm)

Total surface area of 3 cuboids = 3[2(75×30+30×20+20×75)] – 3(75×30)

= 6(2250+600+1500) – 6750 = 19350 cm^2

∴ cost of painting at rate 10p per cm^2 = $19350\times\frac{10}{100}=Rs.1935$

Hence, total expense = Rs.(4340+1935) = Rs. 6275

19. Question

The paint in a certain container is sufficient to paint on area equal to 9.375 m^2. How many bricks of dimension 22.5 cm×10cm ×7.5 cm can be painted out of this container?

Answer

Given,

Dimension of a brick = 22.5 cm×10cm×7.5 cm

Total surface area = 2(lb + bh + hl)

= 2[22.5× 10 + 10× 7.5 + 22.5 × 7.5] cm^2

= 2(225 + 75 + 168.75) cm^2

= 2(468.75) cm^2

= 937.5 cm^2

Area that can be painted = 9.375 m^2

As 1 m^2 = 10000 cm^2

9.375 m^2 = 9.375 × 10000

= 93750 cm^2

Number of bricks = $\frac{\textit{total area that get paint}}{\textit{surface area of one brick}} = \frac{93750}{937.5}$

$= \frac{937500}{93750}$

= 100 bricks

Exercise 18.2

1. Question

A cuboidal water tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water can it hold?

Answer

Given,

Length of cuboidal tank = 6m

Breadth of cuboidal tank = 5 m

Depth of cuboidal tank = 4.5 m

So, capacity of cuboid = l×b×h = 6×5×4.5 = 135m^3= 135000 litre

2. Question

A cubical vessel is 10 m long and 8 m wide. How high must it be made to hold 380 cubic metres of a liquid?

Answer

Given,

Length of cuboidal vessel = 10m

Width of cuboidal vessel = 8m

Let height of vessel = h m

So,

volume of vessel = l×b×h = 10×8×h m^3

= 10×8×h = 380

= h = $\frac{380}{10\times8} = 4.75\ m$

3. Question

Find the cost of digging a cuboidal pit 8 m long, 6 m broad and 3 m deep at the rate of Rs. 30 per m^3.

Answer

Given,

Dimension of cuboidal pit = 8m×6m×3m

So, volume of pit = 8×6×3 = 144 m^3

Cost of digging pit at rate Rs.30 per m^3 = 30×144 = Rs.4320

4. Question

If V is the volumw of cuboid of dimensions a,b,c and S is its surface area, then prove that

$\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

Answer

Given,

V= volume of cuboid

S = surface area of cuboid

=a = length of cuboid

=b =breadth of cuboid

= c = height of cuboid

V = abc

S = 2(ab+bc+ca)

Dividing surface area by volume

$= \frac{s}{v} = \frac{2(ab+bc+ca)}{abc}$

$= \frac{s}{v} = 2\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right) = \frac{1}{v} = \frac{2}{s}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ *Proved*

5. Question

The areas of three adjacent faces of a cuboid are *x, y,* and *z.* If the volumes is *V,* prove that $V^2 = xyz$.

Answer

Given,

Area of 3 adjacent faces of a cuboid = x, y, z

V = volume of cuboid

Let , a,b,c are respectively length , breadth, height of each faces of cuboid

So, x = ab

= y = bc

= z = ca

V = abc

Hence , xyz = ab×bc×ca = $(abc)^2$ = v^2 (v=abc)

= v^2 = xyz Proved.

6. Question

If the areas of three adjacent faces of a cuboid are 8 cm^2, 18 cm^2 and 25 cm^3. Find the vlume of the cuboid.

Answer

Given,

Area of 3 adjacent faces of cuboid = x= 8 cm^2 , y = 18 cm^2 , z = 25 cm^2

From previous question we get ,

= v^2 =xyz

= v^2= 8×18×25 = 3600

=v =$\sqrt{3600} = 60\ cm^3$

7. Question

The breadth of a room is twice its height, one half of its length and the volume of the room is 512 cu. Dm. Find its dimensions.

Answer

Given,

Breadth of room = 2×height of room = b = 2×h = h = $\frac{b}{2}$

Breadth of room = $\frac{1}{2} length\ of\ room = b = \frac{1}{2} l\ = l = 2b$

Volume of room = 512 dm^3

= l×b×h = 512

= 2b×b×$\frac{b}{2} = 512$

= b^3 = 512 = b = $\sqrt[3]{512} = 8\ dm$

So, breadth of room = 8 dm

Length of room = 2b = 2×8 = 16 dm

Height of room = $\frac{b}{2} = \frac{8}{2} = 4\ dm$

8. Question

A river 3 m deep and 40 m wide is flowing at the rate of 2 km per hour. How much water will fall into the sea ina minute?

Answer

Given,

Depth of river = 3m

Width of river = 40 m

Rate of flow of river = 20km/h

Length of river = distance travelled by water in 1 minute at speed 2km/h

=$\frac{2000}{60} = \frac{100}{3}\ m$

So, volume of water = $\frac{100}{3}$×40×3 = $4000\ m^3 = 4000000\ litre$

9. Question

Water in a canal 30 dm wide and 12 dm deep, is flowing with a velocity of 100 km per hour. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

Answer

Given,

Width of canal = 30 dm = 3m

Depth of canal = 12 dm = 1.2 m

Velocity of flow = 100 km/h

Length of cuboid thus formed = distance travelled by water in 30 minute at speed 100km/h

=$100 \times \frac{30}{60} = 50\ km = 50000m$

Volume of water = 50000×3×1.2 m^3

Water accumulated in field forms a cuboid of base area equals to area of field and height = $\frac{8}{100} m$

= hence , area of field = $\frac{5000 \times 3 \times 1.2 \times 100}{8} = 2250000\ m^2$

10. Question

Three metal cubes with edges 6 cm, 8 cm and 10 cm respectively are melted together and formed into a single cube. Find the volume, surface area and diagonal of the new cube.

Answer

Given,

Edges of 3 metal cubes = 6cm, 8cm, 10 cm

Volume of these cubes together = $6^3+8^3+10^3$ = 216+512+1000 = 1728 cm^3

Let volume of new cube formed = V

So, V = 1728

= a^3 = 1728 = a = $\sqrt[3]{1728} = 12\ cm$

Surface area of new cube = $6a^2 = 6\times12^2$ = 864 cm^2

Diagonal of new cube = $\sqrt{3}a = \sqrt{3}\times12 = 12\sqrt{3}\ cm$

11. Question

Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Answer

Given,

Volume of 2 equal cubes = 512 cm^3

= a^3 = 512 = a = $\sqrt[3]{512} = 8\ cm$

When these cubes joined end to end a cuboid is formed so,

Length of cuboid = 8+8 = 16 cm

Breadth of cuboid = 8 cm

Height of cuboid = 8 cm

So, surface area of cuboid = 2(lb+bh+hl) =2(16×8+8×8+8×16)

= 2×320 = 640 cm^2

12. Question

Half cubic metre of gold-sheet is extended by hammering so as to cover an area of 1 hectare. Find the thickness of the gold-sheet.

Answer

Given,

Volume of the gold sheet = $\frac{1}{2}\ m^3$

Area of sheet = 1 hectare = 10000 m^2

So, thickness of gold sheet = $\frac{volume\ of\ gold\ sheet}{area\ of\ gold\ sheet} = \frac{\frac{1}{2}m^3}{10000m^2} = \frac{1}{200}cm$

13. Question

A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube.

Answer

Given,

Edge of metal cube = 12 cm

Volume of cube = $a^3 = 12^3 = 1728\ cm^3$

Edge of 2 smaller cube = 6cm , 8 cm

Let edge of 3rd cube = x cm

So, $6^3+8^3+ x^3 = 1728$

$= x^3 = 1728 - (216+512) = 1000$

$= x = \sqrt[3]{1000} = 10\ cm$

14. Question

The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the hall, if each person requires 150 m^3 of air?

Answer

Given,

Dimension of cinema hall = 100m×50m×18m

Volume of cinema hall = $100×50×18\ m^3$

Number of persons can sit in cinema hall = $\frac{volume\ of\ hall}{one\ person\ air\ volume}$

$=\frac{100×50×18}{150} = 600$

15. Question

Given that 1 cubic cm of marble weighs 0.25 kg, the weight of marble block 28 cm in width and 5 cm thick is 112 kg. Find the length of the block.

Answer

Given,

Weight of $1m^3$ marble = 0.25 kg

Weight of marble 28 cm wide and 5 cm thick = 112 kg

Let length of block = l cm

Then volume of block = $l×28×5\ cm^3 = 140\ l\ cm^3$

Weight of block = 140 l ×0.25

= 112 = 140 l ×0.25

$= l = \frac{112×100}{140×0.25} = 3.2\ cm$

16. Question

A box with lid is made of 2 cm thick wood. Its external length, breadth and height are 25 cm, 18 cm and 15 cm respectively. How much cubic cm of liquid can be places in it> Also, find the volume of the wood used in it.

Answer

Given,

External dimensions of box = 25cm×18cm×15cm

Thickness of wood = 2 cm

So, internal dimensions of box = (25-4) cm×(18-4) cm×(15-4) cm

= 21cm×14cm×11cm

Volume of liquid can place in it = 21 ×14×11 = 3234 cm^3

Volume of wood use in it = external volume – internal volume

= $6750 - 3234 = 3516\ cm^3$

17. Question

The external dimensions of a closed wooden box are 48 cm, 36 cm, 30 cm. The box is made of 1.5 cm thick wood. How many bricks of size 6 cm × 3 cm×0.75 cm can be put in this box?

Answer

Given,

External dimensions of closed wooden box = 48cm×36cm×30cm

Thickness of wood = 1.5 cm

So inner dimensions of box would be = [(48-3)×(36-3)×(30-3)]

= 45cm×33cm×27cm

Volume of box = 45×33×27 = 38880 cm^3

Volume of a brick = 6×3×0.75 = 13.5 cm^3

So, number of bricks can put in box = $\frac{38880}{13.5} = 2970$

18. Question

How many cubic centimetres of iron are there in an open box whose external dimension are 36 cm, 25 cm and 16.5 cm, the iron being 1.5 cm thick throughout? If 1 cubic cm of iron weighs 15 g, find the weight of the empty box in kg.

Answer

Given,

External dimensions of box = 36cm×25cm×16.5cm

Thickness of iron = 1.5 cm

Inner dimensions of box = [(36-3)×(25-3)×(16.5-1.5)]

=33cm×22cm×15cm

Volume of iron used = external volume – internal volume

= 36×25×16.5 – 33×22×15 = 3960 cm^3

∵ weight of 1 cm^3 iron = 15 gm (given)

∴ weight of 3960 cm^3 iron = 3960×15 = 59400 gm = 59.4 kg

19. Question

A cube of 9 cm egde is immersed completely in a rectangular vessel containing water. If the dimensions of the base are 15 cm and 12 cm, find the rise in water level in the vessel.

Answer

Given,

Edge of cube = 9 cm

Dimension of base of rectangular vessel = 15cm×12cm

Volume of cube = 9×9×9 = 729 cm^3

Area of base of vessel = 15×12 = 180 cm^2

So, rise of water in vessel = $\frac{volume\ of\ cube}{area\ of\ base} = \frac{729}{180} = 4.05\ cm$

20. Question

A rectangular container, whose base is a square of side 5 cm, stands on a horizontal table, and holds water upto 1 cm from the top. When a cube is placed in the water it is completely submerged, the water rises to the top and 2 cubic cm of water overflows. Calculate the volume of the cube and also the length of its edge.

Answer

Given,

Side of square base of container = 5 cm

Height of water level in container = 1 cm from top

Water overflow after submerging of cube = 2 cm^3

Let edge of cube = a cm

Volume of cube = a^3 cm^3

= $a^3 = 5^2 \times 1 + 2 = 27\ cm^3$

= a $\sqrt[3]{27} = 3\ cm$

21. Question

A field is 200 m long and 150 m broad. There is a plot, 50 m long and 40 m broad, near the field. The plot is dug 7 m deep and the earth taken out is spread evenly on the field. By how many metres is the level of the field raised? Give the answer to the second place of decimal.

Answer

Given,

Length of field = 200 m

Breadth of field = 150 m

Dimension of plot = 50m×40m

Depth up to which plot is dug = 7 m

Volume of earth dug out = 50×40×7 = 14000 m^3

Let level of earth rises in field = h meter

Hence,

= 200×150×h = 14000

= h = $\frac{14000}{200 \times 150} = 0.47\ m$

22. Question

A field is in the form of a rectangle of length 18 m and width 15 m. A pit, 7.5 m long, 6 m broad and 0.8 m deep, is dug in a corner of the field and the earth taken out is spread over the remaining area of the field. Find out the extent to which the level of the field has been raised.

Answer

Given,

Dimension of rectangular field = 18m ×15m

Dimension of pit = 7.5m×6m×0.8m

So, area on which earth taken out is to be spread = 18×15 – 7.5×6=225 m^2

Let level of earth rises in field = h meter

So,

=225×h = 7.5×6×0.8

= h = $\frac{7.5\times6\times0.8}{225} = 0.16\ m = 16\ cm$

23. Question

A rectangular tank is 80 m long and 25 m broad. Water flows into it through a pipe whose cross-section is 25 cm^2, at the rate of 16 km per hour. How much the level of the water rise in the tank in 45 minutes.

Answer

Given,

Dimension of rectangular tank = 80m×25m = 8000cm×2500cm

Cross section area of pipe = 25 cm^2

Rate of flow of water through pipe = 16 km/h

Length of pipe = flow of water through it in 45 minute at speed 16 km/h

= $\frac{45}{60}\times16\times1000\times100\ cm$

Volume of pipe = $\frac{45}{60}\times25\times16000\times100\ cm^3$

Let level of water rises in tank in 45 minute = h cm

= 8000×2500×h = $\frac{45}{60}\times25\times16000\times100$

= h = $\frac{16000\times100\times45\times25}{60\times8000\times2500} = 1.5\ cm$

24. Question

Water in a rectangular reservoir having base 80 m by 60 m is 6.5 m deep. In what time can the water be emptied by a pipe of which the cross-section is a square of side 20 cm, if the water runs through the pipe at the rate of 15 km/hr.

Answer

Given,

Dimension of rectangular reservoir = 80m×60m

Depth of water in reservoir = 6.5 m

Side of square pipe = 20cm

Rate of flow of water through pipe = 15 km/h

Volume of reservoir = 80×60×6.5 m^3

Volume of pipe = $\frac{20}{100}\times\frac{20}{100}\times15000\ m^3$

Let reservoir be emptied in t hours

= $\frac{20}{100}\times\frac{20}{100}\times15000\times t = 80\times60\times6.5$

= t = $\frac{80\times60\times6.5\times100\times100}{20\times20\times15000} = 52\ hours$

25. Question

A village having a population of 4000 requires 150 litres of water per head per day. It has a tank measuring 20 m×15 m×6 m. For how many days will the water of this tank last?

Answer

Given,

Population of village = 4000

Requirement of water per head per day = 150 litre

Dimension of tank = 20m×15m×6m

Let the water in tank last for x days ..

$= x \times 4000 \times 150 = 20 \times 15 \times 6 \times 1000$

$= x = \frac{20 \times 15 \times 6 \times 1000}{4000 \times 150} = 3\ days$

26. Question

A child playing with building blocks, which are of the shape of the cubes, has built a structure as shown in Fig. 18.12. If the edge of each cube is 3 cm, find the volume of the structure built by the child.

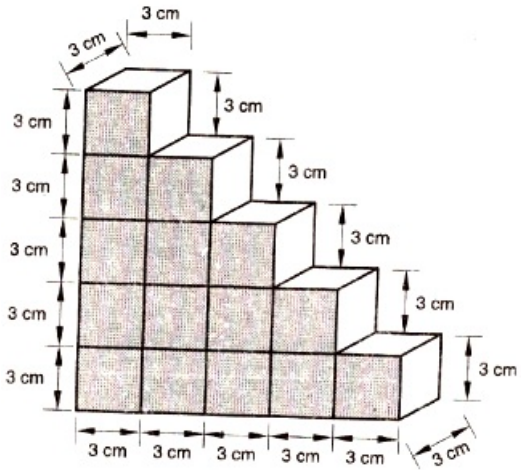

Fig. 18.12

Answer

Given,

Edge of cube = 3 cm

Volume of cube = 3^3 = 27 cm^3

Number of cubes = 15

Volume of structure = number of cubes × volume of one cube

= 15×27 = 405 cm^3

27. Question

A godown measures 40 m×25 m×10 m. Find the maximum number of wooden crates each measuring 1.5 m×1.25 m×0.5 m that can be stored in the godown.

Answer

Given,

Dimension of godown = 40m×25m×10m

Dimension of wooden crates = 1.5m×1.25m×0.5m

Number of wooden crates = $\frac{40 \times 25 \times 10}{1.5 \times 1.25 \times 0.5} = 10666.66$

28. Question

A wall of length 10 m was to be built across an open ground. The height of the wall is 4 m and thickness of

the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm×12 cm×8 cm, how many bricks would be required?

Answer

Given,

Length of wall = 10m

Height of wall = 4m

Thickness of wall = 24 cm

Dimension of bricks = 24cm×12cm×8cm

So, number of bricks required = $\frac{volume\ of\ wall}{volume\ of\ one\ brick} = \frac{1000\times400\times24}{24\times12\times8} = 4167.$

CCE - Formative Assessment

1. Question

If two cubes each of side 6 cm are joined face to face, then find the volume of the resulting cuboid.

Answer

Given,

Side of two equal cubes = 6 cm

(after joining them face to face a cuboid is formed)

Length of cuboid = 6+6 = 12 cm

Breadth of cuboid = 6 cm

Height of cuboid = 6 cm

So, volume of cuboid = l×b×h = 12×6×6 = 432 cm^3

2. Question

Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted down into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of three cubes.

Answer

Given,

Ratio of edge of 3 cubes = 3:4:5

Diagonal of new cube formed = 12√3 cm

Let edges are = 3x ,4x 5x

Comined volume = $(3x)^3 + (4x)^3 + (5x)^3$

= 216 x^3

New diagonal = √3a = 12√3 = a = 12 cm

So, a^3 = 216 x^3

= 12×12×12 = 216 x^3

= $x^3 = \frac{216}{12\times12\times12} = 8$

=x = $\sqrt[3]{8} = 2$

So , edges are = 3x = 3×2 = 6 cm

=4x = 4×2 = 8cm

= 5x = 5×2 = 10 cm

3. Question

If the perimeter of each face of a cube is 32 cm, find its lateral surface area. Note that four faces which meet the base of a cube are called its leteral faces.

Answer

Given,

Perimeter of each face of a cube = 32 cm

Let each edge will be = a cm

So,

4a = 32

= a = $\frac{32}{4} = 8\,cm$

Lateral surface area of cube = $4a^2 = 4\times8^2 = 256\ cm^2$

4. Question

Find the edge of a cube whose surface area is 432 m^2.

Answer

Given,

Surface area of cube = 432 m^2

Let edge of cube = a m

= 6 a^2 = 432

= $a^2 = \frac{432}{6} = 72$

= a = $\sqrt{36\times2} = 6\sqrt{2}\,m$

5. Question

A cuboid has total surface area of 372 cm^2 and its lateral surface area is 180 cm^2, find the area of its base.

Answer

Given,

Total surface area of cuboid = 372 cm^2

Lateral surface area = 180 cm^2

= 2(l+b) h = 180

= lh +bh = $\frac{180}{2} = 90$ .....(i)

And ,

= 2(lb+bh+hl) = 372

= lb+bh+hl = $\frac{372}{2} = 186$ .....(ii)

Putting value of lh+bh from equation (i) to (ii)

= lb +90 = 186

= lb = 186-90 = 96

=l×b = area of base = 96 cm^2

6. Question

Three cubes of each side 4 cm are joined end to end. Find the surface area of the resulting cuboid.

Answer

Given,

Edge of each cube = 4cm

After joining 3 cubes end to end a cuboid is formed

Length of cuboid = 4+4+4 = 12 cm

Breadth of cuboid = 4 cm

Height of cuboid = 4cm

Surface area of cuboid = 2(lb+bh+hl)

= 2(48+16+48)

= 224 cm^2

7. Question

The surface area of a cuboid is 1300 cm^2. If its breadth is 10 cm and height is 20 cm^2, find its length.

Answer

Given,

Surface area of cuboid = 1300 cm^2

Breadth of cuboid = 10 cm

Height of cuboid = 20 cm

Let length of cuboid = $l\ cm$

= $2(lb \times bh \times hl) = 1300$

=$10l + 200 + 20l = \frac{1300}{2} = 650$

=$30l = 650 - 200 = 450$

=$l = \frac{450}{30} = 15\ cm$

1. Question

If A_1, A_2 and A_3 denote the areas of three adjacent faces of a cuboid, then its volume is

A. $A_1A_2A_3$

B. 2 $A_1A_2A_3$

C. $\sqrt{A_1A_2A_3}$

D. $\sqrt[3]{A_1A_2A_3}$

Answer

Given,

A_1A_2,A_3 = areas of 3 adjacent faces of cuboid

V = volume of cuboid

Let dimensions of cuboid = $l \times b \times h$

$A_1 = l \times b$

$A_2 = b \times h$

$A_3 = h \times l$

$V = lbh$

So, $A_1A_2A_3 = lb \times bh \times hl = l^2b^2h^2 = (lbh)^2 = v^2$

$= v = \sqrt{A_1 A_2 A_3}$

2. Question

The length of the longest rod that can be fitted in a cuboid vessel of edge 10 cm long, is

A. 10 cm

B. $10\sqrt{2}$ cm

C. $10\sqrt{3}$ cm

D. 20 cm

Answer

Given,

Edge of cube = 10 cm

Longest rod that can be fitted in cube = diagonal of cube = $\sqrt{3}a$

$= \sqrt{3}a = 10\sqrt{3}$ cm

3. Question

If *l* is the length of a diagonal of a cube of volume *V*, then

A. $3V = l^3$

B. $\sqrt{3}\, V = l^3$

C. $3\sqrt{3}\, V = 2\, l^3$

D. $3\sqrt{3}\, V = l^3$

Answer

Given ,

$= l = length\ of\ diagonal\ of\ cube$

= v = volume of cube

So, $\sqrt{3}a = l$

$= a = \frac{l}{\sqrt{3}} = a^3 = \frac{l^3}{3\sqrt{3}}$

$\because v = a^3$

$= v = \frac{l^3}{3\sqrt{3}}$

$= 3\sqrt{3}v = l^3$

4. Question

Three equal cubes are placed adjacently in a row. The ratio of the total surface area of the resulting cuboid to that of the sum of the surface areas of three cubes, is

A. 7 : 9

B. 49 : 81

C. 9 : 7

D. 27 : 23

Answer

Given,

Three equal cubes are placed adjacently in a row

Let edge of cubes = a

Hence , sum of surface area of 3 cubes = $3\times6a^2 = 18\ a^2$

So, length of cuboid thus formed = a+a+a = 3a

Breadth of cuboid = a

Height of cuboid = a

Total surface area of cuboid = $2(lb+bh+hl) = 2(3a^2+a^2+3a^2) = 14a^2$

Hence , $\frac{sum\ of\ surfaces\ area\ of\ 3\ cubes}{total\ surface\ area\ of\ cuboid} = \frac{14a^2}{18a^2} = \frac{7}{9}$.

5. Question

If *V* is the volume of a cuboid of dimensions *x*, *y*, *z* and *A* is its surface area, then A/V

A. $x^2y^2z^2$

B. $\frac{1}{2}\left(\frac{1}{xy}+\frac{1}{yz}+\frac{1}{zx}\right)$

C. $2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

D. $\frac{1}{xyz}$

Answer

Volume of a Cuboid = Length × Breadth × Height

Hence, Volume of a Cuboid V = xyz

Surface Area of a Cuboid = 2 × (Length × Breadth + Breadth × Height + Length × Height)

∴ Surface Area of a Cuboid A = 2 × (xy + yz + zx)

$\Rightarrow A = 2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$\Rightarrow \frac{A}{V} = \frac{2xyz}{xyz}\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

$\Rightarrow \frac{A}{V} = 2\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$

6. Question

The sum of the length, breadth and depth of a cuboid is 19 cm and its diagonal is $5\sqrt{5}$ cm. Its surface area is

A. 361 cm^2

B. 125 cm^2

C. 236 cm^2

D. 486 cm^2

Answer

Given,

Sum of length ,breadth, height of cuboid = 19 cm

Diagonal of cuboid = 5√5 cm

= $\sqrt{l^2+b^2+h^2} = 5\sqrt{5}$

(Squaring both side)

= $l^2+b^2+h^2 = 125 \ldots\ldots\ldots\ldots\ldots\ldots (i)$

$\because (l+b+h)^2 = l^2+b^2+h^2+2\,(lb+bh+hl)\ by\ formula$

Putting values ...

= 19^2 = 125 + $2(lb+bh+hl)$

= $2(lb+bh+hl) = 361-125 = 236$

= surface area of cuboid = 236 cm^2

7. Question

If the length of a diagonal of a cube is 8 $\sqrt{3}$ cm, then its surface area is

A. 512 cm^2

B. 384 cm^2

C. 192 cm^2

D. 768 cm^2

Answer

Given,

Length of diagonal of cube = 8√3 cm

∵ √3 a = 8√3

= $a = \frac{8\sqrt{3}}{\sqrt{3}} = 8\ cm$

= surface area of cube = 6 a^2 = 6× 8×8 = 384 cm^2

8. Question

If each edge of a cube is increased by 50%, the percentage increase in its surface area is

A. 50%

B. 75%

C. 100%

D. 125%

Answer

Given,

Increment in each edge of a cube = 50%

Let original length of edge of cube = a

Original surface area of cube = 6 a^2

After increment ,

New length of edge = a + a$\times\frac{50}{100} = a + \frac{a}{2} = \frac{3a}{2}$

New surface area of cube = $6a^2 = 6\times\left(\frac{3a}{2}\right)^2 = 6\times\frac{9a^2}{4} = \frac{27}{2}a^2$

Increase in area = $\frac{27}{2}a^2 - 6a^2 = \frac{15a^2}{2}$

% increase in area = $\frac{increased\ area}{original\ area}\times 100 = \frac{\frac{15a^2}{2}}{6a^2}\times 100 = 125\%$

9. Question

If the volumes of two cubes are in the ratio 8 : 1, then the ratio of their edges is

A. 8 : 1

B. 2 $\sqrt{2}$: 1

C. 2 : 1

D. none of these

Answer

Given,

Ratio of volumes of two cubes = $\frac{V_1}{V_2} = \frac{8}{1}$

$\therefore \frac{a_1^3}{a_2^3} = \frac{8}{1}$

$= \frac{a_1}{a_2} = \sqrt{\frac{8}{1}} = \frac{2}{1}$.

10. Question

The volume of a cube whose surface area os 96 cm^2, is

A. 16 $\sqrt{2}$ cm^3

B. 32 cm^3

C. 64 cm^3

D. 216 cm^3

Answer

Given,

Surface area of cube = 96 cm^2

= $6a^2 = 96$

= $a^2 = \frac{96}{6} = 16$

= a = $\sqrt{16} = 4\ cm$

Volume of cube = $a^3 = 4\times4\times4 = 64\ cm^3$

11. Question

The length, width and height of a rectangular solid are in the ratio of 3 : 2 : 1. If the volume of the box is 48 cm^3, the total surface area of the box is

A. 27 cm^2

B. 32 cm^2

C. 64 cm^2

D. 88 cm^2

Answer

Given,

Ratio of length , width and height of rectangular solid = 3:2:1

Volume of box = 48 cm^3

Let length of box = 3x cm

Breadth of box = 2x cm

Height of box = x cm

Volume of box = v = $3x \times 2x \times x = 48$

= $6x^3 = 48$

= $x^3 = \frac{48}{6} = 8$

= $x = \sqrt[3]{8} = 2\ cm$

Hence . length = 3x = 3×2 = 6 cm

= bredth = 2x = 2×2 = 4 cm

= height = x = 2 cm

Total surface area of box = $2(lb + bh + hl) = 2(24 + 8 + 12) = 88cm^2$

12. Question

A cube whose volume is 1/8 cubic centimetre is placed on top of a cube whose volume is 1 cm^3. The two cubes are then placed on top of a third cube whose volume is 8 cm^3. The height of the stacked cubes is

A. 3.5 cm

B. 3 cm

C. 7 cm

D. none of these

Answer

Given,

Volume of 1st cube = $\frac{1}{8}\ cm^3$

Volume of second cube = 1 cm^3

Volume of third cube = 8 cm^3

∴edge of first cube = $a_1 = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}\ cm$

∴ edge of second cube = $a_2 = \sqrt[3]{1} = 1\ cm$

∴ edge of third cube = $a_3 = \sqrt[3]{8} = 2\ cm$

Hence height of cubes together = $a_1+a_2+a_3 = \frac{1}{2} + 1 + 2 = \frac{7}{2} = 3.5\ cm$

13. Question

If the areas of the adjacent faces of a rectangular block are in the ratio 2 : 3 : 4 and its volume is 9000 cm^3, then the length of the shortest edge is

A. 30 cm

B. 20 cm

C. 15 cm

D. 10 cm

Answer

Given,

Ratio of areas of adjacent faces of cube = 2:3:4

Volume of block = 9000 cm^3

= A_1:A_2:A_3 = 2:3:4

= bh:lb:lh = 2:3:4

= b:l = 2:3

=h:l = 2:4

= h:b = 3:4 and v = lbh

Assume that , l = 6x , b= 4x , h = 3x

= $6x \times 4x \times 3x = 9000$

= $x^3 = \frac{9000}{72} = 125$

= x = $\sqrt[3]{125} = 5$

So, smallest edge would be 3x = 3×5 = 15 cm

14. Question

If each edge of a cube, of volume *V,* is doubled, then the volume of the new cube is

A. 2 V

B. 4 V

C. 6 V

D. 8 V

Answer

Given,

Let original edge of cube = a

Then original volume of cube = a^3

New edge of cube = 2a

New volume of cube = $(2a)^3 = 8a^3 = 8v$ ∵$v=a^3$

15. Question

If each edge of a cuboid of surface area *S* is doubled, then surface area of the new cuboid is

A. 2 S

B. 4 S

C. 6 S

D. 8 S

Answer

Given,

Let edges of cuboid = l , b , h

Surface area of cuboid = $2(lb + bh + hl)$

New edges of cuboid = 2l , 2b , 2h

New surface area of cuboid = $2(4lb + 4bh + 4hl) = 4\times2(lb + bh + hl)$

= 4S

16. Question

The area of the floor of a room is 15 m^2. If its height is 4 m, then the volume of the air contained in the room is

A. 60 dm^3

B. 600 dm^3

C. 6000 dm^3

D. 60000 dm^3

Answer

Given,

Area of floor of room = 15 m^2

Height of room = 4 m

Volume of air contained in room = area of floor ×height of room

= 15×4 = 60 m^3 = 60000 dm^3

17. Question

The cost of constructing a wall 8 m long, 4 m high and 20 cm thick at the rate of Rs. 25 per m^3 is

A. Rs. 16

B. Rs. 80

C. Rs. 160

D. Rs. 320

Answer

Given,

Length of wall = 8m

Breadth of wall = 4 m

Thickness of wall = 20 cm = $\frac{20}{100}m$

Volume of wall = $l\times b\times h = 8\times4\times\frac{20}{100} = \frac{32}{5}\ m^3$

∵ cost of construction of 1m^3 wall = Rs. 25

∴ cost of construction of $\frac{32}{5}\ m^3\ wall = \frac{32}{5}\times25\ = Rs.160$

18. Question

10 cubic metres clay is uniformly spread on a land of area 10 ares. The rise in the level of the ground is

A. 1 cm

B. 10 cm

C. 100 cm

D. 1000 cm

Answer

Given,

Volume of clay = 10 m^3

Area of earth = 10 acre = 1000000 cm^2

Rise in height of earth level = $\frac{volume\ of\ clay}{area\ on\ which\ it\ spread} = \frac{1000000cm^3}{1000000cm^2} = 1\ cm$

19. Question

Volumeof a cuboid is 12 cm^3. The volume (in cm^3) of a cuboid whose sides are double of the above cuboid is

A. 24

B. 48

C. 72

D. 96

Answer

Given,

Volume of cuboid =12 cm^3

Let dimensions of cuboid = l×b×h

So, l×b×h = 12 .....(i)

Dimensions of one another cuboid = 2l ×2b×2h

Volume of cuboid = 8lbh = 8×12 = 96cm^3

20. Question

If the sum of all the edges of acube is 36 cm, then the volume (in cm^3) of that cube is

A. 9

B. 27

C. 219

D. 729

Answer

Given,

Sum of all edges of cube = 36 cm

= 12 a = 36

= a = $\frac{36}{12} = 3\ cm$

Volume of cube = a^3 = 3×3×3 = 27 cm^3

21. Question

The number of cubes of side 3 cm that can be cut from a cuboid of dimensions 10 cm×9 cm×6 cm, is

A. 9

B. 10

C. 18

D. 20

Answer

Given,

Side of cube = 3cm

Dimension of cuboid = 10 cm×9cm ×6 cm

Volume of cuboid = 10×9×6 = 540 cm^3

Volume of cube = $a^3 = 3^3 = 27\ cm^3$

Number of cubes can be formed = $\frac{volume\ of\ cuboid}{volume\ of\ cube} = \frac{540}{27} = 20$

22. Question

On a particular day, the rain fall recorded in a terrace 6 m long and 5 m board is 15 cm. The quantity of water collected in the terrace is

A. 300 litres

B. 450 litres

C. 3000 litres

D. 4500 litres

Answer

Given ,

Length of terrace = 6m = 600 cm

Breadth of terrace = 5m = 500 cm

Height of rainfall = 15 cm

Quantity of water collected on roof = 600×500×15 = 450000 cm^3

$=\frac{450000}{1000} = 450\ litre$