17. Understanding Shapes-III (Special Types of Quadrilaterales)

Exercise 17.1

1. Question

Given below is a parallelogram *ABCD.* Complete each statement along with the definition or property used.

(i) $AD =$ (ii) $\angle DCB =$

(iii) $OC =$ (iv) $\angle DAB + \angle CDA =$

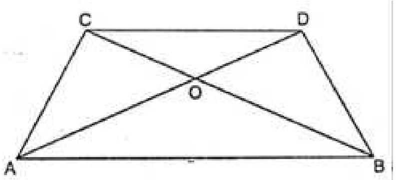


Fig. 17.21

Answer

(i) $AD =$ BC [In a parallelogram diagonals bisect each other]

(ii) $\angle DCB = \angle BAD$ [alternate interior angles are equal]

(iii) $OC =$ OA [In a parallelogram diagonals bisect each other]

(iv) $\angle DAB + \angle CDA =$ 180° [Sum of adjacent angles in a parallelogram is 180°]

2. Question

The following figures are parallelograms. Find the degree values of the unknowns *x, y, z.*

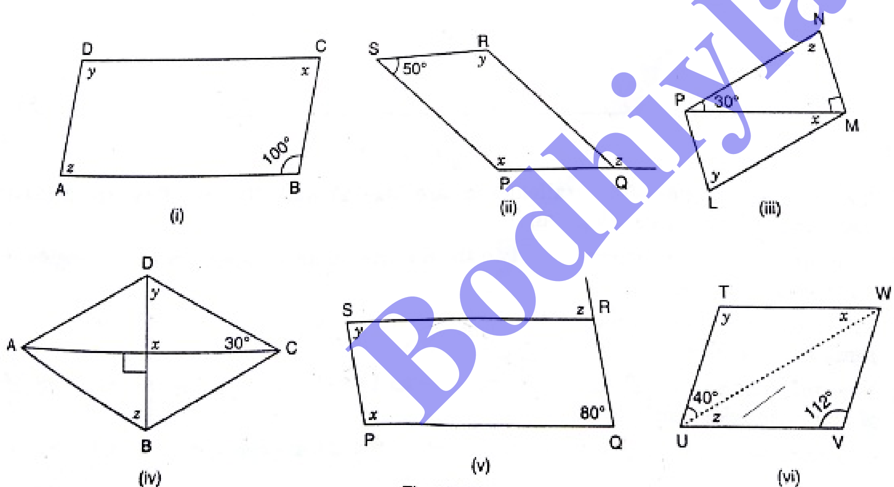


Fig. 17.22

Answer

(i) $\angle ABC = \angle Y = 100°$ [In a parallelogram opposite angles are equal]

$\angle x + \angle Y = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle x + 100° = 180°$

$\angle x = 180°-100°$

$\angle x = 80°$

$\angle x = \angle z = 80°$ [In a parallelogram opposite angles are equal]

(ii) $\angle PSR + \angle Y = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle Y + 50° = 180°$

$\angle Y = 180°-50°$

$\angle Y = 130°$

$\angle x = \angle Y = 130°$ [In a parallelogram opposite angles are equal]

$\angle PSR = \angle PQR = 50°$ [In a parallelogram opposite angles are equal]

$\angle PQR + \angle Z = 180°$ [Linear pair]

$50° + \angle Z = 180°$

$\angle Z = 180°\text{-}50°$

$\angle Z = 130°$

(iii) **In ΔPMN**

$\angle MPN + \angle PMN + \angle PNM = 180°$ [Sum of all the angles of a triangle is 180°]

$30° + 90° + \angle z = 180°$

$\angle z = 180°\text{-}120°$

$\angle z = 60°$

$\angle y = \angle z = 60°$ [In a parallelogram opposite angles are equal]

$\angle z = 180°\text{-}120°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle z = 60°$

$\angle z + \angle NML = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$60° + 90° + \angle x = 180°$

$\angle x = 180°\text{-}150°$

$\angle x = 30°$

(iv) $\angle x = 90°$ [vertically opposite angles are equal]

In ΔDOC

$\angle x + \angle y + 30° = 180°$ [Sum of all the angles of a triangle is 180°]

$90° + 30° + \angle y = 180°$

$\angle y = 180°\text{-}120°$

$\angle y = 60°$

$\angle y = \angle z = 60°$ [alternate interior angles are equal]

(v) $\angle x + \angle POR = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle x + 80° = 180°$

$\angle x = 180°\text{-}80°$

$\angle x = 100°$

$\angle y = 80°$ [In a parallelogram opposite angles are equal]

$\angle QRS = \angle x = 100°$

$\angle QRS + \angle Z = 180°$ [Linear pair]

$100° + \angle Z = 180°$

$\angle Z = 180°\text{-}100°$

$\angle Z = 80°$

(vi) $\angle y = 112°$ [In a parallelogram opposite angles are equal]

$\angle y + \angle TUV = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$\angle z + 40° + 112° = 180°$

$\angle z = 180° - 152°$

$\angle z = 28°$

$\angle z = \angle x = 28°$ [alternate interior angles are equal]

3. Question

Can the following figures be parallelograms? Justify your answer.

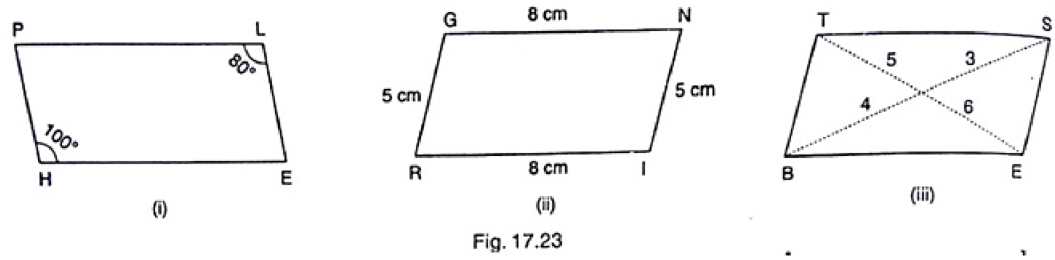


Fig. 17.23

Answer

(i) No, In a parallelogram opposite angles are equal.

(ii) Yes, In a parallelogram opposite sides are equal and parallel.

(iii) No, In a parallelogram diagonals bisect each other.

4. Question

In the adjacent figure *HOPE* is a parallelogram. Find the angle measures *x, y* and *z*. State the geometrical truths you use to find them.

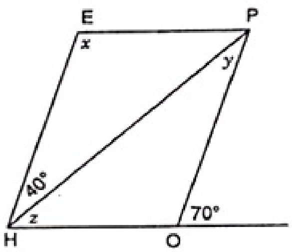


Fig. 17.24

Answer

$\angle HOP + 70° = 180°$ [Linear pair]

$\angle HOP = 180° - 70°$

$\angle HOP = 110°$

$\angle HOP = \angle x = 110°$ [In a parallelogram opposite angles are equal]

$\angle x + \angle z + 40° = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$110° + \angle z + 40° = 180°$

$\angle z = 180° - 150°$

$\angle z = 30°$

$\angle z + \angle y = 70°$

$\angle y + 30° = 70°$

$\angle y = 70° - 30°$

$\angle y = 40°$

5. Question

In the following figures *GUNS* and *RUNS* are parallelograms. Find *x* and *y.*

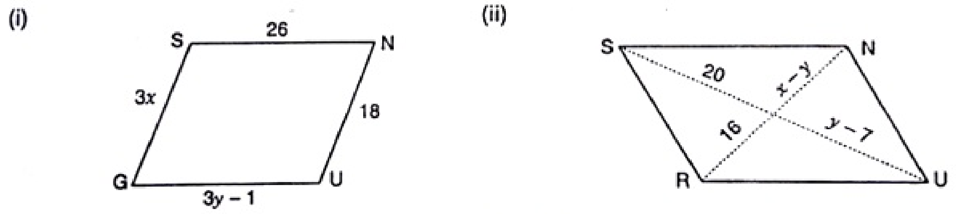


Fig. 17.25

Answer

(i) 3y – 1 = 26 [In a parallelogram opposite sides are of equal length]

3y = 26 + 1

Y = $\frac{27}{3} = 9$

Y = 9 units

3x = 18 [In a parallelogram opposite sides are of equal length]

x = $\frac{18}{3} = 6$

x = 6 units

(ii) y – 7 = 20 [In a parallelogram diagonals bisect each other]

y = 20 + 7

Y = 27 units

x-y = 16 [In a parallelogram diagonals bisect each other]

x -27 = 16

x = 16 + 27 = 43 units

6. Question

In the following figure *RISK* and *CLUE* are parallelograms. Find the measure of *x*.

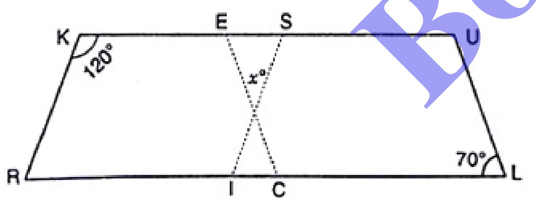


Fig. 17.26

Answer

In parallelogram RISK

$\angle SKR + \angle ISK = 180°$ [In a parallelogram sum of the adjacent angles is equal to 180°]

$120°\ + \angle ISK = 180°$

$\angle ISK = 180°$ - 120°

$\angle z = 60°$

In parallelogram CLUE

$\angle UEC = \angle z = 70°$ [In a parallelogram opposite angles are equal]

In ΔEOS

$70° + \angle x + 60° = 180°$ [Sum of angles of a triangles is 180°]

$\angle x = 180°$ - 130°

$\angle x = 50°$

7. Question

Two opposite angles of a parallelogram are (3*x* - 2)° and (50 - *x*)°. Find the measure of each angle of the parallelogram.

Answer

We know that opposite angles of a parallelogram are equal.

Therefore (3x - 2)° = (50 - x)°

3x - 2° = 50° - x

3x° + x = 50° + 2°

4x = 52°

x = $\frac{52°}{4} = 13°$

Measure of opposite angles are: 3x - 2 = 3 × 13°-2 = 37°

(50 - x)° = 50 - 13 = 37°

Sum of adjacent angles = 180°

Other two angles are 180° - 37° = 143° each

8. Question

If an angle of a parallelogram is two-third of its adjacent angle, find the angles of the parallelogram.

Answer

Let one of the adjacent angle is x°

Therfore other adjacent angle = $\frac{2x°}{3}$

Sum of adjacent angles = 180°

$$x° + \frac{2x°}{3} = 180°$$

$$\frac{3x° + 2x°}{3} = 180°$$

$$\frac{5x°}{3} = 180°$$

$$x° = \frac{180° \times 3}{5}$$

$$x° = 108°$$

Other angle = 180° - 108° = 72°

Therefore angles of the parallelograms are 72°, 72°, 108° and 108°

9. Question

The measure of one angle of a parallelogram is 70°. What are the measures of the remaining angles?

Answer

Let one of the adjacent angle is x°

Therfore other adjacent angle = 70°

Sum of adjacent angles = 180°

$x° + 70° = 180°$

$x° = 180° - 70°$

$x° = 110°$

Therefore angles of the parallelograms are 70°, 70°, 110° and 110°

10. Question

Two adjacent angles of a parallelogram are as 1 : 2. Find the measures of all the angles of the parallelogram.

Answer

Let one of the adjacent angles are x°

Therfore adjacent angles are = x° and 2x°

Sum of adjacent angles = 180°

$x° + 2x° = 180°$

$3x° = 180°$

$x° = \frac{180°}{3}$

$x° = 60°$

Other angle = 180° - 60° = 120°

Therefore angles of the parallelograms are 60°, 60°, 120° and 120°

11. Question

In a parallelogram *ABCD*, $\angle D$= 135°, determine the measure of $\angle A$ and $\angle B$.

Answer

Let one of the adjacent angle $\angle D = 135°$

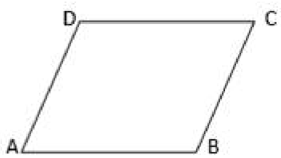
Therfore other adjacent angle $\angle A$ = x°

Sum of adjacent angles = 180°

$x° + 135° = 180°$

$x° = 180° - 135°$

$\angle A = x° = 45°$

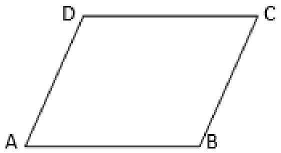


$\angle A = \angle C$ =45° and $\angle D = \angle B$ =135° [Measure of opposite angles of a parallelogram are equal]

12. Question

ABCD is a parallelogram in which $\angle A$ = 70°. Compute $\angle B$, $\angle C$ and $\angle D$.

Answer



Let one of the adjacent angle $\angle A = 70°$

Therfore other adjacent angle $\angle B = x°$

Sum of adjacent angles = 180°

$x° + 70° = 180°$

$x° = 180° - 70°$

$\angle B = x° = 110°$

$\angle A = \angle C = 70°$ and $\angle D = \angle B = 110°$ [Measure of opposite angles of a parallelogram are equal]

13. Question

The sum of two opposite angles of a parallelogram is 130°. Find all the angles of the parallelogram.

Answer

Let one of the adjacent angle $\angle A = 130°$

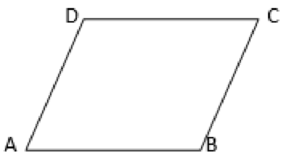
Therfore other adjacent angle $\angle B = x°$

Sum of adjacent angles = 180°

$x° + 130° = 180°$

$x° = 180° - 130°$

$\angle B = x° = 50°$

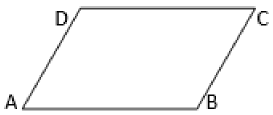


$\angle A = \angle C = 130°$ and $\angle D = \angle B = 70°$ [Measure of opposite angles of a parallelogram are equal]

14. Question

All the angles of a quadrilateral are equal to each other. Find the measure of each. Is the quadrilateral a parallelogram? What special type of parallelogram is it?

Answer



Let each angle of parallelogram ABCD = x°

Sum of all the angles = 360°

$x° + x° + x° + x° = 360°$

$4x° = 360°$

$x° = \frac{360°}{4} = 90°$

Therfore each angle of the parallelogram is equal to 90°

Yes, this quadrilateral is a parallelogram. Aparallelogram with each angle equal to 90° is a rectangle.

15. Question

Two adjacent sides of a parallelogram are 4 cm and 3 cm respectively. Find its perimeter.

Answer

We know that opposite sides of a parallelogram are equal and parallel.

Perimeter = Sum of all sides

Perimeter = 4 + 3 + 4 + 3 = 14 cm

16. Question

The perimeter of a parallelogram is 150 cm. One of its sides is greater than the other by 25 cm. Find the length of the sides of the parallelogram.

Answer

Perimeter of the parallelogram = 150 cm

Let one of the sides = x cm

Other side = (x + 25) cm

We know that opposite sides of a parallelogram are equal and parallel.

Perimeter = Sum of all sides

x + x + 25 + x + x + 25 = 150

4x + 50 = 150

4x = 150 – 50

x = $\frac{100}{4} = 25$

Therefore sides of the parallelogram are 25 cm and 50 cm.

17. Question

The shorter side of a parallelogram is 4.8 cm and the longer side is half as much again as the shorter side. Find the perimeter of the parallelogram.

Answer

Shorter side of the parallelogram = 4.8 cm

Longer side of the parallelogram = $4.8 + \frac{4.8}{2}$

= $4.8 + 2.4 = 7.2\ cm$

We know that opposite sides of a parallelogram are equal and parallel.

Perimeter = Sum of all sides

Perimeter of the parallelogram = 4.8 + 7.2 + 4.8 + 7.2 = 24cm

Therefore perimeter of the parallelogram 24 cm.

18. Question

Two adjacent angles of a parallelogram are (3*x*-4)° and (3*x*+10)°. Find the angles of the parallelogram.

Answer

We know that sum of the adjacent angles of a parallelogram = 180°

(3*x*-4)° + (3*x*+10)° = 180°

3x°- 4° + 3x° + 10° = 180°

6x° = 180°- 6°

x = $\frac{174°}{6}$ = 29°

Measure of one angle: 3x-4 = 3 × 29°-4 = 83°

Measure of other angle = (3x + 10)°= 3 × 29 + 10 = 97°

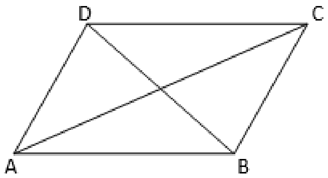
Therefore angles of the parallelogram are 83° 83°, 97° and 97°

19. Question

In a parallelogram *ABCD,* the diagonals bisect each other at *O.* If ∠*ABC* =30°, ∠*BDC*= 10° and ∠*CAB* =70°. Find:

∠*DAB*, ∠*ADC*, ∠*BCD*, ∠*AOD*, ∠*DOC*, ∠*BOC*, ∠*AOB*, ∠*ACD*, ∠*CAB*, ∠*ADB*, ∠*ACB*, ∠*DBC*, and ∠*DBA*.

Answer



∠*ABC* = ∠*ADC* = 30° [Measure of opposite angles is equal in a parallelogram]

∠*BDC* = 10°............. given

∠BDA = 30° - 10° = 20°

∠DAB = 180° - 30° = 150°

∠*BCD* = ∠*DAB* = 150° [Measure of opposite angles is equal in a parallelogram]

∠*DBA* = ∠*BDC* = *10°* [Alternate interior angles are equal]

In ΔDOC

∠*BDC* + ∠*ACD* + ∠*DOC* = *180°* [Sum of all angles og a triangle is 180°]

10° + 70° + ∠*DOC* = *180°*

∠*DOC* = *180°- 80°*

∠*DOC* = *100°*

∠*DOC* = ∠*AOB* = *100°* [Vertically opposite angles are equal]

∠*DOC* + ∠*AOD* = *180°* [Linear pair]

100° + ∠*AOD* = *180°*

∠*AOD* = *180°- 100°*

∠*AOD* = *80°*

∠*AOD* = ∠*BOC* = *80°* [Vertically opposite angles are equal]

∠*ABC* + ∠*BCD* = 180° [In a parallelogram sum of adjacent angles is 180°]

30° + ∠*ACB* + ∠*ACD* = 180°

30° + ∠*ACB* + 70° = 180°

∠*ACB* = 180° - 100°

∠*ACB* = 80°

∠*ACB* = ∠*ACB* = 80° [Alternate interior angles are equal]

20. Question

Find the angles marked with a question mark shown in Fig. 17.27

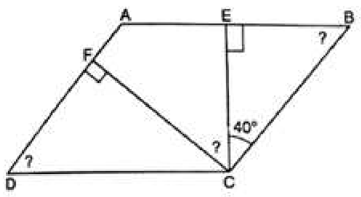


Fig. 17.27

Answer

In ΔBEC

$\angle BEC + \angle ECB + \angle CBC = 180°$ *[Sum of angles of a triangle is 180°]*

$90° + 40° + \angle CBC = 180°$

$\angle CBC = 180° - 130°$

$\angle CBC = 50°$

$\angle B = \angle D = 50°$ *[Opposite angles of a parallelogram are equal]*

$\angle A + \angle B = 180°$ *[Sum of adjacent angles of a triangle is 180°]*

$\angle A + 50° = 180°$

$\angle A = 180° - 50°$

$\angle A = 130°$

In ΔDFC

$\angle DFC + \angle FCD + \angle CDF = 180°$ *[Sum of angles of a triangle is 180°]*

$90° + \angle FCD + 50° = 180°$

$\angle FCD = 180° - 140°$

$\angle FCD = 40°$

$\angle A = \angle C = 130°$ *[Opposite angles of a parallelogram are equal]*

$\angle C = \angle FCE + \angle BCE + \angle FCD$

$\angle DCF + 40° + 40° = 130°$

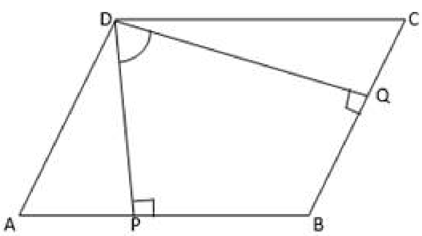
$\angle DCF = 130° - 80°$

$\angle DCF = 50°$

21. Question

The angle between the altitudes of a parallelogram, through the same vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

Answer



Given ABCD is a parallelogram in which DP⊥AB and AQ ⊥BC.Given ∠PDQ = 60°In quad. DPBQ∠PDQ + ∠DPB + ∠B + ∠BQD = 360° [Sum of all the angles of a Quad is 360°]60° + 90° + ∠B + 90° = 360°∠B = 360° –

240°Therefore, ∠B = 120°But ∠B = ∠D = 120° [Opposite angles of parallelogram are equal]∠B + ∠C = 180° [Sum of adjacent interior angles in a parallelogram is 180°]120° + ∠C = 180°∠C = 180° – 120° = 60°Therefore, ∠A = ∠C = 70° (Opposite angles of parallelogram are equal)

22. Question

In Fig. 17.28, *ABCD* and *AEFG* are parallelograms. If $\angle C = 55°$, what is the measure of $\angle F$? Figure

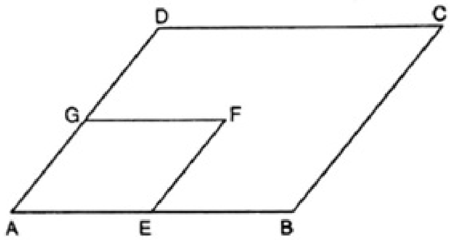


Fig. 17.28

Answer

In parallelogram ABCD

$\angle C = \angle A = 55°$ [In a parallelogram opposite angles are equal]

In parallelogram AEFG

$\angle A = \angle F = 55°$ [In a parallelogram opposite angles are equal]

23. Question

In Fig. 17.29, *BDEF* and *DCEF* are each a parallelogram. Is it true that $BD = DC$? Why or why not?

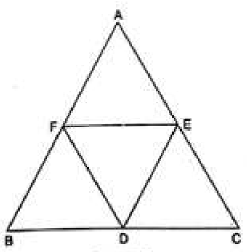


Fig. 17.29

Answer

In parallelogram BDEF

BD = EF ....(i) [In a parallelogram opposite sides are equal]

In parallelogram DCEF

DC = EF ....(ii) [In a parallelogram opposite sides are equal]

From equations (i) and (ii), we get

BD = EF = DC

Hence, BD = DC Proved

24. Question

In Fig. 17.29, suppose it is known that $DE = DF$. Then, is ΔABC isosceles? Why or why not?

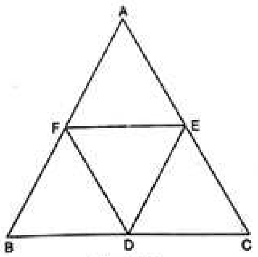


Fig. 17.29

Answer

In parallelogram BDEF

BD = EF and BF = DE ....(i) [In a parallelogram opposite sides are equal]

In parallelogram DCEF

DC = EF and DF = CE ....(ii) [In a parallelogram opposite sides are equal]

In parallelogram AFDE

AF = DE and DF = AE ....(ii) [In a parallelogram opposite sides are equal]

Therefore DE = AF =BF ....(iv)

Similarly: DF = CE = AE ....(v)

But, DE = DF ......given

From equations (iv) and (v), we get

AF + BF = AE + EC

AB = AC

Therefore ΔABC is an isosceles triangle.

25. Question

Diagonals of parallelogram *ABCD* intersect at *O* as shown in Fig. 17.30. *XY* contains *O*, and *x,Y* are points on opposite sides of the parallelogram. Give reasons for each of the following:

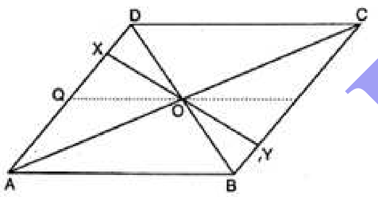


Fig. 17.30

Answer

(i) *OB = OD*

OB = OD [In a parallelogram diagonals bisect each other]

(ii) *∠OBY =∠ODX [Alternate interior angles are equal]*

(iii) *∠BOY= ∠DOX [Vertically opposite angles are equal]*

(iv) Δ*BOY* ≅ Δ*DOX*

In ΔBOY and ΔDOX

OB = OD [In a parallelogram diagonals bisect each other]

∠OBY =∠ODX [Alternate interior angles are equal]

∠BOY= ∠DOX [Vertically opposite angles are equal]

$\Delta BOY \cong \Delta DOX$ *[ASA rule]*

Now, state if XY is bisected at O.

Hence OX = OY [Corresponding parts of congruent triangles]

26. Question

In Fig. 17.31, $ABCD$ is a parallelogram, CE bisects $\angle C$ and AF bisects $\angle A$. In each of the following, if the statement is true, give a reason for the same.

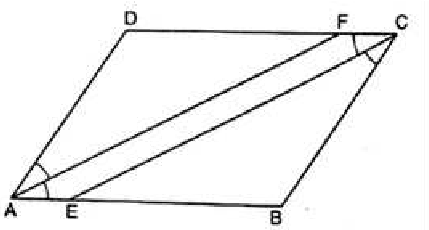


Fig. 17.31

Answer

(i) $\angle A = \angle C$

True,

$\angle C = \angle A = 55°$ [In a parallelogram opposite angles are equal]

(ii) $\angle FAB = \frac{1}{2} \angle A$

True,

AF is the angle bisectoe of angle A

(iii) $\angle DCE = \frac{1}{2} \angle C$

True,

CE is the angle bisectoe of angle A

(iv) $\angle CEB = \angle FAB$

True,

$\angle C = \angle A$ *[In a parallelogram opposite angles are equal]*

$\frac{1}{2} \angle C = \frac{1}{2} \angle A$ [AF and CE are angle bisectors]

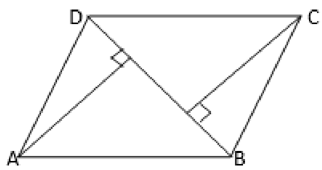
(v) $CE \parallel AF$

True, Since one pair of opposite angles are equal, therefore Quad AEFC is aparallelogram.

27. Question

Diagonals of a parallelogram $ABCD$ intersect at O. AL and CM are drawn perpendiculars to BD such that L and M lie on BD. Is $AL = CM$? Why or why not?

Answer



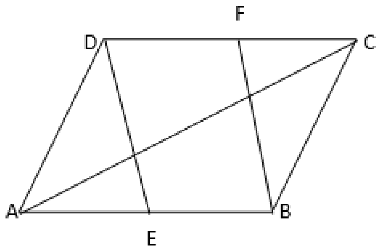
AL and CM are perpendiculars on diagonal BD.

AL = CM [In a parallelogram length of perpendiculars drawn on diagonal from opposite vertices are equal]

28. Question

Points *E* and *F* lie on diagonals *AC* of a parallelogram *ABCD* such that *AE* = *CF*. What type of quadrilateral is *BFDE*?

Answer



In parallelogram ABCD:

AB = CD…………….(i) [In aparallelogram opposite sides are equal and parallel]

AE = CF…….. (ii) given

On subtracting (ii) from (i)

AB-AE = CD-CF

BE = DF

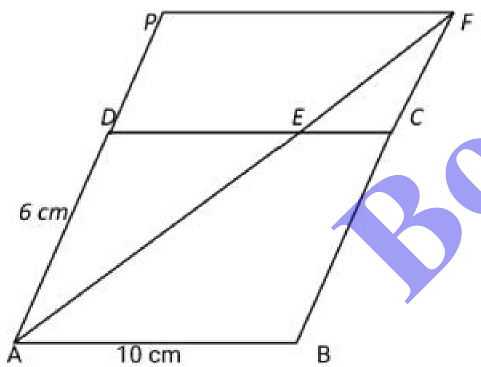
BE parallel to DF

Therefore quad BFDE is aparallelogram, since one pair of opposite sides are equal and parallel.

29. Question

In a parallelogram *ABCD, AB =* 10cm, *AD* = 6 cm. The bisector of ∠*A* meets *DC* in *E, AE* and *BC* produced meet at *F.* Find the length *CF.*

Answer



In a parallelogram ABCD

AB = 10 cm, AD = 6 cm⇒ DC = AB = 10 cm and AD = BC = 6 cm [In a parallelogram opposite sides are equal] Given that bisector of ∠A intersects DE at E and BC produced at F.Draw PF || CDFrom the figure, CD || FP and CF || DPHence PDCF is a parallelogram. [Since one pair of opposite sides are equal and parallel]AB || FP and AP || BF⇒ ABFP is also a parallelogramConsider ΔAPF and ΔABF∠APF = ∠ABF [Since opposite angles of a parallelogram are equal]AF = AF (Common side)∠PAF = ∠AFB (Alternate angles)ΔAPF ≅ ΔABF (By ASA congruence criterion)⇒ AB = AP (CPCT)⇒ AB = AD + DP= AD + CF [Since DCFP is a parallelogram]∴ CF = AB – ADCF = (10 – 6) cm = 4 cm

Exercise 17.2

1. Question

Which of the following statements are true for a rhombus?

(i) It has two pairs of parallel sides.

(ii) It has two pairs of equal sides.

(iii) It has only two pairs of equal sides.

(iv) Two of its angles are at right angles.

(v) Its diagonals bisect each other at right angles.

(vi) Its diagonals are equal and perpendicular.

(vii) It has all its sides of equal lengths.

(viii) It is a parallelogram.

(ix) It is a quadrilateral.

(x) It can be a square.

(xi) It is a square.

Answer

(i) True, Rhombus is a parallelogram.

(ii) True, Rhombus has all four sides equal.

(iii) False, Rhombus has all four sides equal.

(iv) False, In rhombus no angle is right angle.

(v) True, in rhombus diagonals bisect each other at right angles.

(vi) False, in rhombus diagonals are of unequal length.

(vii) True, Rhombus has all four sides equal.

(viii) True, Rhombus is a parallelogram since opposite sides equal and parallel.

(ix) True, Rhombus is a quadrilateral since it has four sides.

(x) True, Rhombus becomes square when any one angle is 90°.

(xi) False, Rhombus is never a square. Since in a square each angle is 90°.

2. Question

Fill in the blanks, in each of the following, so as to make the statement true:

(i) A rhombus is a parallelogram in which _______.

(ii) A square is a rhombus in which _________.

(iii) A rhombus has all its sides of ______ length.

(iv) The diagonals of a rhombus _____ each other at ______ angles.

(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a ______.

Answer

(i) A rhombus is a parallelogram in which opposite sides are equal and parallel.

(ii) A square is a rhombus in which all four sides are equal.

(iii) A rhombus has all its sides of equal length.

(iv) The diagonals of a rhombus bisect each other at right angles.

(v) If the diagonals of a parallelogram bisect each other at right angles, then it is a rhombus.

3. Question

The diagonals of a parallelogram are not perpendicular. Is it a rhombus? Why or why not?

Answer

No, Diagonals of a rhombus bisect each other at 90°.

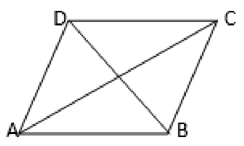
A parallelogram is rhombus only when its diagonals bisect each other at right angles.

4. Question

The diagonals of a quadrilateral are perpendicular to each other. Is such a quadrilateral always a rhombus? If your answer is ‘No’, draw a figure to justify your answer.

Answer

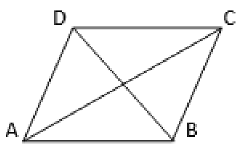
No it is not always a rhombus.



5. Question

ABCD is a rhombus. If $\angle ACB = 40°$, find $\angle ADB$.

Answer



In rhombus ABCD

$\angle ACB = 40°$ given

$\angle ACB = \angle CAD = 40°$ [Altermate interior angles are equal]

In ΔAOD

$\angle AOD = 90°$ [In rhombus diagonals bisect eact other at right angles]

$\angle AOD + \angle CAD + \angle ADB = 180°$ [Angle sum property of a triangle]

$90° + 40° + \angle ADB = 180°$

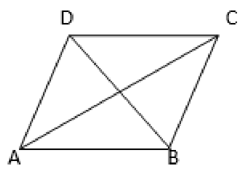
$\angle ADB = 180° - 130°$

$\angle ADB = 50°$

6. Question

If the diagonals of a rhombus are 12 cm and 16 cm, find the length of each side.

Answer



We know in rhombus diagonals bisect each other at right angle.

In ΔAOB

AO = $\frac{12}{2} = 6cm$, BO = $\frac{16}{2} = 8cm$

Using pythagorous theorem in ΔAOB

$AB^2 = AO^2 + BO^2$

$AB^2 = 6^2 + 8^2$

$AB^2 = 36 + 64$

$AB^2 = 100$

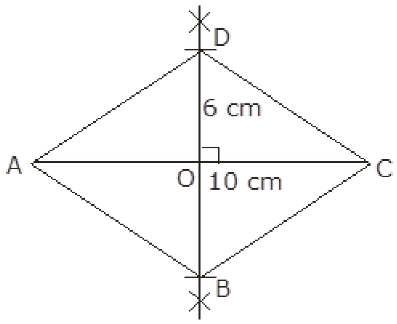
$AB = \sqrt{100} = 10cm$

Therefore each side of a rhombus is 10cm.

7. Question

Construct a rhombus whose diagonals are of length 10 cm and 6 cm.

Answer



Steps of Construction:

(i) Draw diagonal AC of length 10 cm.

(ii) Draw perpendicular bisector of AC at point O.

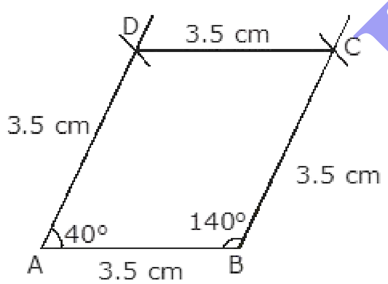
(iii) From point ‘O’ out two arcs of length 3cm to get points B and D.

(iv) Join AD and DC to get rhombus ABCD.

8. Question

Draw a rhombus, having each side of lengfth 3.5 cm and one of the angles as 40°.

Answer



Steps of construction:

(i) Draw a line segment AB of length 3.5 cm

(ii) From point A and B draw angles of 40 and 140 respectively.

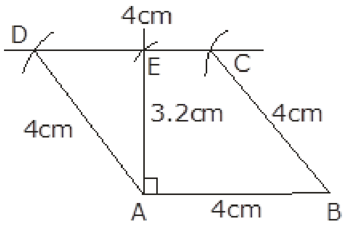
(iii) From points A and B draw two arcs of length 3.5 cm each is get points D and C.

(iv) Join ABCD to get rhombus ABCD.

9. Question

One side of a rhombus is of length 4 cm and the length of an altitude is 3.2 cm. Draw the rhombus.

Answer



Steps of construction:

(i) Draw a line segment of 4 cm

(ii) From point A draw a perpendicular from point A and cut a length of 3.2 cm to get point E.

(iii) From point E and a line parallel to AB.

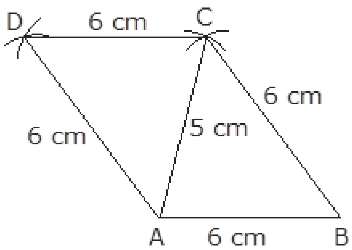
(iv) From points A and B cut two arcs of length 4 cm on the drawn parallel line to get points D and C.

(v) Join AD and BC to get rhombus ABCD.

10. Question

Draw a rhombus *ABCD*, if *AB* = 6 cm and *AC* = 5 cm.

Answer



Steps of construction:

(i) Draw a line segment AB of length 6 cm.

(ii) From point 'A' draw an arc of length 5 cm and from point B draw an arc of length 6 cm. Such that both the arcs intersect at 'C'.

(iii) Join AC and BC.

(iv) From point A draw an arc of length 6 cm and from point C draw an arc of 6cm, so that both the arcs intersect at point D.

(v) Joint AD and DC to get rhombus ABCD.

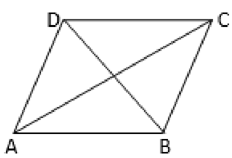
11. Question

ABCD is a rhombus and its diagonals intersect at *O*.

(i) Is $\Delta BOC \cong \Delta DOC$? State the condruence condition used?

(ii) Also state, if $\angle BCO = \angle DCO$.

Answer



(i) In ΔBOC and ΔDOC

BO = DO [In a rhombus diagonals bisect each other]

CO = CO Common

BC = CD [All sides of a rhombus are equal]

ΔBOC≅ΔDOC [SSS Congurency]

(ii) *∠BCO =∠DCO from above [corresponding parts of congruent triangles]*

12. Question

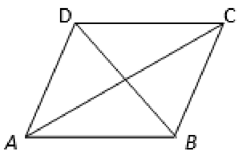
Show that each diagonal of a rhombus bisects the angle through which it passes.

Answer

(i) In Δ*BOC* and Δ*DOC*

BO = DO [In a rhombus diagonals bisect each other]

CO = CO Common



BC = CD [All sides of a rhombus are equal]

ΔBOC≅ΔDOC [SSS Congurency]

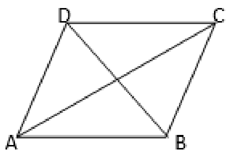
∠BCO =∠DCO from above [corresponding parts of congruent triangles]

Hence, each diagonal of a rhombus bisect the angle through which it passes.

13. Question

ABCD is a rhombus whose diagonals interesct at *O*. If *AB*=10 cm, diagonal *BD* = 16 cm, find the length of diagonal *AC*.

Answer



We know in rhombus diagonals bisect each other at right angle.

In ΔAOB

BO = $\frac{BD}{2} = \frac{16}{2} = 8cm$

Using pythagorous theorem in ΔAOB

$AB^2 = AO^2 + BO^2$

$10^2 = AO^2 + 8^2$

$100-64 = AO^2$

$AO^2 = 36$

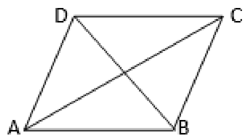
AO = $\sqrt{36}$ = 6cm

Therefore length of diagonal AC of rhombus ABCD is 6 × 2 = 12cm.

14. Question

The diagonal of a quadrilateral are of lengths 6 cm and 8 cm. If the diagonals bisect each other at right angles, what is the length of each side of the quadrilateral?

Answer



We know in rhombus diagonals bisect each other at right angle.

In ΔAOB

BO = $\frac{BD}{2} = \frac{6}{2} = 3cm$

AO = $\frac{AC}{2} = \frac{8}{2} = 4cm$

Using pythagorous theorem in ΔAOB

$AB^2 = AO^2 + BO^2$

$AB^2 = 4^2 + 3^2$

$AB^2 = 16 + 9$

$AB^2 = 25$

AB = $\sqrt{25}$ = 6cm

Therefore length of each side of a rhombus ABCD is 5cm.

Exercise 17.3

1. Question

Which of the following statements are true for a rectangle?

(i) It has two pairs of equal sides.

(ii) It has all its sides of equal length.

(iii) Its diagonals are equal.

(iv) Its diagonals bisect each other.

(v) Its diagonals are perpendicular.

(vi) Its diagonals are perpendicular and bisect each other.

(vii) Its diagonals are equal and bisect each other.

(viii) Its diagonals are equal and perpendicular, and bisect each other.

(ix) All rectangles are squares.

(x) All rhombuses are parallelograms.

(xi) All squares are rhombuses and also rectangles.

(xii) All squares are not parallelograms.

Answer

(i) True, In a rectangle two pairs of sides are equal.

(ii) False, In a rectangle two pairs of sides are equal.

(iii) True, In a rectangle diagonals are of equal length.

(iv) True, In a rectangle diagonals bisect each other.

(v) False, Diagonals of a rectangle need not be perpendicular.

(vi) False, Diagonals of a rectangle need not be perpendicular. Diagonals only bisect each other.

(vii) True, Diagonals are of equal length and bisect each other.

(viii) False, Diagonals are of equal length and bisect each other. Diagonals of a rectangle need not be perpendicular

(ix) False, In a square all sides are of equal length.

(x) True, All rhombuses are parallelograms, since opposite sides are equal and parallel.

(xi) True, All squares are rhombuses, since all sides are equal in a square and rhombus. All squares are rectangles, since opposite sides are equal and parallel.

(xii) False, All squares are parallelograms, since opposite sides are parallel and equal.

2. Question

Which of the following statements are true for a square?

(i) It is a rectangle.

(ii) It has all its sides of equal length.

(iii) Its diagonals bisect each other at right angle.

(v) Its diagonals are equal to its sides.

Answer

(i) True, square is a rectangle, since opposite sides are equal and parallel and each angle is right angle.

(ii) True, In a square all sides are of equal length.

(iii) True, in a square diagonals bisect each other at right angle.

(v) False, in a square diagonals are of equal length. Length of diagonals is not equal to the length of sides

3. Question

Fill in the blanks in each of the following, so as to make the statement true :

(i) A rectangle is a parallelogram in which ________.

(ii) A square is a rhombus in which __________.

(iii) A square is a rectangle in which ___________.

Answer

(i) A rectangle is a parallelogram in which opposite sides are parallel and equal.

(ii) A square is a rhombus in which all the sides are of equal length.

(iii) A square is a rectangle in which opposite sides are equal and parallel and each angle is a right angle.

4. Question

A window frame has one diagonal longer then the other. Is the window frame a rectangle? Why or why not?

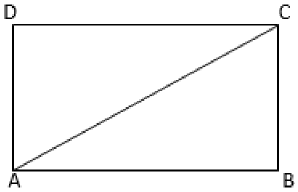
Answer

No, diagonals of a rectangle are of equal length equal.

5. Question

In a rectangle *ABCD*, prove that $\Delta ACB \cong \Delta CAD$.

Answer



In ΔACB and ΔCAD

AB = CD [Opposite sides of a rectangle are equal]

BC = DA

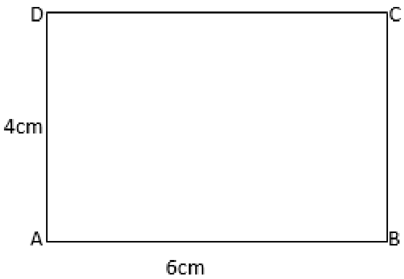
AC = CA Common

$\Delta ACB \cong \Delta CAD$ (SSS Congurency)

6. Question

The sides of a rectangle are in the ratio 2 : 3, and its perimeter is 20 cm. Draw the rectangle.

Answer



ABCD is a rectangle

Let the side is x

Length of rectangle = 3x

Breadth of the rectangle = 2x

Given perimeter of rectangle = 20 cm

Perimeter of the rectangle = 2(length + breadth)

20 = 2(3x + 2x)

10x = 20

x = 2

Therefore Length of the rectangle = 3×2 = 6cm

Therefore breadth of the rectangle = 2×2 = 4cm

7. Question

The sides of a rectangle are in the ratio 4 : 5. Find its sides if the perimeter is 90 cm.

Answer

ABCD is a rectangle

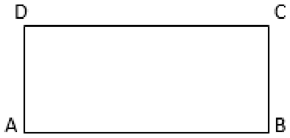
Let the side is x

Length of rectangle = 5x

Breadth of the rectangle = 4x

Given perimeter of rectangle = 90 cm

Perimeter of the rectangle = 2(length + breadth)



90 = 2(5x + 4x)

18x = 90

x = 5

Therefore Length of the rectangle = 5×5 = 25cm

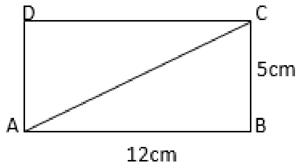
Therefore breadth of the rectangle = 4×5 = 20cm

8. Question

Find the length of the diagonal of a rectangle whose sides are 12 cm and 5 cm.

Answer

ABCD is a rectangle



In ΔABC using pythagorous theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 12^2 + 5^2$

$AC^2 = 144 + 25$

$AC^2 = 169$

$AC = \sqrt{169}$

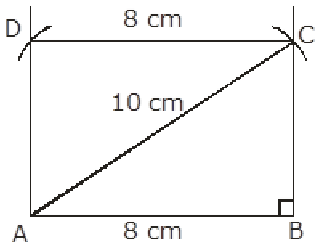
AC = 13cm

Therefore length of diagonal is 13cm.

9. Question

Draw a rectangle whose one side measures 8 cm and the length of each of whose diagonals is 10 cm.

Answer



Steps of construction:

(i) Draw a lien segment AB of length 8 cm

(ii) From point 'A' draw an arc of length 10 cm.

(iii) From point B draw an angle of 90°, and the arc from point A cuts it at point C.

(iv) Join Ac

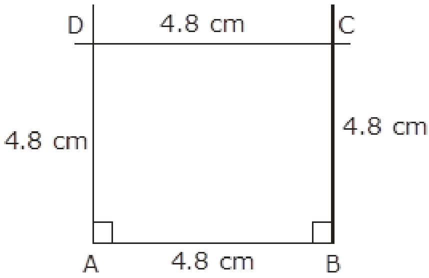
(v) From point A draw an angle of 90° and point C drawn an arc of length 8 cm to get point D.

(vi) Join CD and AD to get required rectangle.

10. Question

Draw a square whose each side measures 4.8 cm.

Answer



Steps of construction:

(i) Draw a line segment AB of length 4.8 cm.

(ii) From points A and B draw perpendiculars.

(iii) Cut and arc of 4.8 cm from point A and B on the perpendiculars to get point D and C.

(iv) Join DC and AD to get required rectangle.

11. Question

Identify all the quadrilaterals that have:

Answer

(i) Four sides of equal length

Rhombus and square are the quadrilaterals that have all four sides of equal length.

(ii) Four right angles

Rectangle and square have all four angles right angles.

12. Question

Explain how a square is

(i) a quadrilateral?

(ii) a parallelogram?

(iii) a rhombus?

(iv) a rectangle?

Answer

(i) a quadrilateral?

A square is a quadrilateral because it has four equal sides.

(ii) a parallelogram?

A square is a parallelogram since it has opposite sides equal and parallel.

(iii) a rhombus?

A square is a rhombus because it has all four sides of equal length.

(iv) a rectangle?

A square is a rectangle because its opposite sides are equal and parallel and each angle is right angle.

13. Question

Name the quadrilaterals whose diagonals:

(i) bisect each other

(ii) are perpendicular bisector of each other

(iii) are equal.

Answer

(i) bisect each other

In a Parallelogram, rectangle, rhombus and square diagonals bisect each other.

(ii) are perpendicular bisector of each other

In a Rhombus and square diagonals are perpendicular bisector of each other

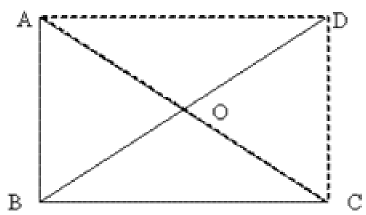
(iii) are equal.

In a square and rectangle diagonals are of equal length.

14. Question

ABC is a right angled triangle and *O* is the mid-point of the side opposite to the right angle. Explain why *O* is equidistant from *A*, *B*, and *C*.

Answer



ABC is a right angled triangle. O is the mid point of hypotenuse AC, such that OA = OC

Draw CD||AB and join AD, such that AB = CD and AD = BC

Now quad ABCD is a rectangle, since each angle is a right angle and opposite sides are equal and

parallel.

We know in a rectangle diagonals are of equal length and they bisect each other.

Therefore, AC = BD

And also, AO = OC =BO =OD

Hence, O is equidistant from A, B and C.

15. Question

A mason has made a concrete slab. He needs it to be rectangular. In what different ways can he make sure that it is rectangular?

Answer

a. By measuring each angle, because in a rectangle each angle is a right ange.

b. By measuring opposite sides. Since in a rectangle opposite sides are of equal length.

c. By measuring the lengths of diagonals. Since in a rectangle diagonals are of equal length.