17. Combinations

Exercise 17.1

1 A. Question

Evaluate the following:

$^{14}C_3$

Answer

We know that $^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)...........2.1

Given that we need to find the value of $^{14}C_3$

From (1)

$\Rightarrow {}^{14}C_3 = \frac{14!}{(14-3)!3!}$

$\Rightarrow {}^{14}C_3 = \frac{14!}{11!3!}$

$\Rightarrow {}^{14}C_3 = \frac{14 \times 13 \times 12}{3 \times 2 \times 1}$

$\Rightarrow {}^{14}C_3 = 364$

∴ The value of $^{14}C_3$ is 364.

1 B. Question

Evaluate the following:

$^{12}C_{10}$

Answer

We know that $^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)...........2.1

Given that we need to find the value of $^{12}C_{10}$

From (1)

$\Rightarrow {}^{12}C_{10} = \frac{12!}{(12-10)!10!}$

$\Rightarrow {}^{12}C_{10} = \frac{12!}{2!10!}$

$\Rightarrow {}^{12}C_{10} = \frac{12 \times 11}{2 \times 1}$

$\Rightarrow {}^{12}C_{10} = 66$

∴ The value of $^{12}C_{10}$ is 66.

1 C. Question

Evaluate the following:

$^{35}C_{35}$

Answer

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)...........2.1

Given that we need to find the value of ${}^{35}C_{35}$

From (1)

$\Rightarrow {}^{35}C_{35} = \frac{35!}{(35-35)!35!}$

$\Rightarrow {}^{35}C_{35} = \frac{35!}{0!35!}$

We know that 0! = 1

$\Rightarrow {}^{35}C_{35} = \frac{35!}{1 \times 35!}$

$\Rightarrow {}^{35}C_{35} = 1$

∴ The value of ${}^{35}C_{35}$ is 1.

1 D. Question

Evaluate the following:

$${}^{n+1}C_{n}$$

Answer

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)...........2.1

Given that we need to find the value of ${}^{n+1}C_{n}$

From (1)

$\Rightarrow {}^{n+1}C_{n} = \frac{(n+1)!}{(n+1-n)!n!}$

$\Rightarrow {}^{n+1}C_{n} = \frac{(n+1)!}{1!n!}$

$\Rightarrow {}^{n+1}C_{n} = \frac{n+1}{1}$

$\Rightarrow {}^{n+1}C_{n} = n + 1$

∴ The value of ${}^{n+1}C_{n}$ is n + 1.

1 E. Question

Evaluate the following:

$$\sum_{r=1}^{5} {}^{5}C_{r}$$

Answer

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)...........2.1

Given that we need to find the value of $\sum_{r=1}^{5} {}^{5}C_{r}$

From (1)

$\Rightarrow \sum_{r=1}^{5} {}^{5}C_r = {}^{5}C_1 + {}^{5}C_2 + {}^{5}C_3 + {}^{5}C_4 + {}^{5}C_5$

$\Rightarrow \sum_{r=1}^{5} {}^{5}C_r = \frac{5!}{(5-1)!1!} + \frac{5!}{(5-2)!2!} + \frac{5!}{(5-3)!3!} + \frac{5!}{(5-4)!4!} + \frac{5!}{(5-5)!5!}$

$\Rightarrow \sum_{r=1}^{5} {}^{5}C_r = \frac{5!}{4!1!} + \frac{5!}{3!2!} + \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!}$

We know that 0! = 1

$\Rightarrow \sum_{r=1}^{5} {}^{5}C_r = \frac{5}{1} + \frac{5 \times 4}{2 \times 1} + \frac{5 \times 4}{2 \times 1} + \frac{5}{1} + \frac{1}{1}$

$\Rightarrow \sum_{r=1}^{5} {}^{5}C_r = 5 + 10 + 10 + 5 + 1$

$\Rightarrow \sum_{r=1}^{5} {}^{5}C_r = 31$

∴ The value of $\sum_{r=1}^{5} {}^{5}C_r$ is 31.

2. Question

If ${}^{n}C_{12} = {}^{n}C_5$, find the value of n.

Answer

We know that if ${}^{n}C_p = {}^{n}C_q$, then one of the following conditions need to be satisfied:

i. p = q

ii. n = p + q

From the problem ${}^{n}C_{12} = {}^{n}C_5$ we can say that,

$\Rightarrow 12 \neq 5$

So, the condition(ii) must be satisfied,

$\Rightarrow n = 12 + 5$

$\Rightarrow n = 17.$

∴ The value of n is 17.

3. Question

If ${}^{n}C_4 = {}^{n}C_6$, find ${}^{12}C_n$.

Answer

We know that if ${}^{n}C_p = {}^{n}C_q$, then one of the following conditions need to be satisfied:

i. p = q

ii. n = p + q

From the problem ${}^{n}C_4 = {}^{n}C_6$ we can say that,

$\Rightarrow 4 \neq 6$

So, the condition(ii) must be satisfied,

$\Rightarrow n = 4 + 6$

$\Rightarrow n = 10.$

We need to find the value of ${}^{12}C_n = {}^{12}C_{10}$

We know that ${}^{n}C_r = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)............2.1

From (1)

$\Rightarrow {}^{12}C_{10} = \frac{12!}{(12-10)!10!}$

$\Rightarrow {}^{12}C_{10} = \frac{12!}{2!10!}$

$\Rightarrow {}^{12}C_{10} = \frac{12 \times 11}{2 \times 1}$

$\Rightarrow {}^{12}C_{10} = 66$

∴ The value of ${}^{12}C_{10}$ is 66.

4. Question

If ${}^{n}C_{10} = {}^{n}C_{12}$, find ${}^{23}C_{n}$.

Answer

We know that if ${}^{n}C_{p} = {}^{n}C_{q}$, then one of the following conditions need to be satisfied:

i. p = q

ii. n = p + q

From the problem ${}^{n}C_{10} = {}^{n}C_{12}$ we can say that,

⇒ 10≠12

So, the condition(ii) must be satisfied,

⇒ n = 10 + 12

⇒ n = 22.

We need to find the value of ${}^{23}C_{n} = {}^{23}C_{22}$

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ ..... (1)

And also n! = n(n – 1)(n – 2)...........2.1

From (1)

$\Rightarrow {}^{23}C_{22} = \frac{23!}{(23-22)!22!}$

$\Rightarrow {}^{23}C_{22} = \frac{23!}{1!22!}$

$\Rightarrow {}^{23}C_{22} = \frac{23}{1}$

$\Rightarrow {}^{23}C_{22} = 23$

∴ The value of ${}^{23}C_{22}$ is 23.

5. Question

If ${}^{24}C_{x} = {}^{24}C_{2x+3}$, find x.

Answer

We know that if ${}^{n}C_{p} = {}^{n}C_{q}$, then one of the following conditions need to be satisfied:

i. p = q

ii. n = p + q

Let us use condition (i),

$\Rightarrow x = 2x + 3$

$\Rightarrow x = -3$

We know that for a combination ${}^{n}C_{r}$,r≥0 and r should be an integer which is not satisfies here,

So, the condition (ii) must be satisfied,

$\Rightarrow 24 = x + 2x + 3$

$\Rightarrow 3x = 21$

$\Rightarrow x = \frac{21}{3}$

$\Rightarrow x = 7.$

∴ The value of x is 7.

6. Question

If ${}^{18}C_{x} = {}^{18}C_{x+2}$, find x.

Answer

We know that if ${}^{n}C_{p} = {}^{n}C_{q}$, then one of the following conditions need to be satisfied:

i. $p = q$

ii. $n = p + q$

From the problem ${}^{18}C_{x} = {}^{18}C_{x+2}$ we can say that,

$\Rightarrow x \neq x + 2$

So, the condition(ii) must be satisfied,

$\Rightarrow 18 = x + x + 2$

$\Rightarrow 2x = 16$

$\Rightarrow x = \frac{16}{2}$

$\Rightarrow x = 8.$

∴ The value of x is 8.

7. Question

If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r.

Answer

We know that if ${}^{n}C_{p} = {}^{n}C_{q}$, then one of the following conditions need to be satisfied:

i. $p = q$

ii. $n = p + q$

Let us use condition (i),

$\Rightarrow 3r = r + 3$

$\Rightarrow 2r = 3$

$\Rightarrow r = \frac{3}{2}$

$\Rightarrow r + 3 = \frac{3}{2} + 3$

$\Rightarrow r + 3 = \frac{9}{2}$

We know that for a combination ^nC_r,r≥0 and r should be an integer, which is not satisfies here,

So, the condition (ii) must be satisfied,

⇒ 15 = 3r + r + 3

⇒ 4r = 12

⇒ $r = \frac{12}{4}$

⇒ r = 4.

∴ The value of r is 4.

8. Question

If $^8C_r - {}^7C_3 = {}^7C_2$, find r.

Answer

Given:

⇒ $^8C_r - {}^7C_3 = {}^7C_2$

⇒ $^8C_r = {}^7C_2 + {}^7C_3$

We know that $^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

⇒ $^8C_r = {}^{7+1}C_{2+1}$

⇒ $^8C_r = {}^8C_3$

We know that if $^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:

i. p = q

ii. n = p + q

Let us use condition (i),

⇒ r = 3

Let us also check condition (ii),

⇒ 8 = 3 + r

⇒ r = 5

∴ The values of ‘r’ are 3 and 5.

9. Question

If $^{15}C_r : {}^{15}C_{r-1} = 11 : 5$, find r.

Answer

Given:

⇒ $\frac{^{15}C_r}{^{15}C_{r-1}} = \frac{11}{5}$

We know that $^nC_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)…………2.1

⇒ $\frac{\frac{15!}{(15-r)!r!}}{\frac{15!}{(15-(r-1))!(r-1)!}} = \frac{11}{5}$

$\Rightarrow \frac{(16-r)!}{(15-r)!r} = \frac{11}{5}$

$\Rightarrow \frac{16-r}{r} = \frac{11}{5}$

⇒ 5(16 – r) = 11r

⇒ 80 – 5r = 11r

⇒ 16r = 80

$\Rightarrow r = \frac{80}{16}$

⇒ r = 5

∴ The value of r is 5.

10. Question

If $^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, find n.

Answer

Given:

$\Rightarrow \frac{^{n+2}C_8}{^{n-2}P_4} = \frac{57}{16}$

We know that $^nC_r = \frac{n!}{r!(n-r)!}$, $^nP_r = \frac{n!}{(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow \frac{\frac{(n+2)!}{(n+2-8)!8!}}{\frac{(n-2)!}{(n-2-4)!}} = \frac{57}{16}$

$\Rightarrow \frac{(n+2)!(n-6)!}{(n-6)!(n-2)!8!} = \frac{57}{16}$

$\Rightarrow \frac{(n+2)(n+1)(n)(n-1)}{8!} = \frac{57}{16}$

$\Rightarrow (n + 2)(n + 1)(n)(n - 1) = \frac{19 \times 3 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{16}$

⇒ (n + 2)(n + 1)(n)(n – 1) = 21 × 20 × 19 × 18

Equating the corresponding terms on both sides we get,

⇒ n = 19

∴ The value of n is 19.

11. Question

If $^{28}C_{2r} : {}^{24}C_{2r-4} : 225 : 11$, find r.

Answer

Given:

$\Rightarrow \frac{^{28}C_{2r}}{^{24}C_{2r-4}} = \frac{225}{11}$

We know that $^nC_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow \frac{\frac{28!}{(28-2r)!2r!}}{\frac{24!}{(24-(2r-4))!(2r-4)!}} = \frac{225}{11}$

$\Rightarrow \frac{28!(28-2r)!(2r-4)!}{(28-2r)!24!2r!} = \frac{225}{11}$

$\Rightarrow \frac{28 \times 27 \times 26 \times 25}{(2r)(2r-1)(2r-2)(2r-3)} = \frac{225}{11}$

$\Rightarrow (2r)(2r-1)(2r-2)(2r-3) = \frac{28 \times 27 \times 26 \times 25 \times 11}{225}$

$\Rightarrow (2r)(2r - 1)(2r - 2)(2r - 3) = 11 \times 12 \times 13 \times 14$

Equating the corresponding terms on both sides we get,

$\Rightarrow 2r = 14$

$\Rightarrow r = \frac{14}{2}$

$\Rightarrow r = 7$

∴ The value of r is 7.

12. Question

If ${}^nC_4$, ${}^nC_5$, and ${}^nC_6$ are in A.P., then find n.

Answer

Given that ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in A.P.

We know that for three numbers a, b, c are in A.P, the following condition holds,

$\Rightarrow 2b = a + c$

So,

$\Rightarrow 2{}^nC_5 = {}^nC_4 + {}^nC_6$

Adding $2{}^nC_5$ on both sides we get,

$\Rightarrow 4{}^nC_5 = {}^nC_4 + {}^nC_5 + {}^nC_5 + {}^nC_6$

We know that ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$

$\Rightarrow 4{}^nC_5 = {}^{n+1}C_5 + {}^{n+1}C_6$

$\Rightarrow 4{}^nC_5 = {}^{n+2}C_6$

We know that ${}^nC_r = \frac{n!}{r!(n-r)!}$, ${}^nP_r = \frac{n!}{(n-r)!}$

And also n! = n(n – 1)(n – 2)…………2.1

$\Rightarrow 4\left(\frac{n!}{(n-5)!5!}\right) = \frac{(n+2)!}{(n+2-6)!6!}$

$\Rightarrow \frac{4}{(n-5)!} = \frac{(n+2)(n+1)}{(n-4)!6}$

$\Rightarrow 24 = \frac{(n+2)(n+1)}{n-4}$

$\Rightarrow 24(n - 4) = n^2 + 2n + n + 2$

$\Rightarrow 24n - 96 = n^2 + 3n + 2$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow n^2 - 14n - 7n + 98 = 0$

$\Rightarrow n(n - 14) - 7(n - 14) = 0$

⇒ (n – 7)(n – 14) = 0

⇒ n – 7 = 0 or n – 14 = 0

⇒ n = 7 or n = 14

∴ The values of n are 7 and 14.

13. Question

If ${}^{2n}C_3 : {}^{n}C_2 = 44 : 3$, find n.

Answer

Given:

$\Rightarrow \frac{{}^{2n}C_3}{{}^{n}C_2} = \frac{44}{3}$

We know that ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow \frac{\frac{2n!}{(2n-3)!3!}}{\frac{n!}{(n-2)!2!}} = \frac{44}{3}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{(n)(n-1)3} = \frac{44}{3}$

⇒ 2 × (2n – 1) × 2 = 44

⇒ 2n – 1 = 11

⇒ 2n = 12

$\Rightarrow n = \frac{12}{2}$

⇒ n = 6

∴ The value of n is 6.

14. Question

If ${}^{16}C_r = {}^{16}C_{r+2}$, find ${}^{r}C_4$.

Answer

We know that if ${}^{n}C_p = {}^{n}C_q$, then one of the following conditions need to be satisfied:

iii. p = q

iv. n = p + q

From the problem ${}^{16}C_r = {}^{16}C_{r+2}$ we can say that,

⇒ r≠r + 2

So, the condition(ii) must be satisfied,

⇒ 16 = r + r + 2

⇒ 2r = 14

$\Rightarrow r = \frac{14}{2}$

⇒ r = 7.

∴ The value of r is 7.

15. Question

If $\alpha = {}^{m}C_2$, then find the value of ${}^{\alpha}C_2$.

Answer

$\Rightarrow \alpha = {}^{m}C_2$

We know that ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow \alpha = \frac{m!}{(m-2)!2!}$

$\Rightarrow \alpha = \frac{(m)(m-1)}{2}$ ..... (1)

We need to find ${}^{\alpha}C_2$

$\Rightarrow {}^{\alpha}C_2 = \frac{\alpha!}{(\alpha-2)!2!}$

$\Rightarrow {}^{\alpha}C_2 = \frac{(\alpha)(\alpha-1)}{2}$

From (1)

$\Rightarrow {}^{\alpha}C_2 = \frac{\left(\frac{(m)(m-1)}{2}\right)\left(\frac{(m)(m-1)}{2}-1\right)}{2}$

$\Rightarrow {}^{\alpha}C_2 = \frac{(m)(m-1)(m^2-m-2)}{8}$

$\Rightarrow {}^{\alpha}C_2 = \frac{(m)(m-1)(m^2-2m+m-2)}{8}$

$\Rightarrow {}^{\alpha}C_2 = \frac{(m)(m-1)\left(m(m-2)+1(m-2)\right)}{8}$

$\Rightarrow {}^{\alpha}C_2 = \frac{(m+1)(m)(m-1)(m-2)}{8}$

∴ The value of ${}^{\alpha}C_2$ is $\frac{(m+1)(m)(m-1)(m-2)}{8}$.

16. Question

Prove that the product of 2n consecutive negative integers is divisible by (2n)!

Answer

Let us assume the negative consecutive integers are –1, – 2,......, – (2n)

Let M be the product of the negative integers,

$\Rightarrow$ M = (– 1).(– 2)(– 3)......(– 2n + 1).(– 2n)

$\Rightarrow$ M = $(- 1)^{2n}$(1.2.3.......(2n – 1).(2n))

$\Rightarrow$ M = 1.2.3...........(2n – 1).(2n)

We know that, n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow$ M = (2n)!

$\Rightarrow \frac{M}{(2n)!} = \frac{(2n)!}{(2n)!}$

$\Rightarrow \frac{M}{(2n)!} = 1$

∴ M is divisible by (2n)!.

17. Question

For all positive integers n, show that ${}^{2n}C_n + {}^{2n}C_{n-1} = \frac{1}{2}\left({}^{2n+2}C_{n+1}\right)$

Answer

Given that we need to prove ${}^{2n}C_n + {}^{2n}C_{n-1} = \frac{1}{2}({}^{2n+2}C_{n+1})$.

Consider L.H.S:

We know that ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = {}^{2n+1}C_n$

We know that ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{(2n+1)!}{(2n+1-n)!n!}$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{(2n+1)!}{(n+1)!n!}$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{(2n+1)!}{(n+1)!n!} \times \frac{2n+2}{2n+2}$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{(2n+2)!}{2 \times (n+1)! \times (n+1) \times n!}$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{1}{2}\left(\frac{(2n+2)!}{(n+1)!(n+1)!}\right)$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{1}{2}\left(\frac{(2n+2)!}{(2n+2-(n+1))!(n+1)!}\right)$

$\Rightarrow {}^{2n}C_n + {}^{2n}C_{n-1} = \frac{1}{2}({}^{2n+2}C_{n+1})$

= R.H.S

∴ L.H.S = R.H.S, thus proved.

18. Question

Prove that : ${}^{4n}C_{2n} : {}^{2n}C_n = [1 . 3 . 5 .... (4n – 1)] : [1.3.5..... (2n – 1)]^2$.

Answer

Given that we need to prove:

${}^{4n}C_{2n} : {}^{2n}C_n = [1 . 3 . 5 .... (4n – 1)] : [1.3.5..... (2n – 1)]^2$.

Consider L.H.S:

We know that ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{\frac{(4n)!}{(4n-2n)!(2n)!}}{\frac{(2n)!}{(2n-n)!n!}}$

$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{(4n)!(n)!(n)!}{(2n)!(2n)!(2n)!}$

$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{((4n).(4n-1).(4n-2)..........2.1).(n)!.(n)!}{(2n)!(2n)!(2n)!}$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{(1.3.5.......(4n-1)).(2.4.6.......(4n-2).(4n)).(n)!(n)!}{(2n)!(2n)!(2n)!}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{2^{2n}(1.2.3........(2n-1).(2n)).(1.3.5......(4n-1)).(n)!(n)!}{(2n)!(2n)!(2n)!}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{2^{2n}(2n)!(1.3.5..............(4n-1)).(n!)(n!)}{(2n)!(2n)!(2n)!}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{2^{2n}(1.3.5...........(4n-1))(n)!(n)!}{(1.2.3...........(2n-1)(2n))(1.2.3...........(2n-1)(2n))}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{2^{2n}(1.3.5..............(4n-1))(n)!(n)!}{(1.3.5...........(2n-1))(1.3.5.........(2n-1))(2.4.6.........(2n-2)(2n))(2.4.6.........(2n-2)(2n))}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{2^{2n}(1.3.5.................(4n-1))(n)!(n)!}{(1.3.5.........(2n-1))^2.2^n(1.2.3......(n-1)(n)).2^n(1.2.3........(n-1)(n))}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{2^{2n}(1.3.5...........(4n-1))(n)!(n)!}{(1.3.5.........(2n-1))^2 2^{2n}.(n)!(n)!}$$

$$\Rightarrow \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{(1.3.5..............(4n-1))}{(1.3.5............(2n-1))^2}$$

= R.H.S

∴ L.H.S = R.H.S, thus proved.

19. Question

Evaluate ${}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4$

Answer

Given that we need to find the value of ${}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4$.

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = {}^{20}C_5 + {}^{23}C_4 + {}^{22}C_4 + {}^{21}C_4 + {}^{20}C_4$

We know that ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = ({}^{20}C_5 + {}^{20}C_4) + {}^{21}C_4 + {}^{22}C_4 + {}^{23}C_4$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = ({}^{21}C_5 + {}^{21}C_4) + {}^{22}C_4 + {}^{23}C_4$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = ({}^{22}C_5 + {}^{22}C_4) + {}^{23}C_4$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = {}^{23}C_5 + {}^{23}C_4$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = {}^{24}C_5$

We know that ${}^{n}C_r = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)…………2.1

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = \frac{24!}{(24-5)!5!}$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = \frac{24!}{19!5!}$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = \frac{24 \times 23 \times 22 \times 21 \times 20}{5 \times 4 \times 3 \times 2 \times 1}$

$\Rightarrow {}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4 = 42504$

∴ The value of ${}^{20}C_5 + \sum_{r=2}^{5} {}^{25-r}C_4$ is 42504.

20 A. Question

Let r and n be positive integers such that 1 ≤ r ≤ n. Then prove the following:

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$$

Answer

Given that we need to prove $\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$

Consider L.H.S,

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{\frac{n!}{(n-r)!r!}}{\frac{n!}{(n-(r-1))!(r-1)!}}$$

$$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{(n-r+1)!(r-1)!}{(n-r)!(r)!}$$

$$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$$

= R.H.S

∴ L.H.S = R.H.S, thus proved.

20 B. Question

Let r and n be positive integers such that 1 ≤ r ≤ n. Then prove the following:

$n.{}^{n-1}C_{r-1} = (n-r+1)({}^{n}C_{r-1})$

Answer

Given that we need to prove $n.{}^{n-1}C_{r-1} = (n-r+1).{}^{n}C_{r-1}$

Consider L.H.S,

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)...........2.1

$$\Rightarrow n.{}^{n-1}C_{r-1} = n.\frac{(n-1)!}{(n-1-(r-1))!(r-1)!}$$

$$\Rightarrow n.{}^{n-1}C_{r-1} = \frac{n!}{(n-r)!(r-1)!}$$

$$\Rightarrow n.{}^{(n-1)}C_{r-1} = \frac{n!}{(n-r)!(r-1)!} \times \frac{n-r+1}{n-r+1}$$

$$\Rightarrow n.{}^{n-1}C_{r-1} = (n-r+1) \times \frac{n!}{(n-r+1)!(r-1)!}$$

$$\Rightarrow n.{}^{n-1}C_{r-1} = (n-r+1) \times \frac{n!}{(n-(r-1))!(r-1)!}$$

$$\Rightarrow n.{}^{n-1}C_{r-1} = (n-r+1) \times {}^{n}C_{r-1}$$

= R.H.S

∴ L.H.S = R.H.S, thus proved.

20 C. Question

Let r and n be positive integers such that 1 ≤ r ≤ n. Then prove the following:

$$\frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{n}{r}$$

Answer

Given that we need to prove $\frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{n}{r}$

Consider L.H.S,

We know that ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$

And also n! = n(n – 1)(n – 2)………..2.1

$$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{\frac{n!}{r!(n-r)!}}{\frac{(n-1)!}{(r-1)!(n-1-(r-1))!}}$$

$$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{n(n-r)!}{r(n-r)!}$$

$$\Rightarrow \frac{{}^{n}C_{r}}{{}^{n-1}C_{r-1}} = \frac{n}{r}$$

= R.H.S

∴ L.H.S = R.H.S, thus proved.

20 D. Question

Let r and n be positive integers such that 1 ≤ r ≤ n. Then prove the following:

${}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+2}Cr$

Answer

Given that we need to prove ${}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+2}C_{r}$

Consider L.H.S,

We know that ${}^{n}C_{r} + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

$\Rightarrow {}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = ({}^{n}C_{r} + {}^{n}C_{r-1}) + ({}^{n}C_{r-1} + {}^{n}C_{r-2})$

$\Rightarrow {}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+1}C_{r} + {}^{n+1}C_{r-1}$

$\Rightarrow {}^{n}C_{r} + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+2}C_{r}$

= R.H.S

∴ L.H.S = R.H.S, thus proved.

Exercise 17.2

1. Question

From a group of 15 cricket players, a team of 11 players is to be chosen. In how many ways can this be done?

Answer

Given that we need to choose 11 players for a team out of available 15 players,

It is similar to choosing ‘r’ combinations out of ‘n’ items.

i.e., ${}^{n}C_{r}$ ways.

Let us assume the choosing the no. of ways be N,

⇒ N = choosing 11 players out of 15 players

⇒ N = ${}^{15}C_{11}$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

⇒ $N = \frac{15!}{(15-11)!11!}$

⇒ $N = \frac{15!}{4!11!}$

⇒ $N = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$

⇒ N = 1365 ways

∴ The total no. of ways of choosing 11 players out of 15 is 1365 ways.

2. Question

How many different boat parties of 8, consisting of 5 boys and 3 girls, can be made from 25 boys and 10 girls?

Answer

Given we need to find the different boat parties of 8, consisting of 5 boys and 3 girls.

The selections of girls and boys are to be made from 25 boys and 10 girls.

Let us assume the number of ways of choosing to be N.

⇒ N = (no. of ways of choosing 5 boys) × (No. of ways of choosing 3 girls)

⇒ N = $({}^{25}C_{5}) \times ({}^{10}C_{3})$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)(n – 2)...........2.1

⇒ $N = \frac{25!}{(25-5)!5!} \times \frac{10!}{(10-3)!3!}$

⇒ $N = \frac{25!}{20!5!} \times \frac{10!}{7!3!}$

⇒ $N = \frac{25 \times 24 \times 23 \times 22 \times 21}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{10 \times 9 \times 8}{3 \times 2 \times 1}$

⇒ N = (53130) × (120)

⇒ N = 6375600

∴ The total no. of different boat parties are 6375600 ways.

3. Question

In how many ways can a student choose 5 courses out of 9 courses if 2 courses are compulsory for every student?

Answer

Given that we need to find no. of ways of choosing 5 courses out of 9 courses if 2 courses are compulsory.

This is similar to choosing 3 subjects out of the remaining 7 subjects as 2 subjects are compulsory.

Let us assume the total no. of ways of choosing courses is N.

⇒ N = No. of ways of choosing 3 subjects out of 7 subjects.

⇒ N = $^7C_3$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)(n – 2)……2.1

⇒ $N = \frac{7!}{(7-3)!3!}$

⇒ $N = \frac{7!}{4!3!}$

⇒ $N = \frac{7\times6\times5}{3\times2\times1}$

⇒ N = 35

∴ The total no. of ways of choosing 5 subjects out of 9 subjects in which 2 are compulsory is 35 ways.

4. Question

In how many ways can a football team of 11 players be selected from 16 players? How many of these will (i) include 2 particular players? (ii) exclude 2 particular players.

Answer

Given that we need to choose 11 players for a team out of available 16 players,

Let us assume the choosing the no. of ways be N,

⇒ N = choosing 11 players out of 16 players

⇒ N = $^{16}C_{11}$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)…………2.1

⇒ $N = \frac{16!}{(16-11)!11!}$

⇒ $N = \frac{16!}{5!11!}$

⇒ $N = \frac{16\times15\times14\times13\times12}{5\times4\times3\times2\times1}$

⇒ N = 4368 ways

(i) It is told that two players are always included.

It is similar to selecting 9 players out of the remaining 14 players as 2 players are already selected.

Let us assume the choosing the no. of ways be N_1,

⇒ N_1 = choosing 9 players out of 14 players

⇒ $N_1 = {}^{14}C_9$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)…………2.1

⇒ $N_1 = \frac{14!}{(14-9)!9!}$

⇒ $N_1 = \frac{14!}{5!9!}$

⇒ $N_1 = \frac{14\times13\times12\times11\times10}{5\times4\times3\times2\times1}$

$\Rightarrow N_1 = 2002$ ways

(ii) It is told that two players are always excluded.

It is similar to selecting 11 players out of the remaining 14 players as 2 players are already removed.

Let us assume the choosing the no. of ways be N_2,

$\Rightarrow N_2$ = choosing 11 players out of 14 players

$\Rightarrow N_2 = {}^{14}C_{11}$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)............2.1

$\Rightarrow N_2 = \frac{14!}{(14-11)!11!}$

$\Rightarrow N_2 = \frac{14!}{3!11!}$

$\Rightarrow N_2 = \frac{14 \times 13 \times 12}{3 \times 2 \times 1}$

$\Rightarrow N_2 = 364$ ways

∴ The required no. of ways are 4368, 2002, 364.

5 A. Question

There are 10 professors and 20 students out of whom a committee of 2 professors and 3 students is to be formed. Find the number of ways in which this can be done. Further, find in how many of these committees:

a particular professor is included

Answer

Given that we need to choose 2 professors and 3 students out of 10 professors and 20 students,

Let us assume the choosing the no. of ways be N,

$\Rightarrow$ N = (choosing 2 professors out of 10 professors) × (choosing 3 students out of 20 students)

$\Rightarrow N = ({}^{10}C_2) \times ({}^{20}C_3)$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)............2.1

$\Rightarrow N = \left(\frac{10!}{(10-2)!2!}\right) \times \left(\frac{20!}{(20-3)!3!}\right)$

$\Rightarrow N = \left(\frac{10!}{8!2!}\right) \times \left(\frac{20!}{17!3!}\right)$

$\Rightarrow N = \left(\frac{10 \times 9}{2 \times 1}\right) \times \left(\frac{20 \times 19 \times 18}{3 \times 2 \times 1}\right)$

$\Rightarrow N = 45 \times 1140$

$\Rightarrow N = 51300$ ways

It is told that a particular is always included.

It is similar to selecting 1 professor and 3 students out of the remaining 9 professors and 20 students as 1 professor is already selected.

Let us assume the choosing the no. of ways be N_1,

$\Rightarrow N_1$ = (choosing 1 professor out of 9 professors) × (choosing 3 students out of 20 students)

$\Rightarrow N_1 = {}^9C_1 \times {}^{20}C_3$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N_1 = \left(\frac{9!}{(9-1)!1!}\right) \times \left(\frac{20!}{(20-3)!3!}\right)$

$\Rightarrow N_1 = \left(\frac{9!}{8!1!}\right) \times \left(\frac{20!}{17!3!}\right)$

$\Rightarrow N_1 = \left(\frac{9}{1}\right) \times \left(\frac{20 \times 19 \times 18}{3 \times 2 \times 1}\right)$

$\Rightarrow N_1 = 9 \times 1140$ ways

$\Rightarrow N_1 = 10260$ ways

5 B. Question

There are 10 professors and 20 students out of whom a committee of 2 professors and 3 students is to be formed. Find the number of ways in which this can be done. Further, find in how many of these committees:

a particular student is included

Answer

Given that we need to choose 2 professors and 3 students out of 10 professors and 20 students,

Let us assume the choosing the no. of ways be N,

⇒ N = (choosing 2 professors out of 10 professors) × (choosing 3 students out of 20 students)

$\Rightarrow N = ({}^{10}C_2) \times ({}^{20}C_3)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N = \left(\frac{10!}{(10-2)!2!}\right) \times \left(\frac{20!}{(20-3)!3!}\right)$

$\Rightarrow N = \left(\frac{10!}{8!2!}\right) \times \left(\frac{20!}{17!3!}\right)$

$\Rightarrow N = \left(\frac{10 \times 9}{2 \times 1}\right) \times \left(\frac{20 \times 19 \times 18}{3 \times 2 \times 1}\right)$

$\Rightarrow N = 45 \times 1140$

⇒ N = 51300 ways

It is told that one student is always included.

It is similar to selecting 2 professors and 2 students out of remaining 10 professors and 19 students as 1 student is already selected.

Let us assume the choosing the no. of ways be N_2,

⇒ N_2 = (choosing 2 professors out of 10 professors) × (choosing 2 students out of 19 students)

$\Rightarrow N_2 = {}^{10}C_2 \times {}^{19}C_2$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N_2 = \left(\frac{10!}{(10-2)!2!}\right) \times \left(\frac{19!}{(19-2)!2!}\right)$

$\Rightarrow N_2 = \left(\frac{10!}{8!2!}\right) \times \left(\frac{19!}{17!2!}\right)$

$\Rightarrow N_2 = \left(\frac{10 \times 9}{2 \times 1}\right) \times \left(\frac{19 \times 18}{2 \times 1}\right)$

$\Rightarrow N_2 = 45 \times 171$

$\Rightarrow N_2 = 7695$ ways

5 C. Question

There are 10 professors and 20 students out of whom a committee of 2 professors and 3 students is to be formed. Find the number of ways in which this can be done. Further, find in how many of these committees:

a particular student is excluded.

Answer

Given that we need to choose 2 professors and 3 students out of 10 professors and 20 students,

Let us assume the choosing the no. of ways be N,

$\Rightarrow$ N = (choosing 2 professors out of 10 professors) $\times$ (choosing 3 students out of 20 students)

$\Rightarrow N = ({}^{10}C_2) \times ({}^{20}C_3)$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N = \left(\frac{10!}{(10-2)!2!}\right) \times \left(\frac{20!}{(20-3)!3!}\right)$

$\Rightarrow N = \left(\frac{10!}{8!2!}\right) \times \left(\frac{20!}{17!3!}\right)$

$\Rightarrow N = \left(\frac{10 \times 9}{2 \times 1}\right) \times \left(\frac{20 \times 19 \times 18}{3 \times 2 \times 1}\right)$

$\Rightarrow N = 45 \times 1140$

$\Rightarrow N = 51300$ ways

It is told that one student is always excluded.

It is similar to selecting 2 professors and 3 students out of remaining 10 professors and 19 students as 1 student are already removed.

Let us assume the choosing the no. of ways be N_3,

$\Rightarrow N_3$ = (choosing 2 professors out of 10 professors) $\times$ (choosing 3 students out of 19 students)

$\Rightarrow N_3 = {}^{10}C_2 \times {}^{19}C_3$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N_3 = \left(\frac{10!}{(10-2)!2!}\right) \times \left(\frac{19!}{(19-3)!3!}\right)$

$\Rightarrow N_3 = \left(\frac{10!}{8!2!}\right) \times \left(\frac{19!}{16!3!}\right)$

$\Rightarrow N_3 = \left(\frac{10 \times 9}{2 \times 1}\right) \times \left(\frac{19 \times 18 \times 17}{3 \times 2 \times 1}\right)$

$\Rightarrow N_3 = 45 \times 969$

$\Rightarrow N_3 = 43605$ ways.

∴ The required no. of ways are 51300, 10260, 7695, 43605.

6. Question

How many different products can be obtained by multiplying two or more of the numbers 3, 5, 7, 11 (without repetition)?

Answer

Given that we need to find the no. of ways of obtaining a product by multiplying two or more from the numbers 3, 5, 7, 11.

The following are the cases the product can be done,

i. Multiplying two numbers

ii. Multiplying three numbers

iii. Multiplying four numbers

Let us assume the total numbers of ways of the product be N

⇒ N = (no. of ways of multiplying two numbers) + (no. of ways of multiplying three numbers) + (no. of multiplying four numbers)

⇒ $N = {}^4C_2 + {}^4C_3 + {}^4C_4$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N = \left(\frac{4!}{(4-2)!2!}\right) + \left(\frac{4!}{(4-3)!3!}\right) + \left(\frac{4!}{(4-4)!4!}\right)$

$\Rightarrow N = \left(\frac{4!}{2!2!}\right) + \left(\frac{4!}{1!3!}\right) + \left(\frac{4!}{0!4!}\right)$

$\Rightarrow N = \left(\frac{4 \times 3}{2 \times 1}\right) + \left(\frac{4}{1}\right) + \left(\frac{1}{1}\right)$

⇒ N = 6 + 4 + 1

⇒ N = 11

∴ The total number of ways of product is 11 ways.

7. Question

From a class of 12 boys and 10 girls, 10 students are to be chosen for the competition, at least including 4 boys and 4 girls. The 2 girls who won the prizes last year should be included. In how many ways can the selection be made?

Answer

Given that 10 students need to be selected from 12 boys and 10 girls including at least 4 boys and 4 girls.

It is also told that the two girls must be includes who won prizes last year.

The cases that satisfy these conditions are:

i. Selecting 6 boys and 4 girls (in which 2 girls are already included)

ii. Selecting 5 boys and 5 girls (in which 2 girls are already included)

iii. Selecting 4 boys and 6 girls. (in which 2 girls are already included)

Let us assume the total no. of ways of selection be N,

⇒ N = (no. of ways of selecting 6 boys and 2 girls from remaining 12 boys and 8 girls) + (no. of ways of selecting 5 boys and 3 girls from remaining 12 boys and 8 girls) + (no. of ways of selecting 4 boys and 4 girls from remaining 12 boys and 8 girls)

Since, two girls are already selected,

$\Rightarrow N = (^{12}C_6 \times {^8C_2}) + (^{12}C_5 \times {^8C_3}) + (^{12}C_4 \times {^8C_4})$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N = \left(\left(\frac{12!}{(12-6)!6!}\right) \times \left(\frac{8!}{(8-2)!2!}\right)\right) + \left(\left(\frac{12!}{(12-5)!5!}\right) \times \left(\frac{8!}{(8-3)!3!}\right)\right) + \left(\left(\frac{12!}{(12-4)!4!}\right) \times \left(\frac{8!}{(8-4)!4!}\right)\right)$

$\Rightarrow N = \left(\left(\frac{12!}{6!6!}\right) \times \left(\frac{8!}{6!2!}\right)\right) + \left(\left(\frac{12!}{7!5!}\right) \times \left(\frac{8!}{5!3!}\right)\right) + \left(\left(\frac{12!}{8!4!}\right) \times \left(\frac{8!}{4!4!}\right)\right)$

$\Rightarrow N = \left(\left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 32 \times 1}\right) \times \left(\frac{8 \times 7}{2 \times 1}\right)\right) + \left(\left(\frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}\right) \times \left(\frac{8 \times 7 \times 6}{3 \times 2 \times 1}\right)\right) + \left(\left(\frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1}\right) \times \left(\frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}\right)\right)$

$\Rightarrow N = (924 \times 28) + (792 \times 56) + (495 \times 70)$

$\Rightarrow N = 25872 + 44352 + 34650$

$\Rightarrow N = 104874$

∴ The total number of ways of product is 11 ways.

8 A. Question

How many different selections of 4 books can be made from 10 different books, if there is no restriction

Answer

Given that we need to choose 4 books out of available 10 different books,

Let us assume the choosing the no. of ways be N,

⇒ N = choosing 4 books out of 10 books

$\Rightarrow N = {^{10}C_4}$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N = \frac{10!}{(10-4)!4!}$

$\Rightarrow N = \frac{10!}{6!4!}$

$\Rightarrow N = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$

⇒ N = 210 ways

8 B. Question

How many different selections of 4 books can be made from 10 different books, if two particular books are always selected

Answer

It is told that two books are always selected.

It is similar to selecting 2 books out of the remaining 8 books as 2 books are already selected.

Let us assume the choosing the no. of ways be N_1,

$\Rightarrow N_1$ = choosing 2 books out of 8 books

$\Rightarrow N_1 = {}^8C_2$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N_1 = \frac{8!}{(8-2)!2!}$

$\Rightarrow N_1 = \frac{8!}{6!2!}$

$\Rightarrow N_1 = \frac{8 \times 7}{2 \times 1}$

$\Rightarrow N_1$ = 28 ways

8 C. Question

How many different selections of 4 books can be made from 10 different books, if

two particular books are never selected.

Answer

It is told that two books are never selected.

It is similar to selecting 4 books out of remaining 8books as 2 books are already removed.

Let us assume the choosing the no. of ways be N_2,

$\Rightarrow N_2$ = choosing 4 books out of 8 books

$\Rightarrow N_2 = {}^8C_4$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)...........2.1

$\Rightarrow N_2 = \frac{8!}{(8-4)!4!}$

$\Rightarrow N_2 = \frac{8!}{4!4!}$

$\Rightarrow N_2 = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$

$\Rightarrow N_2$ = 70 ways

∴ The required no. of ways are 210, 28, 70.

9. Question

From 4 officers and 8 jawans in how many ways can 6 be chosen (i) to include exactly one officer (ii) to include at least one officer?

Answer

Given that we have 4 officers and 8 jawans, we need to choose 6 persons with the following conditions,

i. To include exactly one officer:

ii. To include at least one officer.

(i) It is told that we need to choose 6 persons with exactly one officer.

Let us assume the no. of ways of choosing to be N.

⇒ N = (no. of ways of choosing 1 officer and 5 jawans from 4 officers and 8 jawans)

⇒ N = (no. of ways of choosing 1 officer from 4 officers) × (no. of ways of choosing 5 jawans from 8 jawans)

⇒ N = $({}^4C_1) \times ({}^8C_5)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N = \left(\frac{4!}{(4-1)!1!}\right) \times \left(\frac{8!}{(8-5)!5!}\right)$

⇒ $N = \left(\frac{4!}{3!1!}\right) \times \left(\frac{8!}{3!5!}\right)$

⇒ $N = \left(\frac{4}{1}\right) \times \left(\frac{8 \times 7 \times 6}{3 \times 2 \times 1}\right)$

⇒ N = 4 × 56

⇒ N = 224 ways.

(ii) It is told we need to choose 6 persons with at least 1 officers.

Let us assume the total no. of ways be N_1

⇒ N_1 = (No. of ways of choosing 6 persons with at least one officer)

⇒ N_1 = (total no. of ways of choosing 6 persons from all 12 persons) – (no. of ways of choosing 6 persons without any officer)

⇒ $N_1 = {}^{12}C_6 - {}^8C_6$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_1 = \frac{12!}{(12-6)!6!} - \frac{8!}{(8-6)!6!}$

⇒ $N_1 = \frac{12!}{6!6!} - \frac{8!}{6!2!}$

⇒ $N_1 = \left(\frac{12 \times 11 \times 10 \times 9 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2 \times 1}\right) - \left(\frac{8 \times 7}{2 \times 1}\right)$

⇒ N_1 = 924 – 28

⇒ N_1 = 896 ways

∴ The required no. of ways are 224 and 896.

10. Question

A sports team of 11 students is to be constituted, choosing at least 5 from class XI and at least 5 from class XII. If there are 20 students in each of these classes, in how many ways can the teams be constituted?

Answer

Given that we need to choose a team of 11 students with at least 5 from class XI and 5 from class XII.

It is also mentioned that each class constitutes 20 students.

There are two cases of selecting a team:

i. 6 from class XI and 5 from class XII

ii. 5 from class XI and 6 from class XII

Let us assume the total no. of ways of selecting 11 students to be N.

⇒ N = no. of ways of selecting 11 students from both classes

⇒ N = (No. of ways of selecting 6 students from class XI and 5 students from class XII) + (No. of ways of selecting 5 students from class XI and 6 students from class XII)

⇒ N = $({}^{20}C_6 \times {}^{20}C_5) + ({}^{20}C_5 \times {}^{20}C_6)$

We know that $^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N = \left(\left(\frac{20!}{(20-6)!6!}\right) \times \left(\frac{20!}{(20-5)!5!}\right)\right) + \left(\left(\frac{20!}{(20-5)!5!}\right) \times \left(\frac{20!}{(20-6)!6!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{20!}{14!6!}\right) \times \left(\frac{20!}{15!5!}\right)\right) + \left(\left(\frac{20!}{15!5!}\right) \times \left(\frac{20!}{14!6!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{6 \times 5 \times 4 \times 3 \times 2 \times 1}\right) \times \left(\frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1}\right)\right) + \left(\left(\frac{20 \times 19 \times 18 \times 17 \times 16}{5 \times 4 \times 3 \times 2 \times 1}\right) \times \left(\frac{20 \times 19 \times 18 \times 17 \times 16 \times 15}{6 \times 5 \times 4 \times 3 \times 2 \times 1}\right)\right)$$

⇒ N = ((38760) × (15504)) + ((15504) × (38760))

⇒ N = 600935040 + 600935040

⇒ N = 1201870080 ways

11. Question

A student has to answer 10 questions, choosing at least 4 from each of part A and part B. If there are 6 questions in part A and 7 in part B, in how many ways can the student choose 10 questions?

Answer

Given that 10 questions are to be answered by part A and part B by choosing at least 4 from each part.

It is also mentioned that there are 6 questions in part A and 7 in part B.

There are 3 cases to answer 10 questions:

i. 4 from part A and 6 from part B

ii. 5 from part A and 5 from part B

iii. 6 from part A and 4 from part B

Let us assume the total no. of ways of answering 10 questions be N.

⇒ N = no. of ways of answering 10 questions from both parts

⇒ N = (No. of ways of answering 4 questions from part A and 6 from part B) + (No. of ways of answering 5 questions from part A and 5 questions from part B) + (No. of ways of answering 6 questions from part A and 4 from part B)

⇒ N = $({}^{6}C_4 \times {}^{7}C_6) + ({}^{6}C_5 \times {}^{7}C_5) + ({}^{6}C_6 \times {}^{7}C_4)$

We know that $^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N = \left(\left(\frac{6!}{(6-4)!4!}\right) \times \left(\frac{7!}{(7-6)!6!}\right)\right) + \left(\left(\frac{6!}{(6-5)!5!}\right) \times \left(\frac{7!}{(7-5)!5!}\right)\right) + \left(\left(\frac{6!}{(6-6)!6!}\right) \times \left(\frac{7!}{(7-4)!4!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{6!}{2!4!}\right) \times \left(\frac{7!}{1!6!}\right)\right) + \left(\left(\frac{6!}{1!5!}\right) \times \left(\frac{7!}{2!5!}\right)\right) + \left(\left(\frac{6!}{0!6!}\right) \times \left(\frac{7!}{3!4!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{6\times5}{2\times1}\right)\times\left(\frac{7}{1}\right)\right)+\left(\left(\frac{6}{1}\right)\times\left(\frac{7\times6}{2\times1}\right)\right)+\left(\left(\frac{1}{1}\right)\times\left(\frac{7\times6\times5}{3\times2\times1}\right)\right)$$

$\Rightarrow N = (15 \times 7) + (6 \times 21) + (1 \times 35)$

$\Rightarrow N = 105 + 126 + 35$

$\Rightarrow N = 266$

∴ The total no. of ways of answering 10 questions is 266 ways.

12. Question

In an examination, a student to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make a choice.

Answer

Given that student have to answer 4 questions from 5 questions.

It is also told that questions 1 and 2 are compulsory.

It is similar to answering the 2 questions out of the remaining 3 questions as 1 and 2 are compulsory.

Let us assume the no. of ways of answering the questions be N.

⇒ N = No. of ways of answering 2 questions from remaining 3 questions.

$\Rightarrow N = {}^{3}C_{2}$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \frac{3!}{(3-2)!2!}$

$\Rightarrow N = \frac{3!}{1!2!}$

$\Rightarrow N = \frac{3}{1}$

⇒ N = 3

∴ The no. of answering the questions is 3.

13. Question

A candidate is required to answer 7 questions out of 12 questions which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 questions from either group. In how many ways can he choose the 7 questions?

Answer

Given that we need to answer 7 questions from 2 groups which consist of 6 questions each.

It is also told the candidate is permitted to answer the utmost 5 questions from any group.

The cases for answering the 7 questions:

i. 5 questions from group 1 and 2 from group 2

ii. 4 questions from group 1 and 3 from group 2

iii. 3 questions from group 1 and 4 from group 2

iv. 4 questions from group 1 and 5 from group 2

Let us assume the total no. of ways of answering 7 questions be N.

⇒ N = no. of ways of answering 7 questions from both groups

⇒ N = (No. of ways of answering 5 questions from group 1 and 2 from group 2) + (No. of ways of answering 4 questions from group 1 and 3 from group 2) + (No. of ways of answering 3 questions from group 1 and 4 from group 2) + (No. of ways of answering 2 questions from group 1 and 5 from group 2)

⇒ N = $({}^6C_5 \times {}^6C_2) + ({}^6C_4 \times {}^6C_3) + ({}^6C_3 \times {}^6C_4) + ({}^6C_2 \times {}^6C_5)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N = \left(\left(\frac{6!}{(6-5)!5!}\right) \times \left(\frac{6}{(6-2)!2!}\right)\right) + \left(\left(\frac{6!}{(6-4)!4!}\right) \times \left(\frac{6!}{(6-3)!3!}\right)\right) + \left(\left(\frac{6!}{(6-3)!3!}\right) \times \left(\frac{6!}{(6-4)!4!}\right)\right) + \left(\left(\frac{6}{(6-2)!2!}\right) \times \left(\frac{6!}{(6-5)!5!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{6!}{1!5!}\right) \times \left(\frac{6!}{2!4!}\right)\right) + \left(\left(\frac{6!}{2!4!}\right) \times \left(\frac{6!}{3!3!}\right)\right) + \left(\left(\frac{6!}{3!3!}\right) \times \left(\frac{6!}{2!4!}\right)\right) + \left(\left(\frac{6!}{2!4!}\right) \times \left(\frac{6!}{1!5!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{6}{1}\right) \times \left(\frac{6 \times 5}{2 \times 1}\right)\right) + \left(\left(\frac{6 \times 5}{2 \times 1}\right) \times \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1}\right)\right) + \left(\left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1}\right) \times \left(\frac{6 \times 5}{2 \times 1}\right)\right) + \left(\left(\frac{6 \times 5}{2 \times 1}\right) \times \left(\frac{6}{1}\right)\right)$$

⇒ N = (6 × 15) + (15 × 20) + (20 × 15) + (15 × 6)

⇒ N = 90 + 300 + 300 + 90

⇒ N = 780

∴ The total no. of ways of choosing 7 questions is 780 ways.

14. Question

There are 10 points in a plane of which 4 are collinear. How many different straight lines can be drawn by joining these points.

Answer

Given that we need to find the no. of different straight lines that can be drawn from the 10 points in which 4 are collinear.

We know that 2 points are required to draw a line and the collinear points will lie on the same line, and only one line can be drawn by joining any two points of these collinear points.

Let us assume the no. of lines formed be N,

⇒ N = (total no. of lines formed by all 10 points) – (no. of lines formed by collinear points) + 1

Here 1 is added because only 1 line can be formed by the four collinear points.

⇒ N = ${}^{10}C_2 - {}^4C_2 + 1$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N = \left(\frac{10!}{(10-2)!2!}\right) - \left(\frac{4!}{(4-2)!2!}\right) + 1$$

$$\Rightarrow N = \left(\frac{10!}{8!2!}\right) - \left(\frac{4!}{2!2!}\right) + 1$$

$$\Rightarrow N = \left(\frac{10 \times 9}{2 \times 1}\right) - \left(\frac{4 \times 3}{2 \times 1}\right) + 1$$

⇒ N = 45 – 6 + 1

⇒ N = 40

∴ The total no. of ways of different lines formed are 40.

15 A. Question

Find the number of diagonals of

a hexagon

Answer

Given that we need to find the no. of diagonals of

A hexagon

We know that the hexagon has 6 vertices and each side and diagonal can be formed by joining two vertices of a hexagon,

We know that hexagon has 6 sides,

Let us assume the no. of diagonals of the hexagon are N,

⇒ N = (no. of lines formed on joining any two vertices) – (no. of sides of the hexagon)

⇒ N = $^{6}C_{2}$ – 6

We know that $^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N = \left(\frac{6!}{(6-2)!2!}\right) - 6$

⇒ $N = \left(\frac{6!}{4!2!}\right) - 6$

⇒ $N = \left(\frac{6 \times 5}{2 \times 1}\right) - 6$

⇒ N = 15 – 6

⇒ N = 9

∴ The total no. of diagonals formed is 9.

15 B. Question

Find the number of diagonals of

a polygon of 16 sides

Answer

a polygon of 16 sides

We have given that polygon has 16 vertices and each side and diagonal can be formed by joining two vertices of a polygon,

Let us assume the no. of diagonals of the polygon are N,

⇒ N = (no. of lines formed on joining any two vertices) – (no. of sides of the polygon)

⇒ N = $^{16}C_{2}$ – 16

We know that $^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N = \left(\frac{16!}{(16-2)!2!}\right) - 16$

$\Rightarrow N = \left(\frac{16!}{14!2!}\right) - 16$

$\Rightarrow N = \left(\frac{16 \times 15}{2 \times 1}\right) - 16$

⇒ N = 120 – 16

⇒ N = 104

∴ The total no. of diagonals formed are 104.

16. Question

How many triangles can be obtained by joining 12 points, five of which are collinear?

Answer

Given that we need to find the no. of triangles that can be drawn from the 12 points in which 5 are collinear.

We know that 3 points are required to draw a triangle and the collinear points will lie on the same line, and no triangle can be drawn by joining any three points of these collinear points.

Let us assume the no. of triangles formed be N,

⇒ N = (total no. of triangles formed by all 12 points) – (no. of triangles formed by collinear points)

⇒ N = $^{12}C_3$ – $^{5}C_3$

We know that $^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{12!}{(12-3)!3!}\right) - \left(\frac{5!}{(5-3)!3!}\right)$

$\Rightarrow N = \left(\frac{12!}{9!3!}\right) - \left(\frac{5!}{2!3!}\right)$

$\Rightarrow N = \left(\frac{12 \times 11 \times 10}{3 \times 2 \times 1}\right) - \left(\frac{5 \times 4}{2 \times 1}\right)$

⇒ N = 220 – 10

⇒ N = 210

∴ The total no. of triangles formed are 210.

17. Question

In how many ways can a committee of 5 persons be formed out of 6 men and 4 women when at least one woman has to be necessarily selected?

Answer

Given that we need to find no. of ways of selecting 5 persons out of 6 men and 4 women in which at least one woman is necessary.

Let us assume the no. of ways of selection be N.

⇒ N = (total no. of ways of selecting 5 persons out of all 10 persons) – (No. of ways of selecting 5 persons without any women)

⇒ N = ($^{10}C_5$) – ($^{6}C_5$)

We know that $^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{10!}{(10-5)!5!}\right) - \left(\frac{6!}{(6-5)!5!}\right)$

$\Rightarrow N = \left(\frac{10!}{5!5!}\right) - \left(\frac{6!}{1!5!}\right)$

$\Rightarrow N = \left(\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}\right) - \left(\frac{6}{1}\right)$

⇒ N = 252 – 6

⇒ N = 246

∴ The total no. of choosing 5 persons with at least one woman is 246.

18. Question

In a village, there are 87 families of which 52 families have at most 2 children. In a rural development programme, 20 families are to be helped chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

Answer

Given that 52 families out of 82 families have at most 2 children.

It is told that 20 families need to be selected with at least 18 families having utmost 2 children.

The following are the cases:

i. 18 families having at most 2 children and 2 from other families

ii. 19 families are having at most 2 children and 1 from other families

iii. 20 families having at most 2 children

Let us assume that the no. of ways choosing to be N.

⇒ N = (Selecting 18 families having at most 2 children and 2 from other families) + (Selecting 19 families having at most 2 children and 1 from other families) + (Selecting 20 families having at most 2 children)

$\Rightarrow N = ({}^{52}C_{18} \times {}^{35}C_2) + ({}^{52}C_{19} \times {}^{35}C_1) + ({}^{52}C_{20})$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\left(\frac{52!}{(52-18)!18!}\right) \times \left(\frac{35!}{(35-2)!2!}\right)\right) + \left(\left(\frac{52!}{(52-19)!19!}\right) \times \left(\frac{35!}{(35-1)!1!}\right)\right) + \left(\frac{52!}{(52-20)!20!}\right)$

$\Rightarrow N = \left(\left(\frac{52!}{34!18!}\right) \times \left(\frac{35!}{33!2!}\right)\right) + \left(\left(\frac{52!}{33!19!}\right) \times \left(\frac{35!}{34!1!}\right)\right) + \left(\frac{52!}{32!20!}\right)$

$\Rightarrow N = \left(\frac{52!}{32!18!}\right) \times \left(\left(\left(\frac{1}{34 \times 33}\right)\left(\frac{35 \times 34}{2 \times 1}\right)\right) + \left(\left(\frac{1}{33 \times 19}\right)\left(\frac{35}{1}\right)\right) + \left(\frac{1}{20 \times 19}\right)\right)$

$\Rightarrow N = \left(\frac{52!}{32!18!}\right) \times \left(\frac{35}{66} + \frac{35}{627} + \frac{1}{380}\right)$

$\Rightarrow N = \left(\frac{52!}{32!18!}\right) \times \left(\frac{2461}{4180}\right)$

∴ The no. of ways of choosing 18 families are $\left(\frac{52!}{32!18!}\right) \times \left(\frac{2461}{4180}\right)$.

19. Question

A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl? (ii) at least one boy and one girl? (ii) at least 3 girls?

Answer

Given that a group consists of 4 girls and 7 boys.

We need to select a team of 5 members with the following conditions:

i. If a team has no girl

ii. If a team has at least one boy and one girl

iii. If a team has at least 3 girls.

i. Given that we need to select a team of 5 members with no girl present in it out of 4 girls and 7 boys.

Let us assume the no. of ways of selection be N

⇒ N = (selecting 5 members out of 7 boys without any girl)

⇒ N = ${}^{7}C_{5}$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \frac{7!}{(7-5)!5!}$

$\Rightarrow N = \frac{7!}{2!5!}$

$\Rightarrow N = \frac{7 \times 6}{2 \times 1}$

⇒ N = 21 ways

The no. of ways of selecting 5 members without a girl is 21 ways.

ii. Given that we need to select team of 5 members with at least 1 boy and 1 girl.

Let us assume the no. of ways of selection be N_1.

⇒ N_1 = (Total ways of selecting 5 members out of all 11 members) – (No. of ways of selecting 5 members without any girl)

⇒ $N_1 = ({}^{11}C_5) - ({}^{7}C_5)$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_1 = \left(\frac{11!}{(11-5)!5!}\right) - \left(\frac{7!}{(7-5)!5!}\right)$

$\Rightarrow N_1 = \left(\frac{11!}{6!5!}\right) - \left(\frac{7!}{2!5!}\right)$

$\Rightarrow N_1 = \left(\frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1}\right) - \left(\frac{7 \times 6}{2 \times 1}\right)$

⇒ $N_1 = 462 - 21$

⇒ $N_1 = 441$

The no. of ways of selecting 5 members with at least 1 girl and 1 boy is 441.

iii. Given that we need to find the no. of ways to select 5 members with at least 3 girls out of 7 boys and 4 girls.

The following are the possible cases:

i. Selecting 3 girls and 2 boys

ii. Selecting 4 girls and 1 boy

Let us assume the no. of ways of selection be N_2.

⇒ N_2 = (No. of ways of selecting 3 girls and 2 boys out of 7 boys and 4 girls) + (No. of ways of selecting 4 girls and 1 boy out of 7 boys and 4 girls)

⇒ $N_2 = ((^4C_3) \times (^7C_2)) + ((^4C_4) \times (^7C_1))$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)……2.1

$$\Rightarrow N_2 = \left(\left(\frac{4!}{(4-3)!3!}\right) \times \left(\frac{7!}{(7-2)!2!}\right)\right) + \left(\left(\frac{4!}{(4-4)!4!}\right) \times \left(\frac{7!}{(7-1)!1!}\right)\right)$$

$$\Rightarrow N_2 = \left(\left(\frac{4!}{1!3!}\right) \times \left(\frac{7!}{5!2!}\right)\right) + \left(\left(\frac{4!}{0!4!}\right) \times \left(\frac{7!}{6!1!}\right)\right)$$

$$\Rightarrow N_2 = \left(\left(\frac{4}{1}\right) \times \left(\frac{7 \times 6}{2 \times 1}\right)\right) + \left(\left(\frac{1}{1}\right) \times \left(\frac{7}{1}\right)\right)$$

⇒ $N_2 = (4 \times 21) + (1 \times 7)$

⇒ $N_2 = 84 + 7$

⇒ $N_2 = 91$

The no. of ways of selecting 5 members with at least 3 girls is 91.

20. Question

A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of this committee would consist of 1 man and 2 women?

Answer

Given that we need to select 3 persons out of 2 men and 3 women,

Let us assume the no. of ways of selecting be N,

⇒ N = selecting 3 persons out of total 5 persons

⇒ $N = {^5C_3}$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)…………2.1

$$\Rightarrow N = \frac{5!}{(5-3)!3!}$$

$$\Rightarrow N = \frac{5!}{2!3!}$$

$$\Rightarrow N = \frac{5 \times 4}{2 \times 1}$$

⇒ N = 10 ways

The no. of ways of selecting 3 persons out of 2 men and 3 women is 10.

(ii) It is told that 1 man and 2 women should be selected out of 2 men and 3 women.

Let us assume the no. of ways of selection be N_1,

⇒ N_1 = (selecting one man out of 2 men) × (selecting 2 women out of 3 women)

⇒ $N_1 = (^2C_1) \times (^3C_2)$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$

And also n! = (n)(n – 1)(n – 2)…………2.1

$\Rightarrow N_1 = \left(\frac{2!}{(2-1)!1!}\right) \times \left(\frac{3!}{(3-2)!2!}\right)$

$\Rightarrow N_1 = \left(\frac{2!}{1!1!}\right) \times \left(\frac{3!}{1!2!}\right)$

$\Rightarrow N_1 = \left(\frac{2}{1}\right) \times \left(\frac{3}{1}\right)$

$\Rightarrow N_1 = 6$ ways

The no. of ways of selecting 1 man and 2 women is 6.

21. Question

Find the number of (i) diagonals (ii) triangles formed in a decagon.

Answer

i. We know that the decagon has 10 vertices and each side and diagonal can be formed by joining two vertices of a hexagon,

We know that decagon has 10 sides,

Let us assume the no. of diagonals of the hexagon are N,

$\Rightarrow$ N = (no. of lines formed on joining any two vertices) – (no. of sides of the hexagon)

$\Rightarrow N = {}^{10}C_2 - 10$

We know that ${}^{n}C_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{10!}{(10-2)!2!}\right) - 10$

$\Rightarrow N = \left(\frac{10!}{8!2!}\right) - 10$

$\Rightarrow N = \left(\frac{10 \times 9}{2 \times 1}\right) - 10$

$\Rightarrow N = 45 - 10$

$\Rightarrow N = 35$

$\therefore$ The total no. of diagonals formed is 35.

ii. Given that we need to find the no. of triangles that can be drawn in a decagon.

We know that 3 points are required to draw a triangle.

We know that decagon has 10 sides

Let us assume the no. of triangles formed be N_1,

$\Rightarrow N_1$ = (total no. of triangles formed by all 10 points)

$\Rightarrow N_1 = {}^{10}C_3$

We know that ${}^{n}C_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_1 = \left(\frac{10!}{(10-3)!3!}\right)$

$\Rightarrow N_1 = \left(\frac{10!}{7!3!}\right)$

$\Rightarrow N_1 = \left(\frac{10 \times 9 \times 8}{3 \times 2 \times 1}\right)$

$\Rightarrow N_1 = 120$

∴ The total no. of ways of different lines formed are 120.

22. Question

Determine the number of 5 cards combinations out of a deck of 52 cards if at least one of the 5 cards has to be a king?

Answer

Given that we need to draw 5 cards from a deck of 52 cards.

We need to find the no. of ways that at least one of the 5 cards has to be a king.

We know that there are 4 kings present in a deck.

Let us assume the no. of ways of drawing cards be N.

⇒ N = (Total no. of ways of drawing 5 cards out of 52 cards) – (No. of ways of drawing 5 cards without a king from remaining 48 cards)

$\Rightarrow N = ({}^{52}C_5) - ({}^{48}C_5)$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{52!}{(52-5)!5!}\right) - \left(\frac{48!}{(48-5)!5!}\right)$

$\Rightarrow N = \left(\frac{52!}{47!5!}\right) - \left(\frac{48!}{43!5!}\right)$

$\Rightarrow N = \left(\frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1}\right) - \left(\frac{48 \times 47 \times 46 \times 45 \times 44}{5 \times 4 \times 3 \times 2 \times 1}\right)$

⇒ N = 2598960 – 1712304

⇒ N = 886656

∴ The no. of ways of drawing 5 cards with at least 1 king is 886656.

23. Question

We wish to select 6 persons from 8, but if the person A is chosen, then B must be chosen. In how many ways can the selection be made?

Answer

Given that we need to select 6 persons out of 8 persons, it is also mentioned that person B must be selected in the case of selection of A.

We need to find the no. of ways of selecting 6 persons.

The possible cases for the selection of 6 persons are:

i. selecting person A and person B and 4 others(similar to selecting 4 persons out of remaining 6 persons).

ii. selecting 6 persons other than person A and person B

Let us assume the no. of ways of selection be N.

N = (Selecting 4 persons from remaining 6 persons) + (Selecting 6 persons leaving Person A and B)

$N = ({}^{6}C_4) + ({}^{6}C_6)$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{6!}{(6-4)!4!}\right) + \left(\frac{6!}{(6-6)!6!}\right)$

$\Rightarrow N = \left(\frac{6!}{2!4!}\right) + \left(\frac{6!}{0!6!}\right)$

$\Rightarrow N = \left(\frac{6 \times 5}{2 \times 1}\right) + \left(\frac{1}{1}\right)$

⇒ N = 15 + 1

⇒ N = 16

∴ The no. of ways of selecting 6 persons is 16.

24. Question

In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Answer

Given that we need to select a team consisting of 3 boys and 3 girls out of 5 boys and 4 girls.

Let us assume the no. of ways of selection be N.

⇒ N = (Selecting 3 boys out of 5 boys) × (selecting 3 girls out of 4 girls)

⇒ N = ($^5C_3$) × ($^4C_3$)

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{5!}{(5-3)!3!}\right) \times \left(\frac{4!}{(4-3)!3!}\right)$

$\Rightarrow N = \left(\frac{5!}{2!3!}\right) \times \left(\frac{4!}{1!3!}\right)$

$\Rightarrow N = \left(\frac{5 \times 4}{2 \times 1}\right) \times \left(\frac{4}{1}\right)$

⇒ N = (10) × (4)

⇒ N = 40

∴ The no. of ways of selecting 3 boys and 3 girls is 40.

25. Question

Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls, and 5 blue balls if each selection consists of 3 balls of each color.

Answer

Given that we need to select the 3 red, 3 white, and 3 blue balls out of 6 red, 5 white and 5 blue balls.

Let us assume the no. of ways of selection be N.

⇒ N = (no. of ways of selection of 3 red balls out of 6 red balls) × (no. of ways of selection of 3 white balls out of 5 white balls) × (no. of ways of selection of 3 blue balls out of 5 blue balls)

⇒ N = ($^6C_3$) × ($^5C_3$) × ($^5C_3$)

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{6!}{(6-3)!3!}\right) \times \left(\frac{5!}{(5-3)!3!}\right) \times \left(\frac{5!}{(5-3)!3!}\right)$

$\Rightarrow N = \left(\frac{6!}{3!3!}\right) \times \left(\frac{5!}{2!3!}\right) \times \left(\frac{5!}{2!3!}\right)$

$\Rightarrow N = \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1}\right) \times \left(\frac{5 \times 4}{2 \times 1}\right) \times \left(\frac{5 \times 4}{2 \times 1}\right)$

⇒ N = 20 × 10 × 10

⇒ N = 2000

∴ The no. of ways of selection is 2000.

26. Question

Determine the number of 5 cards combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Answer

Given that 5 cards are drawn out of 52 cards. We know that there are 4 aces present in a deck of 52 cards.

We need to find the no. of ways of drawing 5 cards with exactly one ace out of a deck of 52 cards.

Let us assume the no. of ways of drawing cards be N.

⇒ N = (Drawing 1 ace out of 4 aces) × (Drawing 4 cards out of remaining 48 cards)

⇒ N = ($^4C_1$) × ($^{48}C_4$)

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{4!}{(4-1)!1!}\right) \times \left(\frac{48!}{(48-4)!4!}\right)$

$\Rightarrow N = \left(\frac{4!}{3!1!}\right) \times \left(\frac{48!}{44!4!}\right)$

$\Rightarrow N = \left(\frac{4}{1}\right) \times \left(\frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1}\right)$

⇒ N = 4 × 194580

⇒ N = 778320

∴ The no. of ways of drawing 5 cards with exactly one ace is 778320.

27. Question

In how many ways can one select a cricket team of eleven from 17 players in which only 5 persons can bowl if each cricket team of 11 must include exactly 4 bowlers?

Answer

Given that we need to select 11 players out of available 17 players in which 5 players are bowlers.

It is also mentioned that each team should contain exactly 4 bowlers.

Let us assume the no. of ways of selection be N.

⇒ N = (no. of ways of selecting 4 bowlers out of 5) × (no. of ways of selecting 7 players from remaining 12 players)

⇒ N = ($^5C_4$) × ($^{12}C_7$)

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{5!}{(5-4)!4!}\right) \times \left(\frac{12!}{(12-7)!7!}\right)$

$\Rightarrow N = \left(\frac{5!}{1!4!}\right) \times \left(\frac{12!}{5!7!}\right)$

$\Rightarrow N = \left(\frac{5}{1}\right) \times \left(\frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1}\right)$

⇒ N = 5 × 792

⇒ N = 3960

28. Question

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Answer

Given that we need to draw 2 black and 3 red balls from a bag of 5 black and 6 red balls.

Let us assume the no. of ways of drawing be N.

⇒ N = (no. of ways of selecting 2 black balls from 5 black balls) × (no. of ways of selecting 3 red balls from 6 red balls)

⇒ N = ($^5C_2$) × ($^6C_3$)

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{5!}{(5-2)!2!}\right) \times \left(\frac{6!}{(6-3)!3!}\right)$

$\Rightarrow N = \left(\frac{5!}{3!2!}\right) \times \left(\frac{6!}{3!3!}\right)$

$\Rightarrow N = \left(\frac{5 \times 4}{2 \times 1}\right) \times \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1}\right)$

⇒ N = 10 × 20

⇒ N = 200

∴ The no. of ways of drawing 2 black and 3 red balls is 200.

29. Question

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Answer

Given that a student needs to choose 5 courses out of 9 courses. It is also told 2 courses are compulsory to the student.

It is similar to choosing 3 courses of remaining 7 courses since 2 courses are already chosen.

Let us assume the no. of ways of choosing to be N,

⇒ N = (no. of ways of choosing 3 courses out of remaining 7 courses)

⇒ N = $^7C_3$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \frac{7!}{(7-3)!3!}$

$\Rightarrow N = \frac{7!}{4!3!}$

$\Rightarrow N = \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$

$\Rightarrow N = 35$

∴ The no. of ways of choosing 5 courses is 35.

30 A. Question

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of :

exactly 3 girls?

Answer

Given that we need to select 7 members out of 9 boys and 4 girls by following the conditions:

i. exactly 3 girls

ii. at least 3 girls

iii. at most 3 girls.

i. It is told we need to select 7 members out of 9 boys and 4 girls with exactly 3 girls.

Let us assume the no. of ways of selecting is N.

⇒ N = (no. of ways of selecting 3 girls out of 4 girls) × (no. of ways of selecting 4 boys out of 9 boys)

$\Rightarrow N = ({}^4C_3) \times ({}^9C_4)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{4!}{(4-3)!3!}\right) \times \left(\frac{9!}{(9-4)!4!}\right)$

$\Rightarrow N = \left(\frac{4!}{1!3!}\right) \times \left(\frac{9!}{5!4!}\right)$

$\Rightarrow N = \left(\frac{4}{1}\right) \times \left(\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}\right)$

$\Rightarrow N = 4 \times 126$

$\Rightarrow N = 504$

The no. of ways of selecting 7 members with exactly 3 girls is 504.

30 B. Question

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of :

at least 3 girls?

Answer

Given that we need to select 7 members out of 9 boys and 4 girls by following the conditions:

i. exactly 3 girls

ii. at least 3 girls

iii. at most 3 girls.

It is told we need to select 7 members out of 9 boys and 4 girls with at least 3 girls.

The possible cases are the following:

i. Selecting 3 girls and 4 boys

ii. Selecting 4 girls and 3 boys

Let us assume the no. of ways of selecting is N_1.

$\Rightarrow N_1$ = ((no. of ways of selecting 3 girls out of 4 girls) × (no. of ways of selecting 4 boys out of 9 boys)) × ((no. of ways of selecting 4 girls out of 4 girls) × (no. of ways of selecting 3 boys out of 9 boys))

$\Rightarrow N_1 = ((^4C_3) \times (^9C_4)) + ((^4C_4) \times (^9C_3))$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N_1 = \left(\left(\frac{4!}{(4-3)!3!}\right) \times \left(\frac{9!}{(9-4)!4!}\right)\right) + \left(\left(\frac{4!}{(4-4)!4!}\right) \times \left(\frac{9!}{(9-3)!3!}\right)\right)$$

$$\Rightarrow N_1 = \left(\left(\frac{4!}{1!3!}\right) \times \left(\frac{9!}{5!4!}\right)\right) + \left(\left(\frac{4!}{0!4!}\right) \times \left(\frac{9!}{6!3!}\right)\right)$$

$$\Rightarrow N_1 = \left(\left(\frac{4}{1}\right) \times \left(\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}\right)\right) + \left(\left(\frac{1}{1}\right) \times \left(\frac{9 \times 8 \times 7}{3 \times 2 \times 1}\right)\right)$$

$\Rightarrow N_1 = (4 \times 126) + (1 \times 84)$

$\Rightarrow N_1 = 504 + 84$

$\Rightarrow N_1 = 588$

The no. of ways of selecting 7 members with at least 3 girls is 588.

30 C. Question

A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of :

at most 3 girls?

Answer

Given that we need to select 7 members out of 9 boys and 4 girls by following the conditions:

i. exactly 3 girls

ii. at least 3 girls

iii. at most 3 girls.

It is told we need to select 7 members out of 9 boys and 4 girls with at most 3 girls.

The possible cases are the following:

i. Selecting 3 girls and 4 boys

ii. Selecting 2 girls and 5 boys

iii. Selecting 1 girl and 6 boys

iv. Selecting 7 boys

Let us assume the no. of ways of selecting is N_2.

$\Rightarrow N_2$ = ((no. of ways of selecting 3 girls out of 4 girls) × (no. of ways of selecting 4 boys out of 9 boys)) × ((no. of ways of selecting 2 girls out of 4 girls) × (no. of ways of selecting 5 boys out of 9 boys)) × ((no. of ways of selecting 1 girls out of 4 girls) × (no. of ways of selecting 6 boys out of 9 boys))

$\Rightarrow N_2 = ((^4C_3) \times (^9C_4)) + ((^4C_2) \times (^9C_5)) + ((^4C_1) \times (^9C_6)) + (^9C_7)$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N_2 = \left(\left(\frac{4!}{(4-3)!3!}\right) \times \left(\frac{9!}{(9-4)!4!}\right)\right) + \left(\left(\frac{4!}{(4-2)!2!}\right) \times \left(\frac{9!}{(9-5)!5!}\right)\right) + \left(\left(\frac{4!}{(4-1)!1!}\right) \times \left(\frac{9!}{(9-6)!6!}\right)\right) + \left(\frac{9!}{(9-7)!7!}\right)$$

$$\Rightarrow N_2 = \left(\left(\frac{4!}{1!3!}\right) \times \left(\frac{9!}{5!4!}\right)\right) + \left(\left(\frac{4!}{2!2!}\right) \times \left(\frac{9!}{4!5!}\right)\right) + \left(\left(\frac{4!}{3!1!}\right) \times \left(\frac{9!}{3!6!}\right)\right) + \left(\frac{9!}{2!7!}\right)$$

$$\Rightarrow N_2 = \left(\left(\frac{4}{1}\right) \times \left(\frac{9\times8\times7\times6}{4\times3\times2\times1}\right)\right) + \left(\left(\frac{4\times3}{2\times1}\right) \times \left(\frac{9\times8\times7\times6}{4\times3\times2\times1}\right)\right) + \left(\left(\frac{4}{1}\right) \times \left(\frac{9\times8\times7}{3\times2\times1}\right)\right) + \left(\frac{9\times}{2\times1}\right)$$

$\Rightarrow N_2 = (4 \times 126) + (6 \times 126) + (4 \times 84) + (36)$

$\Rightarrow N_2 = 504 + 756 + 336 + 36$

$\Rightarrow N_2 = 1632$

The no. of ways of selecting 7 members with at most 3 girls is 1632.

31. Question

In an examination, a question paper consists of 12 questions divided into two parts, i.e., Part I and Part II, containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many ways can help a student select the questions?

Answer

Given that a student needs to answer 8 questions out of 12 questions in which 5 from part I and 7 from part II.

It is also told that student needs to answer at least 3 questions from each part.

The possible cases are the following:

i. 3 from part I and 5 from part II

ii. 4 from part I and 4 from part II

iii. 5 from part I and 3 from part II

Let us assume the no. of ways of answering be N.

⇒ N = (no. of ways of answering 3 questions from part I and 5 from part II) + (no. of ways of answering 4 questions from part I and 4 from part II) + (no. of ways of answering 5 questions from part I and 3 from part II)

$\Rightarrow N = ((^5C_3) \times (^7C_5)) + ((^5C_4) \times (^7C_4)) + ((^5C_5) \times (^7C_3))$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$$\Rightarrow N = \left(\left(\frac{5!}{(5-3)!3!}\right) \times \left(\frac{7!}{(7-5)!5!}\right)\right) + \left(\left(\frac{5!}{(5-4)!4!}\right) \times \left(\frac{7!}{(7-4)!4!}\right)\right) + \left(\left(\frac{5!}{(5-5)!5!}\right) \times \left(\frac{7!}{(7-3)!3!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{5!}{2!3!}\right) \times \left(\frac{7!}{2!5!}\right)\right) + \left(\left(\frac{5!}{1!4!}\right) \times \left(\frac{7!}{3!4!}\right)\right) + \left(\left(\frac{5!}{0!5!}\right) \times \left(\frac{7!}{4!3!}\right)\right)$$

$$\Rightarrow N = \left(\left(\frac{5\times4}{2\times1}\right) \times \left(\frac{7\times6}{2\times1}\right)\right) + \left(\left(\frac{5}{1}\right) \times \left(\frac{7\times6\times5}{3\times2\times1}\right)\right) + \left(\left(\frac{1}{1}\right) \times \left(\frac{7\times6\times5}{3\times2\times1}\right)\right)$$

$\Rightarrow N = (10 \times 21) + (5 \times 35) + (35)$

⇒ N = 420

∴ The no. of ways of answering the question paper is 420.

32. Question

A parallelogram is cut by two sets of m lines parallel to its sides. Find the number of parallelograms thus formed.

Answer

We know that parallelogram has 4 lines in which 2 sides are parallel to each other which means 2 pairs of lines are parallel lines

It is told that parallelogram is cut by two sets of m lines parallel to its sides.

This means there will be two sets of (m + 2) lines parallel to each other.

We need two sets of parallel lines to form a parallelogram in which the lines need to be chosen from these two sets of (m + 2) parallel lines.

Let us assume that the no. of parallelograms formed be N.

⇒ N = (choosing 2 parallel lines from (m + 2) parallel lines which are parallel to one side) × (choosing 2 parallel lines from (m + 2) parallel lines which are parallel to the side which is not parallel to the first side)

⇒ N = $({}^{m+2}C_2) \times ({}^{m+2}C_2)$

⇒ N = $({}^{m+2}C_2)^2$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{(m+2)!}{(m+2-2)!2!}\right)^2$

$\Rightarrow N = \left(\frac{(m+2)!}{m!2!}\right)^2$

$\Rightarrow N = \left(\frac{(m+2)(m+1)}{2 \times 1}\right)^2$

$\Rightarrow N = \frac{(m+2)^2(m+1)^2}{4}$

∴ The total no. of parallelograms formed are $\frac{(m+2)^2(m+1)^2}{4}$.

33. Question

Out of 18 points in a plane, no three are in the same straight line except five points which are collinear. How many (i) straight lines (ii) triangles can be formed by joining them?

Answer

Given that we need to find the no. of different straight lines that can be drawn from the 18 points in which 5 are collinear.

We know that 2 points are required to draw a line and the collinear points will lie on the same line, and only one line can be drawn by joining any two points of these collinear points.

Let us assume the no. of lines formed be N,

⇒ N = (total no. of lines formed by all 18 points) – (no. of lines formed by collinear points) + 1

Here 1 is added because only 1 line can be formed by the four collinear points.

⇒ N = ${}^{18}C_2 - {}^{5}C_2 + 1$

We know that ${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{18!}{(18-2)!2!}\right) - \left(\frac{5!}{(5-2)!2!}\right) + 1$

$\Rightarrow N = \left(\frac{18!}{16!2!}\right) - \left(\frac{5!}{3!2!}\right) + 1$

$\Rightarrow N = \left(\frac{18 \times 17}{2 \times 1}\right) - \left(\frac{5 \times 4}{2 \times 1}\right) + 1$

⇒ N = 153 – 10 + 1

⇒ N = 144

∴ The total no. of ways of different lines formed are 144.

Given that we need to find the no. of triangles that can be drawn from the 18 points in which 5 are collinear.

We know that 3 points are required to draw a triangle and the collinear points will lie on the same line, and no triangle can be drawn by joining any three points of these collinear points.

Let us assume the no. of triangles formed be N,

⇒ N = (total no. of triangles formed by all 18 points) – (no. of triangles formed by collinear points)

⇒ N = $^{18}C_3$ – $^5C_3$

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N = \left(\frac{18!}{(18-3)!3!}\right) - \left(\frac{5!}{(5-3)!3!}\right)$

$\Rightarrow N = \left(\frac{18!}{15!3!}\right) - \left(\frac{5!}{2!3!}\right)$

$\Rightarrow N = \left(\frac{18 \times 17 \times 16}{3 \times 2 \times 1}\right) - \left(\frac{5 \times 4}{2 \times 1}\right)$

⇒ N = 816 – 10

⇒ N = 806

∴ The total no. of triangles formed are 806.

Exercise 17.3

1. Question

How many different words, each containing 2 vowels and 3 consonants can be formed with 5 vowels and 17 consonants?

Answer

Given that we need to find the no. of words formed by 2 vowels and 3 consonants which were taken from 5 vowels and 17 consonants.

Let us find the no. of ways of choosing 2 vowels and 3 consonants and assume it to be N_1.

⇒ N_1 = (No. of ways of choosing 2 vowels from 5 vowels) × (No. of ways of choosing 3 consonants from 17 consonants)

⇒ N_1 = ($^5C_2$) × ($^{17}C_3$)

We know that $^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_1 = \left(\frac{5!}{(5-2)!2!}\right) \times \left(\frac{17!}{(17-3)!3!}\right)$

$\Rightarrow N_1 = \left(\frac{5!}{3!2!}\right) \times \left(\frac{17!}{14!3!}\right)$

$\Rightarrow N_1 = \left(\frac{5 \times 4}{2 \times 1}\right) \times \left(\frac{17 \times 16 \times 15}{3 \times 2 \times 1}\right)$

$\Rightarrow N_1 = 10 \times 680$

$\Rightarrow N_1 = 6800$

Now we need to find the no. of words that can be formed by 2 vowels and 3 consonants.

Now we need to arrange the chosen 5 letters. Since every letter differs from other.

The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 5!.

Let us the total no. of words formed be N.

$\Rightarrow N = N_1 \times 5!$

$\Rightarrow N = 6800 \times 120$

$\Rightarrow N = 816000$

∴ The no. of words that can be formed containing 2 vowels and 3 consonants are 816000.

2. Question

There are 10 persons named P_1, P_2, P_3,......P_{10}. Out of 10 persons, 5 persons are to be arranged in a line such that is each arrangement P_1 must occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

Answer

Given that 5 persons need to be selected from 10 person P_1, P_2, P_3,......P_{10}.

It is also told that P_1 should be present and P_4 and P_5 should not be present.

It is similar to choosing 4 persons from remaining 7 persons as P_1 is selected and P_4 and P_5 are already removed.

Let us first find the no. of ways to choose persons and assume it to be N_1.

$\Rightarrow N_1$ = Selecting 4 persons from remaining 7 persons

$\Rightarrow N_1 = {}^7C_4$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_1 = \frac{7!}{(7-4)!4!}$

$\Rightarrow N_1 = \frac{7!}{3!4!}$

$\Rightarrow N_1 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1}$

$\Rightarrow N_1 = 35$

Now we need to arrange the chosen 5 people. Since 1 person differs from other.

The arrangement is similar to that of arranging n people in n places which are n! Ways to arrange. So, the persons can be arranged in 5! Ways.

Let us assume the total possible arrangements be N.

$\Rightarrow N = N_1 \times 5!$

$\Rightarrow N = 35 \times 120$

$\Rightarrow N = 4200$

∴ The total no. of possible arrangement can be done is 4200.

3 A. Question

How many words, with or without meaning can be formed from the letters of the word 'MONDAY', assuming that no letter is repeated, if

4 letters are used at a time

Answer

Given that we need to find the no. of words formed by 4 letters which were taken from word 'MONDAY.'

Let us find the no. of ways of choosing 4 letters and assume it to be N_1.

$\Rightarrow N_1$ = (No. of ways of choosing 4 letters from MONDAY)

$\Rightarrow N_1 = ({}^6C_4)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_1 = \left(\frac{6!}{(6-4)!4!}\right)$

$\Rightarrow N_1 = \left(\frac{6!}{2!4!}\right)$

$\Rightarrow N_1 = \left(\frac{6 \times 5}{2 \times 1}\right)$

$\Rightarrow N_1 = 15$

Now we need to find the no. of words that can be formed by 4 letters.

Now we need to arrange the chosen 4 letters. Since 1 person differs from other.

The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 4!.

Let us the total no. of words formed be N.

$\Rightarrow N = N_1 \times 4!$

$\Rightarrow N = 15 \times 24$

$\Rightarrow N = 360$

∴ The no. of words that can be formed by 4 letters of MONDAY is 360.

3 B. Question

How many words, with or without meaning can be formed from the letters of the word 'MONDAY', assuming that no letter is repeated, if

all letters are used at a time

Answer

Given that we need to find the no. of words formed by all letters of MONDAY.

Now we need to arrange the 6 letters. Since every letter differs from other.

The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 6!.

Let us the total no. of words formed be N.

⇒ N = 6!

⇒ N = 360

∴ The no. of words that can be formed by 6 letters of MONDAY is 360.

3 C. Question

How many words, with or without meaning can be formed from the letters of the word 'MONDAY', assuming that no letter is repeated, if

all letters are used but the first letter is a vowel?

Answer

Given that we need to find the no. of words formed by all letters from MONDAY in which the first letter should be a vowel.

In MONDAY the vowels are O and A. We need to choose one vowel from these 2 vowels for the first place of the word.

Let us find the no. of ways of choosing vowel and assume it to be N_1.

⇒ N_1 = (No. of ways of choosing a vowel from 2 vowels)

⇒ $N_1 = ({}^2C_1)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_1 = \left(\frac{2!}{(2-1)!1!}\right)$

⇒ $N_1 = \left(\frac{2!}{1!1!}\right)$

⇒ $N_1 = \left(\frac{2}{1}\right)$

⇒ $N_1 = 2$

Now we need to find the no. of words that can be formed by remaining 5 letters.

Now we need to arrange the remaining 5 letters. Since every letter differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 5!.

Let us the total no. of words formed be N.

⇒ $N = N_1 \times 5!$

⇒ N = 2 × 120

⇒ N = 240

∴ The no. of words that can be formed by all letters of MONDAY in which the first letter is a vowel is 240.

4. Question

Find the number of permutations of n distinct things taken r together, in which 3 particular things must occur together.

Answer

Given that we need to find the no. of permutations formed by r things which were taken from n distinct things in which 3 particular things must occur together.

Here, it is clear that 3 things are already selected and we need to choose (r – 3) things from the remaining (n – 3) things.

Let us find the no. of ways of choosing (r – 3) things and assume it to be N_1.

⇒ N_1 = (No. of ways of choosing (r – 3) things from remaining (n – 3) things)

⇒ $N_1 = {}^{n-3}C_{r-3}$

Now we need to find the no. of permutations than can be formed using 3 things which are together.

Now we need to arrange the chosen 3 things. Since every thing differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 3!.

Now let us assume the together things as a single thing this gives us total (r – 2) things which were present now.

Now, we need to arrange these (r – 2) things. Since every thing differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is (r – 2)!.

Let us the total no. of words formed be N.

⇒ $N = N_1 \times 3! \times (r-2)!$

⇒ $N = {}^{n-3}C_{r-3} \times 3! \times (r-2)!$

∴ The no. of permutations that can be formed by r things which are chosen from n things in which 3 things are always together is ${}^{n-3}C_{r-3} \times 3! \times (r-2)!$.

5. Question

How many words each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?

Answer

Given the word is INVOLUTE. We have 4 vowels namely I,O,U,E, and consonants namely N,V,L,T.

We need to find the no. of words that can be formed using 3 vowels and 2 consonants which were chosen from the letters of involute.

Let us find the no. of ways of choosing 3 vowels and 2 consonants and assume it to be N_1.

⇒ N_1 = (No. of ways of choosing 3 vowels from 4 vowels) × (No. of ways of choosing 2 consonants from 4 consonants)

⇒ $N_1 = ({}^4C_3) \times ({}^4C_2)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_1 = \left(\frac{4!}{(4-3)!3!}\right) \times \left(\frac{4!}{(4-2)!2!}\right)$

⇒ $N_1 = \left(\frac{4!}{1!3!}\right) \times \left(\frac{4!}{2!2!}\right)$

⇒ $N_1 = \left(\frac{4}{1}\right) \times \left(\frac{4 \times 3}{2 \times 1}\right)$

⇒ $N_1 = 4 \times 6$

⇒ $N_1 = 24$

Now we need to find the no. of words that can be formed by 3 vowels and 2 consonants.

Now we need to arrange the chosen 5 letters. Since every letter differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 5!.

Let us the total no. of words formed be N.

$\Rightarrow N = N_1 \times 5!$

$\Rightarrow N = 24 \times 120$

$\Rightarrow N = 2880$

∴ The no. of words that can be formed containing 3 vowels and 2 consonants chosen from INVOLUTE is 2880.

6. Question

Find the number of permutations of n different things r at a time such that two specified things occur together?

Answer

Given that we need to find the no. of permutations formed by r things which were taken from n distinct things in which 2 specified things must occur together.

Here, it is clear that 2 things are already selected and we need to choose (r – 2) things from the remaining (n – 2) things.

Let us find the no. of ways of choosing (r – 2) things and assume it to be N_1.

$\Rightarrow N_1 =$ (No. of ways of choosing (r – 2) things from remaining (n – 2) things)

$\Rightarrow N_1 = {}^{n-2}C_{r-2}$

Now we need to find the no. of permutations than can be formed using 2 things which are together.

Now we need to arrange the chosen 2 things. Since every thing differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is 2!.

Now let us assume the together things as a single thing this gives us total (r – 1) things which were present now.

Now, we need to arrange these (r – 1) things. Since every thing differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of words that can be formed is (r – 1)!.

Let us the total no. of words formed be N.

$\Rightarrow N = N_1 \times 2! \times (r - 1)!$

$\Rightarrow N = {}^{n-2}C_{r-2} \times 2 \times (r - 1)!$

∴ The no. of permutations that can be formed by r things which are chosen from n things in which 3 things are always together are ${}^{n-2}C_{r-2} \times 2 \times (r - 1)!$.

7. Question

Find the number of ways in which: (a) a selection (b) an arrangement, of four letters, can be made from the letters of the word ‘PROPORTION’?

Answer

Given the word is PROPORTION. The letters present in it are:

P: 2 in number

R: 2 in number

O: 3 in number

T: 1 in number

I: 1 in number

N: 1 in number

a. We need to find the no. of ways of selecting 4 letters from the word proportion:

The possible cases are the following:

i. 4 distinct letters

ii. 2 alike letters and 2 distinct letters.

iii. 2 alike letters of one type and 2 alike letters of another type

iv. 3 alike letters and 1 distinct letter

i. There are 6 different letters from which we need to select 4 letters. Let us assume no. of ways of selection be N_1

⇒ N_1 = no. of ways of selecting 4 letters from 6 letters

⇒ $N_1 = {}^6C_4$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_1 = \frac{6!}{(6-4)!4!}$

⇒ $N_1 = \frac{6!}{2!4!}$

⇒ $N_1 = \frac{6 \times 5}{2 \times 1}$

⇒ $N_1 = 15$

ii. There are 3 letters which occurred more than once. So, we need to select 1 letter from these 3 and 2 distinct letters from the remaining 5 distinct letters. Let us assume no. of ways of selection be N_2

⇒ N_2 = (no. of ways of selecting 2 alike letters from the 3 types of alike letters) × (no. of ways of selecting 2 distinct letters from remaining 5 distinct letters)

⇒ $N_2 = ({}^3C_1) \times ({}^5C_2)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_2 = \left(\frac{3!}{(3-1)!1!}\right) \times \left(\frac{5!}{(5-2)!2!}\right)$

⇒ $N_2 = \left(\frac{3!}{2!1!}\right) \times \left(\frac{5!}{3!2!}\right)$

⇒ $N_2 = \left(\frac{3}{1}\right) \times \left(\frac{5 \times 4}{2 \times 1}\right)$

⇒ $N_2 = 3 \times 10$

⇒ $N_2 = 30$

iii. There are 3 letters which occurred more than once from which we need to select 2. Let us assume no. of ways of selection be N_3

⇒ N_3 = no. of ways of selecting 2 alike letters of one type and 2 alike letters of another type

⇒ $N_3 = {}^3C_2$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_3 = \frac{3!}{(3-2)!2!}$

$\Rightarrow N_3 = \frac{3!}{1!2!}$

$\Rightarrow N_3 = \frac{3}{1}$

$\Rightarrow N_3 = 3$

iv. There is only 1 letter which occurred thrice, and 1 letter needs to be selected from the remaining 5 distinct letters.

Let us assume no. of ways of selection be N_4

$\Rightarrow N_4$ = no. of ways of selecting 1 letter from 5 letters

$\Rightarrow N_4 = {}^5C_1$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_4 = \frac{5!}{(5-1)!1!}$

$\Rightarrow N_4 = \frac{5!}{4!1!}$

$\Rightarrow N_4 = \frac{5}{1}$

$\Rightarrow N_4 = 5$

Total no. of ways of selection = $N_1 + N_2 + N_3 + N_4$

Total no. of ways of selection = 15 + 30 + 3 + 5

Total no. of ways of selection = 53

b. We need to find the no. of ways of arranging 4 letters from the word proportion:

The possible cases are the following:

i. 4 distinct letters

ii. 2 alike letters and 2 distinct letters.

iii. 2 alike letters of one type and 2 alike letters of another type

iv. 3 alike letters and 1 distinct letter

i. Now we need to arrange the chosen 4 different letters. Since every word differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of arrangements that can be made is 4!.

Let us assume no. of ways of arrangement be N_5.

$\Rightarrow N_5 = N_1 \times 4!$

$\Rightarrow N_5 = 15 \times 24$

$\Rightarrow N_5 = 360$

ii. Now we need to arrange the chosen 2 different letters and 2 alike letters. The arrangement is similar to that of arranging n people in n places in which r are similar which are $\frac{n!}{r!}$ ways to arrange. So, the total no. of arrangements that can be made are $\frac{4!}{2!}$.

Let us assume no. of ways of arrangement be N_6.

$\Rightarrow N_6 = N_2 \times \frac{4!}{2!}$

$\Rightarrow N_6 = 30 \times 4 \times 3$

$\Rightarrow N_6 = 360$

iii. Now we need to arrange the chosen 2 alike letters of one type and 2 alike letters of another type. The arrangement is similar to that of arranging n people in n places in which r are similar of one type and m are similar of another type which are $\frac{n!}{r!m!}$ ways too arrange. So, the total no. of arrangements that can be made are $\frac{4!}{2!2!}$.

Let us assume no. of ways of arrangement be N_7.

$\Rightarrow N_7 = N_3 \times \frac{4!}{2!2!}$

$\Rightarrow N_7 = 3 \times \frac{4 \times 3}{2 \times 1}$

$\Rightarrow N_7 = 18$

iv. Now we need to arrange the chosen 1 different letters and 3 alike letters. The arrangement is similar to that of arranging n people in n places in which r are similar which are $\frac{n!}{r!}$ ways to arrange. So, the total no. of arrangements that can be made are $\frac{4!}{3!}$.

Let us assume no. of ways of arrangement be N_8.

$\Rightarrow N_8 = N_4 \times \frac{4!}{3!}$

$\Rightarrow N_8 = 5 \times 4$

$\Rightarrow N_8 = 20$

Total no. of ways of arrangement = $N_5 + N_6 + N_7 + N_8$

Total no. of ways of arrangement = 360 + 360 + 18 + 20

Total no. of ways of arrangement = 758

8. Question

How many words can be formed by taking 4 letters at a time from the letters of the word 'MORADABAD'?

Answer

Given the word is MORADABAD. The letters present in it are:

M: 1 in number

O: 1 in number

R: 1 in number

A: 3 in number

D: 2 in number

B: 1 in number

a. We need to find the no. of words formed by 4 letters from the word MORADABAD:

The possible cases are the following:

i. 4 distinct letters

ii. 2 alike letters and 2 distinct letters.

iii. 2 alike letters of one type and 2 alike letters of another type

iv. 3 alike letters and 1 distinct letter

i. There are 6 different letters from which we need to select 4 letters. Let us assume no. of ways of selection be N_1

⇒ N_1 = no. of ways of selecting 4 letters from 6 letters

⇒ $N_1 = {}^6C_4$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_1 = \frac{6!}{(6-4)!4!}$

⇒ $N_1 = \frac{6!}{2!4!}$

⇒ $N_1 = \frac{6 \times 5}{2 \times 1}$

⇒ $N_1 = 15$

Now we need to arrange the chosen 4 different letters. Since every word differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of arrangements that can be made is 4!.

Let us assume no. of ways of arrangement be N_2.

⇒ $N_2 = N_1 \times 4!$

⇒ $N_2 = 15 \times 24$

⇒ $N_2 = 360$

ii. There are 2 letters which occurred more than once. So, we need to select 1 letter from these 2 and 2 distinct letters from the remaining 5 distinct letters. Let us assume no. of ways of selection be N_3

⇒ N_3 = (no. of ways of selecting 2 alike letters from the 2 types of alike letters) × (no. of ways of selecting 2 distinct letters from remaining 5 distinct letters)

⇒ $N_3 = ({}^2C_1) \times ({}^5C_2)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_3 = \left(\frac{2!}{(2-1)!1!}\right) \times \left(\frac{5!}{(5-2)!2!}\right)$

⇒ $N_3 = \left(\frac{2!}{1!1!}\right) \times \left(\frac{5!}{3!2!}\right)$

⇒ $N_3 = \left(\frac{2}{1}\right) \times \left(\frac{5 \times 4}{2 \times 1}\right)$

⇒ $N_3 = 2 \times 10$

⇒ $N_3 = 20$

Now we need to arrange the chosen 2 different letters and 2 alike letters. The arrangement is similar to that of arranging n people in n places in which r are similar which are $\frac{n!}{r!}$ ways to arrange. So, the total no. of arrangements that can be made are $\frac{4!}{2!}$.

Let us assume no. of ways of arrangement be N_4.

⇒ $N_4 = N_3 \times \frac{4!}{2!}$

⇒ $N_4 = 20 \times 4 \times 3$

$\Rightarrow N_4 = 240$

iii. There are 2 letters which occurred more than once from which we need to select 2. Let us assume no. of ways of selection be N_5

$\Rightarrow N_5$ = no. of ways of selecting 2 alike letters of one type and 2 alike letters of another type

$\Rightarrow N_5 = {}^2C_2$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_5 = \frac{2!}{(2-2)!2!}$

$\Rightarrow N_5 = \frac{2!}{0!2!}$

$\Rightarrow N_5 = \frac{1}{1}$

$\Rightarrow N_5 = 1$

Now we need to arrange the chosen 2 alike letters of one type and 2 alike letters of another type. The arrangement is similar to that of arranging n people in n places in which r are similar of one type and m are similar of another type which are $\frac{n!}{r!m!}$ ways too arrange. So, the total no. of arrangements that can be made are $\frac{4!}{2!2!}$.

Let us assume no. of ways of arrangement be N_6.

$\Rightarrow N_6 = N_5 \times \frac{4!}{2!2!}$

$\Rightarrow N_6 = 1 \times \frac{4 \times 3}{2 \times 1}$

$\Rightarrow N_6 = 6$

iv. There is only 1 letter which occurred thrice, and 1 letter needs to be selected from the remaining 5 distinct letters.

Let us assume no. of ways of selection be N_7

$\Rightarrow N_7$ = no. of ways of selecting 1 letter from 5 letters

$\Rightarrow N_7 = {}^5C_1$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_7 = \frac{5!}{(5-1)!1!}$

$\Rightarrow N_7 = \frac{5!}{4!1!}$

$\Rightarrow N_7 = \frac{5}{1}$

$\Rightarrow N_7 = 5$

Now we need to arrange the chosen 1 different letters and 3 alike letters. The arrangement is similar to that of arranging n people in n places in which r are similar which are $\frac{n!}{r!}$ ways to arrange. So, the total no. of arrangements that can be made are $\frac{4!}{3!}$.

Let us assume no. of ways of arrangements be N_8.

$\Rightarrow N_8 = N_7 \times \frac{4!}{3!}$

$\Rightarrow N_8 = 5 \times 4$

$\Rightarrow N_8 = 20$

Total no. of ways of words formed = $N_2 + N_4 + N_6 + N_8$

Total no. of ways of words formed = 360 + 240 + 6 + 20

Total no. of ways of words formed = 626

9. Question

A businessman hosts a dinner to 21 guests. He has 2 round tables which can accommodate 15 and 6 persons each. In how many ways can he arrange the guests?

Answer

Given that we need to accommodate 21 guests to two round tables which can accommodate 15 and 6 persons each.

We need to select 6 members first and arrange 6 and 15 members accordingly in the respective tables.

Let us assume the no. of ways of choosing 6 members to be N_1

$\Rightarrow N_1$ = No. of ways of choosing 6 members out of 21 guests.

$\Rightarrow N_1 = {}^{21}C_6$

Now we need to arrange the 6 members in a round table. By fixing a guest at a single seat, We arrange the remaining 5 members. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of arrangements that can be made is 5!.

Now we need to arrange the 15 members in a round table. By fixing a guest at a single seat, We arrange the remaining 5 members. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of arrangements that can be made is 14!.

Let us assume total no. of ways of arranging the guests in the table be N

$\Rightarrow N = N_1 \times 5! \times 14!$

$\Rightarrow N = {}^{21}C_6 \times 5! \times 14!$

$\therefore$ The no. of ways of accommodating guests is ${}^{21}C_6 \times 5! \times 14!$.

10. Question

Find the number of combinations and permutations of 4 letters taken from the word 'EXAMINATION.'

Answer

Given the word is EXAMINATION. The letters present in it are:

E: 1 in number

X: 1 in number

A: 2 in number

M: 1 in number

I: 2 in number

N: 2 in number

T: 1 in number

O: I in number

a. We need to find the no. of words formed by 4 letters from the word EXAMINATION:

The possible cases are the following:

i. 4 distinct letters

ii. 2 alike letters and 2 distinct letters.

iii. 2 alike letters of one type and 2 alike letters of another type

i. There are 8 different letters from which we need to select 4 letters. Let us assume no. of ways of selection be N_1

$\Rightarrow N_1$ = no. of ways of selecting 4 letters from 8 letters

$\Rightarrow N_1 = {}^8C_4$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_1 = \frac{8!}{(8-4)!4!}$

$\Rightarrow N_1 = \frac{8!}{4!4!}$

$\Rightarrow N_1 = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1}$

$\Rightarrow N_1 = 70$

Now we need to arrange the chosen 4 different letters. Since every word differs from other. The arrangement is similar to that of arranging n people in n places which are n! ways to arrange. So, the total no. of arrangements that can be made is 4!.

Let us assume no. of ways of arrangement be N_2.

$\Rightarrow N_2 = N_1 \times 4!$

$\Rightarrow N_2 = 70 \times 24$

$\Rightarrow N_2 = 1680$

ii. There are 3 letters which occurred more than once. So, we need to select 1 letter from these 2 and 2 distinct letters from the remaining 7 distinct letters. Let us assume no. of ways of selection be N_3

$\Rightarrow N_3$ = (no. of ways of selecting 2 alike letters from the 3 types of alike letters) × (no. of ways of selecting 2 distinct letters from remaining 5 distinct letters)

$\Rightarrow N_3 = ({}^3C_1) \times ({}^7C_2)$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_3 = \left(\frac{3!}{(3-1)!1!}\right) \times \left(\frac{7!}{(7-2)!2!}\right)$

$\Rightarrow N_3 = \left(\frac{3!}{2!1!}\right) \times \left(\frac{7!}{5!2!}\right)$

$\Rightarrow N_3 = \left(\frac{3}{1}\right) \times \left(\frac{7 \times 6}{2 \times 1}\right)$

$\Rightarrow N_3 = 3 \times 21$

$\Rightarrow N_3 = 63$

Now we need to arrange the chosen 2 different letters and 2 alike letters. The arrangement is similar to that of arranging n people in n places in which r are similar which are $\frac{n!}{r!}$ ways to arrange. So, the total no. of arrangements that can be made are $\frac{4!}{2!}$.

Let us assume no. of ways of arrangement be N_4.

$\Rightarrow N_4 = N_3 \times \frac{4!}{2!}$

$\Rightarrow N_4 = 63 \times 4 \times 3$

$\Rightarrow N_4 = 756$

iii. There are 3 letters which occurred more than once from which we need to select 2. Let us assume no. of ways of selection be N_5

$\Rightarrow N_5$ = no. of ways of selecting 2 alike letters of one type and 2 alike letters of other type

$\Rightarrow N_5 = {}^3C_2$

We know that ${}^nC_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

$\Rightarrow N_5 = \frac{3!}{(3-2)!2!}$

$\Rightarrow N_5 = \frac{3!}{1!2!}$

$\Rightarrow N_5 = \frac{3}{1}$

$\Rightarrow N_5 = 3$

Now we need to arrange the chosen 2 alike letters of one type and 2 alike letters of another type. The arrangement is similar to that of arranging n people in n places in which r are similar of one type and m are similar of another type which are $\frac{n!}{r!m!}$ ways too arrange. So, the total no. of arrangements that can be made are $\frac{4!}{2!2!}$.

Let us assume no. of ways of arrangement be N_6.

$\Rightarrow N_6 = N_5 \times \frac{4!}{2!2!}$

$\Rightarrow N_6 = 3 \times \frac{4 \times 3}{2 \times 1}$

$\Rightarrow N_6 = 18$

Total no. of combinations = $N_1 + N_3 + N_5$

Total no. of combinations = 70 + 63 + 3

Total no. of combinations = 136

Total no. of ways of permutations = $N_2 + N_4 + N_6$

Total no. of ways of permutations = 1680 + 756 + 18

Total no. of ways of permutations = 2454

11. Question

A tea party is arranged for 16 persons along two sides of a long table with 8 chairs on each side. Four persons wish to sit on one particular side and two on the other side. In how many ways can they are seated?

Answer

Given that 16 persons need to be seated along two sides of a long table with 8 persons on each side.

It is also told that 4 persons sit on a particular side and 2 on the other side.

We need to choose 4 members to sit on one side from the remaining 10 members as the 6 members are already fixed about their seatings and arrange 8 members on both sides accordingly.

Let us first find the no. of ways to choose 4 members and assume it to be N_1.

⇒ N_1 = Selecting 4 members from remaining 7 person members

⇒ $N_1 = {}^{10}C_4$

We know that ${}^{n}C_r = \frac{n!}{(n-r)!r!}$,

And also n! = (n)(n – 1)......2.1

⇒ $N_1 = \frac{10!}{(10-4)!4!}$

⇒ $N_1 = \frac{10!}{6!4!}$

⇒ $N_1 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1}$

⇒ $N_1 = 210$

Now we need to arrange the chosen 8 members. Since 1 person differs from other.

The arrangement is similar to that of arranging n people in n places which are n! Ways to arrange. So, the persons can be arranged in 8! Ways.

This will be the same for both the tables.

Let us assume the total possible arrangements be N.

⇒ $N = N_1 \times 8! \times 8!$

⇒ $N = 210 \times 8! \times 8!$

∴ The total no. of ways of seating arrangements can be done 210 × 8! × 8!.

Very Short Answer

1. Question

Write $\sum_{r=0}^{m} {}^{n+r}C_r$ in the simplified form.

Answer

We know:

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$ ⇒ (1)

$\sum_{r=0}^{m} {}^{n+r}C_r = {}^{n}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + {}^{n+3}C_3 + \ldots\ldots + {}^{n+m}C_{n+1}$

$\sum_{r=0}^{m} {}^{n+r}C_r = {}^{n+1}C_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + {}^{n+3}C_3 + \ldots\ldots + {}^{n+m}C_{n+1}$ ⇒ $({}^{n}C_0 = {}^{n+1}C_0)$

Using equation (1),

$\sum_{r=0}^{m} {}^{n+r}C_r = {}^{n+2}C_1 + {}^{n+2}C_2 + {}^{n+3}C_3 + \ldots\ldots + {}^{n+m}C_{n+1}$

$\sum_{r=0}^{m} {}^{n+r}C_r = {}^{n+3}C_2 + {}^{n+3}C_3 + \ldots\ldots + {}^{n+m}C_{n+1}$

Proceeding in the same way :

$\sum_{r=0}^{m} {}^{n+r}C_r = {}^{n+m}C_{m-1} + {}^{n+m}C_m = {}^{n+m+1}C_{n+1}$

$\sum_{r=0}^{m} {}^{n+r}C_r = {}^{n+m+1}C_{n+1}$

2. Question

If ${}^{35}C_{n+7} = {}^{35}C_{4n} - 2$, then write the values of *n*.

Answer

${}^{35}C_{n+7} = {}^{35}C_{4n-2}$

n + 7 + 4n – 2 = 35 (${}^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

5n + 5 = 35

5n = 30

n = 6

And,

n + 7 = 4n – 2

3n = 9

n = 3

3. Question

Write the number of diagonals of an n-sided polygon.

Answer

An n-sided polygon has n vertices.

By joining any two vertices of the polygon, we obtain either a side or a diagonal of the polygon.

Number of line segments obtained by joining the vertices of an n-sided polygon if we take two vertices at a time, number of

Ways of selection 2 out of n = ${}^nC_2$

Out of these lines, n lines are sides of the polygon.

Number of diagonals of the polygon =

${}^nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$

4. Question

Write the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ in the simplest form.

Answer

${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$

$= ({}^nC_{r+1} + {}^nC_r) + ({}^nC_r + {}^nC_{r-1})$

$\Rightarrow ({}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$

$\Rightarrow ({}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

$= {}^{n+2}C_{r+1}$

5. Question

Write the value of $\sum_{r=1}^{6} {}^{56-r}C_3 + {}^{50}C_4$.

Answer

We know,

${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

Now, we have,

$\sum_{r=1}^{6} {}^{56-r}C_3 + {}^{50}C_4$

$= {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{50}C_3 + {}^{50}C_4$

$= {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{51}C_3 + {}^{51}C_4$

$= {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{52}C_3 + {}^{52}C_4$

$= {}^{55}C_3 + {}^{54}C_3 + {}^{53}C_3 + {}^{53}C_4$

$= {}^{55}C_3 + {}^{54}C_3 + {}^{54}C_4$

$= {}^{55}C_3 + {}^{55}C_4$

$= {}^{56}C_4$

6. Question

There are 3 letters and 3 directed envelopes. Write the number of ways in which no letter is put in the correct envelope.

Answer

Total number of ways in which the letters can be put = 3! = 6

Suppose, out of the three letters, one has been put in the correct envelope.

This can be done in ${}^{3}C_1$ ways. (3 ways)

Now, out of three, if two letters have been put in the current envelope, then the last one has been put in the correct envelope as well. This can be done in ${}^{3}C_3$ ways. (1 way)

∴ Number of ways = 3 + 1 = 4

∴ Number of ways in which no letter is put in correct envelope = 6 – 4 = 2

7. Question

Write the maximum number of points of intersection of 8 straight lines in a plane.

Answer

We know that two lines are required for one point of intersection.

∴ Number of points of intersection =

${}^{8}C_2 = \frac{8}{2} \times \frac{7}{1}$

= 28

8. Question

Write the number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines.

Answer

A parallelogram can be formed by choosing two parallel lines from the set of four parallel lines and two parallel lines from the set of three parallel lines.

Two parallel lines from the set of four parallel lines can be chosen in $^4C_2$ ways.

Two parallel lines from the set of three parallel lines can be chosen in $^3C_2$ ways.

∴ Number of parallelograms that can be formed =

$^4C_2 \times {}^3C_2 = \frac{4!}{2!2!} \times \frac{3!}{2!1!}$

$= 6 \times 3$

$= 18$

9. Question

Write the number of ways in which 5 red and 4 white balls can be drawn from a bag containing 10 red and 8 white balls.

Answer

4 white and 5 red balls are to be selected from 8 white and 10 red balls.

∴ Required number of ways = $^8C_4 \times {}^{10}C_5 = \frac{8!}{4!4!} \times \frac{10!}{5!5!}$

$= 70 \times 252$

$= 17640$

10. Question

Write the number of ways in which 12 boys may be divide into three groups of 4 boys each.

Answer

Number of groups in which 12 boys are to be divided = 3

Now 4 boys can be chosen out of 12 boys in $({}^{12}C_4 \times {}^8C_4 \times {}^4C_4)$ ways.

$= = \frac{12!}{4!8!} \times \frac{8!}{4!4!} \times 1 = \frac{12!}{4!4!4!} = 34650$

These groups can be arranged in 3! ways.

∴ Total number of ways =

$\frac{34650}{3!} = \frac{34650}{6}$

$= 5775$

11. Question

Write the total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants.

Answer

2 out of 4 vowels and 3 out of 5 consonants can be chosen in $^4C_2 \times {}^5C_3$ ways.

The total number of letters is 5. These letters can be arranged in 5! ways.

∴ Total number of words =

$^4C_2 \times {}^5C_3 \times 5!$

$= \frac{4!}{2!\,2!} \times \frac{5!}{2!\,3!} \times 120$

$= 6 \times 10 \times 120$

$= 7200$

MCQ

1. Question

Mark the correct alternative in the following:

If ${}^{20}C_r = {}^{20}C_{r-10}$, then ${}^{18}C_r$ is equal to

A. 4896

B. 816

C. 1632

D. none of these

Answer

$r + r - 10 = 20$ (${}^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

$2r - 10 = 20$

$2r = 30$

$r = 15$

Now,

${}^{18}C_r = {}^{18}C_{15}$

$\therefore {}^{18}C_{15} = {}^{18}C_3$

$\therefore {}^{18}C_3 = \frac{18}{3} \times \frac{17}{2} \times 16 = 816$

2. Question

Mark the correct alternative in the following:

If ${}^{20}C_r = {}^{20}C_{r+4}$, then ${}^rC_3$ is equal to

A. 54

B. 56

C. 58

D. none of these

Answer

$r + r + 4 = 20$ (${}^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

$2r + 4 = 20$

$2r = 16$

$r = 8$

Now,

${}^rC_3 = {}^8C_3$

$^8C_3 = \frac{8!}{3!5!} = \frac{8\times7\times6}{3\times2\times1} = 56$

3. Question

Mark the correct alternative in the following:

If $^{15}C_{3r} = {}^{15}C_{r+3}$, then *r* is equal to

A. 5

B. 4

C. 3

D. 2

Answer

3r + r + 3 = 15 ($^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

4r + 3 = 15

4r = 12

r = 3

4. Question

Mark the correct alternative in the following:

If $^{20}C_{r+1} = {}^{20}C_{r-1}$, then *r* is equal to

A. 10

B. 11

C. 19

D. 12

Answer

r + 1 + r – 1 = 20 ($^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

2r = 20

r = 10

5. Question

Mark the correct alternative in the following:

If $C(n, 12) = C(n, 8)$, then $C(22, n)$ is equal to

A. 231

B. 210

C. 252

D. 303

Answer

$^nC_{12} = {}^nC_8$

n = 12 + 8 ($^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

n = 20

Now,

$^{22}C_n = {}^{22}C_{20}$

$= \frac{22}{2} \times \frac{21}{1}$

= 231

6. Question

Mark the correct alternative in the following:

If $^{m}C_1 = {}^{n}C_2$, then

A. 2 m = n

B. 2 m = n (n + 1)

C. 2 m = n (n– 1)

D. 2 n = m (m– 1)

Answer

$^{m}C_1 = {}^{n}C_2$

$$\frac{m!}{1!\,(m-1)!} = \frac{n!}{2!\,(n-2)!}$$

$$\frac{m(m-1)!}{(m-1)!} = \frac{n(n-1)(n-2)!}{2(n-2)!}$$

2m = n(n - 1)

7. Question

Mark the correct alternative in the following:

If $^{n}C_{12} = {}^{n}C_8$, then n =

A. 20

B. 12

C. 6

D. 30

Answer

n = 12 + 8 ($^{n}C_x = {}^{n}C_y \Rightarrow n = x + y$ or $x = y$)

n = 20

8. Question

Mark the correct alternative in the following:

If $^{n}C_r = {}^{n}C_{r+1} = {}^{n+1}C_x$, then x =

A. r

B. r– 1

C. n

D. r + 1

Answer

${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_x$ (Given)

Now, we have ⇒

${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ ⇒ (1)

From (1) and (Given) we have,

${}^{n+1}C_{r+1} = {}^{n+1}C_x$

r + 1 = x (${}^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

9. Question

Mark the correct alternative in the following:

If ${}^{(a^2-a)}C_2 = {}^{(a^2-a)}C_4$, then a =

A. 2

B. 3

C. 4

D. none of these

Answer

$a^2 - a = 2 + 4$ (${}^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

$a^2 - a - 6 = 0$

$a^2 - 3a - 2a - 6 = 0$

$a(a - 3) + 2(a - 3) = 0$

$(a + 2)(a - 3) = 0$

a = -2 or a = 3

But, a = -2 is not possible since it's negative.

So, a = 3.

10. Question

Mark the correct alternative in the following:

${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$ is equal to

A. 30

B. 31

C. 32

D. 33

Answer

${}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$

$= {}^5C_1 + {}^5C_2 + {}^5C_2 + {}^5C_1 + {}^5C_5$ (${}^5C_1 = {}^5C_4$ and ${}^5C_2 = {}^5C_3$)

$= 2 \times {}^5C_1 + 2 \times {}^5C_2 + {}^5C_5$

$= 2 \times 5 + 2 \times \frac{5!}{2!3!} + 1$

$= 10 + 20 + 1$

$= 31$

11. Question

Mark the correct alternative in the following:

Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to

A. 60

B. 120

C. 7200

D. none of these

Answer

2 out of 4 vowels can be chosen in ${}^4C_2$ ways.

3 out of 5 consonants can be chosen in ${}^5C_3$ ways.

Thus, there are $({}^4C_2 \times {}^5C_3)$ groups, each containing 2 vowels and 3 consonants.

Each group contains 5 letters that can be arranged in 5! Ways.

∴ Required number of words =

$({}^4C_2 \times {}^5C_3) \times 5!$

$= 60 \times 120$

$= 7200$

12. Question

Mark the correct alternative in the following:

There are 12 points in a plane. The number of the straight lines joining any two of them when 3 of them are collinear, is

A. 62

B. 63

C. 64

D. 65

Answer

Number of straight lines joining 12 points if we take 2 points at a time $= {}^{12}C_2$

$= \frac{12!}{2!\,10!}$

$= 66$

Number of straight lines joining 3 points if we take 2 points at a time $= {}^3C_2$

$= 3$

But, 3 collinear points, when joined in pairs gives only one line.

∴ Required number of straight lines =

66 – 3 + 1

= 64

13. Question

Mark the correct alternative in the following:

Three persons enter a railway compartment. If there are 5 seats vacant, in how many ways can they take these seats?

A. 60

B. 20

C. 15

D. 125

Answer

Three persons can take 5 seats in ${}^5C_3$ ways.

Also, 3 persons can sit in 3! ways.

∴ Required number of ways =

${}^5C_3 \times 3!$

$= 10 \times 6$

= 60

14. Question

Mark the correct alternative in the following:

In how many was can a committee of 5 be made out of 6 men and 4 women containing at least one women?

A. 246

B. 222

C. 186

D. none of these

Answer

Required number of ways =

${}^4C_1 \times {}^6C_4 + {}^4C_2 \times {}^6C_3 + {}^4C_3 \times {}^6C_2 + {}^4C_4 \times {}^6C_1$

= 60 + 120 + 60 + 6

= 246

15. Question

Mark the correct alternative in the following:

There are 10 points in a plane and 4 of them are collinear. The number of straight lines joining any two of them is

A. 45

B. 40

C. 39

D. 38

Answer

Number of straight lines formed by joining the 10 points if we take 2 points at a time = ${}^{10}C_2$

$= \frac{10}{2} \times \frac{9}{1} = 45$

Number of straight lines formed by joining the 4 points if we take 2 points at a time = ${}^{4}C_2$

$= \frac{4}{2} \times \frac{3}{1} = 6$

But, 4 collinear points, when joined in pairs give only one line.

∴ Required number of straight lines =

45 – 6 + 1

= 40

16. Question

Mark the correct alternative in the following:

There are 13 players of cricket, out of which 4 are bowlers. In how many ways a team eleven be selected from them so as to include at least two bowlers?

A. 72

B. 78

C. 42

D. none of these

Answer

4 out of 13 players are bowlers.

In other words, 9 players are not bowlers.

A team of 11 is to be selected so as to include at least 2 bowlers.

∴ Number of ways = ${}^{4}C_2 \times {}^{9}C_9 + {}^{4}C_3 \times {}^{9}C_8 + {}^{4}C_4 \times {}^{9}C_7$

= 6 + 36 + 36

= 78

17. Question

Mark the correct alternative in the following:

If $C_0 + C_1 + C_2 + \ldots C_n = 256$, then ${}^{2n}C_2$ is equal to

A. 56

B. 120

C. 28

D. 91

Answer

If set S has n elements, then C(n,k) is the number of ways of choosing k elements from S.

Thus, the number of subsets of S of all possible values is given by,

C(n,0) + C(n,1) + C(n,2) + ... + C(n,n) = 2^n

Comparing the given equation with the above equation we get,

$2^n = 256$

$2^n = 2^8$

n = 8

$\therefore {}^{2n}C_2 = {}^{16}C_2$

$= \frac{16!}{2!\,14!} = \frac{16 \times 15}{2}$

= 120

18. Question

Mark the correct alternative in the following:

The number of ways in which a host lady can invite for a party of 8 out of 12 people of whom two do not want to attend the party together is

A. $2 \times {}^{11}C_7 + {}^{10}C_8$

B. ${}^{10}C_8 + {}^{11}C_7$

C. ${}^{12}C_8 - {}^{10}C_6$

D. none of these

Answer

A host lady can invite 8 out of 12 people in ${}^{12}C_8$ ways.

Two out of these 12 people do not want to attend the party together.

∴ Number of ways =

${}^{12}C_8 - {}^{10}C_6$

19. Question

Mark the correct alternative in the following:

Given 11 points, of which 5 lie on one circle, other than these 5, no 4 lie on one circle. Then the number of circles that can be drawn so that each contains at least 3 of the given point is

A. 216

B. 156

C. 172

D. none of these

Answer

We need at least three points to draw a circle that passes through them.

Now, number of circles formed out of 11 points by taking three points at a time = ${}^{11}C_3$

= 165

Number of circles formed out of 5 points by taking three points at a time = ${}^{5}C_3$

= 10

It is given that 5 points lie on once circle.

∴ Required number of circles =

165 – 10 + 1

= 156

20. Question

Mark the correct alternative in the following:

How many different committees of 5 can be formed from 6 men and 4 women on which exact 3 men and 2 women serve?

A. 6

B. 20

C. 60

D. 120

Answer

Number of committees that can be formed ⇒

$= {}^6C_3 \times {}^4C_2$

$= \frac{6!}{3!\,3!} \times \frac{4!}{2!\,2!}$

$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{4 \times 3}{2}$

= 120

21. Question

Mark the correct alternative in the following:

If $^{43}C_{r-6} - {}^{43}C_{3r+1}$, then the value of *r* is

A. 12

B. 8

C. 6

D. 10

E. 14

Answer

r – 6 + 3r + 1 = 43 ($^nC_x = {}^nC_y \Rightarrow n = x + y$ or $x = y$)

4r – 5 = 43

4r = 48

r = 12

22. Question

Mark the correct alternative in the following:

The number of diagonals that can be drawn by joining the vertices of an octagon is

A. 20

B. 28

C. 8

D. 16

Answer

An octagon has 8 vertices.

The number of diagonals of a polygon is given by $\frac{n(n-3)}{2}$. (Where n = number of vertices)

∴ Number of diagonals of an octagon =

$$\frac{8(8-3)}{2}=\frac{8\times 5}{2}=\frac{40}{2}=20$$

23. Question

Mark the correct alternative in the following:

The value of $\left({}^7C_0+{}^7C_1\right)+\left({}^7C_1+{}^7C_2\right)+\ldots+\left({}^7C_0+{}^7C_7\right)$ is

A. 2^7-1

B. 2^8-2

C. 2^8-1`

D. 2^8

Answer

$({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_2) + ({}^7C_2 + {}^7C_3) + ({}^7C_3 + {}^7C_4) + ({}^7C_4 + {}^7C_5) + ({}^7C_5 + {}^7C_6) + ({}^7C_6 + {}^7C_7)$

$= 1 + 2\times{}^7C_1 + 2\times{}^7C_2 + 2\times{}^7C_3 + 2\times{}^7C_4 + 2\times{}^7C_5 + 2\times{}^7C_6 + 1$

$= 1 + 2\times{}^7C_1 + 2\times{}^7C_2 + 2\times{}^7C_3 + 2\times{}^7C_3 + 2\times{}^7C_2 + 2\times{}^7C_6 + 1 \Rightarrow ({}^7C_3 = {}^7C_4$ and ${}^7C_2 = {}^7C_5)$

$= 2 + 2^2\,({}^7C_1 + {}^7C_2 + {}^7C_3)$

$= 2 + 2^2\,(7 + \frac{7}{2}\times 6 + \frac{7}{3}\times\frac{6}{2}\times 5)$

= 2 + 252

= 254

$= 2^8 - 2$

24. Question

Mark the correct alternative in the following:

Among 14 players, 5 are bowlers. In how many ways a team of 11 may be formed with at least 4 bowlers?

A. 265

B. 263

C. 264

D. 275

Answer

5 out of 14 players are bowlers.

In other words, 9 players are not bowlers.

A team of 11 is to be selected so as to include at least 4 bowlers.

∴ Number of ways = $^5C_4 \times {}^9C_7 + {}^5C_5 \times {}^9C_6$

= 180 + 84

= 264

25. Question

Mark the correct alternative in the following:

A lady gives a dinner party for six guests. The number of ways in which they may be selected from among tem friends if two of the friends will not attend the party together is

A. 112

B. 140

C. 164

D. none of these

Answer

Suppose there are two friends, A and B, who do not attend the party together.

If both of them do not attend the party, then the number of ways selecting 6 guests $= {}^8C_6 = 28$

If one of them attends the party, then the number of ways of selecting 6 guests $= 2 \times {}^8C_5 = 112$

∴ Total number of ways = 112 + 28 = 140

26. Question

Mark the correct alternative in the following:

If ${}^{n+1}C_3 = 2 \cdot {}^nC_2$, then $n =$

A. 3

B. 4

C. 5

D. 6

Answer

${}^{n+1}C_3 = 2 \times {}^nC_2$

$$\frac{(n+1)!}{3!\,(n-2)!} = 2 \times \frac{n!}{2!\,(n-2)!}$$

$$\frac{(n+1)n!}{3 \times 2!\,(n-2)!} = 2 \times \frac{n!}{(n-2)!}$$

n + 1 = 6

n = 5

27. Question

Mark the correct alternative in the following:

The number of parallelogram that can be formed from a set of four parallel lines intersecting another set of three parallel lines is

A. 6

B. 9

C. 12

D. 18

Answer

A parallelogram can be formed by choosing two parallel lines from the set of four parallel lines and two parallel lines from the set of three parallel lines.

Two parallel lines from the set of four parallel lines can be chosen in ${}^4C_2$ ways.

Two parallel lines from the set of three parallel lines can be chosen in ${}^3C_2$ ways.

∴ Number of parallelograms that can be formed =

${}^4C_2 \times {}^3C_2 = \frac{4!}{2!2!} \times \frac{3!}{2!1!}$

$= 6 \times 3$

$= 18$