16. Understanding Shapes-II (Quadrilaterals)

Exercise 16.1

1. Question

Define the following terms:

(i) Quadrilateral

(ii) Convex Quadrilateral

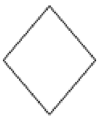
Answer

(i) Quadrilateral

A quadrilateral is a four sided enclosed figure.

(ii) Convex Quadrilateral

In a convex quadrilateral all the vertices are pointing outward.



2. Question

In a quadrilateral, define each of the following:

(i) Sides

(ii) Vertices

(iii) Angles

(iv) Diagonals

(v) Adjacent angles

(vi) Adjacents sides

(vii) Opposite sides

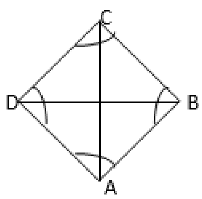
(viii) Opposite angles

(ix) Interior

(x) Exterior

Answer

Example:



(i) Sides: Sides are the edges of a quadrilateral. All the sides may have same of different length.

In the above figure AB, BC, CD and DA are sides.

(ii) Vertices

Vertices are the angular points where two sides or edges meet.

In the above figure vertices are A, B, C and D

(iii) Angles

Angle is the inclination inclination between two sides of a quadrilateral.

In the above figure angles are: ABC, BCA, CDA and DAB

(iv) Diagonals

Diagonals are the lines joining two opposite vertices of a quadrilateral.

In the above figure diagonals are: BD and AC

(v) Adjacent angles

Adjacent angles have one common arm.

In the above figure angles ABC, BCD are adjacent anges.

(vi) Adjacents sides

Adjacent sides make an angle.

In the above figure AB BC, BC CA, CD DA, DA AB are pairs of adjacent sides.

(vii) Opposite sides: Opposite sides don't have anything in common like sides orv angles.

In the above figure AB CD, BC DA are the pairs of opposite sides.

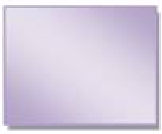
(viii) Opposite angles

Opposite angles are made by non adjacent sides.

In the above figure angles A and C, angles B and D are opposite angles.

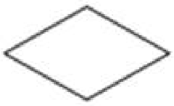
(ix) Interior

Interior means within the quadrilateral.



(x) Exterior

Exterior means outside of a quadrilateral. For example point B is exterior of quadrilateral.



3. Question

Complete each of the following, so as to make a true statement:

(i) A quadrilateral has ________ sides.

(ii) A quadrilateral has ________angles.

(iii) A quadrilateral has ________, no three of which are ________.

(iv) A quadrilateral has ________diagonals.

(v) The number of pairs of adjacent angles of a quadrilateral is ________.

(vi) The number of pairs of opposite angles of a quadrilateral is ________.

(vii) The sum of the angles of a quadrilateral is ________.

(viii) A diagonal of a quadrilateral is a line segment that joins two ________ vertices of the quadrilateral.

(ix) The sum of the angles of a quadrilateral is ________ right angles.

(x) The measure of each angle of a convex quadrilateral is ________ 180°.

(xi) In a quadrilateral the point of intersection of the diagonals lies in ________ of the quadrilateral.

(xii) A point os in the interior of a convex quadrilateral, if it is in the ________ of its two opposite angles.

(xiii) A quadrilateral is convex if for each side, the remaining ________ lie on the same side of the line containing the side.

Answer

(i) A quadrilateral has *Four* sides.

(ii) A quadrilateral has *Four* angles.

(iii) A quadrilateral has *Four* vertices, no three of which are *collinear.*

(iv) A quadrilateral has *two* diagonals.

(v) The number of pairs of adjacent angles of a quadrilateral is *two.*

(vi) The number of pairs of opposite angles of a quadrilateral is *two.*

(vii) The sum of the angles of a quadrilateral is 360°.

(viii) A diagonal of a quadrilateral is a line segment that joins two *opposite* vertices of the quadrilateral.

(ix) The sum of the angles of a quadrilateral is *four* right angles.

(x) The measure of each angle of a convex quadrilateral is *less than* 180°.

(xi) In a quadrilateral the point of intersection of the diagonals lies in interior of the quadrilateral.

(xii) A point os in the interior of a convex quadrilateral, if it is in the interiors of its two opposite angles.

(xiii) A quadrilateral is convex if for each side, the remaining vertices lie on the same side of the line containing the side.

4. Question

In Fig. 16.19, *ABCD* is a quadrilateral.

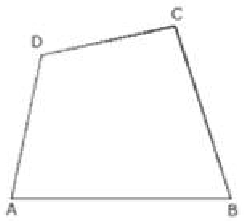


Fig. 16.19

(i) Name a pair of adjacent sides.

(ii) Name a pair of opposite sides.

(iii) How many pairs of adjacent sides are there?

(iv) How many pairs of opposite sides are there?

(v) Name a pair of adjacent angles.

(vi) Name a pair of opposite angles.

(vii) How many pairs of adjacent angles are there?

(viii) How many pairs of opposite angles are there?

Answer

(i) Name a pair of adjacent sides.

Adjacent sides are: AB, BC, CD and DA

(ii) Name a pair of opposite sides.

Adjacent sides are: AB CD and BC DA

(iii) How many pairs of adjacent sides are there?

Four pairs of adjacent sides.

AB BC, BC CD, CD DA and DA AB

(iv) How many pairs of opposite sides are there?

Two pairs of opposite sides.

AB DC and DA BC

(v) Name a pair of adjacent angles.

Four pairs of Adjacent angles are: DAB ABC, ABC BCA, BCA CDA and CDA DAB

(vi) Name a pair of opposite angles.

Pair of opposite angles are: DAB BCA and ABC CDA

(vii) How many pairs of adjacent angles are there?

Four pairs of adjacent angles. DAB ABC, ABC BCA, BCA CDA and CDA DAB

(viii) How many pairs of opposite angles are there?

Two pairs of opposite angles. DAB BCA and ABC CDA

5. Question

The angles of a quadrilateral are 110°, 72°, 55° and x°. Find the value of x.

Answer

Sum of angles of a quadrilateral is 360°

$110° + 72° + 55° + x° = 360°$

$x° = 360° - 237°$

$x° = 23°$

6. Question

The three angles of a quadrilateral are respectively equal to 110°, 50° and 40°. Find its fourth angle.

Answer

Sum of angles of a quadrilateral is 360°

Let the fourth angle is x°

$110° + 50° + 40° + x° = 360°$

$x° = 360° - 200°$

$x° = 160°$

7. Question

A quadrilateral has three acute angles each measures 80°. What is the measure of the fourth angle?

Answer

Sum of angles of a quadrilateral is 360°

Let the fourth angle is x°

80° + 80° + 80° + x° = 360°

x° = 360° - 240°

x° = 120°

8. Question

A quadrilateral has all its four angles of the same measure. What is the measure of each?

Answer

Sum of angles of a quadrilateral is 360°

Let each angle is x°

x° + x° + x° + x° = 360°

x° = $\frac{360^\circ}{4}$

x° = 90°

9. Question

Two angles of a quadrilateral are of measure 65° and the other two angles are equal. What is the measure of each of these two angles?

Answer

Sum of angles of a quadrilateral is 360°

Let each equal angle is x°

65° + 65° + x° + x° = 360°

2x° = 360° - 130°

x° = $\frac{230^\circ}{2}$

x° = 115°

Therefore other equal angles are 115° each.

10. Question

Three angles of a quadrilateral are equal. Fourth angle is of measure 150°. What is the measure of equal angles.

Answer

Sum of angles of a quadrilateral is 360°

Let each equal angle is x°

150° + x° + x° + x° = 360°

3x° = $360^\circ - 150^\circ$

x° = $\frac{210^\circ}{3}$

x° = 70°

Therefore other equal angles are 70° each.

11. Question

The four angles of a quadrilateral are as 3 ; 5 : 7 : 9. Find the angles.

Answer

Sum of angles of a quadrilateral is 360°

Let angle is x°

Therefore each angle is 3x, 5x, 7x and 9x

$3x° + 5x° + 7x° + 9x° = 360°$

$24x° = 360°$

$x° = \frac{360°}{24}$

$x° = 15°$

Therefore angles are: $3x = 3 \times 15 = 45°$

$5x = 5 \times 15 = 75°$

$7x = 7 \times 15 = 105°$

$9x = 3\times 15 = 45°$

12. Question

If the sum of the two angles of a quadrilateral is 180°. What is the sum of the remaining two angles?

Answer

Sum of angles of a quadrilateral is 360°

Let the sum of remaining two angles is x°

$180° + x° = 360°$

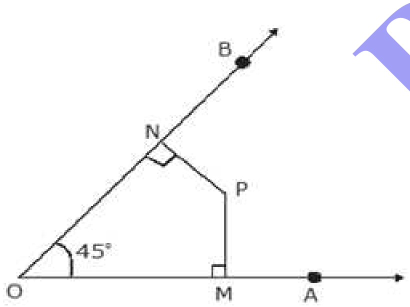
$x° = 360° - 180°$

$x° = 180°$

Therefore the sum of other two angles is 180°

13. Question

In Fig. 16.20, find the measure of $\angle MPN$.



Answer

Sum of angles of a quadrilateral is 360°

In the quadrilateral MPNO

$\angle NOP = 45°, \angle OMP = \angle PNO = 90°,$

Let angle $\angle MPN$ is x°

$\angle NOP + \angle OMP + \angle PNO + \angle MPN = 360°$

$45° + 90° + 90° + x° = 360°$

$x° = 360° - 225°$

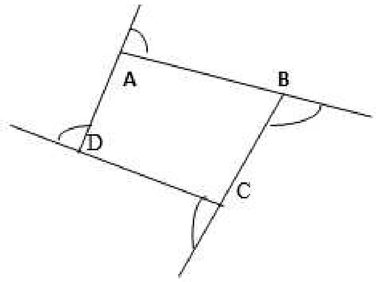
x° = 135°

Therefore ∠MPN is 135°

14. Question

The sides of a quadrilateral are produced in order. What is the sum of the four exterior angles?

Answer



We know that, exterior angle + interior adjacent angle = 180° [Linear pair]

Applying relation for polygon having n sides

Sum of all exterior angles + Sum of all interior angles = n × 180°

Therfore sum of all exterior angles = n × 180° - Sum of all interior angles

Sum of all exterior angles = n × 180° - (n -2) × 180° [Sum of interior angles is = (n - 2) x 180°]

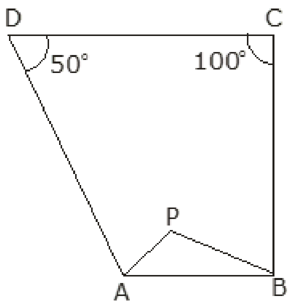
Sum of all exterior angles = n × 180° - n × 180° + 2 × 180°

Sum of all exterior angles = 180°n - 180°n + 360°

Sum of all exterior angles = 360°

15. Question

In Fig. 16.21, the bisectors of $\angle A$ and $\angle B$ meet at a point P. If $\angle C$ =100° and $\angle D$ = 50°, find the measure of $\angle APB$.



Answer

Sum of angles of a quadrilateral is 360°

In the quadrilateral ABCD

$\angle D = 50°, \angle C = 100°,$

$\angle A + \angle B + \angle C + \angle D = 360°$

$\angle A + \angle B + 100° + 50° = 360°$

$\angle A + \angle B = 360° - 150°$

$\angle A + \angle B = 210°$ ....(i)

Now in Δ APB

$\frac{1}{2}\angle A + \frac{1}{2}\angle B + \angle APB = 180°$ [Sum of angles of a triangle is 180°]

$\frac{1}{2}(\angle A + \angle B) + \angle APB = 180°$ ....(ii)

On substituting value of $\angle A + \angle B = 210$ from equation (i) in equation (ii)

$\frac{1}{2} \times 210° + \angle APB = 180°$

$105° + \angle APB = 180°$

$\angle APB = 180° - 105°$

$\boldsymbol{\angle APB = 75°}$

16. Question

In a quadrilateral *ABCD,* the angles *A, B, C* and *D* are in the ratio 1 : 2 : 4 : 5. Find the measure of each angle of the quadrilateral.

Answer

Sum of angles of a quadrilateral is 360°

Let angle is x°

Therefore each angle is x°, 2x°, 4x° and 5x°

x° + 2x° + 4x° + 5x° = 360°

12x° = $360°$

x° = $\frac{360°}{12}$

x° = $30°$

Therefore angles are: x = 30°

2x = 2 × 30° = 60°

4x = 4 × 30 = 120°

9x = 5 × 30 = 150°

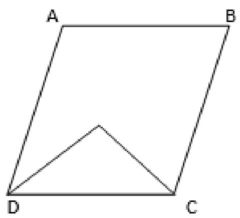
17. Question

In a quadrilateral *ABCD, CO* and *DO* are the bisectors of $\angle C$ and $\angle D$ respectively. Prove that $\angle COD = \frac{1}{2}$ ($\angle A$ $+ \angle B$).

Answer

Sum of angles of a quadrilateral is 360°

In the quadrilateral ABCD



$\angle A + \angle B + \angle C + \angle D = 360°$

$\angle A + \angle B = 360° - (\angle C + \angle D)$

$\frac{1}{2}(\angle A + \angle B) = \frac{1}{2}\{360° - (\angle C + \angle D)\}$

$\frac{1}{2}(\angle A + \angle B) = 180° - \frac{1}{2}(\angle C + \angle D)$ ....(i)

Now in Δ DOC

$\frac{1}{2}\angle D + \frac{1}{2}\angle C + \angle \text{COD} = 180°$ [Sum of angles of a triangle is 180°]

$\frac{1}{2}(\angle D + \angle C) + \angle \text{COD} = 180°$

$\angle \text{COD} = 180° - \frac{1}{2}(\angle C + \angle D)$ ....(ii)

From above equations (i) and (ii) RHS is equal therefore LHS will also be equal.

Therefore $\angle \text{COD} = \frac{1}{2}(\angle A + \angle B)$

18. Question

Find the number of sides of a regular polygon, when each of its angles has a measure of

(i) 160°

(ii) 135°

(iii) 175°

(iv) 162°

(v) 150°

Answer

(i) 160°

The measure of interior angle A of a polygon of n sides is given by $A = \frac{\{(n-2)\times 180°\}}{n}$

Angle of quadrilateral is 160°

$160° = \frac{\{(n-2)\times 180°\}}{n}$

$(n-2)\times 180° = 160° n$

$180° n - 360° = 160° n$

$180° n - 160° n = 360°$

$20° n = 360°$

$n = \frac{360°}{20} = 18$

Therfore number of sides are 18

(ii) 135°

The measure of interior angle A of a polygon of n sides is given by $A = \frac{\{(n-2)\times 180°\}}{n}$

Angle of quadrilateral is 135°

$135° = \frac{\{(n-2)\times 180°\}}{n}$

$(n-2)\times 180° = 135° n$

$180° n - 360° = 135° n$

$180° n - 135° n = 360°$

$45°n = 360°$

$n = \frac{360°}{45°} = 8$

Therfore numbers of sides are 8

(iii) 175°

The measure of interior angle A of a polygon of n sides is given by $A = \frac{\{(n-2)\times 180°\}}{n}$

Angle of quadrilateral is 175°

$175° = \frac{\{(n-2)\times 180°\}}{n}$

$(n-2)\times 180° = 175°n$

$180°n - 360° = 175°n$

$180°n - 175°n = 360°$

$5°n = 360°$

$n = \frac{360°}{5°} = 72$

Therfore numbers of sides are 72

(iv) 162°

The measure of interior angle A of a polygon of n sides is given by $A = \frac{\{(n-2)\times 180°\}}{n}$

Angle of quadrilateral is 162°

$162° = \frac{\{(n-2)\times 180°\}}{n}$

$(n-2)\times 180° = 162°n$

$180°n - 360° = 162°n$

$180°n - 162°n = 360°$

$18°n = 360°$

$n = \frac{360°}{18°} = 20$

Therfore numbers of sides are 20

(v) 150°

The measure of interior angle A of a polygon of n sides is given by $A = \frac{\{(n-2)\times 180°\}}{n}$

Angle of quadrilateral is 150°

$150° = \frac{\{(n-2)\times 180°\}}{n}$

$(n-2)\times 180° = 150°n$

$180°n - 360° = 150°n$

$180°n - 150°n = 360°$

$30°n = 360°$

$$n = \frac{360°}{30°} = 12$$

Therfore numbers of sides are 12

19. Question

Find the numbers of degrees in each exterior angle of a regular pentagon.

Answer

The sum of exterior angles of a polygon is 360°

Measure of each exterior angle of a polygon is $= \frac{360°}{n}$ where n is the number of sides

Number of sides in a pentagon is 5

Measure of each exterior angle of a pentagon is $= \frac{360°}{5} = 72°$

Measure of each exterior angle of a pentagon is $72°$

20. Question

The measure of angles of a hexagon are $x°$, $(x\text{-}5)°$, $(x\text{-}5)°$, $(2x\text{-}5)°$, $(2x\text{-}5)°$, $(2x+20)°$. Find value of x.

Answer

The sum of interior angles of a polygon = (n – 2) × 180°

where n = number of sides of polygon.Now, we know, a hexagon has 6 sides. So,

The sum of interior angles of a hexagon = (6 – 2) × 180° = 4 × 180° = 720°

therefore, we have

$x° + (x\text{-}5)° + (x\text{-}5)° + (2x\text{-}5)° + (2x\text{-}5)° + (2x+20)° = 720°$

$x° + x° \text{-} 5° + x° \text{-} 5° + 2x° \text{-} 5° + 2x° \text{-} 5° + 2x° + 20° = 720°$

$9x° = 720°$

$$x = \frac{720°}{9}$$

x = 80°

21. Question

In a convex hexagon, prove that the sum of all interior angle is equal to twice the sum of its exterior angles formed by producing the sides in the same order.

Answer

The sum of interior angles of a polygon = (n – 2) × 180°

The sum of interior angles of a hexagon = (6 – 2) × 180° = 4 × 180° = 720°

The Sum of exterior angle of a plygon is 360°

Therefoe sum of interior angles of a hexagon = twice the sum of interior angles.

22. Question

The sum of the interior angles of a polygon is three times the sum of its exterior angles. Determine the number of sides of the polygon.

Answer

The sum of interior angles of a polygon = (n – 2) × 180° …..(i)

The Sum of exterior angle of a plygon is 360°

According to the question:

Sum of interior angles = 3 × sum of exterior angles

Sum of interior angles = 3 × 360° = 1080°

Now applying relation as per equation (i)

(n – 2) × 180° = 1080°

n – 2 = $\frac{1080}{180}$

n – 2 = 6

n = 6 + 2 = 8

Therfore numbers of sides in the polygon are 8.

23. Question

Determine the number of sides of a polygon whose exterior and interior angles are in the ratio 1 : 5.

Answer

The sum of interior angles of a polygon = (n – 2) × 180° ....(i)

The Sum of exterior angle of a plygon is 360°

According to the question:

$$\frac{Sum\ of\ exterior\ angles}{Sum\ of\ interior\ angles} = \frac{1}{5}$$

$$\frac{360°}{(n-2)\times180°} = \frac{1}{5}$$

On cross multiplication we get

$$(n-2)\times180° = 360°\times5$$

$$(n-2) = \frac{360°\times5}{180°}$$

(n - 2) = 10

n = 12

Therefore the numbers of sides in the polygon are 12.

24. Question

PQRSTU is a regular hexagon, Determine each angle of Δ*PQT*.

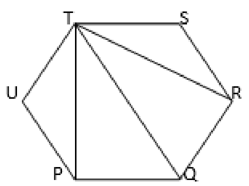
Answer

The sum of interior angles of a polygon = (n – 2) × 180°

The sum of interior angles of a hexagon = (6 – 2) × 180° = 4 × 180° = 720°

Measure of each angle of hexagon = $\frac{720°}{6} = 120°$

$\angle PUT = 120°$ Proved above



In Δ PUT

$\angle PUT + \angle UTP + \angle TPU = 180°$ [Angle sum property of a triangle]

$120° + 2\angle UTP = 180°$ [Since ΔPUT is isosceles triangle]

$2\angle UTP = 180° - 120°$

$\angle UTP = \frac{60°}{2} = 30°$

$\angle UTP = \angle TPU = 30°$

Similarly $\angle RTS = 30°$

Therefore $\angle PTR = \angle UTS - \angle UTP - \angle RTS$

$\angle PTR = 120° - 30° - 30° = 120° - 60° = 60°$

$\angle TPQ = \angle UPQ - \angle UPT$

$\angle TPQ = 120° - 30° = 90°$

$\angle TQP = 180° - 150° = 30°$ [Using angle sum property of triangle in ΔPQT]