16. Tangents and Normals

Exercise 16.1

1 A. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$y = \sqrt{x^3}$ at x = 4

Answer

Let y = f(x) be a continuous function and $P(x_0, y_0)$ be point on the curve, then,

The The Slope of the tangent at P(x,y) is f'(x) or $\frac{dy}{dx}$

Since the normal is perpendicular to tangent,

The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

Given:

$y = \sqrt{x^3}$ at x = 4

First, we have to find $\frac{dy}{dx}$ of given function, f(x),i.e, to find the derivative of f(x)

$y = \sqrt{x^3}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow y = (x^3)^{\frac{1}{2}}$

$\Rightarrow y = (x)^{\frac{3}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3}{2}(x)^{\frac{3}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}(x)^{\frac{1}{2}}$$

Since, x = 4

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2}(4)^{\frac{1}{2}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times \sqrt{4}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times 2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = 3$

$\therefore$The Slope of the tangent at x = 4 is 3

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{x=4}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{3}$

1 B. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$y = \sqrt{x}$ at x = 9

Answer

Given:

$y = \sqrt{x}$ at x = 9

First, we have to find $\frac{dy}{dx}$ of given function, f(x),i.e, to find the derivative of f(x)

$\Rightarrow y = \sqrt{x}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow y = (x)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = (x)^{\frac{1}{2}}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{1}{2}-1}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}(x)^{\frac{-1}{2}}$

Since, x = 9

$\left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2}(9)^{\frac{-1}{2}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{(9)^{\frac{1}{2}}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{\sqrt{9}}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{2} \times \frac{1}{3}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=9} = \frac{1}{6}$

$\therefore$The Slope of the tangent at x = 9 is $\frac{1}{6}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{x=9}}$

⇒ The Slope of the normal $= \frac{-1}{\frac{1}{6}}$

⇒ The Slope of the normal = – 6

1 C. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$y = x^3 - x$ at $x = 2$

Answer

Given:

$y = x^3 - x$ at $x = 2$

First, we have to find $\frac{dy}{dx}$ of given function, f(x),i.e, to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = x^3 - x$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dx}(x^3) + 3\times \frac{dy}{dx}(x)$

$\Rightarrow \frac{dy}{dx} = 3.x^{3-1} - 1.x^{1-0}$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 1$

Since, x = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 3\times(2)^2 - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = (3\times4) - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 12 - 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=2} = 11$

∴The Slope of the tangent at x = 2 is 11

⇒ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{x=2}}$

⇒ The Slope of the normal $= \frac{-1}{11}$

1 D. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$y = 2x^2 + 3 \sin x$ at $x = 0$

Answer

Given:

$y = 2x^2 + 3\sin x$ at $x = 0$

First, we have to find $\frac{dy}{dx}$ of given function, f(x),i.e, to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = 2x^2 + 3\sin x$

$\Rightarrow \frac{dy}{dx} = 2\times\frac{dy}{dx}(x^2) + 3\times\frac{dy}{dx}(\sin x)$

$\Rightarrow \frac{dy}{dx} = 2\times2x^{2-1} + 3\times(\cos x)$

$\therefore \frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow \frac{dy}{dx} = 4x + 3\cos x$

Since, x = 2

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 4\times0 + 3\cos(0)$

$\therefore \cos(0) = 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 0 + 3\times1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=0} = 3$

$\therefore$The Slope of the tangent at x = 0 is 3

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{x=0}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{3}$

1 E. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

x = a (θ – sin θ), y = a(1 + cos θ) at

θ = – π/2

Answer

Given:

$x = a(\theta - \sin\theta)$ & $y = a(1 + \cos\theta)$ at $\theta = \frac{-\pi}{2}$

Here, To find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get our desired $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

$\Rightarrow x = a(\theta - \sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(\frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin\theta))$

$\Rightarrow \frac{dx}{d\theta} = a(1 - \cos\theta)$ ...(1)

$\therefore \frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow y = a(1 + \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a(\frac{dx}{d\theta}(1) + \frac{dx}{d\theta}(\cos\theta))$

$\therefore \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{d}{dx}(\text{Constant}) = 0$

$\Rightarrow \frac{dy}{d\theta} = a(0 + (-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = a(-\sin\theta)$

$\Rightarrow \frac{dy}{d\theta} = -a\sin\theta$ ...(2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{(1-\cos\theta)}$

The Slope of the tangent is $\frac{-\sin\theta}{(1-\cos\theta)}$

Since, $\theta = \frac{-\pi}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{-\sin\frac{-\pi}{2}}{(1-\cos\frac{-\pi}{2})}$

$\therefore \sin(\frac{\pi}{2}) = 1$

$\therefore \cos(\frac{\pi}{2}) = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{-(-1)}{(1-(-0))}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = \frac{1}{(1-0)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}} = 1$

$\therefore$The Slope of the tangent at $x = -\frac{\pi}{2}$ is 1

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{-\pi}{2}}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{1}$

$\Rightarrow$ The Slope of the normal $= -1$

1 F. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \pi/4$

Answer

Given:

$x = a\cos^3\theta$ & $y = a\sin^3\theta$ at $\theta = \frac{\pi}{4}$

Here, To find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get our desired $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

$\Rightarrow x = a\cos^3\theta$

$\Rightarrow \frac{dx}{d\theta} = a(\frac{dx}{d\theta}(\cos^3\theta))$

$\therefore \frac{d}{dx}(\cos x) = -\sin x$

$\Rightarrow \frac{dx}{d\theta} = a(3\cos^{3-1}\theta \times -\sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(3\cos^2\theta \times -\sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = -3a\cos^2\theta\sin\theta$ ...(1)

$\Rightarrow y = a\sin^3\theta$

$\Rightarrow \frac{dy}{d\theta} = a(\frac{dy}{d\theta}(\sin^3\theta))$

$\therefore \frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow \frac{dy}{d\theta} = a(3\sin^{3-1}\theta \times \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a(3\sin^2\theta \times \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$ ...(2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{-\cos\theta}{\sin\theta}$

$\Rightarrow \frac{dy}{dx} = -\tan\theta$

The Slope of the tangent is $-\tan\theta$

Since, $\theta = \frac{\pi}{4}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -\tan(\frac{\pi}{4})$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}} = -1$

$\therefore \tan(\frac{\pi}{4}) = 1$

$\therefore$The Slope of the tangent at $x = \frac{\pi}{4}$ is – 1

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{4}}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{-1}$

$\Rightarrow$ The Slope of the normal = 1

1 A. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$y = \sqrt{x^3}$ at x = 4

Answer

Let y = f(x) be a continuous function and $P(x_0,y_0)$ be point on the curve, then, The The Slope of the tangent at P(x,y) is f'(x) or $\frac{dy}{dx}$ Since the normal is perpendicular to tangent, The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

Given:

$y = \sqrt{x^3}$ at x = 4

First, we have to find $\frac{dy}{dx}$ of given function, f(x),i.e, to find the derivative of f(x)

$y = \sqrt{x^3}$

$\therefore \sqrt[n]{x} = x^{\frac{1}{n}}$

$\Rightarrow y = (x^3)^{\frac{1}{2}}$

$\Rightarrow y = (x)^{\frac{3}{2}}$

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{3}{2}(x)^{\frac{3}{2}-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3}{2}(x)^{\frac{1}{2}}$$

Since, x = 4

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2}(4)^{\frac{1}{2}}$$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times \sqrt{4}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = \frac{3}{2} \times 2$

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=4} = 3$

∴The Slope of the tangent at x = 4 is 3

⇒ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{x=4}}$

⇒ The Slope of the normal $= \frac{-1}{3}$

1 G. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

x = a(θ – sin θ), y = a(1 – cos θ) at θ = π/2

Answer

Given:

$x = a(\theta - \sin\theta)$ & $y = a(1 - \cos\theta)$ at $\theta = \frac{\pi}{2}$

Here, To find $\frac{dy}{dx}$, we have to find $\frac{dy}{d\theta}$ & $\frac{dx}{d\theta}$ and and divide $\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ and we get our desired $\frac{dy}{dx}$.

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

$\Rightarrow x = a(\theta - \sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(\frac{dx}{d\theta}(\theta) - \frac{dx}{d\theta}(\sin\theta))$

$\Rightarrow \frac{dx}{d\theta} = a(1 - \cos\theta)$ ...(1)

$\therefore \frac{d}{dx}(\sin x) = \cos x$

$\Rightarrow y = a(1 - \cos\theta)$

$\Rightarrow \frac{dy}{d\theta} = a(\frac{dx}{d\theta}(1) - \frac{dx}{d\theta}(\cos\theta))$

$\therefore \frac{d}{dx}(\cos x) = -\sin x$

$\therefore \frac{d}{dx}(\text{Constant}) = 0$

$\Rightarrow \frac{dy}{d\theta} = a(0 - (-\sin\theta))$

$\Rightarrow \frac{dy}{d\theta} = a\sin\theta$ ...(2)

$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin\theta}{(1-\cos\theta)}$

The Slope of the tangent is $\frac{-\sin\theta}{(1-\cos\theta)}$

Since, $\theta = \frac{\pi}{2}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{\sin\frac{\pi}{2}}{(1-\cos\frac{\pi}{2})}$

$\therefore \sin(\frac{\pi}{2}) = 1$

$\therefore \cos(\frac{\pi}{2}) = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{(1)}{(1-(-0))}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = \frac{1}{(1-0)}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}} = 1$

$\therefore$The Slope of the tangent at $x = \frac{\pi}{2}$ is 1

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{2}}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{1}$

$\Rightarrow$ The Slope of the normal = – 1

1 H. Question

$y = (\sin 2x + \cot x + 2)^2$ at $x = \pi/2$

Answer

Given:

$y = (\sin 2x + \cot x + 2)^2$ at $x = \frac{\pi}{2}$

First, we have to find $\frac{dy}{dx}$ of given function, f(x),i.e, to find the derivative of f(x)

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

The Slope of the tangent is $\frac{dy}{dx}$

$\Rightarrow y = (\sin 2x + \cot x + 2)^2$

$\frac{dy}{dx} = 2\times(\sin 2x + \cot x + 2)^{2-1}\{\frac{dy}{dx}(\sin 2x) + \frac{dy}{dx}(\cot x) + \frac{dy}{dx}(2)\}$

$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)\{(\cos 2x) \times 2 + (-\mathrm{cosec}^2 x) + (0)\}$

$\therefore \frac{d}{dx}(\sin x) = \cos x$

$\therefore \frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$

$\Rightarrow \frac{dy}{dx} = 2(\sin 2x + \cot x + 2)(2\cos 2x - \mathrm{cosec}^2 x)$

Since, $x = \frac{\pi}{2}$

$\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2\times(\sin 2(\frac{\pi}{2}) + \cot(\frac{\pi}{2}) + 2)(2\cos 2(\frac{\pi}{2}) - \mathrm{cosec}^2(\frac{\pi}{2}))$

$\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2\times(\sin(\pi) + \cot(\frac{\pi}{2}) + 2)\times(2\cos(\pi) - \mathrm{cosec}^2(\frac{\pi}{2}))$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2\times(0 + 0 + 2)\times(2(-1) - 1)$

$\therefore \sin(\pi) = 0, \cos(\pi) = -1$

$\therefore \cot(\frac{\pi}{2}) = 0, \mathrm{cosec}(\frac{\pi}{2}) = 1$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 2(2)\times(-2 - 1)$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = 4\times -3$

$\Rightarrow \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}} = -12$

$\therefore$The Slope of the tangent at $x = \frac{\pi}{2}$ is -12

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{2}}}$

$\Rightarrow$ The Slope of the normal $= \frac{-1}{-12}$

$\Rightarrow$ The Slope of the normal $= \frac{1}{12}$

1 I. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

$x^2 + 3y + y^2 = 5$ at (1, 1)

Answer

Given:

$x^2 + 3y + y^2 = 5$ at (1,1)

Here we have to differentiate the above equation with respect to x.

$\Rightarrow \frac{d}{dx}(x^2 + 3y + y^2) = \frac{d}{dx}(5)$

$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(3y) + \frac{d}{dx}(y^2) = \frac{d}{dx}(5)$

$\therefore \frac{dy}{dx}(x^n) = n.x^{n-1}$

$\Rightarrow 2x + 3\times\frac{dy}{dx} + 2y\times\frac{dy}{dx} = 0$

$\Rightarrow 2x + \frac{dy}{dx}(3 + 2y) = 0$

$\Rightarrow \frac{dy}{dx}(3 + 2y) = - 2x$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{(3+2y)}$

The Slope of the tangent at (1,1)is

$\Rightarrow \frac{dy}{dx} = \frac{-2 \times 1}{(3 + 2 \times 1)}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{(3+2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-2}{5}$

∴The Slope of the tangent at (1,1) is $\frac{-2}{5}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\text{The Slope of the tangent}}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow$ The Slope of the normal = $\frac{-1}{\frac{-2}{5}}$

$\Rightarrow$ The Slope of the normal = $\frac{5}{2}$

1 J. Question

Find the The Slopes of the tangent and the normal to the following curves at the indicated points :

xy = 6 at (1, 6)

Answer

Given:

xy = 56 at (1,6)

Here we have to use the product rule for above equation.

If u and v are differentiable function, then

$\frac{d}{dx}(UV) = U\times \frac{dV}{dx} + V\times \frac{dU}{dx}$

$\frac{d}{dx}(xy) = \frac{d}{dx}(6)$

$\Rightarrow x\times \frac{d}{dx}(y) + y\times \frac{d}{dx}(x) = \frac{d}{dx}(5)$

$\therefore \frac{d}{dx}$ (Constant) = 0

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow x\frac{dy}{dx} = - y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

The Slope of the tangent at (1,6)is

$\Rightarrow \frac{dy}{dx} = \frac{-6}{1}$

$\Rightarrow \frac{dy}{dx} = -6$

∴The Slope of the tangent at (1,6) is – 6

⇒ The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

⇒ The Slope of the normal $= \frac{-1}{\left(\frac{dy}{dx}\right)}$

⇒ The Slope of the normal $= \frac{-1}{-6}$

⇒ The Slope of the normal $= \frac{1}{6}$

2. Question

Find the values of a and b if the The Slope of the tangent to the curve xy + ax + by = 2 at (1, 1) is 2.

Answer

Given:

The Slope of the tangent to the curve xy + ax + by = 2 at (1,1) is 2

First, we will find The Slope of tangent

we use product rule here,

$\therefore \frac{d}{dx}(UV) = U\times \frac{dV}{dx} + V\times \frac{dU}{dx}$

$\Rightarrow xy + ax + by = 2$

$\Rightarrow x\times \frac{d}{dx}(y) + y\times \frac{d}{dx}(x) + a\frac{d}{dx}(x) + b\frac{d}{dx}(y) + = \frac{d}{dx}(2)$

$\Rightarrow x\frac{dy}{dx} + y + a + b\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(x + b) + y + a = 0$

$\Rightarrow \frac{dy}{dx}(x + b) = -(a + y)$

$\Rightarrow \frac{dy}{dx} = \frac{-(a+y)}{x+b}$

since, The Slope of the tangent to the curve xy + ax + by = 2 at (1,1) is 2

i.e, $\frac{dy}{dx} = 2$

$\Rightarrow \left\{\frac{-(a+y)}{x+b}\right\}_{(x=1, y=1)} = 2$

$\Rightarrow \frac{-(a+1)}{1+b} = 2$

$\Rightarrow -a - 1 = 2(1 + b)$

$\Rightarrow -a - 1 = 2 + 2b$

$\Rightarrow a + 2b = -3$ ...(1)

Also, the point (1,1) lies on the curve xy + ax + by = 2,we have

$1\times1 + a\times1 + b\times1 = 2$

$\Rightarrow 1 + a + b = 2$

$\Rightarrow a + b = 1$ ...(2)

from (1) & (2),we get

$$\begin{array}{l} a + 2b = -3 \\ a + b = 1 \\ \underline{-\quad -\quad -} \\ b = -4 \end{array}$$

substitute b = – 4 in a + b = 1

a – 4 = 1

⇒ a = 5

So the value of a = 5 & b = – 4

3. Question

If the tangent to the curve $y = x^3 + ax + b$ at (1, – 6) is parallel to the line x – y + 5 = 0, find a and b

Answer

Given:

The Slope of the tangent to the curve $y = x^3 + ax + b$ at

(1, – 6)

First, we will find The Slope of tangent

$y = x^3 + ax + b$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(ax) + \frac{d}{dx}(b)$$

$$\Rightarrow \frac{dy}{dx} = 3x^{3-1} + a\left(\frac{dx}{dx}\right) + 0$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + a$$

The Slope of the tangent to the curve $y = x^3 + ax + b$ at

(1, – 6) is

$$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3(1)^2 + a$$

$$\Rightarrow \frac{dy}{dx}_{(x=1, y=-6)} = 3 + a \ldots(1)$$

The given line is x – y + 5 = 0

y = x + 5 is the form of equation of a straight line y = mx + c,where m is the The Slope of the line.

so the The Slope of the line is $y = 1 \times x + 5$

so The Slope is 1. ...(2)

Also the point (1, – 6) lie on the tangent, so

x = 1 & y = – 6 satisfies the equation, $y = x^3 + ax + b$

i.e, $-6 = 1^3 + a \times 1 + b$

⇒ – 6 = 1 + a + b

⇒ a + b = – 7 ...(3)

Since, the tangent is parallel to the line, from (1) & (2)

Hence,

$3 + a = 1$

$\Rightarrow a = -2$

From (3)

$a + b = -7$

$\Rightarrow -2 + b = -7$

$\Rightarrow b = -5$

So the value is $a = -2$ & $b = -5$

4. Question

Find a point on the curve $y = x^3 - 3x$ where the tangent is parallel to the chord joining (1, – 2) and (2, 2).

Answer

Given:

The curve $y = x^3 - 3x$

First, we will find the Slope of the tangent

$y = x^3 - 3x$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(3x)$

$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 3(\frac{dx}{dx})$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 3$ ...(1)

The equation of line passing through (x_0,y_0) and The Slope m is $y - y_0 = m(x - x_0)$.

so The Slope, $m = \frac{y-y_0}{x-x_0}$

The Slope of the chord joining (1, – 2) & (2,2)

$\Rightarrow \frac{dy}{dx} = \frac{2-(-2)}{2-1}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{1}$

$\Rightarrow \frac{dy}{dx} = 4$ ...(2)

From (1) & (2)

$3x^2 - 3 = 4$

$\Rightarrow 3x^2 = 7$

$\Rightarrow x^2 = \frac{7}{3}$

$\Rightarrow x = \pm\sqrt{\frac{7}{3}}$

$y = x^3 - 3x$

$\Rightarrow y = x(x^2 - 3)$

$\Rightarrow y = \pm\sqrt{\frac{7}{3}}((\pm\sqrt{\frac{7}{3}})^2 - 3)$

$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\left(\frac{7}{3} - 3\right)\right.$

$\Rightarrow y = \pm\sqrt{\frac{7}{3}}\left(\frac{-2}{3}\right)$

we know that, $(\pm \times -) = \mp$

$\Rightarrow y = \mp\left(\frac{-2}{3}\right)\sqrt{\frac{7}{3}}$

Thus, the required point is $x = \pm\sqrt{\frac{7}{3}}$ & $y = \mp\left(\frac{-2}{3}\right)\sqrt{\frac{7}{3}}$

5. Question

Find a point on the curve $y = x^3 - 2x^2 - 2x$ at which the tangent lines are parallel to the line $y = 2x - 3$.

Answer

Given:

The curve $y = x^3 - 2x^2 - 2x$ and a line $y = 2x - 3$

First, we will find The Slope of tangent

$y = x^3 - 2x^2 - 2x$

$\frac{dy}{dx} = \frac{d}{dx}(x^3) - \frac{d}{dx}(2x^2) - \frac{d}{dx}(2x)$

$\Rightarrow \frac{dy}{dx} = 3x^{3-1} - 2 \times 2(x^{2-1}) - 2 \times x^{1-1}$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x - 2$ ...(1)

$y = 2x - 3$ is the form of equation of a straight line $y = mx + c$,where m is the The Slope of the line.

so the The Slope of the line is $y = 2 \times (x) - 3$

Thus, The Slope = 2. ...(2)

From (1) & (2)

$\Rightarrow 3x^2 - 4x - 2 = 2$

$\Rightarrow 3x^2 - 4x = 4$

$\Rightarrow 3x^2 - 4x - 4 = 0$

We will use factorization method to solve the above Quadratic equation.

$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$

$\Rightarrow 3x(x - 2) + 2(x - 2) = 0$

$\Rightarrow (x - 2)(3x + 2) = 0$

$\Rightarrow (x - 2) = 0$ & $(3x + 2) = 0$

$\Rightarrow x = 2$ or

$x = \frac{-2}{3}$

Substitute $x = 2$ & $x = \frac{-2}{3}$ in $y = x^3 - 2x^2 - 2x$

when $x = 2$

$\Rightarrow y = (2)^3 - 2\times (2)^2 - 2\times(2)$

$\Rightarrow y = 8 - (2\times4) - 4$

$\Rightarrow y = 8 - 8 - 4$

$\Rightarrow y = -4$

when $x = \frac{-2}{3}$

$\Rightarrow y = (\frac{-2}{3})^3 - 2\times (\frac{-2}{3})^2 - 2\times(\frac{-2}{3})$

$\Rightarrow y = (\frac{-8}{27}) - 2\times (\frac{4}{9}) + (\frac{4}{3})$

$\Rightarrow y = (\frac{-8}{27}) - (\frac{8}{9}) + (\frac{4}{3})$

taking lcm

$\Rightarrow y = \frac{(-8\times1)-(8\times3)+(4\times9)}{27}$

$\Rightarrow y = \frac{-8-24+36}{27}$

$\Rightarrow y = \frac{4}{27}$

Thus, the points are $(2, -4)$ & $(\frac{-2}{3}, \frac{4}{27})$

6. Question

Find a point on the curve $y^2 = 2x^3$ at which the Slope of the tangent is 3

Answer

Given:

The curve $y^2 = 2x^3$ and The Slope of tangent is 3

$y^2 = 2x^3$

Differentiating the above w.r.t x

$\Rightarrow 2y^{2-1}\times \frac{dy}{dx} = 2\times 3x^{3-1}$

$\Rightarrow y\frac{dy}{dx} = 3x^2$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{y}$

Since, The Slope of tangent is 3

$\frac{3x^2}{y} = 3$

$\Rightarrow \frac{x^2}{y} = 1$

$\Rightarrow x^2 = y$

Substituting $x^2 = y$ in $y^2 = 2x^3$,

$(x^2)^2 = 2x^3$

$x^4 - 2x^3 = 0$

$x^3(x - 2) = 0$

$x^3 = 0$ or $(x - 2) = 0$

$x = 0$ or $x = 2$

If $x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{3(0)^2}{y}$

$\Rightarrow \frac{dy}{dx} = \mathbf{0}$, which is not possible.

So we take $x = 2$ and substitute it in $y^2 = 2x^3$,we get

$y^2 = 2(2)^3$

$y^2 = 2\times 8$

$y^2 = 16$

$y = 4$

Thus, the required point is (2,4)

7. Question

Find a point on the curve $xy + 4 = 0$ at which the tangents are inclined at an angle of 45^o with the x–axis.

Answer

Given:

The curve is $xy + 4 = 0$

If a tangent line to the curve $y = f(x)$ makes an angle θ with x – axis in the positive direction, then

$\frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

$xy + 4 = 0$

Differentiating the above w.r.t x

$\Rightarrow x\times \frac{d}{dx}(y) + y\times \frac{d}{dx}(x) + \frac{d}{dx}(4) = 0$

$\Rightarrow x\frac{dy}{dx} + y = 0$

$\Rightarrow x\frac{dy}{dx} = - y$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$ ...(1)

Also, $\frac{dy}{dx} = \tan 45° = 1$ ...(2)

From (1) & (2),we get,

$\Rightarrow \frac{-y}{x} = 1$

$\Rightarrow x = - y$

Substitute in $xy + 4 = 0$,we get

$\Rightarrow x(- x) + 4 = 0$

$\Rightarrow - x^2 + 4 = 0$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

so when x = 2,y = – 2

& when x = – 2,y = 2

Thus, the points are (2, – 2) & (– 2,2)

8. Question

Find a point on the curve $y = x^2$ where the Slope of the tangent is equal to the x – coordinate of the point.

Answer

Given:

The curve is $y = x^2$

$y = x^2$

Differentiating the above w.r.t x

$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$

$\Rightarrow \frac{dy}{dx} = 2x$ ...(1)

Also given the Slope of the tangent is equal to the x – coordinate,

$\frac{dy}{dx} = x$ ...(2)

From (1) & (2),we get,

i.e,2x = x

$\Rightarrow x = 0.$

Substituting this in $y = x^2$, we get,

$y = 0^2$

$\Rightarrow y = 0$

Thus, the required point is (0,0)

9. Question

At what point on the circle $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangent is parallel to x – axis.

Answer

Given:

The curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Differentiating the above w.r.t x

$\Rightarrow x^2 + y^2 - 2x - 4y + 1 = 0$

$\Rightarrow 2x^{2-1} + 2y^{2-1} \times \frac{dy}{dx} - 2 - 4 \times \frac{dy}{dx} + 0 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(2y - 4) = -2x + 2$

$\Rightarrow \frac{dy}{dx} = \frac{-2(x-1)}{2(y-2)}$

$\Rightarrow \frac{dy}{dx} = \frac{-(x-1)}{(y-2)}$ ...(1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

Since, the tangent is parallel to x – axis

i.e,

$\Rightarrow \frac{dy}{dx} = \tan(0) = 0$ ...(2)

$\therefore \tan(0) = 0$

From (1) & (2),we get,

$\Rightarrow \frac{-(x-1)}{(y-2)} = 0$

$\Rightarrow -(x - 1) = 0$

$\Rightarrow x = 1$

Substituting x = 1 in $x^2 + y^2 - 2x - 4y + 1 = 0$,we get,

$\Rightarrow 1^2 + y^2 - 2\times1 - 4y + 1 = 0$

$\Rightarrow 1 - y^2 - 2 - 4y + 1 = 0$

$\Rightarrow y^2 - 4y = 0$

$\Rightarrow y(y - 4) = 0$

$\Rightarrow y = 0$ & $y = 4$

Thus, the required point is (1,0) & (1,4)

10. Question

At what point of the curve $y = x^2$ does the tangent make an angle of 45^o with the x–axis?

Answer

Given:

The curve is $y = x^2$

Differentiating the above w.r.t x

$\Rightarrow y = x^2$

$\Rightarrow \frac{dy}{dx} = 2x^{2-1}$

$\Rightarrow \frac{dy}{dx} = 2x$ ...(1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

Since, the tangent make an angle of 45^o with x – axis

i.e,

$\Rightarrow \frac{dy}{dx} = \tan(45°) = 1$ ...(2)

$\therefore \tan(45°) = 1$

From (1) & (2), we get,

⇒ $2x = 1$

⇒ $x = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ in $y = x^2$, we get,

⇒ $y = (\frac{1}{2})^2$

⇒ $y = \frac{1}{4}$

Thus, the required point is $(\frac{1}{2}, \frac{1}{4})$

11. Question

Find a point on the curve $y = 3x^2 - 9x + 8$ at which the tangents are equally inclined with the axes.

Answer

Given:

The curve is $y = 3x^2 - 9x + 8$

Differentiating the above w.r.t x

⇒ $y = 3x^2 - 9x + 8$

⇒ $\frac{dy}{dx} = 2 \times 3x^{2-1} - 9 + 0$

⇒ $\frac{dy}{dx} = 6x - 9$ ...(1)

Since, the tangent are equally inclined with axes

i.e, $\theta = \frac{\pi}{4}$ or $\theta = \frac{-\pi}{4}$

∴ $\frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

⇒ $\frac{dy}{dx} = \tan(\frac{\pi}{4})$ or $\tan(\frac{-\pi}{4})$

⇒ $\frac{dy}{dx} = 1$or -1 ...(2)

∴ $\tan(\frac{\pi}{4}) = 1$

From (1) & (2),we get,

⇒ $6x - 9 = 1$ 0r $6x - 9 = -1$

⇒ $6x = 10$ 0r $6x = 8$

⇒ $x = \frac{10}{6}$ or $x = \frac{8}{6}$

⇒ $x = \frac{5}{3}$ or $x = \frac{4}{3}$

Substituting $x = \frac{5}{3}$ or $x = \frac{4}{3}$ in $y = 3x^2 - 9x + 8$,we get,

When $x = \frac{5}{3}$

⇒ $y = 3(\frac{5}{3})^2 - 9(\frac{5}{3}) + 8$

$\Rightarrow y = 3(\frac{25}{9}) - (\frac{45}{3}) + 8$

$\Rightarrow y = (\frac{75}{9}) - (\frac{45}{3}) + 8$

taking LCM = 9

$\Rightarrow y = (\frac{(75\times1)-(45\times3)+(8\times9)}{9})$

$\Rightarrow y = (\frac{75-135+72}{9})$

$\Rightarrow y = (\frac{12}{9})$

$\Rightarrow y = (\frac{4}{3})$

when $x = \frac{4}{3}$

$\Rightarrow y = 3(\frac{4}{3})^2 - 9(\frac{4}{3}) + 8$

$\Rightarrow y = 3(\frac{16}{9}) - (\frac{36}{3}) + 8$

$\Rightarrow y = (\frac{48}{9}) - (\frac{36}{3}) + 8$

taking LCM = 9

$\Rightarrow y = (\frac{(48\times1)-(36\times3)+(8\times9)}{9})$

$\Rightarrow y = (\frac{48-108+72}{9})$

$\Rightarrow y = (\frac{12}{9})$

$\Rightarrow y = (\frac{4}{3})$

Thus, the required point is $(\frac{5}{3}, \frac{4}{3})$ & $(\frac{4}{3}, \frac{4}{3})$

12. Question

At what points on the curve $y = 2x^2 - x + 1$ is the tangent parallel to the line $y = 3x + 4$?

Answer

Given:

The curve is $y = 2x^2 - x + 1$and the line $y = 3x + 4$

First, we will find The Slope of tangent

$y = 2x^2 - x + 1$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(2x^2) - \frac{d}{dx}(x) + \frac{d}{dx}(1)$

$\Rightarrow \frac{dy}{dx} = 4x - 1$ ...(1)

$y = 3x + 4$ is the form of equation of a straight line $y = mx + c$,where m is the The Slope of the line.

so the The Slope of the line is $y = 3\times(x) + 4$

Thus, The Slope = 3. ...(2)

From (1) & (2),we get,

$4x - 1 = 3$

$\Rightarrow 4x = 4$

$\Rightarrow x = 1$

Substituting $x = 1$in $y = 2x^2 - x + 1$,we get,

$\Rightarrow y = 2(1)^2 - (1) + 1$

$\Rightarrow y = 2 - 1 + 1$

$\Rightarrow y = 2$

Thus, the required point is (1,2)

13. Question

Find a point on the curve $y = 3x^2 + 4$ at which the tangent is perpendicular to the line whose slope is $-\frac{1}{6}$.

Answer

Given:

The curve $y = 3x^2 + 4$ and the Slope of the tangent is $\frac{-1}{6}$

$y = 3x^2 + 4$

Differentiating the above w.r.t x

$\Rightarrow \frac{dy}{dx} = 2\times 3x^{2-1} + 0$

$\Rightarrow \frac{dy}{dx} = 6x$ ...(1)

Since, tangent is perpendicular to the line,

$\therefore$The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

i.e, $\frac{-1}{6} = \frac{-1}{6x}$

$\Rightarrow \frac{1}{6} = \frac{1}{6x}$

$\Rightarrow x = 1$

Substituting $x = 1$ in $y = 3x^2 + 4$,

$\Rightarrow y = 3(1)^2 + 4$

$\Rightarrow y = 3 + 4$

$\Rightarrow y = 7$

Thus, the required point is (1,7).

14. Question

Find the point on the curve $x^2 + y^2 = 13$, the tangent at each one of which is parallel to the line $2x + 3y = 7$.

Answer

Given:

The curve $x^2 + y^2 = 13$ and the line $2x + 3y = 7$

$x^2 + y^2 = 13$

Differentiating the above w.r.t x

$\Rightarrow 2x^{2-1} + 2y^{2-1}\frac{dy}{dx} = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow 2(x + y\frac{dy}{dx}) = 0$

$\Rightarrow (x + y\frac{dy}{dx}) = 0$

$\Rightarrow y\frac{dy}{dx} = -x$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y}$ ...(1)

Since, line is $2x + 3y = 7$

$\Rightarrow 3y = -2x + 7$

$\Rightarrow y = \frac{-2x+7}{3}$

$\Rightarrow y = \frac{-2x}{3} + \frac{7}{3}$

∴The equation of a straight line is $y = mx + c$, where m is the The Slope of the line.

Thus, the The Slope of the line is $\frac{-2}{3}$ ...(2)

Since, tangent is parallel to the line,

∴ the The Slope of the tangent = The Slope of the normal

$\frac{-x}{y} = \frac{-2}{3}$

$\Rightarrow -x = \frac{-2y}{3}$

$\Rightarrow x = \frac{2y}{3}$

Substituting $x = \frac{2y}{3}$ in $x^2 + y^2 = 13$,

$\Rightarrow (\frac{2y}{3})^2 + y^2 = 13$

$\Rightarrow (\frac{4y^2}{9}) + y^2 = 13$

$\Rightarrow y^2(\frac{4}{9} + 1) = 13$

$\Rightarrow y^2(\frac{13}{9}) = 13$

$\Rightarrow y^2(\frac{1}{9}) = 1$

$\Rightarrow y^2 = 9$

$\Rightarrow y = \pm 3$

Substituting $y = \pm 3$ in $x = \frac{2y}{3}$,we get,

$x = \frac{2\times(\pm 3)}{3}$

$x = \pm 2$

Thus, the required point is (2, 3) & (– 2, – 3)

15. Question

Find the point on the curve $2a^2y = x^3 - 3ax^2$ where the tangent is parallel to the x – axis.

Answer

Given:

The curve is $2a^2y = x^3 - 3ax^2$

Differentiating the above w.r.t x

$\Rightarrow 2a^2 \times \frac{dy}{dx} = 3x^{3-1} - 3 \times 2ax^{2-1}$

$\Rightarrow 2a^2 \frac{dy}{dx} = 3x^2 - 6ax$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 6ax}{2a^2}$ ...(1)

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

Since, the tangent is parallel to x – axis

i.e,

$\Rightarrow \frac{dy}{dx} = \tan(0) = 0$ ...(2)

$\therefore \tan(0) = 0$

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

From (1) & (2),we get,

$\Rightarrow \frac{3x^2 - 6ax}{2a^2} = 0$

$\Rightarrow 3x^2 - 6ax = 0$

$\Rightarrow 3x(x - 2a) = 0$

$\Rightarrow 3x = 0$ or $(x - 2a) = 0$

$\Rightarrow x = 0$ or $x = 2a$

Substituting $x = 0$ or $x = 2a$ in $2a^2y = x^3 - 3ax^2$,

when $x = 0$

$\Rightarrow 2a^2y = (0)^3 - 3a(0)^2$

$\Rightarrow y = 0$

when $x = 2$

$\Rightarrow 2a^2y = (2a)^3 - 3a(2a)^2$

$\Rightarrow 2a^2y = 8a^3 - 12a^3$

$\Rightarrow 2a^2y = -4a^3$

$\Rightarrow y = -2a$

Thus, the required point is (0,0) & (2a, – 2a)

16. Question

At what points on the curve $y = x^2 - 4x + 5$ is the tangent perpendicular to the line $2y + x = 7$?

Answer

Given:

The curve $y = x^2 - 4x + 5$ and line is $2y + x = 7$

$y = x^2 - 4x + 5$

Differentiating the above w.r.t x,

we get the Slope of the tangent,

$\Rightarrow \frac{dy}{dx} = 2x^{2-1} - 4 + 0$

$\Rightarrow \frac{dy}{dx} = 2x - 4$ ...(1)

Since, line is $2y + x = 7$

$\Rightarrow 2y = -x + 7$

$\Rightarrow y = \frac{-x+7}{2}$

$\Rightarrow y = \frac{-x}{2} + \frac{7}{2}$

∴The equation of a straight line is $y = mx + c$, where m is the The Slope of the line.

Thus, the The Slope of the line is $\frac{-1}{2}$ ...(2)

Since, tangent is perpendicular to the line,

∴The Slope of the normal $= \frac{-1}{\text{The Slope of the tangent}}$

From (1) & (2),we get

i.e, $\frac{-1}{2} = \frac{-1}{2x-4}$

$\Rightarrow 1 = \frac{1}{x-2}$

$\Rightarrow x - 2 = 1$

$\Rightarrow x = 3$

Substituting $x = 3$ in $y = x^2 - 4x + 5$,

$\Rightarrow y = y = 3^2 - 4 \times 3 + 5$

$\Rightarrow y = 9 - 12 + 5$

$\Rightarrow y = 2$

Thus, the required point is (3,2)

17 A. Question

Find the point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to the

x – axis

Answer

Given:

The curve is $\frac{x^2}{4} + \frac{y^2}{25} = 1$

Differentiating the above w.r.t x, we get the The Slope of a tangent,

$\Rightarrow \frac{2x^{2-1}}{4} + \frac{2y^{2-1} \times \frac{dy}{dx}}{25} = 0$

Cross multiplying we get,

$\Rightarrow \frac{25 \times 2x + 4 \times 2y \times \frac{dy}{dx}}{100} = 0$

$\Rightarrow 50x + 8y\frac{dy}{dx} = 0$

$\Rightarrow 8y\frac{dy}{dx} = -50x$

$\Rightarrow \frac{dy}{dx} = \frac{-50x}{8y}$

$\Rightarrow \frac{dy}{dx} = \frac{-25x}{4y}$ ...(1)

(i)

Since, the tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx} = \tan(0) = 0$ ...(2)

$\therefore \tan(0) = 0$

$\therefore \frac{dy}{dx} =$ The Slope of the tangent $= \tan\theta$

From (1) & (2),we get,

$\Rightarrow \frac{-25x}{4y} = 0$

$\Rightarrow -25x = 0$

$\Rightarrow x = 0$

Substituting x = 0 in $\frac{x^2}{4} + \frac{y^2}{25} = 1$,

$\Rightarrow \frac{0^2}{4} + \frac{y^2}{25} = 1$

$\Rightarrow y^2 = 25$

$\Rightarrow y = \pm 5$

Thus, the required point is (0,5) & (0, – 5)

17 B. Question

Find the point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are parallel to the y – axis.

Answer

Since, the tangent is parallel to y – axis, its The Slope is not defined, then the normal is parallel to x – axis whose The Slope is zero.

i.e, $\frac{-1}{\frac{dy}{dx}} = 0$

$\Rightarrow \frac{-1}{\frac{-25x}{4y}} = 0$

$\Rightarrow \frac{-4y}{25x} = 0$

$\Rightarrow y = 0$

Substituting y = 0 in $\frac{x^2}{4} + \frac{y^2}{25} = 1$,

$\Rightarrow \frac{x^2}{4} + \frac{0^2}{25} = 1$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Thus, the required point is (2,0) & (– 2,0)

18 A. Question

Find the point on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the x – axis

Answer

Given:

The curve is $x^2 + y^2 - 2x - 3 = 0$

Differentiating the above w.r.t x, we get The Slope of tangent,

$\Rightarrow 2x^{2-1} + 2y^{2-1}\frac{dy}{dx} - 2 - 0 = 0$

$\Rightarrow 2x + 2y\frac{dy}{dx} - 2 = 0$

$\Rightarrow 2y\frac{dy}{dx} = 2 - 2x$

$\Rightarrow \frac{dy}{dx} = \frac{2-2x}{2y}$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$ ...(1)

(i) Since, the tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx} = \tan(0) = 0$ ...(2)

$\therefore \tan(0) = 0$

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

From (1) & (2),we get,

$\Rightarrow \frac{1-x}{y} = 0$

$\Rightarrow 1 - x = 0$

$\Rightarrow x = 1$

Substituting x = 1 in $x^2 + y^2 - 2x - 3 = 0$,

$\Rightarrow 1^2 + y^2 - 2\times1 - 3 = 0$

$\Rightarrow 1 + y^2 - 2 - 3 = 0$

$\Rightarrow\ y^2 - 4 = 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

Thus, the required point is (1,2) & (1, – 2)

18 B. Question

Find the point on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to the y – axis.

Answer

Since, the tangent is parallel to y – axis, its slope is not defined, then the normal is parallel to x – axis whose slope is zero.

i.e, $\frac{-1}{\frac{dy}{dx}} = 0$

$\Rightarrow \frac{-1}{\frac{1-x}{y}} = 0$

$\Rightarrow \frac{-y}{1-x} = 0$

$\Rightarrow y = 0$

Substituting y = 0 in $x^2 + y^2 - 2x - 3 = 0$,

$\Rightarrow\ x^2 + 0^2 - 2\times x - 3 = 0$

$\Rightarrow\ x^2 - 2x - 3 = 0$

Using factorization method, we can solve above quadratic equation

$\Rightarrow\ x^2 - 3x + x - 3 = 0$

$\Rightarrow\ x(x - 3) + 1(x - 3) = 0$

$\Rightarrow\ (x - 3)(x + 1) = 0$

$\Rightarrow x = 3\ \&\ x = -1$

Thus, the required point is (3,0) & (– 1,0)

19 A. Question

Find the point on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to x – axis

Answer

Given:

The curve is $\frac{x^2}{9} + \frac{y^2}{16} = 1$

Differentiating the above w.r.t x, we get the Slope of tangent,

$\Rightarrow \frac{2x^{2-1}}{9} + \frac{2y^{2-1}\times\frac{dy}{dx}}{16} = 0$

$\Rightarrow \frac{2x}{9} + \frac{y\times\frac{dy}{dx}}{8} = 0$

Cross multiplying we get,

$\Rightarrow \frac{(8\times 2x)+(9\times y)\times\frac{dy}{dx}}{72} = 0$

$\Rightarrow 16x + 9y\frac{dy}{dx} = 0$

$\Rightarrow 9y\frac{dy}{dx} = -16x$

$\Rightarrow \frac{dy}{dx} = \frac{-16x}{9y}$ ...(1)

(i)

Since, the tangent is parallel to x – axis

$\Rightarrow \frac{dy}{dx} = \tan(0) = 0$ ...(2)

$\therefore \tan(0) = 0$

$\therefore \frac{dy}{dx}$ = The Slope of the tangent = $\tan\theta$

From (1) & (2),we get,

$\Rightarrow \frac{-16x}{9y} = 0$

$\Rightarrow -16x = 0$

$\Rightarrow x = 0$

Substituting x = 0 in $\frac{x^2}{9} + \frac{y^2}{16} = 1$,

$\Rightarrow \frac{0^2}{9} + \frac{y^2}{16} = 1$

$\Rightarrow y^2 = 16$

$\Rightarrow y = \pm 4$

Thus, the required point is (0,4) & (0, – 4)

19 B. Question

Find the point on the curve $\frac{x^2}{9} + \frac{y^2}{16} = 1$ at which the tangents are parallel to y – axis

Answer

Since the tangent is parallel to y–axis, its slope is not defined, then the normal is parallel to x–axis whose The Slope is zero.

i.e., $\frac{-1}{\frac{dy}{dx}} = 0$

$\Rightarrow \frac{-1}{\frac{-16x}{9y}} = 0$

$\Rightarrow \frac{-9y}{16x} = 0$

$\Rightarrow y = 0$

Substituting y = 0 in $\frac{x^2}{9} + \frac{y^2}{16} = 1$,

$\Rightarrow \frac{x^2}{9} + \frac{0^2}{16} = 1$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$

Thus, the required point is (3,0) & (– 3,0)

20. Question

Show that the tangents to the curve $y = 7x^3 + 11$ at the points x = 2 and x = – 2 are parallel.

Answer

Given:

The curve $y = 7x^3 + 11$

Differentiating the above w.r.t x

$\Rightarrow \frac{dy}{dx} = 3 \times 7x^{3-1} + 0$

$\Rightarrow \frac{dy}{dx} = 21x^2$

when x = 2

$\Rightarrow \frac{dy}{dx}_{x=2} = 21 \times (2)^2$

$\Rightarrow \frac{dy}{dx}_{x=2} = 21 \times 4$

$\Rightarrow \frac{dy}{dx}_{x=2} = 84$

when x = – 2

$\Rightarrow \frac{dy}{dx}_{x=2} = 21 \times (-2)^2$

$\Rightarrow \frac{dy}{dx}_{x=2} = 21 \times 4$

$\Rightarrow \frac{dy}{dx}_{x=2} = 84$

Let y = f(x) be a continuous function and $P(x_0, y_0)$ be point on the curve, then,

The Slope of the tangent at P(x,y) is f'(x) or $\frac{dy}{dx}$

Since, the Slope of the tangent is at x = 2 and x = – 2 are equal, the tangents at x = 2 and x = – 2 are parallel.

21. Question

Find the point on the curve $y = x^3$ where the Slope of the tangent is equal to x – coordinate of the point.

Answer

Given:

The curve is $y = x^3$

$y = x^3$

Differentiating the above w.r.t x

$\Rightarrow \frac{dy}{dx} = 3x^{2-1}$

$\Rightarrow \frac{dy}{dx} = 3x^2$ ...(1)

Also given the The Slope of the tangent is equal to the x – coordinate,

$\frac{dy}{dx} = x$ ...(2)

From (1) & (2),we get,

i.e, $3x^2 = x$

$\Rightarrow x(3x - 1) = 0$

$\Rightarrow x = 0$ or $x = \frac{1}{3}$

Substituting $x = 0$ or $x = \frac{1}{3}$ this in $y = x^3$,we get,

when $x = 0$

$\Rightarrow y = 0^3$

$\Rightarrow y = 0$

when $x = \frac{1}{3}$

$\Rightarrow y = (\frac{1}{3})^3$

$\Rightarrow y = \frac{1}{27}$

Thus, the required point is (0,0) & $(\frac{1}{3}, \frac{1}{27})$

Exercise 16.2

1. Question

Find the equation of the tangent to the curve $\sqrt{x} + \sqrt{y} = a$, at the point $(a^2/4, a^2/4)$

Answer

finding slope of the tangent by differentiating the curve

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = -\frac{\sqrt{x}}{\sqrt{y}}$$

at $\left(\frac{a^2}{4}, \frac{a^2}{4}\right)$ slope m, is – 1

the equation of the tangent is given by $y - y_1 = m(x - x_1)$

$$y - \frac{a^2}{4} = -1\left(x - \frac{a^2}{4}\right)$$

$$x + y = \frac{a^2}{2}$$

2. Question

Find the equation of the normal toy $= 2x^3 - x^2 + 3$ at (1, 4).

Answer

finding the slope of the tangent by differentiating the curve

$$m = \frac{dy}{dx} = 6x^2 - 2x$$

m = 4 at (1,4)

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(normal) = -\frac{1}{4}$$

equation of normal is given by $y - y_1 = m(normal)(x - x_1)$

$$y - 4 = \left(-\frac{1}{4}\right)(x - 1)$$

x + 4y = 17

3 A. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at (0, 5)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

m(tangent) at (0,5) = – 10

$$m(normal)\text{ at }(0,5) = \frac{1}{10}$$

equation of tangent is given by $y - y_1 = m(tangent)(x - x_1)$

y – 5 = – 10x

y + 10x = 5

equation of normal is given by $y - y_1 = m(normal)(x - x_1)$

$$y - 5 = \frac{1}{10}x$$

3 B. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y = x^4 - 6x^3 + 13x^2 - 10x + 5$ at x = 1 y = 3

Answer

finding slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$

m(tangent) at (x = 1) = 2

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(normal)\text{ at }(x = 1) = -\frac{1}{2}$$

equation of tangent is given by $y - y_1 = m(tangent)(x - x_1)$

y – 3 = 2(x – 1)

y = 2x + 1

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - 3 = -\frac{1}{2}(x - 1)$$

$$2y = 7 - x$$

3 C. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y = x^2$ at (0, 0)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 2x$$

m(tangent) at (x = 0) = 0

normal is perpendicular to tangent so, $m_1 m_2 = -1$

$$m(\text{normal}) \text{ at } (x = 0) = \frac{1}{0}$$

We can see that the slope of normal is not defined

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y = 0

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

x = 0

3 D. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y = 2x^2 - 3x - 1$ at (1, – 2)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 4x - 3$$

m(tangent) at (1, – 2) = 1

normal is perpendicular to tangent so, $m_1 m_2 = -1$

m(normal) at (1, – 2) = – 1

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y + 2 = 1(x – 1)

y = x – 3

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

y + 2 = – 1(x – 1)

y + x + 1 = 0

3 E. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y^2 = \frac{x^3}{4-x}$ at (2, – 2)

Answer

finding the slope of the tangent by differentiating the curve

$$2y\frac{dy}{dx} = \frac{(4-x)3x^2 + x^4}{(4-x)^2}$$

$$\frac{dy}{dx} = \frac{(4-x)3x^2 + x^4}{2y(4-x)^2}$$

m(tangent) at (2, – 2) = – 2

$$m(\text{normal})\text{ at }(2,-2) = \frac{1}{2}$$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y + 2 = – 2(x – 2)

y + 2x = 2

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y + 2 = \frac{1}{2}(x-2)$$

2y + 4 = x – 2

2y – x + 6 = 0

3 F. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y = x^2 + 4x + 1$ at x = 3

Answer

finding slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 2x + 4$$

m(tangent) at (3,0) = 10

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal})\text{ at }(3,0) = -\frac{1}{10}$$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y at x = 3

$y = 3^2 + 4×3 + 1$

y = 22

y – 22 = 10(x – 3)

y = 10x – 8

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - 22 = -\frac{1}{10}(x - 3)$$

x + 10y = 223

3 G. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at (a cos θ, b sin θ)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{x}{a^2} + \frac{y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{xa^2}{yb^2}$$

$$m(\text{tangent})\text{at }(a\cos\theta, b\sin\theta) = -\frac{\cot\theta\, a^2}{b^2}$$

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal})\text{ at }(a\cos\theta, b\sin\theta) = \frac{b^2}{\cot\theta\, a^2}$$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - b\sin\theta = -\frac{\cot\theta\, a^2}{b^2}(x - a\cos\theta)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - b\sin\theta = -\frac{b^2}{\cot\theta\, a^2}(x - a\cos\theta)$$

3 H. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (a sec θ, b tan θ)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{x}{a^2} - \frac{y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$m(\text{tangent})\text{ at }(a\sec\theta, b\tan\theta) = \frac{b}{a\sin\theta}$$

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal})\text{ at }(a\sec\theta, b\tan\theta) = -\frac{a\sin\theta}{b}$$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - b\tan\theta = \frac{b}{a\sin\theta}(x - a\sec\theta)$$

equation of normal is given by $y - y_1 = m(normal)(x - x_1)$

$$y - b\tan\theta = -\frac{a\sin\theta}{b}(x - a\sec\theta)$$

3 I. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y^2 = 4a\,x$ at $(a/m^2, 2a/m)$

Answer

finding the slope of the tangent by differentiating the curve

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

m(tangent) at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

m(tangent) = m

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(normal) = -\frac{1}{m}$$

equation of tangent is given by $y - y_1 = m(tangent)(x - x_1)$

$$y - \frac{2a}{m} = m\left(x - \frac{a}{m^2}\right)$$

equation of normal is given by $y - y_1 = m(normal)(x - x_1)$

$$y - \frac{2a}{m} = -\frac{1}{m}\left(x - \frac{a}{m^2}\right)$$

3 J. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$$c^2\left(x^2 + y^2\right) = x^2y^2 \text{ at } \left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right)$$

Answer

finding the slope of the tangent by differentiating the curve

$$c^2\left(2x + 2y\frac{dy}{dx}\right) = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$2xc^2 - 2xy^2 = 2x^2y\frac{dy}{dx} - 2yc^2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{xc^2 - xy^2}{x^2y - yc^2}$$

m(tangent) at $\left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right) = -\frac{\cos^3\theta}{\sin^3\theta}$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at $\left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right) = \frac{\sin^3\theta}{\cos^3\theta}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - \frac{c}{\sin\theta} = -\frac{\cos^3\theta}{\sin^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - \frac{c}{\sin\theta} = \frac{\sin^3\theta}{\cos^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

3 K. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$xy = c^2$ at (ct, c/t)

Answer

finding slope of the tangent by differentiating the curve

$$y + x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

m(tangent) at $\left(ct, \frac{c}{t}\right) = -\frac{1}{t^2}$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at $\left(ct, \frac{c}{t}\right) = t^2$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - \frac{c}{t} = t^2(x - ct)$$

3 L. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_1, y_1)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{x}{a^2} - \frac{y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{b^2x}{ya^2}$$

m(tangent) at $(x_1, y_1) = \frac{b^2x_1}{y_1a^2}$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at $(x_1, y_1) = -\frac{a^2 y_1}{x_1 b^2}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - y_1 = \frac{b^2 x_1}{y_1 a^2}(x - x_1)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - y_1 = -\frac{a^2 y_1}{x_1 b^2}(x - x_1)$$

3 M. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at (x_0, y_0)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{x}{a^2} - \frac{y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{b^2 x}{y a^2}$$

m(tangent) at $(x_0, y_0) = \frac{b^2 x_0}{y_0 a^2}$

normal is perpendicular to tangent so, $m_1 m_2 = -1$

m(normal) at $(x_1, y_1) = -\frac{a^2 y_0}{x_0 b^2}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - y_1 = \frac{b^2 x_0}{y_0 a^2}(x - x_1)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - y_1 = -\frac{a^2 y_0}{x_0 b^2}(x - x_1)$$

3 N. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$x^{2/3} + y^{2/3} = 2$ at (1, 1)

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{2}{3x^{1/3}} + \frac{2}{3y^{1/3}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{y^{1/3}}{x^{1/3}}$$

m(tangent) at (1,1) = – 1

normal is perpendicular to tangent so, $m_1 m_2 = -1$

m(normal) at (1,1) = 1

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y – 1 = – 1(x – 1)

x + y = 2

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

y – 1 = 1(x – 1)

y = x

3 O. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$x^2 = 4y$ at (2, 1)

Answer

finding the slope of the tangent by differentiating the curve

$$2x = 4\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x}{2}$$

m(tangent) at (2,1) = 1

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at (2,1) = – 1

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y – 1 = 1(x – 2)

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

y – 1 = – 1(x – 2)

3 P. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y^2 = 4x$ at (1, 2)

Answer

finding the slope of the tangent by differentiating the curve

$$2y\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y}$$

m(tangent) at (1,2) = 1

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at (1,2) = – 1

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y – 2 = 1(x – 1)

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$y - 2 = -1(x - 1)$

3 Q. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$4x^2 + 9y^2 = 36$ at $(3\cos\theta, 2\sin\theta)$

Answer

finding the slope of the tangent by differentiating the curve

$$8x + 18y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{9y}$$

m(tangent) at $(3\cos\theta, 2\sin\theta) = -\frac{2\cos\theta}{3\sin\theta}$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at $(3\cos\theta, 2\sin\theta) = \frac{3\sin\theta}{2\cos\theta}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - 2\sin\theta = -\frac{2\cos\theta}{3\sin\theta}(x - 3\cos\theta)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - 2\sin\theta = \frac{3\sin\theta}{2\cos\theta}(x - 3\cos\theta)$$

3 P. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$y^2 = 4ax$ at (x_1, y_1)

Answer

finding slope of the tangent by differentiating the curve

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{2a}{y}$$

m(tangent) at $(x_1, y_1) = \frac{2a}{y_1}$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at $(x_1, y_1) = -\frac{y_1}{2a}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - y_1 = \frac{2a}{y_1}(x - x_1)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - y_1 = -\frac{y_1}{2a}(x - x_1)$$

3 S. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $\left(\sqrt{2}a, b\right)$

Answer

finding slope of the tangent by differentiating the curve

$$\frac{x}{a^2} - \frac{y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$m(\text{tangent}) \text{ at } \left(\sqrt{2}a, b\right) = \frac{\sqrt{2}ab^2}{ba^2}$$

normal is perpendicular to tangent so, $m_1m_2 = -1$

$$m(\text{normal}) \text{ at } \left(\sqrt{2}a, b\right) = -\frac{ba^2}{\sqrt{2}ab^2}$$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - b = \frac{\sqrt{2}ab^2}{ba^2}(x - \sqrt{2}a)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - b = -\frac{ba^2}{\sqrt{2}ab^2}(x - \sqrt{2}a)$$

4. Question

Find the equation of the tangent to the curve x = θ + sin θ, y = 1 + cos θ at θ = π/4.

Answer

finding slope of the tangent by differentiating x and y with respect to theta

$$\frac{dx}{d\theta} = 1 + \cos\theta$$

$$\frac{dy}{d\theta} = -\sin\theta$$

Dividing both the above equations

$$\frac{dy}{dx} = -\frac{\sin\theta}{1 + \cos\theta}$$

m at theta ($\pi/4$) = $-1 + \frac{1}{\sqrt{2}}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - 1 - \frac{1}{\sqrt{2}} = \left(-1 + \frac{1}{\sqrt{2}}\right)\left(x - \frac{\pi}{4} - \frac{1}{\sqrt{2}}\right)$$

5 A. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

x = θ + sin θ, y = 1 + cos θ at θ = π/2.

Answer

finding slope of the tangent by differentiating x and y with respect to theta

$$\frac{dx}{d\theta} = 1 + \cos\theta$$

$$\frac{dy}{d\theta} = -\sin\theta$$

Dividing both the above equations

$$\frac{dy}{dx} = -\frac{\sin\theta}{1 + \cos\theta}$$

m(tangent) at theta ($\pi/2$) = – 1

normal is perpendicular to tangent so, $m_1 m_2$ = – 1

m(normal) at theta ($\pi/2$) = 1

equation of tangent is given by y – y_1 = m(tangent)(x – x_1)

$$y - 1 = -1\left(x - \frac{\pi}{2} - 1\right)$$

equation of normal is given by y – y_1 = m(normal)(x – x_1)

$$y - 1 = 1\left(x - \frac{\pi}{2} - 1\right)$$

5 B. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

$x = \frac{2\,at^2}{1+t^2}, y = \frac{2\,at^3}{1+t^2}$ at t = 1/2

Answer

finding slope of the tangent by differentiating x and y with respect to t

$$\frac{dx}{dt} = \frac{(1 + t^2)4at - 2at^2(2t)}{(1 + t^2)^2}$$

$$\frac{dx}{dt} = \frac{4at}{(1 + t^2)^2}$$

$$\frac{dy}{dt} = \frac{(1 + t^2)6at^2 - 2at^3(2t)}{(1 + t^2)^2}$$

$$\frac{dy}{dt} = \frac{6at^2 + 2at^4}{(1 + t^2)^2}$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{6at^2 + 2at^4}{4at}$$

m(tangent) at t = $\frac{1}{2}$ is $\frac{13}{16}$

normal is perpendicular to tangent so, $m_1 m_2$ = – 1

m(normal) at t = $\frac{1}{2}$ is $-\frac{16}{13}$

equation of tangent is given by y – y_1 = m(tangent)(x – x_1)

$$y-\frac{a}{5}=\frac{13}{16}\left(x-\frac{2a}{5}\right)$$

equation of normal is given by y – y_1 = m(normal)(x – x_1)

$$y-\frac{a}{5}=-\frac{16}{13}\left(x-\frac{2a}{5}\right)$$

5 C. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

x = at^2, y = 2at at t = 1.

Answer

finding slope of the tangent by differentiating x and y with respect to t

$$\frac{dx}{dt}=2at$$

$$\frac{dy}{dt}=2a$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx}=\frac{1}{t}$$

m(tangent) at t = 1 is 1

normal is perpendicular to tangent so, m_1m_2 = – 1

m(normal) at t = 1 is – 1

equation of tangent is given by y – y_1 = m(tangent)(x – x_1)

y – 2a = 1(x – a)

equation of normal is given by y – y_1 = m(normal)(x – x_1)

y – 2a = – 1(x – a)

5 D. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

x = a sec t, y = b tan t at t.

Answer

finding slope of the tangent by differentiating x and y with respect to t

$$\frac{dx}{dt}=a\sec t\tan t$$

$$\frac{dy}{dt}=b\sec^2 t$$

Now dividing $\frac{dy}{dt}$ and $\frac{dx}{dt}$ to obtain the slope of tangent

$$\frac{dy}{dx}=\frac{b\operatorname{cosec} t}{a}$$

m(tangent) at t = $\frac{b\operatorname{cosec} t}{a}$

normal is perpendicular to tangent so, $m_1 m_2 = -1$

m(normal) at t = $-\frac{a}{b}\sin t$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - b\tan t = \frac{b\,\mathrm{cosec}\,t}{a}(x - a\sec t)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - b\tan t = -\frac{a\sin t}{b}(x - a\sec t)$$

5 E. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

x = a (θ + sin θ), y = a (1 – cos θ) at θ

Answer

finding slope of the tangent by differentiating x and y with respect to theta

$$\frac{dx}{d\theta} = a(1 + \cos\theta)$$

$$\frac{dy}{d\theta} = a(\sin\theta)$$

Now dividing $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta}$$

m(tangent) at theta is $\frac{\sin\theta}{1 + \cos\theta}$

normal is perpendicular to tangent so, $m_1 m_2 = -1$

m(normal) at theta is $-\frac{\sin\theta}{1 + \cos\theta}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - a(1 - \cos\theta) = \frac{\sin\theta}{1 + \cos\theta}(x - a(\theta + \sin\theta))$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - a(1 - \cos\theta) = \frac{1 + \cos\theta}{-\sin\theta}(x - a(\theta + \sin\theta))$$

5 F. Question

Find the equation of the tangent and the normal to the following curves at the indicated points:

x = 3 cos θ – $\cos^3$ θ, y = 3 sin θ – $\sin^3\theta$

Answer

finding slope of the tangent by differentiating x and y with respect to theta

$$\frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta\sin\theta$$

$$\frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta\cos\theta$$

Now dividing $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ to obtain the slope of tangent

$$\frac{dy}{dx} = \frac{3\cos\theta - 3\sin^2\theta\cos\theta}{-3\sin\theta + 3\cos^2\theta\sin\theta} = -\tan^3\theta$$

m(tangent) at theta is $-\tan^3\theta$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at theta is $\cot^3\theta$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

$$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta(x - 3\cos\theta + 3\cos^3\theta)$$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - 3\sin\theta + \sin^3\theta = \cot^3\theta(x - 3\cos\theta + 3\cos^3\theta)$$

6. Question

Find the equation of the normal to the curve $x^2 + 2y^2 - 4x - 6y + 8 = 0$ at the point whose abscissa is 2

Answer

finding slope of the tangent by differentiating the curve

$$2x + 4y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{4 - 2x}{4y - 6}$$

Finding y co – ordinate by substituting x in the given curve

$2y^2 - 6y + 4 = 0$

$y^2 - 3y + 2 = 0$

$y = 2$ or $y = 1$

m(tangent) at $x = 2$ is 0

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at $x = 2$ is $\frac{1}{0}$, which is undefined

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$x = 2$

7. Question

Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3).

Answer

finding the slope of the tangent by differentiating the curve

$$2ay\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2ay}$$

m(tangent) at (am^2, am^3) is $\frac{3m}{2}$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) at (am^2, am^3) is $-\frac{2}{3m}$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

$$y - am^3 = -\frac{2}{3m}(x - am^2)$$

8. Question

The equation of the tangent at (2, 3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 5$. Find the values of a and b.

Answer

finding the slope of the tangent by differentiating the curve

$$2y\frac{dy}{dx} = 3ax^2$$

$$\frac{dy}{dx} = \frac{3ax^2}{2y}$$

m(tangent) at (2,3) = 2a

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

now comparing the slope of a tangent with the given equation

$2a = 4$

$a = 2$

now (2,3) lies on the curve, these points must satisfy

$3^2 = 2 \times 2^3 + b$

$b = -7$

9. Question

Find the equation of the tangent line to the curve $y = x^2 + 4x - 16$ which is parallel to the line $3x - y + 1 = 0$.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 2x + 4$$

m(tangent) = 2x + 4

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

now comparing the slope of a tangent with the given equation

$2x + 4 = 3$

$$x = -\frac{1}{2}$$

Now substituting the value of x in the curve to find y

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

Therefore, the equation of tangent parallel to the given line is

$$y + \frac{71}{4} = 3\left(x + \frac{1}{2}\right)$$

10. Question

Find the equation of normal line to the curve y = x^3 + 2x + 6 which is parallel to the line x + 14y + 4 = 0.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 3x^2 + 2$$

m(tangent) = $3x^2 + 2$

normal is perpendicular to tangent so, $m_1m_2 = -1$

m(normal) = $\frac{-1}{3x^2+2}$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

now comparing the slope of normal with the given equation

m(normal) = $-\frac{1}{14}$

$$-\frac{1}{14} = -\frac{1}{3x^2 + 2}$$

x = 2 or – 2

hence the corresponding value of y is 18 or – 6

so, equations of normal are

$$y - 18 = -\frac{1}{14}(x - 2)$$

Or

$$y + 6 = -\frac{1}{14}(x + 2)$$

11. Question

Determine the equation (s) of tangent (s) line to the curve y = $4x^3$ – 3x + 5 which are perpendicular to the line 9y + x + 3 = 0.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 12x^2 - 3$$

m(tangent) = $12x^2 - 3$

the slope of given line is $-\frac{1}{9}$, so the slope of line perpendicular to it is 9

$$12x^2 - 3 = 9$$

x = 1 or – 1

since this point lies on the curve, we can find y by substituting x

y = 6 or 4

therefore, the equation of the tangent is given by

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y – 6 = 9(x – 1)

or

y – 4 = 9(x + 1)

12. Question

Find the equation of a normal to the curve $y = x \log_e x$ which is parallel to the line2x – 2y + 3 = 0.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = \ln x + 1$$

m(tangent) = $\ln x + 1$

normal is perpendicular to tangent so, $m_1 m_2 = -1$

m(normal) = $-\frac{1}{\ln x + 1}$

equation of normal is given by $y - y_1 = m(\text{normal})(x - x_1)$

now comparing the slope of normal with the given equation

m(normal) = 1

$$-\frac{1}{\ln x + 1} = 1$$

$$x = \frac{1}{e^2}$$

since this point lies on the curve, we can find y by substituting x

$$y = -\frac{2}{e^2}$$

The equation of normal is given by

$$y + \frac{2}{e^2} = x - \frac{1}{e^2}$$

13 A. Question

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

parallel to the line 2x – y + 9 = 0

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 2x - 2$$

m(tangent) = 2x – 2

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

now comparing the slope of a tangent with the given equation

m(tangent) = 2

2x – 2 = 2

x = 2

since this point lies on the curve, we can find y by substituting x

$y = 2^2 - 2 \times 2 + 7$

$y = 7$

therefore, the equation of the tangent is

$y - 7 = 2(x - 2)$

13 B. Question

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

perpendicular to the line $5y - 15x = 13$.

Answer

slope of given line is 3

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 2x - 2$$

m(tangent) = 2x – 2

since both lines are perpendicular to each other

(2x – 2) × 3 = – 1

$$x = \frac{5}{6}$$

since this point lies on the curve, we can find y by substituting x

$$y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{217}{36}$$

therefore, the equation of the tangent is

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$

14. Question

Find the equation of all lines having slope 2 and that are tangent to the curve $y = \frac{1}{x-3}, x \neq 3.$

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = -\frac{1}{(x-3)^2}$$

Now according to question, the slope of all tangents is equal to 2, so

$$-\frac{1}{(x-3)^2} = 2$$

$$(x-3)^2 = -\frac{1}{2}$$

We can see that LHS is always greater than or equal to 0, while RHS is always negative. Hence no tangent is possible

15. Question

Find the equation of all lines of slope zero and that is tangent to the curve $y = \frac{1}{x^2 - 2x + 3}$.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = -\frac{(2x - 2)}{(x^2 - 2x + 3)}$$

Now according to question, the slope of all tangents is equal to 0, so

$$-\frac{(2x - 2)}{(x^2 - 2x + 3)} = 0$$

Therefore the only possible solution is x = 1

since this point lies on the curve, we can find y by substituting x

$$y = \frac{1}{1 - 2 + 3}$$

$$y = \frac{1}{2}$$

equation of tangent is given by $y – y_1 = m(\text{tangent})(x – x_1)$

$$y - \frac{1}{2} = 0(x - 1)$$

$$y = \frac{1}{2}$$

16. Question

Find the equation of the tangent to the curve $y = \sqrt{3x - 2}$ which is parallel to the line 4x – 2y + 5 = 0.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = \frac{3}{2\sqrt{3x - 2}}$$

equation of tangent is given by $y – y_1 = m(\text{tangent})(x – x_1)$

now comparing the slope of a tangent with the given equation

m(tangent) = 2

$$\frac{3}{2\sqrt{3x - 2}} = 2$$

$$\frac{9}{16} = 3x - 2$$

$$x = \frac{41}{48}$$

since this point lies on the curve, we can find y by substituting x

$$y = \sqrt{\frac{41}{16} - 2}$$

$$y = \frac{3}{4}$$

therefore, the equation of the tangent is

$$y-\frac{3}{4} = 2\left(x-\frac{41}{48}\right)$$

17. Question

Find the equation of the tangent to the curve $x^2 + 3y - 3 = 0$, which is parallel to the line $y = 4x - 5$.

Answer

finding the slope of the tangent by differentiating the curve

$$3\frac{dy}{dx} + 2x = 0$$

$$\frac{dy}{dx} = -\frac{2x}{3}$$

m(tangent) $= -\frac{2x}{3}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

now comparing the slope of a tangent with the given equation

m(tangent) = 4

$$-\frac{2x}{3} = 4$$

$x = -6$

since this point lies on the curve, we can find y by substituting x

$6^2 + 3y - 3 = 0$

$y = -11$

therefore, the equation of the tangent is

$y + 11 = 4(x + 6)$

18. Question

Prove that $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ for all $n \in \mathrm{N}$, at the point (a, b).

Answer

finding the slope of the tangent by differentiating the curve

$$n\left(\frac{x}{a}\right)^{n-1} + n\left(\frac{y}{b}\right)^{n-1}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1}\left(\frac{b}{a}\right)^{n}$$

m(tangent) at (a,b) is $-\frac{b}{a}$

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

therefore, the equation of the tangent is

$$y-b = -\frac{b}{a}(x-a)$$

$$\frac{x}{a} + \frac{y}{b} = 2$$

Hence, proved

19. Question

Find the equation of the tangent to the curve x = sin 3t, y = cos 2t at $t = \frac{\pi}{4}$.

Answer

finding the slope of the tangent by differentiating x and y with respect to t

$$\frac{dx}{dt} = 3\cos 3t$$

$$\frac{dy}{dt} = -2\sin 2t$$

Dividing the above equations to obtain the slope of the given tangent

$$\frac{dy}{dx} = \frac{-2\sin 2t}{3\cos 3t}$$

m(tangent) at $\frac{\pi}{4}$ is $\frac{2\sqrt{2}}{3}$

equation of tangent is given by y – y_1 = m(tangent)(x – x_1)

therefore, equation of tangent is

$$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

20. Question

At what points will be tangents to the curve y = $2x^3$ – $15x^2$ + 36x – 21 be parallel to the x – axis? Also, find the equations of the tangents to the curve at these points.

Answer

finding the slope of the tangent by differentiating the curve

$$\frac{dy}{dx} = 6x^2 - 30x + 36$$

According to the question, tangent is parallel to the x – axis , which implies m = 0

$$6x^2 - 30x + 36 = 0$$

$$x^2 - 5x + 6 = 0$$

x = 3 or x = 2

since this point lies on the curve, we can find y by substituting x

y = $2(3)^3$ – $15(3)^2$ + 36(3) – 21

y = 6

or

y = $2(2)^3$ – $15(2)^2$ + 36(2) – 21

y = 7

equation of tangent is given by y – y_1 = m(tangent)(x – x_1)

y – 6 = 0(x – 3)

y = 6

or

y – 7 = 0(x – 2)

y = 7

21. Question

Find the equation of the tangents to the curve $3x^2 - y^2 = 8$, which passes through the point (4/3, 0).

Answer

assume point (a, b) which lies on the given curve

finding the slope of the tangent by differentiating the curve

$$6x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{3x}{y}$$

m(tangent) at (a,b) is $\frac{3a}{b}$

Since this tangent passes through $\left(\frac{4}{3}, 0\right)$, its slope can also be written as

$$\frac{b-0}{a-\frac{4}{3}}$$

Equating both the slopes as they are of the same tangent

$$\frac{b}{a-\frac{4}{3}} = \frac{3a}{b}$$

$b^2 = 3a^2 - 4a$ ...(i)

Since points (a,b) lies on this curve

$3a^2 - b^2 = 8$ ...(ii)

Solving (i) and (ii) we get

$3a^2 - 8 = 3a^2 - 4a$

a = 2

b = 2 or – 2

therefore points are (2,2) or (2, – 2)

equation of tangent is given by $y - y_1 = m(\text{tangent})(x - x_1)$

y – 2 = 3(x – 2)

or

y + 2 = – 3(x – 3)

Exercise 16.3

1 A. Question

Find the angle to intersection of the following curves :

$y^2 = x$ and $x^2 = y$

Answer

Given:

Curves $y^2 = x$ ...(1)

& $x^2 = y$ ...(2)

First curve is $y^2 = x$

Differentiating above w.r.t x,

$\Rightarrow 2y.\frac{dy}{dx} = 1$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{1}{2x}$ ...(3)

The second curve is $x^2 = y$

$\Rightarrow 2x = \frac{dy}{dx}$

$\Rightarrow m_2 = \frac{dy}{dx} = 2x$ ...(4)

Substituting (1) in (2),we get

$\Rightarrow x^2 = y$

$\Rightarrow (y^2)^2 = y$

$\Rightarrow y^4 - y = 0$

$\Rightarrow y(y^3 - 1) = 0$

$\Rightarrow y = 0$ or $y = 1$

Substituting $y = 0$ & $y = 1$ in (1) in (2),

$x = y^2$

when $y = 0, x = 0$

when $y = 1, x = 1$

Substituting above values for m_1 & m_2 we get,

when $x = 0$,

$m_1 = \frac{dy}{dx} = \frac{1}{2\times0} = \infty$

when $x = 1$,

$m_1 = \frac{dy}{dx} = \frac{1}{2\times1} = \frac{1}{2}$

Values of m_1 is ∞ & $\frac{1}{2}$

when $y = 0$,

$m_2 = \frac{dy}{dx} = 2x = 2\times0 = 0$

when $x = 1$,

$m_2 = \frac{dy}{dx} = 3x = 2\times1 = 2$

Values of m_2 is 0 & 2

when $m_1 = \infty$ & $m_2 = 0$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan\theta = \left|\frac{0-\infty}{1+\infty\times0}\right|$

$\tan\theta = \infty$

$\theta = \tan^{-1}(\infty)$

$\therefore \tan^{-1}(\infty) = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

when $m_1 = \frac{1}{2}$ & $m_2 = 2$

> Angle of intersection of two curve is given by
> $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$
> where m_1 & m_2 are slopes of the curves.

$\tan\theta = \left|\frac{2-\frac{1}{2}}{1+\frac{1}{2}\times2}\right|$

$\tan\theta = \left|\frac{\frac{3}{2}}{2}\right|$

$\tan\theta = \left|\frac{3}{4}\right|$

$\theta = \tan^{-1}(\frac{3}{4})$

$\theta \cong 36.86$

1 B. Question

Find the angle to intersection of the following curves :

$y = x^2$ and $x^2 + y^2 = 20$

Answer

Given:

Curves $y = x^2$ ...(1)

& $x^2 + y^2 = 20$ ...(2)

First curve $y = x^2$

$\Rightarrow m_1 = \frac{dy}{dx} = 2x$ ...(3)

Second curve is $x^2 + y^2 = 20$

Differentiating above w.r.t x,

$\Rightarrow 2x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow y.\frac{dy}{dx} = -x$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y}$ ...(4)

Substituting (1) in (2),we get

$\Rightarrow y + y^2 = 20$

$\Rightarrow y^2 + y - 20 = 0$

We will use factorization method to solve the above Quadratic equation

$\Rightarrow y^2 + 5y - 4y - 20 = 0$

$\Rightarrow y(y + 5) - 4(y + 5) = 0$

$\Rightarrow (y + 5)(y - 4) = 0$

$\Rightarrow y = -5$ & $y = 4$

Substituting $y = -5$ & $y = 4$ in (1) in (2),

$y = x^2$

when $y = -5$,

$\Rightarrow -5 = x^2$

$\Rightarrow x = \sqrt{-5}$

when $y = 4$,

$\Rightarrow 4 = x^2$

$\Rightarrow x = \pm 2$

Substituting above values for m_1 & m_2,we get,

when $x = 2$,

$m_1 = \frac{dy}{dx} = 2 \times 2 = 4$

when $x = 1$,

$m_1 = \frac{dy}{dx} = 2 \times -2 = -4$

Values of m_1 is 4 & – 4

when $y = 4$ & $x = 2$

$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-2}{4} = \frac{-1}{2}$

when $y = 4$ & $x = -2$

$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{2}{4} = \frac{1}{2}$

Values of m_2 is $\frac{-1}{2}$ & $\frac{1}{2}$

when $m_1 = \infty$ & $m_2 = 0$

Angle of intersection of two curve is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ where m_1 & m_2 are slopes of the curves.

$\tan\theta = \left|\frac{\frac{-1}{2} - 4}{1 + 2 \times 4}\right|$

$\tan\theta = \left|\frac{\frac{-9}{2}}{1 - 2}\right|$

$\tan\theta = \left|\frac{9}{2}\right|$

$\theta = \tan^{-1}\left(\frac{9}{2}\right)$

$\theta \cong 77.47$

1 C. Question

Find the angle to intersection of the following curves :

$2y^2 = x^3$ and $y^2 = 32x$

Answer

Given:

Curves $2y^2 = x^3$ ...(1)

& $y^2 = 32x$ ...(2)

First curve is $2y^2 = x^3$

Differentiating above w.r.t x,

$\Rightarrow 4y.\frac{dy}{dx} = 3x^2$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{3x^2}{4y}$ ...(3)

Second curve is $y^2 = 32x$

$\Rightarrow 2y.\frac{dy}{dx} = 32$

$\Rightarrow y.\frac{dy}{dx} = 16$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{16}{y}$ ...(4)

Substituting (2) in (1),we get

$\Rightarrow 2y^2 = x^3$

$\Rightarrow 2(32x) = x^3$

$\Rightarrow 64x = x^3$

$\Rightarrow x^3 - 64x = 0$

$\Rightarrow x(x^2 - 64) = 0$

$\Rightarrow x = 0$ & $(x^2 - 64) = 0$

$\Rightarrow x = 0$ & ± 8

Substituting $x = 0$ & $x = \pm 8$ in (1) in (2),

$y^2 = 32x$

when $x = 0, y = 0$

when $x = 8$

$\Rightarrow y^2 = 32 \times 8$

$\Rightarrow y^2 = 256$

$\Rightarrow y = \pm 16$

Substituting above values for m_1 & m_2,we get,

when $x = 0, y = 16$

$m_1 = \frac{dy}{dx}$

$\Rightarrow \frac{3 \times 0^2}{4 \times 8} = 0$

when x = 8,y = 16

$m_1 = \frac{dy}{dx}$

$\Rightarrow \frac{3\times8^2}{4\times16} = 3$

Values of m_1 is 0 & 3

when x = 0,y = 0,

$m_2 = \frac{dy}{dx}$

$\Rightarrow \frac{16}{y} = \frac{16}{0} = \infty$

when y = 16,

$m_2 = \frac{dy}{dx}$

$\Rightarrow \frac{16}{y} = \frac{16}{16} = 1$

Values of m_2 is ∞ & 1

when $m_1 = 0$ & $m_2 = \infty$

$\Rightarrow \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{\infty - 0}{1 + \infty\times0}\right|$

$\Rightarrow \tan\theta = \infty$

$\Rightarrow \theta = \tan^{-1}(\infty)$

$\therefore \tan^{-1}(\infty) = \frac{\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{2}$

when $m_1 = \frac{1}{2}$ & $m_2 = 2$

Angle of intersection of two curve is given by
$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$
where m_1 & m_2 are slopes of the curves.

$\Rightarrow \tan\theta = \left|\frac{3 - 1}{1 + 3\times1}\right|$

$\Rightarrow \tan\theta = \left|\frac{2}{4}\right|$

$\Rightarrow \tan\theta = \left|\frac{1}{2}\right|$

$\Rightarrow \theta = \tan^{-1}(\frac{1}{2})$

$\Rightarrow \theta \cong 25.516$

1 D. Question

Find the angle to intersection of the following curves :

$x^2 + y^2 - 4x - 1 = 0$ and $x^2 + y^2 - 2y - 9 = 0$

Answer

Given:

Curves $x^2 + y^2 - 4x - 1 = 0$ ...(1)

& $x^2 + y^2 - 2y - 9 = 0$ ...(2)

First curve is $x^2 + y^2 - 4x - 1 = 0$

$\Rightarrow x^2 - 4x + 4 + y^2 - 4 - 1 = 0$

$\Rightarrow (x - 2)^2 + y^2 - 5 = 0$

Now ,Subtracting (2) from (1),we get

$\Rightarrow x^2 + y^2 - 4x - 1 - (x^2 + y^2 - 2y - 9) = 0$

$\Rightarrow x^2 + y^2 - 4x - 1 - x^2 - y^2 + 2y + 9 = 0$

$\Rightarrow - 4x - 1 + 2y + 9 = 0$

$\Rightarrow - 4x + 2y + 8 = 0$

$\Rightarrow 2y = 4x - 8$

$\Rightarrow y = 2x - 4$

Substituting $y = 2x - 4$ in (3),we get,

$\Rightarrow (x - 2)^2 + (2x - 4)^2 - 5 = 0$

$\Rightarrow (x - 2)^2 + 4(x - 2)^2 - 5 = 0$

$\Rightarrow (x - 2)^2(1 + 4) - 5 = 0$

$\Rightarrow 5(x - 2)^2 - 5 = 0$

$\Rightarrow (x - 2)^2 - 1 = 0$

$\Rightarrow (x - 2)^2 = 1$

$\Rightarrow (x - 2) = \pm 1$

$\Rightarrow x = 1 + 2$ or $x = - 1 + 2$

$\Rightarrow x = 3$ or $x = 1$

So ,when $x = 3$

$y = 2\times 3 - 4$

$\Rightarrow y = 6 - 4 = 2$

So ,when $x = 3$

$y = 2\times 1 - 4$

$\Rightarrow y = 2 - 4 = - 2$

The point of intersection of two curves are (3,2) & (1, – 2)

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow x^2 + y^2 - 4x - 1 = 0$

$\Rightarrow 2x + 2y.\frac{dy}{dx} - 4 - 0 = 0$

$\Rightarrow x + y.\frac{dy}{dx} - 2 = 0$

$\Rightarrow y.\frac{dy}{dx} = 2 - x$

$\Rightarrow \frac{dy}{dx} = \frac{2-x}{y}$ ...(3)

$\Rightarrow x^2 + y^2 - 2y - 9 = 0$

$\Rightarrow 2x + 2y.\frac{dy}{dx} - 2\frac{dy}{dx} - 0 = 0$

$\Rightarrow x + y.\frac{dy}{dx} - \frac{dy}{dx} = 0$

$\Rightarrow x + (y - 1)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{y-1}$ ...(4)

At (3,2) in equation(3),we get

$$\Rightarrow \frac{dy}{dx} = \frac{2-3}{2}$$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-1}{2}$

At (3,2) in equation(4),we get

$$\Rightarrow \frac{dy}{dx} = \frac{-3}{2-1}$$

$\Rightarrow \frac{dy}{dx} = -3$

$\Rightarrow m_2 = \frac{dy}{dx} = -3$

when $m_1 = \frac{-1}{2}$ & $m_2 = 0$

Angle of intersection of two curve is given by $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ where m_1 & m_2 are slopes of the curves.

$\Rightarrow \tan\theta = \left|\frac{\frac{-1}{2} - 3}{1 + \frac{-1}{2} \times 3}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{-7}{2}}{1 + \frac{-3}{2}}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{-7}{2}}{\frac{-1}{2}}\right|$

$\Rightarrow \tan\theta = 7$

$\Rightarrow \theta = \tan^{-1}(7)$

$\Rightarrow \theta \cong 81.86$

1 E. Question

Find the angle to intersection of the following curves :

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab$

Answer

Given:

Curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ...(1)

& $x^2 + y^2 = ab$ ...(2)

Second curve is $x^2 + y^2 = ab$

$y^2 = ab - x^2$

Substituting this in equation (1),

$\Rightarrow \frac{x^2}{a^2} + \frac{ab-x^2}{b^2} = 1$

$\Rightarrow \frac{x^2b^2 + a^2(ab-x^2)}{a^2b^2} = 1$

$\Rightarrow x^2b^2 + a^3b - a^2x^2 = a^2b^2$

$\Rightarrow x^2b^2 - a^2x^2 = a^2b^2 - a^3b$

$\Rightarrow x^2(b^2 - a^2) = a^2b(b - a)$

$\Rightarrow x^2 = \frac{a^2b(b-a)}{x^2\,(b^2-a^2)}$

$\Rightarrow x^2 = \frac{a^2b(b-a)}{x^2\,(b-a)(b+a)}$

$\Rightarrow x^2 = \frac{a^2b}{(b+a)}$

$\therefore a^2 - b^2 = (a + b)(a - b)$

$\Rightarrow x = \pm\sqrt{\frac{a^2b}{(b+a)}}$ ...(3)

since , $y^2 = ab - x^2$

$\Rightarrow y^2 = ab - (\frac{a^2b}{(b+a)})$

$\Rightarrow y^2 = \frac{ab^2 + a^2b - a^2b}{(b+a)}$

$\Rightarrow y^2 = \frac{ab^2}{(b+a)}$

$\Rightarrow y = \pm\sqrt{\frac{ab^2}{(b+a)}}$ ...(4)

since ,curves are $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ & $x^2 + y^2 = ab$

Differentiating above w.r.t x,

$\Rightarrow \frac{2x}{a^2} + \frac{2y}{b^2}.\frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{b^2}.\frac{dy}{dx} = -\frac{x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-\frac{x}{a^2}}{\frac{y}{b^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{-b^2x}{a^2y}$

$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b^2x}{a^2y}$ ...(5)

Second curve is $x^2 + y^2 = ab$

$\Rightarrow 2x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y}$ ...(6)

Substituting (3) in (4), above values for m_1 & m_2,we get,

At ($\sqrt{\frac{a^2b}{(b+a)}}$, $\sqrt{\frac{ab^2}{(b+a)}}$) in equation(5),we get

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times \sqrt{\frac{a^2b}{(b\ +\ a)}}}{a^2 \times \sqrt{\frac{ab^2}{(b\ +\ a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times a\sqrt{\frac{b}{(b\ +\ a)}}}{a^2 \times b\sqrt{\frac{a}{(b\ +\ a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2a\sqrt{b}}{a^2b\sqrt{a}}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b\sqrt{b}}{a\sqrt{a}}$$

At ($\sqrt{\frac{a^2b}{(b+a)}}$, $\sqrt{\frac{ab^2}{(b+a)}}$) in equation(6),we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{\frac{a^2b}{(b\ +\ a)}}}{\sqrt{\frac{ab^2}{(b\ +\ a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a\sqrt{\frac{b}{(b\ +\ a)}}}{b\sqrt{\frac{a}{(b\ +\ a)}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-a\sqrt{b}}{b\sqrt{a}}$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -\sqrt{\frac{a}{b}}$$

when $m_1 = \frac{-b\sqrt{b}}{a\sqrt{a}}$ & $m_2 = -\sqrt{\frac{a}{b}}$

Angle of intersection of two curve is given by $\tan\theta = \left\lvert\frac{m_1 - m_2}{1 + m_1m_2}\right\rvert$ where m_1 & m_2 are slopes of the curves.

$$\Rightarrow \tan\theta = \left\lvert\frac{\frac{-b\sqrt{b}}{a\sqrt{a}} - -\sqrt{\frac{a}{b}}}{1 + \frac{-b\sqrt{b}}{a\sqrt{a}} \times -\sqrt{\frac{a}{b}}}\right\rvert$$

$$\Rightarrow \tan\theta = \left\lvert\frac{\frac{-b\sqrt{b}}{a\sqrt{a}} + \sqrt{\frac{a}{b}}}{1 + \frac{b}{a}}\right\rvert$$

$$\Rightarrow \tan\theta = \left\lvert\frac{\frac{-b\sqrt{b}\times\sqrt{b} + a\sqrt{a}\times\sqrt{a}}{a\sqrt{a}\times\sqrt{b}}}{1 + \frac{b}{a}}\right\rvert$$

$$\Rightarrow \tan\theta = \left\lvert\frac{\frac{-b\times b + a\times a}{a\sqrt{ab}}}{1 + \frac{b}{a}}\right\rvert$$

$\Rightarrow \tan\theta = \left|\frac{\frac{a^2-b^2}{a\sqrt{ab}}}{\frac{a+b}{a}}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{(a+b)(a-b)}{\sqrt{ab}}}{a+b}\right|$

$\Rightarrow \tan\theta = \left|\frac{(a-b)}{\sqrt{ab}}\right|$

$\Rightarrow \theta = \tan^{-1}\left(\frac{(a-b)}{\sqrt{ab}}\right)$

1 F. Question

Find the angle to intersection of the following curves :

$x^2 + 4y^2 = 8$ and $x^2 - 2y^2 = 2$

Answer

Given:

Curves $x^2 + 4y^2 = 8$ ...(1)

& $x^2 - 2y^2 = 2$ ...(2)

Solving (1) & (2),we get,

from 2nd curve,

$x^2 = 2 + 2y^2$

Substituting on $x^2 + 4y^2 = 8$,

$\Rightarrow 2 + 2y^2 + 4y^2 = 8$

$\Rightarrow 6y^2 = 6$

$\Rightarrow y^2 = 1$

$\Rightarrow y = \pm 1$

Substituting on $y = \pm 1$,we get,

$\Rightarrow x^2 = 2 + 2(\pm 1)^2$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

∴ The point of intersection of two curves (2,1) & (– 2, – 1)

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow x^2 + 4y^2 = 8$

$\Rightarrow 2x + 8y.\frac{dy}{dx} = 0$

$\Rightarrow 8y.\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{4y}$ ...(3)

$\Rightarrow x^2 - 2y^2 = 2$

$\Rightarrow 2x - 4y.\frac{dy}{dx} = 0$

$\Rightarrow x - 2y.\frac{dy}{dx} = 0$

$\Rightarrow 4y\frac{dy}{dx} = x$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2y}$ ...(4)

At (2,1) in equation(3),we get

$\Rightarrow \frac{dy}{dx} = \frac{-2}{4 \times 1}$

$\Rightarrow m_1 = \frac{-1}{2}$

At (2,1) in equation(4),we get

$\Rightarrow \frac{dy}{dx} = \frac{2}{2 \times 1}$

$\Rightarrow \frac{dy}{dx} = 1$

$\Rightarrow m_2 = 1$

when $m_1 = \frac{-1}{2}$ & $m_2 = 1$

Angle of intersection of two curve is given by $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ where m_1 & m_2 are slopes of the curves.

$\Rightarrow \tan\theta = \left|\frac{\frac{-1}{2} - 1}{1 + \frac{-1}{2} \times 1}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{-3}{2}}{1 - \frac{1}{2}}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{-3}{2}}{\frac{1}{2}}\right|$

$\Rightarrow \tan\theta = |-3|$

$\Rightarrow \theta = \tan^{-1}(3)$

$\Rightarrow \theta \cong 71.56$

1 G. Question

Find the angle to intersection of the following curves :

$x^2 = 27y$ and $y^2 = 8x$

Answer

Given:

Curves $x^2 = 27y$ ...(1)

& $y^2 = 8x$ ...(2)

Solving (1) & (2),we get,

From $y^2 = 8x$,we get,

$\Rightarrow x = \frac{y^2}{8}$

Substituting $x = \frac{y^2}{8}$ on $x^2 = 27y$,

$\Rightarrow (\frac{y^2}{8})^2 = 27y$

$\Rightarrow (\frac{y^4}{64}) = 27y$

$\Rightarrow y^4 = 1728y$

$\Rightarrow y(y^3 - 1728) = 0$

$\Rightarrow y = 0$ or $(y^3 - 1728) = 0$

$\Rightarrow y = 0$ or $y = \sqrt[3]{1728}$

$\therefore \sqrt[3]{1728} = 12$

$\Rightarrow y = 0$ or $y = 12$

Substituting $y = 0$ or $y = 12$ on $x = \frac{y^2}{8}$

when $y = 0$,

$\Rightarrow x = \frac{0^2}{8}$

$\Rightarrow x = 0$

when $y = 12$,

$\Rightarrow x = \frac{12^2}{8}$

$\Rightarrow x = 18$

$\therefore$ The point of intersection of two curves (0,0) & (18,12)

First curve is $x^2 = 27y$

Differentiating above w.r.t x,

$\Rightarrow 2x = 27.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{27}$

$\Rightarrow m_1 = \frac{2x}{27}$ ...(3)

Second curve is $y^2 = 8x$

$\Rightarrow 2y.\frac{dy}{dx} = 8$

$\Rightarrow y.\frac{dy}{dx} = 4$

$\Rightarrow m_2 = \frac{4}{y}$ ...(4)

Substituting (18,12) for m_1 & m_2, we get,

$m_1 = \frac{2x}{27}$

$\Rightarrow \frac{2\times18}{27} = \frac{36}{27}$

$m_1 = \frac{4}{3}$ ...(5)

$m_2 = \frac{4}{y}$

$\Rightarrow \frac{4}{y} = \frac{4}{12}$

$m_2 = \frac{1}{3}$ ...(6)

when $m_1 = \frac{4}{3}$ & $m_2 = \frac{1}{3}$

Angle of intersection of two curve is given by $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\|$ where m_1 & m_2 are slopes of the curves.

$\Rightarrow \tan\theta = \left|\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{3}{3}}{1 + \frac{4}{9}}\right|$

$\Rightarrow \tan\theta = \left|\frac{1}{\frac{13}{9}}\right|$

$\Rightarrow \tan\theta = \left|\frac{9}{13}\right|$

$\Rightarrow \theta = \tan^{-1}\left(\frac{9}{13}\right)$

$\Rightarrow \theta \cong 34.69$

1 H. Question

Find the angle to intersection of the following curves :

$x^2 + y^2 = 2x$ and $y^2 = x$

Answer

Given:

Curves $x^2 + y^2 = 2x$ ...(1)

& $y^2 = x$ ...(2)

Solving (1) & (2),we get

Substituting $y^2 = x$ in $x^2 + y^2 = 2x$

$\Rightarrow x^2 + x = 2x$

$\Rightarrow x^2 - x = 0$

$\Rightarrow x(x - 1) = 0$

$\Rightarrow x = 0$ or $(x - 1) = 0$

$\Rightarrow x = 0$ or $x = 1$

Substituting $x = 0$ or $x = 1$in $y^2 = x$,we get,

when $x = 0$,

$\Rightarrow y^2 = 0$

$\Rightarrow y = 0$

when $x = 1$,

$\Rightarrow y^2 = 1$

$\Rightarrow y = 1$

The point of intersection of two curves are (0,0) & (1,1)

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow x^2 + y^2 = 2x$

$\Rightarrow 2x + 2y.\frac{dy}{dx} = 2$

$\Rightarrow x + y.\frac{dy}{dx} = 1$

$\Rightarrow y.\frac{dy}{dx} = 1 - x$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$ ...(3)

$\Rightarrow y^2 = x$

$\Rightarrow 2y.\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$ ...(4)

At (1,1) in equation(3),we get

$$\Rightarrow \frac{dy}{dx} = \frac{1-x}{y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1-1}{1}$$

$\Rightarrow m_1 = 0$

At (1,1) in equation(4),we get

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2 \times 1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

$\Rightarrow m_2 = \frac{1}{2}$

when $m_1 = 0$ & $m_2 = \frac{1}{2}$

Angle of intersection of two curve is given by $\tan\theta = \left\lvert\frac{m_1 - m_2}{1 + m_1 m_2}\right\rvert$ where m_1 & m_2 are slopes of the curves.

$$\Rightarrow \tan\theta = \left\lvert\frac{0-\frac{1}{2}}{1 + 0 \times \frac{1}{2}}\right\rvert$$

$$\Rightarrow \tan\theta = \left\lvert\frac{\frac{-1}{2}}{1+0}\right\rvert$$

$$\Rightarrow \tan\theta = \left\lvert\frac{-1}{2}\right\rvert$$

$\Rightarrow \tan\theta = \frac{1}{2}$

$\Rightarrow \theta = \tan^{-1}(\frac{1}{2})$

$\Rightarrow \theta \cong 26.56$

1 I. Question

Find the angle to intersection of the following curves :

$y = 4 - x^2$ and $y = x^2$

Answer

Given:

Curves $y = 4 - x^2$ ...(1)

& $y = x^2$ ...(2)

Solving (1) & (2),we get

$\Rightarrow y = 4 - x^2$

$\Rightarrow x^2 = 4 - x^2$

$\Rightarrow 2x^2 = 4$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

Substituting $\sqrt{2}$ in $y = x^2$,we get

$y = (\sqrt{2})^2$

$y = 2$

The point of intersection of two curves are $(\sqrt{2},2)$ & $(-\sqrt{2}, -2)$

First curve $y = 4 - x^2$

Differentiating above w.r.t x,

$\Rightarrow \frac{dy}{dx} = 0 - 2x$

$\Rightarrow m_1 = -2x$ ...(3)

Second curve $y = x^2$

Differentiating above w.r.t x,

$\Rightarrow \frac{dy}{dx} = 2x$

$m_2 = 2x$ ...(4)

At $(\sqrt{2},2)$,we have,

$m_1 = \frac{dy}{dx} = -2x$

$\Rightarrow -2 \times \sqrt{2}$

$\Rightarrow m_1 = -2\sqrt{2}$

At $(-\sqrt{2},2)$,we have,

$m_2 = \frac{dy}{dx} = -2x$

$(-1) \times -\sqrt{2} \times 2 = 2\sqrt{2}$

When $m_1 = -2\sqrt{2}$ & $m_2 = 2\sqrt{2}$

Angle of intersection of two curve is given by
$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$
where m_1 & m_2 are slopes of the curves.

$\tan\theta = \left|\frac{-2\sqrt{2}-2\sqrt{2}}{1-2\sqrt{2}\times 2\sqrt{2}}\right|$

$\tan\theta = \left|\frac{-4\sqrt{2}}{1-8}\right|$

$\tan\theta = \left|\frac{-4\sqrt{2}}{-7}\right|$

$\tan\theta = \frac{4\sqrt{2}}{7}$

$\theta = \tan^{-1}(\frac{4\sqrt{2}}{7})$

$\theta \cong 38.94$

2 A. Question

Show that the following set of curves intersect orthogonally :

$y = x^3$ and $6y = 7 - x^2$

Answer

Given:

Curves $y = x^3$ ...(1)

& $6y = 7 - x^2$ ...(2)

Solving (1) & (2),we get

$\Rightarrow 6y = 7 - x^2$

$\Rightarrow 6(x^3) = 7 - x^2$

$\Rightarrow 6x^3 + x^2 - 7 = 0$

Since $f(x) = 6x^3 + x^2 - 7$,

we have to find f(x) = 0,so that x is a factor of f(x).

when x = 1

$f(1) = 6(1)^3 + (1)^2 - 7$

$f(1) = 6 + 1 - 7$

$f(1) = 0$

Hence, x = 1 is a factor of f(x).

Substituting x = 1 in $y = x^3$,we get

$y = 1^3$

$y = 1$

The point of intersection of two curves is (1,1)

First curve $y = x^3$

Differentiating above w.r.t x,

$\Rightarrow m_1 = \frac{dy}{dx} = 3x^2$ ...(3)

Second curve $6y = 7 - x^2$

Differentiating above w.r.t x,

$\Rightarrow 6\frac{dy}{dx} = 0 - 2x$

$\Rightarrow m_2 = \frac{-2x}{6}$

$\Rightarrow m_2 = \frac{-x}{3}$ ...(4)

At (1,1),we have,

$m_1 = 3x^2$

$\Rightarrow 3\times(1)^2$

$m_1 = 3$

At (1,1),we have,

$\Rightarrow m_2 = \frac{-x}{3}$

$\Rightarrow \frac{-1}{3}$

$\Rightarrow m_2 = \frac{-1}{3}$

When $m_1 = 3$ & $m_2 = \frac{-1}{3}$

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow 3\times\frac{-1}{3} = -1$

∴ Two curves $y = x^3$ & $6y = 7 - x^2$ intersect orthogonally.

2 B. Question

Show that the following set of curves intersect orthogonally :

$x^3 - 3xy^2 = -2$ and $3x^2y - y^3 = 2$

Answer

Given:

Curves $x^3 - 3xy^2 = -2$ ...(1)

& $3x^2y - y^3 = 2$ ...(2)

Adding (1) & (2),we get

$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = -2 + 2$

$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = -0$

$\Rightarrow (x - y)^3 = 0$

$\Rightarrow (x - y) = 0$

$\Rightarrow x = y$

Substituting x = y on $x^3 - 3xy^2 = -2$

$\Rightarrow x^3 - 3\times x\times x^2 = -2$

$\Rightarrow x^3 - 3x^3 = -2$

$\Rightarrow -2x^3 = -2$

$\Rightarrow x^3 = 1$

$\Rightarrow x = 1$

Since $x = y$

$y = 1$

The point of intersection of two curves is (1,1)

First curve $x^3 - 3xy^2 = -2$

Differentiating above w.r.t x,

$\Rightarrow 3x^2 - 3(1\times y^2 + x\times 2y\frac{dy}{dx}) = 0$

$\Rightarrow 3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 3y^2 = 6xy\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2-3y^2}{6xy}$

$\Rightarrow \frac{dy}{dx} = \frac{3(x^2-y^2)}{6xy}$

$\Rightarrow m_1 = \frac{(x^2-y^2)}{2xy}$ ...(3)

Second curve $3x^2y - y^3 = 2$

Differentiating above w.r.t x,

$\Rightarrow 3(2x\times y + x^2\times\frac{dy}{dx}) - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$

$\Rightarrow 6xy + (3x^2 - 3y^2)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-6xy}{3x^2-3y^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-2xy}{x^2-y^2}$

$\Rightarrow m_2 = \frac{-2xy}{x^2-y^2}$ ...(4)

When $m_1 = \frac{(x^2-y^2)}{2xy}$ & $m_2 = \frac{-2xy}{x^2-y^2}$

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \frac{(x^2-y^2)}{2xy}\times\frac{-2xy}{x^2-y^2} = -1$

∴ Two curves $x^3 - 3xy^2 = -2$ & $3x^2y - y^3 = 2$ intersect orthogonally.

2 C. Question

Show that the following set of curves intersect orthogonally :

$x^2 + 4y^2 = 8$ and $x^2 - 2y^2 = 4$.

Answer

Given:

Curves $x^2 + 4y^2 = 8$ ...(1)

& $x^2 - 2y^2 = 4$ ...(2)

Solving (1) & (2),we get,

from 2nd curve,

$x^2 = 4 + 2y^2$

Substituting on $x^2 + 4y^2 = 8$,

$\Rightarrow 4 + 2y^2 + 4y^2 = 8$

$\Rightarrow 6y^2 = 4$

$\Rightarrow y^2 = \frac{4}{6}$

$\Rightarrow y = \pm\sqrt{\frac{2}{3}}$

Substituting on $y = \pm\sqrt{\frac{2}{3}}$, we get,

$\Rightarrow x^2 = 4 + 2(\pm\sqrt{\frac{2}{3}})^2$

$\Rightarrow x^2 = 4 + 2(\frac{2}{3})$

$\Rightarrow x^2 = 4 + \frac{4}{3}$

$\Rightarrow x^2 = \frac{16}{3}$

$\Rightarrow x = \pm\sqrt{\frac{16}{3}}$

$\Rightarrow x = \pm\frac{4}{\sqrt{3}}$

$\therefore$ The point of intersection of two curves $(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}})$ & $(-\frac{4}{\sqrt{3}}, -\sqrt{\frac{2}{3}})$

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow x^2 + 4y^2 = 8$

$\Rightarrow 2x + 8y.\frac{dy}{dx} = 0$

$\Rightarrow 8y.\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{4y}$ ...(3)

$\Rightarrow x^2 - 2y^2 = 4$

$\Rightarrow 2x - 4y.\frac{dy}{dx} = 0$

$\Rightarrow x - 2y.\frac{dy}{dx} = 0$

$\Rightarrow 4y\frac{dy}{dx} = x$

$\Rightarrow \frac{dy}{dx} = \frac{x}{2y}$ ...(4)

At $\left(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$ in equation(3),we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{4}{\sqrt{3}}}{4 \times \sqrt{\frac{2}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\frac{1}{\sqrt{3}}}{\sqrt{\frac{2}{3}}}$$

$\Rightarrow m_1 = \frac{-1}{\sqrt{2}}$

At $\left(\frac{4}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right)$ in equation(4),we get

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{4}{\sqrt{3}}}{2 \times \sqrt{\frac{2}{3}})}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{2}{\sqrt{3}}}{\sqrt{\frac{2}{3}})}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{2}}$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{2}$$

$\Rightarrow m_2 = 1$

when $m_1 = \frac{-1}{\sqrt{2}}$ & $m_2 = \sqrt{2}$

Two curves intersect orthogonally if $m_1 m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \frac{-1}{\sqrt{2}} \times \sqrt{2} = -1$

∴ Two curves $x^2 + 4y^2 = 8$ & $x^2 - 2y^2 = 4$ intersect orthogonally.

3 A. Question

Show that the following curves intersect orthogonally at the indicated points :

$x^2 = 4y$ and $4y + x^2 = 8$ at (2, 1)

Answer

Given:

Curves $x^2 = 4y$ ...(1)

& $4y + x^2 = 8$ ...(2)

The point of intersection of two curves (2,1)

Solving (1) & (2),we get,

First curve is $x^2 = 4y$

Differentiating above w.r.t x,

$\Rightarrow 2x = 4.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{4}$

$\Rightarrow m_1 = \frac{x}{2}$ ...(3)

Second curve is $4y + x^2 = 8$

$\Rightarrow 4.\frac{dy}{dx} + 2x = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{4}$

$\Rightarrow m_2 = \frac{-x}{2}$ ...(4)

Substituting (2,1) for m_1 & m_2,we get,

$m_1 = \frac{x}{2}$

$\Rightarrow \frac{2}{2}$

$m_1 = 1$ ...(5)

$m_2 = \frac{-x}{2}$

$\Rightarrow \frac{-2}{2}$

$m_2 = -1$ ...(6)

when $m_1 = 1$ & $m_2 = -1$

Two curves intersect orthogonally if $m_1 m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow 1 \times -1 = -1$

$\therefore$ Two curves $x^2 = 4y$ & $4y + x^2 = 8$ intersect orthogonally.

3 B. Question

Show that the following curves intersect orthogonally at the indicated points :

$x^2 = y$ and $x^3 + 6y = 7$ at (1, 1)

Answer

Given:

Curves $x^2 = y$ ...(1)

& $x^3 + 6y = 7$ ...(2)

The point of intersection of two curves (1,1)

Solving (1) & (2),we get,

First curve is $x^2 = y$

Differentiating above w.r.t x,

$\Rightarrow 2x = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = 2x$

$\Rightarrow m_1 = 2x$ ...(3)

Second curve is $x^3 + 6y = 7$

Differentiating above w.r.t x,

$\Rightarrow 3x^2 + 6.\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-3x^2}{6}$

$\Rightarrow \frac{dy}{dx} = \frac{-x^2}{2}$

$\Rightarrow m_2 = \frac{-x^2}{2}$ ...(4)

Substituting (1,1) for m_1 & m_2, we get,

$m_1 = 2x$

$\Rightarrow 2\times1$

$m_1 = 2$ ...(5)

$m_2 = \frac{-x^2}{2}$

$\Rightarrow \frac{-1^2}{2}$

$m_2 = -\frac{-1}{2}$ ...(6)

when $m_1 = 2$ & $m_2 = -\frac{-1}{2}$

Two curves intersect orthogonally if $m_1m_2 = -1$, where m_1 and m_2 the slopes of the two curves.

$\Rightarrow 2\times\frac{-1}{2} = -1$

∴ Two curves $x^2 = y$ & $x^3 + 6y = 7$ intersect orthogonally.

3 C. Question

Show that the following curves intersect orthogonally at the indicated points :

$y^2 = 8x$ and $2x^2 + y^2 = 10$ at $(1, 2\sqrt{2})$

Answer

Given:

Curves $y^2 = 8x$ ...(1)

& $2x^2 + y^2 = 10$ ...(2)

The point of intersection of two curves are (0,0) & $(1,2\sqrt{2})$

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow y^2 = 8x$

$\Rightarrow 2y.\frac{dy}{dx} = 8$

$\Rightarrow \frac{dy}{dx} = \frac{8}{2y}$

$\Rightarrow \frac{dy}{dx} = \frac{4}{y}$ ...(3)

$\Rightarrow 2x^2 + y^2 = 10$

Differentiating above w.r.t x,

$\Rightarrow 4x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow 2x + y.\frac{dy}{dx} = 0$

$\Rightarrow y.\frac{dy}{dx} = -2x$

$\Rightarrow \frac{dy}{dx} = \frac{-2x}{y}$ ...(4)

Substituting $(1,2\sqrt{2})$for m_1 & m_2,we get,

$m_1 = \frac{4}{y}$

$\Rightarrow \frac{4}{2\sqrt{2}}$

$m_1 = \sqrt{2}$ ...(5)

$m_2 = \frac{-2x}{y}$

$\Rightarrow \frac{-2\times1}{2\sqrt{2}}$

$m_2 = - \frac{-1}{\sqrt{2}}$ ...(6)

when $m_1 = \sqrt{2}$ & $m_2 = \frac{-1}{\sqrt{2}}$

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \sqrt{2}\times\frac{-1}{\sqrt{2}} = -1$

∴ Two curves $y^2 = 8x$ & $2x^2 + y^2 = 10$ intersect orthogonally.

4. Question

Show that the curves $4x = y^2$ and $4xy = k$ cut at right angles, if $k^2 = 512$.

Answer

Given:

Curves $4x = y^2$ ...(1)

& $4xy = k$ ...(2)

We have to prove that two curves cut at right angles if $k^2 = 512$

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow 4x = y^2$

$\Rightarrow 4 = 2y.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$m_1 = \frac{2}{y}$ ...(3)

$\Rightarrow 4xy = k$

Differentiating above w.r.t x,

$\Rightarrow 4(1\times y + x\frac{dy}{dx}) = 0$

$\Rightarrow y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$

$\Rightarrow m_2 = \frac{-y}{x}$ ...(4)

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

Since m_1 and m_2 cuts orthogonally,

$\Rightarrow \frac{2}{y}\times\frac{-y}{x} = -1$

$\Rightarrow \frac{-2}{x} = -1$

$\Rightarrow x = 2$

Now , Solving (1) & (2),we get,

$4xy = k$ & $4x = y^2$

$\Rightarrow (y^2)y = k$

$\Rightarrow y^3 = k$

$\Rightarrow y = k^{\frac{1}{3}}$

Substituting $y = k^{\frac{1}{3}}$ in $4x = y^2$,we get,

$\Rightarrow 4x = (k^{\frac{1}{3}})^2$

$\Rightarrow 4\times2 = k^{\frac{2}{3}}$

$\Rightarrow k^{\frac{2}{3}} = 8$

$\Rightarrow k^2 = 8^3$

$\Rightarrow k^2 = 512$

5. Question

Show that the curves $2x = y^2$ and $2xy = k$ cut at right angles, if $k^2 = 8$.

Answer

Given:

Curves $2x = y^2$ ...(1)

& $2xy = k$ ...(2)

We have to prove that two curves cut at right angles if $k^2 = 8$

Now ,Differentiating curves (1) & (2) w.r.t x, we get

$\Rightarrow 2x = y^2$

$\Rightarrow 2 = 2y.\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{y}$

$m_1 = \frac{1}{y}$ ...(3)

$\Rightarrow 2xy = k$

Differentiating above w.r.t x,

$\Rightarrow 2(1\times y + x\frac{dy}{dx}) = 0$

$\Rightarrow y + x\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

$\Rightarrow m_2 = \frac{-y}{x}$ ...(4)

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

Since m_1 and m_2 cuts orthogonally,

$\Rightarrow \frac{1}{y}\times\frac{-y}{x} = -1$

$\Rightarrow \frac{-1}{x} = -1$

$\Rightarrow x = 1$

Now , Solving (1) & (2),we get,

$2xy = k$ & $2x = y^2$

$\Rightarrow (y^2)y = k$

$\Rightarrow y^3 = k$

$\Rightarrow y = k^{\frac{1}{3}}$

Substituting $y = k^{\frac{1}{3}}$ in $2x = y^2$,we get,

$\Rightarrow 2x = (k^{\frac{1}{3}})^2$

$\Rightarrow 2\times1 = k^{\frac{2}{3}}$

$\Rightarrow k^{\frac{2}{3}} = 2$

$\Rightarrow k^2 = 2^3$

$\Rightarrow k^2 = 8$

6. Question

Prove that the curves $xy = 4$ and $x^2 + y^2 = 8$ touch each other.

Answer

Given:

Curves $xy = 4$ ...(1)

& $x^2 + y^2 = 8$ ...(2)

Solving (1) & (2),we get

$\Rightarrow xy = 4$

$\Rightarrow x = \frac{4}{y}$

Substituting $x = \frac{4}{y}$ in $x^2 + y^2 = 8$,we get,

$\Rightarrow (\frac{4}{y})^2 + y^2 = 8$

$\Rightarrow \frac{16}{y^2} + y^2 = 8$

$\Rightarrow 16 + y^4 = 8y^2$

$\Rightarrow y^4 - 8y^2 + 16 = 0$

We will use factorization method to solve the above equation

$\Rightarrow y^4 - 4y^2 - 4y^2 + 16 = 0$

$\Rightarrow y^2(y^2 - 4) - 4(y^2 - 4) = 0$

$\Rightarrow (y^2 - 4)(y^2 - 4) = 0$

$\Rightarrow y^2 - 4 = 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

Substituting $y = \pm 2$ in $x = \frac{4}{y}$,we get,

$\Rightarrow x = \frac{4}{\pm 2}$

$\Rightarrow x = \pm 2$

$\therefore$ The point of intersection of two curves (2,2) &

(– 2, – 2)

First curve $xy = 4$

$\Rightarrow 1 \times y + x.\frac{dy}{dx} = 0$

$\Rightarrow x.\frac{dy}{dx} = -y$

$\Rightarrow m_1 = \frac{-y}{x}$ ...(3)

Second curve is $x^2 + y^2 = 8$

Differentiating above w.r.t x,

$\Rightarrow 2x + 2y.\frac{dy}{dx} = 0$

$\Rightarrow y.\frac{dy}{dx} = -x$

$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y}$ ...(4)

At (2,2),we have,

$m_1 = \frac{-y}{x}$

$\Rightarrow \frac{-2}{2}$

$m_1 = -1$

At (2,2),we have,

$\Rightarrow m_2 = \frac{-x}{y}$

$\Rightarrow \frac{-2}{2}$

$\Rightarrow m_2 = -1$

Clearly, $m_1 = m_2 = -1$ at (2,2)

So, given curve touch each other at (2,2)

7. Question

Prove that the curves $y^2 = 4x$ and

$x^2 + y^2 - 6x + 1 = 0$ touch each other at the point (1, 2).

Answer

Given:

Curves $y^2 = 4x$ ...(1)

& $x^2 + y^2 - 6x + 1 = 0$ ...(2)

∴The point of intersection of two curves is (1,2)

First curve is $y^2 = 4x$

Differentiating above w.r.t x,

$\Rightarrow 2y.\frac{dy}{dx} = 4$

$\Rightarrow y.\frac{dy}{dx} = 2$

$\Rightarrow m_1 = \frac{2}{y}$ ...(3)

Second curve is $x^2 + y^2 - 6x + 1 = 0$

$\Rightarrow 2x + 2y.\frac{dy}{dx} - 6 - 0 = 0$

$\Rightarrow x + y.\frac{dy}{dx} - 3 = 0$

$\Rightarrow y.\frac{dy}{dx} = 3 - x$

$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$ ...(4)

At (1,2),we have,

$m_1 = \frac{2}{y}$

$\Rightarrow \frac{2}{2}$

$m_1 = 1$

At (1,2),we have,

$\Rightarrow m_2 = \frac{3-x}{y}$

$\Rightarrow \frac{3-1}{2}$

$\Rightarrow m_2 = 1$

Clearly, $m_1 = m_2 = 1$ at (1,2)

So, given curve touch each other at (1,2)

8 A. Question

Find the condition for the following set of curves to interest orthogonally.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $xy = c^2$

Answer

Given:

Curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(1)

& $xy = c^2$ ...(2)

First curve is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Differentiating above w.r.t x,

$\Rightarrow \frac{2x}{a^2} - \frac{2y}{b^2}.\frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{b^2}.\frac{dy}{dx} = \frac{x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{x}{a^2}}{\frac{y}{b^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{b^2x}{a^2y}$

$\Rightarrow m_1 = \frac{b^2x}{a^2y}$ ...(3)

Second curve is $xy = c^2$

$\Rightarrow \Rightarrow 1 \times y + x.\frac{dy}{dx} = 0$

$\Rightarrow x.\frac{dy}{dx} = -y$

$\Rightarrow m_2 = \frac{-y}{x}$ ...(4)

When $m_1 = \frac{b^2x}{a^2y}$ & $m_2 = \frac{-y}{x}$

Since ,two curves intersect orthogonally,

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \frac{-b^2x}{a^2y} \times \frac{-y}{x} = -1$

$\Rightarrow \frac{b^2}{a^2} = 1$

$\Rightarrow \therefore a^2 = b^2$

8 B. Question

Find the condition for the following set of curves to interest orthogonally.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ and } \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1.$$

Answer

Given:

Curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(1)

& $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ ...(2)

First curve is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Differentiating above w.r.t x,

$\Rightarrow \frac{2x}{a^2} - \frac{2y}{b^2}.\frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{b^2}.\frac{dy}{dx} = \frac{x}{a^2}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{x}{a^2}}{\frac{y}{b^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{b^2x}{a^2y}$

$\Rightarrow m_1 = \frac{b^2x}{a^2y}$ ...(3)

Second curve is $\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$

Differentiating above w.r.t x,

$\Rightarrow \frac{2x}{A^2} - \frac{2y}{B^2}.\frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{B^2}.\frac{dy}{dx} = \frac{x}{A}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{x}{A}}{\frac{y}{B^2}}$

$\Rightarrow \frac{dy}{dx} = \frac{B^2x}{A^2y}$

$\Rightarrow m_1 = \frac{B^2x}{A^2y}$ ...(4)

When $m_1 = \frac{b^2x}{a^2y}$ & $m_2 = \frac{B^2x}{A^2y}$

Since ,two curves intersect orthogonally,

Two curves intersect orthogonally if $m_1m_2 = -1$,where m_1 and m_2 the slopes of the two curves.

$\Rightarrow \frac{b^2x}{a^2y} \times \frac{B^2x}{A^2y} = -1$

$\Rightarrow \frac{b^2B^2}{a^2A^2} \times \frac{x^2}{y^2} = -1$

$\Rightarrow \frac{x^2}{y^2} = \frac{-a^2A^2}{b^2B^2}$ ...(5)

Now equation (1) – (2) gives

$$\Rightarrow \left(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\right) - \left(\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1\right)$$

$\Rightarrow x^2(\frac{1}{a^2} - \frac{1}{A^2}) - y^2(\frac{1}{b^2} - \frac{1}{B^2}) = 0$

$\Rightarrow x^2(\frac{1}{a^2} - \frac{1}{A^2}) = y^2(\frac{1}{b^2} - \frac{1}{B^2})$

$\Rightarrow \frac{x^2}{y^2} = \frac{(\frac{1}{b^2} - \frac{1}{B^2})}{(\frac{1}{a^2} - \frac{1}{A^2})}$

$\Rightarrow \frac{x^2}{y^2} = \frac{(\frac{B^2 - b^2}{b^2 B^2})}{(\frac{A^2 - a^2}{A^2 a^2})}$

$\Rightarrow \frac{x^2}{y^2} = \frac{(B^2 - b^2)(A^2 a^2)}{(A^2 - a^2)(b^2 B^2)}$

Substituting $\frac{x^2}{y^2}$ from equation (5),we get

$\Rightarrow \frac{-a^2 A^2}{b^2 B^2} = \frac{(B^2 - b^2)(A^2 a^2)}{(A^2 - a^2)(b^2 B^2)}$

$\Rightarrow -1 = \frac{(B^2 - b^2)}{(A^2 - a^2)}$

$\Rightarrow (-1)(A^2 - a^2) = (B^2 - b^2)$

$\Rightarrow a^2 - A^2 = B^2 - b^2$

$\Rightarrow a^2 + b^2 = B^2 + A^2$

9. Question

Show that the curves $\frac{x^2}{a^2 + \lambda_1} + \frac{y^2}{b^2 + \lambda_1} = 1$ and $\frac{x^2}{a^2 + \lambda_2} + \frac{y^2}{b^2 + \lambda_2} = 1$ interest at right angles

Answer

Given:

Curves $\frac{x^2}{a^2 + \lambda_1} + \frac{y^2}{b^2 + \lambda_1} = 1$ ...(1)

& $\frac{x^2}{a^2 + \lambda_2} + \frac{y^2}{b^2 + \lambda_2} = 1$ ...(2)

First curve is $\frac{x^2}{a^2 + \lambda_1} + \frac{y^2}{b^2 + \lambda_1} = 1$

Differentiating above w.r.t x,

$\Rightarrow \frac{2x}{a^2 + \lambda_1} + \frac{2y}{b^2 + \lambda_1} . \frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{b^2 + \lambda_1} . \frac{dy}{dx} = \frac{-x}{a^2 + \lambda_1}$

$\Rightarrow \frac{dy}{dx} = \frac{\frac{-x}{a^2 + \lambda_1}}{\frac{y}{b^2 + \lambda_1}}$

$\Rightarrow m_1 = \frac{-x(b^2 + \lambda_1)}{y(a^2 + \lambda_1)}$ ...(3)

Second curve is $\frac{x^2}{a^2 + \lambda_2} + \frac{y^2}{b^2 + \lambda_2} = 1$

Differentiating above w.r.t x,

$$\Rightarrow \frac{2x}{a^2+\lambda_2} + \frac{2y}{b^2+\lambda_2}.\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{y}{b^2+\lambda_2}.\frac{dy}{dx} = \frac{-x}{a^2+\lambda_2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{-x}{a^2+\lambda_2}}{\frac{y}{b^2+\lambda_2}}$$

$$\Rightarrow m_2 = \frac{-x(b^2+\lambda_2)}{y(a^2+\lambda_2)} \ldots(4)$$

Now equation (1) – (2) gives

$$\Rightarrow \left(\frac{x^2}{a^2+\lambda_1} + \frac{y^2}{b^2+\lambda_1} = 1\right) - \left(\frac{x^2}{a^2+\lambda_2} + \frac{y^2}{b^2+\lambda_2} = 1\right)$$

$$\Rightarrow x^2\left(\frac{1}{a^2+\lambda_1} - \frac{1}{a^2+\lambda_2}\right) + y^2\left(\frac{1}{b^2+\lambda_1} - \frac{1}{b^2+\lambda_2}\right) = 0$$

$$\Rightarrow x^2\left(\frac{1}{a^2+\lambda_1} - \frac{1}{a^2+\lambda_2}\right) = -\, y^2\left(\frac{1}{b^2+\lambda_1} - \frac{1}{b^2+\lambda_2}\right)$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{-\left(\frac{1}{b^2+\lambda_1} - \frac{1}{b^2+\lambda_2}\right)}{\left(\frac{1}{a^2+\lambda_1} - \frac{1}{a^2+\lambda_2}\right)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{-\left(\frac{b^2+\lambda_2-(b^2+\lambda_1)}{(b^2+\lambda_1)(b^2+\lambda_1)}\right)}{\left(\frac{(a^2+\lambda_2)-a^2+\lambda_1}{(a^2+\lambda_1)(a^2+\lambda_2)}\right)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{-\left(\frac{b^2+\lambda_2-b^2-\lambda_1}{(b^2+\lambda_1)(b^2+\lambda_1)}\right)}{\left(\frac{a^2+\lambda_2-a^2-\lambda_1}{(a^2+\lambda_1)(a^2+\lambda_2)}\right)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{\left(\frac{-(\lambda_2-\lambda_1)}{(b^2+\lambda_1)(b^2+\lambda_1)}\right)}{\left(\frac{(\lambda_2-\lambda_1}{(a^2+\lambda_1)(a^2+\lambda_2)}\right)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{\left(\frac{(\lambda_1-\lambda_2)}{(b^2+\lambda_1)(b^2+\lambda_1)}\right)}{\left(\frac{(\lambda_2-\lambda_1}{(a^2+\lambda_1)(a^2+\lambda_2)}\right)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{(\lambda_1-\lambda_2)(a^2+\lambda_1)(a^2+\lambda_2)}{(\lambda_2-\lambda_1)(b^2+\lambda_1)(b^2+\lambda_1)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{-(\lambda_2-\lambda_1)(a^2+\lambda_1)(a^2+\lambda_2)}{(\lambda_2-\lambda_1)(b^2+\lambda_1)(b^2+\lambda_1)}$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{-(a^2+\lambda_1)(a^2+\lambda_2)}{(b^2+\lambda_1)(b^2+\lambda_1)} \ldots(5)$$

When $m_1 = \frac{-x(b^2+\lambda_1)}{y(a^2+\lambda_1)}$ & $m_2 = \frac{-x(b^2+\lambda_2)}{y(a^2+\lambda_2)}$

Two curves intersect orthogonally if $m_1m_2 = -\ 1$,where m_1 and m_2 the slopes of the two curves.

$$\Rightarrow \frac{-x(b^2+\lambda_1)}{y(a^2+\lambda_1)} \times \frac{-x(b^2+\lambda_2)}{y(a^2+\lambda_2)}$$

$$\Rightarrow \frac{x^2}{y^2} \times \frac{(b^2+\lambda_1)(b^2+\lambda_2)}{(a^2+\lambda_1)(a^2+\lambda_2)}$$

Substituting $\frac{x^2}{y^2}$ from equation (5),we get

$$\Rightarrow \frac{-(a^2+\lambda_1)(a^2+\lambda_2)}{(b^2+\lambda_1)(b^2+\lambda_1)} \times \frac{(b^2+\lambda_1)(b^2+\lambda_2)}{(a^2+\lambda_1)(a^2+\lambda_2)}$$

$\Rightarrow$ – 1

∴ The two curves intersect orthogonally,

10. Question

If the straight line xcosα + ysinα = p touches the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then prove that

$a^2\cos^2\alpha - b^2\sin^2\alpha = \rho^2$.

Answer

Given:

The straight line xcosα + ysinα = p touches the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Suppose the straight line xcosα + ysinα = p touches the curve at (x_1, y_1).

But the equation of tangent to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at (x_1, y_1) is

$\Rightarrow \frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

Thus ,equation $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$ and xcosα + ysinα = p represent the same line.

$\therefore \frac{\frac{x_1}{a^2}}{\cos\alpha}+\frac{\frac{y_1}{b^2}}{\sin\alpha}=\frac{1}{p}$

$\Rightarrow x_1=\frac{a^2\cos\alpha}{p}, y_1=\frac{b^2\sin\alpha}{p}$

Since the point (x_1, y_1) lies on the curve $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow \frac{\left(\frac{a^2\cos\alpha}{p}\right)^2}{a^2}-\frac{\left(\frac{b^2\sin\alpha}{p}\right)^2}{b^2}=1$

$\Rightarrow \frac{a^4\cos\alpha^2}{p^2a^2}-\frac{b^4\sin\alpha^2}{p^2b^2}=1$

$\Rightarrow \frac{a^2\cos\alpha^2}{p^2}-\frac{b^2\sin\alpha^2}{p^2}=1$

$\Rightarrow a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$

Thus proved.

MCQ

1. Question

The equation to the normal to the curve y = sinx at (0, 0) is

A. x = 0

B. y = 0

C. x + y = 0

D. x – y = 0

Answer

Given that y = sinx

Slope of the tangent $\frac{dy}{dx}=\cos x$

Slope at origin = cos 0 = 1

Equation of normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow (y - 0) = \frac{-1}{1}(x - 0)$$

⇒ y + x=0

Hence option C is correct.

2. Question

The equation of the normal to the curve y = x + sin x cos x at $x = \frac{\pi}{2}$ is

A. x = 2

B. x = π

C. x + π = 0

D. 2x = π

Answer

Given that the curve y = x + sin x cos x

Differentiating both the sides w.r.t. x,

$$\frac{dy}{dx} = 1 + \cos^2 x - \sin^2 x$$

Now,

Slope of the tangent $\frac{dy}{dx}\left(x = \frac{\pi}{2}\right) = 1 + \cos^2\frac{\pi}{2} - \sin^2\frac{\pi}{2}$

$$\Rightarrow \frac{dy}{dx} = 1 - 1 + 0 = 0$$

When $x = \frac{\pi}{2}, y = \frac{\pi}{2}$

Equation of the normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow \left(y - \frac{\pi}{2}\right) = \frac{-1}{0}\left(x - \frac{\pi}{2}\right)$$

⇒ 2x = π

Hence option D is correct.

3. Question

The equation of the normal to the curve y = x (2 – x) at the point (2, 0) is

A. x – 2y - 2

B. x – 2y + 2 = 0

C. 2x + y = 4

D. 2x + y – 4 = 0

Answer

Given that y = x (2 – x)

⇒ y = 2x – x^2

Slope of the tangent $\frac{dy}{dx} = 2 - 2x$

Slope at (2, 0) = 2 – 4 = -2

Equation of normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow (y - 0) = \frac{-1}{-2}(x - 2)$$

⇒2y=x-2

⇒ x – 2y – 2 = 0

Hence option A is correct.

4. Question

The point on the curve $y^2 = x$ where tangent makes 45° angle with x-axis is

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$

B. $\left(\frac{1}{4}, \frac{1}{2}\right)$

C. (4, 2)

D. (1, 1)

Answer

Given that $y^2 = x$

The tangent makes 45° angle with x-axis.

So, slope of tangent = tan 45° = 1

∵ the point lies on the curve

∴ Slope of the curve at that point must be 1

$$2y\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$\Rightarrow \frac{1}{2y} = 1$$

$$\Rightarrow y = \frac{1}{2}$$

And $x = \frac{1}{4}$

So, the correct option is B.

5. Question

If the tangent to the curve $x = at^2$, y = 2at is perpendicular to x-axis, then its point of contact is

A. (a, a)

B. (0, a)

C. (0, 0)

D. (a, 0)

Answer

Given that the tangent to the curve $x = at^2$, $y = 2at$ is perpendicular to x-axis.

Differentiating both w.r.t. t,

$$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

From $y = 2at$, $t = \frac{y}{2a}$

$$\Rightarrow \text{Slope of the curve} = \frac{2a}{y}$$

Slope of x axis = 0

$$\Rightarrow \frac{2a}{y} = 0$$

$\Rightarrow a = 0$

Then point of contact is (0, 0).

6. Question

The point on the curve $y = x^2 - 3x + 2$ where tangent is perpendicular to $y = x$ is

A. (0, 2)

B. (1, 0)

C. (–1, 6)

D. (2, –2)

Answer

Given that the curve $y = x^2 - 3x + 2$ where tangent is perpendicular to $y = x$

Differentiating both w.r.t. x,

$$\frac{dy}{dx} = 1 \text{ and } \frac{dy}{dx} = 2x - 3$$

∵ the point lies on the curve and line both

Slope of the tangent = -1

$\Rightarrow 2x - 3 = -1$

$\Rightarrow x = 1$

And $y = 1-3+2$

$\Rightarrow y = 0$

So, the required point is (1, 0).

7. Question

The point on the curve $y^2 = x$ where tangent makes 45° angle with x-axis is

A. $\left(\frac{1}{2}, \frac{1}{4}\right)$

B. $\left(\frac{1}{4}, \frac{1}{2}\right)$

C. (4, 2)

D. (1, 1)

Answer

Given that $y^2 = x$

The tangent makes 45° angle with x-axis.

So, slope of tangent = tan 45° = 1

∵ the point lies on the curve

∴ Slope of the curve at that point must be 1

$2y\frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

$\Rightarrow \frac{1}{2y} = 1$

$\Rightarrow y = \frac{1}{2}$

And $x = \frac{1}{4}$

So, the correct option is B

8. Question

The point on the curve $y = 12x - x^2$ where the slope of the tangent is zero will be

A. (0, 0)

B. (2, 16)

C. (3, 9)

D. (6, 36)

Answer

Given that the curve $y = 12x - x^2$

The slope of the curve $\frac{dy}{dx} = 12 - 2x$

Given that the slope of the tangent = 0

⇒ 12 – 2x = 0

⇒ x = 6

So, y = 72 – 36

⇒ y = 36

So, the correct option is D.

9. Question

The angle between the curves $y^2 = x$ and $x^2 = y$ at (1, 1) is

A. $\tan^{-1}\frac{4}{3}$

B. $\tan^{-1}\frac{3}{4}$

C. 90°

D. 45°

Answer

Given two curves $y^2 = x$ and $x^2 = y$

Differentiating both the equations w.r.t. x,

$$\Rightarrow 2y\frac{dy}{dx} = 1 \text{ and } 2x = \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y} \text{ and } \frac{dy}{dx} = 2x$$

For (1, 1):

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \text{ and } \frac{dy}{dx} = 2$$

Thus we get

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{\frac{1}{2} - 2}{1 + 1}\right|$$

$$\Rightarrow \tan\theta = \frac{3}{4}$$

$$\Rightarrow \theta = \tan^{-1}\frac{3}{4}$$

10. Question

The equation of the normal to the curve $3x^2 - y^2 = 8$ which is parallel to $x + 3y = k$ is

A. x – 3y = 8

B. x – 3y + 8 = 0

C. x + 3y ± 8 = 0

D. x = 3y = 0

Answer

Given that the normal to the curve $3x^2 - y^2 = 8$ which is parallel to $x + 3y = k$.

Let (a, b) be the point of intersection of both the curve.

$\Rightarrow 3a^2 - b^2 = 8$ ....(1)

and a + 3b = k ....(2)

Now, $3x^2 - y^2 = 8$

On differentiating w.r.t. x,

$$6x - 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3x}{y}$$

Slope of the tangent at (a, b)$= \frac{3a}{b}$

Slope of the normal at (a, b)$= \frac{-b}{3a}$

Slope of normal = Slope of the line

$$\Rightarrow \frac{-b}{3a} = \frac{-1}{3}$$

⇒ b = a ....(3)

Put (3) in (1),

$3a^2 - a^2 = 8$

⇒ $2a^2=8$

⇒ a = ±2

Case: 1

When a = 2, b = 2

⇒ x + 3y =k

⇒ k = 8

Case: 2

When a = -2, b = -2

⇒ x + 3y =k

⇒ k = -8

From both the cases,

The equation of the normal to the curve $3x^2 - y^2 = 8$ which is parallel to x + 3y = k is x + 3y = ±8.

11. Question

The equation of tangent at those points where the curve $y = x^2 - 3x + 2$ meets x-axis are

A. x – y + 2 = 0 = x – y – 1

B. x + y – 1 = 0 = x – y – 2

C. x – y – 1 = 0 = x – y

D. x – y = 0 = x + y

Answer

Given that the curve $y = x^2 - 3x + 2$

$$\Rightarrow \frac{dy}{dx} = 2x - 3$$

The tangent passes through point (x, 0)

⇒ $0 = x^2 - 3x + 2$

⇒ (x-2)(x-1)=0

⇒ x = 1 or 2

Equation of the tangent:

$(y-y_1)$=Slope of tangent×$(x-x_1)$

Case: 1

When x = 2

Slope of tangent = 1

Equation of tangent:

y = 1× (x – 2)

⇒ x – y – 2 = 0

Case: 2

When x = 1

Slope of tangent = -1

Equation of tangent:

y = -1× (x – 1)

⇒ x + y – 1 = 0

Hence, option B is correct.

12. Question

The slope of the tangent to the curve x = t^2 + 3t – 8, y = $2t^2$ – 2t – 5 at point (2, –1) is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. –6

D. $\frac{7}{6}$

Answer

Given that x = t^2 + 3t – 8, y = $2t^2$ – 2t – 5

Differentiating both the sides,

$$\frac{dx}{dt} = 2t + 3, \frac{dy}{dt} = 4t - 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{4t - 2}{2t + 3}$$

The given point is (2, -1)

2 = t^2 + 3t – 8, -1 = $2t^2$ – 2t – 5

On solving we get,

t = 2 or -5 and t = 2 or -1

∵ t =2 is the common solution

So, $\frac{dy}{dx} = \frac{8-2}{4+3}$

$= \frac{6}{7}$

13. Question

At what points the slope of the tangent to the curve $x^2 + y^2 - 2x - 3 = 0$ is zero.

A. (3, 0), (–1, 0)

B. (3, 0), (1, 2)

C. (–1, 0), (1, 2)

D. (1, 2), (1, –2)

Answer

Given that the curve $x^2 + y^2 - 2x - 3 = 0$

Differentiation on both the sides,

$$2x + 2y\frac{dy}{dx} - 2 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - x}{y}$$

According to the question,

Slope of the tangent = 0

$$\Rightarrow \frac{1 - x}{y} = 0$$

⇒ x = 1

Putting this in equation of curve,

$1 + y^2 - 2 - 3 = 0$

$\Rightarrow y^2 = 4$

⇒ y = ±2

So, the required points are (1, 2) and (1, -2)

14. Question

The angle of intersection of the curves $xy = a^2$ and $x^2 - y^2 = 2a^2$ is:

A. 0°

B. 45°

C. 90°

D. 30°

Answer

Given that the curves $xy = a^2$ and $x^2 - y^2 = 2a^2$

Differentiating both of them w.r.t. x,

$$x\frac{dy}{dx}+y=0 \text{ and } 2x-2y\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-y}{x} \text{ and } \frac{dy}{dx}=\frac{x}{y}$$

Let $m_1=\frac{-y}{x}$ and $m_2=\frac{x}{y}$

$m_1 \times m_2=-1$

So, the angle between the curves is 90°.

15. Question

If the curve ay + x^2= 7 and x^3 = y cut orthogonally at (1, 1), then a is equal to

A. 1

B. –6

C. 6

D. 0

Answer

Given that the curves ay + x^2= 7 and x^3 = y

Differentiating both of them w.r.t. x,

$$a\frac{dy}{dx}+2x=0 \text{ and } 3x^2=\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2x}{a} \text{ and } \frac{dy}{dx}=3x^2$$

For (1, 1)

$$\frac{dy}{dx}=\frac{-2}{a} \text{ and } \frac{dy}{dx}=3$$

Let $m_1=\frac{-2}{a}$ and $m_2=3$

$m_1 \times m_2=-1$

(because curves cut each other orthogonally)

$$\Rightarrow \frac{-6}{a}=-1$$

⇒ a = 6

16. Question

If the line y = x touches the curve y = x^2 + bx + c at a point (1, 1) then

A. b = 1, c = 2

B. b = –1, c = 1

C. b = 2, c = 1

D. b = –2, c = 1

Answer

Given that line y = x touches the curve y = x^2 + bx + c at a point (1, 1)

Slope of line = 1

Slope of tangent to the curve = 1

$\Rightarrow \frac{dy}{dx} = 2x + b$

⇒ 2x+b = 1

⇒ 2 + b = 1

⇒ b = -1

Putting this and x =1 and y = 1 in the equation of the curve,

1 = 1 -1 + c

⇒ c = 1

17. Question

The slope of the tangent to the curve $x = 3t^2 + 1$, $y = t^3 - 1$ at x = 1 is

A. $\frac{1}{2}$

B. 0

C. –2

D. ∞

Answer

Given that $x = 3t^2 + 1$, $y = t^3 - 1$

For x = 1,

$3t^2 + 1=1$

$\Rightarrow 3t^2 = 0$

⇒ t =0

Now, differentiating both the equations w.r.t. t, we get

$\frac{dx}{dt} = 6t \text{ and } \frac{dy}{dt} = 3t^2$

⇒Slope of the curve:

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$= \frac{3t^2}{6t}$

$= \frac{1}{2}t$

For t =0,

Slope of the curve =0

Hence, option B is correct.

18. Question

The curves $y = ae^x$ and $y = be^{-x}$ cut orthogonally, if

A. a = b

B. a = –b

C. ab = 1

D. ab = 2

Answer

Given that the curves y = ae^{x} and y = be^{-x}

Differentiating both of them w.r.t. x,

$$\frac{dy}{dx} = ae^{x} \text{ and } \frac{dy}{dx} = -be^{-x}$$

Let m_1=ae^{x} and m_2=-be^{-x}

$m_1 \times m_2$=-1

(Because curves cut each other orthogonally)

⇒ -ab = -1

⇒ ab = 1

19. Question

The equation of the normal to the curve x = acos^{3} θ, y = a sin^{3}θ at the point $\theta = \frac{\pi}{4}$ is

A. x = 0

B. y = 0

C. x = y

D. x + y = a

Answer

Given that the curve x = acos^{3} θ, y = a sin^{3}θ have a normal at the point $\theta = \frac{\pi}{4}$

Differentiating both w.r.t. θ,

$$\frac{dx}{d\theta} = -3\cos^2\theta\sin\theta, \frac{dy}{d\theta} = 3a\sin^2\theta\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = -\tan\theta$$

For $\theta = \frac{\pi}{4}$

Slope of the tangent = -1

$$x = \frac{a}{2\sqrt{2}}, y = \frac{a}{2\sqrt{2}}$$

Equation of normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

x = y

20. Question

If the curves y = 2 e^{x} and y = ae^{-x} interest orthogonally, then a =

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C.2

D. $2e^2$

Answer

Given that the curves y = 2 e^x and y = ae^{-x}

Differentiating both of them w.r.t. x,

$\frac{dy}{dx} = 2e^x$ and $\frac{dy}{dx} = -ae^{-x}$

Let $m_1 = 2e^x$ and $m_2 = -ae^{-x}$

$m_1 \times m_2 = -1$

(Because curves cut each other orthogonally)

⇒ -2a = -1

$\Rightarrow a = \frac{1}{2}$

21. Question

The point on the curve y = 6x – x^2 at which the tangent to the curve is inclined at $\frac{\pi}{4}$ to the line x + y = 0 is

A. (–3, – 27)

B. (3, 9)

C. $\left(\frac{7}{2}, \frac{35}{4}\right)$

D. (0, 0)

Answer

The curve y = 6x – x^2 has a point at which the tangent to the curve is inclined at to $\frac{\pi}{4}$ the line x + y = 0.

Differentiating w.r.t. x,

$\frac{dy}{dx} = 6 - 2x = m_1$ and $\frac{dy}{dx} = -1 = m_2$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \tan\frac{\pi}{4} = \left|\frac{6 - 2x + 1}{1 + 2x - 6}\right|$

On solving we get x = 3

Thus y = 9

Hence, option B is correct.

22. Question

The angle of intersection of the parabolas $y^2 = 4\ ax$ and $x^2 = 4ay$ at the origin is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{4}$

Answer

Given that the the parabolas $y^2 = 4\ ax$ and $x^2 = 4ay$

Differentiating both w.r.t. x,

$2y\frac{dy}{dx} = 4a \text{ and } 2x = 4a\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{2a}{y} = m_1 \text{ and } \frac{dy}{dx} = \frac{x}{2a} = m_2$

At origin,

m_1=infinity and $m_2 = 0$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{\infty - 0}{1 + 0 \times \infty}\right| = \infty$

$\Rightarrow \theta = 90°$

23. Question

The angle of intersection of the curves $y = 2\sin^2 x$ and $y = \cos^2 x$ at $x = \frac{\pi}{6}$ is

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer

Given that the curve $y = 2\sin^2 x$ and $y = \cos^2 x$

Differentiating both w.r.t. x,

$\frac{dy}{dx} = 4\sin x\ \cos x \text{ and } \frac{dy}{dx} = -2\cos x \sin x$

$m_1 = 4\sin x\cos x$ and $m_2 = -2\cos x\sin x$

At $x = \frac{\pi}{6}$,

$m_1 = \sqrt{3}$ and $m_2 = -\frac{\sqrt{3}}{2}$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{\sqrt{3} + \frac{\sqrt{3}}{2}}{1 - \sqrt{3} \times \frac{\sqrt{3}}{2}}\right| = \frac{\frac{3\sqrt{3}}{2}}{\frac{1}{2}} = 3\sqrt{3}$$

$\Rightarrow \theta = \tan^{-1} 3\sqrt{3}$

24. Question

Any tangent to the curve $y = 2x^7 + 3x + 5$.

A. is parallel to x-axis

B. is parallel to y-axis

C. makes an acute angle with x-axis

D. makes an obtuse angle with x-axis

Answer

Given curve $y = 2x^7 + 3x + 5$.

Differentiating w.r.t. x,

$$\frac{dy}{dx} = 14x^6 + 3$$

Here $\frac{dy}{dx} \geq 3$

$$\Rightarrow \frac{dy}{dx} > 0$$

So, $\tan\theta > 0$

Hence, θ lies in first quadrant.

So, any tangent to this curve makes an acute angle with x-axis.

25. Question

The point on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts with the axes is

A. $\left(4, \pm\frac{8}{3}\right)$

B. $\left(-4, \frac{8}{3}\right)$

C. $\left(-4, -\frac{8}{3}\right)$

D. $\left(\frac{8}{3}, 4\right)$

Answer

Given curve $9y^2 = x^3$ ....(1)

Differentiate w.r.t. x,

$$18y\frac{dy}{dx} = 3x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{6y}$$

Equation of normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

∵ it makes equal intercepts with the axes

∴ slope of the normal = ±1

$\Rightarrow x^2 = \pm 6y$

Squaring both the sides,

$x^4 = \pm\ 36y^2$

From (1),

x= 0, 4

and $y = 0, \pm\frac{8}{3}$

But the line making equal intercept cannot pass through origin.

So, the required points are $\left(4, \pm\frac{8}{3}\right)$.

26. Question

The slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at the point (2, –1) is

A. $\frac{22}{7}$

B. $\frac{6}{7}$

C. $\frac{7}{6}$

D. $-\frac{6}{7}$

Answer

Given that $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$

Differentiating w.r.t. t,

$$\frac{dx}{dt} = 2t + 3, \frac{dy}{dt} = 4t - 2$$

$$\frac{dy}{dx} = \frac{4t - 2}{2t + 3}$$

For (2, -1),

The given point is (2, -1)

$2 = t^2 + 3t - 8$, $-1 = 2t^2 - 2t - 5$

On solving we get,

t = 2 or -5 and t = 2 or -1

∵ t =2 is the common solution

So, $\frac{dy}{dx} = \frac{8-2}{4+3} = \frac{6}{7}$

27. Question

The line y = mx + 1 is a tangent to the curve $y^2 = 4x$, if the value of m is

A. 1

B. 2

C. 3

D. $\frac{1}{2}$

Answer

It is given that the line y = mx + 1 is a tangent to the curve $y^2 = 4x$.

Slope of the line = m

Slope of the curve $\frac{dy}{dx}$,

Differentiating the curve we get

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

$$\Rightarrow \frac{2}{y} = m$$

$$\Rightarrow y = \frac{2}{m}$$

∵ The given line is a tangent to the curve so the point passes through both line and curve.

$$\Rightarrow y = mx + 1 \text{ and } y^2 = 4x$$

$$\Rightarrow \frac{2}{m} = mx + 1 \text{ and } \frac{4}{m^2} = 4x$$

$$\Rightarrow mx = \frac{2-m}{m} \text{ and } x = \frac{1}{m^2}$$

$$\Rightarrow x = \frac{2-m}{m^2} \text{ and } x = \frac{1}{m^2}$$

$$\Rightarrow \frac{2-m}{m^2} = \frac{1}{m^2}$$

⇒ 2 – m =1

⇒ m = 1

Hence, the correct option is A.

28. Question

The normal at the point (1, 1) on the curve $2y + x^2 = 3$ is

A. x + y = 0

B. x – y = 0

C. x + y + 1 = 0

D. x – y = 1

Answer

Given that the curve $2y + x^2 = 3$ has a normal passing through point (1, 1).

Differentiating both the sides w.r.t. x,

$$2\frac{dy}{dx} + 2x = 0$$

Slope of the tangent $\frac{dy}{dx} = -x$

For (1, 1):

$$\Rightarrow \frac{dy}{dx} = -1$$

Equation of the normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow (y - 1) = \frac{-1}{-1}(x - 1)$$

⇒ y – 1 = x - 1

⇒ y - x = 0

⇒ x – y = 0

Hence, option B is correct.

29. Question

The normal to the curve $x^2 = 4y$ passing through (1, 2) is

A. 2x + y = 4

B. x – y = 3

C. x + y = 1

D. x – y = 1

Answer

Given that the curve $x^2 = 4y$

Differentiating both the sides w.r.t. x,

$$4\frac{dy}{dx} = 2x$$

Slope of the tangent $\frac{dy}{dx} = \frac{1}{2}x$

For (1, 2):

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$$

Equation of the normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow (y - 2) = \frac{-2}{1}(x - 1)$$

⇒ y – 2 = -2x + 2

⇒ y + 2x = 4

No option matches the answer.

Very short answer

1. Question

Find the point on the curve $y = x^2 - 2x + 3$, where the tangent is parallel to x-axis.

Answer

Given curve $y = x^2 - 2x + 3$

We know that the slope of the x-axis is 0.

Let the required point be (a, b).

∵ the point lies on the given curve

∴ $b = a^2 - 2a + 3$ ….(1)

Now, $y = x^2 - 2x + 3$

$$\frac{dy}{dx} = 2x - 2$$

Slope of the tangent at (a, b) = 2a – 2

According to the question,

2a – 2 = 0

⇒ a = 1

Putting this in (1),

b = 1 – 2 +3

⇒ b = 2

So, the required point is (1, 2)

2. Question

Find the slope of the tangent to the curve $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$ at t = 2.

Answer

Given that $x = t^2 + 3t - 8$, $y = 2t^2 - 2t - 5$

$$\Rightarrow \frac{dx}{dt} = 2t + 3, \frac{dy}{dt} = 4t - 2$$

$$\therefore \frac{dy}{dx} = \frac{4t - 2}{2t + 3}$$

Now,

$$\text{Slope of the tangent (at } t = 2) = \frac{8 - 2}{4 + 3} = \frac{6}{7}$$

3. Question

If the tangent line at a point (x, y) on the curve y = f(x) is parallel to x-axis, then write the value of $\frac{dy}{dx}$.

Answer

Given curve y = f(x) has a point (x, y) which is parallel to x-axis.

We know that the slope of the x-axis is 0.

∵ the point lies on the given curve

∴ Slope of the tangent $\frac{dy}{dx} = 0$

4. Question

Write the value of $\frac{dy}{dx}$, if the normal to the curve y = f(x) at (x, y) is parallel to y-axis.

Answer

Given that the normal to the curve y = f(x) at (x, y) is parallel to y-axis.

We know that the slope of the y-axis is ∞.

∵ Slope of the normal = Slope of the y-axis = ∞

∴ Slope of the tangent $\frac{dy}{dx} = \frac{-1}{\text{Slope of the normal}} = \frac{1}{\infty} = 0$

5. Question

If the tangent to a curve at a point (x, y) is equally inclined to the coordinate axes, then write the value of $\frac{dy}{dx}$.

Answer

Given that the tangent to a curve at a point (x, y) is equally inclined to the coordinate axes.

⇒The angle made by the tangent with the axes can be ±45°.

∴ Slope of the tangent $\frac{dy}{dx} = \tan \pm 45° = \pm 1$

6. Question

If the tangent line at a point (x, y) on the curve y = f(x) is parallel to y-axis, find the value of $\frac{dx}{dy}$.

Answer

Given that the tangent line at a point (x, y) on the curve y = f(x) is || to y-axis.

Slope of the y-axis = ∞

∴ Slope of the tangent $\frac{dy}{dx} = \infty$

$$\frac{dx}{dy} = \frac{1}{\infty} = 0$$

7. Question

Find the slope of the normal at the point ‘t’ on the curve $x = \frac{1}{t}, y = t$.

Answer

Given that the curve $x = \frac{1}{t}, y = t$

$$\frac{dx}{dt} = \frac{-1}{t^2}, \frac{dy}{dt} = 1$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{\frac{-1}{t^2}} = -t^2$$

Now, $\text{Slope of tangent} = -t^2$

$$\text{Slope of normal} = \frac{-1}{\text{Slope of tangent}} = \frac{1}{t^2}$$

8. Question

Write the coordinates of the point on the curve $y^2 = x$ where the tangent line makes an angle $\frac{\pi}{4}$ with x-axis.

Answer

Given that the curve $y^2 = x$ has a point where the tangent line makes an angle $\frac{\pi}{4}$ with x-axis.

$\therefore$ Slope of the tangent $\frac{dy}{dx} = \tan 45° = 1$

$\because$ the point lies on the curve.

$y^2 = x$

$$\Rightarrow 2y\frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2y}$$

$$\Rightarrow \frac{1}{2y} = 1$$

$$\Rightarrow y = \frac{1}{2}$$

So, $x = \frac{1}{4}$

Hence, the required point is $(\frac{1}{4}, \frac{1}{2})$

9. Question

Write the angle made by the tangent to the curve $x = e^t \cos t$, $y = e^t \sin t$ at $t = \frac{\pi}{4}$ with the x-axis.

Answer

Given that the curve $x = e^t\cos t$, $y = e^t \sin t$

$$\frac{dx}{dt} = e^t \cos t - e^t \sin t \text{ and } \frac{dy}{dt} = e^t \sin t + e^t \cos t$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t \sin t + e^t \cos t}{e^t \cos t - e^t \sin t} = \frac{\sin t + \cos t}{\cos t - \sin t}$$

Now, for $t = \frac{\pi}{4}$

$$\frac{dy}{dx} = \frac{\sin\frac{\pi}{4} + \cos\frac{\pi}{4}}{\cos\frac{\pi}{4} - \sin\frac{\pi}{4}} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}} = \infty$$

Let θ be the angle made by the tangent with the x-axis.

∴ $\tan\theta = \infty$

$$\Rightarrow \theta = \frac{\pi}{2}$$

10. Question

Write the equation of the normal to the curve y = x + sin x cos x at $x = \frac{\pi}{2}$.

Answer

Given that the curve y = x + sin x cos x

Differentiating both the sides w.r.t. x,

$$\frac{dy}{dx} = 1 + \cos^2 x - \sin^2 x$$

Now,

Slope of the tangent $\frac{dy}{dx}\left(x = \frac{\pi}{2}\right) = 1 + \cos^2\frac{\pi}{2} - \sin^2\frac{\pi}{2}$

$$\Rightarrow \frac{dy}{dx} = 1 - 1 + 0 = 0$$

When $x = \frac{\pi}{2}, y = \frac{\pi}{2}$

Equation of the normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow \left(y - \frac{\pi}{2}\right) = \frac{-1}{0}\left(x - \frac{\pi}{2}\right)$$

⇒ 2x = π

11. Question

Find the coordinates of the point on the curve $y^2 = 3 - 4x$ where tangent is parallel to the line 2x + y – 2 = 0.

Answer

Given that the curve $y^2 = 3 - 4x$ has a point where tangent is || to the line 2x + y – 2 =0.

Slope of the given line is -2.

∵ the point lies on the curve

∴ $y^2 = 3 - 4x$

$$\Rightarrow 2y\frac{dy}{dx} = -4$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2}{y}$$

Now, the slope of the curve = slope of the line

$$\Rightarrow \frac{-2}{y} = -2$$

⇒ y = 1

Putting above value in the equation of the line,

2x + 1 – 2 =0

⇒ 2x – 1 =0

$\Rightarrow\ x = \frac{1}{2}$

So, the required coordinate is $\left(\frac{1}{2}, 1\right)$.

12. Question

Write the equation of the tangent to the curve y = x^2 – x + 2 at the point where it crosses the y-axis.

Answer

Given that the curve y = x^2 – x + 2 has a point crosses the y-axis.

The curve will be in the form of (0, y)

⇒ y = 0- 0 + 2

⇒ y = 2

So, the point at which curve crosses the y-axis is (0, 2).

Now, differentiating the equation of curve w.r.t. x

$\frac{dy}{dx} = x - 1$

For (0, 2),

$\frac{dy}{dx} = -1$

Equation of the tangent:

$(y - y_1) = \text{Slope of tangent} \times (x - x_1)$

⇒ (y – 2) = -1× (x – 0)

⇒ y – 2 = -x

⇒ x + y = 2

13. Question

Write the angle between the curves y^2 = 4x and x^2 = 2y – 3 at the point (1, 2).

Answer

Given two curves y^2 = 4x and x^2 = 2y – 3

Differentiating both the equations w.r.t. x,

$\Rightarrow 2y\frac{dy}{dx} = 4 \text{ and } 2x = 2\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y} \text{ and } \frac{dy}{dx} = x$

For (1, 2):

$\Rightarrow \frac{dy}{dx} = \frac{2}{2} = 1 \text{ and } \frac{dy}{dx} = 1$

Thus we get

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{1-1}{1+1}\right|$$

⇒ tan θ=0

⇒ θ=0°

14. Question

Write the angle between the curves y = e^{-x} and y = e^{x} at their point of intersection.

Answer

Given that y = e^{-x} ...(1) and y = e^{x} ....(2)

Substituting the value of y in (1),

$e^{-x} = e^{x}$

⇒ x = 0

And y = 1 (from 2)

On differentiating (1) w.r.t. x, we get

$$\frac{dy}{dx} = -e^{-x}$$

$$\Rightarrow\ m_1 = \frac{dy}{dx} = -1$$

On differentiating (2) w.r.t. x, we get

$$\frac{dy}{dx} = e^{-x}$$

$$\Rightarrow\ m_2 = \frac{dy}{dx} = 1$$

∵ $m_1 \times m_2$ = -1

Since the multiplication of both the slopes is -1 so the slopes are perpendicular to each other.

∴ Required angle = 90°

15. Question

Write the slope of the normal to the curve $y = \frac{1}{x}$ at the point $\left(3, \frac{1}{3}\right)$.

Answer

Given that $y = \frac{1}{x}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = -\frac{1}{x^2}$$

Now, slope of the tangent at $(3, \frac{1}{3})$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{9}$$

Slope of normal = 9

16. Question

Write the coordinates of the point at which the tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line $y = 3x + 9$.

Answer

Let (a, b) be the required coordinate.

Given that the tangent to the curve $y = 2x^2 - x + 1$ is parallel to the line $y = 3x + 9$.

Slope of the line = 3

∵ the point lies on the curve

$\Rightarrow b = 2a^2 - a + 1$ … (1)

Now, $y = 2x^2 - x + 1$

$$\Rightarrow \frac{dy}{dx} = 4x - 1$$

Now value of slope at (a, b)

$$\Rightarrow \frac{dy}{dx} = 4a - 1$$

Given that Slope of tangent = Slope of line

$\Rightarrow 4a - 1 = 3$

$\Rightarrow 4a = 4$

$\Rightarrow a = 1$

From (1),

$b = 2 - 1 + 1$

$\Rightarrow b = 2$

17. Question

Write the equation of the normal to the curve $y = \cos x$ at (0, 1).

Answer

Given that $y = \cos x$

On differentiating both the sides w.r.t. x

$$\frac{dy}{dx} = -\sin x$$

Now,

Slope of tangent at (0, 1) = 0

Equation of normal:

$$(y - y_1) = \frac{-1}{\text{Slope of tangent}}(x - x_1)$$

$$\Rightarrow (y - 1) = \frac{-1}{0}(x - 0)$$

$\Rightarrow x = 0$

18. Question

Write the equation of the tangent drawn to the curve $y = \sin x$ at the point (0, 0).

Answer

Given that y = sin x

The slope of the tangent:

$$\frac{dy}{dx} = \cos x$$

For origin (a, b) slope = cos 0 = 1

Equation of the tangent:

$$(y - y_1) = \text{Slope of tangent} \times (x - x_1)$$

So, the equation of the tangent at the point (0, 0)

y-0 = 1(x-0)

⇒ y = x