16. Surface Areas and Volumes

Exercise 16.1

1. Question

How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm?

Answer

Sum of volumes of balls that can be made from a large sphere will have same volume, as that of the sphere.

As we know that, volume of a sphere = $\frac{4}{3}\pi r^3$. Given that, radius of large sphere, r_1 is 8 cm and that of the small spheres, r_2 is 1 cm.

Therefore, number of small balls that can be made from the big sphere

$$= \frac{volume\ of\ big\ sphere}{volume\ of\ one\ small\ sphere}$$

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{8^3}{1^3} = 512 \text{ spheres}$$

2. Question

How many spherical bullets each of 5 cm in diameter can be cast from a rectangular block of metal 11 dm × 1 m × 5 dm?

Answer

Sum of volumes of spherical bullets that can be cast from a rectangular block of metal will have same volume, as that of the block.

As we know that, volume of the rectangular block of metal=lbh. Also, volume of spherical bullets = $\frac{4}{3}\pi r^3$

Given that, dimension of rectangular block of metal is 11 dm × 1 m × 5 dm = 110 cm × 100 cm × 50 cm and diameter of spherical bullets = 5cm (radius = $\frac{d}{2} = \frac{5}{2}$cm)

Therefore, number of small bullets that can be made from the block

$$= \frac{volume\ of\ rectangular\ block}{volume\ of\ one\ small\ spherical\ bullet}$$

$$= \frac{110 \times 100 \times 50}{\frac{4}{3}\pi\left(\frac{5}{2}\right)^3}$$

$$= \frac{110 \times 25 \times 50 \times 3 \times 8 \times 7}{22 \times 125}$$

$$= 50 \times 3 \times 8 \times 7 = 8400$$

3. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of the two of the balls are 1.5 cm and 2 cm respectively. Determine the diameter of the third ball.

Answer

Given that, radius of spherical ball = 3 cm

We know that, Volume of the sphere = $\frac{4}{3}\pi r^3$

So, its volume, v = $\frac{4}{3}\pi(3)^3$

The spherical ball of radius 3 cm is melted and recast into three spherical balls.

Let the radius of the third ball be r.

Then, volume of third spherical ball = $\frac{4}{3}\pi(r)^3$

Volume of first ball = $\frac{4}{3}\pi(1.5)^3$

Volume of second ball = $\frac{4}{3}\pi(2)^3$

The volume of the original spherical ball is equal to that of the total volumes of the three balls.

$$\Rightarrow \frac{4}{3}\pi(1.5)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi(r)^3 = \frac{4}{3}\pi(3)^3$$

$$\Rightarrow (1.5)^3 + (2)^3 + (r)^3 = (3)^3$$

$$\Rightarrow (r)^3 = (3)^3 - (1.5)^3 - (2)^3$$

$$\Rightarrow r = \sqrt[3]{27 - 3.375 - 8}$$

$$\Rightarrow r = \sqrt[3]{15.625}$$

$$\Rightarrow r = 2.5 \text{ cm}$$

Thus, diameter = 5 cm

4. Question

2.2 cubic dm of brass is to be drawn into a cylindrical wire 0.25 cm in diameter. Find the length of the wire.

Answer

Given: 2.2 dm^3 of brass is to be drawn into a cylindrical wire 0.25 cm in diameter.

Diameter of cylindrical wire = 0.25cm

Radius of wire, r = $\frac{d}{2} = \frac{0.25}{2} = 0.125cm$

Let length of wire be h

Volume of the cylinder = пr^{2}h

Volume of brass of 2.2dm^3 is equal to volume of cylindrical wire

$$\frac{22}{7}(0.125 \times 10^{-3}$$

$$\Rightarrow h = \frac{2.2 \times 10^{-3} \times 7}{22(0.125 \times 10^{-2})^2}$$

⇒ h = 448m

5. Question

What length of a solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of length 16 cm, external diameter 20 cm and thickness 2.5 mm?

Answer

The solid cylinder should be to hollow cylinder

Given that, diameter of solid cylinder = 2 cm

Length of hollow cylinder = 16cm

External diameter = 20cm

Thickness = 2.5mm=0.25cm

Volume of solid cylinder = пr^{2}h ------(i)

Radius of the cylinder = 1cm

So, volume of the solid cylinder = п(1)^{2}h = п h

Let length of the solid cylinder be h

Volume of hollow cylinder = пh(R^2 – r^2)

Thickness = R – r

⇒ r = 10 – 0.25

⇒ Internal radius = 9.75cm

So, volume of hollow cylinder = п × 16(100 − 95.0625)

= п × 16 × 4.9375

≈ п × 16 × 4.94 -------(2)

As we know that, volume of both solid and hollow cylinder must be the same, therefore

п h = п × 16 × 4.94

⇒ h ≈ 79cm

∴ Length of the solid cylinder = 79cm

6. Question

A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm which are filled completely. Find the diameter of the cylindrical vessel.

Answer

Given: diameter of the cylindrical vessel = height of the vessel

∴ h = 2r

Given: water filled in the cylindrical vessel is poured in two cylindrical vessels

Volume of a cylindrical vessel = п r^2 h

So, volume = пr^2(2r) = 2 п r^3

Volume of vessel with diameter 42 cm = $\pi \times \left(\frac{42}{2}\right)^2 \times 21$

= п $(21)^3$ cm^3

Since cylinders are filled by water completely, so volume of large cylinder = sum of these small cylinders

∴ 2 п r^3 = 2 × п $(21)^3$

⇒ r = 21cm

∴ diameter of the cylindrical vessel = 42cm

7. Question

50 circular plates each of diameter 14 cm and thickness 0.5 cm are placed one above the other to form a right circular cylinder. Find its total surface area.

Answer

Diameter of each circular plate = 14 cm

∴ their radius, r = 7 cm

Thickness of each plate = 0.5 cm

50 plates are placed one above the other to form a right circular cylinder

∴ height of the cylinder formed, h = 50 × 0.5 cm

Or h = 25cm

Total surface area of the cylinder = 2πr(r + h)

$$= 2 \times \frac{22}{7} \times 7(7 + 25)$$

= 1408 cm²

8. Question

25 circular plates, each of radius 10.5 cm and thickness 1.6 cm, are placed one above the other to form a solid circular cylinder. Find the curved surface area and the volume of the cylinder so formed.

Answer

Radius of each circular plate = 10.5 cm

Thickness of each plate = 1.6 cm

25 plates are placed one above the other to form a solid circular cylinder

∴ height of the cylinder formed, h = 25 × 1.6 cm = 40cm

Total curved surface area of the cylinder = 2πrh

$$= 2 \times \frac{22}{7} \times 10.5 \times 40$$

= 2640 cm²

Volume of cylinder = π r² h

= π (10.5)² × 40

= 13860 cm³

9. Question

A path 2 m wide surrounds a circular pond of diameter 40 m. How many cubic metres of gravel are required to grave the path to a depth of 20 cm?

Answer

Diameter of circular pond is given as 40 cm

∴ Radius of the pond, r = 20cm

Also, thickness, t = R − r

⇒ 2 = R − r

⇒ 2 = R − 20

⇒ R = 22cm

Volume of a hollow cylinder = π (R^2 – r^2)h

= 3.14 × (22^2 – 20^2) × 0.2

≈ 52 .8

∴ 52.8 cubic metres of gravel are required to grave the path to a depth of 20 cm

10. Question

A 16 m deep well with diameter 3.5 m is dug up and the earth from it is spread evenly to form a platform 27.5 m by 7 m. Find the height of the platform.

Answer

Let us assume shape of well is like a solid right circular cylinder

Radius of cylinder, r = 3.5/2 = 1.75m

Depth of well, h = 16m

Volume of right circular cylinder, V' = π r^2 h

$$= \frac{22}{7} \times 1.75^2 \times 16$$ ---------(I)

Given that, length of platform, l = 27.5 cm

Breadth of the platform, b = 7cm

Let height of the platform be x m

Volume of the rectangle, V'' = lbh

= 27.5 × 7 × x

= 192.5x -----------(II)

Since well is spread evenly to form the platform, so volume V' = V''

$$\Rightarrow \frac{22}{7} (1.75)^2 \times 16 = 192.5x$$

⇒ x = 0.8m

∴ Height of the platform = 80cm

11. Question

A well of diameter 2 m is dug 14 m deep. The earth taken out of it is spread evenly all around it to form and embankment of height 40 cm. Find the width of the embankment.

Answer

Let us assume shape of well is like a solid circular cylinder

Radius of cylinder, r = 2/2 = 1m

Depth(height) of well, h = 14m

Volume of right circular cylinder, V' = π r² h

$$= \pi \times 1^2 \times 14 \text{ ---------(I)}$$

Given that, length of embankment, l = 40 cm

Let width of the embankment be x m

Volume of the embankment, V'' = π r² h

$$= \pi ((1+x)^2 - 1)^2 \times 0.4 \text{ -----------(II)}$$

Since well is spread evenly to form the embankment, so their volumes, V' = V''

$$\Rightarrow \pi \times 14 = \pi ((1+x)^2 - 1)^2 \times 0.4$$

$$\Rightarrow x = 5m$$

∴ Height of the embankment, x = 5cm

12. Question

Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9 cm.

Answer

Given that, side of the cube = 9cmLet radius of cone be x.

Since largest cone is curved from cube.

Diameter of base of cone = side of cube

$$\Rightarrow 2x = 9$$

$$\Rightarrow x = \frac{9}{2} \text{ cm}$$

since sides of cube are equal and cone is cut from the cube.Maximum height of the cone can be 9 cm.So,

Height of cone is: h = 9 cm

Volume of largest cone = $\frac{1}{3}\pi r^2 h$

$$V = \frac{1}{3}\pi \left(\frac{9}{2}\right)^2 \times 9$$

$$V = \frac{\pi}{12} \times 9^3$$

$$\Rightarrow V = \frac{22 \times 9 \times 9 \times 9}{7 \times 12}$$

$$\Rightarrow V = \frac{22^{11} \times 9 \times 9 \times 9}{7 \times 12_6}$$

$$\Rightarrow V = \frac{11 \times 9 \times 9 \times 9}{7 \times 6}$$

$$\Rightarrow V = \frac{11 \times \cancel{9}^3 \times 9 \times 9}{7 \times \cancel{6}_2}$$

$$\Rightarrow V = \frac{11 \times 3 \times 9 \times 9}{7 \times 2}$$

$$\Rightarrow V = \frac{2673}{14}$$

V = 190.92 cm³

∴ volume of largest cone, v = 190.92 cm³

13. Question

A cylindrical bucket, 32 cm high and 18 cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer

Volume of cylindrical bucket = π r²h

Here, r = 18cm and h = 32cm

So, volume of cylindrical bucket = π (18)²(32) cm²

When emptied on ground, volume of conical heap of sand = volume of cylindrical bucket

$$\therefore \frac{1}{3} \pi r^2 \times 24 = \pi \times 18^2 \times 32$$

$$\Rightarrow r = \sqrt{\frac{3 \times 18^2 \times 32}{24}} = 36cm$$

Now, $l = \sqrt{h^2 + r^2} = \sqrt{24^2 + 36^2} \approx 43.27$

14. Question

Rain water, which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm. What will be the height of water in the cylindrical vessel if a rainfall of 1 cm has fallen? $\left[Use \ \pi = 22/7 \right]$

Answer

Volume of rainfall = lbh

= 6 × 4 × 0.01

= 0.24 m³

∵ Volume of water in cylindrical vessel = volume of rainfall

∴ πr²h = 0.24

$$\Rightarrow h = \frac{0.24 \times 7}{22 \times 0.2^2}$$

⇒ h ≈ 1.91m = 191cm

15. Question

A conical flask is full of water. The flask has base-radius r and height h. The water is poured into a cylindrical flask of base-radius rm. Find the height of water in the cylindrical flask.

Answer

Volume of conical flask = $\frac{1}{3}$ πr²h

∵ Volume of water in cylindrical flask = volume of conical flask

∴ π(rm)²h′ = $\frac{1}{3}$ πr²h

$$\Rightarrow h' = \frac{1}{3}\left(\frac{r}{rm}\right)^2 h$$

$$\Rightarrow h' = \frac{h}{3m^2}$$

16. Question

A rectangular tank 15 m long and 11 m broad is required to receive entire liquid contents from a full cylindrical tank of internal diameter 21 m and length 5 m. find the least height of the tank that will serve the purpose.

Answer

Volume of liquid in the new rectangular tank = volume of liquid filled in full cylindrical tank

lbh = π r² h′

$$\Rightarrow h = \frac{\pi r^2 h'}{lb}$$

Here, r = $\frac{21}{2}$ m, h′ = 5m, l = 15m and b = 11m, then

$$h = \frac{\frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 5}{15 \times 11} = 10.5m$$

17. Question

A hemispherical bowl of internal radius 9 cm is full of liquid. This liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are necessary to

empty the bowl?

Answer

Number of bottles = (volume of cylindrical shaped small bottles/ Internal volume of hemispherical bowl)

$$\Rightarrow n = \frac{\frac{2}{3}\pi r'^3}{\pi r''^2 h}$$

$$\Rightarrow n = \frac{2}{3} \times \frac{r'^3}{r''^2 h}$$

$$\Rightarrow n = \frac{2}{3} \times \frac{9^3}{\left(\frac{3}{2}\right)^2 \times 4} = 54$$

18. Question

The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder.

Answer

Internal radius of the hollow spherical shell = 6/2 = 3cm

And external radius of that shell = 10/2 = 5cm

Radius of the cylinder = 14/2 = 7

Let height of the cylinder be x cm

According to the question,

Volume of the cylinder = volume of the spherical shell

$$\pi(7)^2 \times x = \frac{4}{3} \times \pi \times (5^3 - 3^3)$$

$$\Rightarrow 49 \times x = \frac{4}{3}(125 - 27)$$

$$\Rightarrow 49 \times x = \frac{4}{3} \times 98$$

$$\Rightarrow x = \frac{4 \times 98}{3 \times 49} = \frac{8}{3} cm$$

∴ height of cylinder = $\frac{8}{3}$ cm

19. Question

A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Calculate the height of the cone.

Answer

Given, internal radius of hollow sphere, r = (4/2) = 2cm

External radius = 8/2 = 4cm

Volume of hollow sphere = $\frac{4}{3}\pi(R^2 - r^2)$

$= \frac{4}{3}\pi(8^2 - 4^2)$ --------(i)

Given, diameter of the cone = 8cm

∴ Radius of the cone = 4 cm

Let height of the cone be h

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi 4^2 h$ --------(ii)

Since hollow sphere is melted into a cone so their volumes are equal,

$\Rightarrow \frac{4}{3}\pi(64 - 16) = \frac{1}{3}\pi 4^2 h$

∴ h = 12cm

20. Question

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical ball is dropped into the tub and the level of the water is raised by 6.75 cm. Find the radius of the ball.

Answer

Given, the radius of the cylindrical tube, r = 12cm

Level of water raised in tube, h = 6.75cm

Volume of cylinder = πr² h

= π (12)² × 6.75 cm³

$= \pi \times 12^2 \times 6.75\ cm^3$ ----------(i)

let r be the radius of a spherical shell balls

volume of the sphere = $\frac{4}{3}\pi r^3$ -------(ii)

∵ volume of cylinder = volume of spherical ball

∴ $\pi \times 12^2 \times 6.75 = \frac{4}{3}\pi r^3$

$$\Rightarrow r^3 = \frac{12^2 \times 6.75}{\frac{4}{3} \times \pi}$$

$\Rightarrow$ r = 9cm

$\therefore$ radius of spherical ball, r = 9cm

21. Question

500 persons have to dip in a rectangular tank which is 80 m long and 50 m broad. What is the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m³?

Answer

To find: the rise in the level of water in the tank, if the average displacement of water by a person is 0.04 m³ and 500 persons have to dip in a rectangular tank which is 80 m long and 50 m broad.**Solution:** Length of rectangular tank, l = 80m

Breadth, b = 50m

displacement of water in rectangular tank by 1 person = 0.04 m³

Total displacement of water in rectangular tank by 500 persons = 500 × 0.04 m³ = 20m³ -----(i)

Let depth of the rectangular tank be hThe amount of water displaced is equal to the volume of the tank.

Volume of rectangular tank = lbh

Volume of rectangular tank = 80 × 50 × h m³ -----------(ii)

Equating (i) and (ii), we get

20 = 80 × 50 × h

$\Rightarrow$ h = 0.005 m As 1 m = 100 cmSo 0.005 m = 0.005 × 100 cm$\Rightarrow$ h = 0.5 cm

$\therefore$ rise in level of water due to angular displacement = 0.5cm

22. Question

A cylindrical jar of radius 6 cm contains oil. Iron spheres each of radius 1.5 cm are immersed in the oil. How many spheres are necessary to raise the level of the oil by two centimeters?

Answer

Given that, radius of cylindrical jar, r = 6cm

Depth/height of the jar, h = 2cm

Volume of the jar, V' = π r² h

$\Rightarrow$ V' = π (6)² (2) cm³

Radius of the sphere = 1.5cm

So, volume of the sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.5)^3$

Volume of oil in cylindrical jar = volume of n spheres needed to raise the level by 2 cm

$$\therefore \pi (6)^2 (2) = n \times \frac{4}{3}\pi(1.5)^3$$

$\Rightarrow n = 16$

$\therefore$ number of iron sphere needed = 16

23. Question

A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

Answer

Volume of hollow sphere before melting = volume of cone

$$\Rightarrow \frac{4}{3}\pi(R^3 - r'^3) = \frac{1}{3} \times \pi \times r''^2 \times h''$$

Here, R = 4cm, r = 2cm, r'' = 4cm, therefore

$$\Rightarrow \frac{4}{3}\pi(4^3 - 2^3) = \frac{1}{3} \times \pi \times 4^2 \times h''$$

$\Rightarrow h'' = 14$cm = height of the cone

Now, slant height, l = $\sqrt{h''^2 + r''^2}$

$= \sqrt{14^2 + 4^2}$

= 14.56 cm

24. Question

The internal and external diameters of a hollow hemispherical vessel are 21 cm and 25.2 cm respectively. The cost of painting 1 cm^2 of the surface is 10 paise. Find the total cost to paint the vessel all over.

Answer

Given that, internal diameter of hollow hemisphere, r = 10.5cm

External radius, R = 12.6cm

Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$

$= 2\pi(12.6)^2 + 2\pi(10.5)^2 + \pi((12.6)^2 - (10.5)^2)$

$= 1843.38$ cm^2

Given that, cost of painting 1 cm^2 of surface = 10paise

Therefore, total cost to paint 1843.38 cm^2 = $\frac{1843.38 \times 10}{100} = Rs184.34$

25. Question

A cylindrical tub of radius 12 cm contains water to a depth of 20 cm. A spherical form ball of radius 9 cm is dropped into the tub and thus the level of water is raised by h cm. What is the value of h?

Answer

Given that, radius of cylindrical tube = 12cm

Let height of cylindrical tube be h

Volume of the cylinder = $\pi r_1^2 h = \pi (12)^2 h$ --------(i)

Also given that, spherical ball radius (r_2) = 9cm

Volume of sphere = $\frac{4}{3}\pi r_1^3$

$= \frac{4}{3}\pi 9^3$ ---------(ii)

Equating (i) and (ii), we get

$\Rightarrow \pi 12^2 h = \frac{4}{3}\pi 9^3$

$\Rightarrow$ h = 6.75cm

∴ level of water raised in tube = 6.75cm

26. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

Answer

Let radius of the third ball be r.

Volume of the larger ball = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi 3^3$

Volume of larger ball = sum of volume of smaller balls

$\Rightarrow \frac{4}{3}\pi 3^3 = \frac{4}{3}\pi(1.5)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = 27 - 3.375 - 8$

$\Rightarrow$ r = 2.5cm

27. Question

A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

Answer

Let the radius of third sphere be r_3

Volume of big spherical ball=volume of all three balls

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1{}^3 + \frac{4}{3}\pi r_2{}^3 + \frac{4}{3}\pi r_3{}^3$$

$\Rightarrow R^3 = r_1{}^3 + r_2{}^3 + r_3{}^3$

$\Rightarrow 3 \times 3 \times 3 = (1.5 \times 1.5 \times 1.5) + (2 \times 2 \times 2) + r_3{}^3$

$\Rightarrow r_3{}^3 = 27 - (3.375 + 8)$

$\Rightarrow r_3{}^3 = 15.625$

$\Rightarrow r_3 = \sqrt[3]{15.625}$

$\Rightarrow r_3 = 2.5$ cm

27. Question

Prove that the surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

Answer

Curved surface area of sphere$= 4\pi r^2$

Let 'r' be the radius of sphere

Height of cylinder='2r'

Curved surface area of cylinder$= 2\pi r\, h$

$\Rightarrow 2\pi r(2r)$

$\Rightarrow 4\pi r^2$

∴ Surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

28. Question

The diameter of a metallic sphere is equal to 9 cm. It is melted and drawn into a long wire of diameter 2 mm having uniform cross-section. Find the length of the wire.

Answer

Volume of the metallic sphere=volume of the wire(cylinder in shape)

Radius of sphere $= \frac{9}{2}$ cm

=4.5 cm

Volume of sphere $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi 4.5^3$$

Radius of cylinder $= \frac{2}{2} = 1mm = 0.1cm$

Volume of cylinder $= \pi r^2h$

$\Rightarrow \pi r^2h = \frac{4}{3}\pi R^3$

$\Rightarrow r^2h = \frac{4}{3}R^3$

$\Rightarrow 0.1 \times 0.1 \times h = \frac{4}{3} \times 4.5 \times 4.5 \times 4.5$

$\Rightarrow 0.1 \times 0.1 \times h = 4 \times 1.5 \times 4.5 \times 4.5$

$\Rightarrow h = 12150$ cm

29. Question

An iron spherical ball has been melted and recast into smaller balls of equal size. If the radius of each of the smaller balls is 1/4 of the radius of the original ball, how many such balls are made? Compare the surface area, of all the smaller balls combined together with that of the original ball.

Answer

Let the radius of bigger spherical be R

Volume of bigger spherical ball $= \frac{4}{3}\pi R^3$

Radius of smaller spherical ball $= \frac{1}{4}R$

Volume of smaller ball $= \frac{4}{3}\pi \left(\frac{1}{4}R\right)^3$

Let number of equal size spherical balls be n

Volume of n equal spherical ball = Volume of bigger spherical ball

$$\Rightarrow n \times \frac{4}{3}\pi \left(\frac{1}{4}R\right)^3 = \frac{4}{3}\pi R^3$$

$\Rightarrow n \times (R/4)^3 = R^3$

$\Rightarrow n = 4^3$

$\Rightarrow n = 64$ balls

Surface area of bigger spherical ball $= 4\pi R^2$

Surface area of smaller spherical ball $= 4\pi \left(\frac{1}{4}R\right)^2$

Ratio between the surface area of bigger and 64 smaller spherical ball

$$\Rightarrow \frac{64 \times 4\pi(\frac{1}{4}R)^2}{4\pi R^2} = 4$$

30. Question

A tent of height 77 dm is in the form a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m². $\left[Use\ \pi = 22\ /\ 7 \right]$

Answer

Height of tent =77dm

=7.7m

Radius of the cylinder $= \frac{36}{2}$ =18 m

Height of cylinder =44dm =4.4m

Height of cone = (7.7-4.4) m

=3.3m

Curved Surface area of cylinder portion of the tent $= 2\pi \times r \times h$

= 2 × (22/7) × 18 × 4.4

= 497.83 m²

Curved surface area of conical tent =п r l

Slant height of cone $= \sqrt{(3.3^2 + 18^2)}$

= 18.3m

Curved surface area of cone = (22/7) × 18 × 18.3

= 1035.26m²

Total surface area of the tent = 497.83m² + 1035.26m²

= 1533.09m²

Total cost of canvas = 3.50 × 1533.09

= Rs 5365.82

31. Question

Metal spheres, each of radius 2 cm, are packed into a rectangular box of internal dimension 16 cm × 8 cm × 8 cm when 16 spheres are packed the box is filled with preservative liquid. Find the volume of this liquid. $\left[Use\ \pi = 669\ /\ 213 \right]$

Answer

Volume of sphere $= \frac{4}{3} \pi R^3$

$$= \frac{4 \times 669 \times 8}{3 \times 213}$$

= 33.50 cm^3

Volume of 16 spheres = 16 × 33.50

= 536cm^3

Volume of rectangle = 16cm × 8cm × 8cm

= 1024cm^3

Volume of this liquid = 1024cm^3 – 536cm^3

= 488cm^3

32. Question

The largest sphere is to be curved out of a right circular cylinder of radius 7 cm and height 14 cm. Find the volume of the sphere.

Answer

Diameter of the sphere =Height of the cylinder

Diameter of the sphere =14 cm

Radius of the sphere $= \frac{14}{2}$ cm

=7 cm

Volume of the sphere $= \frac{4}{3} \pi R^3$

$$= \frac{4 \times 22 \times 7 \times 7 \times 7}{3 \times 7}$$

= 1437.3cm^3

33. Question

A copper sphere of radius 3 cm is melted and recast into a right circular cone of height 3 cm. Find the radius of the base of the cone.

Answer

Volume of the sphere $= \frac{4}{3} \pi R^3$

$$= \frac{4}{3} \pi 3^3$$

Volume of the cone $= \frac{1}{3} \pi r^2 h$

$$= \frac{1}{3} \pi r^2 \times 3$$

Volume of the cone = volume of the sphere

$$\frac{1}{3}\pi r^2 \times 3 = \frac{4}{3}\pi \, 3^3$$

$$\Rightarrow \frac{1}{3} \times r^2 \times 3 = \frac{4}{3} \times 3^3$$

$$\Rightarrow r^2 = \frac{4 \times 3 \times 3 \times 3 \times 3}{3 \times 3}$$

$\Rightarrow r^2 = 36$ cm

$\Rightarrow r = 6$ cm

34. Question

A vessel in the shape of a cuboid contains some water. If three identical spheres are immersed in the water, the level of water is increased by 2 cm. If the area of the base of the cuboid is 160 cm^2 and its height 12 cm, determine the radius of any of the spheres.

Answer

Given that area of cuboid =160 cm^2

Level of water is increased = 2 cm

Volume of vessel =160× 2 cm^3 ……. (1)

Volume of each sphere $= \frac{4}{3}\pi R^3$

Volume of 3 sphere $= 3 \times \frac{4}{3}\pi R^3$ ……. (2)

From eq 1 and eq 2

$\Rightarrow 3 \times (4/3)\, \pi R^3 = 160 \times 2$

$$\Rightarrow R^3 = \frac{160 \times 2}{3 \times \frac{4}{3}\pi}$$

$\Rightarrow R = 2.94$ cm

35. Question

A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

Answer

Given: A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness.

To find: the thickness of the wire.

Solution: Diameter of copper wire =1 cm

Radius $= \frac{1}{2}$ cm

Height of copper rod =Length of copper rod =8 cm

Volume of rod $= \pi r^2 h$

$$= \pi \times (\frac{1}{2})^2 \times 8$$

Let the radius of wire be 'r' cm

As 1 m = 100 cm

length of the wire=Height of the wire =18 m =1800 cmAs rod is converted into wire,Therefore,

Volume of cylinder wire = Volume of cylindrical rod

$$\Rightarrow \pi r^2 h = \pi \times \left(\frac{1}{2}\right)^2 \times 8$$

$$\Rightarrow r^2 = \frac{1}{900}$$

$$\Rightarrow r^2 = \frac{8}{2 \times 2 \times 1800}$$

$$\Rightarrow r^2 = \frac{\cancel{8}^2}{\cancel{4}_1 \times 1800}$$

$$\Rightarrow r^2 = \frac{\cancel{8}\cancel{2}^1}{\cancel{4}_1 \times \cancel{1800}_{900}}$$

$$\Rightarrow r = \sqrt{\frac{1}{900}}$$

$$\Rightarrow r = \pm \frac{1}{30}$$

As radius cannot be negative,

$$\Rightarrow r = \frac{1}{30}$$

⇒ r = 0.033 cm

Hence thickness of the wire is 0.033 cm.

$$\Rightarrow \pi r^2 \times 0.18 = \pi \times (\frac{1}{2})^2 \times 8$$

$$\Rightarrow r^2 = \frac{1 \times 8}{0.18 \times 2 \times 2}$$

$$\Rightarrow r = \frac{1}{30} \text{ cm}$$

$$\Rightarrow r = 1/3 \text{ mm}$$

∴ Diameter of cross section = (1/3) × 2 mm

= 0.67 mm

NOTE: When one figure is converted into another figure, the volume of two figures remain same.

36. Question

The diameters of internal and external surfaces of a hollow spherical shell are 10 cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $2\frac{2}{3}$ cm, find the diameter of the cylinder.

Answer

Diameter of internal surface =10 cm

∴ Radius of internal surface $= \frac{10}{2}$ cm =5 cm

Diameter of external surface =6 cm

∴ Radius of external surface $= \frac{6}{2}$ cm =3 cm

Volume of spherical shell hollow $= (\frac{4}{3}) \pi (R^3 - r^3)$

$$= (\frac{4}{3}) \times \pi \times (5^3 - 3^3)$$

Height of solid cylinder $= \frac{8}{3}$ cm

Let the radius of the solid cylinder be 'r' cm

Volume of the solid cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times r \times r \times \frac{8}{3} \text{ cm}^3$$

Volume of the solid cylinder = Volume of spherical shell hollow

$$\Rightarrow \pi \times r \times r \times \frac{8}{3} \text{ cm}^3 = (\frac{4}{3}) \times \pi \times (5^3 - 3^3)$$

$$\Rightarrow \pi \times r \times r \times \frac{8}{3} = (\frac{4}{3}) \times \pi \times (5^3 - 3^3)$$

$$\Rightarrow r^2 \times \frac{8}{3} = \left(\frac{4}{3}\right) \times (125 - 27)$$

⇒ r² × (8/3) = (4/3) × 98

⇒ r = $7\sqrt{3}$

Diameter of cylinder = $14\sqrt{3}$ cm

37. Question

A right angled triangle whose sides are 3 cm, 4 cm and 5 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two cones so formed. Also, find their curved surfaces.

Answer

Height of the cone (h_1)=3 cm

Radius of the cone (r_1) = 4 cm

Slant height of the cone (l_1) = 5 cm

Volume of cone (V_1)= $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 4^2 \times 3$ cm³

$= 16\pi$ cm³

Again after rotating

Height of the cone (h_2)=4 cm

Radius of the cone (r_2)=3 cm

Slant height of the cone (l_2)=5 cm

Volume of cone (V_2)= $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 3^2 \times 4$ cm³

$= 12\pi$ cm³

Difference between the volume two cones =16п – 12п

= 4п cm³

Curved surface area of first cone =п × r_1× l_1

= п × 4 × 5

= 20 π cm²

Curved surface area of second cone $= \pi \times r_2 \times l_2 = \pi \times 3 \times 5 = 15 \pi$ cm^2

38. Question

How many coins 1.75 cm in diameter and 2 mm thick must be melted to form a cuboid 11 cm × 10 cm × 7 cm?

Answer

Volume of cuboid = l × b × h = 11 × 10 × 7

Diameter of coin = 1.75 cm

Radius $= \dfrac{1.75}{2}$ cm = 0.875 cm

Thickness of coin =2 mm = 0.2 cm

Volume of coin (cylinder) $= \pi r^2 h$

= (22/7) × 0.875 × 0.875 × 0.2 cm^3

Let number of coins be n

⇒ n × volume of coins = volume of cuboid

⇒ n × (22/7) × 0.875 × 0.875 × 0.2 = 11 × 10 × 7

$$\Rightarrow n = \frac{11 \times 10 \times 7 \times 7}{22 \times 0.875 \times 0.875 \times 0.2}$$

⇒ n = 1600

39. Question

A well with inner radius 4 m is dug 14 m deep. Earth taken out of it has been spread evenly all around a width of 3 m it to form an embankment. Find the height of the embankment.

Answer

Let the height of the embankment be 'h' m

Radius of well = 4 m

Height of the well = 14m

Volume of earth dug out (volume of cylinder)$= \pi r^2 h$

$$= \frac{22}{7} \times 4 \times 4 \times 14 \, m^3$$

= 704 m^3

Outer Radius of the embankment(R) = 4 m + 3 m = 7 m

Inner radius of the embankment(r) = 4 m

Volume of embankment =Volume of earth dug out

Volume of embankment $= \pi R^2 h - \pi r^2 h$

$= \pi h (R^2 - r^2)$

$= \pi h (7^2 - 4^2)$

$= 33\pi h$

Volume of embankment = Volume of earth dug out

$\Rightarrow 33\pi h = 704$

$\Rightarrow h = \dfrac{704 \times 7}{33 \times 22}$

$\Rightarrow$ h = 6.78 m

40. Question

Water in a canal 1.5 m wide and 6 m deep is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes if 8 cm of standing water is desired?

Answer

Speed of flowing water =10 km/hr

In 30 minutes length of flowing water $=10 \times \dfrac{30}{60}$ km

= 5 km = 5000 m

Volume of flowing water in 30 minutes =5000 × width × depth

Width of canal = 1.5 m

Depth of canal = 6 m

Volume of canal =5000 × 1.5 × 6 m^3

= 45000 m^3

Irrigated area in 30 minutes if 8 cm of flowing water is required $= \dfrac{45000}{0.08}$

= 562500 m^2

41. Question

A farmer runs a pipe of internal diameter 20 cm from the canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km / h, in how much time will the tank be filled?

Answer

Let the time-taken by the pipe to fill the tank be x hrs.

Since the water is flowing at a rate of 3 kmph, therefore

Length of water column in x hrs is

3 × x = 3x km = 3000x m

Therefore the length of the pipe is $\frac{20}{2} = 10cm = 0.1m$

Therefore, the volume of water flowing through the pipe in x hours, V' = п r^2 h

= п (0.1)^2 (3000x) m^3

Also, volume of water that falls into the tank in x hours = п r^2 h

= п (5)^2 (2)

The volume of water flowing through the pipe in x hours = volume of water that falls into the tank in x hours

∴ п (0.1)^2 (3000x) = п (5)^2 (2)

⇒ 30x = 50

$$\Rightarrow x = \frac{50}{30} = \frac{100}{60} hrs$$

= 1hr 40min

42. Question

A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.

Answer

Let the height of the embankment be 'h' m

Radius of well $= \frac{3}{2}m$

=1.5 m

Height of the well =14m

$R = \frac{11}{2}$ m

$r = \frac{3}{2}$ m

4 m
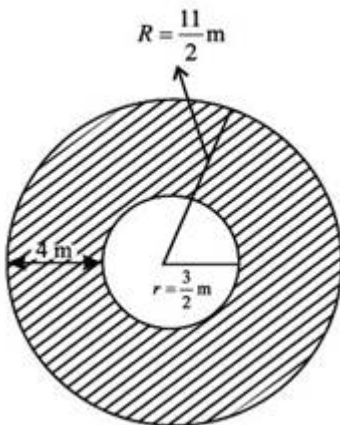
Volume of earth dug out (volume of cylinder)$= \pi r^2 h$

$$= \frac{22}{7} \times 1.5 \times 1.5 \times 14 \, m^3$$

$$= 99 \, m^3$$

Outer Radius of the embankment(R) =1.5 m + 4 m

=5.5 m

Inner radius of the embankment(r) =1.5 m

Volume of embankment =Volume of earth dug out

Volume of embankment $= \pi R^2 h - \pi r^2 h$

$$= \pi h (5.5^2 - 1.5^2)$$

$$= 27.5 \pi h$$

Volume of embankment =Volume of earth dug out

$\Rightarrow 27.5 \pi h = 99$

$\Rightarrow h = \frac{99 \times 7}{27.5 \times 22} \, m$ h = 1.145

$\Rightarrow h = 9/8 = 1.125 \, m$

43. Question

The surface area of a solid metallic sphere is 616 cm². It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed $\left(Use \; \pi = 22 \, / \, 7 \right)$.

Answer

Surface area of the sphere $= 4 \pi r^2$

$\Rightarrow 4 \pi r^2 = 616$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 49$

$\Rightarrow r = \sqrt{49}$

$\Rightarrow r = 7 \, cm$

Diameter of base = 7 × 2

= 14 cm

44. Question

The difference between the outer and inner curved surface areas of a hollow right circular cylinder 14 cm long is 88 cm². If the volume of metal used in making the cylinder is 176 cm³, find the outer and inner diameters of the cylinder. (Use π = 22 / 7)

Answer

Height of hollow cylinder =14 cm

Let the radius of outer radius be 'R' cm

Let the radius of inner radius be 'r' cm

Difference between inner and outer Curved surface area= 88 cm

Curved surface area of hollow cylinder= $2\pi(R - r)h$

⇒ 2π(R – r)h = 88

⇒ $R - r = \dfrac{88 \times 7}{2 \times 22 \times 14}$

⇒ R – r = 1 .... (1)

Volume of metal = 176

Height of hollow cylinder = 14 cm

Volume of hollow cylinder (metal) = $\pi(R^2 - r^2)h$

⇒ $\pi(R^2 - r^2)h = 176$

⇒ $(R^2 - r^2) = \dfrac{176 \times 7}{22 \times 14}$

⇒ $R^2 - r^2 = 4$

⇒ (R + r)(R – r) = 4

⇒ (R + r) = 4 .... (2)

Solving (1) and (2) we get,

R = 2.5cm

r = 1.5cm

∴ Outer diameter $= 2.5 \times 2$ cm

= 5 cm

Inner diameter $= 1.5 \times 2$ cm

= 3 cm

45. Question

The volume of a hemi-sphere is $2425\frac{1}{2}cm^3$. Find its curved surface area. (Use π = 22 / 7)

Answer

Volume of hemi-sphere $= \frac{2}{3}\pi r^3$

$\Rightarrow \frac{2}{3}\pi r^3 = 4851$

$\Rightarrow r^3 = \frac{4851 \times 3 \times 7}{22 \times 2 \times 2}$

$\Rightarrow$ r = 10.5 cm

Curved surface area$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 10.5^2$

= 693 cm^2

46. Question

A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied out on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Answer

Height of the cylindrical bucket =32 cm

Radius of the cylindrical bucket = 18 cm

Volume of the bucket = пr^2 h

=п × 18^2 × 32 cm^3

$= \pi \times 18 \times 18 \times 32$

Height of the conical heap =24 cm

Let Radius of the conical heap =r cm

Volume of conical heap $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times 24$ cub. cm

Volume of conical heap=Volume of cylindrical bucket

$\Rightarrow \frac{1}{3}\pi \times r^2 \times 32 \ cm^3 = \pi \times 18 \times 18 \times 32$

$\Rightarrow r^2 = \frac{18 \times 18 \times 32 \times 3}{24}$

$\Rightarrow$ r^2 = 1296 cm

$\Rightarrow r = \sqrt{1296}\, cm$

⇒ r = 36 cm

Slant height $= \sqrt{h^2 + r^2}$

∴ slant height $= \sqrt{24^2 + 36^2}$

$= 12\sqrt{13}\, cm$

47. Question

If the total surface area of a solid hemisphere is 462 cm^2, find its volume. (*Take* π = 22 / 7)

Answer

Total surface area of solid hemisphere $= 3\pi r^2$

⇒ 3πr^2 = 462

⇒3× (22/7) × r^2 = 462

$\Rightarrow r^2 = \dfrac{462 \times 7}{22 \times 3}$

⇒ r^2 =49

⇒ r=7 cm

Volume of solid hemisphere $= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3}\pi 7^3$ cm^3

$= \dfrac{2}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 7$

= 718.67 cm^3

48. Question

150 spherical marbles, each of diameter 1.4 cm are dropped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.

Answer

Diameter of spherical marbles $= \dfrac{1.4}{2}$ cm

=0.7 cm

Diameter of cylinder vessel $= \dfrac{7}{2} cm$

=3.5 cm

Volume of 150 spherical balls $= 150 \times \frac{4}{3} \pi R^3$

$= \frac{150 \times 4 \times 22 \times 0.7 \times 0.7 \times 0.7}{3 \times 7}$ cm^3

$= 215.6$ cm^3

Let the rise in level of water be h cm

Volume of rise in level of water (Volume of cylinder) $= \pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times h$

Volume of Rise in level of water in the vessel = volume of 150 spherical balls

$\Rightarrow$ (22/7) × 3.5 × 3.5 × h = 215.6

$\Rightarrow h = \frac{215.6 \times 7}{22 \times 3.5 \times 3.5}$ cm

$\Rightarrow$ h = 5.6 cm

49. Question

A cylindrical tank full of water is emptied by a pipe at the rate of 225 liters per minute. How much time will it take to empty half the tank, if the diameter of its base is 3 m and its height is 3.5 m? $\left[Use\ \pi = 22/7 \right]$

Answer

Diameter of the cylindrical base = 3 m

$\therefore$ Radius of cylindrical tank $= \frac{3}{2}$ m

= 1.5 m

Height of the tank = 3.5 m

Volume of the tank $= \pi r^2 h$

$= \pi \times 1.5^2 \times 3.5 = \left(\frac{22}{7} \right) \times 1.5 \times 1.5 \times 3.5$

= 24.75 m^3

Now, 1 m^3 = 1000 liters

24.75 m^3 = 1000 × 24.75 liters

= 24750 liters

Full quantity of the water when it is full = 24750 m^3

Quantity of water when it is half filled $= \frac{24750}{2}$ liters

= 12375 liters

Time taken by it to empty 225 liters 0f water = 1 minute

∴ Time taken by it empty 12375 litersof water = $\dfrac{12375}{225}$ minutes

= 55 minutes

50. Question

A solid metallic sphere of radius 5.6 cm is melted and solid cones each of radius 2.8 cm and height 3.2 cm are made. Find the number of such cone formed.

Answer

Volume of the sphere = $\dfrac{4}{3}\pi R^3$

$= \dfrac{4}{3}\pi\, 5.6^3$

Let number of cones be n

⇒n × volume of cone = volume of sphere

$\Rightarrow n\times\dfrac{1}{3}\pi 2.8^2\times 3.2 = \dfrac{4}{3}\pi\, 5.6^3$

$\Rightarrow n\times\dfrac{1}{3}\times 2.8\times 2.8\times 3.2 = \dfrac{4}{3}\times 5.6\times 5.6\times 5.6$

$\Rightarrow n = \dfrac{4\times 5.6\times 5.6\times 5.6\times 3}{3\times 2.8\times 2.8\times 3.2}$

⇒ n = 28

52. Question

Sushant has a vessel of the form of an inverted cone, open at the top of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\left(\dfrac{2}{5}\right)^{th}$ of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?

Answer

Height of the conical vessel, h' = 11cm

It's radius, r' and r'' resp.: r' = 2.5cm r'' = 0.25cm

Assume the number of spherical balls that were dropped in the vessel as 'n'

Volume of water replaces by the spherical ball = volume of spherical balls dropped into the vessel

(2/5) × Volume of the cone = n × each spherical ball

$$\Rightarrow \frac{2}{5} \times \frac{1}{3}\pi r'^2 h = n \times \frac{4}{3}\pi r''^3$$

$\Rightarrow (2.5)^2 \times 11 = n \times 10 \times (0.25)^3$

$\Rightarrow n = 440$

53. Question

A solid cuboid of iron with dimensions 53 cm × 40 cm × 15 cm is melted and recast into a cylindrical pipe. The outer and inner diameters of pipe are 8 cm and 7 cm respectively. Find the length of pipe.

Answer

Outer and inner radius of the pipe: r' = 8/2 = 4cm and r'' = $\left(\frac{7}{2}\right)$ cm

Volume of cuboid = volume of pipe

L b h = п h[R² – r²]

$$\Rightarrow 53 \times 40 \times 15 = \frac{22}{7} \times h\left[4^2 - \left(\frac{7}{2}\right)^2\right]$$

$$\Rightarrow 31800 = \frac{22}{7} \times h\left[\frac{15}{4}\right]$$

$\Rightarrow$ h = 2698.18cm $\approx$ 27m

54. Question

Water is flowing at the rate of 2.52 km / h through a cylindrical pipe into a cylindrical tank, the radius of the base is 40 cm. If the increase in the level of water in the tank, in half and hour is 3.15 m, find the internal diameter of the pipe.

Answer

Speed of water = 2.52kmph = 2520 m/hr

Length, h' = 2520 m

Let the radius of the pipe be r' and radius of the tank, r'' = 40 cm = 0.4 m

Level of water in the tank in half and hour = 3.15 m

Level of water in the tank in an hour, h'' = 6.30 m

Now, the volume of pipe = volume of the tank

$\Rightarrow$ п r'² h' = п r''² h''

$\Rightarrow$ r'² (2520) = (0.4)² (6.3)

$$\Rightarrow r = \sqrt{\frac{(0.4)^2(6.3)}{2520}} = 0.02m = 2cm$$

∴ diameter = 2r' = 2 × 2 = 4 cm

Exercise 16.2

1. Question

A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.

Answer

Diameter= 24 m

r =24/2= 12

Height of cylinder, H =1cm

Height of cone, h = 16-11= 5 m

Slant height (l) = $\sqrt{r^2 + h^2}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{169}$ = 13 m

Surface area of tent = curved surface area of cylinder + curved surface area of cone

= 2πrH + πrl

= 2 × (22/7) × 12 × 11 + (22/7) × 12 × 13

= (22/7) × 12 (22 +13)

= (22/7) × 12 × 35

= 1320 m^2

2. Question

A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area and the volume of the rocket.

Answer
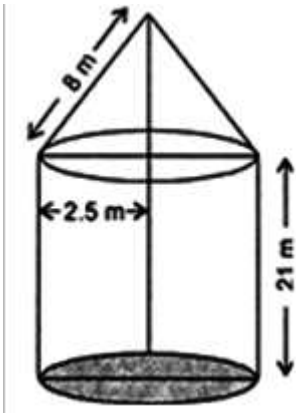
Radius of the cylinder portion (r) = 2.5 m

Height of the cylinder portion (h) = 21 m

∴ Surface area of the cylindrical portion = 2πrh

$$= 2 \times \frac{22}{7} \times 2.5 \times 21 \text{ m}$$

= 330 m ....(1)

Radius of the conical portion (r) = 2.5 m

Slant height of the conical portion (l) = 8 m

∴ Curved surface area of the conical portion = πrl

= (22/7) × 2.5 × 8 m

= 62.86 m ....(2)

Area of the circular top $= \pi r^2$

= (22/7) × 2.5 × 2.5 m²

= 19.64 m² ....(3)

∴ Total surface area of the rocket

= (330 + 62.86 + 19.64) m² [From (1) , (2), (3)]

= 412.5 m²

Volume of the cylindrical portion $= \pi r^2 h$

$$= \frac{22}{7} \times (2.5)^2 \times 21$$

= 412.5 m²

3. Question

A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m^2 (*Use π = 22 / 7*).

Answer

Given: height of the tent, H = 77dm = 7.7 m

Height of the cylindrical portion, h = 44dm = 4.4m

∴ Height of the conical portion, h' = 7.7 − 4.4 = 3.3 m

Radius of the cylinder, r = 36/2 = 18m

Curved surface area of the cylindrical portion of the tent = $2 \pi r^2 h$

= $2 \times \pi \times (18)^2 \times 4.4$

= 158.4 π m^2

Slant height of the conical portion of the tent = $\sqrt{3.3^2 + 18^2}$

= 18.3 m

∴ Curved surface area of the conical portion = $\pi \times 18 \times 18.3$

= 329.4 π m^2

∴ Total surface area of the tent = 158.4 π + 329.4 π

= 487.8 π

= 1533.09 m^2

∴ Canvas required to make the tent = 1533.09 m^2

Total cost of the canvas = 1533.09 × 3.50

= Rs5365.80

4. Question

A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm, respectively. Determine the surface area of the toy. (*Use π = 3.14*)

Answer

Height of the cone =4 cm

Radius of the cone = radius of hemisphere = 3 cm

Volume of toy = Volume of conical part + Volume of hemispherical part

Volume of cone $= \dfrac{\pi r^2}{3}$

∴ Volume of conical part $= \dfrac{3.14 \times 3^2 \times 4}{3}$

= 37.6 cm^2

Volume of hemisphere $=\dfrac{2\pi r^2}{3}$

Volume of toy = 37.6 +56.52 = 94.12 cm^2

And total surface area of toy = Curved surface area of conical part + Curved surface area of hemispherical part

Curved surface area of cone $=\pi$r**l**, Where l = $\sqrt{h^2 + r^2}$

= 3.14 × 3 × $\sqrt{4^2 + 3^2}$

= 47.1 cm^2

And curved surface area of hemisphere =2пr^2

= 2 × 3.14 × 3 × 3

= 56.52 c m^2

Then, total surface area of the toy = 47.1 + 56.52 = 103.62 c m^2

5. Question

A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the heights of the cylindrical and conical portions are 10 cm and 6 cm, respectively. Find the total surface area of the solid. (*Use* π = 22 / 7)

Answer

Height of cylinder (h) = 10 cm

Height of conical part = 6 cm

Slant height of cone (l) = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{3.5^2 + 6^2}$

⇒ l = 48.25 cm

Curved surface area of cone $=\pi$rl

= п (3.5) (48.25)

= 76.408 cm^2 ...(1)

Curved surface area of cylinder = 2п rh

=2п (3.5) (10)

=220 cm^2 ...(2)

Curved surface area of hemisphere = 2πr^2

$= 2\pi (3.5)^2$

$= 77 \text{ cm}^2 \dots (3)$

∴ Total curved surface area = Curved surface area of(cone + cylinder + hemisphere)

$= 76.408 + 220 + 77$

$= 373.408$

∴ Total surface area of solid = 373.408 cm^2

6. Question

A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical part are 5 cm and 13 cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.

Answer

Total surface area of toy = C.S.A. of cylinder + C.S.A. of hemisphere + C.S.A. of cone

$= (2 \times \frac{22}{7} \times 5 \times 13) + (2 \times \frac{22}{7} \times 5 \times 5) + (\frac{22}{7} \times 5 \times 13)$

$= \frac{22}{7} \times 5[(2 \times 13) + (2 \times 5)13]$

$= \frac{22}{7} \times 5[26+10+13]$

$= \frac{22}{7} \times 5 \times 49$

$= 770 \text{ cm}^2$

7. Question

A cylindrical tub of radius 5 cm and length 9.8 cm is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed in the tub. If the radius of the hemisphere is immersed in the tub. If the radius of the hemi-sphere is 3.5 cm and height of the cone outside the hemisphere is 5 cm, find the volume of the water left into hr tub. (Take $\pi = 22/7$)

Answer

Given radius of cylindrical tube(r) = 5 cm

Height of cylindrical tube (h) = 9.8 cm

Volume of cylindrical = $\pi r^2 h$

V1 = $\pi 5^2 (9.8)$

$= 770$ cm

Given of hemisphere = 3.5 cm

Height of cone (h) = 5 cm

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi(3.5)r^3$

$= 89.79$ cm^3

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r(3.5)^2 5$

$= 64.14$ cm^3

Volume of cone + Volume of hemisphere (v2) $= 89.79 + 64.14 = 154$ cm^3

8. Question

A circus tent has cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 m. The heights of the cylindrical and conical portions are 4.2 m and 2.1 m respectively. Find the volume of the tent.

Answer

Given radius of Cylindrical base = 20 m

Height of Cylindrical part (h) = 4.2

Volume of Cylindrical $= \pi r^2 h$ ....(1)

V1 $= \pi(20)^2$ 4.2

$= 5280$ m^3

Volume 0f Cone $= \frac{1}{3}\pi r^2 h$

Height of conical part (h) = 2.1 m .....(2)

V2 $= \frac{\pi}{3}(20)^2 (2.1)$

$= 880$ m^3

Volume of tent (v) = V1 + V2

V = 5280 + 880

$= 6160$ m^3

9. Question

A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with conical ends each of axis length 9 cm. Determine the capacity of the tank.

Answer

Diameter =21 cm

∴ Radius, r =10.5 cm

Length of the cylindrical part, l =18 cm

Height of the conical part, h = 9 cm

Volume of the tank = Volume of the cylindrical part + 2 × Volume of the conical part

$$= \pi r^2 L + 2 \times \frac{1}{3} \times \pi r^2 h$$

$$= \pi r^2 \left(L + \frac{2}{3} \times h \right)$$

= (22/7) × 10.5 × 10.5 × 24

= 8316 cm^3

10. Question

A conical hole is drilled in a circular cylinder of height 12 cm and base radius 5 cm. The height and the base radius of the cone are also the same. Find the whole surface and volume of the remaining cylinder.

Answer

Surface area = 210 π cm^2

Radius of cylinder(r) = 5 cm

Height of cylinder, (h) = 12 cm

Let, l be slant height of cone

$$l = \sqrt{r^2 + h^2}$$

Slant height of the cone = $\sqrt{5^2 + 12^2}$ =13 cm

Volume of cylinder = п × 5^2 × 12 = 300 cm^3

Volume of the conical hole = $\frac{1}{3}$ × п × 5 × 12 = 100 cm^3

Therefore, Volume of the remaining solid

= Volume of the cylinder – Volume of the removed conical part

= 300 п -100 п = 200 п cm^3

Curved surface of the cylinder

= 2 × п r h = 2 × п × 5 × 12 = 120п cm

Curved surface of cone = п r l = п × 5 × 13 = 65п cm

Base area of cylinder = п × 5^2 = 25cm^2

The whole surface area of the remaining solid includes the curved surface of the cylinder and cylinder and the cone and area of the base

Therefore, Whole surface area = 120 п + 65п + 25п =210п cm^2

11. Question

A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m. find the capacity of the tent and the cost of the canvas at Rs. 100 per square metre.

Answer

Given: A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m.

To find: the capacity of the tent and the cost of the canvas at Rs. 100 per square metre.

Solution:
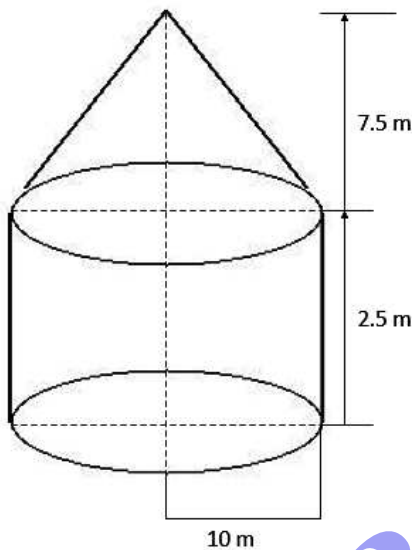
height of Cylinder (h) = 2.5 m

Radius of cylinder = 20/2=10 m

height of cone(h$_1$)=7.5 m

Let "l" be the slant height of the cone,As we know l^2 = $\sqrt{r^2 + h^2}$

⇒ l^2 = $\sqrt{10^2 + 7.5^2}$

⇒ l^2 = $\sqrt{100+ 56.25}$

⇒ l^2 = $\sqrt{156.25}$

⇒ l = 12.5 m

Volume of a cylinder (v)= пr^{2}h

$V_1 = \pi(10)^2(2.5)$ cm .....(1)

Volume of cone $(V_1) = \dfrac{1}{3}\Pi r^2 h_1$

Volume of cone $(V_1) = \dfrac{1}{3}\Pi(10)^2 \times 7.5$ ..... (2)

Volume of boiler = (1) + (2)

$V = V_1 + V_2$

$$V = \left(\Pi(10)^2(2.5)\right) + \left(\dfrac{1}{3}\Pi(10)^2 \times 7.5\right)$$

$\Rightarrow V = 250\,\Pi + 250\,\Pi \Rightarrow V = 500\,\Pi$ m^3

Now to find the cost find the curved surface of the tent.CSA of tent = CSA of cylinder + CSA of cone= $2\Pi rh + \Pi rl = \Pi r\,(2h + l) = \dfrac{22}{7} \times 10[2(2.5) + 12.5] = \dfrac{22}{7} \times 10[5 + 12.5]$

$$= \dfrac{22}{7} \times 10(17.5)$$

$$= \dfrac{3850}{7}$$

= 550 m^2

Cost of 1 m^2 canva = Rs 100

cost of 550 m^2 canva = 550× 100

= Rs 55000

12. Question

A boiler is in the form of a cylinder 2 m long with hemispherical ends each of 2 metre diameter. Find the volume of the boiler.

Answer

Given: radius of cylinder (r) = $\dfrac{20}{2}$ = 10 cm

Height of cylinder (h_1) = 2.5 m

Height of cone (h_2) = 7.5 m

Let l be slant height of cone

$l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{10^2 + 7.5^2}$

$\Rightarrow$ l = 12.5 m

Volume of cylinder (V_1) = πr²h

$\Rightarrow V_1 = \pi \, r(10)^2(2.5)$ .... (1)

Volume of cone (V_2) = $\frac{1}{3}$πr²h

= $\frac{1}{3}$π(10)² (7.5) .... (2)

Total capacity of tent = (1) + (2)

V = $V_1 + V_2$

$\Rightarrow$ V = π(10)²(2.5) + $\frac{1}{3}$π(10)² (7.5)

$\Rightarrow$ V = 250π + 250π

$\Rightarrow$ V = 500 π m³

∴Total capacity of tent = 500 π m³

13. Question

A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is $\frac{14}{3}m$ and the diameter of hemisphere is 3.5 m. Calculate the volume and the internal surface area of the solid.

Answer

Radius of the hemispherical part of the hollow cylinder = $\frac{3.5}{2} = \frac{7}{4}m$

Hollow cylinder and the hemisphere have the same base

∴ Radius of the cylinder = $\frac{7}{4}m$

Depth of the cylinder, h = $\frac{14}{3}m$

Volume of the vessel = Volume of cylindrical part + Volume of the hemispherical part

$= \pi r^2 h + \frac{2}{3}\pi r^3$

$= \pi r^2 \left(h + \frac{2}{3}r \right)$

$= \left(\frac{22}{7}\right)\left(\frac{7}{4}\right)^2 \left(\frac{14}{3} + \frac{2}{3}\left(\frac{7}{4}\right)\right)$

$\approx$ 56.15 m^3

Internal surface area of the vessel = CSA of (cylinder + hemisphere)

= 2πrh + 2π r^2

= 2π r(h+r)

$$= 2\pi \left(\frac{7}{4}\right)\left(\frac{14}{3}+\frac{7}{4}\right)$$

$= 70.58 \approx 70\frac{7}{12}$ m^2

14. Question

A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Rs. 10 per dm^2.

Answer

Radius of hemispherical ends, r = 7 cm

Height of body (h + 2r) = 104 cm

Curved surface area of cylinder = 2πrh

= 2π (7)h ……….(1)

Given: h + (2r) = 104 cm

⇒ h = 104 − 2 (7)

⇒ h =90 cm

Substitute the value of h in eq. (1),

Curved surface area of cylinder = 2π (7) (90)

= 3958.40 cm …..(2)

Curved area of 2 hemisphere = 2(2πr^2)

= 2(2 ×π ×7^2)

= 615.75 cm ……….. (3)

Total curved surface area = 2+3

= 3958.40 + 615.75

= 4574.15 cm^2

Cost of polishing for 1 dm^2 = Rs.10

Cost of polishing for 54.75 dm^2 = 45.74×10

= Rs.457.60

15. Question

A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16 cm and height 42 cm. The total space between the two vessels is filled with cork dust for heat insulation purposes. How many cubic centimeters of cork dust will be required?

Answer

Given height of cylindrical vessel (h) = 42 cm

Inner radius of vessel (r') = $\frac{14}{2}$ cm = 7 cm

Outer radius of vessel (r'') = $\frac{16}{2}$ = 8 cm

Volume of cylinder = π (r''^{2} – r'^{2}) h

= π (8^2 - 7^2) 42

= 1980 cm^3

16. Question

A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54 cm and the thickness of the iron sheet used in making the roller is 9 cm. find the mass of the roller, if 1 cm^3 of iron has 7.8 gm mass (*Use* π = 3.14)

Answer

Given that, internal radius of cylinder road roller (r1) = $\frac{54}{2}$ = 27cm

Thickness of road roller (t) = 9 cm

Radius of cylinder road roller be R

t = R – r

t = R-27

R = 9 + 27 = 36 cm

Given height of cylindrical road roller (h) = 1m

h = 100 cm

Volume of iron = πh(R^2-r^2)

= π(36^2 - 27^2) × 100

= 1780.38 cm

Mass of 1 cm of iron = 7.8 gm

Mass of 1780.38 cm of iron = 1780.38 × 7.8

= 1388696.4 gm

$\therefore$ Mass of roller (m) = 1388.7 kg

17. Question

A vessel in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. find the inner surface area of the vessel.

Answer

Given radius of hemisphere & cylindrical (r)

$$= \frac{14}{2}$$

=7 cm

Given total height of vessel = 13 cm

$\therefore$ (h +r) = 13 cm

Inner surface area of vessel = 2п r (h +r)

= 2 ×п × 7 (13)

= 182п

=575 cm^2

18. Question

A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of the same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Answer

Given radius of cone (r) = 3.5 cm

Total height of toy (h) = 15.5 cm

height of cone (l) = 15.5 – 3.5

= 12 cm

As we know,

l^2 = r^2 + h^2

= 3.5^2 + 12^2

$$= \left(\frac{7}{2}\right)^2 + 144$$

$$= \frac{49}{4} + 144$$

$$= \frac{49 + 576}{4}$$

$$= \frac{625}{4}$$

$$l = \sqrt{\frac{625}{4}}$$

$$l = \frac{25}{2} \quad \text{l = 12.5}$$

The curved surface area of a cone,S′ = пrl

= п (3.5) (12.5)

=137.5 cm² …….. (1)

The curved surface area of the hemisphere, S″ = 2пr²

= 2п(3.5)²

= 77 cm² ………… (2)

∴ Total surface area of the toy = 137.5 + 77 = 214.5 cm²

19. Question

The difference between outside and inside surface areas of cylindrical metallic pipe 14 cm long is 44 m². If the pipe is made of 99 cm³ of metal, find the outer and inner radii of the pipe.

Answer

Given: The difference between outside and inside surface areas of cylindrical metallic pipe 14 cm long is 44 m² and the pipe is made of 99 cm³ of metal.

To find: the outer and inner radii of the pipe.

Solution:
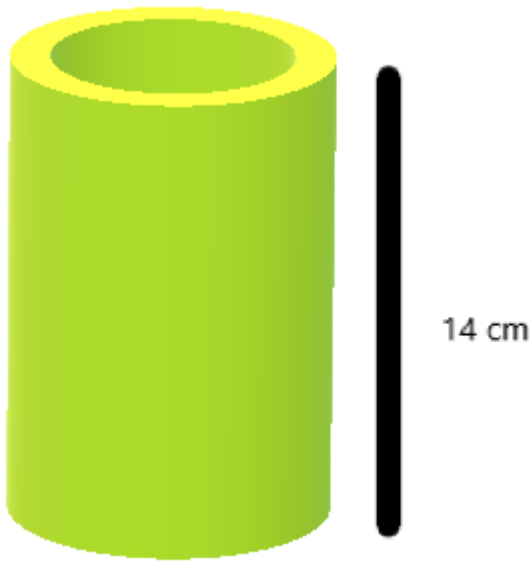
14 cm

Let the inner radius of pipe = r1

And the radius of outer cylinder = r2

Length of cylinder (h) = 14 cm

Surface area of hollow cylinder =2πh (r2 −r1)

Given surface area of cylinder = 44 m^2.

⇒ 2π h(r2 −r1) = 44

⇒ 2π (14) (r2 −r1) = 44

⇒ (r2 −r1) = $\frac{44}{28\pi}$

⇒ (r2 −r1) = $\frac{44 \times 7}{28 \times 22}$

⇒ (r2 −r1) = $\frac{44^2 \times 7^1}{28^4 \times 22^1}$

⇒ (r2 −r1) = 1/2 ……. (1)

Given volume of a hollow cylinder = 99 cm^3

Volume of a hollow cylinder = π h($r2^2 - r1^2$)

⇒ π h (r2^2 - r1^2) = 99

⇒ 14π (r2^2 - r1^2) = 99Apply the formula a^2 - b^2 = (a - b) (a + b) in (r2^2 - r1^2).

⇒ 14π (r2 + r1) (r2 − r1) = 99Put the value of r2 - r1 from (1).

⇒ 14π (r2 + r1) $\frac{1}{2}$ = 99

$$\Rightarrow 14 \times \frac{22}{7} \times (r2+r1) \times \frac{1}{2} = 99$$

$$\Rightarrow 14^2 \times \frac{22^{11}}{7_1} \times (r2+r1) \times \frac{1}{2_1} = 99$$

$\Rightarrow$ 22 (r2+r1) = 99

$$\Rightarrow (r2+r1) = \frac{99}{22}$$

$\Rightarrow$ (r2+r1) = $\frac{9}{2}$ .... (2)

Equating (1) & (2) equations we get

r2 = 5/2 cm

Substituting r2 value in (1)

$$\frac{5}{2} - r1 = \frac{1}{2}$$

$$\Rightarrow \frac{5}{2} - \frac{1}{2} = r1$$

$$\Rightarrow r1 = \frac{5-1}{2}$$

$$\Rightarrow r1 = \frac{4}{2}$$

$\Rightarrow$ r1 = 2 cm

∴ Inner radius of pipe (r1) = 2 cm

Radius of outer cylinder (r2) = 5/2 cm

20. Question

A right circular cylinder having diameter 12 cm and height 15 cm is full ice-cream. The ice-cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice-cream.

Answer

Given radius of cylinder (r') = $\frac{12}{2}$ = 6 cm

Given radius of hemisphere (r'') = $\frac{6}{2}$ = 3 cm

Given height of cylinder (h) = 15 cm

Height of cone (l) = 12 cm

Volume of cylinder = пr²h

= пr(6)²(15) cm ………. (1)

Volume of each cone = Volume of cone + Volume of hemisphere

V = п(r)²(l) + $\frac{2}{3}$п(r)³

V = п(3)²(12) + $\frac{2}{3}$п(3)³ ………… (2)

Let number of cone be n

n (volume of each cone) = volume of cylinder

n = [($\frac{1}{3}$п(3)² (12) + $\frac{2}{3}$п(3)³] = п(16)² 15

⇒ n = 10

21. Question

A solid iron pole having cylindrical portion 110 cm high and of base diameter 12 cm is surmounted by a cone 9 cm high. Find the mass of the pole, given that the mass of 1 cm³ of iron is 8 gm.

Answer

Given radius of cylindrical part(r)= 12/2= 6 cm

Height of cylinder (h) = 110 cm

Length of cone (l)= 9 cm

Volume of cylinder= πr^2h

V1= п (6)² 110 cm³

Volume of cone=$\frac{1}{3}\pi r^2 l$

V2=$\frac{1}{3}\pi(6)^2 9$

=108 п cm³

Volume of pole =v1 + v2

= $\pi(6)^2 110 + 108\pi$

= 12785.14 $\overline{108 \pi}$ cm³

Given mass of 1 cm³ of iron = 8 gm

Mass of 1275.14 cm³ of iron = 12785.14 × 8

= 102281.12

=102.2 kg

∴ Mass of pole for 12785.14 cm³ of iron is 102.2 kg

22. Question

A solid toy is in the form of a hemisphere surmounted by a right circular come. Height of the cone is 2 cm and the diameter of the base is 4 cm. /if a right circular cylinder circumscribes the toy, find how much more space it will cover.

Answer

Given radius of cone, cylinder & hemisphere (r) = 4/2 = 2 cm

Height of cone (l) = 2 cm

Height of cylinder (h) = 4 cm

Volume of cylinder = $\pi r^2 h = \pi (2)^2 (4)$ cm …….. (1)

Volume of cone = $\frac{1}{3} \pi r^2 l$

$= \frac{1}{3} \pi (2)^2 \times 2$

$= \frac{2}{3} \times 4 \times 2$ cm ………………. (2)

Volume of hemisphere = $\frac{2}{3} \pi r^3$

$= \frac{2}{3} \pi (2)^3$

$= \frac{2}{3} \pi (8)^3$ ……………. (3)

So remaining volume of cylinder when toy is instead to it

$= \pi r^2 h - (\frac{1}{3} \pi r^2 l + \frac{2}{3} \pi r^3)$

= eq.(1) − eq.(2) + eq.(3)

$= \pi (2)^2 (4) - (\frac{\pi}{3} \times \frac{2}{3} \times \pi \times 8)$

$= 16\pi - \frac{2}{3} \pi (4+8)$

= 16π − 8π

= 8π cm³

∴ So remaining volume of cylinder when toy is instead to it = 8π cm³

23. Question

A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottoms. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

Answer

Given: radius of circular cone (r) = 60 cm

Height of a cone (l) = 120 cm

Volume of a cone = $\frac{1}{3}$ πr² l

= $\frac{1}{3}$ π(60)²(120) cm …….. (1)

Volume of a hemisphere = $\frac{2}{3} \pi r^3$

Given radius of hemisphere = 60 cm

= $\frac{2}{3} \pi (60)^3$ …………… (2)

Radius of cylinder (r) = 60 cm

Height of cylinder (h) = 180 cm

Volume of cylinder = $\pi r^2 h$

= π(60)²(180) cm³ ……… (3)

Volume of water left in cylinder = eq.(3) – eq.(1) + eq.(2)

$$= \frac{1}{3} \pi (60)^2 (120) - \left[\frac{2}{3} \pi (60)^3 + \pi (60)^2 (180) \right]$$

= 113.1 cm³ = 1.131m³

24. Question

A cylindrical vessel with internal diameter 10 cm and height 10.5 cm is full of water. A solid cone of base diameter 7 cm and height 6 cm is completely immersed in water. Find the value of water (i) displaced out of the cylinder. (ii) left in the cylinder. (*Take* π = 22 / 7)

Answer

Given internal radius (r1) = 10/2 = 5 cm

Height of cylindrical vessel (h) = 10.5 cm

Outer radius of cylindrical vessel (r2) = 7/2 = 3.5 cm

Length of cone (l) = 6 cm

(i) Volume of water displaced = Volume of cone

Volume of cone = $\frac{1}{3}\pi r^2 l$

$= \frac{1}{3}\pi (3.5)^2 \times 6$

= 76.9 cm^3

$\approx$ 77 cm^3

Volume of water displaced = 77 cm^3

Volume of cylinder = пr^{2}h

= п(5)^{2}10.5

= 824.6

$\approx$ 825 cm^3

(ii) Volume of water left in cylinder = Volume of cylinder – Volume of cone

= 825 -77 = 748 cm^3

25. Question

A hemispherical depression is cut out from one face of a cubical wooden block of edge 21 cm, such that the diameter of the hemisphere is equal to the edge of the cube. Determine the volume and total surface area of the remaining block.

Answer

Edge of wooden block (a) = 21 cm

Diameter of hemisphere = edge of cube

Radius = 21/2 = 10.5 cm

Volume of remaining block = Volume of box – Volume of hemisphere

$= a^3 - \frac{2}{3}\pi r^3$

$= (2)^3 - \frac{2}{3}\pi (10.5)^3$

= 6835.5 cm^3

Surface area of box = 6a^2 .... (1)

Curved surface area of hemisphere = 2пr^3 .... (2)

Area of base of hemisphere $= \pi r^2$ .... (3)

So remaining surface area of box = (1) − (2) + (3)

$$= 6a^2 - \pi r^2\ 2\pi r^2$$

$$= 6(21)^2 - \pi(10.5)^2 + 2\pi(10.5)^2$$

$$= 2992.5\ \text{cm}^2$$

∴Remaining surface area of box=2992.5 cm²

Volume of remaining block=6835.5 cm³

26. Question

A toy is in the form of a hemisphere surmounted by a right circular cone of the same base radius as that of the hemisphere. If the radius of the base of the cone is 21 cm and its volume is 2/3 of the volume of the hemisphere, calculate the height of the cone and the surface area of the toy. (*Use π = 22 / 7*)

Answer

Radius of base = 21cm and its volume = 2/3 × volume of the hemisphere

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3}\pi r^3$$

$$\Rightarrow h = \frac{4}{3}r = \frac{4}{3} \times 21 = 28cm$$

Hence surface area of the toy = Surface area of the cone + Surface area of hemisphere

$$= \pi r\ (\sqrt{21^2 + 28^2}) + 2\ \pi\ r^2$$

$$= \pi\ (21)[35 + 2 \times 21]$$

$$= 5082\ \text{cm}^2$$

27. Question

A solid is in the shape of a cone surmounted on a hemi-sphere, the radius of each of them is being 3.5 cm and the total height of solid is 9.5 cm. Find the volume of the solid. (*Use π = 22 / 7*)

Answer

Height of the cylinder = height of solid − radius of the cone

= 9.5 − 3.5 = 6cm

Volume of the solid = volume of the cone + volume of the cone

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi(3.5)^2(6) + \frac{2}{3}\pi(3.5)^3$$

= 166.83 cm³

28. Question

An wooden toy is made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. (Use π = 22 / 7)

Answer

the height of the cylinder = 10 cm, and its base is of radius = 3.5 cm

Volume of wood in the toy = volume of cylinder − 2 × volume of hemisphere

$$= \pi r^2 h - 2 \times \frac{2}{3} \pi r^3$$

$$= \pi (3.5)^2 (10) - 2 \times \frac{2}{3} \pi (3.5)^3$$

= 205.33 cm³

29. Question

The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. (Use π = 22 / 7)

Answer

To be the largest, diameter of the sphere = edge of the cube

∴ its radius = 1/2 × edge

Volume of wood left = volume of cube − volume of sphere

$$= (7)^3 - \frac{4}{3} \pi \left(\frac{7}{2}\right)^3$$

= 343 − 179.67

= 163.33 cm³

30. Question

From a solid cylinder of height 2.8 cm and diameter 4.2 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid. (Take π = 22 / 7).

Answer

Slant height of the cone, l = $\sqrt{(2.8)^2 + (2.1)^2} = 3.5 cm$

Remaining area of the cylinder = Curved surface area of the cylindrical part + Area of the conical part + Area of cylindrical base

= 2πrh + πrl + πr²

$= 2\pi \times (2.1) \times (2.8) + \pi \times 2.1 \times (3.5) + \pi (2.1)^2$

$= 36.96 + 23.1 + 13.86$

$= 73.92$ cm^2

31. Question

The largest cone is curved out from one face of solid cube of side 21 cm. Find the volume of the remaining solid.

Answer

Largest cone is the right circular cone with base diameter and height = edge of the cube

∴ radius of the cone = (21/2) cm

and height of the cone = 21 cm we know, volume of cube is (side)^3

and $volume\ of\ cone = \dfrac{1}{3}\pi r^2 h$

where 'r' is radius and 'h' is height of the cone. Volume of the remaining cube = volume of the cube – volume of cone cone)

$= (21)^3 - \left(\dfrac{1}{3}\right)\left(\dfrac{22}{7}\right)\left(\dfrac{21}{2}\right)^2 (21)$

$= 9261 - \dfrac{4851}{2}$

$= 9261 - 2425.5$

$= 6835.5\ cm^3$

32. Question

A solid wooden toy is in the form of a hemisphere surmounted by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}\ cm^3$. Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of ` 10 per cm^2. (Take $\pi = 22/7$)

Answer

Let h be the height of the cone and r be radius of its base

Now, volume of the wooden toy $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \dfrac{77}{6}(h+7)$

According to the question, $\dfrac{77}{6}(h+6) = 166\frac{5}{6}$

⇒ h = 6cm

The height of the wooden toy = 6cm + 3.5 cm

= 9.5cm
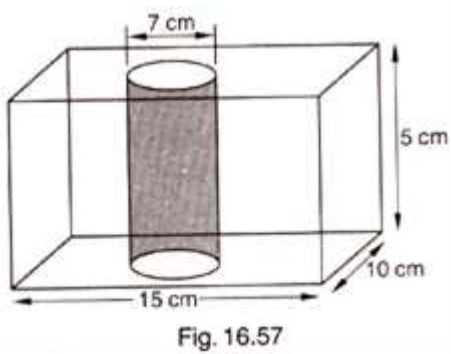
Now, curved surface area of the hemispherical part = 2 п r²

= 2п × (3.5)²

=77cm²

Hence the cost of painting the hemispherical part of the toy at the rate of ` 10 per cm² = 77 × 10 = Rs.770

33. Question

In Fig. 16.57, from a cuboidal solid metallic block, of dimensions 15 cm × 10 cm × 5 cm, a cylindrical hole of diameter 7 cm is drilled out. Find the surface area of the remaining block. (*Take $\pi = 22/7$*)

Fig. 16.57

Answer

Here, dimension of cuboid = 15 cm × 10 cm × 5 cm

Radius of cylindrical hole = $\frac{7}{2}$ cm

Surface area of the remaining block = surface area of cuboidal solid metallic block – 2 × base area of cylindrical hole

= 2(15 × 10 + 10 × 5 + 5 × 15) - 2 × $\frac{22}{7}$ × $\left(\frac{7}{2}\right)^2$

= 583 cm²

Exercise 16.3

1. Question

A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of Rs. 1.20 per dm². (*Use $\pi = 3.14$*)

Answer

Radius of top of the bucket, r' = 40/2 = 20 cm

Radius of bottom of the bucket, r″ = 20/2 = 10 cm

Depth, h = 12cm

Volume of the bucket = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$

$= \frac{1}{3}\pi(20^2 + 10^2 + 20\times10)\times12$

= 8800 cub. cm

Let l be slant height of the bucket

$\Rightarrow 1 = \sqrt{(r'^2 - r''^2) + h^2}$

$\Rightarrow 1 = \sqrt{(20^2 - 10^2) + 12^2}$

$\Rightarrow 1 = 15.62$ cm

Total surface area of bucket = ⊓ (r' + r″) × l + ⊓ r″²

= ⊓ (20 + 10) × 15.620 + ⊓ (10)²

= 17.81 dm²

Cost of tin sheet = 1.20 × 17.87 = Rs21.40

2. Question

A frustum of a right circular cone has a diameter of base 20 cm, of top 12 cm, and height 3 cm. Find the area of its whole surface and volume.

Answer

For frustum:

Base radius, r' = 20/2 = 10cm

Top radius, r″ = 12/2 = 6cm

Height, h = 3cm

Volume = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$

$= \frac{1}{3}\pi(10^2 + 6^2 + 10\times6)\times3$

= 616 cub. cm

Let l be the slant height of the cone, then

$\Rightarrow 1 = \sqrt{(r' - r'')^2 + h^2}$

$\Rightarrow 1 = \sqrt{(10 - 6)^2 + 3^2}$

⇒ l = 5cm

Total surface area of the frustum = п (r' + r'') × l + п r'² + п r''²

= п (20 + 10) × 15.620 + п (10)² + п (6)²
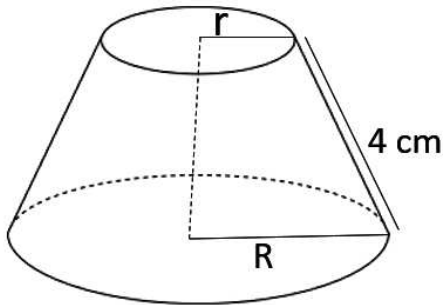
= 678.85 cm²

3. Question

The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface of the frustum.

Answer

Given: The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm.

To find: the curved surface of the frustum.

Solution: For the given frustum:

Slant height, l = 4 cm

Perimeter of its circular ends = 18 cm and 6 cm

Let radius of larger and shorter ends be 'r' and 'R' resp., then

Perimeter of larger circular end = 2 п r = 18 cm

⇒ п r = 9

Similarly, 2п R = 6

⇒ п R = 3

Curved surface of the frustum = (п r + п R) × l

= (9 + 3) × 4 = 12 × 4

= 48 cm²

4. Question

The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.

Answer

For the frustum,

Perimeter of the larger end = 44cm

$\Rightarrow 2\pi r' = 44$

$\Rightarrow r' = \dfrac{44}{2 \times \dfrac{22}{7}} = 7cm$

Perimeter of the smaller end = 33cm

$\Rightarrow 2\pi r'' = 33$

$\Rightarrow r'' = \dfrac{33}{2 \times \dfrac{22}{7}} = \dfrac{21}{4} cm \ or \ 5.25cm$

Height = 16cm

Its volume $= \dfrac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$= \dfrac{1}{3}\pi\left(7^2 + \left(\dfrac{21}{4}\right)^2 + 7 \times \dfrac{21}{4}\right) \times 16$

$= \dfrac{1}{3} \times \dfrac{22}{7}(784 + 441 + 588)$

≈ 1900 cub. cm

Let l be the slant height of the cone, then

$\Rightarrow l = \sqrt{(r' - r'')^2 + h^2}$

$\Rightarrow l = \sqrt{(7 - 5.25)^2 + 16^2}$

$\Rightarrow l = 16.1cm$

Curved surface area of frustum = π (r' + r'') × l

= 3.14(7 + 5.25) × 16.1

= 619.65 cm²

Total surface area of the frustum = π (r' + r'') × l + π r'² + π r''²

= 619.65 + 3.14 × (7² + 5.25²)

= 860.275 cm²

5. Question

If the radii of the circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm, find the capacity of the bucket. (*Use* π = 22 / 7)

Answer

For the bucket:

Height = 45cm

Radius of top = 28cm

Radius of bottom = 7cm

Volume of the bucket = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$

$= \frac{1}{3}\pi(28^2 + 7^2 + 28\times7)\times45$

$= \frac{22}{7}\times15(784 + 49 + 196)$

= 48510 cub. cm

6. Question

The height of a cone is 20 cm. A small cone is cut off from the top by a plane parallel to the base. If its volume be 1/125 of the volume of the original cone, determine at what height above the base the section is made.

Answer

Height of the cone = 20cm

Let the small cone was cut at the height x from the top then height of the small cone = x cm

From cones ABC and AEF,

$\frac{h'}{h''} = \frac{r'}{r''}$ (h′, r′ and h″, r″ are heights and radius of larger and smaller cones resp.)

$\Rightarrow \frac{20}{x} = \frac{r'}{r''}$ -------(i)

Given: Volume of the small cone = $\frac{1}{125}$ × volume of the large cone

And Volume of a cone = $\frac{1}{3}\pi(r^2)h$
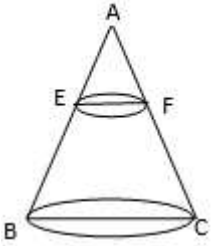
$\therefore \frac{1}{3}\pi(r''^2)x = \frac{1}{125}\left(\frac{1}{3}\pi(r'^2)(20)\right)$

$\Rightarrow \frac{r''^2}{r'^2} = \frac{1}{125}\left(\frac{20}{x}\right)$

$\Rightarrow \frac{x^2}{20^2} = \frac{1}{125}\left(\frac{20}{x}\right)$

$$\Rightarrow x^3 = \left(\frac{20}{5}\right)^3$$

$\Rightarrow$ x = 4cm

Height at which section is made = 20 − 4 = 16cm

7. Question

If the radii of the circular ends of a conical bucket 45 cm high are 5 cm and 15 cm respectively, find the surface area of the bucket.

Answer

Given: for the bucket,

Height = 45cm

r′ = 5cm

r″ = 15cm

Curved surface area = п(r′ + r″)l + п r″^{2}

= п(5 + 15)(26) + п (15)^2

= 745 п cm^2

Curved surface area of bucket = 745п cm^2

8. Question

The radii of the circular bases of a frustum of a right circular cone are 12 cm and 3 cm and the height is 12 cm. Find the total surface area and the volume of the frustum.

Answer

Given: radius of bases of frustum = 12cm and 3cm and its height = 12cm

Let l be the slant height of the cone, then

$$\Rightarrow l = \sqrt{(r' - r'')^2 + h^2}$$

$$\Rightarrow l = \sqrt{(12 - 3)^2 + 12^2}$$

$\Rightarrow$ l = 15cm

Total surface area of the frustum = п (r′ + r″) × l + п r′^{2} + п r″^{2}

$= \text{п} (12 + 3) + \text{п} \times (12^2 + 3^2)$

$= 378 \text{ cm}^2$

Volume $= \frac{1}{3}\pi\left(r'^2 + r''^2 + r'r''\right)h$

$= \frac{1}{3}\pi(12^2 + 3^2 + 12\times3)\times12$

$= 756 \text{ п cm}^3$

9. Question

A tent consists of a frustum of a cone capped by a cone. If the radii of the ends of the frustum be 13 m and 7 m, the height of the frustum be 8 m and the slant height of the conical cap be 12 m, find the canvas required for the tent.

(Take : π = 22 / 7)

Answer

Height of the frustum, h = 8m

Radii of frustum are: r' = 13m and r'' = 7m

Let l be the slant height of the frustum

$\Rightarrow l = \sqrt{(r' - r'')^2 + h^2}$

$\Rightarrow l = \sqrt{(13 - 7)^2 + 8^2}$

⇒ l = 10cm

Curved surface area of frustum, A' = п (r' + r'') × l

= п (13 + 7) × 10

= 628.57 cm²

Slant height of the conical cap = 12m

Base radius of the cap = 7m

Curved surface of the cap, A'' = пrl

= п × 7 × 12

= 264m²

Total canvas required = A' + A''

= 628.57 + 264

= 892.57 m²

∴ Total canvas = 892.57 m²

10. Question

A reservoir in the form of the frustum of a right circular cone contains 44 × 10⁷ litres of water which fills it completely. The radii of the bottom and top of the reservoir are 50 metres and 100 metres respectively. Find the depth of water and the lateral surface area of the reservoir. (*Take : π = 22 / 7*)

Answer

Let height and slant height of the frustum be h and l.

Radius, r' = 100m, r'' = 50m

Volume of frustum = 44 × 10⁷ litres = 44 × 10⁴ m³

$$\frac{1}{3} \times \frac{22}{7} (100^2 + 50^2 + 100 \times 50)h = 44 \times 10^7$$

$$h = \frac{44 \times 10^4 \times 7 \times 3}{22 \times 17500} = 24m$$

Slant height, $l = \sqrt{(r' - r'')^2 + h^2}$

$$\Rightarrow l = \sqrt{(100 - 50)^2 + 24^2}$$

$$\Rightarrow l = 55.46m$$

Lateral surface area of the frustum = п l (r' + r'')

$$= \frac{22}{7} \times 55.46 \times 150$$

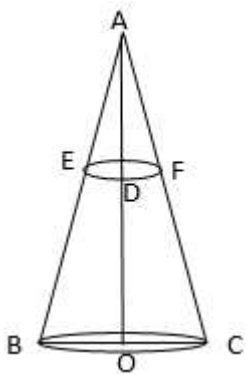$$= 26145.4 \ m^2$$

11. Question

A metallic right circular cone 20 cm high and whose vertical angle is 90° is cut into two parts at the middle point of its axis by a plane parallel to the base. If the frustum so obtained be drawn into a wire of diameter (1/16) cm, find the length of the wire.

Answer

let ABC be the given cone.

Here, the height of the metallic cone AO = 20cm

Cone is cut into two pieces at the middle point of the axis.

hence, the height of the frustum cone AD = 10cm

Since ∠A is right-angled, so ∠B = ∠C = 45°

From triangle ADE,

$$\Rightarrow \frac{DE}{AD} = cot\ 45°$$

$$\Rightarrow \frac{r'}{10} = 1$$

⇒ r'=10cm

Similarly, from triangle AOB,

$$\Rightarrow \frac{OB}{OA} = cot45°$$

$$\Rightarrow \frac{r''}{20} = 1$$

⇒ r''=20cmThe volume of the frustum of a cone

$$= \frac{1}{3}\pi\left(r_1^{\ 2} + r_2^{\ 2} + r_1 r_2\right) \times h$$

$$= \frac{1}{3}\pi\left(10^2 + 20^2 + 10 \times 20\right) \times 10$$

$$= \frac{1}{3} \times \frac{22}{7}(100 + 400 + 200) \times 10$$

$$= \frac{1}{3} \times \frac{22}{7} \times 700 \times 10$$

$$= \frac{22000}{3}$$

12. Question

A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. (Use π = 3.14)

Answer

Volume of the frustum = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$ = 12308.8

$$\Rightarrow \frac{1}{3}\pi(20^2 + 12^2 + (20)(12))h = 12308.8$$

⇒ h = 15cm

Let l be the slant height of the bucket

⇒ Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

⇒ $l = \sqrt{(20 - 12)^2 + 15^2}$

⇒ l = 17cm

∴ length of the bucket, l = 17cm

Curved surface area = π(r' + r'')l + π r''²

= π(20 + 12)(17) + π (12)²

= 2160.32 cm²

13. Question

A bucket made of aluminium sheet is of height 20 cm and its upper and lower ends are of radius 25 cm and 10 cm respectively. Find the cost of making the bucket if the aluminium sheet costs Rs. 70 per 100 cm². (Use π = 3.14)

Answer

Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

⇒ $l = \sqrt{(25 - 10)^2 + 20^2}$

⇒ l = 25cm

∴ Slant height of the bucket, l = 25cm

Curved surface area = π(r' + r'')l + π r''²

= π(25 + 10)(25) + π (10)²

=3061.5 cm²

Cost of making bucket per 100 cm² = Rs70

Cost of making bucket per 3061.5 cm² = $\frac{3061.5}{100} \times 70 = Rs2143.05$

∴ Total cost = Rs2143.05

14. Question

The radii of the circular ends of a solid frustum of a cone are 33 cm and 27 cm and its slant height is 10 cm. Find its total surface area.

Answer

Slant height of frustum cone, l = 10cm

Radii of circular ends of frustum cone: r' = 33cm and r'' = 27cm

Total surface area = π(r' + r'')l + π r'² + π r''²

= π(33 + 27)10 + π (33)² + π (27)²

= 7599.42cm²

15. Question

A bucket made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with diameters of its lower and upper ends as 16 cm and 40 cm respectively. Find the volume of the bucket. Also, find the cost of the bucket if the cost of metal sheet used is Rs. 20 per 100 cm^2. (Use π = 3.14)

Answer

Given:

Height of frustum, h = 16cm

Radius of lower and upper end of bucket, r' = 8cm and r'' = 20cm

Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

$\Rightarrow l = \sqrt{(20 - 8)^2 + 16^2}$

$\Rightarrow$ l = 20cm

Volume = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$

$= \frac{1}{3}\pi(8^2 + 20^2 + 8\times20)\times16$

= 10449.92 cm³

Curved surface area = π(r' + r'')l + π r''²

= π(20 + 8)(20) + π (8)²

= 624π cm²

Cost of making bucket per 100 cm² = Rs20

Cost of making bucket per 3061.5 cm² = $\frac{624\pi}{100}\times20 = Rs391.9$

∴ Total cost = Rs.391.9

16. Question

A solid is in the shape of a frustum of a cone. The diameters of the two circular ends are 60 cm and 36 cm and the height id 9 cm. Find the area of its whole surface and the volume.

Answer

Given: height of the frustum = 9cm

Radius of its lower and upper ends: r' = 30cm and r'' = 18cm

Let slant height be l

Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

⇒ l = $\sqrt{(30 - 18)^2 + 9^2}$

⇒ l = 15cm

Volume = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$

= $\frac{1}{3}\pi(30^2 + 18^2 + 30 \times 18) \times 9$

= 5292 π cm^3

Total surface area = π(r' + r'')l + π r'^{2} + π r''^{2}

= π(30 + 18)15 + π (30)^2 + π (18)^2

= 1944π cm^2

17. Question

A milk container is made of metal sheet in the shape of frustum of a cone whose volume is $10459\frac{3}{7} cm^3$. The radii of its lower and upper circular ends are 8 cm and 20 cm respectively. Find the cost of metal sheet used in making the container at the rate of Rs. 1.40 per cm^2 .

Answer

Given: volume of the frustum = $10459\frac{3}{7}$ cm^3

Radii of lower and upper ends resp.: r' = 8cm and r'' = 20cm

Volume = $\frac{1}{3}\pi(r'^2 + r''^2 + r'r'')h$

⇒ $\frac{1}{3}\pi(8^2 + 20^2 + 8 \times 20) \times h = 10459\frac{3}{7}$

⇒ h = 16cm

Let slant height be l

Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

⇒ l = $\sqrt{(20 - 8)^2 + 16^2}$

⇒ l = 20cm

Total surface area of the frustum = π(r' + r'')l + π r'^{2} + π r''^{2}

= π(20 + 8)20 + π (20)^2 + π (8)^2

= 3218.29 cm^2

cost of metal sheet used in making the container at the rate of Rs. 1.40 per cm^2 = 3218.28 × 1.40
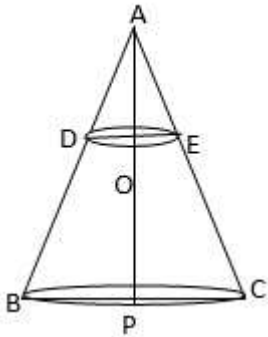
= Rs4506

18. Question

A solid cone of base radius 10 cm is cut into two parts through the mid-point of its height, by a plane parallel to its base. Find the ratio on the volumes of two parts of the cone.

Answer

Given: let the height of the cone be H and its base radius be R

This cone is divided into two parts through the mid-point of the height of the cone such that

ED||BC

Therefore triangle AED is similar to triangle ABC

By the condition of similarity,

$$\frac{OE}{PC} = \frac{AO}{AP} = \frac{AO}{2AO}$$

$$\Rightarrow \frac{OE}{R} = \frac{1}{2} \Rightarrow OE = \frac{R}{2}$$

Volume of a cone = 1/3 пr^{2}h

Volume of the frustum = Volume of the cone ABC – Volume of the cone AED

$$= \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi\frac{1}{3}\pi\left(\frac{R}{2}\right)^2\left(\frac{H}{2}\right)$$

$$= \frac{1}{3}\pi R^2 H\left(1 - \frac{1}{8}\right) = \frac{1}{3}\pi R^2 H\left(\frac{1}{7}\right)$$

$$\therefore \frac{(Ratio\ of\ part\ taken\ out)}{Ratio\ of\ remaining\ part} = \frac{\frac{1}{8}\times\frac{1}{3}\pi R^2 H}{\frac{7}{8}\times\frac{1}{3}\pi R^2 H} = \frac{1}{7}$$

19. Question

A bucket open at the top and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of Rs. 10 per 100 cm^2. (Use π = 3.14)

Answer

Given, h = 24cm, r' = 15cm and r'' = 5cm

Let slant height be l

Slant height, l = $\sqrt{(r' - r'')^2 + h^2}$

$\Rightarrow$ l = $\sqrt{(15 - 5)^2 + 24^2}$

$\Rightarrow$ l = 26cm

Total surface area of the frustum = п(r' + r'')l + п r''²

= п(15 + 5)(26) + п (5)²

= 1711.3 cm²

Cost of metal sheet used at the rate of Rs. 10 per 100 cm^2 = $\dfrac{1711.3}{100}$ × 10

= Rs171

20. Question

In Fig. 16.74, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid. $\left(Use\ \pi = 22\ /\ 7\ and\ \sqrt{5} = 2.236\right)$
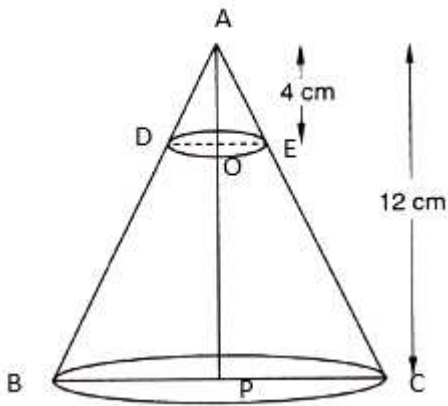
Answer

In triangle ABC and ADE,

DE||BC

So, ΔABC ~ ΔADE

$\therefore \dfrac{OE}{PC} = \dfrac{AO}{AP}$

$\Rightarrow \dfrac{OE}{6} = \dfrac{4}{12}$

$\Rightarrow$ OE = 2cm

let slant height be l

Slant height, l = $\sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{12^2 + 6^2}$

$\Rightarrow$ l = 13.42cm

Total surface area of the frustum = пr(r + l)

= п × 6(6 + 13.42)

= 366.21 cm^2

Now, for smaller cone:

Slant height, l' = $\sqrt{4^2 + 2^2}$

$\Rightarrow$ l = 4.472 cm

So, curved surface area of smaller cone = пrl

= п × 2 × 4.472 = 28.11 cm^2

Now, total surface area of the remaining frustum = total surface area of the bigger cone – curved surface area of smaller cone + area of base of smaller cone

= 366.21 – 28.11 + п (2)^2

= 350.52 cm^2