16. Circles

Exercise 16.1

1. Question

Fill in the blanks:

(i) All points lying inside/outside a circle are called …….points/………points.

(ii) Circles having the same centre and different radii are called ……. Circles.

(iii) A point whose distance from the centre of a circle is greater than its radius lies in …… of the circle.

(iv) A continuous piece of a circle is ……. of the circle.

(v) The longest chord of a circle is a …. of the circle.

(vi) An arc is a ……when its ends are the ends of a diameter.

(vii) Segment of a circle is the region between an arc and ….of the circle.

(viii) A circle divides the plane, on which it lies, in …..parts.

Answer

(i) Interior/exterior

(ii) Concentric

(iii) Exterior

(iv) Arc

(v) Diameter

(vi) Semi-circle

(vii) Centre

(viii) Three

2. Question

Write the truth value (T/F) of the following with suitable reasons:

(i) A circle is a plane figure.

(ii) Line segment joining the centre to any point on the circle is a radius of the circle.

(iii) If a circle is divided into three equal arcs each is a major arc.

(iv) A circle has only finite number of equal chords.

(v) A chord of a circle, which is twice as long is its radius is a diameter of the circle.

(vi) Sector is the region between the chord and its corresponding arc.

(vii) The degree measure of an arc is the complement of the central angle containing the arc.

(viii) The degree measure of a semi-circle is 180°.

Answer

(i) True: Because it is a one dimensional figure

(ii) True: Since, line segment joining the centre to any point on the circle is a radius of the circle

(iii) True: Because each arc measures equal

(iv) False: Since, a circle has only infinite number of equal chords

(v) True: Because, radius equal to $\frac{1}{2}$ times of its diameter

(vi) True: Yes, sector is the region between the chord and its corresponding arc

(vii) False: The degree measure of an arc is half of the central angle containing the arc

(viii) True: Yes, The degree measure of a semi-circle is 180°

Exercise 16.2

1. Question

The radius of a circle is 8 cm and the length of one of its chords is 12 cm. Find the distance of the chord from the centre.

Answer

Given that,

Radius of circle (OA) = 8 cm

Chord (AB) = 12 cm

Draw OC perpendicular to AB

We know that,

The perpendicular from centre to chord bisects the chord

Therefore,

AC = BC = $\frac{12}{2}$

= 6 cm

Now,

In ΔOCA, by using Pythagoras theorem

$AC^2 + OC^2 = OA^2$

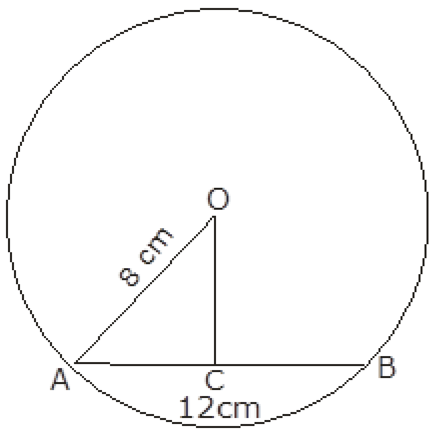
$6^2 + OC^2 = 8^2$

$36 + OC^2 = 64$

$OC^2 = 64 - 36$

$OC^2 = 28$

$OC = 5.291$ cm



2. Question

Find the length of a chord which is at a distance of 5 cm from the centre of a circle of radius 10 cm.

Answer

Given that,

Distance (OC) = 5 cm

Radius of circle (OA) = 10 cm

In ΔOCA, by using Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$AC^2 + 5^2 = 10^2$

$AC^2 = 100 - 25$

$AC^2 = 75$

$AC = 8.66$ cm

We know that,

The perpendicular from centre to chord bisects the chord

Therefore,

$AC = BC = 8.66$ cm

Then,

Chord $AB = 8.66 + 8.66$

= 17.32 cm

3. Question

Find the length of a chord which is at a distance of 4 cm from the centre of the circle of radius 6 cm.

Answer

Radius of circle (OA) = 6 cm

Distance (OC) = 4 cm

In ΔOCA, by using Pythagoras theorem

$AC^2 + OC^2 = OA^2$

$AC^2 + 4^2 = 6^2$

$AC^2 = 36 - 16$

$AC^2 = 20$

AC = 4.47 cm

We know that,

The perpendicular distance from centre to chord bisects the chord

AC = BC = 4.47 cm

Then,

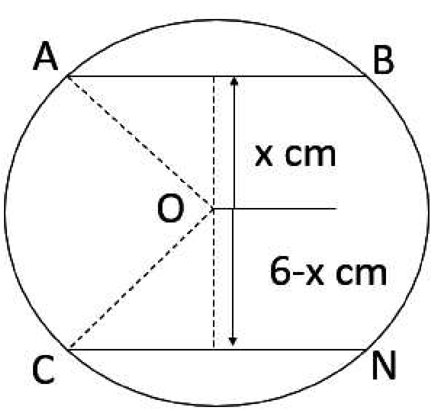
AB = 4.47 + 4.47

= 8.94 cm

4. Question

Two chords *AB, CD* of lengths 5 cm, 11 cm respectively of a circle are parallel. If the distance between *AB* and *CD* is 3 cm, find the radius of the circle.

Answer



Let r be the radius of the given circle and its center be O. Draw OM ⊥ AB and ON⊥ CD. Since, OM perpendicular AB, ON perpendicular CD.

and AB||CD

Therefore, points M, O and N are collinear.

So, MN = 6cm

Let, OM = x cm.

Then,ON = (6 - x)cm.

Join OA and OC.

Then OA = OC = r

As the perpendicular from the centre to a chord of the circle bisects the chord.

∴ AM = BM = 1/2 AB

= 1/2 x 5 = 2.5cm

CN = DN = 1/2CD

= 1/2 x 11 = 5.5cm

In right triangles OAM and OCN, we have,

$OA^2 = OM^2 + AM^2$ and $OC^2 = ON^2 + CN^2$

$$r^2 = x^2 + \left(\frac{5}{2}\right)^2 \quad \ldots\ldots (i)$$

$$r^2 = (6-x)^2 + \left(\frac{11}{2}\right)^2 \quad \ldots\ldots (ii)$$

From (i) and (ii), we have

$$x^2 + \left(\frac{5}{2}\right)^2 = (6-x)^2 + \left(\frac{11}{2}\right)^2$$

$$x^2 + \frac{25}{4} = (6-x)^2 + \frac{121}{4}$$

⇒ $4x^2 + 25 = 144 + 4x^2 - 48x + 121$

⇒ 48x = 240

⇒ x = 240/48

⇒ x = 5

Putting the value of x in euation (i), we get

$r^2 = 5^2 + (5/2)^2$

$\Rightarrow r^2 = 25 + 25/4$

$\Rightarrow r^2 = 125/4$

$\Rightarrow r = 5\sqrt{5}/2$ cm

5. Question

Give a method to find the centre of a given circle.

Answer

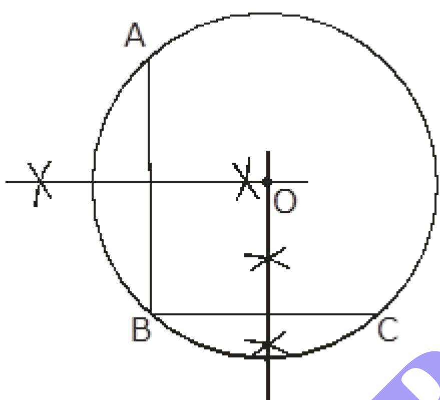
Steps of construction:

(i) Take three points A, B and C on the given circle

(ii) Join AB and BC

(iii) Draw the perpendicular bisectors of chord AB and BC which intersect each other at O

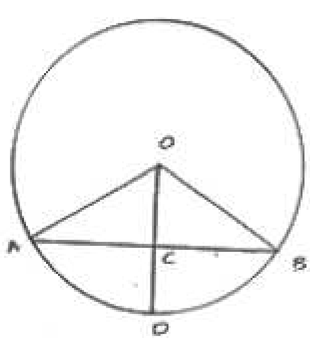
(iv) Point O will be required circle because we know that perpendicular bisector of chord always passes through centre.



6. Question

Prove that the line joining the mid-point of a chord to the centre of the circle passes through the mid-point of the corresponding minor arc.

Answer



Given that,

C is the mid-point of chord AB

To prove: D is the mid-point of arc AB

Proof: In ΔOAC and ΔOBC,

OA = OB (Radius of circle)

AC = OC (Common)

AC = BC (C is the mid-point of AB)

Then,

$\Delta OAC \cong \Delta OBC$ (By SSS congruence rule)

∠AOC = ∠BOC (By c.p.c.t)

m (A$\bar{D}$) = m (B$\bar{D}$)

A$\bar{D}$ ≅ B$\bar{D}$

Here, D is the mid-point of arc AB.

7. Question

Prove that a diameter of a circle which bisects a chord of the circle also bisects the angle subtended by the chord at the centre of the circle.

Answer

Given that,

PQ is a diameter of circle which bisects chord AB to C

To prove: PQ bisects ∠AOB

Proof: In ΔAOC and ΔBOC,

OA = OB (Radius of circle)

OC = OC (Common)

AC = BC (Given)

Then,

$\Delta ADC \cong \Delta BOC$ (By SSS congruence rule)

∠AOC = ∠BOC (By c.p.c.t)

Hence, PQ bisects ∠AOB.

8. Question

Given an arc of a circle, show how to complete the circle.

Answer

Steps of construction:

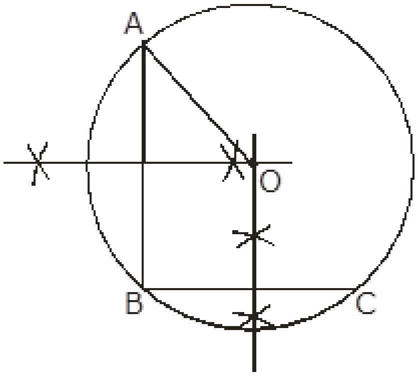
(i) Take three points A, B and C on the given arc

(ii) Join AB and BC

(iii) Draw the perpendicular bisectors of chord AB and BC which intersect each other at point O. Then, O will be the required centre of the required circle.

(iv) Join OA

(v) With centre O and radius OA, complete the circle



9. Question

Prove that two different circles cannot intersect each other at more than two points.

Answer

Suppose two circles intersect in three points A, B and C. Then A, B, C are non-collinear. So, a unique circle passes through these three points. This is contradiction to the face that two given circles are passing through A, B, C. Hence, two circles cannot intersect each other at more than two points.

10. Question

A line segment *AB* is length 5 cm. Draw a circle of radius 4 cm passing through *A* and *B.* Can you draw a circle of radius 2 cm passing through *A* and *B*? Give reason in support of your answer.

Answer

(i) Draw a line segment AB of 5 cm

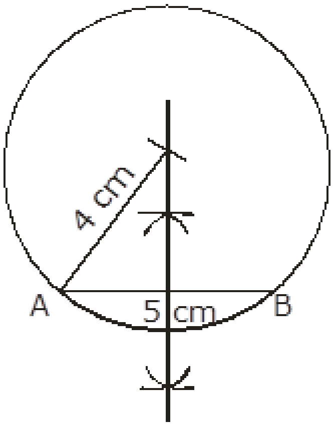
(ii) Draw the perpendicular bisector of AB

(iii) With centre A and radius of 4 cm draw an arc which intersects the perpendicular bisector at point O. O will be the required centre.

(iv) Join OA

(v) With centre O and radius OA draw a circle.

No, we cannot draw a circle of radius 2 cm passing through A and B because when we draw an arc of radius 2 cm with centre A, the arc will not intersect the perpendicular bisector and we will not find the centre.



11. Question

An equilateral triangle of side 9 cm is inscribed in a circle. Find the radius of the circle.

Answer

Let ABC be an equilateral triangle of side 9 cm

Let, AD be one of its medians and G be the centroids of the triangle ABC

Then,

AG: GD = 2: 1

We know that,

In an equilateral triangle centroid coincides with the circumcentre

Therefore,

G is the centre of the circumference with circum radius GA

Also, G is the centre and GD is perpendicular to BC

Therefore,

In right triangle ADB, we have

$AB^2 = AD^2 + DB^2$

$9^2 = AD^2 + DB^2$

$AD = \frac{9\sqrt{3}}{2}$ cm

Therefore,

Radius = AG = $\frac{2}{3}$ AD

$= 3\sqrt{3}$ cm

12. Question

Given an arc of a circle, complete the circle.

Answer

Steps of construction:

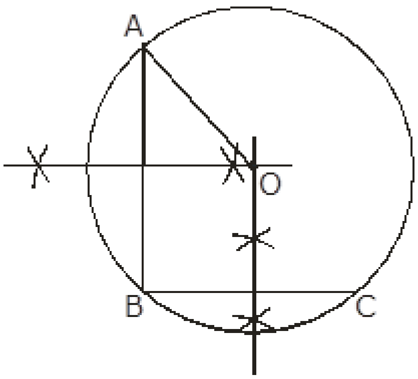
(i) Take three points A, B, C on the given arc

(ii) Join AB and BC

(iii) Draw the perpendicular bisectors of chords AB and BC which intersect each other at point O. Then, O will be the required centre of the required circle.

(iv) Join OA

(v) With centre O and radius OA, complete the circle



13. Question

Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?

Answer

Each pair of circles have 0, 1 or 2 points in common. The maximum number of points in common is 2.

14. Question

Suppose you are given a circle. Give a construction to find its centre.

Answer

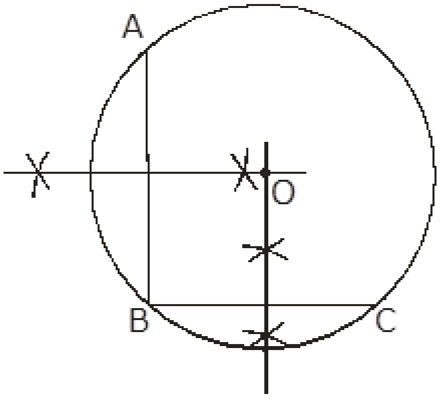
Steps of construction:

(i) Take three points A, B and C in the given circle.

(ii) Join AB and BC

(iii) Draw the perpendicular bisector of chord AB and BC which intersect each other at O

(iv) Point O will be the required centre of the circle because we know that, the perpendicular bisector of the chord always passes through the centre



15. Question

Two chords *AB* and *CD* of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are opposite side of its centre. If the distance between *AB* and *CD* is 6 cm, find the radius of the circle.

Answer

Draw OM perpendicular to AB and ON perpendicular to CD

Join OB and OC

BM = $\frac{AB}{2} = \frac{5}{2}$ (Perpendicular from centre bisects the chord)

ND = $\frac{CD}{2} = \frac{11}{2}$

Let,

ON be r so OM will be (6 − x)

In ΔMOB,

$OM^2 + MB^2 = OB^2$

$(6 - x)^2 + (\frac{5}{2})^2 = OB^2$

$36 + x^2 - 12x + \frac{25}{4} = OB^2$ (i)

In ΔNOD,

$ON^2 + ND^2 = OD^2$

$x^2 + (\frac{11}{2})^2 = OD^2$

$x^2 + \frac{121}{4} = OD^2$ (ii)

We have,

OB = OD (Radii of same circle)

So from (i) and (ii), we get

$36 + x^2 + 2x + \frac{25}{4} = x^2 + \frac{121}{4}$

$12x = 36 + \frac{25}{4} - \frac{121}{4}$

$= \frac{144+25-121}{4} = \frac{48}{4}$

$= 12$

From (ii), we get

$(1)^2 + (\frac{121}{4}) = OD^2$

$OD^2 = 1 + \frac{121}{4}$

$= \frac{125}{4}$

$OD = \frac{5\sqrt{5}}{2}$

So, radius of circle is found to be $\frac{5}{2}\sqrt{5}$ cm

16. Question

The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre?

Answer

Distance of smaller chord AB from centre of circle = 4 cm

OM = 4 cm

$MB = \frac{AB}{2} = \frac{6}{2}$

= 3 cm

In $\Delta OMB,$

$OM^2 + MB^2 = OB^2$

$(4)^2 + (3)^2 = OB^2$

$16 + 9 = OB^2$

$OB^2 = 25$

OB = 5 cm

In $\Delta OND,$

OD = OB = 5 cm (Radii of same circle)

$ND = \frac{CD}{2} = \frac{8}{2}$

= 4 cm

$ON^2 + ND^2 = OD^2$

$ON^2 + (4)^2 = (5)^2$

$ON^2 = 25 - 16$

= 9

ON = 3

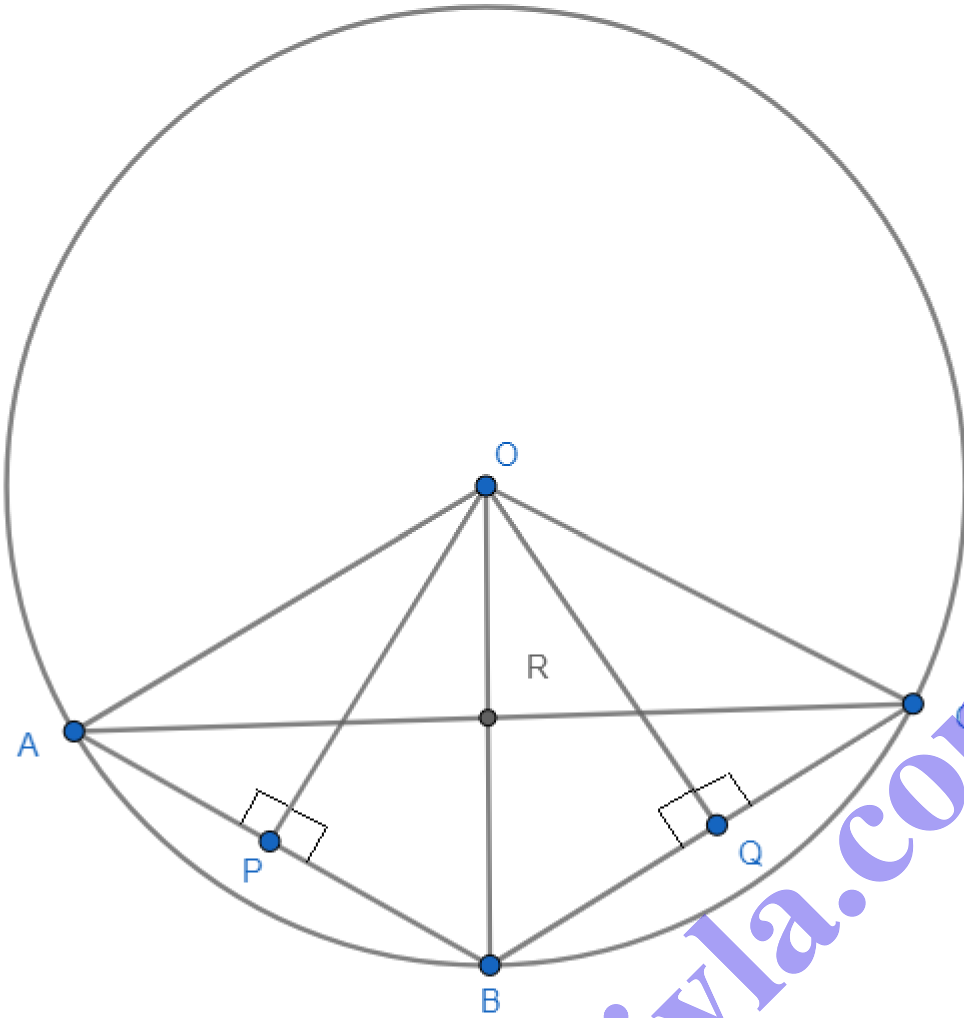
SO, distance of bigger chord from circle is 3 cm.

Exercise 16.3

1. Question

Three girls Ishita, Isha and Nisha are playing a game by standing on a circle of radius 20 m drawn in a park. Ishita throws a ball to Isha, Isha to Nisha and Nisha to Ishita. If the distance between Ishita and Isha and between Isha and Nisha is 24 m each, what is distance between Ishita and Nisha?

Answer



Let A be the position of Ishita, B be the position of Isha and C be the position of NishaGiven AB = BC = 24 m OA = OB = OC = 20 m [Radii of circle]Draw perpendiculars OP and OQ on AB and BC respectivelyAP = PB = 12 mIn right ΔOPA,$OP^2 + AP^2 = OA^2 OP^2 + (12)^2 = (20)^2 OP^2 = 256$ sq m

Therefore, OP = 16 mFrom the figure, OABC is a kite since OA = OC and AB = BC.Recall that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.Therefore, ∠ARB = 90° and AR = RCArea of ΔOAB

$= \frac{1}{2}$ x OP x AB

$= \frac{1}{2}$ x 16 x 24 = 192 sq m

Also area of ΔOAB = $\frac{1}{2}$ x OB x AR

Hence, $\frac{1}{2}$ x OB x AR = 192

$\frac{1}{2}$ x 20 x AR = 192

Therefore, AR = 19.2 mBut AC = 2AR = 2(19.2) = 38.4 mThus the distance between Ishita and Nisha is 38.4 m

2. Question

A circular park of radius 40 m is situated in a colony. Three boys Ankur, Amit and Anand are sitting at equal distance on its boundary each having a toy telephone in his hands to talk to each other. Find the length of the string of each phone.

Answer

Given that,

AB = BC = CA

So, ABC is an equilateral triangle

OA = 40 cm (Radius)

Medians of equilateral triangle pass through the circumference (O) of the equilateral triangle ABC

We also know that,

Median intersects each other at 2: 1 as AD is the median of equilateral triangle ABC, we can write:

$\frac{OA}{OD} = \frac{2}{7}$

$\frac{40}{OD} = \frac{2}{7}$

OD = 20 m

Therefore,

AO = OA + OD

= 40 + 20

= 60 m

In $\Delta ADC,$

By using Pythagoras theorem

$AC^2 = AO^2 + DC^2$

$AC^2 = (60)^2 + (\frac{AC}{2})^2$

$AC^2 = 3600 + \frac{AC*AC}{4}$

$\frac{3}{4}AC^2 = 3600$

$AC^2 = 4800$

$AC = 40\sqrt{3}$ m

So, length of string of each phone will be $40\sqrt{3}$ m

Exercise 16.4

1. Question

In Fig. 16.120, *O* is the centre of the circle. If $\angle APB = 50°$, find $\angle AOB$ and $\angle OAB$.

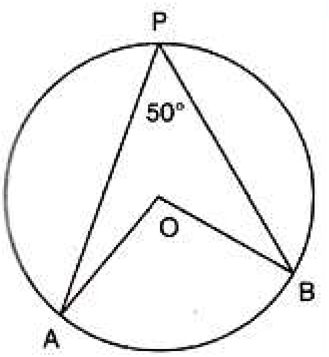


Fig. 16.120

Answer

$\angle APB = 50^o$ (Given)

By degree measure theorem,

$\angle AOB = \angle APB$

$\angle APB = 2 * 50$

$= 100^o$

Since,

OA = OB (Radii)

Hence,

$\angle OAB = \angle OBA$ (Angle opposite to equal sides are equal)

Let,

$\angle OAB = x$

In Triangle OAB,

$\angle OAB + \angle OBA + \angle AOB = 180^o$

$x + x + 100^o = 180^o$

$2x = 80^o$

$x = 40^o$

$\angle OAB = \angle OBA = 40^o$

2. Question

In Fig. 16.121, it is given that *O* is the centre of the circle and $\angle AOC = 150°$. Find $\angle ABC$.

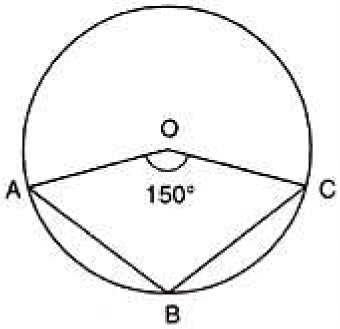


Fig. 16.121

Answer

We have,

$\angle AOC = 150^o$

Therefore,

$\angle AOC$ + Reflex $\angle AOC = 360^o$

Reflex $\angle AOC = 210^o$

$2\ \angle ABC = 210^o$ (By degree measure theorem)

$\angle ABC = 105^o$

3. Question

In Fig. 16.122, *O* is the centre of the circle. Find $\angle BAC$.

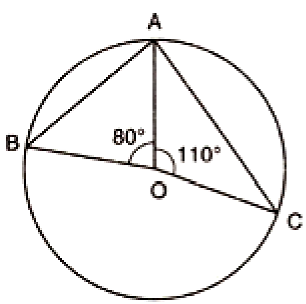


Fig. 16.122

Answer

We have,

$\angle AOB = 80^o$

$\angle AOC = 110^o$

$\angle AOB + \angle AOC + \angle BOC = 360^o$ (Complete angle)

$80^\circ + 110^\circ + \angle BOC = 360^\circ$

$\angle BOC = 170^\circ$

By degree measure theorem,

$\angle BOC = 2\angle BAC$

$170^\circ = 2\angle BAC$

$\angle BAC = 85^\circ$

4. Question

If *O* is the centre of the circle, find the value of *x* in each of the following figures :

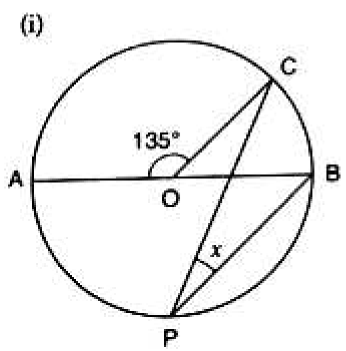


Fig. 16.123

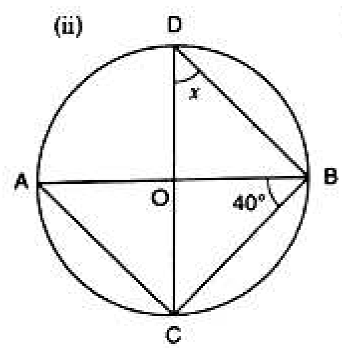


Fig. 16.124

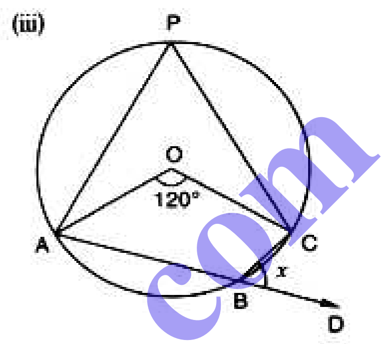


Fig. 16.125

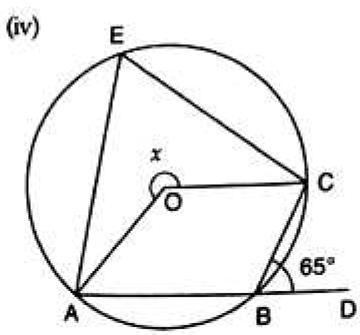


Fig. 16.126

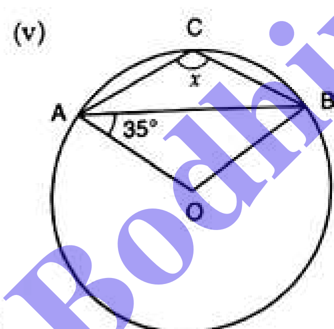


Fig. 16.127

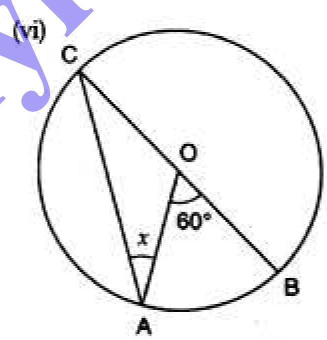


Fig. 16.128

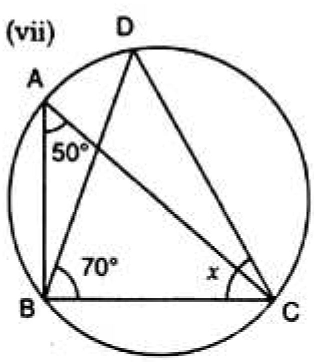


Fig. 16.129

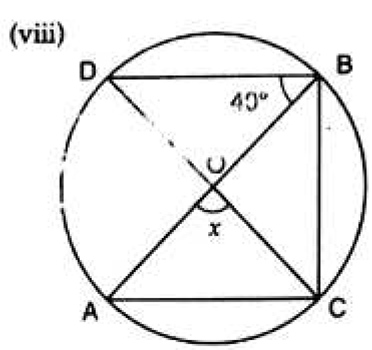


Fig. 16.130

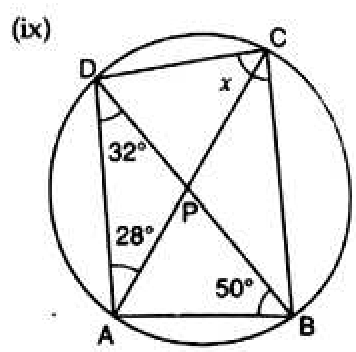


Fig. 16.131

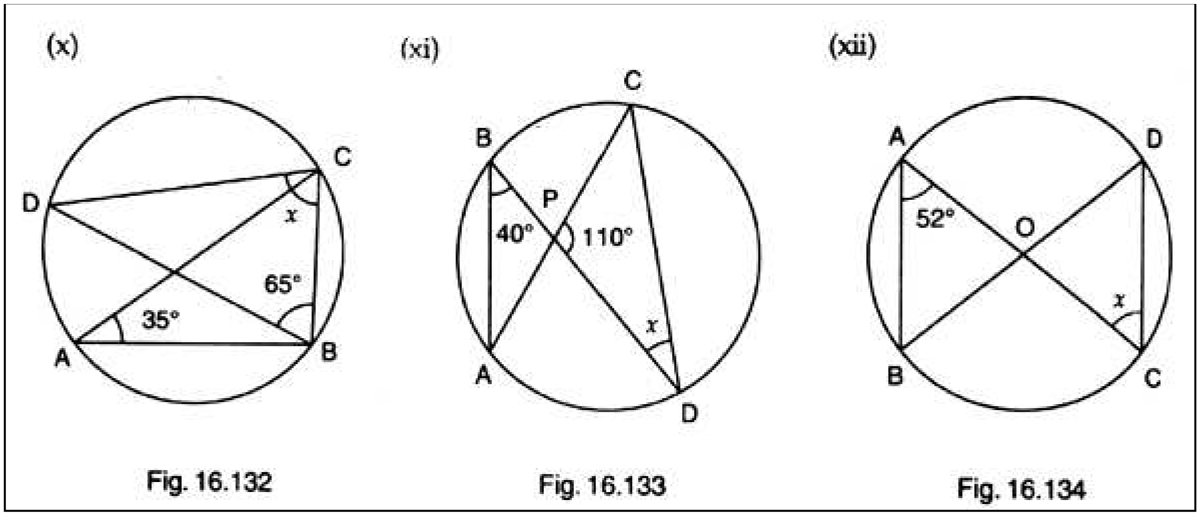


Fig. 16.132 Fig. 16.133 Fig. 16.134

Answer

(i) $\angle AOC = 135^o$

Therefore,

$\angle AOC + \angle BOC = 180^o$ (Linear pair)

$135^o + \angle BOC = 180^o$

$\angle BOC = 45^o$

By degree measure theorem,

$\angle BOC = 2 \angle COB$

$45^o = 2x$

$x = 22\frac{1}{2}{}^o$

(ii) We have,

$\angle ABC = 40^o$

$\angle ACB = 90^o$ (Angle in semi-circle)

In triangle ABC, by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180^o$

$\angle CAB + 90^o + 40^o = 180^o$

$\angle CAB = 50^o$

Now,

$\angle COB = \angle CAB$ (Angle on same segment)

$x = 50^o$

(iii) We have,

$\angle AOC = 120^o$

BY degree measure theorem,

$\angle AOC = 2\ \angle APC$

$120^o = 2\ \angle APC$

$\angle APC = 60^o$

Therefore,

$\angle APC + \angle ABC = 180^o$ (Opposite angles of cyclic quadrilateral)

$60^o + \angle ABC = 180^o$

$\angle ABC = 120^o$

$\angle ABC + \angle DBC = 180^o$ (Linear pair)

$120^o + x = 180^o$

$x = 60^o$

(iv) We have,

$\angle CBD = 65^o$

Therefore,

$\angle ABC + \angle CBD = 180^o$ (Linear pair)

$\angle ABC + 65^o = 180^0$

$\angle ABC = 115^o$

Therefore,

Reflex $\angle AOC = 2\ \angle ABC$ (By degree measure theorem)

$x = 2 * 115^o$

$= 230^o$

(v) We have,

$\angle OAB = 35^o$

Then,

$\angle OBA = \angle OAB = 35^o$ (Angle opposite to equal sides are equal)

In triangle AOB, by angle sum property

$\angle AOB + \angle OAB + \angle OBA = 180^o$

$\angle AOB + 35^o + 35^o = 180^o$

$\angle AOB = 110^o$

Therefore,

$\angle AOB$ + Reflex $\angle AOB = 360^o$ (Complete angle)

110^o + Reflex $\angle AOB = 360^o$

Reflex $\angle AOB = 250^o$

By degree measure theorem,

Reflex $\angle AOB = 2 \angle ACB$

$250^o = 2x$

$x = 125^o$

(vi) We have,

$\angle AOB = 60^o$

By degree measure theorem,

$\angle AOB = 2 \angle ACB$

$60^o = 2 \angle ACB$

$\angle ACB = 30^o$

$x = 30^o$

(vii) We have,

$\angle BAC = 50^o$

$\angle DBC = 70^o$

Therefore,

$\angle BDC = \angle BAC = 50^o$ (Angles on same segment)

In triangle BDC, by angle sum property

$\angle BDC + \angle BCD + \angle DBC = 180^o$

$50^o + x + 70^o = 180^o$

$120^o + x = 180^o$

$x = 60^o$

(viii) We have,

$\angle DBO = 40^o$

$\angle DBC = 90^o$ (Angle in semi-circle)

Therefore,

$\angle DBO + \angle OBC = 90^o$

$40^o + \angle OBC = 90^o$

$\angle OBC = 50^o$

By degree measure theorem,

$\angle AOC = 2 \angle OBC$

$x = 2 * 50^o$

$x = 100^o$

(ix) In triangle DAB, by angle sum property

$\angle ADB + \angle DAB + \angle ABD = 180^o$

$32^o + \angle DAB + 50^o= 180^o$

$\angle DAB = 98^o$

Now,

$\angle DAB + \angle DCB = 180^o$ (Opposite angle of cyclic quadrilateral)

$98^o + x = 180^o$

$x = 180^o – 98^o$

$= 82^o$

(x) We have,

$\angle BAC = 35^o$

$\angle BDC = \angle BAC = 35^o$ (Angle on same segment)

In triangle BCD, by angle sum property

$\angle BDC + \angle BCD + \angle DBC = 180^o$

$30^o + x + 65^o = 180^o$

$x = 80^o$

(xi) We have,

$\angle ABD = 40^o$

$\angle ACD = \angle ABD = 40^o$ (Angle on same segment)

In triangle PCD, by angle sum property

$\angle PCD + \angle CPD + \angle PDC = 180^o$

$40^o + 110^o + x = 180^o$

$x = 30^o$

(xii) Given that,

$\angle BAC = 52^o$

$\angle BDC = \angle BAC = 52^o$ (Angle on same segment)

Since, OD = OC

Then,

$\angle ODC = \angle OCD$ (Opposite angles to equal radii)

$x = 52^o$

5. Question

O is the circumcentre of the triangle *ANC* and *OD* is perpendicular on *BC*. Prove that $\angle BOD = \angle A$

Answer

Given that,

O is the circumcentre of triangle ABC and OD perpendicular BC

To prove: $\angle BOD = \angle A$

Proof: In triangle OBD and triangle OCD, we have

$\angle ODB = \angle ODC$ (Each 90^o)

OB = OC (Radii)

OD = OD (Common)

By R.H.S rule,

$\Delta ODB \cong \Delta ODC$

$\angle BOD = \angle COD$ (By c.p.c.t) (i)

By degree measure theorem,

$\angle BOC = 2\angle BAC$

$2\angle BOD = 2\angle BAC$ [From (i)]

$\angle BOD = \angle BAC$

Hence, proved

6. Question

In Fig. 16.135, *O* is the centre of the circle, *Bo* is the bisector of $\angle ABC$. Show that *AB=AC.*

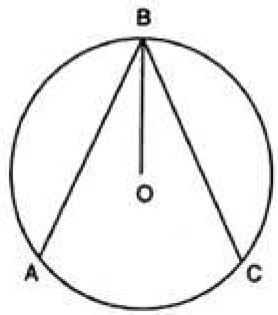


Fig. 16.135

Answer

Given that,

BO is the bisector of ∠ABC

To prove: AB = BC

Proof: ∠ABO = ∠CBO (BO bisector of ∠ABC) (i)

OB = OA (Radii)

Therefore,

∠ABO = ∠DAB (Opposite angle to equal sides are equal) (ii)

OB = OC (Radii)

Therefore,

∠CBO = ∠OCB (Opposite angles to equal sides are equal) (iii)

Compare (i), (ii) and (iii)

∠OAB = ∠OCB (iv)

In triangle OAB and OCB, we have

∠OAB = ∠OCB [From (iv)]

∠OBA = ∠OBC (Given)

OB = OB (Common)

By AAS congruence rule

$\Delta OAB \cong \Delta OCB$

$AB = BC$ (By c.p.c.t)

Hence, proved

7. Question

In Fig. 16. 136, *O* is the centre of the circle, prove that $\angle x = \angle y + \angle z$.

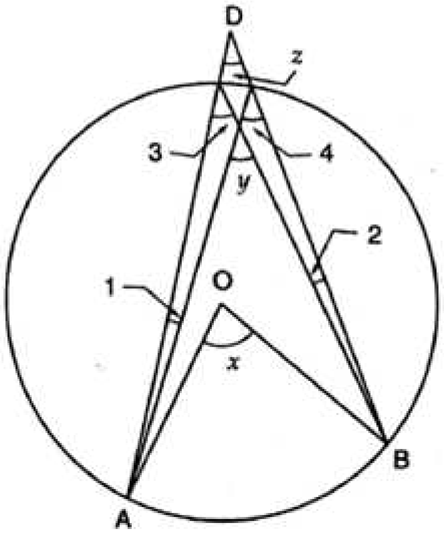


Fig. 16.136

Answer

We have,

$\angle 3 = \angle 4$ (Angle on same segment)

By degree measure theorem,

$\angle x = 2\angle 3$

$\angle x = \angle 3 + \angle 3$

$\angle x = \angle 3 + \angle 4$ (i) (Therefore, $\angle 3 = \angle 4$)

But,

$\angle y = \angle 3 + \angle 1$ (By exterior angle property)

$\angle 3 = \angle y - \angle 1$ (ii)

From (i) and (ii),

$\angle x = \angle y - \angle 1 + \angle 4$

$\angle x = \angle y + \angle 4 - \angle 1$

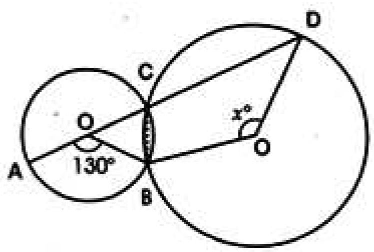
$\angle x = \angle y + \angle z + \angle 1 - \angle 1$ (By exterior angle property)

$\angle x = \angle y + \angle z$

Hence, proved

8. Question

In Fig. 16.137, *O* and *O′* are centres of two circles intersecting at *B* and *C. ACD* is a straight line, find *x*.



Answer

By degree measure theorem,

∠AOB = 2 ∠ACB

130^o = 2 ∠ACB

∠ACB = 65^o

Therefore,

∠ACB + ∠BCD = 180^o (Linear pair)

65^o + ∠BCD = 180^o

∠BCD = 115^o

By degree measure theorem,

Reflex ∠BOD = 2 ∠BCD

Reflex ∠BOD = 2 * 115^o

= 230^o

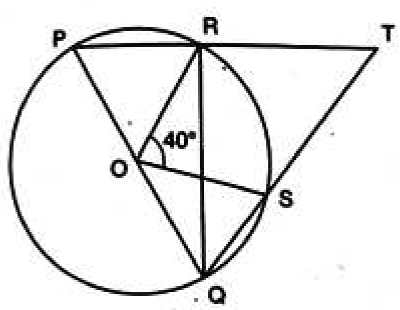
∠BOD + Reflex ∠BOD = 360^o (Complete angle)

230^o + x = 360^o

x = 130^o

9. Question

In Fig. 16.138, *O* is the centre of a circle and *PQ* is a diameter. If ∠*ROS*=40°, find ∠*RTS*.



Answer

Since,

PQ is diameter

Then,

$\angle PRQ = 90^o$ (Angle in semi-circle)

Therefore,

$\angle PRQ + \angle TRQ = 180^o$ (Linear pair)

$90^o + \angle TRQ = 180^o$

$\angle TRQ = 90^o$

By degree measure theorem,

$\angle ROS = 2\ \angle RQS$

$\angle RQS = 20^o$

In triangle RQT, we have

$\angle RQT + \angle QRT + \angle RTS = 180^o$ (By angle sum property)

$20^o + 90^o + \angle RTS = 180^o$

$\angle RTS = 70^o$

10. Question

In Fig. 16.139, if $\angle ACB$=40°, $\angle DPB$=120°, find $\angle CBD$.

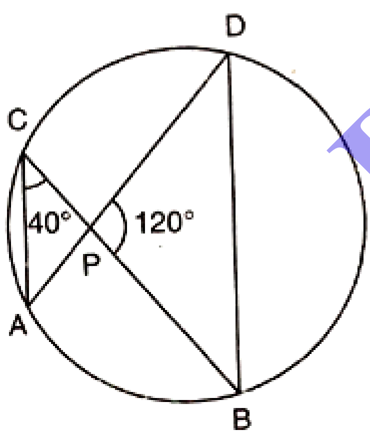


Fig. 16.139

Answer

We have,

$\angle ACB = 40^o$

$\angle DPB = 120^o$

$\angle ADB = \angle ACB = 40^o$ (Angle on same segment)

In triangle PDB, by angle sum property

$\angle PDB + \angle PBD + \angle BPD = 180^o$

$40^o + \angle PBD + 120^o = 180^o$

$\angle PBD = 20^o$

Therefore,

$\angle CBD = 20^o$

11. Question

A chord of a circle is equal to the radius of the circle. Find the angle substended by the chords at a point on the minor arc and also at a point on the major arc.

Answer

We have,

Radius OA = Chord AB

OA = OB = AB

Then, triangle OAB is an equilateral triangle

Therefore,

$\angle AOB = 60^o$ (Angle of an equilateral triangle)

By degree measure theorem,

$\angle AOB = 2\,\angle APB$

$60^o = 2\,\angle APB$

$\angle APB = 30^o$

Now,

$\angle APB + \angle AQB = 180^o$ (Opposite angle of cyclic quadrilateral)

$30^o + \angle AQB = 180^o$

$\angle AQB = 150^o$

Therefore,

Angle by chord AB at minor arc = 150^o

And, by major arc = 30^o

Exercise 16.5

1. Question

In Fig. 16.176, Δ *ABC* is an equilateral triangle. Find *m∠BEC.*

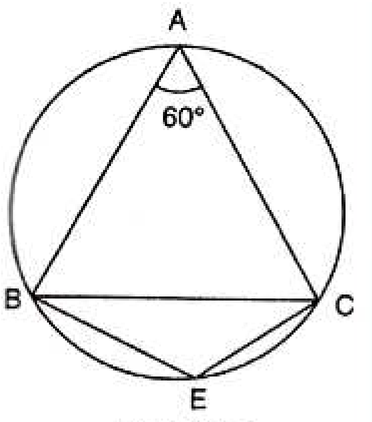


Fig. 16.176

Answer

Since,

Triangle ABC is an equilateral triangle

$\angle BAC = 60^o$

$\angle BAC + \angle BEC = 180^o$ (Opposite angles of quadrilateral)

$60^o + \angle BEC = 180^o$

$\angle BEC = 120^o$

2. Question

In Fig. 16.177, Δ *PQR* is an isosceles triangle with *PQ*=*PR* and *m∠PQR*=35°. Find *m∠QSR* and *m∠QTR*.

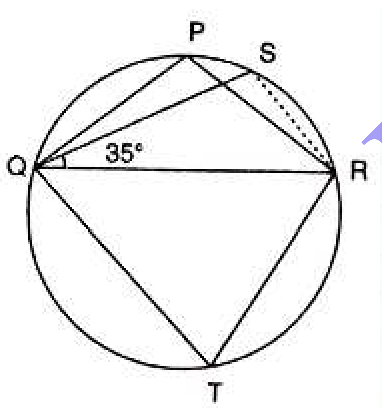


Fig. 16.177

Answer

We have,

$\angle PQR = 35^o$

$\angle PQR + \angle PRQ = 35^o$ (Angle opposite to equal sides)

In triangle PQR, by angle sum property

$\angle P + \angle Q + \angle R = 180^o$

$\angle P + 35^o + 35^o = 180^o$

$\angle P = 110^o$

Now,

$\angle QSR + \angle QTR = 180^o$

$110^o + \angle QTR = 180^o$

$\angle QTR = 70^o$

3. Question

In Fig. 16.178, *O* is the centre of the circle. If $\angle BOD = 160°$, find the values of *x* and y*.*

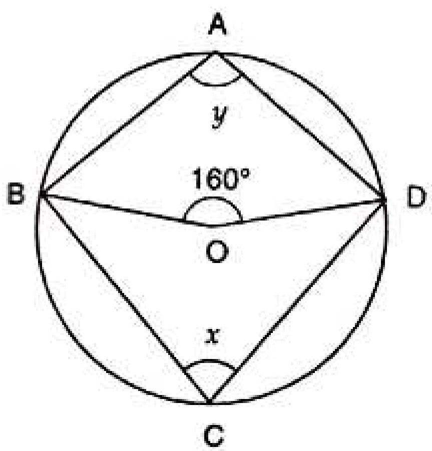


Fig. 16.178

Answer

Given that,

O is the centre of the circle

We have,

$\angle BOD = 160^o$

By degree measure theorem,

$\angle BOD = 2\ \angle BCD$

$160^o = 2x$

$x = 80^o$

Therefore,

$\angle BAD + \angle BCD = 180^o$ (Opposite angles of Cyclic quadrilateral)

$y + x = 180^o$

y + 80° = 180°

y = 100°

4. Question

In Fig. 16.179 *ABCD* is a cyclic quadrilateral. If ∠*BCD*=100° and ∠*ABD*=70°, find ∠*ADB.*

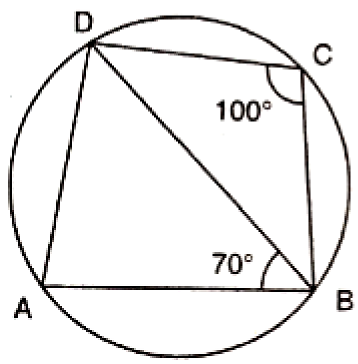


Fig. 16.179

Answer

We have,

∠BCD = 100°

∠ABD = 70°

Therefore,

∠DAB + ∠BCD = 180° (Opposite angles of cyclic quadrilateral)

∠DAB + 100° = 180°

∠DAB = 180° – 100°

= 80°

In triangle DAB, by angle sum property

∠ADB + ∠DAB + ∠ABD = 180°

∠ABD + 80° + 70° = 180°

∠ABD = 30°

5. Question

If *ABCD* is a cyclic quadrilateral in which *AD*||*BC* (fig. 16.180). Prove that ∠*B*=∠*C*.

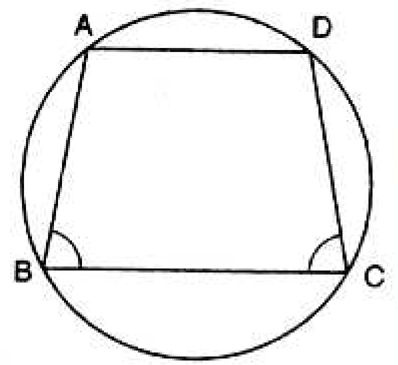


Fig. 16.180

Answer

Since, ABCD is a cyclic quadrilateral with AD ∥ BC

Then,

$\angle A + \angle C = 180^{\circ}$ (i) (Opposite angles of cyclic quadrilateral)

And,

$\angle A + \angle B = 180^{\circ}$ (ii) (Co. interior angles)

Comparing (i) and (ii), we get

$\angle B = \angle C$

Hence, proved

6. Question

In Fig. 16.181, *O* is the centre of the circle. Find $\angle CBD$.

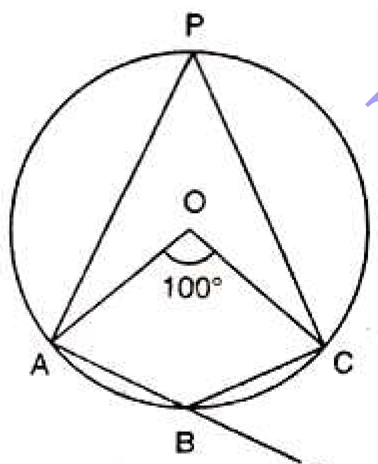


Fig. 16.181

Answer

Given that,

$\angle BOC = 100^{\circ}$

By degree measure theorem,

$\angle AOC = 2\ \angle APC$

$100^o = 2\ \angle APC$

$\angle APC = 50^o$

Therefore,

$\angle APC + \angle ABC = 180^o$ (Opposite angles of a cyclic quadrilateral)

$50^o + \angle ABC = 180^o$

$\angle ABC = 130^o$

Therefore,

$\angle ABC + \angle CBD = 180^o$ (Linear pair)

$130^o + \angle CBD = 180^o$

$\angle CBD = 50^o$

7. Question

In Fig. 16.182, *AB* and *CD* are diameters of a circle with centre *O.* If $\angle OBD$=50°, find $\angle AOC$.

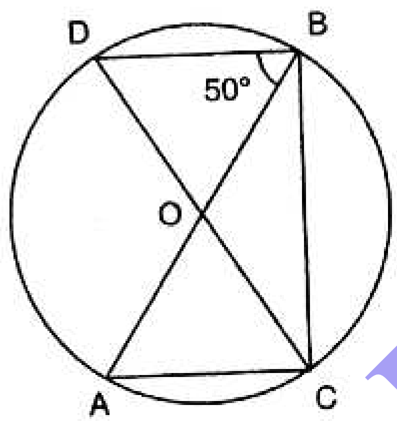


Fig. 16.182

Answer

Given that,

$\angle OBD = 50^o$

Since,

AB and CD are the diameters of the circles and O is the centre of the circle

Therefore,

$\angle PBC = 90^o$ (Angle in the semi-circle)

$\angle OBD + \angle DBC = 90^o$

$50^o + \angle DBC = 90^o$

$\angle DBC = 40^o$

By degree measure theorem,

$\angle AOC = 2\ \angle ABC$

$\angle AOC = 2 * 40^o$

$= 80^o$

8. Question

On a semi-circle with *AB* as diameter, a point *C* is taken, so that $m(\angle CAB) = 30°$. Find $m(\angle ACB)$ and $m(\angle ABC)$.

Answer

We have,

$\angle CAB = 30^o$

$\angle ACB = 90^o$ (Angle in semi-circle)

IN triangle ABC, by angle sum property

$\angle CAB + \angle ACB + \angle ABC = 180^o$

$30^o + 90^o + \angle ABC = 180^o$

$\angle ABC = 60^o$

9. Question

In a cyclic quadrilateral *ABCD* if $AB || CD$ and $B = 70°$, find the remaining angles.

Answer

Given that,

$\angle B = 70^o$

Since, ABCD is a cyclic quadrilateral

Then,

$\angle B + \angle D = 180^o$

$70^o + \angle D = 180^o$

$\angle D = 110^o$

Since, AB ∥ DC

Then,

$\angle B + \angle C = 180^o$ (Co. interior angle)

$70^o + \angle C = 180^o$

$\angle C = 110^o$

Now,

$\angle A + \angle C = 180^o$ (Opposite angles of cyclic quadrilateral)

$\angle A + 110^o = 180^o$

$\angle A = 70^o$

10. Question

In a cyclic quadrilateral *ABCD,* if $m \angle A = 3(m\angle C)$. Find $m\angle A.$

Answer

WE have,

$\angle A = 3 \angle C$

Let, $\angle C = x$

Therefore,

$\angle A + \angle C = 180^o$ (Opposite angles of cyclic quadrilateral)

$3x + x = 180^o$

$4x = 180^o$

$x = 45^o$

$\angle A = 3x$

$= 3 * 45^o$

$= 135^o$

Therefore,

$\angle A = 135^o$

11. Question

In Fig. 16.183, *O* is the centre of the circle $\angle DAB = 50°$. Calculate the values of *x* and *y*.

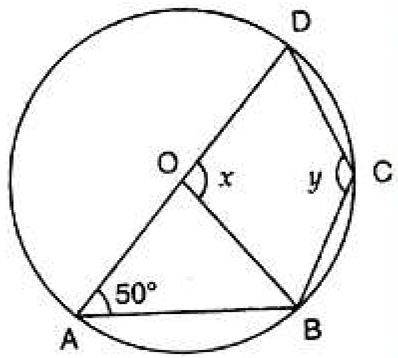


Fig. 16.183

Answer

We have,

$\angle DAB = 50^o$

By degree measure theorem

$\angle BOD = 2\ \angle BAD$

$x = 2 * 50^o$

$= 100^o$

Since, ABCD is a cyclic quadrilateral

Then,

$\angle A + \angle C = 180^o$

$50^o + y = 180^o$

$y = 130^o$

12. Question

In Fig. 16.184, if $\angle BAC=60°$ and $\angle BCA=20°$, find $\angle ADC$.

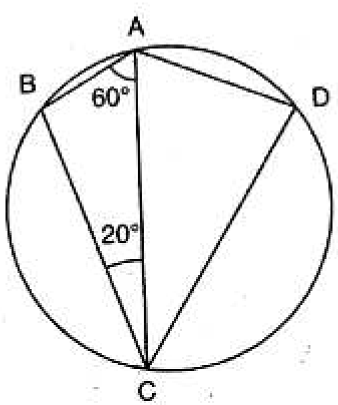


Fig. 16.184

Answer

By using angle sum property in triangle ABC,

$\angle B = 180^o - (60^o + 20^o)$

$= 100^o$

In cyclic quadrilateral ABCD, we have

$\angle B + \angle D = 180^o$

$\angle D = 180^o - 100^o$

$= 100^o$

13. Question

In Fig. 16.185, if *ABC* is an equilateral triangle. Find $\angle BDC$ and $\angle BEC$

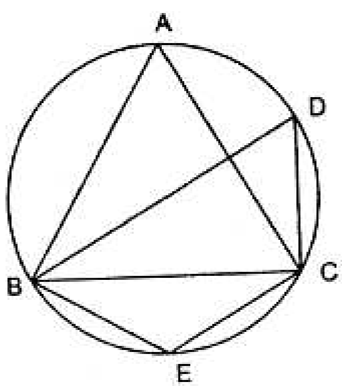


Fig. 16.185

Answer

Since, ABC is an equilateral triangle

Then,

$\angle BAC = 60^o$

Therefore,

$\angle BDC = \angle BAC = 60^o$ (Angles in the same segment)

Since, quadrilateral ABEC is a cyclic quadrilateral

Then,

$\angle BAC + \angle BEC = 180^o$

$60^o + \angle BEC = 180^o$

$\angle BEC = 180^o - 60^o$

$= 120^o$

14. Question

In Fig. 16.186, *O* is the centre of the circle. If ∠*CEA*=30°, find the values of *x, y* and *z*.

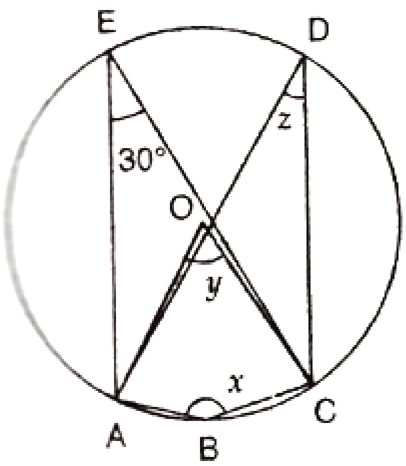


Fig. 16.186

Answer

We have,

∠AEC = 30^o

Since, quadrilateral ABCE is a cyclic quadrilateral

Then,

∠BAC + ∠AEC = 180^o

x + 30^o= 180^o

x = 150^o

By degree measure theorem,

∠AOC = 2 ∠AEC

y = 2 * 30^o

= 60^o

Therefore,

∠ADC = ∠AEC (Angles in same segment)

z = 30^o

15. Question

In Fig. 16.187, ∠*BAD*=78°, ∠*DCF*=*x*° and ∠*DEF*=*y*°. Find the values of *x* and *y*.

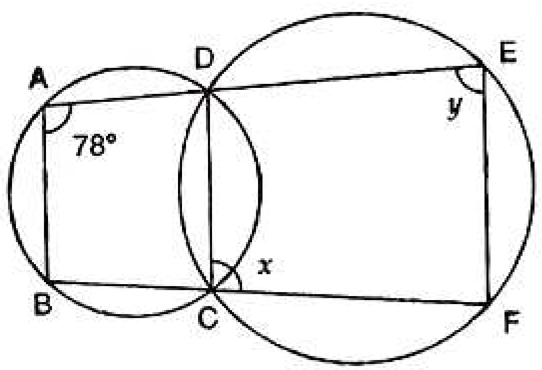


Fig. 16.187

Answer

We have,

$\angle BAD = 78^o$

$\angle DCF = x^o$

$\angle DEF = y^o$

Since, ABCD is a cyclic quadrilateral

$\angle BAD + \angle BCD = 180^o$

$78^o + \angle BCD = 180^o$

$\angle BCD = 102^o$

Now,

$\angle BCD + \angle DCF = 180^o$ (Linear pair)

$102^o = x - 180^o$

$x = 78^o$

Since,

DCEF is a cyclic quadrilateral

$x + y = 180^o$

$78^o + y = 180^o$

$y = 102^o$

16. Question

In a cyclic quadrilateral *ABCD,* if $\angle A$-$\angle C$=60°, prove that the smaller of two is 60°.

Answer

WE have,

$\angle A - \angle C = 60^o$ (i)

Since, ABCD is a cyclic quadrilateral

Then,

$\angle A + \angle C = 180^o$ (ii)

Adding (i) and (ii), we get

$\angle A - \angle C + \angle A + \angle C = 60^o + 180^o$

$2 \angle A = 240^o$

$\angle A = 120^o$

Put value of $\angle A$ in (ii), we get

$120^o + \angle C = 180^o$

$\angle C = 60^o$

17. Question

In Fig. 16.188, *ABCD* is a cyclic quadrilateral. Find the value of *x*.

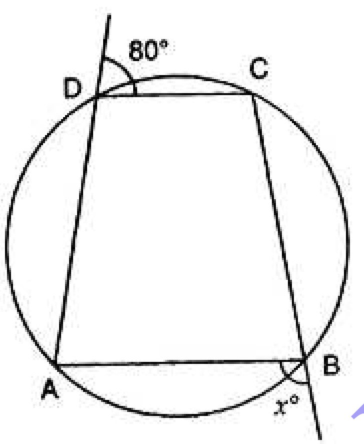


Fig. 16.188

Answer

$\angle FDC + \angle CDA = 180^o$ (Linear pair)

$80^o + \angle CDA = 180^o$

$\angle CDA = 100^o$

Since, ABCD is a cyclic quadrilateral

$\angle ADC + \angle ABC = 180^o$

$100^o + \angle ABC = 180^o$

$\angle ABC = 80^o$

Now,

$\angle ABC + \angle ABF = 180^o$ (Linear pair)

$80^o + x = 180^o$

$x = 180^o - 80^o$

$= 100^o$

18. Question

ABCD is a cyclic quadrilateral in which:

(i) BC||AD, $\angle ADC = 110°$ and $\angle BAC = 50°$. Find $\angle DAC$.

(ii) $\angle DBC = 80°$ and $\angle BAC = 40°$ Find $\angle BCD$.

(iii) $\angle BCD = 100°$ and $\angle ABD = 70°$. Find $\angle ADB$.

Answer

(i) Since, ABCD is a cyclic quadrilateral

Then,

$\angle ABC + \angle ADC = 180^o$

$\angle ABC + 110^o = 180^o$

$\angle ABC = 70^o$

Since, AD ∥ BC

Then,

$\angle DAB + \angle ABC = 180^o$ (Co. interior angle)

$\angle DAC + 50^o + 70^o = 180^o$

$\angle DAC = 180^o - 120^o$

$= 60^o$

(ii) $\angle BAC = \angle BDC = 40^o$ (Angle in the same segment)

In $\Delta BDC,$ by angle sum property

$\angle DBC + \angle BCD + \angle BDC = 180^o$

$80^o + \angle BCD + 40^o = 180^o$

$\angle BCD = 60^o$

(iii) Given that,

Quadrilateral ABCD is a cyclic quadrilateral

Then,

$\angle BAD + \angle BCD = 180^o$

$\angle BAD = 80^o$

In triangle ABD, by angle sum property

$\angle ABD + \angle ADB + \angle BAD = 180^o$

$70^o + \angle ADB + 80^o = 180^o$

$\angle ADB = 30^o$

19. Question

Prove that the perpendicular bisectors of the sides of a cyclic quadrilateral are concurrent.

Answer

Let ABCD be a cyclic quadrilateral and let O be the centre of the corresponding circle

Then, each side of the equilateral ABCD is a chord of the circle and the perpendicular bisector of a chord always passes through the centre of the circle

So, right bisectors of the sides of the quadrilateral ABCD will pass through the centre O of the corresponding circle.

20. Question

Prove that the centre of the circle circumscribing the cyclic rectangle *ABCD* is the point of intersection of its diagonals.

Answer

Let O be the circle circumscribing the cyclic rectangle ABCD.

Since, $\angle ABC = 90^o$ and AC is the chord of the circle. Similarly, BD is a diameter

Hence, point of intersection of AC and BD is the centre of the circle.

21. Question

Prove that the circles described on the four sides of a rhombus as diameters, pass through the point of intersection of its diagonals.

Answer

Let ABCD be a rhombus such that its diagonals AC and BD intersects at O

Since, the diagonals of a rhombus intersect at right angle

Therefore,

$\angle ACB = \angle BOC = \angle COD = \angle DOA = 90^o$

Now,

$\angle AOB = 90^o$ = circle described on AB as diameter will pass through O

Similarly, all the circles described on BC, AD and CD as diameter pass through O.

22. Question

If the two sides of a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are equal.

Answer

Given that,

ABCD is cyclic quadrilateral in which AB = DC

To prove: AC = BD

Proof: In ΔPAB and ΔPDC,

AB = DC (Given)

∠BAP = ∠CDP (Angles in the same segment)

∠PBA = ∠PCD (Angles in the same segment)

Then,

$\Delta PAB = \Delta PDC$ (i) (By c.p.c.t)

$PC = PB$ (ii) (By c.p.c.t)

Adding (i) and (ii), we get

PA + PC = PD + PB

AC = BD

23. Question

ABCD is a cyclic quadrilateral in which *BA* and *CD* when produced meet in *E* and *EA=ED.* Prove that:

(i) *AD*||*BC* (ii) *EB=EC*

Answer

Given that, ABCD is a cyclic quadrilateral in which

(i) Since,

EA = ED

Then,

∠EAD = ∠EDA (i) (Opposite angles to equal sides)

Since, ABCD is a cyclic quadrilateral

Then,

∠ABC + ∠ADC = 180^{o}

But,

$\angle ABC + \angle EBC = 180^o$ (Linear pair)

Then,

$\angle ADC = \angle EBC$ (ii)

Compare (i) and (ii), we get

$\angle EAD = \angle EBC$ (iii)

Since, corresponding angles are equal

Then,

$BC \parallel AD$

(ii) From (iii), we have

$\angle EAD = \angle EBC$

Similarly,

$\angle EDA = \angle ECB$ (iv)

Compare equations (i), (iii) and (iv), we get

$\angle EBC = \angle ECB$

EB = EC (Opposite angles to equal sides)

24. Question

Circles are described on the sides of a triangle as diameters. Prove that the circles on any two sides intersect each other on the third side (or third side produced).

Answer

Since,

AB is a diameter

Then,

$\angle ADB = 90^o$ (i) (Angle in semi-circle)

Since,'

AC is a diameter

Then,

$\angle ADC = 90^o$ (ii) (Angle in semi-circle)

Adding (i) and (ii), we get

$\angle ADB + \angle ADC = 90^o + 90^o$

$\angle BDC = 180^o$

Then, BDC is a line

Hence, the circles on any two sides intersect each other on the third side.

25. Question

Prove that the angle in a segment shorter than a semicircle is greater than a right angle.

Answer

Given that,

$\angle ACB$ is an angle in minor segment

To prove: $\angle ACB > 90^{o}$

Proof: By degree measure theorem

Reflex $\angle AOB = 2 \angle ACB$

And,

Reflex $\angle AOB > 180^{o}$

Then,

$2 \angle ACB > 180^{o}$

$\angle ACB > \frac{180}{2}$

$\angle ACB > 90^{o}$

Hence, proved

26. Question

Prove that the angle in a segment greater than a semi-circle is less than a right angle.

Answer

Given that,

$\angle ACB$ is an angle in major segment

To prove: $\angle ACB > 90^{o}$

Proof: By degree measure theorem,

$\angle AOB = 2 \angle ACB$

And,

$\angle AOB < 180^{o}$

Then,

$2 \angle ACB < 180^{o}$

$\angle ACB < 90^{o}$

Hence, proved

27. Question

ABCD is a cyclic trapezium with *AD*||*BC.* If $\angle B$=70°, determine other three angles of the trapezium.

Answer

Given that,

ABCD is a cyclic trapezium with AD ∥ BC and $\angle B = 70^o$

Since, ABCD is a quadrilateral

Then,

$\angle B + \angle D = 180^o$

$70^o + \angle D = 180^o$

$\angle D = 110^o$

Since, AD ∥ BC

Then,

$\angle A + \angle B = 180^o$ (Co. interior angle)

$\angle A + 70^o = 180^o$

$\angle A = 110^o$

Since, ABCD is a cyclic quadrilateral

Then, $\angle A + \angle C = 180^o$

$110^o + \angle C = 180^o$

$\angle C = 70^o$

28. Question

Prove that the line segment joining the mid-point of the hypotenuse of a right triangle to its opposite vertex is half of the hypotenuse.

Answer

Let, triangle ABC be a right angle triangle at $\angle B$

Let P be the mid-point of hypotenuse AC

Draw a circle with centre P and AC as diameter

Since,

$\angle ABC = 90^o$

Therefore, the circle passes through B

Therefore,

BP = Radius

Also,

AP = CP = Radius

Therefore,

AP = BP = CP

Hence, BP = $\frac{1}{2}$ AC

29. Question

In Fig. 16.189, *ABCD* is a cyclic quadrilateral in which *AC* and *BD* are its diagonals. If $\angle DBC = 55°$ and $\angle BAC = 45°$, find $\angle BCD$.

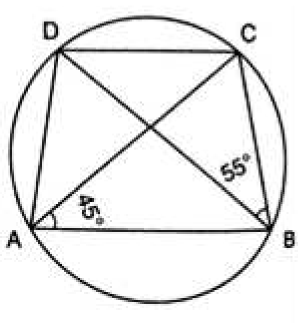


Fig. 16.189

Answer

Since angles on the same segment of a circle are equal

Therefore,

$\angle CAD = \angle DBC = 55^{o}$

$\angle DAB = \angle CAD + \angle BAC$

$= 55^{o} + 45^{o}$

$= 100^{o}$

But,

$\angle DAB + \angle BCD = 180^{o}$ (Opposite angles of a cyclic quadrilateral)

Therefore,

$\angle BCD = 180^{o} - 100^{o}$

$\angle BCD = 80^{o}$

CCE - Formative Assessment

1. Question

In Fig. 16.193, two circles intersect at *A* and *B.* The centre of the smaller circle is *O* and it lies on the circumference of the larger circle. If ∠*APB*=70°, find ∠*ACB*.

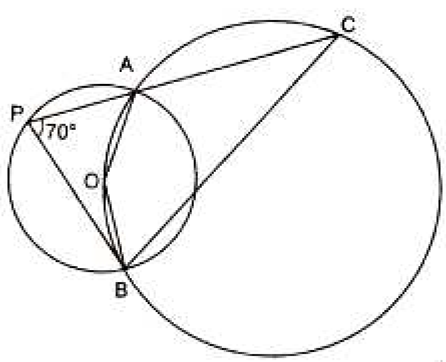


Fig. 16.193

Answer

O is the centre of the smaller circle.

∠APB = 70°

By degree measure theorem,

∠AOB = 2 ∠APB

∠AOB = 2 × 70°

= 140°

Therefore,

AOBC is a cyclic quadrilateral

∠ACB + ∠AOB = 180°

∠ACB + 140° = 180°

∠ACB = 40°

2. Question

In Fig. 16.194, two congruent circles with centres *O* and *O′* intersect at *A* and *B*. If ∠*AO′B*=50°, then find ∠*APB*.

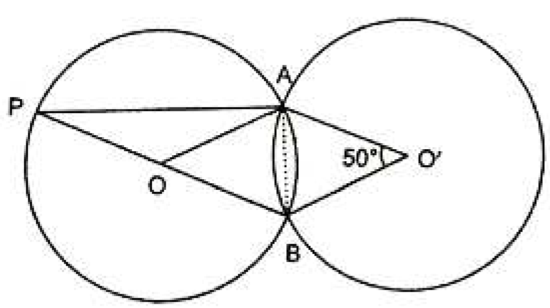


Fig. 16.194

Answer

$\angle AO'B = 50^o$

Since, both the triangle are congruent so their corresponding angles are equal.

$\angle AOB = AO'B = 50°$

Now,

$\angle APB = \frac{AOB}{2}$

$\angle APB = \frac{50}{2}$

$= 25°$

3. Question

In Fig. 16.195, *ABCD* is a cyclic quadrilateral in which $\angle BAD=75°$, $\angle ABD=58°$ and $\angle ADC=77°$, *AC* and *BD* intersect at *P.* Then, find $\angle DPC$.

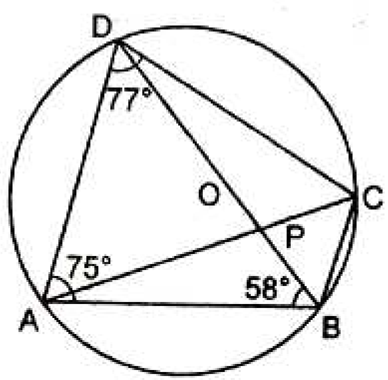


Fig. 16.195

Answer

$\angle DBA = \angle DCA = 58°$ (Angles on the same segment)

In triangle DCA

$\angle DCA + \angle CDA + \angle DAC = 180°$

$58° + 77° + \angle DAC = 180°$

$\angle DAC = 45°$

$\angle DPC = 180° - 58° - 30°$

$= 92°$

4. Question

In Fig. 16.196, if $\angle AOB = 80°$ and $\angle ABC=30°$, then find $\angle CAO$.

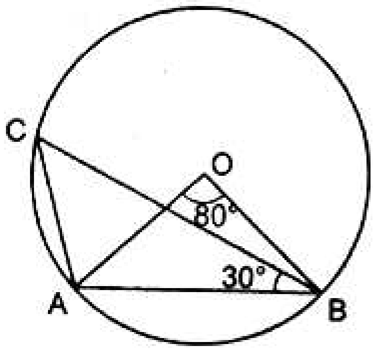


Fig. 16.196

Answer

2 ∠OAB = 100°

∠OAB = 50°

Therefore,

∠OAB = ∠OBA = 50°

∠AOB = 2 ∠BCA (Angle subtended by any point on circle)

80° = 2 ∠BCA

∠BCA = 40°

Now,

In triangle ABC

∠A + ∠B + ∠C = 180°

∠A + 30° + 40° = 180°

∠A = 110°

∠CAB = ∠CAO + ∠OAB

110° = ∠CAO + 50°

∠CAO = 60°

5. Question

In Fig. 16.196, if *O* is the circumcentre of Δ *ABC,* then find the value of ∠*OBC* + ∠*BAC*.

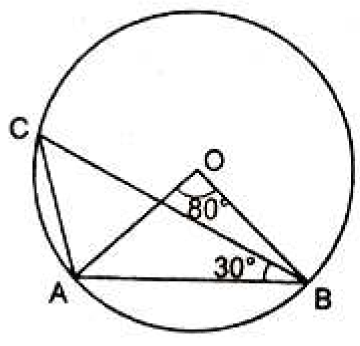


Fig. 16.196

Answer

∠OBC + ∠CBA = ∠OBA

∠OBC + 30° = 50°

∠OBC = 20°

∠OBC + ∠BAC = ∠OBC + ∠CAB

= 20° + 110°

= 130°

6. Question

In Fig. 16.197, *AOC* is a diameter if the circle and arc *AXB*= $\frac{1}{2}$ arc *BYC.* Find ∠*BOC*.

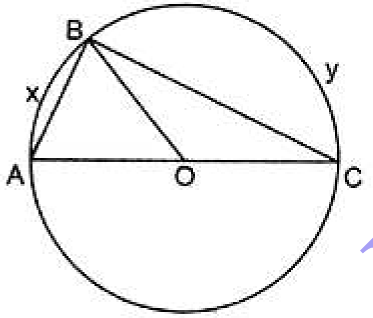


Fig. 16.197

Answer

Given that,

Arc AXB = $\frac{1}{2}$ Arc BYC (i)

Since,

Arc AXBYC is the arc equal to half circumference

And,

Angle subtended by half circumference at centre is 180°

Arc AXBYC = Arc AXB + Arc BYC

Arc AXBYC = $\frac{1}{2}$ Arc BYC + Arc BYC

Arc AXBYC = $\frac{2}{3}$ Arc AXBYC

Now,

∠BOC = $\frac{2}{3}$ ∠AOC

∠BOC = $\frac{2}{3}$ * 180°

∠BOC = 120°

7. Question

In Fig. 16.198, *A* is the centre of the circle. *ABCD* is a parallelogram and *CDE* is a straight line. Find ∠*BCD*:∠*ABE*

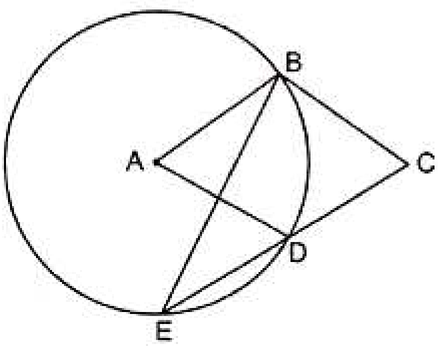


Fig. 16.198

Answer

Given that,

A is the centre of the circle, then

AB = AD

ABCD is a parallelogram, then

AD ∥ BC, AB ∥ CD

CDE is a straight line, then

AB ∥ CE

Let,

∠BEC = ∠ABE = x' (Alternate angle)

We know that,

The angle substended by an arc of a circle at the centre double the angle are angle substended by it at any point on the remaining part of circle

∠BAD = 2 ∠BEC

∠BAD = 2x′

In a rhombus opposite angles are equal to each other

∠BAD = ∠BCD = 2x′

Now, we have to find

$\frac{\angle BCD}{\angle ABE} = \frac{2x\prime}{x\prime}$

$\frac{\angle BCD}{\angle ABE} = \frac{2}{1}$

$\frac{\angle BCD}{\angle ABE} = \frac{2x\prime}{x\prime}$

Hence,

∠BCD: ∠ABE is 2: 1

8. Question

In Fig. 16.199, *AB* is a diameter of the circle such that ∠*A*=35° and ∠*Q*=25°, find ∠*PBR*.

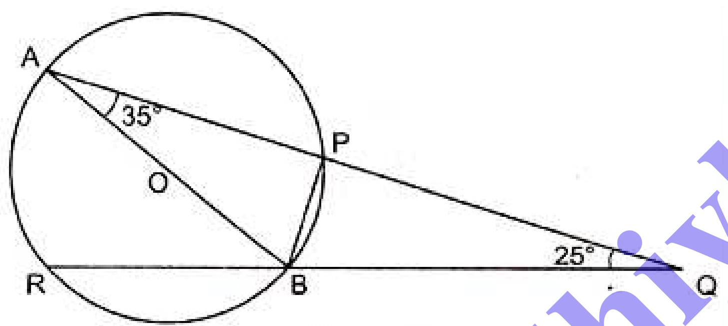


Fig. 16.199

Answer

In triangle ABQ,

∠ABQ + ∠AQB + ∠BAQ = 180^o

∠ABQ + 25^o + 35^o = 180^o

∠ABQ = 120^o

∠APB = 90^o (Angle in the semi-circle)

In triangle APB,

∠APB + ∠PBA + ∠PAB = 180^o

90^o + ∠PBA + 35^o = 180^o

∠PBA = 55^o

Now,

∠PBR = ∠PBA + ∠PBR

$\angle PBR = 55^o + (180^o - 120^o)$

$\angle PBR = 115^o$

Thus,

$\angle PBR = 115^o$

9. Question

In Fig. 16.200, *P* and *Q* are centres of two circles intersecting at *B* and *C. ACD* is a straight line. Then, $\angle BQD$=

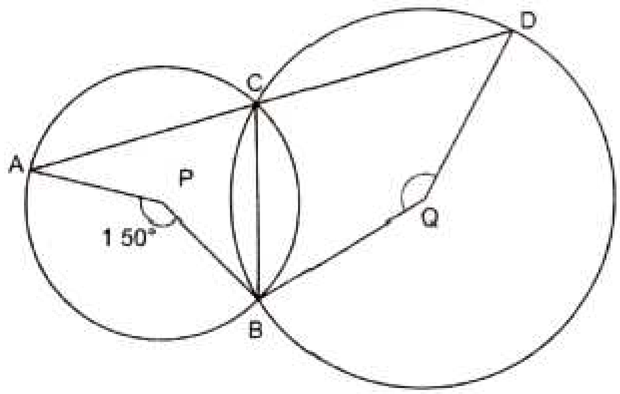


Fig. 16.200

Answer

We know that,

$\angle ACB = \frac{\angle APB}{2}$

$\angle ACB = \frac{150}{2}$

$\angle ACB = 75^o$

Since,

ACD is a straight line, so

$\angle ACB + \angle BCD = 180^o$

$75^o + \angle BCD = 180^o$

$\angle BCD = 180^o - 75^o$

$= 105^o$

Now,

$\angle BCD = \frac{1}{2}$ Reflex $\angle BQD$

$105^o = \frac{1}{2}(360^o - \angle BQD)$

$210^o = 360^o - \angle BQD$

$\angle BQD = 360^o – 210^o$

Therefore,

$\angle BQD = 150^o$

10. Question

In Fig. 16.201, *ABCD* is a quadrilateral inscribed in a circle with centre *O. CD* is produced to *E* such that $\angle AED$ = 95° and $\angle OBA$ = 30°. Find $\angle OAC$.

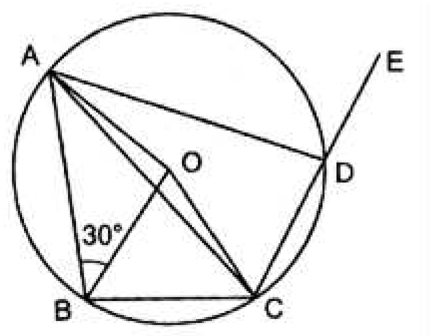


Fig. 16.201

Answer

$\angle ADE = 95^o$ (Given)

Since,

OA = OB, so

$\angle OAB = \angle OBA$

$\angle OAB = 30^o$

$\angle ADE + \angle ADC = 180^o$ (Linear pair)

$95^o + \angle ADC = 180^o$

$\angle ADC = 85^o$

We know that,

$\angle ADC = 2\ \angle ADC$

$\angle ADC = 2 * 85^o$

$\angle ADC = 170^o$

Since,

AO = OC (Radii of circle)

$\angle OAC = \angle OCA$ (Sides opposite to equal angle) (i)

In triangle OAC,

$\angle OAC + \angle OCA + \angle AOC = 180^o$

$\angle OAC + \angle OAC + 170^o = 180^o$ [From (i)]

$2\angle OAC = 10^o$

$\angle OAC = 5^o$

Thus,

$\angle OAC = 5^o$

1. Question

If the length of a chord of a circle is 16 cm and is at a distance of 15 cm from the centre of the circle, then the radius of the circle is

A. 15 cm

B. 16 cm

C. 17 cm

D. 34 cm

Answer

Let AB be the chord of length 16cm.

Given that,

Distance from centre to the chord AB is OC = 15 cm

Now,

$OC \perp AB$

Therefore,

AC = CB (Since perpendicular drawn from centre of the circle bisects the chord)

Therefore,

AC = CB = 8 cm

In right ΔOCA,

$OA^2 = AC^2 + OC^2$

$= 8^2 + 15^2$

$= 225 + 64$

$= 289$

OA = 17 cm

Thus, the radius of the circle is 17 cm

2. Question

The radius of a circle is 6 cm. The perpendicular distance from the centre of the circle to the chord which is 8 cm in length, is

A. $\sqrt{5}$ cm

B. $2\sqrt{5}$ cm

C. $2\sqrt{7}$ cm

D. $\sqrt{7}$ cm

Answer

Let, O be the centre of the circle with chord AB = 8cm

And,

OC be the perpendicular bisector of AC

AO = 6cm

AC = 4cm

In ΔAOC,

$OA^2 = AC^2 + OC^2$

$6^2 = 4^2 + OC^2$

$OC^2 = 20$

$OC = 2\sqrt{5}$

3. Question

If *O* is the centre of a circle of radius *r* and *AB* is chord of the circle at a distance *r*/2 from *O,* then ∠*BAO* =

A. 60°

B. 45°

C. 30°

D. 15°

Answer

Let, O be the centre of the circle and r be the radius

Sin A = $\frac{OA}{AC}$

$= \frac{\frac{r}{2}}{r}$

$= \frac{1}{2}$

Sin A $= \frac{1}{2}$

Sin A = Sin 30°

A = 30°

Therefore,

∠BAO = ∠CAO = 30°

4. Question

ABCD is a cyclic quadrilateral such that ∠*ADB* =30° and ∠*DCA* =80°, then ∠*DAB* =

A. 70°

B. 100°

C. 125°

D. 150°

Answer

ABCD is a cyclic quadrilateral

∠ADB = 30°

∠DCA = 80°

∠ADB = ∠ACB = 30° (Angle on the same segment)

Now,

∠BCD = ∠ACB + ∠DCA

= 30° + 80°

= 110°

∠OAB + ∠BCD = 180°

∠OAB + 110° = 180°

∠OAB = 70°

5. Question

A chord of length 14 cm is at a distance of 6 cm from the centre of a circle. The length of another chord at a distance of 2 cm from the centre of the circle is

A. 12 cm

B. 14 cm

C. 16 cm

D. 18 cm

Answer

Let AB and CD be two chords of the circle.

Draw OM perpendicular to AB and ON = CD

AB = 14 cm

OM = 6 cm

ON = 2 cm

Let,

CD = x

In ΔAOM,

$AO^2 = AM^2 + OM^2$

$= 7^2 + 6^2$

$AO^2 = 85$ (i)

In ΔCON,

$CO^2 = ON^2 + CN^2$

$CO^2 = 4 + \frac{x*x}{4}$ (ii)

We Know,

AO = CO

$AO^2 = CO^2$

$85 = 4 + \frac{x*x}{4}$

$x^2 = 324$

x = 18 cm

6. Question

One chord of a circle is known to be 10 cm. The radius of this circle must be

A. 5 cm

B. Greater than 5 cm

C. Greater than or equal to 5 cm

D. Less than 5 cm

Answer

It must be greater than 5cm.

7. Question

ABC is a triangle with *B* as right angle, *AC*=5 cm and *AB* = 4 cm. A circle is drawn with *A* as centre and *AC* as radius. The length of the chord of this circle passing through *C* and *B* is

A. 3 cm

B. 4 cm

C. 5 cm

D. 6 cm

Answer

Given: AC = radius = 5 cm

AB = 4 cm

DC is a chord passing B and C

In $\triangle$ABC

$AC^2 = AB^2 + BC^2$

$BC^2 = 9$

BC = 3 cm

CD = 2 BC

= 6 cm

8. Question

If *AB, BC* and *CD* are equal chords of a circle with *O* as a centre and *AD* diameter, than ∠*AOB* =

A. 60°

B. 90°

C. 120°

D. None of these

Answer

We can't say that,

∠AOB = 60°, 90° or 120°

So, angle AOB is none of these.

9. Question

Let *C* be the mid-point of an arc *AB* of a circle such that $m\,\widehat{AB}$ =183°. If the region bounded by the arc *ACB* and line segment *AB* is denoted by *S,* then the centre *O* of the circle lies

A. In the interior of *S*

B. In the exterior of *S*

C. On the segment *AB*

D. On *AB* and bisects *AB*

Answer

The centre O lies in the interior of S

10. Question

In a circle, the major arc is 3 times the minor arc. The corresponding central angles and the degree measures of two arcs are

A. 90° and 270°

B. 90° and 90°

C. 270° and 90°

D. 60° and 210°

Answer

Arc ACB = 3 arc AB (Given)

Central angle = 270°

Degree measures of the two arcs are 90°

11. Question

If *A* and *B* are two points on a circle such that $m\left(\widehat{AB}\right)$ =260°. A possible value for the angle subtended by arc *BA* at a point on the circle is

A. 100°

B. 75°

C. 50°

D. 25°

Answer

Arc AB = 260° (Given)

Let a point C on the circle

We Know that,

An angle subtended by an arc at the centre of the circle is double the angle subtended at any point on the circle.

$\angle ACB = \frac{1}{2} \angle AOB$

$\angle ACB = \frac{1}{2} * 100$

$= 50°$

12. Question

An equilateral triangle *ABC* is inscribed in a circle with centre *O*. The measures of ∠*BOC* is

A. 30°

B. 60°

C. 90°

D. 120°

Answer

Given that O is the centre of circle.

Triangle ABC is an equilateral triangle

$\angle A = \angle B = \angle C = 60°$

We Know that,

An angle subtended by an arc at the centre of the circle is double the angle subtended at any point on the circle.

$\angle BOC = 2\ \angle BAC$

$\angle BOC = 2\ \angle A$

$\angle BOC = 120°$

13. Question

In a circle with centre *O, AB* and *CD* are two diameters perpendicular to each other. The length of chord *AC* is

A. 2*AB*

B. $\sqrt{2}$

C. $\frac{1}{2}AB$

D. $\frac{1}{\sqrt{2}}AB$

Answer

Given: O is the centre of circle

AB and CD are diameters of the circle

AO = BO = CO = DO (Radius of the circle)

In right angle ΔAOC,

Cos A = $\frac{AM}{OA}$

Cos A = $\frac{\frac{1}{2}AC}{\frac{1}{2}AB}$

Cos A = $\frac{AC}{AB}$ (i)

$\angle$OMA = 90°

$\angle$AOM = $\angle$MAO = 45°

Using value of angle A in (i)

Cos 45° = $\frac{AC}{AB}$

$\frac{1}{\sqrt{2}} = \frac{AC}{AB}$

AC = $\frac{1}{\sqrt{2}}$ AB

14. Question

Two equal circles of radius *r* intersect such that each passes through the centre of the other. The length of the common chord of the circles is

A. $\sqrt{r}$

B. $\sqrt{2}\, r\, AB$

C. $\sqrt{3}\, r$

D. $\frac{\sqrt{3}}{2} r$

Answer

Let O and O' be the centre of two circles

OA and O'A = Radius of the circles

AB be the common chord of both the circles

OM perpendicular to AB

And,

O'M perpendicular to AB

ΔAOO' is an equilateral triangle.

AM = Altitude of AOO'

Height of ΔAOO' = $\frac{\sqrt{3}}{2}$ r

AB = 2 AM

$= 2\frac{\sqrt{3}}{2}r$

$= \sqrt{3}r$

15. Question

If *AB* is a chord of a circle, *P* and *Q* are the two points on the circle different from *A* and *B,* then

A. $\angle APB=\angle AQB$

B. $\angle APB+\angle AQB = 180°$ or $\angle APB=\angle AQB$

C. $\angle APB+\angle AQB=90°$

D. $\angle APB+\angle AQB=180°$

Answer

AB is a chord of circle P and Q are two points on circle

$\angle APB = \angle AQB$ (Angles on the same segment)

16. Question

If two diameters of a circle intersect each other at right angles, then quadrilateral formed by joining their end points is a

A. Rhombus

B. Rectangle

C. Parallelogram

D. Square

Answer

Let AB and CD are two diameters of circle

$\angle AOD = \angle BOD = \angle BOC = \angle AOC = 90°$

AB and CD are diagonals of quadrilateral ABCD

They intersect each other at right angles

And

AB = BC = CD = DA

We know that,

Sides of a square are equal and diagonals intersect at 90°

Therefore, ABCD is a square

17. Question

If *ABC* is an arc of a circle and $\angle ABC=135°$, then the ratio of arc $\widehat{ABC}$ to the circumference is

A. 1 : 4

B. 3 : 4

C. 3 : 8

D. 1 : 2

Answer

∠ABC= 135°

ABC is an arc

Circumference= 360°

Arc = 135°

$$\frac{Arc\ ABC}{Circumference} = \frac{135}{360}$$

$= \frac{3}{8}$

18. Question

The chord of a circle is equal to its radius. The angle substended by this chord at the minor arc of the circle is

A. 60°

B. 75°

C. 120°

D. 150°

Answer

Let AB be the chord of circle equal to radius r

OA = OB = r (Radii)

Therefore,

OA = OB = AB

OBC is equilateral triangle

Each angle = 60°

Hence,

Angle surrounded by AB at minor arc = $\frac{1}{2}$ (Reflex * ∠AOD)

$= \frac{1}{2}(360^o - 60^o)$

$= 150^o$

19. Question

PQRS is a cyclic quadrilateral such that *PR* is a diameter of the circle. If $\angle QPR = 67°$ and $\angle SPR = 72°$, then $\angle QRS =$

A. 41°

B. 23°

C. 67°

D. 18°

Answer

Given that,

PQRS is a cyclic quadrilateral

$\angle QPR = 67°$

$\angle SPR = 72°$

$\angle SPQ = \angle QPR + \angle SPR$

$= 67^{o} + 72^{o}$

$= 139°$

$\angle SPQ + \angle QRS = 180°$ (Opposite angles of cyclic quadrilateral)

$139^{o} + \angle QRS = 180°$

$\angle QRS = 41°$

20. Question

If *A, B, C* are three points on a circle with centre *O* such that $\angle AOB = 90°$ and $\angle BOC = 120°$, then $\angle ABC =$

A. 60°

B. 75°

C. 90°

D. 135°

Answer

$\angle BOC = 120°$

$\angle AOC = \angle AOB + \angle BOC$

$= 90^{o} + 120^{o}$

$= 210°$

Now,

$\angle ABC = \frac{1}{2} *$ (Reflex $\angle AOC$)

$= \frac{1}{2}(360^o - 210^o)$

= 75°

21. Question

AB and *CD* are two parallel chords of a circle with centre *O* such that *AB*=6 cm and *CD*= 12 cm. The chords are on the same side of the centre and the distance between them in 3 cm. The radius of the circle is

A. 6 cm

B. $5\sqrt{2}$ cm

C. 7 cm

D. $3\sqrt{5}$ cm

Answer

Given that,

AB || CD (Chords on same side of centre)

AO = CO (Radii)

OL and OM perpendicular bisector of CD and AB respectively

CL = LD = 6 cm

AM = MB = 3 cm

LM = 3 cm (Given)

In ΔCOL,

$CO^2 = OL^2 + 6^2$ (i)

In AOM,

$AO^2 = AM^2 + OM^2$

$= 3^2 + (OL + LM)^2$

$= 9 + OL^2 + 9 + 6O\,L$

$OL^2 = AO^2 - 18 - 6O\,L$ (ii)

Using (ii) in (i),

OL = 3 cm

Putting OL in (i),

$AO^2 = \sqrt{45}$

$AO = 3\sqrt{5}$

22. Question

In a circle of radius 17 cm, two parallel chords are drawn on opposite side of a diameter. The distance between the chords is 23 cm. If the length of one chord is 16 cm, then the length of the other is

A. 34 cm

B. 15 cm

C. 23 cm

D. 30 cm

Answer

Given that,

AB || CD (Chords on opposite side of centre)

DO = BO (Radii)

OL and OM perpendicular bisector of CD and AB respectively

LM = 23 cm

AB = 16 cm

In ΔOLB,

$OB^2 = OL^2 + LB^2$

$OL^2 = 225$

OL = 15 cm

OM = LM - OL

= 8 cm

In ΔOMD,

$OD^2 = OM^2 + MD^2$

$MD^2 = 225$

MD = 15 cm

Now,

CD = 2 MD = 30 cm

23. Question

The greatest chord of a circle is called its

A. Radius

B. Secant

C. Diameter

D. None of these

Answer

The largest chord in any circle is its diameter.

24. Question

Angle formed in minor segment of a circle is

A. Acute

B. Obtuse

C. Right angle

D. None of these

Answer

The minor segment in a circle always forms an obtuse angle.

25. Question

Number of circles that can be drawn through three non-collinear points is

A. 1

B. 0

C. 2

D. 3

Answer

Only and only a single circle can be drawn passing through any three non collinear points.

26. Question

In Fig. 16.202, O is the centre of the circle such that $\angle AOC = 130°$, then $\angle ABC =$

A. 130°

B. 115°

C. 65°

D. 165°

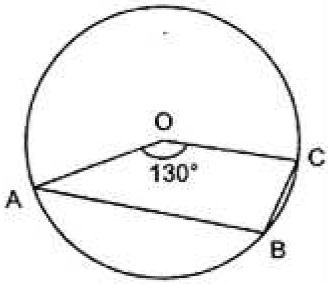


Fig. 16.202

Answer

We have,

∠AOC = 130°

∠ABC = $\frac{1}{2}$ * (Reflex of AOC)

= $\frac{1}{2}$ * (360° – 130°)

= $\frac{1}{2}$ * 230

= 115°

27. Question

In Fig. 16.203, if chords *AB* and *CD* of the circle intersect each other at right angles, then *x* + *y* =

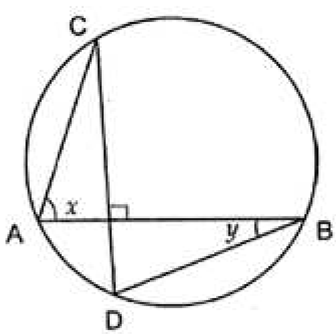


Fig. 16.203

A. 45°

B. 60°

C. 75°

D. 90°

Answer

Given: AB and CD are two chords of the circle.

∠APC = 90°

∠ACP = ∠PBD = y (Angles on the same segment)

In ΔACP,

∠ACP + ∠APC + ∠PAC = 180°

y + 90° + y = 180°

x + y = 90°

28. Question

In Fig. 16.204, if ∠*ABC*= 45°, then ∠*AOC*=

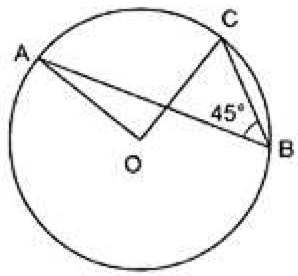


Fig. 16.204

A. 45°

B. 60°

C. 75°

D. 90°

Answer

We know that,

An angle subtended by an arc at the centre of the circle is double the angle subtended at any point on the circle

∠AOC = 2 ∠ABC

= 2 * 45°

= 90°

29. Question

In Fig. 16.205, chords *AD* and *BC* intersect each other at right angles at a point *P.* If ∠*DAB*=35°, then ∠*ADC*=

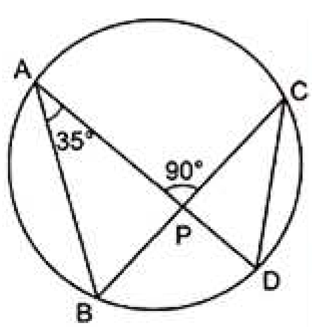


Fig. 16.205

A. 35°

B. 45°

C. 55°

D. 65°

Answer

Given that,

Chords AD and BC intersect at right angles,

∠DAB = 35°

∠APC = 90°

∠APC + ∠CPD = 180°

90° + ∠CPD = 180°

∠CPD = 90°

∠DAB = ∠PCD = 35° (Angles on the same segment)

In triangle PCD,

∠PCD + ∠PDC + ∠CPD = 180°

35° + ∠PDC + 90° = 180°

∠PDC = 45°

∠ADC = 45°

30. Question

In Fig. 16.206, *O* is the centre of the circle and ∠*BDC*=42°. The measure of ∠*ACB* is

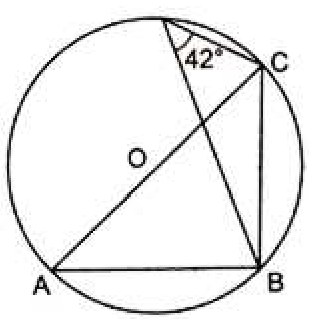


Fig. 16.206

A. 42°

B. 48°

C. 58°

D. 52°

Answer

$\angle BDC = 42°$

$\angle ABC = 90°$ (Angle in a semi-circle)

In $\triangle ABC$,

$\angle ABC + \angle BAC = 42°$ (Angles on the same segment)

$90° + 42° + \angle ACB = 180°$

$\angle ACB = 48°$