15. Areas of Parallelograms

Exercise 15.1

1. Question

Which of the following figures lie on the same base and between the same parallel. In such a case, write the common base and two parallel:

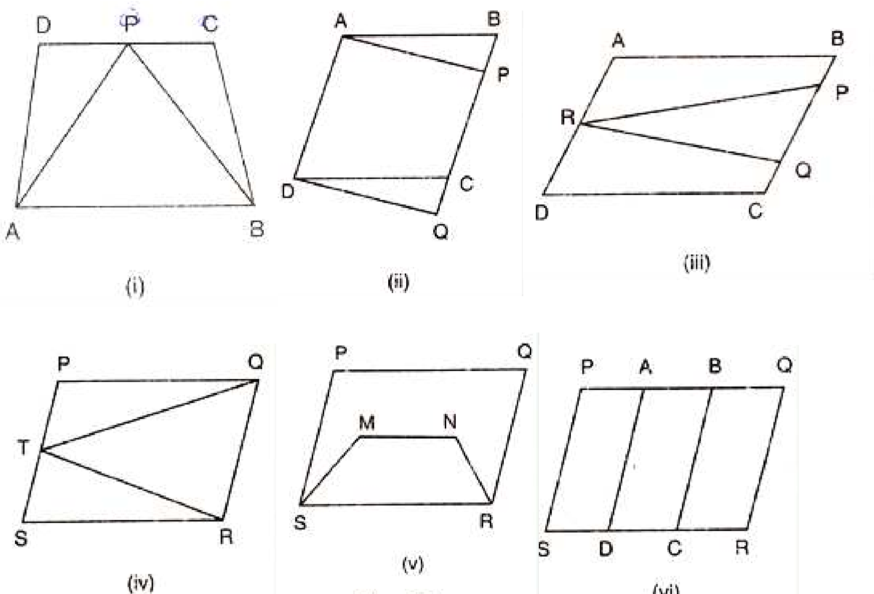

Fig. 15.4

Answer

(i) ΔPCD and trapezium ABCD are on the same base CD and between the same parallels AB and DC.

(ii) Parallelogram ABCD and APQD are on the same base AD and between the same parallel AD and BQ.

(iii) Parallelogram ABCD and ΔPQR are between the same parallels AD and BQ.

(iv) ΔQRT and Parallelogram PQRS are on the same base QR and between the same parallels QR and PS.

(v) Parallelogram PQRS and trapezium SMNR are on the same base SR but they are not between the same parallel.

(vi) Parallelograms PQRS, AQRD, BCQR are between the same parallels also parallelograms PQRS, BPSC and APSD are between the same parallels.

Exercise 15.2

1. Question

If Fig. 15.26, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB=16 cm, AE=8 cm and CF=10 cm, find AD.

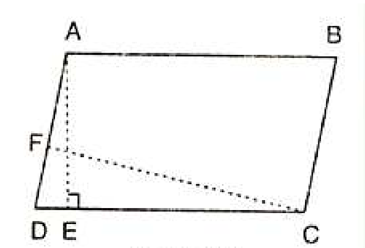

Fig. 15.26

Answer

Given that,

In a parallelogram ABCD:

CD = AB = 16cm (Opposite sides of parallelogram are equal)

We know that,

Area of parallelogram = Base * Corresponding altitude

Area of parallelogram ABCD:

CD * AE = AD * CF

16 cm * 18 cm = AD * 10 cm

AD = $\frac{16\times 8}{10}$

AD = 12.8 cm

2. Question

In Q. No. 1, if AD=6 cm, CF=10 cm and AE=8 cm, find AB.

Answer

Area of parallelogram ABCD = AD * CF (i)

Again,

Area of parallelogram ABCD = DC * AE (ii)

From (i) and (ii), we get

AD * CF = DC * AF

6 * 10 = CD * 8

CD = $\frac{60}{8}$

= 7.5 cm

Therefore,

AB = DC = 7.5 cm (Opposite sides of parallelogram are equal)

3. Question

Let ABCD be a parallelogram of area 124 cm^2. If E and F are the mid-points of sides AB and CD respectively, then find the area of parallelogram AEFD.

Answer

Given that,

Area of parallelogram ABCD = 124 cm^2

Construction: Draw AP perpendicular to DC

Proof: Area of parallelogram AFED = DF * AP (i)

Area of parallelogram EBCF = FC * AP (ii)

And,

DF = FC (iii) (F is the mid-point of DC)

Compare (i), (ii) and (iii), we get

Area of parallelogram AEFD = Area of parallelogram EBCF

Therefore,

Area of parallelogram AEFD = $\frac{\text{Area of parallelogram ABCD}}{2}$

= $\frac{124}{2}$ = 62 cm^2

4. Question

If ABCD is a parallelogram, then prove that

ar(Δ ABD) = ar(Δ BCD) = ar(Δ ABC)=ar(Δ ACD) = $\frac{1}{2}$ar($||^{gm}$ ABCD)

Answer

We know that,

Diagonal of parallelogram divides it into two quadrilaterals.

Since,

AC is the diagonal

Then, Area (ΔABC) = Area (ΔACD)

= $\frac{1}{2}$ Area of parallelogram ABCD (i)

Since,

BD is the diagonal

Then, Area (ΔABD) = Area (ΔBCD)

= $\frac{1}{2}$ Area of parallelogram ABCD (ii)

Compare (i) and (ii), we get

Therefore,

Area (ΔABC) = Area (ΔACD) = Area (ΔABD) = Area (ΔBCD) = $\frac{1}{2}$ Area of parallelogram ABCD

Exercise 15.3

1. Question

In Fig. 15.74, compute the aresa of quadrilateral ABCD.

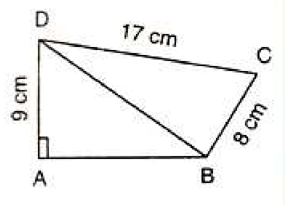

Fig. 15.74

Answer

Given that,

DC = 17 cm

AD = 9 cm

And,

BC = 8 cm

In Δ BCD, we have

$CD^2 = BD^2 + BC^2$

$(17)^2 = BD^2 + (8)^2$

$BD^2 = 289 - 64$

=15

In Δ ABD, we have

$BD^2 = AB^2 + AD^2$

$(15)^2 = AB^2 + (9)^2$

$AB^2 = 225 - 81$

= 144

=12

Therefore,

Area of Quadrilateral ABCD = Area (Δ ABD) + Area (Δ BCD)

Area of quadrilateral ABCD = $\frac{1}{2}(12 \times 9) + \frac{1}{2}(8 \times 17)$

= 54 + 68

= 112 cm^2

2. Question

In Fig. 15.75, PQRS is a square and T and U are respectively, the mid-points of PS and QR. Find the area of Δ OTS if PQ=8 cm

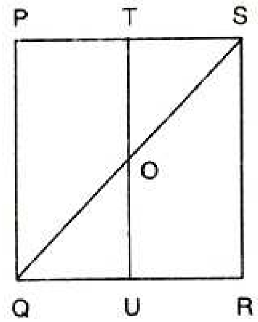

Fig. 15.75

Answer

From the figure,

T and U are the mid points of PS and QR respectively

Therefore,

TU || PQ

TO||PQ

Thus,

In Δ PQS and T is the mid-point of PS and TO||PQ

Therefore,

TO $= \frac{1}{2} *$ PQ

= 4 cm

Also,

TS $= \frac{1}{2} *$ PS

=4 cm

Therefore,

Area (Δ OTS) = $\frac{1}{2}$(TO * TS)

=$\frac{1}{2}$(4 * 4)

=8 cm^2

3. Question

Compute the area of trapezium PQRS in Fig. 15.76.

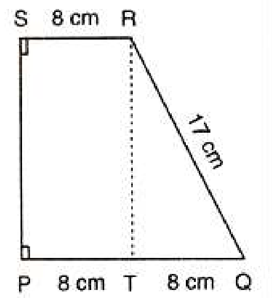

Fig. 15.76

Answer

We have,

Area of trapezium PQRS = Area of rectangle PSRT + Area (Δ QRT)

Area of trapezium PQRS = PT * RT + $\frac{1}{2}$(QT * RT)

= 8 * RT + $\frac{1}{2}$(8 * RT)

= 12 * RT

In Δ QRT, we have

$QR^2 = QT^2 + RT^2$

$RT^2 = QR^2 - QT^2$

$RT^2 = (17)^2 - (8)^2$

= 225

= 15

Hence,

Area of trapezium PQRS = 12 * 15

= 180 cm^2

4. Question

In Fig. 15.77, $\angle$AOB=90°, AC=BC, OA=12 cm and OC=6,5 cm. Find the area of Δ AOB.

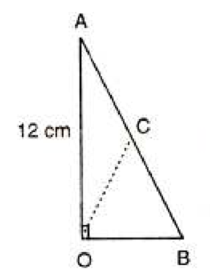

Fig. 15.77

Answer

Since,

The mid-point of the hypotenuse of a right triangle is equidistant from the vertices

Therefore,

CA = CB = OC

CA = CB = 6.5 cm

AB = 13 cm

In right (ΔOAB)

We have,

$AB^2 = OB^2 - OA^2$

$13^2 = OB^2 + 12^2$

OB = 5 cm

Therefore,

Area (Δ AOB) = $\frac{1}{2}$(OA * OB)

= $\frac{1}{2}$ (12 * 5)

= 30 cm^2

5. Question

In Fig. 15.78, ABCD is a trapezium in which AB=7 cm, AD=BC=5 cm, DC=x cm, and distance between AB and DC is 4 cm. Find the value of x and area of trapezium ABCD.

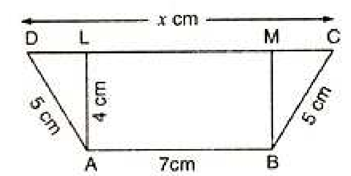

Fig. 15.78

Answer

Draw AL perpendicular to DC

And,

BM perpendicular DC

Then,

AL = BM = 4 cm

And,

LM = 7 cm

In Δ ADL, we have

$AD^2 = AL^2 + DL^2$

$25 = 16 + DL^2$

DL = 3 cm

Similarly,

$MC = \sqrt{BC^2 - BM^2}$

$= \sqrt{25 - 16}$

= 3 cm

Therefore,

x = CD = CM + ML + LD

= 3 + 7 + 3

= 13 cm

Area of trapezium ABCD = $\frac{1}{2}$ (AB + CD) * AL

= $\frac{1}{2}$(7 + 13) * 4

= 40 cm^2

6. Question

In Fig. 15.79, OCDE is a rectangle inscribed in a quadrilateral of a circle of radium 10 cm. If OE=2 $\sqrt{5}$, find the area of the rectangle.

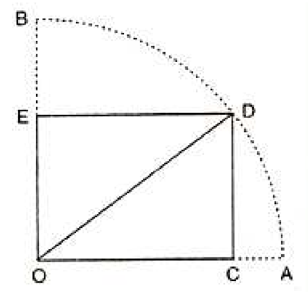

Fig. 15.79

Answer

We have,

OD = 10 cm

And,

OE = 2$\sqrt{5}$ cm

Therefore,

OD^2 = OE^2 + DE^2

DE = $\sqrt{OD^2 - OE^2}$

= $\sqrt{(10)^2 - (2\sqrt{5})^2}$

= 4$\sqrt{5}$ cm

Therefore,

Area of trapezium OCDE = OE * DE

= 2$\sqrt{5}$ * 4$\sqrt{5}$

= 40 cm^2

7. Question

In Fig. 15.80, ABCD is a trapezium in which AB||DC. Prove that ar(Δ AOD) = ar(Δ BOC).

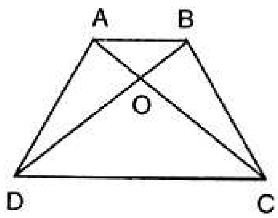

Fig. 15.80

Answer

Given that,

ABCD is a trapezium with AB ∥ DC

To prove: Area (ΔAOD) = Area (ΔBOC)

Proof: Since,

ΔABC and ΔABD are on the same base AB and between the same parallels AB and DC

Therefore,

Area (Δ ABC) = Area (Δ ABD)

Area (Δ ABC) – Area (Δ AOB) = Area (ΔABD) – Area (Δ AOB)

Area (Δ AOD) = Area (Δ BOC)

Hence, proved

8. Question

In Fig. 15.81, ABCD and CDEF are parallelograms. Prove that

ar(Δ ADE) = ar(Δ BCF).

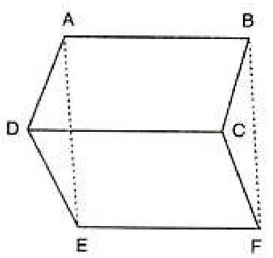

Fig. 15.81

Answer

Given that,

ABCD is a parallelogram

So,

AD = BC

CDEF is a parallelogram

So,

DE = CF

ABFE is a parallelogram

So,

AE = BF

Thus,

In ΔADE and ΔBCF, we have

AD = BC

DE = CF

And,

AE = BF

So, by SSS congruence rule, we have

Δ ADE ≅ ΔBCF

Therefore,

Area (Δ ADE) = Area (Δ BCF)

9. Question

Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:

ar(Δ APB) × ar(Δ CPD) = ar(Δ APD) × ar(Δ BPC).

Answer

Construction: Draw BQ perpendicular to AC

And,

DR perpendicular to AC

Proof: We have,

L.H.S = Area (ΔAPB) * Area (ΔCPD)

$= \frac{1}{2}(AP * BQ) * \frac{1}{2}(PC * DR)$

$= (\frac{1}{2} * PC * BQ) * (\frac{1}{2} * AP * DR)$

= Area (ΔBPC) * Area (ΔAPD)

= R.H.S

Therefore,

L.H.S = R.H.S

Hence, proved

10. Question

In Fig. 15.82, ABC and ABD are two triangles on the base AB. If line segment CD is bisected by AB at O, Show that ar(Δ ABC) = ar(Δ ABD).

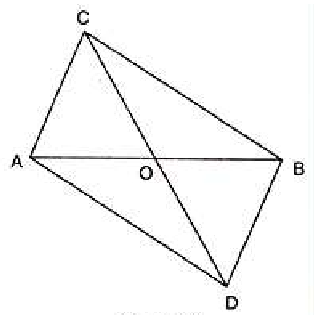

Fig. 15.82

Answer

Given that,

CD bisected AB at O

To prove: Area (ΔABC) = Area (ΔABD)

Construction: CP perpendicular to AB and DQ perpendicular to AB

Proof: Area (ΔABC) = $\frac{1}{2}$ (AB * CP) (i)

Area (ΔABD) = $\frac{1}{2}$ (AB * DQ) (ii)

In ΔCPO and ΔDQO, we have

$\angle CPO = \angle DQO$ (Each 90^o)

Given that,

CO = DO

$\angle COP = \angle DOQ$ (Vertically opposite angle)

Then, by AAS congruence rule

$\Delta CPO \cong \Delta DQO$

Therefore,

CP = DQ (By c.p.c.t)

Thus,

Area (ΔABC) = Area (ΔABD)

Hence, proved

11. Question

If P is any point in the interior of a parallelogram ABCD, then prove that area of the triangle APB is less than half the area of parallelogram.

Answer

Construction: Draw DN$\perp$ AB and PM$\perp$ AB.

Proof: Area of parallelogram ABCD = AB * DN

Area (Δ APB) = $\frac{1}{2}$ (AB * PM)

= AB * PM < AB * DN

= $\frac{1}{2}$(AB * PM) < $\frac{1}{2}$(AB * DN)

= Area (Δ APB) < $\frac{1}{2}$ Area of parallelogram ABCD

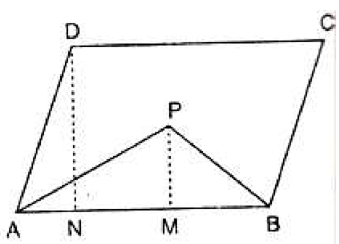

Fig. 15.93

12. Question

If AD is a median of a triangle ABC, then prove that triangles ADB and ADC are equal in area. If G is the mid-point of median AD, prove that ar(Δ BGC) = 2ar(Δ AGC).

Answer

Construction: Draw AM$\perp$ BC

Proof: Since,

AD is the median of ΔABC

Therefore,

BD = DC

BD * AM = DC * AM

$\frac{1}{2}$(BD * AM) = $\frac{1}{2}$(DC * AM)

Area (Δ ABD) = Area (Δ ACD) (i)

Now, in Δ BGC

GD is the median

Therefore,

Area (BGD) = Area (CGD) (ii)

Also,

In Δ ACD, CG is the median

Therefore, Area (ΔAGC) = Area (ΔCGD) (iii)

From (i), (ii) and (iii) we have

Area (ΔBGD) = Area (ΔAGC)

But,

Area (ΔBGC) = 2 Area (ΔBGD)

Therefore,

Area (ΔBGC) = 2 Area (ΔAGC)

Hence, proved

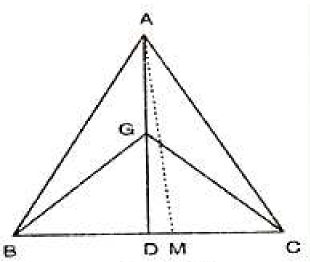

Fig. 15.94

13. Question

A point D is taken on the side BC of a Δ ABC such that BD = 2DC. Prove that

ar(Δ ABD) = 2ar(Δ ADC)

Answer

Given that,

In Δ ABC,

We have

BD = 2DC

To prove: Area (ΔABD) = 2 Area (ΔADC)

Construction: Take a point E on BD such that, BE = ED

Proof: Since,

BE = ED and,

BD = 2DC

Then,

BE = ED = DC

Median of the triangle divides it into two equal triangles

Since,

AE and AD are the medians of ΔABD and ΔAEC respectively

Therefore,

Area (ΔABD) = 2 Area (ΔAED) (i)

And,

Area (ΔADC) = Area (ΔAED) (ii)

Comparing (i) and (ii), we get

Area (ΔABD) = 2 Area (ΔADC)

Hence, proved

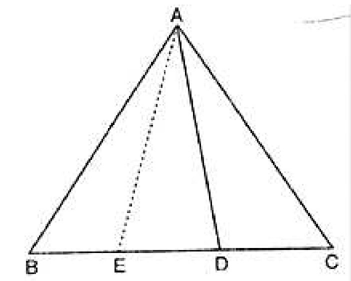

Fig. 15.95

14. Question

ABCD is a parallelogram whose diagonals intersect at O. If P is any point on BO, prove that

(i) ar(Δ ADO) = ar(Δ CDO)

(ii) ar(Δ ABP) = ar(Δ CBP)

Answer

Given that,

ABCD is a parallelogram

To prove: (i) Area (ΔADO) = Area (ΔCDO)

(ii) Area (ΔABP) = Area (ΔCBP)

Proof: We know that,

Diagonals of a parallelogram bisect each other

Therefore,

AO = OC and,

BO = OD

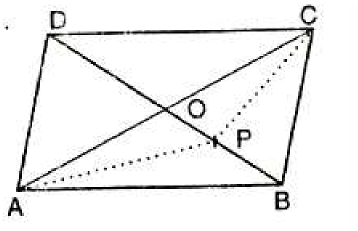

Fig. 15.96

(i) In ΔDAC, DO is a median.

Therefore,

Area (ΔADO) = Area (ΔCDO)

Hence, proved

(ii) In ΔBAC, since BO is a median

Then,

Area (ΔBAO) = Area (ΔBCO) (i)

In a ΔPAC, since PO is the median

Then,

Area (ΔPAO) = Area (ΔPCO) (ii)

Subtract (ii) from (i), we get

Area (ΔBAO) - Area (ΔPAO) = Area (ΔBCO) - Area (ΔPCO)

Area (ΔABP) = Area (ΔCBP)

Hence, proved

15. Question

ABCD is a parallelogram in which BC is produced to E such that CE=BC. AE intersects CD at F.

(i) Prove that ar(Δ ADF) = ar(Δ ECF)

(ii) If the area of Δ DFB=3 cm^2, find the area of $||^{gm}$ ABCD.

Answer

In ΔADF and ΔECF

We have,

∠ADF = ∠ECF

AD = EC

And,

∠DFA = ∠CFA

So, by AAS congruence rule,

Δ ADF ≅ Δ ECF

Area (ΔADF) = Area (ΔECF)

DF = CF

BF is a median in ΔBCD

Area (ΔBCD) = 2 Area (ΔBDF)

Area (ΔBCD) = 2 * 3

$= 6cm^2$

Hence, Area of parallelogram ABCD = 2 Area (ΔBCD)

= 2 * 6

$= 12\ cm^2$

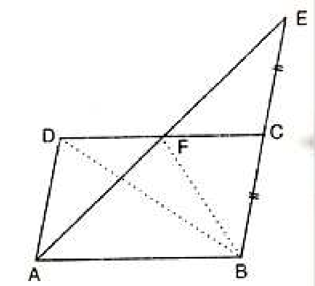

Fig. 15.97

16. Question

ABCD is a parallelogram whose diagonals AC and BD intersect at O. A line through O intersects AB at P and DC at Q. Prove that

ar(Δ POA) = ar(Δ QOC)

Answer

In ΔPOA and ΔQOC, we have

∠AOP = ∠COQ (Vertically opposite angle)

OA = OC (Diagonals of parallelogram bisect each other)

∠PAC = ∠QCA (AB ∥ DC, alternate angles)

So, by ASA congruence rule, we have

ΔPOA ≅ ΔQOC

Area (ΔPOA) = Area (ΔQOC)

Hence, proved

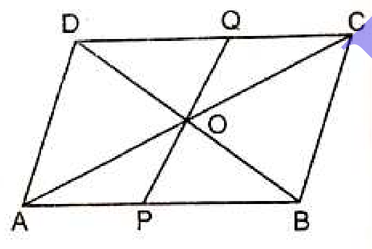

Fig. 15.98

17. Question

ABCD is a parallelogram. E is a point on BA such that BE = 2 EA and F is a point on DC, such that DF=2FC. Prove that AE CF is a parallelogram whose area is one third of the area of parallelogram ABCD.

Answer

Construction: Draw FG perpendicular to AB

Proof: We have,

BE = 2 EA

And,

DF = 2FC

AB - AE =2 AE

And,

DC - FC = 2 FC

AB = 3 AE

And,

DC = 3 FC

AE $= \frac{1}{3}$ AB and FC $= \frac{1}{3}$ DC (i)

But,

AB = DC

Then,

AE = FC (Opposite sides of a parallelogram)

Thus,

AE || FC such that AE = FC

Then,

AECF is a parallelogram

Now, Area of parallelogram (AECF) $= \frac{1}{3}$ (AB * FG) [From (i)

3 Area of parallelogram AECF = AB * FG (ii)

And,

Area of parallelogram ABCD = AB * FG (iii)

Compare equation (ii) and (iii), we get

3 Area of parallelogram AECF = Area of parallelogram ABCD

Area of parallelogram AECF $= \frac{1}{3}$ Area of parallelogram ABCD

Hence, proved

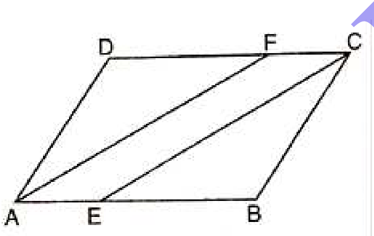

Fig. 15.99

18. Question

In a Δ ABC, P and Q are respectively the mid-point of AB and BC and R is the mid-point of AP. Prove that:

(i) ar(Δ PBQ) = ar(Δ ARC)

(ii) ar(Δ PQR) = $\frac{1}{2}$ar(Δ ARC)

(iii) ar(Δ RQC) = $\frac{3}{8}$ar(Δ ABC)

Answer

(i) We know that each median of a triangle divides it into two triangles of equal area.

Since,

CR is a median of ΔCAP

Therefore,

Area (ΔCRA) = $\frac{1}{2}$ Area (ΔCAP) (i)

Also,

CP is a median of ΔCAB

Therefore,

Area (ΔCAP) = Area (ΔCPB) (ii)

From (i) and (ii), we get

Therefore,

Area (ΔARC) = $\frac{1}{2}$ Area (ΔCPB) (iii)

PQ is a median of ΔPBC

Therefore,

Area (ΔCPB) = 2 Area (ΔPQB) (iv)

From (iii) and (iv), we get

Area (ΔARC) = Area (ΔPBQ) (v)

(ii) Since QP and QR medians of ΔQAB and ΔQAP respectively.

Area (ΔQAP) = Area (ΔQBP) (vi)

And,

Area (ΔQAP) = 2 Area (ΔQRP) (vii)

From (vi) and (vii), we get

Area (ΔPRQ) = $\frac{1}{2}$ Area (ΔPBQ) (viii)

From (v) and (viii), we get

Area (ΔPRQ) = $\frac{1}{2}$ Area (ΔARC) (ix)

(iii) Since CR is a median of ΔCAP

Therefore,

Area (ΔARC) = $\frac{1}{2}$ Area (ΔCAP)

= $\frac{1}{2} * \frac{1}{2}$ Area (ΔABC) (Therefore, CP is a median of Δ ABC)

= $\frac{1}{4}$ Area (ΔABC) (x)

Since,

RQ is a median of Δ RBC.

Therefore,

Area (ΔRQC) = $\frac{1}{2}$ Area (ΔRBC)

= $\frac{1}{2}$[Area (ΔABC) – Area (ΔARC)]

$= \frac{1}{2}$ [Area (ΔABC) - $\frac{1}{4}$ Area (ΔABC)]

= $\frac{3}{4}$ Area (Δ ABC)

19. Question

ABCD is a parallelogram, G is the point on AB such that AG = 2GB, E is a point of DC such that CE = 2DE and F is the point of BC such that BF=2FC. Prove that:

(i) ar(Δ ADGE) = ar(Δ GBCE)

(ii) ar(Δ EGB) = $\frac{1}{6}$ ar(Δ ABCD)

(iii) ar(Δ EFC) = $\frac{1}{2}$ ar(Δ EBF)

(iv) ar(Δ EBG) = $\frac{3}{2}$ ar(Δ EFC)

Answer

Given: ABCD is a parallelogram in which

AG = 2 GB

CE = 2 DE

BF = 2 FC

(i) Since ABCD is a parallelogram, we have AB ∥ CD and AB = CD

Therefore,

BG = $\frac{1}{3}$ AB

And,

DE = $\frac{1}{3}$ CD = $\frac{1}{3}$ AB

Therefore,

BG = DE

ADEH is a parallelogram (Since, AH is parallel to DE and AD is parallel to HE)

Area of parallelogram ADEH = Area of parallelogram BCIG (i)

(Since, DE = BG and AD = BC parallelogram with corresponding sides equal)

Area (ΔHEG) = Area (ΔEGI) (ii)

(Diagonals of a parallelogram divide it into two equal areas)

From (i) and (ii), we get,

Area of parallelogram ADEH + Area (ΔHEG) = Area of parallelogram BCIG + Area (ΔEGI)

Therefore,

Area of parallelogram ADEG = Area of parallelogram GBCE

(ii) Height, h of parallelogram ABCD and ΔEGB is the same

Base of ΔEGB = $\frac{1}{3}$ AB

Area of parallelogram ABCD = h * AB

Area (ΔEGB) = $\frac{1}{2}$ * $\frac{1}{3}$ AB * h

$= \frac{1}{6}$ (h) * AB

$= \frac{1}{6}$ * Area of parallelogram ABCD

(iii) Let the distance between EH and CB = x

Area (ΔEBF) = $\frac{1}{2}$ * BF * x

$= \frac{1}{2} * \frac{2}{3}$ BC * x

$= \frac{1}{3}$ * BC * x

Area (ΔEFC) = $\frac{1}{2}$ * CF * x

$= \frac{1}{2} * \frac{1}{3}$ * BC * x

$= \frac{1}{2}$ * Area (ΔEBF)

Area (ΔEFC) = $\frac{1}{2}$ * Area (ΔEBF)

(iv) As, it has been proved that

Area (ΔEGB) = $= \frac{1}{6}$ * Area of parallelogram ABCD (iii)

Area (ΔEFC) = $\frac{1}{3}$ Area (ΔEBC)

Area (ΔEFC) = $\frac{1}{2} * \frac{1}{3}$ * CE * EP

$= \frac{1}{2} * \frac{1}{3} * \frac{2}{3}$ * CD * EP

$= \frac{1}{6} * \frac{2}{3}$ * Area of parallelogram ABCD

Area (ΔEFC) = $\frac{2}{3}$ * Area (ΔEGB) [By using (iii)]

Area (ΔEGB) = $\frac{3}{2}$ Area (ΔEFC)

20. Question

In Fig. 15.83, CD||AE and CY||BA.

(i) Name a triangle equal in area of Δ CBX

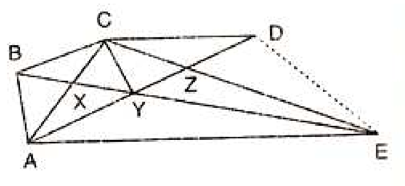

Fig. 15.83

(ii) Prove that ar(Δ ZDE) = ar(Δ CZA)

(iii) Prove that ar(Δ BCYZ) = ar(Δ EDZ)

Answer

(i) ΔAYC and Δ BCY are on the same base CY and between the same parallels

CY || AB

Area (ΔAYC) = Area (ΔBCY)

(Triangles on the same base and between the same parallels are equal in area)

Subtracting ΔCXY from both sides we get,

Area (ΔAYC) – Area (ΔCXY) = Area (ΔBCY) – Area (ΔCXY) (Equals subtracted from equals are equals)

Area (ΔCBX) = Area (ΔAXY)

(ii) Since, ΔACC and ΔADE are on the same base AF and between the same parallels

CD || AF

Then,

Area (ΔACE) = Area (ΔADE)

Area (ΔCEA) + Area (ΔAZE) = Area (ΔAZE) + Area (ΔDZE

Area (ΔCZA) = Area (ΔZDE) (i)

(iii) Since, ΔCBY and ΔCAY are on the same base CY and between the same parallels

CY || BA

Then,

Area (ΔCBY) = Area (ΔCAY)

Adding Area (ΔCYG) on both sides we get

Area (ΔCBY) + Area (ΔCYG) = Area (ΔCAY) + Area (ΔCYG)

Area ($BCYZ$) = Area (ΔCZA) (ii)

Compare (i) and (ii), we get

Area ($BCZY$) = Area (ΔEDZ)

21. Question

In Fig. 15.84, PSDA is a parallelogram in which PQ=QR=RS and AP||BQ||CR. Prove that

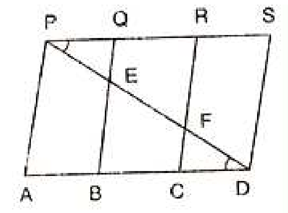

Fig. 15.84

ar(Δ PQE) = ar(Δ CFD)

Answer

Given that,

PSDA is a parallelogram

Since,

AP ∥ BQ ∥ CR ∥ DS and AD ∥ PS

Therefore,

PQ = CD (i)

In ΔBED,

C is the mid-point of BD and CF ∥ BE

Therefore,

F is the mid-point of ED

EF = PE

Similarly,

PE = FD (ii)

In ΔPQE and ΔCFD, we have

PE = FD

∠EPQ = ∠FDC (Alternate angle)

And,

PQ = CD

So, by SAS theorem, we have

$\Delta PQE \cong \Delta CFD$

Area (Δ PQE) = Area (Δ CFD)

Hence, proved

22. Question

In Fig. 15.85, ABCD is a trapezium in which AB||DC and DC=40 cm and AB=60 cm. If X and Y are, respectively, the mid points of AD and BC, Prove that

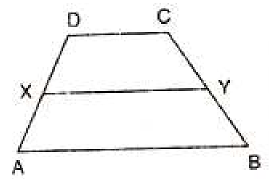

Fig. 15.85

(i) XY =50 cm (ii) DCYX is a trapezium

(iii) ar(trap. DCYX) = $\frac{9}{11}$ar(trap. XYBA)

Answer

(i) Join DY and extend it to meet AB produced at P

∠BYP = ∠CYD (Vertically opposite angles)

∠DCY = ∠PBY (Since DC || AP)

BY = CY (Since Y is the mid-point of BC)

Hence, by A.S.A. congruence rule

ΔBYP ≅ ΔCYD

DY = YP

And,

DC = BP

Also,

X is the mid-point of AD

Therefore,

XY || AP

And,

XY = $\frac{1}{2}$ AP

$XY = \frac{1}{2}(AB + BP)$

$XY = \frac{1}{2}(AB + DC)$

$XY = \frac{1}{2}(60 + 40)$

$= \frac{1}{2} \times 100$

= 50 cm

(ii) We have,

XY || AP

XY || AB and AB || DC

XY || DC

DCYX is a trapezium.

(iii) Since X and Y are the mid-points of AD and BC respectively

Therefore,

Trapezium DCYX and ABYX are of same height and assuming it as 'h' cm

Area (Trapezium DCYX) $= \frac{1}{2}(DC + XY) * h$

$= \frac{1}{2}(40 + 50)\ h$

$= 45h\ cm^2$

Area (Trapezium ABYX) $= \frac{1}{2}(AB + XY) * h$

$= \frac{1}{2}(60 + 50) * h$

$= 55h\ cm^2$

So,

$\frac{\text{Area of trapezium DCYX}}{\text{Area of trapezium ABYX}} = \frac{45h}{55h}$

$= \frac{9}{11}$

Area of trapezium DCYX $= \frac{9}{11}$ Area of trapezium ABXY

23. Question

In Fig. 15.86, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. AE intersects BC in F. Prove that

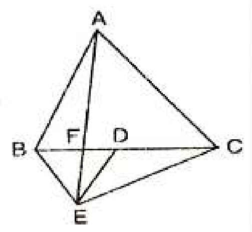

Fig. 15.86

(i) ar(Δ BDE) $= \frac{1}{4}$ar(Δ ABC)

(ii) ar(Δ BDE) $= \frac{1}{2}$ar(Δ BAE)

(iii) ar(Δ BFE) = ar(Δ AFD)

(iv) ar(Δ ABC) = ar(Δ BEC)

(v) ar(Δ FED) = $\frac{1}{8}$ ar(Δ AFC)

(vi) ar(Δ BFE) = 2ar(Δ EFD)

Answer

Given that,

ABC and BDF are two equilateral triangles

Let,

AB = BC = CA = x

Then,

BD = $\frac{x}{2}$ = DE = BF

(i) We have,

Area (ΔABC) = $\frac{\sqrt{3}}{4}$ x^2

Area (ΔBDE) = $\frac{\sqrt{3}}{4}$ $(\frac{x}{2})^2$

= $\frac{1}{4} * \frac{\sqrt{3}}{4}$ x^2

Area (ΔBDE) = $\frac{1}{4}$ Area (ΔABC)

(ii) It is given that triangles ABC and BED are equilateral triangles

∠ACB = ∠DBE = 60^o

BE ∥ AC (Since, alternate angles are equal)

Triangles BAF and BEC re on the same base BE and between the same parallels BE and AC

Therefore,

Area (ΔBAE) = Area (ΔBEC)

Area (ΔBAE) = 2 Area (ΔBDE) (Therefore, ED is the median)

Area (ΔBDE) = $\frac{1}{2}$ Area (ΔBAE)

(iii) Since,

ΔABC and ΔAED are equilateral triangles

Therefore,

∠ABC = 60^o and,

∠BDE = 60^o

∠ABC = ∠BDE

AB ∥ DE

Triangles BED and AED are on the same base ED and between the same parallels AB and DE

Therefore,

Area (ΔBED) = Area (ΔAED)

Area (ΔBED) - Area (ΔEFD) = Area (ΔAED) - Area (ΔEFD)

Area (ΔBEF) = Area (ΔAFD)

(iv) Since,

ED is the median of ΔBEC

Therefore,

Area (ΔBEC) = 2 Area (ΔBDE)

Area (ΔBEC) = 2 * $\frac{1}{4}$ Area (ΔABC)

Area (ΔBEC) = $\frac{1}{2}$ Area (ΔABC)

Area (ΔABC) = 2 Area (ΔBEC)

(v) Let h be the height of vertex E, corresponding to the side BD on ΔBDE

Let H be the vertex A, corresponding to the side BC in ΔABC

From part (i), we have

Area (ΔBDE) = $\frac{1}{4}$ Area (ΔABC)

$\frac{1}{2}$ * BD * h = $\frac{1}{4}$ ($\frac{1}{2}$ * BC * H)

BD * h = $\frac{1}{4}$ (2 BD * H)

h = $\frac{1}{2}$ H (1)

From part (iii), we have

Area (ΔBFC) = Area (ΔAFD)

= $\frac{1}{2}$ * PD * H

= $\frac{1}{2}$ * FD * 2h

= 2 ($\frac{1}{2}$ * FD * h)

= 2 Area (ΔEFD)

(vi) Area (ΔAFC) = Area (ΔAFD) + Area (ΔADC

= Area (ΔBFE) + $\frac{1}{2}$ Area (ΔABC)

[Using part (iii) and AD is the median of Area ΔABC]

= Area (ΔBFE) + $\frac{1}{2}$ * 4 Area (ΔBDE) [Using part (i)]

Area (ΔBFC) = 2 Area (ΔFED) (2)

Area (ΔBDE) = Area (ΔBFE) + Area (ΔFED)

2 Area (ΔFED) + Area (ΔFED)

3 Area (ΔFED) (3)

From above equations,

Area (ΔAFC) = 2 Area (ΔFED) + 2 * 3 Area (ΔFED)

= Area (ΔFED)

Hence,

Area (ΔFED) = $\frac{1}{8}$ Area (ΔAFC)

24. Question

D is the mid-point of side BC of Δ ABC and E is the mid-point of BD. If O is the mid-point of AE, prove that ar(ΔBOE) = $\frac{1}{8}$ ar(Δ ABC)

Answer

Join A and D to get AD median

(Median divides the triangle into two triangles of equal area)

Therefore,

Area (ΔABD) = $\frac{1}{2}$ Area (ΔABC)

Now,

Join A and E to get AE median

Similarly,

We can prove that,

Area (ΔABE) = $\frac{1}{2}$ Area (ΔABD)

Area (ΔABE) = $\frac{1}{4}$ ABC (Area (ΔABD) = $\frac{1}{2}$ Area (ABC) (i)

Join B and O and we get BO median

Now,

Area (ΔBOE) = $\frac{1}{2}$ Area (ΔABE)

Area (ΔBOE) = $\frac{1}{2} * \frac{1}{4}$ Area (ΔABC)

Area (ΔBOE) = $\frac{1}{8}$ Area (ΔABC)

25. Question

In Fig. 15.87, X and Y are the mid-point of AC and AB respectively, QP||BC and CYQ and BXP are straight lines. Prove that:

ar(Δ ABP) = ar(Δ ACQ).

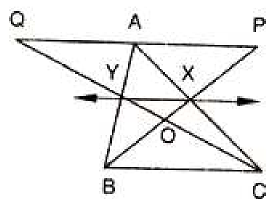

Fig. 15.87

Answer

In a ΔAXP and ΔCXB,

∠PAX = XCB (Alternative angles AP || BC)

AX = CX (Given)

∠AXP = ∠CXB (Vertically opposite angles)

$\Delta AXP \cong \Delta CXB$ (By ASA rule)

AP = BC (By c.p.c.t) (i)

Similarly,

QA = BC (ii)

From (i) and (ii), we get

AP = QA

Now,

AP || BC

And,

AP = QA

Area (ΔAPB) = Area (ΔACQ) (Therefore, Triangles having equal bases and between the same parallels QP and BC)

26. Question

In Fig. 15.88, ABCD and AEFD are two parallelograms. Prove that:

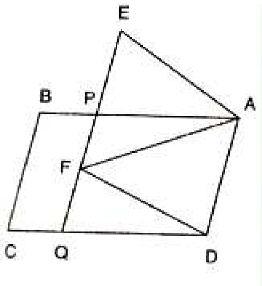

Fig. 15.88

(i) PE=FQ

(ii) ar(ΔAPE):ar(Δ PFA)= ar(Δ QFD)=ar(Δ PFD)

(iii) ar(Δ PEA) = ar(Δ QFD)

Answer

(i) In ΔEPA and ΔFQD

$\angle$PEA = $\angle$QFD (Corresponding angle)

$\angle$EPA = $\angle$FQD (Corresponding angle)

PA = QD (Opposite sides of a parallelogram)

Then,

$\Delta EPA \cong \Delta FQD$ (By AAS congruence rule)

Therefore,

$EP = FQ$ (c.p.c.t)

(ii) Since, ΔPEA and QFD stand on equal bases PE and FQ lies between the same parallel EQ and FQ lies between the same parallel EQ and AD

Therefore,

$\text{Area}\ (\Delta PEA)$ = Area (ΔQFD) (1)

Since,

ΔPEA and ΔPFD stand on the same base PF and lie between the same parallel PF and AD

Therefore,

Area (ΔPFA) = Area (ΔPFD (2)

Divide the equation (1) by (2), we get

$\frac{\text{Area }(\Delta PEA)}{\text{Area }(\Delta PFA)} = \frac{\text{Area }(\Delta QFD)}{\text{Area }(\Delta PFD)}$

(iii) From (i) part,

$\Delta EPA \cong \Delta FQD$

Then,

Area (ΔEPA) = Area (ΔFQD)

27. Question

In Fig. 15.89, ABCD is a $||^{gm}$. O is any point on AC. PQ||AB and LM||AD. Prove that:

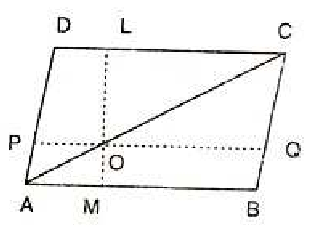

Fig. 15.89

ar($||^{gm}$ DLOP) = ar($||^{gm}$ BMOQ)

Answer

Since,

A diagonal of parallelogram divides it into two triangles of equal area

Therefore,

Area (ΔADC) = Area (ΔABC)

Area (ΔAPO) + Area of parallelogram DLOP + Area (ΔOLC)

Area (ΔAOM) + Area of parallelogram DLOP + Area (ΔOQC) (i)

Since,

AO and CO are diagonals of parallelograms AMOP and OQCL respectively

Therefore,

Area (ΔAPO) = Area (ΔAMO) (ii)

Area (ΔOLC) = Area (ΔOQC) (iii)

Subtracting (ii) from (iii), we get

Area of parallelogram DLOP = Area of parallelogram BMOQ.

28. Question

In a Δ ABC, if L and M are points on AB and AC respectively such that LM||BC. Prove that:

(i) ar(Δ LCM) = ar(Δ LBM)

(ii) ar(Δ LBC) = ar(Δ MBC)

(iii) ar(Δ ABM) = ar(Δ ACL)

(iv) ar(Δ LOB) = ar(Δ MOC)

Answer

(i) Clearly, triangles LMB and LMC are on the same base LM and between the same parallels LM and BC.

Therefore,

Area (ΔLMB) = Area (ΔLMC) (1)

(ii) We observe that triangles LBC and MBC are on the same base BC and between the same parallels LM and BC.

Therefore,

Area (ΔLBC) = Area (ΔMBC) (2)

(iii) We have,

Area (ΔLMB) = Area (ΔLMC) [From (i)]

Area (ΔALM) + Area (ΔLMB) = Area (ΔALM) + Area (ΔLMC)

Area (ΔABM) = Area (ΔACL)

(iv) We have,

Area (ΔLBC) = Area (ΔMBC) [From (ii)]

Area (ΔLBC) - Area (ΔBOC = Area (ΔMBC) - Area (ΔBOC)

Area (ΔLOB) = Area (ΔMOC)

29. Question

In Fig. 15.90, D and E are two points on BC such that BD=DE=EC. Show that ar(Δ ABD) =ar(Δ ADE) =ar(Δ AEC)

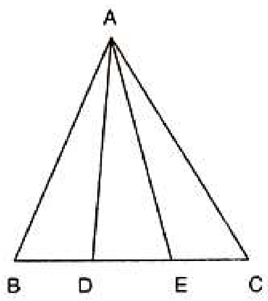

Fig. 15.90

Answer

Draw a line through A parallel to BC

Given that,

BD = BE = EC

We observed that the triangles ABD and AEC are on the same base and between the same parallels l and BC

Therefore, their areas are equal

Hence,

Area (ΔABD) = Area (ΔADE) = Area (ΔACE)

30. Question

In Fig. 15.91, ABC is a right triangle right angled at A, BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX⊥ DE meets BC at Y. Show that:

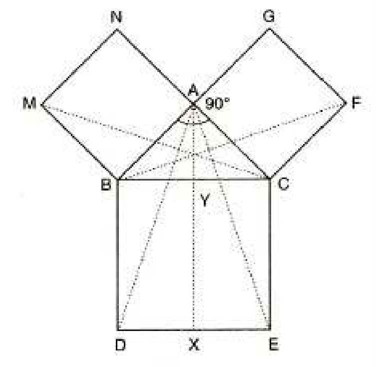

Fig. 15.91

(i) Δ MBC ≅ ABD (ii) ar(BYXD) =2ar(Δ MBC)

(iii) ar(BYXD) = ar(ABMN)

(iv) Δ FCB ≅ Δ ACE

(v) ar(CYXE) = 2ar(Δ FCB)

(vi) ar(CYXE) = ar(ACFG)

(vii) ar(BCED) = ar(ABMN)+ ar(ACFG)

Answer

(i) In ΔMBC and ΔABD, we have

MB = AB

BC = BD

And,

∠ MBC = ∠ ABD (Therefore, ∠MBC and ∠ABC are obtained by adding ∠ABC to right angle)

So, by SAS congruence rule, we have

$\Delta MBC \cong \Delta ABD$

Area (ΔMBC) = Area (ΔABD) (1)

(ii) Clearly, ΔABC and rectangle BYXD are on the same base BD and between the same parallels AX and BD

Therefore,

Area (ΔABD) = $\frac{1}{2}$ Area of rectangle BYXD

Area of rectangle BYXD = 2 Area (ΔABD)

Area of rectangle BYXD = 2 Area (ΔMBC) (2)

[Therefore, Area (ΔABD) = Area (ΔMBC)] From (1)

(iii) Since,

ΔMBC and square MBAN are on the same base MB and between the same parallel MB and NC

Therefore,

2 Area (ΔMBC) = Area of square MBAN (3)

From (2) and (3), we have

Area of square MBAN = Area of rectangle BXYD

(iv) In ΔFCB and ΔACE, we have

FC = AC

CB = CE

And,

∠FCB = ∠ACE (Therefore, ∠FCB and ∠ACE are obtained by adding ∠ACB to a right angle)

So, by SAS congruence rule, we have

$\Delta FCB \cong \Delta ACE$

(v) We have,

$\Delta FCB \cong \Delta ACE$

Area (ΔFCB) = Area (ΔACE)

Clearly,

ΔACE and rectangle CYXE are on the same base CE and between the same parallel CE and AX

Therefore,

2 Area (ΔACE) = Area of rectangle CYXE

2 Area (ΔFCB) = Area of rectangle CYXE (4)

(vi) Clearly,

ΔFCB and rectangle FCAG are on the same base FC and between the same parallels FC and BG

Therefore,

2 Area (ΔFCB) = Area of rectangle FCAG (5)

From (4) and (5), we get

Area of rectangle CYXE = Area of rectangle ACFG

(vii) Applying Pythagoras theorem in ΔACB, we have

$BC^2 = AB^2 + AC^2$

BC * BD = AB * MB + AC * FC

Area of rectangle BCED = Area of rectangle ABMN + Area of rectangle ACFG

CCE - Formative Assessment

1. Question

If ABC and BDE are two equilateral triangles such that D is the mid-point of BC, then find $ar(\Delta ABC):ar(\Delta BDE)$.

Answer

ΔABC and ΔBDE are equilateral triangles

We know that,

Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

D is the mid-point of BC then,

Area (ΔBDE) = $\frac{\sqrt{3}}{4} * (\frac{a}{2})^2$

$= \frac{\sqrt{3}}{4} * \frac{a*a}{4}$

Now,

Area (ΔABC): Area (ΔBDE)

$\frac{\sqrt{3}}{4} * a^2 : \frac{\sqrt{3}}{4} * \frac{a*a}{4}$

$1 : \frac{1}{4}$

4: 1

Hence,

Area (ΔABC): Area (ΔBDE) is 4: 1

2. Question

In Fig. 15.104, ABCD is a rectangle in which CD = 6 cm, AD = 8 cm. Find the area of parallelogram CDEF.

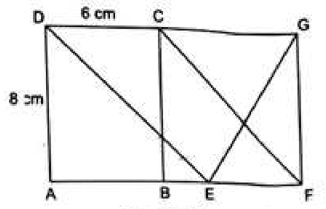

Fig. 15.104

Answer

Given that,

ABCD is a rectangle

CD = 6 cm

AD = 8 cm

We know that,

Area of parallelogram and rectangle on the same base between the same parallels are equal in area

So,

Area of parallelogram CDEF and rectangle ABCD on the same base and between the same parallels, then

We know that,

Area of parallelogram = Base * Height

Area of rectangle ABCD = Area of parallelogram

= AB * AD

= CD * AD (Therefore, AB = CD)

= 6 * 8

= 48 cm^2

Hence,

Area of rectangle ABCD is 48 cm^2

3. Question

In Fig. 15.104, find the area of ΔGEF.

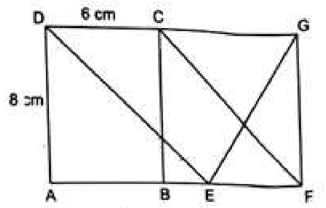

Fig. 15.104

Answer

Given that,

ABCD is rectangle

CD = 6 cm

AD = 8 cm

We know that,

If a rectangle and a parallelogram are on the same base and between the same parallels, then the area of a triangle is equal to the half of parallelogram

So, triangle GEF and parallelogram ABCD are on the same base and between same parallels, then we know

Area of parallelogram = Base * Height

Now,

Area (ΔGEF) = $\frac{1}{2}$ * Area of ABCD

= $\frac{1}{2}$ * AB * AD

= $\frac{1}{2}$ * 6 * 8

= 24 cm^2

Hence,

Area (ΔGEF) is 24 cm^2

4. Question

In Fig. 15.105, ABCD is a rectangle with sides AB = 10 cm and AD = 5 cm. Find the area of ΔEFG.

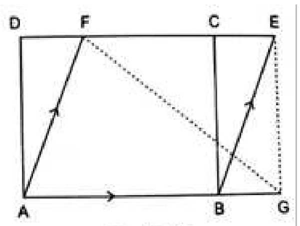

Fig. 15.105

Answer

We know that,

If a triangle and a parallelogram are on the same base and between the same parallel then the angle of triangle is equal to the half of the parallelogram

So, triangle EPG and parallelogram ABCD are on the same base and between same parallels. The

We know that,

Area of parallelogram = Base * Height

Now,

Area (ΔEPG) = $\frac{1}{2}$ * Area of ABCD

= $\frac{1}{2}$ * AB * AD

= $\frac{1}{2}$ * 10 * 5

= 25 cm^2

Hence,

Area (ΔEPG) is 25 cm^2

5. Question

PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then find $ar(\Delta RAS)$.

Answer

Given that,

PQRS is a rectangle

PS = 5 cm

PR = 13 cm

In triangle PSR, by using Pythagoras theorem

SR^2 = PR^2 – PS^2

SR^2 = (13)^2 – (5)^2

SR^2 = 169 – 25

SR^2 = 114

SR = 12 cm

We have to find the area ΔRAS,

Area (ΔRAS) = $\frac{1}{2}$ * Base * Height

= $\frac{1}{2}$ * SR * PS

= $\frac{1}{2}$ * 12 * 5

= 30 cm^2

Hence, Area (ΔRAS) is 30 cm^2

6. Question

In square AB2CD, P and Q are mid-point of AB and CD respectively. If AB = 8 cm and PQ and BD intersect at O, then find area of ΔOPB.

Answer

Given: ABCD is a square

P and Q are the mid points of AB and CD respectively.

AB = 8cm

PQ and BD intersect at O

Now,

AP = BP = $\frac{1}{2}$ AB

AP = BP = $\frac{1}{2}$ * 8

= 4 cm

AB = AD = 8 cm

QP ∥ AD

Then,

AD = QP

So,

OP = $\frac{1}{2}$AD

OP =$\frac{1}{2}$ * 8

= 4 cm

Now,

Area (ΔOPB) = $\frac{1}{2}$ * BP * PO

= $\frac{1}{2}$ * 4 * 4

= 8 cm^2

Hence, Area (ΔOPB) is 8 cm^2

7. Question

ABC is a triangle in which D is the mid-point of BC. E and F are mid-points of DC and AE respectively. If area of ΔABC is 16 cm^2, find the area of ΔDEF.

Answer

Given that,

D, E, F are the mid-points of BC, DC, AE respectively

Let, AD is median of triangle ABC

Area (ΔADC) = $\frac{1}{2}$ Area (ΔABC)

= $\frac{1}{2}$ * 16

= 8 cm^2

Now, AE is a median of ΔADC

Area (ΔAED) = $\frac{1}{2}$ Area (ΔADC)

= $\frac{1}{2}$ * 8

= 4 cm^2

Again,

DE is the median of ΔAED

Area (ΔDEF) = $\frac{1}{2}$ Area (ΔAED)

= $\frac{1}{2}$ * 4

= 2 cm^2

8. Question

PQRS is a trapezium having PS and QR as parallel sides. A is any point on PQ and B is a point on SR such that AB||QR. If area of ΔPBQ is 17 cm^2, find the area of ΔASR.

Answer

Given that,

Area (ΔPBQ) = 17 cm^2

PQRS is a trapezium

PS ∥ QR

A and B are points on PQ and RS respectively

AB ∥ QR

We know that,

If a triangle and a parallelogram are on the same base and between the same parallels, the area of triangle is equal to half area of parallelogram

Here,

Area (ΔABP) = Area (ΔASB) (i)

Area (ΔARQ) = Area (ΔARB) (ii)

We have to find Area (ΔASR),

Area (ΔASR) = Area (ΔASB) + Area (ΔARB)

= Area (ΔABP) + Area (ΔARQ)

= Area (ΔPBQ)

= 17 cm^2

Hence,

Area (ΔASR) is 17 cm^2.

9. Question

ABCD is a parallelogram. P is the mid-point of AB. BD and CP intersect at Q such that CO:OP = 3:1. If $ar(\Delta PBO) = 10cm^2$, Find the area of parallelogram ABCD.

Answer

Given that,

CQ: QP = 3: 1

Let,

CQ = 3x

PQ = x

Area (ΔPBQ) = 10 cm^2

We know that,

Area of triangle = $\frac{1}{2}$ * Base * Height

Area (ΔPBQ) = $\frac{1}{2}$ * x * h

10 = $\frac{1}{2}$ * x * h

x * h = 20

Area (ΔBQC) = $\frac{1}{2}$ * 3x * h

$= \frac{1}{2} * 3 * 20$

$= 30\ cm^2$

Now,

Area (ΔPCB) = $\frac{1}{2}$ * PB * H = 30 cm^2

PB * H = 60 cm^2

We have to find area of parallelogram

We know that,

Area of parallelogram = Base * Height

Area $(ABCD)$ = AB * H

Area $(ABCD)$ = 2 BP * H

Area (ABCD) = 2 (60)

Area (ABCD) = 120 cm^2

Hence,

Area of parallelogram ABCD is 120 cm^2

10. Question

P is any point on base BC of ΔABC and D is the mid-point of BC. DE is drawn parallel to PA to meet AC at E. If $ar(\Delta ABC) = 12cm^2$, then find area of ΔEPC.

Answer

Given that,

Area (ΔABC) = 12 cm^2

D is the mid-point of BC

So,

AD is the median of triangle ABC,

Area (ΔABD) = Area (ΔADC) = $\frac{1}{2}$ * Area (ΔABC)

Area (ΔADB) = Area (ΔADC) = $\frac{1}{2}$ * 12

= 6 cm^2 (i)

We know that,

Area of triangle between the same parallel and on the same base

Area (ΔAPD) = Area (ΔAPE)

Area (ΔAMP) + Area (ΔPDM) = Area (ΔAMP) + Area (ΔAME)

Area (ΔPDM) = Area (ΔAME) (ii)

ME is the median of triangle ADC,

Area (ΔADC) = Area $(MCED)$ + Area (ΔAME)

Area (ΔADC) = Area $(MECD)$ + Area (ΔPDM) [From (ii)]

Area (ΔADC) = Area (ΔPEC)

6 cm^2 = Area (ΔPEC) [From (i)]

Hence,

Area (ΔPEC) is 6 cm^2.

1. Question

The opposite sides of a quadrilateral have

A. No common points

B. One common point

C. Two common points

D. Infinitely many common points

Answer

Since, the two opposite line are joined by two another lines connecting the end points.

2. Question

Two consecutive sides of a quadrilateral have

A. No common points

B. One common point

C. Two common points

D. Infinitely many common points

Answer

Since, quadrilateral is simple closed figure of four line segments.

3. Question

PQRS is a quadrilateral. PR and QS intersect each other at O. In which of the following cases, PQRS is a parallelogram?

A. ∠P=100°, ∠Q=80°, ∠R=100°

B. ∠P=85°, ∠Q=85°, ∠R=95°

C. PQ=7 cm, QR=7 cm, RS=8 cm, SP=8 cm

D. OP=6.5 cm, OQ=6.5 cm, OR=5.2 cm, OS=5.2 cm

Answer

Since, the quadrilateral with opposite angles equal is a parallelogram.

4. Question

Which of the following quadrilateral is not a rhombus?

A. All four sides are equal

B. Diagonals bisect each other

C. Diagonals bisect opposite angles

D. One angle between the diagonals is 60°

Answer

One angle equalling to 60° need not necessarily be a rhombus.

5. Question

Diagonals necessarily bisect opposite angles in a

A. Rectangle

B. Parallelogram

C. Isosceles trapezium

D. Square

Answer

Each angle measures 45° each after the diagonal bisects them.

6. Question

The two diagonals are equal in a

A. Parallelogram

B. Rhombus

C. Rectangle

D. Trapezium

Answer

Let ABCD is a rectangle

AC and BD are the diagonals of rectangle

In ΔABC and ΔBCD, we have

AB = CD (Opposite sides of rectangle are equal)

∠ABC = ∠BCD (Each equal to 90^0)

BC = BC (Common)

Therefore,

ΔABC ≅ ΔBCD (By SAS congruence criterion)

AC = BD (c.p.c.t)

Hence, the diagonals of a rectangle are equal.

7. Question

We get a rhombus by joining the mid-points of the sides of a

A. Parallelogram

B. Rhombus

C. Rectangle

D. Triangle

Answer

Let ABCD is a rectangle such as AB = CD and BC = DA

P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively

Construction: Join AC and BD

In ΔABC,

P and Q are the mid-points of AB and BC respectively

Therefore,

PQ ∥ AC and PQ = $\frac{1}{2}$ AC (Mid-point theorem) (i)

Similarly,

In ΔADC,

SR ∥ AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) (ii)

Clearly, from (i) and (ii)

PQ ∥ SR and PQ = SR

Since, in quadrilateral PQRS one pair of opposite sides is equal and parallel to each other, it is a parallelogram.

Therefore,

PS ∥ QR and PS = QR (Opposite sides of a parallelogram) (iii)

In ΔBCD,

Q and R are the mid-points of side BC and CD respectively

Therefore,

QR ∥ BD and QR = $\frac{1}{2}$ BD (Mid-point theorem) (iv)

However, the diagonals of a rectangle are equal

Therefore,

AC = BD (v)

Now, by using equation (i), (ii), (iii), (iv), and (v), we obtain

PQ = QR = SR = PS

Therefore, PQRS is a rhombus.

8. Question

The bisectors of any two adjacent angles of a parallelogram intersect at

A. 30°

B. 45°

C. 60°

D. 90°

Answer

Let, ABCD is a parallelogram

OA and OD are the bisectors of adjacent angles ∠A and ∠D

As, ABCD is a parallelogram

Therefore,

AB || DC (Opposite sides of the parallelogram are parallel)

AB || DC and AD is the transversal,

Therefore,

∠BAD + ∠CDA = 180^o (Sum of interior angles on the same side of the transversal is 180^o)

∠1 + ∠2 = 90° (AO and DO are angle bisectors ∠A and ∠D) (i)

In ΔAOD,

$\angle 1 + \angle AOD + \angle 2 = 180°$

$\angle AOD + 90° = 180°$ [From (i)]

$\angle AOD = 180° – 90°$

$= 90°$

Therefore,

In a parallelogram, the bisectors of the adjacent angles intersect at right angle.

9. Question

The bisectors of the angle of a parallelogram enclose a

A. Parallelogram

B. Rhombus

C. Rectangle

D. Square

Answer

Let, ABCD is a parallelogram.

AE bisects $\angle BAD$ and BF bisects $\angle ABC$

Also,

CG bisects $\angle BCD$ and DH bisects $\angle ADC$

To prove: LKJI is a rectangle

Proof: $\angle BAD + \angle ABC = 180^o$ (Because adjacent angles of a parallelogram are supplementary)

ΔABJ is a right triangle

Since its acute interior angles are complementary

Similarly,

In ΔCDL, we get

$\angle DLC = 90°$

In ΔADI, we get

$\angle AID = 90°$

Then,

$\angle JIL = 90^o$ because $\angle AID$ and $\angle JIL$ are vertical opposite angles

Since three angles of quadrilateral LKJI are right angles, hence 4th angle is also a right angle

Thus LKJI is a rectangle.

10. Question

The figure formed by joining the mid-points of the adjacent sides of a quadrilateral is a

A. Parallelogram

B. Rectangle

C. Square

D. Rhombus

Answer

Given that,

ABCD is a quadrilateral and P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively

To prove: PQRS is a parallelogram

Construction: Join A with C

Proof: In ΔABC,

P and Q are the mid-points of AB and BC respectively

Therefore,

PQ ∥ AC and PQ = $\frac{1}{2}$ AC (Mid-point theorem) (i)

Again,

In ΔACD,

R and S are mid-points of sides CD and AD respectively

Therefore,

SR ∥ AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) (ii)

From (i) and (ii), we get

PQ ∥ SR and PQ = SR

Hence, PQRS is a parallelogram (One pair of opposite sides is parallel and equal)

11. Question

The figure formed by joining the mid-points of the adjacent sides of a rectangle is a

A. Square

B. Rhombus

C. Trapezium

D. None of these

Answer

Given: ABCD is a rectangle and P, Q, R, S are their midpoints

To Prove: PQRS is a rhombus

Proof: In ΔABC,

P and Q are the mid points

So, PQ is parallel AC

And,

PQ = $\frac{1}{2}$ AC (The line segment joining the mid points of 2 sides of the triangle is parallel to the third side and half of the third side)

Similarly,

RS is parallel AC

And,

RS = $\frac{1}{2}$ AC

Hence, both PQ and RS are parallel to AC and equal to $\frac{1}{2}$ AC.

Hence, PQRS is a parallelogram

In triangles APS & BPQ,

AP = BP (P is the mid-point of side AB)

∠PAS = ∠PBQ (90^o each)

AS = BQ (S and Q are the mid points of AD and BC respectively and since opposite sides of a rectangle are equal, so their halves will also be equal)

$\triangle$APS $\cong$ $\triangle$BPQ (By SAS congruence rule)

PS=PQ (By c.p.c.t.)

PQRS is a parallelogram in which adjacent sides are equal.

Hence, PQRS is a rhombus.

12. Question

The figure formed by joining the mid-points of the adjacent sides of a rhombus is a

A. Square

B. Rectangle

C. Trapezium

D. None of these

Answer

To prove: That the quadrilateral formed by joining the mid points of sides of a rhombus is a rectangle.

ABCD is a rhombus P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.

Construction: Join AC

Proof: In ΔABC, P and Q are the mid points of AB and BC respectively

Therefore,

PQ || AC and PQ = $\frac{1}{2}$ AC (i) (Mid-point theorem)

Similarly,

RS || AC and RS = $\frac{1}{2}$ AC (ii) (Mid-point theorem)

From (i) and (ii), we get

PQ ∥ RS and PQ = RS

Thus, PQRS is a parallelogram (A quadrilateral is a parallelogram, if one pair of opposite sides is parallel and equal)

AB = BC (Given)

Therefore,

$\frac{1}{2}$ AB = $\frac{1}{2}$ BC

PB = BQ (P and Q are mid points of AB and BC respectively)

In ΔPBQ,

PB = BQ

Therefore,

∠BQP = ∠BPQ (iii) (Equal sides have equal angles opposite to them)

In ΔAPS and ΔCQR,

AP = CQ (AB = BC = $\frac{1}{2}$ AB = $\frac{1}{2}$ BC = AP = CQ)

AS = CR (AD = CD = $\frac{1}{2}$ AD = $\frac{1}{2}$ CD = AS = CR)

PS = RQ (Opposite sides of parallelogram are equal)

Therefore,

ΔAPS $\cong$ ΔCQR (By SSS congruence rule)

∠APS = ∠CQR (iv) (By c.p.c.t)

Now,

∠BPQ + ∠SPQ + ∠APS = 180°

∠BQP + ∠PQR + ∠CQR = 180°

Therefore,

∠BPQ + ∠SPQ + ∠APS = ∠BQP + ∠PQR + ∠CQR

∠SPQ = ∠PQR (v) [From (iii) and (iv)]

PS || QR and PQ is the transversal,

Therefore,

∠SPQ + ∠PQR = 180^o (Sum of adjacent interior an angles is 180^o)

∠SPQ + ∠SPQ = 180° [From (v)]

2 ∠SPQ = 180°

∠SPQ = 90°

Thus, PQRS is a parallelogram such that ∠SPQ = 90^o

Hence, PQRS is a rectangle.

13. Question

The figure formed by joining the mid-points of the adjacent sides of a square is a

A. Rhombus

B. Square

C. Rectangle

D. Parallelogram

Answer

Let ABCD is a square such that AB = BC = CD = DA, AC = BD and P, Q, R and S are the mid points of the sides AB, BC, CD and DA respectively.

In ΔABC,

P and Q are the mid-points of AB and BC respectively.

Therefore,

PQ ∥ AC and PQ = $\frac{1}{2}$ AC (Mid-point theorem) (i)

Similarly,

In ΔADC,

SR ∥ AC and SR = $\frac{1}{2}$ AC (Mid-point theorem) (ii)

Clearly,

PQ ∥ SR and PQ = SR

Since, in quadrilateral PQRS, one pair of opposite sides is equal and parallel to each other. Hence, it is a parallelogram.

Therefore,

PS ∥ QR and PS = QR (Opposite sides of a parallelogram) (iii)

In $\Delta BCD,$

Q and R are the mid-points of sides BC and CD respectively

Therefore,

QR ∥ BD and QR = $\frac{1}{2}$ BD (Mid-point theorem) (iv)

However, the diagonals of a square are equal

Therefore,

AC = BD (v)

By using equation (i), (ii), (iii), (iv) and (v), we obtain

PQ = QR = SR = PS

We know that, diagonals of a square are perpendicular bisector of each other

Therefore,

∠AOD = ∠AOB = ∠COD = ∠BOC = 90^o

Now, in quadrilateral EHOS, we have

SE || OH

Therefore,

∠AOD + ∠AES = 180^o (Corresponding angle)

∠AES = 180° - 90°

= 90°

Again,

∠AES + ∠SEO = 180^o (Linear pair)

∠SEO = 180° - 90°

= 90°

Similarly,

SH || EO

Therefore,

∠AOD + ∠DHS = 180^o (Corresponding angle)

∠DHS = 180° - 90° = 90°

Again,

∠DHS + ∠SHO = 180° (Linear pair)

∠SHO = 180° - 90°

= 90°

Again,

In quadrilateral EHOS, we have

∠SEO = ∠SHO = ∠EOH = 90°

Therefore, by angle sum property of quadrilateral in EHOS, we get

∠SEO + ∠SHO + ∠EOH + ∠ESH = 360°

$90^o + 90^o + 90^o$ + ∠ESH = 360°

∠ESH = 90°

In the same manner, in quadrilateral EFOP, FGOQ, GHOR, we get

∠HRG = ∠FQG = ∠EPF = 90°

Therefore, in quadrilateral PQRS, we have

PQ = QR = SR = PS and ∠ESH = ∠HRG = ∠FQG = ∠EPF = 90°

Hence, PQRS is a square.

14. Question

The figure formed by joining the mid-points of the adjacent sides of a parallelogram is a

A. Rectangle

B. Parallelogram

C. Rhombus

D. Square

Answer

Let ABCD be a quadrilateral, with points

E, F, G and H the midpoints of

AB, BC, CD, DA respectively.

(I suggest you draw this, and add segments EF, FG, GH, and HE, along with diagonals AC and BD)

EF = $\frac{1}{2}$ AB (Definition of midpoint)

Similarly,

BF = $\frac{1}{2}$ BC

Thus, triangle BEF is similar to triangle BAC (SAS similarity)

Therefore EF is half the length of diagonal AC, since that's the proportion of the similar triangles.

Similarly, we can show that triangle DHG is similar to triangle DAC,

Therefore,

HG is half the length of diagonal AC

So,

EF = HG

Similarly,

We can use similar triangles to prove that EH and FG are both half the length of diagonal BD, and therefore equal

This means that both pairs of opposite sides of quadrilateral EFGH are equal, so it is a parallelogram.

15. Question

If one side of a parallelogram is 24° less than twice the smallest angle, then the measure of the largest angle of the parallelogram is

A. 176°

B. 68°

C. 112°

D. 102°

Answer

Let the small angle be = x

Then the second angle = 2x - 24°

Since,

Opposite angles are equal the 4 angles will be x, 2x - 24° , x , 2x - 24°

So now by angle sum property:

x + 2x - 24° + x + 2x - 24° = 360°

6x - 48° = 360°

6x = 360° + 48°

6x = 408°

$x = \frac{408}{6}$

x = 68°

Thus, the smallest angle is 68°

The second angle = 2 (68°) - 24°

= 112°

16. Question

In a parallelogram ABCD, if ∠DAB=75° and ∠DBC=60°, then ∠BDC=

A. 75°

B. 60°

C. 45°

D. 55°

Answer

We know that,

The opposite angles of a parallelogram are equal

Therefore,

∠BCD = ∠BAD = 75°

Now, in Δ BCD, we have

∠CDB + ∠DBC + ∠BCD = 180° (Since, sum of the angles of a triangle is 180^o)

∠CDB + 60° + 75° = 180°

∠CDB + 135° = 180°

∠CDB = (180° - 135°) = 45°

17. Question

ABCD is a parallelogram and E and F are the centroids of triangles ABD and BCD respectively, then EF=

A. AE

B. BE

C. CE

D. DE

Answer

Given: ABCD is a parallelogram

E and F are the centroids of triangle ABD and BCD

Since, the diagonals of parallelogram bisect each other

AO is the median of triangle ABD

And,

CO is the median of triangle CBD

EO= $\frac{1}{3}$ AO (Since, centroid divides the median in the ratio 2:1)

Similarly,

FO = $\frac{1}{3}$ CO

EO + FO = $\frac{1}{3}$ AO + $\frac{1}{3}$ CO

= $\frac{1}{3}$ (AO + CO)

EF = $\frac{1}{3}$ AC

AE = $\frac{1}{3}$ AO

= $\frac{2}{3} * \frac{1}{2}$ AC

= $\frac{1}{3}$ AC

Therefore,

EF = AE

18. Question

ABCD is a parallelogram M is the mid-point of BD and BM bisects ∠B. Then, ∠AMB=

A. 45°

B. 60°

C. 90°

D. 75°

Answer

ABCD is a parallelogram. BD is the diagonal and M is the mid-point of BD.

BD is a bisector of ∠B

We know that,

Diagonals of the parallelogram bisect each other

Therefore,

M is the mid-point of AC

AB || CD and BD is the transversal,

Therefore,

∠ ABD = ∠ BDC (i) (Alternate interior angle)

∠ ABD = ∠ DBC (ii) (Given)

From (i) and (ii), we get

∠ BDC = ∠ DBC

In Δ BCD,

∠ BDC = ∠ DBC

BC = CD (iii) (In a triangle, equal angles have equal sides opposite to them)

AB = CD and BC = AD (iv) (Opposite sides of the parallelogram are equal)

From (iii) and (iv), we get

AB = BC = CD = DA

Therefore,

ABCD is a rhombus

∠AMB = 90° (Diagonals of rhombus are perpendicular to each other)

19. Question

ABCD is a parallelogram and E is the mid-point of BC. DE and AB when produced meet at F. Then, AF =

A. $\frac{3}{2}$AB

B. 2 AB

C. 3 AB

D. $\frac{5}{4}$AB

Answer

ABCD is a parallelogram. E is the midpoint of BC. So, BE = CE

DE produced meets the AB produced at F

Consider the triangles CDE and BFE

BE = CE (Given)

∠CED = ∠BEF (Vertically opposite angles)

∠DCE = ∠FBE (Alternate angles)

Therefore,

ΔCDE ≅ ΔBFE

So,

CD = BF (c.p.c.t)

But,

CD = AB

Therefore,

AB = BF

AF = AB + BF

AF = AB + AB

AF = 2 AB

20. Question

If an angle of a parallelogram is two-third of its adjacent angle, the smallest angle of the parallelogram is

A. 108°

B. 54°

C. 72°

D. 81°

Answer

Since the adjacent angle of a parallelogram are supplementary.

Hence,

$x + \frac{2}{3}x = 180°$

$\frac{5}{8}x = 180°$

x = 108°

Now,

$\frac{2}{3}x = \frac{2}{3} * 108°$

= 72°

21. Question

If the degree measures of the angles of quadrilateral are 4x, 7x, 9x and 10x, what is the sum of the measures of the smallest angle and largest angle?

A. 140°

B. 150°

C. 168°

D. 180°

Answer

The total must be equal to 360° (Sum of quadrilaterals)

So,

4x + 7x + 9x + 10x = 360°

30x = 360°

x = 12°

Now,

Substitute x = 12 into 4x (Smallest) + 10x (Biggest)

So,

(4 * 12) + (10 * 12)

= 48 + 120

= 168 so the answer is C

22. Question

In a quadrilateral ABCD, ∠A+∠C is 2 times ∠B+∠D. If ∠A= 140° and f∠D = 60°, then ∠B=

A. 60°

B. 80°

C. 120°

D. None of these

Answer

Given that,

∠A = 140°

∠D = 60°

According to question,

∠A + ∠C = 2 (∠B + ∠D)

140 + ∠C = 2 (∠B + 60°)

∠B = $\frac{1}{2}$(∠C) + 10° (i)

We know,

∠A + ∠B + ∠C + ∠D= 360°

140° + $\frac{1}{2}$(∠C) + 10° + ∠C + 60° = 360°

$\frac{3}{2}$∠C = 150°

∠C = 100°

∠B = $\frac{1}{2}$(100°) + 10°

= 60°

23. Question

If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to

A. 16 cm

B. 15 cm

C. 20 cm

D. 17 cm

Answer

ABCD is a rhombus

AC = 18cm

BD = 24cm

We have to find the sides of the rhombus

In triangle AOB,

AO = 9cm (Diagonals of a parallelogram bisect each other)

BO = 12cm

AOB is a right - triangle right angled at O (Diagonals of a rhombus are perpendicular to each other)

So,

$AB^2 = AO^2 + BO^2$ (By Pythagoras theorem)

$AB^2 = 9^2 + 12^2$

$AB^2 = 81 + 144$

$AB^2 = 225$

AB = 15cm

In a rhombus, all sides are equal

Thus, each side of the rhombus is 15cm.

24. Question

The diagonals AC and BD of a rectangle ABCD intersect each other at P. If ∠ABD =50°, then ∠DPC =

A. 70°

B. 90°

C. 80°

D. 100°

Answer

Given that,

∠ABD = ∠ABP = 50°

∠PBC +∠ABP = 90° (Each angle of a rectangle is a right angle)

∠PBC = 40°

Now,

PB = PC (Diagonals of a rectangle are equal and bisect each other)

Therefore,

∠BCP = 40° (Equal sides has equal angle)

In triangle BPC,

∠BPC + ∠PBC + ∠BCP = 180° (Angle sum property of a triangle)

∠BPC = 100°

∠BPC + ∠DPC = 180° (Angles in a straight line)

$\angle DPC = 180^o - 100^o$

= 80°

25. Question

ABCD is a parallelogram in which diagonal AC bisects ∠BAD. If ∠BAC =35°, then ∠ABC =

A. 70°

B. 110°

C. 90°

D. 120°

Answer

Given: ABCD is a parallelogram

AC is a bisector of angle BAD

∠BAC = 35°

∠A = 2 ∠BAC

∠A = 2 (35°)

∠A = 70°

∠A + ∠B = 180° (Adjacent angles of parallelogram are supplementary)

70 + ∠B = 180°

∠B = 110^o

26. Question

In a rhombus ABCD, if ∠ACB =40°, then ∠ADB =

A. 70°

B. 45°

C. 50°

D. 60°

Answer

The diagonals in a rhombus are perpendicular,

So,

∠BPC = 90^o

From triangle BPC,

The sum of angles is 180°

So,

∠CBP = $180^o - 40^o - 90^o$

= 50°

Since, triangle ABC is isosceles

We have,

AB = BC

So,

∠ACB = ∠CAB = 40^o

Again from triangle APB,

∠PBA = $180^o - 40^o - 90^o$

= 50^o

Again, triangle ADB is isosceles,

So,

∠ADB = ∠DBA = 50^o

∠ADB = 50^o

27. Question

In Δ ABC, ∠A =30°, ∠B=40° and ∠C=110°. The angles of the triangle formed by joining the mid-points of the sides of this triangle are

A. 70°, 70°, 40°

B. 60°, 40°, 80°

C. 30°, 40°, 110°

D. 60°, 70°, 50°

Answer

Since, the triangle formed by joining the mid points of the triangle would be similar to it hence the angles would be equal to the outer triangle's angles.

28. Question

The diagonals of a parallelogram ABCD intersect ay O. If ∠BOC =90° and ∠BDC=50°, then ∠OAB =

A. 40°

B. 50°

C. 10°

D. 90°

Answer

∠BOC is 90°

So,

∠COD and ∠AOB all should be 90° by linear pair

∠BDC is 50°,

So,

Now as in a parallelogram the opposite sides are equal

We say,

AB parallel to CD

∠DCA = 50°

So,

In triangle COA

∠C = 50° (Stated above)

∠COA = 90° (Proved above)

Therefore,

90° + 50° + x° = 180°

x = 40°

29. Question

ABCD is a trapezium in which AB||DC. M and N are the mid-points of AD and BC respectively, If AB=12cm, MN=14 cm, then CD=

A. 10 cm

B. 12 cm

C. 14 cm

D. 16 cm

Answer

Construction: Join A to C

Mark the intersection point of AC and MN as O

Now, M and N are mid points of the non-parallel of a trapezium

Therefore,

MN || AB || DC

So,

MO || BC

And,

M is a mid-point of AD

Therefore,

MO= $\frac{1}{2}$ BC

Similarly,

NO = $\frac{1}{2}$ AB

Therefore,

MN = MO + NO

= $\frac{1}{2}$ (AB + CD)

But,

MN = 14 cm

Hence,

$\frac{1}{2}$ (AB + CD) = 14 cm

12 + CD = 28

CD = 16 cm

30. Question

Diagonals of a quadrilateral ABCD bisect each other. If ∠A =45°, then ∠B =

A. 115°

B. 120°

C. 125°

D. 135°

Answer

Since, diagonals of quadrilateral bisect each other

Hence, it's a parallelogram

We know,

The adjacent angles of parallelogram are supplementary

Therefore,

$\angle A + \angle B = 180°$

$45° + \angle B = 180°$

$\angle B = 135°$

31. Question

P is the mid-point of side BC of a parallelogram ABCD such that $\angle BAP = \angle DAP$. If AD=10 cm, then CD=

A. 5 cm

B. 6 cm

C. 8 cm

D. 10 cm

Answer

Given that,

ABCD is a parallelogram

P is the mid-point of BC

$\angle DAP = \angle PAB = x$

AD=10 cm

To find: The length of CD

$\angle ABP = 180 - 2x$ (Co interior angle of parallelogram)

$\angle APB = 180^o - (180^o - 2x + x) = x$

Therefore,

In triangle ABP,

$\angle APB = \angle PAB = x$

Therefore,

AB = PB (In a triangle sides opposite to equal angles are equal in length)

$CD = AB = PB = \frac{BC}{2} = \frac{AD}{2} = \frac{10}{2} = 5$ cm

32. Question

In Δ ABC, E is the mid-point of median AD such that BE produced meets AC at F. If AC = 10.5 cm, then AF =

A. 3 cm

B. 3.5 cm

C. 2.5 cm

D. 5 cm

Answer

Complete the parallelogram ADCP

So the diagonals DP & AC bisect each other at O

Thus O is the midpoint of AC as well as DP (i)

Since ADCP is a parallelogram,

AP = DC

And,

AP parallel DC

But,

D is mid-point of BC (Given)

AP = BD

And,

AP parallel BD

Hence,

BDPA is also a parallelogram.

So, diagonals AD & BP bisect each other at E (E being given mid-point of AD)

So, BEP is a single straight line intersecting AC at F

In triangle ADP,

E is the mid-point of AD and

O is the midpoint of PD.

Thus, these two medians of triangle ADP intersect at F, which is centroid of triangle ADP

By property of centroid of triangles,

It lies at $\frac{2}{3}$ of the median from vertex

So,

AF = $\frac{2}{3}$ AO (ii)

So,

From (i) and (ii),

AF = $\frac{2}{3} * \frac{1}{2} *$ AC

= $\frac{1}{3}$ AC

= $\frac{10.5}{3}$

= 3.5 cm