15. Areas Related to Circles

Exercise 15.1

1. Question

Find the circumference and area of a circle of radius 4.2cm.

Answer

Given,

Radius of circle = 4.2cm

Circumference of circle = 2πr

$= 2\times\frac{22}{7}\times4.2$

= 26.4cm

Area of circle = $2\pi r^2$

$=\frac{22}{7}\times4.2\times4.2$

$=55.44cm^2$

2. Question

Find the circumference of a circle whose area is $301.84cm^2$.

Answer

Given,

Area of circle = $301.84cm^2$

= πr^2 = 301.84

$= r^2 = \frac{301.84\times7}{22}$ = 96.24

= $r^2 = \sqrt{96.24}$ = 9.81cm

Circumference of the circle = 2πr

$= 2\times\frac{22}{7}\times9.81$

= 61.6 cm

3. Question

Find the area of a circle whose circumference is 44cm.

Answer

Circumference of the circle = 44cm

2πr = 44cm

$$r = \frac{44 \times 7}{2 \times 22} = 7cm$$

Area of circle = $\pi r^2 = \frac{22}{7} \times 7 \times 7$

Area of circle = 154 cm^2

4. Question

The circumference of a circle exceeds the diameter by 16.8cm. Find the circumference of the circle.

Answer

Given : The circumference of a circle exceeds the diameter by 16.8 cm.**To find :** The circumference of the circle.**Solution :**

Let diameter of circle = X cm

So, acc. to given condition

Circumference = x+16.8 cm

Circumference of circle is 2πr.

⇒2πr = x + 16.8 Diameter = 2r

$$\Rightarrow \frac{22}{7} \times x = x + 16.8$$

$$\Rightarrow \frac{22}{7}x - x = 16.8$$

$$\Rightarrow \frac{22x - 7x}{7} = 16.8$$

$$\Rightarrow \frac{15x}{7} = 16.8$$

⇒ 15 x=16.8 × 7⇒15 x=117.6

$$\Rightarrow x = \frac{117.6}{15}$$

$$\Rightarrow x = 7.84$$

Circumference= x + 16.8 (x = 2r)

Circumference=7.84 + 16.8 = 24.64 cm

5. Question

A horse is tied to a pole with 28m long string. Find the area where the horse can graze. $(Take\ \pi = 22/7)$

.

Answer

Length of string = radius of area which horse can graze

r = 28m

so,

Area where the horse can graze = πr^2

$= \frac{22}{7} \times 28 \times 28$ = 2464 m^2

6. Question

A steel wire when bent in the form of square encloses an area of 121cm^2. If the same wire is bent in the form of a circle, find the area of the circle.

Answer

Area of square = 121 cm^2

a^2 = 121

a = $\sqrt{121}$ = 11cm

Perimeter of square = length of wire

4a = 4 ×11 = 44cm

Perimeter of circle = 2πr

2πr = 44

$r = \frac{44 \times 7}{2 \times 22}$ = 7cm

Area of circle = πr^2

Area of circle = $\frac{22}{7} \times 7 \times 7$ = 154cm^2

7. Question

A horse is placed for grazing inside a rectangular field 40m by 36m and is tethered to one corner by a rope 14m long. Over how much area can it graze? $(Take\ \pi = 22/7)$.

Answer

Given,

Length of field = 40m

Breadth of field = 36m

Length of rope (radius) =14m

So,

Area horse can graze = $\frac{\pi r^2}{4}$

Area horse can graze = $\frac{22\times14\times14}{7\times4}$ = 154 m^2

8. Question

A sheet of paper is in the form of a rectangle ABCD in which AB=40cm and AD=28cm. A semi-circular portion with BC as diameter is cut off. Find the area of the remaining paper.

Answer

Area of rectangle = length × breadth

Area of rectangle = 40 × 28

Area of rectangle = 1120 cm^2

Diameter of semi circular portion = 28cm

Radius of semi circular portion = $\frac{28}{2}$ = 14cm

So,

Area of semi circular portion = $\frac{\pi r^2}{2}$

= $\frac{22\times14\times14}{7\times2}$ = 308 cm^2

Area of remaining portion = 1120 – 308 = 812cm^2

9. Question

The circumference of two circles are in the ratio 2:3. Find the ratio of their areas.

Answer

Ratio of circumferences of two circles with radius r_1 and r_2 respectively

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{2}{3}$$

$$= \frac{r_1}{r_2} = \frac{2}{3}$$

Ratio of area = $\frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{2^2}{3^2} = \frac{4}{9}$ = 4:9

10. Question

The side of a square is 10cm. Find the area of circumscribed and inscribed circles.

Answer

Side of square = 10cm

Radius of inscribed circle = $\frac{side}{2}$

Radius of inscribed circle = $\frac{10}{2}$ = 5cm

Area of inscribed circle = $\pi r^2 = \frac{22\times5\times5}{7}$

$= \frac{550}{7}$ = 78.5 cm^2

Radius of circumscribed circle = $\frac{diagonal\ of\ square}{2}$

$$= \frac{\sqrt{2}\,a}{2} = \frac{\sqrt{2}\times10}{2} = 5\sqrt{2}$$

Area of circumscribed circle = πr^2

$$= \frac{22}{7}\times5\sqrt{2}\times5\sqrt{2} = \frac{22\times50}{7}$$

$$= \frac{1100}{7} = 157cm^2$$

11. Question

The sum of the radii of two circles is 140cm and the difference of their circumferences is 88cm. Find the diameters of the circles.

Answer

Let radius of first circle = $r_1$cm

Let radius of second circle = $r_2$cm

So,

$r_1 + r_2$ = 140cm …. (i)

$2\pi r_1 - 2\pi r_2$ = 88cm

$$r_1 - r_2 = \frac{88\times7}{2\times22} = 14cm$$

$r_1 - r_2$ = 14cm …. (ii)

By adding equation 1 & 2

$r_1+r_2 = 140$ cm
$r_1-r_2 = 14$ cm

$2r_1 = 154$

$r_1 = 77$cm

From equation 1

$77 + r_1 = 140$ cm

$r_2 = 140 - 77 = 63$cm

$r_2 = 63$cm

So,

Diameter of first circle = $2\times r_1 = 2\times 77 = 154$cm

Diameter of second circle = $2\times r_2 = 2\times 63 = 126$cm

12. Question

The area of a circle inscribed in an equilateral triangle is $154cm^2$. Find the perimeter of the triangle. $(Use\ \pi = 22/7\ and\ \sqrt{3} = 1.73)$

Answer

Area of inscribed circle = $154cm^2$

$= \pi r^2 = 154cm^2$

$$r^2 = \frac{154\times 7}{22}$$

$$r = \sqrt{49} = 7cm$$

Radius of inscribed circle = 7cm

$$r = \frac{side\ of\ equilateral\ triangle}{2\sqrt{3}}$$

$7 = \frac{a}{2\sqrt{3}}$ (a = side of triangle)

a = $14\sqrt{3}cm$

Perimeter of equilateral triangle = 3a

$3a = 3\times 14\sqrt{3}$

$= 42\times 1.73$ (given)

$= 72.66 = 72.7cm^2$

13. Question

A field is in the form of a circle. A fence is to be erected around the field. The cost of fencing would be Rs.2640 at the rate of Rs.12 per metre. Then, the field is to be thoroughly ploughed at the cost of Re.0.50 per m^2. What is the amount required to plough the field?$(Take\ \pi = 22/7)$

Answer

Total cost of fencing = Rs 2640

Per meter rate of fencing = Rs 12

So,

Circumference of field = $\frac{2640}{12} = 220\ m$

$r = \frac{220\times7}{2\times22} = 35$

Radius of field = 35m

Area of field = $\frac{22}{7}\times35\times35$ = 3850m^2

Cost of plugging 1 m^2 field = 0.50 Rs

Total cost of plugging the field = 3850×0.50 = Rs 1925.00

14. Question

If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.

Answer

When a square inscribed in a circle then,

Diameter of circle = diagonal of square

Let side of the square be = a cm

Diagonal of square be =$\sqrt{2a}\ cm$

Area of square = a^2 cm^2

Diameter of circle = $\sqrt{2}a\ cm$

∴ radius of circle = $\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

Area of circle = $\pi\times\frac{a^2}{2}\ cm$

Ratio of area of circle and square = $\frac{\pi a^2}{2} : a^2$

= π : 2

15. Question

A park is in the form of a rectangle 120m×100m. At the centre of the park there is a circular lawn. The area of park excluding lawn is 8700m^2. Find the radius of the circular lawn. (*Use* $\pi = 22/7$)

Answer

Total area of rectangular park = 120×100 = 12000 m^2

Area of park excluding circular lawn = 8700m^2

So,

Area of circular lawn = 1200 – 8700 = 3300m^2

= πr^2 = 3300

$$r^2 = \frac{3300\times7}{22} = 1050m$$

r = 32.40 m

16. Question

The radii of two circles are 8cm and 6cm respectively. Find the radius of the circle having its area equal to the sum of the areas of the two circles.

Answer

Radius of first circle = 8cm

Area of first circle = πr^2

$$= \frac{22}{7}\times8\times8cm^2$$

Radius of second circle = 6cm

Area of second circle = $\frac{22}{7}\times6\times6cm^2$

Total area = $\frac{22}{7}\times8^2+\frac{22}{7}\times6^2$

$$= \frac{22}{7}(64+36) = \frac{22}{7}\times100\ cm^2$$

$$\pi r^2 = \frac{22}{7}\times100$$

r^2 = 100

r = 10cm

17. Question

The radii of two circles are 19cm and 9cm respectively. Find the radius and area of the circle which has its circumference equal to the sum of the circumferences of the two circles..

Answer

Radius of the first circle = 19cm

Circumference of first circle = 2πr

= 2π × 19cm

Radius of second circle = 2πr

=2π × 9cm

Total circumference = 2π × 19 + 2π × 9

= 2π (19+9)

$$= 2\times\frac{22}{7}\times 28 = 176\ cm$$

2πr = 176

$$r = \frac{176\times 7}{2\times 22} = 28cm$$

Area of circle = $\frac{22}{7}\times 28\times 28 = 2464cm^2$

18. Question

A car travels 1 kilo meter distance in which each wheel makes 450 complete revolutions. Find the radius of its wheels.

Answer

Total distance covered = 1km = 100000cm

Distance covered by circular wheel in 1 revolution = circumference of circle

Circumference of circle = 2πr

Total no. of revolution = 450

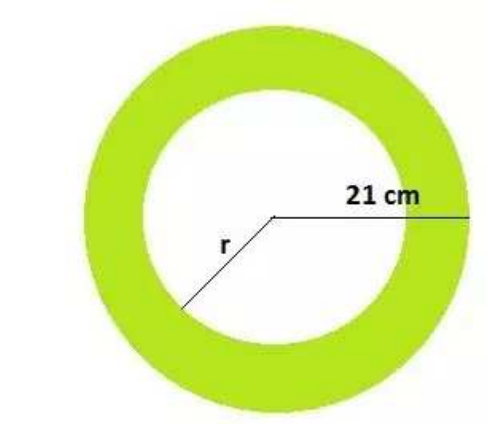
= 2πr × 450 = 100000

$$r = \frac{100000\times 7}{450\times 2\times 22} = 35.35\ cm$$

19. Question

The area enclosed between the concentric circles is 770cm^2. If the radius of the outer circle is 21cm, find the radius of the inner circle.

Answer



Area enclosed between two concentric circle = 770 cm^2

Radius of outer circle = 21cm

Let radius of inner circle = r cm

Area enclosed = area of outer circle – area of inner circle

Area enclosed = 770 п 21^2 – п r^2 = 770

п (441 – r^2) = 770

$$441 - r^2 = \frac{770 \times 7}{22}$$

$$441 - r^2 = 245$$

$$r^2 = 441 - 245$$

$$r^2 = 196$$

$$r = \sqrt{196}$$

$$r = 14$$

r = 14 cm

Exercise 15.2

1. Question

Find, in terms of π, the length of the arc that subtends an angle of 30° at the centre of a circle of radius 4cm.

Answer

Given,

Angle = 30°

Radius of circle = 4cm

180° = п radius

$$1° = \frac{\pi}{180°}$$

$$30° = \frac{30°\pi}{180°} = \frac{\pi}{6} \text{ radius}$$

Arc length = radius × angle subtended by arc at center

$$= 4 \times \frac{\pi}{6} = \frac{2\pi}{3}$$

2. Question

Find the angle subtended at the centre of a circle of radius 5cm by an arc of length $(5\pi / 3)$ cm.

Answer

Arc length = $\frac{5\pi}{3}$ cm

Radius of circle = 5cm

Formula:

Arc length = r×q

r = radius of circle

q = angle subtended by arc at the center

$$= \frac{5\pi}{3} = 5 \times q$$

$$q = \frac{5\pi}{3 \times 5} = \frac{\pi}{3} = \frac{180}{3} = 60°$$

3. Question

An arc of length 20π cm subtends an angle of 144° at the centre of a circle. Find the radius of the circle.

Answer

Arc length = 20π cm

Angle subtend at center = 144°

$$= \frac{\pi \times 144°}{180°} = \frac{4\pi}{5}$$

Arc length = radius × angle

$$\text{radius} = \frac{\text{arc length}}{\text{angle}} = \frac{20\pi \times 5}{4\pi} = 25\text{cm}$$

4. Question

An arc of length 15cm subtends an angle of 45° at the centre of a circle. Find in terms of π, the radius of the circle.

Answer

Arc length = 15cm

Angle subtend = 45°

$$= \frac{45\times\pi}{180^\circ} = \frac{\pi}{4} \text{ radius}$$

$$\text{radius of circle} = \frac{\text{arc length}}{\text{angle subtend at centre}}$$

$$= \frac{15\times4}{\pi} = \frac{60^\circ}{\pi}\text{cm}$$

5. Question

Find the angle subtended at the centre of a circle of a circle of radius 'a' by an arc of length $(a\pi/4)$ cm.

Answer

Radius of circle = a

Length of arc = $\frac{\frac{a\pi}{4}}{a} = \frac{\pi}{4}$

$$= \frac{180^\circ}{4} = 45^\circ$$

So,

Angle subtended at the center = 45°

6. Question

A sector of a circle of radius 4cm contains an angle of 30°. Find the area of the sector.

Answer

Given,

Radius of sector = 4cm

Angle of sector = 30°

Area of sector = $\frac{\theta}{360^\circ}\times\pi r^2$

$$= \frac{30}{360}\times\pi\times16$$

$$= \frac{1}{12}\times\pi\times16 = \frac{4\pi}{3}\text{cm}^2$$

7. Question

A sector of a circle of radius 8 cm contains an angle of 135°. Find the area of the sector.

Answer

Radius of sector = 8cm

Angle = 135°

Area of sector = $\frac{\theta\pi r^2}{360^\circ}$

Area of sector = $\frac{135^\circ}{360^\circ}\times\pi\times8\times8 = 24\pi cm^2$

8. Question

The area of a sector of a circle of radius 2 cm is π cm^2. Find the angle contained by the sector.

Answer

Given,

Area of sector = пcm^2

Radius = 2cm

Area of sector = $\frac{\theta}{360^\circ}\times\pi r^2$

$$\pi = \frac{\theta}{360^\circ}\times\pi\times4$$

$$Q = \frac{360^\circ\times\pi}{\pi\times4} = 90^\circ$$

9. Question

The area of a sector of a circle of radius 5cm is 5π cm^2. Find the angle contained by the sector.

Answer

Area of sector = 5п cm^2

Radius = 5cm

$$5\pi = \frac{\theta}{360^\circ}\times\pi\times25$$

$$Q = \frac{5\pi\times360^\circ}{25\pi} = 72^\circ$$

10. Question

AB is a chord of a circle with centre O and radius 4cm. AB is of length 4 cm. Find the areas of the sector of the circle formed by chord AB.

Answer

Length of the chord = 4cm

Radius of circle = 4cm

(This chord and radius makes an equilateral triangle)

So,

Q = 60° (in equilateral triangle)

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{60^\circ}{360^\circ} \times \pi \times 4 \times 4$

$= \frac{1}{6} \times \pi \times 16 = \frac{8\pi}{3} cm^2$

11. Question

In a circle of radius 35cm, an arc subtends an angle of 72° at the centre. Find the length of the arc and area of the sector.

Answer

Given,

Radius of circle = 35cm

Angle subtend by arc = 72°

Length of arc = r×q

Since,

180° = π radius

$1^\circ = \frac{\pi}{180^\circ}$

$72^\circ = \frac{\pi \times 72^\circ}{180^\circ} = \frac{2\pi}{5}$ radius

Length of the arc = $35 \times 2 \times \frac{22}{7} \times \frac{1}{5} = 44cm$

Area of sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{72^\circ}{360^\circ} \times \frac{22}{7} \times 35 \times 35$

= 770 cm^2

12. Question

The perimeter of a sector of a circle of radius 5.7m is 27.2m. Find the area of the sector.

Answer

Given,

Perimeter of sector of circle = 272m

Radius of sector = 5.7m

Perimeter of sector = $\frac{\theta}{360} \times 2\pi r + 2r = 27.2$

$= \frac{\theta}{360} \times 2\pi r = 27.2 - 11.4$

$= \frac{\theta}{360} = \frac{15.8}{2\pi r}$ (Equation first)

Area of sector = $\frac{\theta}{360} \times 2\pi r^2$ (Second equation)

Put value of $\frac{\theta}{360}$ from equation first to second,

$= \frac{15.8}{2\pi r} \times 2\pi r^2 = \frac{15.8 \times 5.7}{2} = 45.03 cm^2$

13. Question

The perimeter of a certain sector of a circle of radius 5.6m is 27.2m. Find the area of the sector.

Answer

Given,

Perimeter of sector = 27.2m

Radius of sector = 5.6m

$= \frac{\theta}{360} \times 2\pi r + 2r = 27.2$

$= \frac{\theta}{360} \times 2\pi r = 27.2 - 11.2$

$= \frac{\theta}{360} = \frac{16}{2\pi r}$ (Equation first)

Area of sector = $\frac{\theta}{360} \times \pi r^2$ (Equation second)

Put value of $\frac{\theta}{360}$ from equation first to equation second

$= \frac{16}{2\pi r} \times \pi r^2 = \frac{16 \times 5.6}{2} = 44.8\ m^2$

14. Question

A sector is cut-off from a circle of radius 21cm. The angle of the sector is 120°. Find the length of its arc and the area.

Answer

Given,

Radius of sector = 21cm

Angle of sector = 120°

Length of arc = $\frac{120^\circ\pi}{180^\circ}\times 21$

$= \frac{2}{3}\times\frac{22}{7}\times 21 = 44\text{cm}$

Area of sector = $\frac{\theta}{360}\times\pi r^2$

$= \frac{120}{360}\times\frac{22}{7}\times 21\times 21$

$= \frac{1}{3}\times 22\times 3\times 21 = 462\text{cm}^2$

15. Question

The minute hand of a clock is $\sqrt{21}$ cm long. Find the area described by the minute hand on the face of the clock between 7.00AM and 7.05AM.

Answer

Length of minute hand = $\sqrt{21}\text{cm}$

Angle subtend by minute hand in 1 minute = $\frac{360^\circ}{60^\circ} = 6^\circ$

Angle subtend by minute hand in 5 minute (7-7.05) = 5×6 = 30°

So,

Area described by minute hand in 5 minute = $\frac{\theta}{360^\circ}\times\pi r^2$

$= \frac{30}{360}\times\frac{22}{7}\times\sqrt{21}\times\sqrt{21}$

$= \frac{1}{12}\times\frac{22}{7}\times 21$

= 5.5cm^2

16. Question

The minute hand of a clock is 10cm long. Find the area of the face of the clock described by the minute hand between 8AM and 8.25AM.

Answer

Given,

Length of minute hand = 10cm

Angle subtend by minute hand in 25 minute (8-8.25) = 25×6 = 150°

So,

Area described by minute hand between (8-8.25) = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{150}{360} \times \frac{22}{7} \times 10 \times 10 = 130.95 cm^2$$

17. Question

A sector of 56° cut out from a circle contains area 4.4cm^2. Find the radius of the circle.

Answer

Given,

Angle of sector = 56°

Area of sector = 4.4cm^2

From formula,

$$= \frac{56}{360} \times \pi r^2 = 4.4$$

$$r^2 = \frac{4.4 \times 7 \times 360}{22 \times 56} = 9$$

$$r = \sqrt{9} = 3c$$

18. Question

In a circle of radius 6cm, a chord of length 10cm makes an angle of 110° at the centre of the circle. Find:

(i)the circumference of the circle,

(ii)the area of the circle,

(iii)the length of the arc AB,

(iv)the area of the sector OAB.

Answer

Given,

Radius of circle = 6cm

Length of chord = 10cm

Angle subtend by chord = 110°

I. Circumference of circle = 2πr

= 2×3.14×6 = 37.68cm

II. Are of circle = πr^2

= 3.14×6× = 113.1cm^2

III. Length of arc = radius × angle subtend

$$= 6\times\frac{120\pi}{180}$$

$$= 6\times\frac{2}{3}\times\frac{22}{7} = 11.51\text{cm}$$

IV. Area of sector = $\frac{\theta}{360}\times\pi r^2$

$$= \frac{110}{360}\times\frac{22}{7}\times6\times6 = \frac{242}{7} = 34.5\text{cm}^2$$

19. Question

Fig.15.17, shows a sector of a circle, centre O, containing an angle θ°. Prove that:

(i) Perimeter of the shaded region is $r\left(\tan\theta+\sec\theta+\frac{\pi\theta}{180}-1\right)$

(ii) Area of the shaded region is $\frac{r^2}{2}\left(\tan\theta-\frac{\pi\theta}{180}\right)$

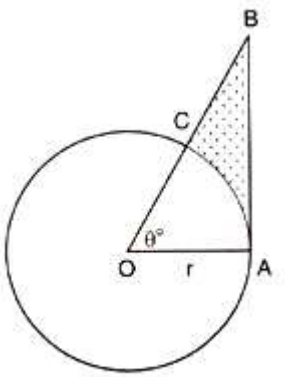


Fig. 15.17

Answer

Angle subtend at centre of circle = θ

Angle OAB = 90°

(At point of contract, tangent is perpendicular to radius)

OAB is right angle triangle

$$\cos\theta = \frac{r}{OB} = OB = r\sec\theta$$

$$\tan\theta = \frac{AB}{r} = AB = r\tan\theta$$

Perimeter of shaded region = AB+ BC+(CA arc)

$$= r\ \tan\theta + (OB - OC) + \frac{\theta}{360} \times 2\pi r$$

$$= r\ \tan\theta + r\ \sec\theta - r + \frac{\pi\theta r}{180}$$

$$= r\left(\tan\theta + \sec\theta + \frac{\pi\theta}{180} - 1\right)$$

Area of shaded region = (area of triangle AOB) – (area of sector)

$$= \left(\frac{1}{2} \times OA \times AB\right) - \frac{\theta}{360} \times \pi r^2$$

$$= \frac{1}{2} \times 2 \times r\tan\theta - \frac{r^2}{2}\left(\frac{\theta}{180} \times \pi\right)$$

$$= \frac{r^2}{2}\left(\tan\theta - \frac{\pi\theta}{180}\right)$$

20. Question

Figure 15.18 shows a sector of a circle of radius r cm containing an angle θ°. The area of the sector is A cm^2 and perimeter of the sector is 50cm.

(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$ (ii) $A = 25r - r^2$

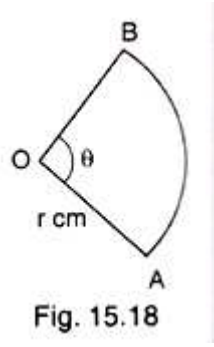


Fig. 15.18

Answer

Given,

Radius of the sector = r cm

Angle subtend = θ

Area of sector = A cm^2

Perimeter of sector = 50cm

Area of sector = $\frac{\theta}{360}\pi r^2$

Perimeter of sector = $\frac{\theta}{360}2\pi r + 2r$

$= \frac{\theta}{360} 2\pi r + 2r = 50$

$= 2r\left(\frac{\pi\theta}{360} + 1\right) = 50$

$= r\times\left(\frac{\pi\theta}{360} + 1\right) = \frac{50}{2} = 25$

$r = \frac{25}{\left(\frac{1+\pi\theta}{360}\right)}$ or $\frac{1+\pi\theta}{360} = \frac{25}{r}$

(i) $\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)$

$= \frac{\pi\theta}{360} = \frac{25}{r} - 1 \rightarrow \left(\theta = \frac{360}{\pi}\left(\frac{25}{r} - 1\right)\right)$

$= \frac{\theta}{360} = \frac{25-r}{\pi r} \rightarrow$ First equation

$\text{area} = \frac{\theta}{360}(\pi r^2) \rightarrow$ Second equation

Put value of $\frac{\theta}{360}$ from equation first to equation second

$\text{area} = \frac{25 - r}{\pi r}(\pi r^2) = (25 - r)r$

Area = $25r-r^2$

21. Question

The length of the minute hand of a clock is 14cm. Find the area swept by the minute hand in 5minutes.

Answer

The length of minute hand = 14cm

Time = 5 minute

Angle subtend by minute hand at center in 60 minute = 360°

In one minute = $\frac{360}{60} = 60°$

In five minute =5×6 = 30°

Area swept in 5 minute = $\frac{\theta}{360}\pi r^2$

$= \frac{30}{360}\times\frac{22}{7}\times 14\times 14 = \frac{154}{3}$

= $51.30cm^2$

22. Question

In a circle of radius 21cm, an arc subtends an angle of 60° at the centre. Find (i)the length of the arc (ii)area of the sector formed by the arc$(Use\ \pi = 22/7)$

Answer

Given,

Radius of circle = 21cm

Angle subtend by arc = 60°

$$= \frac{60\pi}{180} = \frac{\pi}{3}\,\text{radius}$$

Length of the arc = $\frac{\pi}{3} \times 21 = 22\text{cm}$

Area of sector formed by arc = $\frac{\theta}{360}\pi r^2$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 = 1 = 231\text{cm}^2$$

Exercise 15.3

1. Question

AB is a chord of a circle with centre O and radius 4cm. AB is of length 4cm and divides the circle into two segments. Find the area of the minor segment.

Answer

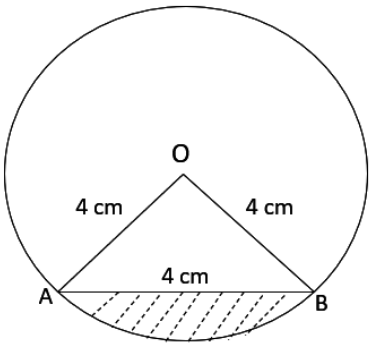
Given: AB is a chord of a circle with centre O and radius 4cm. AB is of length 4cm and divides the circle into two segments.

To find: the area of the minor segment.

Solution:

Radius of circle = 4cm

Length of chord = 4cm



(Hence it makes an equilateral triangle at centre, in which all angle must be = 60°)

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{60}{360}\times\pi\times4\times4$

$= \frac{1}{6}\times\pi\times4\times4 = \frac{8\pi}{3}\,cm^2$

Area of equilateral ΔOAB = $\frac{\sqrt{3}}{4}a^2$ $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times16 = 4\sqrt{3}\,cm^2$

Area of minor segment = area of sector – area of ΔOAB

$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)cm^2$

1. Question

AB is a chord of a circle with centre O and radius 4cm. AB is of length 4cm and divides the circle into two segments. Find the area of the minor segment.

Answer

Given,

Radius of circle = 4cm

Length of chord = 4cm

(Hence it makes an equilateral triangle at centre, in which all angle must be = 60°)

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{60}{360}\times\pi\times4\times4$

$\frac{1}{6}\times\pi\times4\times4 = \frac{8\pi}{3}\,cm^2$

Area of ΔOAB = $\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\times16 = 4\sqrt{3}\,cm^2$

Area of minor segment = area of sector – area of ΔOAB

$= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)cm^2$

2. Question

A chord PQ of length 12 cm subtends an angle of 120° at the centre of a circle. Find the area of the minor segment cut off by the chord PQ.

Answer

Length of chord PQ = 12cm

Angle subtend at the center = 120°

Let radius of circle = r cm

Area of sector = $\frac{120}{360}\pi r^2 = \frac{\pi r^2}{3} cm^2$

Length of triangle POQ = r cos 60

$= r \times \frac{1}{2} = \frac{r}{2} cm$

Length of base PQ = 2×RQ

$= 2 \times r \sin 60 = 2 \times r \times \frac{\sqrt{3}}{2} = \sqrt{3} r$

Put value of r in respective place,

Area of minor segment = area of sector – area of ΔPOQ

$= \frac{\pi r^2}{3} - \frac{1}{2} \times 12 \times \frac{r}{2}$

$= \frac{\pi \times 48}{3} - 3 \times 4\sqrt{3}$

$16\pi - 12\sqrt{3}$

$= 4\left(4\pi - 3\sqrt{3}\right) cm^2$

3. Question

A chord of a circle of radius 14cm makes a right angle at the centre. Find the areas of the minor and major segments of the circle.

Answer

Radius of the circle = 14cm

Angle subtend at center = 90°

By Pythagoras theorem = $AB^2 = OA^2 + OB^2$

$= 14^2 + 14^2$

$AB = 14\sqrt{2}$

Area of sector OAB = $\frac{90}{360} \times \pi r^2$

$= \frac{1}{4} \pi r^2$

$=\frac{1}{4}\times\frac{22}{7}\times14\times14=154cm^2$

Area of triangle AOB $=\frac{1}{2}\times14\times14=98\ cm^2$

So area of minor segment – OACB =area of sector – area of triangle

= 154 – 98 = 56cm^2

Area of major segment = area of circle - area of minor segment

$=\frac{22}{7}\times14\times14-56$

= 44×14 – 56 = 560cm^2

4. Question

A chord 10cm long is drawn in a circle whose radius is $5\sqrt{2}$ cm. Find area of both the segments. $(Take\ \pi=3.14)$

Answer

Length of chord = 10cm

Radius of circle = $5\sqrt{2}cm$

(This triangle POQ satisfy Pythagoras theorem)

= PQ^2 = PO^2 + OQ^2

$=10^2=(5\sqrt{2^2})+(5\sqrt{2}^2)$

= 100 = 50 + 50

So,

Angle AOQ = 90°

Area of sector = $\frac{90}{360}\pi\times50=\frac{25}{2}\pi\ cm^2$

Area of triangle POQ = $\frac{1}{2}\times5\sqrt{2}\times5\sqrt{2}\ =25cm^2$

Area of minor segment = $\frac{25}{2}\pi-25=14.25cm^2$

5. Question

A chord AB of a circle, of radius 14 cm makes an angle of 60° at the centre of the circle. Find the area of the minor segment of the circle. $(Use\ \pi=22/7)$

Answer

Radius of circle = 14cm

Angle = 60°

Area of sector = $\frac{\theta}{360}\pi r^2$

$= \frac{60}{360}\times\pi\times14\times14 = \frac{98}{3}\pi$ = 102.57cm^2

Area of triangle OAB = $\frac{1}{2}r^2\sin\theta$

$= \frac{1}{2}\times14\times14\times\sin\theta$

$= \frac{1}{2}\times14\times14\times\frac{\sqrt{3}}{2} = 49\sqrt{3} = 84.77cm^2$

So,

Area of minor segment = 102.57 – 84.77 = 17.80cm^2

Exercise 15.4

1. Question

A plot is in the form of a rectangle ABCD having semi-circle on BC as shown in Fig.15.64. If AB=60m and BC=28m, find the area of the plot.

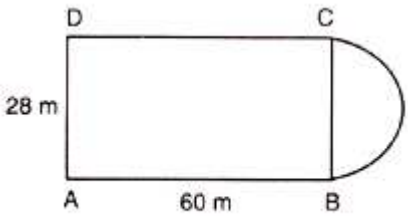


Fig. 15.64

Answer

Given,

AB = 60m

BC = 28m

Area of rectangular portion = 28m×60m = 1680m^2

Diameter of semicircle = length of side BC

Radius = $\frac{28}{2} = 14m$

Area of semicircle = $\frac{\pi r^2}{2} = \frac{22\times14\times14}{7\times2} = 308m^2$

Total area of plot = 1680+308 = 1988m^2

2. Question

A play ground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36m and 24.5m, find the area of the play ground. $(Take\ \pi = 22/7)$.

Answer

Given:

AB = 36m

BC = 24.5m

Area of rectangular portion = 36×24.5 = 882m^2

Radius of semicircular portion = $\frac{24.5}{2} = 12.25m$

Area of both semicircular portion = $2\times\frac{\pi r^2}{2}$

$= \frac{22}{7}\times12.25\times12.25$ = 471.625

Area of play ground = 882+471.625 = 1353.62

3. Question

The outer circumference of a circular race-track is 525m. The track is everywhere 14m wide. Calculate the cost of leveling the track at the rate of 50paise per square meter $(Use\ \pi = 22/7)$

Answer

Given,

Circumference of outer circle = 525m

Let radius of outer circle = R_2m

Let radius of inner circle = R_1m

So,

R_2-R_1 = 14 (equation 1)

= 2пR_2 = 525

$$R_2 = \frac{525}{2}\times\frac{7}{22} = 83.52m$$

Put value of R_1 in equation first

= 83.52 – R_1 = 14

= - R_1 = 14 – 83.52

= R_1 = 69.52m

Area of path = $\pi R_2^2 - \pi R_1^2$

$= \pi(R_2^2 - R_1^2) = \pi(R_2 + R_1)(R_2 - R_1)$

$= \frac{22}{7} \times (83.52 + 69.52)(83.52 - 69.52)$

$= \frac{22}{7} \times 153.04 \times 14 = 6733.76m^2$

Cost of leveling the path = 6733.76×.50 = Rs 3388

4. Question

A rectangular piece is 20m long and 15m wide. From its four corners, quadrants of radii 3.5m have been cut. Find the area of the remaining part.

Answer

Length of rectangle = $20m^2$

Breadth of rectangle = $15m^2$

Area of rectangle = $20 \times 15 = 300m^2$

Radius of quadrant = $3.5m^2$

Area of quadrant = $\frac{1}{4} \times \pi r^2$

Area of quadrant = $\frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 = \frac{19.25}{2} m^2$

Area of 4 quadrant = $4 \times \frac{19.25}{2} m^2 = 2 \times 19.25 = 38.50m^2$

Area of remaining part = (area of rectangle-area of 4 quadrant)

Area of remaining part = $300 - 38.50 = 261.5m^2$

5. Question

Four equal circles, each of radius 5cm, touch each other as showing fig.15.65. Find the area included between them. $(Take\ \pi = 3.14)$

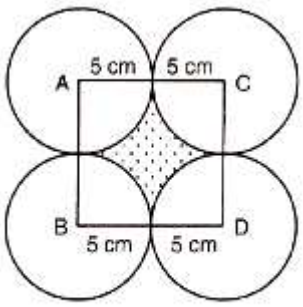


Fig. 15.65

Answer

Given,

Radius of each circle = 5cm

So,

Side of square = 10cm

Area of square = $(10)^2$ = 100cm^2

Area of each quadrant of circle with radius 5cm = $\frac{90}{360}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 25cm^2$

Area of 4 quadrants = $4 \times \frac{1}{4} \times \frac{22}{7} \times 25 = 25\pi cm^2$

Area of remaining portion = 100-25π = 21.5cm^2

6. Question

Four cows are tethered at four corners of a square plot of side 50m, so that they just cannot reach one another. What area will be left un-grazed?

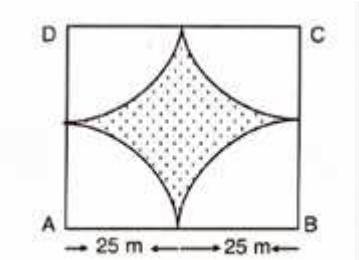


Fig. 15.66

Answer

Side of square = 50m

Area of square = $(5)^2$ = 2500m^2

Radius of quadrant circle = 25m

Area of one quadrant = $\frac{\pi r^2}{4} = \frac{625\pi}{4} m^2$

Area of 4 quadrants = $\frac{625\pi}{4} \times 4 = 625\pi = 1964.28m^2$

So,

Area which left un-grazed = 2500-1964.28 = 535.72m^2

7. Question

A road which is 7m wide surrounds a circular park whose circumference is 352m. Find the area of the road.

Answer

Given,

Circumference of park = 352m

Width of road = 7m

Let radius of park = r

2πr = 352

$$r = \frac{352 \times 7}{2 \times 22} = 56m$$

Area of circle = $\frac{22}{7} \times 56 \times 56 = 9856m^2$

Radius of circle included path of width, 7m = 56+7 = 63m

Area of circle included path = $\frac{22}{7} \times 63 \times 63 = \ 12474m^2$

So,

Area of path = 12474 – 9856 = 2618m^2

8. Question

Four equal circles, each of radius a, touch each other. Show that the area between them is $\frac{6}{7}a^2$ $(Take\ \pi = 3.14)$.

Answer

Radius of each circle = a meter

If we join the centre of each circle it makes a square of side = 2a

Area of square = $(2a)^2 = 4a^2m^2$

Area of each quadrant of circle = $\frac{\pi r^2}{4} = \frac{\pi a^2}{4} m^2$

Area of 4 quadrants = $4 \times \frac{\pi a^2}{4} = \pi a^2 m^2$

So,

Area between circles = $4a^2 - \pi a^2$

$$= 4a^2 - \frac{22}{7}a^2 = \frac{28a^2 - 22a^2}{7} = \frac{6a^2}{7} m^2$$

9. Question

A square water tank has its side equal to 40m. There are four semi-circular grassy plots all round it. Find the cost of surfing the plot at Rs.1.25 per square meter $(Take\ \pi = 3.14)$.

Answer

Side of water tank = 40m

Side of semi circular grassy plots = $\frac{40}{2} = 20m$

Area of one grassy plot = $\frac{\pi r^2}{2}$

$= \frac{22}{7} \times \frac{20 \times 20}{2} = \frac{400\pi}{2} = 200\pi$

Area of grassy plots = 4×200π = 800π

Area of grassy plots = 800 × 3.14 = 2512 cm^2

Cost of surfing 1m^2 plot = 1.25 Rs

Cost of surfing 2512m^2 = 2512 × 1.25 = 3140 Rs

10. Question

A rectangular park is 100m by 50m. It is surrounded by semi-circular flower bed sall round. Find the cost of leveling the semi-circular flower bed sall 60paise per square meter. $(Use\ \pi = 3.14)$

Answer

Length of rectangular park = 100m

Breadth of rectangular park = 50m

Radius of flower bed along length of park = $\frac{100}{2} = 50m$

Area of flower bed along length of park = $\frac{2 \times \pi r^2}{2}$

$= \frac{22}{7} \times 50 \times 50 = 7850m^2$

Radius of flower bed along width = $\frac{50}{2} = 25m$

Area of flower bed along width = $2 \times \frac{22}{7} \times \frac{25 \times 25}{2} = 1962.5m^2$

Total area of flower beds = 7850+1962.50 = 4812.50m^2

So,

Cost of leveling semicircular flower beds = 9812.50×.60 = Rs 5887.50

11. Question

Prove that the area of a circular path of uniform width h surrounding a circular region of radius is $\pi h(2r + h)$.

Answer

Area of inner circle with radius r = πr^2

Radius of outer circle = r+h

Area of outer circle = $\pi(r+h)^2$

Area of circular path with width = h

= $\pi(r+h)^2 - \pi r^2$

By using $(a+b)^2 = a^2 + b^2 + 2ab$

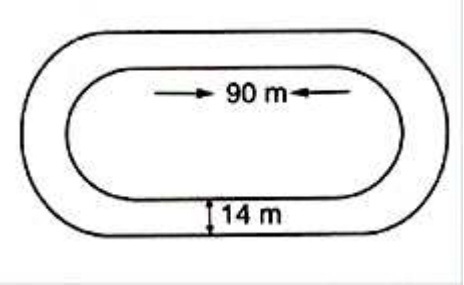
= $\pi(r^2 + h^2 + 2rh) - \pi r^2$

=$\pi r^2 + \pi h^2 + 2\pi rh - \pi r^2$

= $\pi h(2r+h)$… Proved

12. Question

The inside perimeter of a running track (showninFig.15.67) is 400m. The length of each of the straight portion is 90m and the ends are semi-circles. If the track is everywhere 14m wide, find the area of the track. Also find the length of the outer running track.



Answer

Given,

Inside perimeter of track = 400m

Length of straight portion = 90m

Width of path = 14m

Total length of straight path = 90+90 = 180m

Remaining length = 400 – 180 = 220m

This length includes two semi circles or a complete circle.

So,

2πr = 220m

$$= r = \frac{220 \times 7}{2 \times 22} = 35m$$

Then,

Area of path = (area of rectangles ABCD + rectangle EFGH + two semicircles)

= 14×90+14×90+π [$(25+14)^2 - 35^2$]

[$(a^2 - b^2) = (a+b)(a-b)$]

$$= 2520 + \frac{22}{7} \times 84 \times 14^2$$

Area of path = 6216m^2

Length of outer track = 90+90+2πr

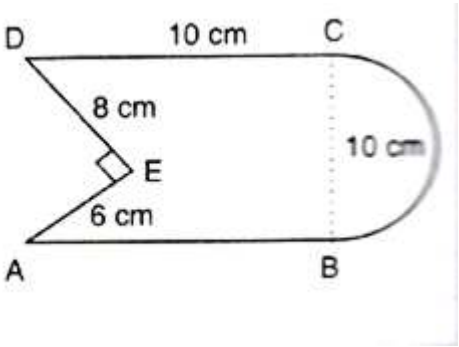
r = 35+14 = 49

$$= 180 + 2\frac{22}{7} \times 49^2$$

= 180 +308 = 488m^2

13. Question

Find the area of Fig15.68, in square cm, correct to one place of decimal. (*Take* $\pi = 22/7$)



Answer

Area of semicircle with diameter = 10cm

$$r = \frac{10}{2} = 5cm$$

$$= \frac{\pi r^2}{2} = \frac{22 \times 5 \times 5}{7 \times 2} = 39.28\ cm^2$$

Area of triangle AED = $\frac{1}{2} \times 8 \times 6 = 24cm^2$

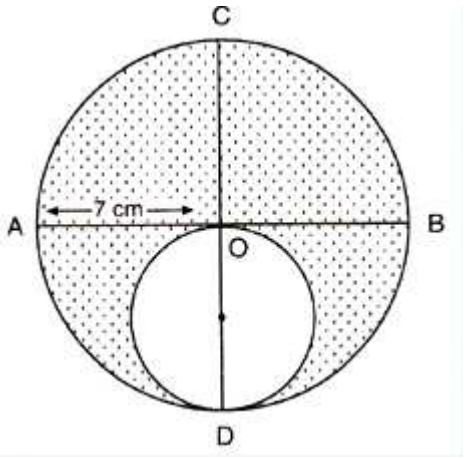
Area of square ABCD = 10×10 = 100cm^2

Area of figure excluded triangle = 100-24 = 76 cm^2

Total area of figure = 39.28+76 = 115.3 cm^2

14. Question

In Fig.15.69, AB and CD are two diameters of a circle perpendicular to each other and OD is the diameter of the smaller circle. If OA=7cm, find the area of the shaded region.



Answer

Area of semicircle ACB = $\frac{\pi r^2}{2}$

$$= \frac{22}{7} \times \frac{7 \times 7}{2} = 77cm^2$$

= area of circle with diameter OD = $\pi r^2 \left(r = \frac{7}{2} = 3.5\right)$

$$= \frac{22}{7} \times 3.5 \times 3.5 = 38.5cm^2$$

Remaining shaded portion in lower semi circle = 77 – 38.5 = 38.5cm^2

Total shaded portion area = 77 + 38.5 = 115.5 cm^2

15. Question

In Fig.15.70, OACB is a quadrant of a circle with centre O and radius 3.5cm. If OD=2cm, find the area of the (i) quadrant OACB (ii) shaded region.

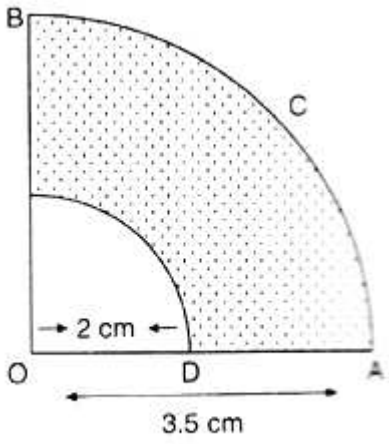


Fig. 15.70

Answer

Given,

Area of quadrant OACB = $\frac{\theta}{360}\pi r^2$

$= \frac{90}{360} \times \frac{22}{7} \times 3.5 \times 3.5$

$= \frac{1}{4} \times 11 \times 3.5 = 9.625 cm^2$

Area of shaded region = area of quadrant OACB – area of quadrant ODEF

$= 9.625 - \frac{90}{360} \times \frac{22}{360} \times 2 \times 2$

$= 9.625 - \frac{1}{4} \times 3.14 \times 4 = 6.482 cm^2$

16. Question

From each of the two opposite corners of a square of side 8cm, a quadrant of a circle of radius 1.4cm is cut. Another circle of radius 4.2cm is also cut from the centre as shown in Fig.15.71. Find the area of the remaining (shaded) portion of the square. (*Use* $\pi = 22/7$).

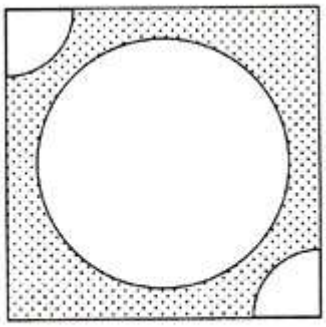


Fig. 15.71

Answer

Given,

Side of square = 8cm

Radius of quadrant circle = 1.4 cm

Radius of inner-circle = 4.2

Area of square = $(side)^2 = 8^2 = 64cm$

Area of one quadrant of circle = $\frac{\theta}{360} \times \pi r^2$

Area of one quadrant of circle = $\frac{90}{360} \times \frac{22}{7} \times 1.4 \times 1.4 = 1.54 cm^2$

So,

Area of 2 quadrant = $2\times1.54 = 3.08cm^2$

Area of inner circle = $\pi r^2 = 3.14\times4.2\times4.2 = 55.44\ cm^2$

Area of shaded portion = area of square – (area of quadrants + area of inner circle)

= 64 – (3.08 + 55.44)

= 64 – 58.52 = $5.48\ cm^2$

17. Question

Find the area of the shaded region in Fig.15.72, if AC=24cm, BC=10cm and O is the centre of the circle. $(Use\ \pi = 3.14)$

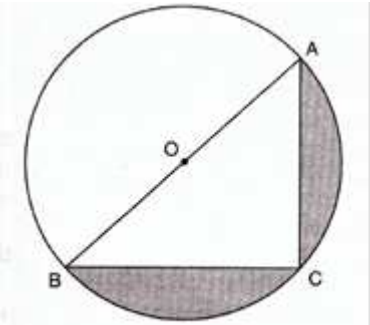


Fig. 15.72

Answer

Given,

AC = 24 cm

BC = 10 cm

By Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$= 24^2+10^2 = 576 + 100 = 676$

AB = $\sqrt{676}$ = 26cm

Radius of semi-circle with diameter AB = $\frac{26}{2}$ = 13cm

Area of semi-circle = $\frac{\pi r^2}{2} = \frac{3.14\times13\times13}{2}$ = $265.33cm^2$

Area of triangle ABC = $\frac{1}{2}\times AC\times BC = \frac{1}{2}\times24\times10$ = $120cm^2$

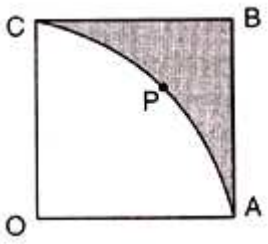
So,

Area of shaded region = area of semi-circle – area of triangle

= 265.33 – 120 = $145.33\ cm^2$

18. Question

In Fig.15.72(a), OABC is a square of side 7cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region. $(Use\ \pi = 22/7)$



Answer

Given,

Side of square = 7cm

Area of square = $(side)^2 = 7^2 = 49cm^2$

Area of quadrant OAPC = $\frac{\theta}{360}\pi r^2$

$$= \frac{90}{360} \times \frac{22}{7} \times 7 \times 7 = \frac{1}{4} \times 154 = 38.5 cm^2$$

Area of shaded region = (area of square – area of quadrant)

$= 49 - 38.5 = 10.5\ cm^2$

19. Question

A circular pond is of diameter 17.5m. It is surrounded by a 2m wide path. Find the cost of constructing the path at the rate of Rs. 25 per square meter $(Use\ \pi = 3.14)$

Answer

Given,

Diameter of circular pond = 17.5m

Radius of circular pond = $\frac{17.5}{2}$ = 8.75m

Radius of outer circle = (radius of inner circle + width of circular path)

= 8.75+2 = 10.25m

Area of circular path = (area of outer circle – area of inner circle)

$= \pi(R^2 - r^2)$

$= \pi(R+r)(R-r)$

$= \frac{22}{7}(10.75+8.75)(10.75-8.75)$

$= \frac{22}{7} \times 19.50 \times 2 = 3061.50\ m^2$

20. Question

A regular hexagon is inscribed in a circle. If the area of hexagon is $24\sqrt{3}\ cm^2$, find the area of the circle.

$(Use\ \pi = 3.14)$

Answer

Given,

Area of regular hexagon = $24\sqrt{3}$ cm^2

From formula

$$\frac{3\sqrt{3}}{2} \times a^2 = 24\sqrt{3}$$

$$a^2 = \frac{24\sqrt{3} \times 2}{3\sqrt{3}} = 16$$

$$a = \sqrt{16} = 4cm$$

So,

Area of circum circle of regular hexagon = π(side)^2

= 3.14×4×4cm^2 = 50.24cm^2

21. Question

A path of width 3.5m runs around a semi-circular grassy plot whose perimeter is 72m. find the area of the path. $(Use\ \pi = 22/7)$

Answer

Given,

Perimeter of semi-circle = 72m

Width of path around it = 3.5m

Perimeter of semi-circle = πr+2r

= $\frac{22}{7}$r +2r = 72

= 22r+14r = 72×7

r = $\frac{72 \times 7}{36}$ = 14cm

Radius including the width of path(R) = r+3.5 = 14+3.5 = 17.5m

So, area of path = $\frac{\pi R^2}{2} - \frac{\pi r^2}{2}$

$= \frac{\pi}{2}\left((17.5^2) - (14^2)\right)$

$= \frac{\pi}{2}\left((17.5 + 14)(17.5 - 14)\right)$

$= \frac{3.14}{2} \times 31.5 \times 3.5$ = 173.25m^2

22. Question

Find the area of a shaded region in the Fig.15.73, where a circular arc of radius 7cm has been drawn with vertex A of an equilateral triangle ABC of side 14cm as centre. $\left(Use\ \pi = 22/7\ and\ \sqrt{3} = 1.73\right)$

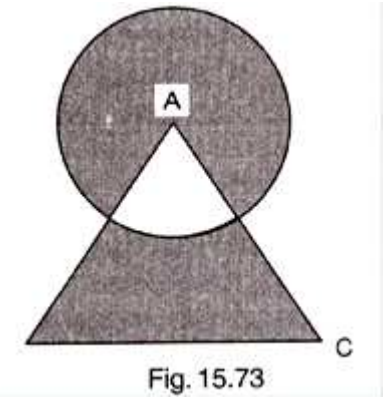


Fig. 15.73

Answer

Given,

Radius = 7cm

Side of equilateral triangle = 14cm

Area of circle = πr^2

Area of circle = $\frac{22}{7} \times 7 \times 7 = 154cm^2$

Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times 14 \times 14$

$= \frac{\sqrt{3}}{4} \times 196 = 84.77cm^2$

We know that an equilateral triangle always subtend an angle of 60 at centre area of sector = $\frac{q}{360} \times \pi r^2$

$= \frac{60}{360} \times \frac{22}{7} \times 7 \times 7$

$= \frac{1}{6} \times 154$ = 25.666cm^2

This area is common in both the figure so,

Area of shaded region = (area of circle + area of equilateral triangle - 2×area of sector)

= (154+84.77-2×25.67)

= (238.77-51.33) = 187.44cm^2

23. Question

A child makes a poster on a chart paper drawing a square ABCD of side 14cm. She draws four circles with centre A,B,C and D in which she suggests different ways to save energy. The circles are drawn in such away that each circle touches externally two of the three remaining circles (Fig.15.74). In the shaded region she writes a message 'Save Energy'. Find the perimeter and area of the shaded region. $(Use\ \pi = 22/7)$

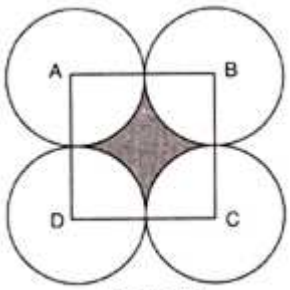


Fig. 15.74

Answer

Given,

Side of square = 14cm

Radius of each circle = $\frac{14}{2}$ = 7cm

Area of square = (side)^2 = 14^2 = 196cm^2

Area of 4 quadrants of circle = $\frac{90}{360}\times\frac{22}{7}\times7\times7$

$$= 4\times\frac{1}{4}\times154 = 154cm^2$$

Area of shaded region = area of square – area of 4 quadrants

= 196 – 154 = 42cm^2

Perimeter of shaded region = $\frac{90}{360}$ ×2πr

$$=\frac{1}{4}\times2\times\frac{22}{7}\times7 = 11cm$$

So, total perimeter of 4 circles = 4×11 = 44cm