14. Differentials, Errors and Approximations

Exercise 14.1

1. Question

If y = sin x and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$, what is the approximate change in y?

Answer

Given y = sin x and x changes from $\frac{\pi}{2}$ to $\frac{22}{14}$.

Let $x = \frac{\pi}{2}$ so that $x + \Delta x = \frac{22}{14}$

$\Rightarrow \frac{\pi}{2} + \Delta x = \frac{22}{14}$

$\therefore \Delta x = \frac{22}{14} - \frac{\pi}{2}$

On differentiating y with respect to x, we get

$\frac{dy}{dx} = \frac{d}{dx}(\sin x)$

We know $\frac{d}{dx}(\sin x) = \cos x$

$\therefore \frac{dy}{dx} = \cos x$

When $x = \frac{\pi}{2}$, we have $\frac{dy}{dx} = \cos\left(\frac{\pi}{2}\right)$.

$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}} = 0$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{dy}{dx} = 0$ and $\Delta x = \frac{22}{14} - \frac{\pi}{2}$

$\Rightarrow \Delta y = (0)\left(\frac{22}{14} - \frac{\pi}{2}\right)$

∴ Δy = 0

Thus, there is approximately no change in y.

2. Question

The radius of a sphere shrinks from 10 to 9.8 cm. Find approximately the decrease in its volume.

Answer

Given the radius of a sphere changes from 10 cm to 9.8 cm.

Let x be the radius of the sphere and Δx be the change in the value of x.

Hence, we have x = 10 and x + Δx = 9.8

⇒ 10 + Δx = 9.8

$\Rightarrow \Delta x = 9.8 - 10$

$\therefore \Delta x = -0.2$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

When x = 10, we have $\frac{dV}{dx} = 4\pi(10)^2$.

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 4\pi \times 100$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=10} = 400\pi$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 400\pi$ and Δx = –0.2

$\Rightarrow \Delta V = (400\pi)(-0.2)$

$\therefore \Delta V = -80\pi$

Thus, the approximate decrease in the volume of the sphere is 80π cm^3.

3. Question

A circular metal plate expands under heating so that its radius increases by k%. Find the approximate increase in the area of the plate, if the radius of the plate before heating is 10 cm.

Answer

Given the radius of a circular plate initially is 10 cm and it increases by k%.

Let x be the radius of the circular plate, and Δx is the change in the value of x.

Hence, we have x = 10 and $\Delta x = \frac{k}{100} \times 10$

$\therefore \Delta x = 0.1k$

The area of a circular plate of radius x is given by

$A = \pi x^2$

On differentiating A with respect to x, we get

$$\frac{dA}{dx} = \frac{d}{dx}(\pi x^2)$$

$$\Rightarrow \frac{dA}{dx} = \pi\frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dA}{dx} = \pi(2x)$$

$$\therefore \frac{dA}{dx} = 2\pi x$$

When x = 10, we have $\frac{dA}{dx} = 2\pi(10)$.

$$\Rightarrow \left(\frac{dA}{dx}\right)_{x=10} = 20\pi$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dA}{dx} = 20\pi$ and Δx = 0.1k

⇒ ΔA = (20π)(0.1k)

∴ ΔA = 2kπ

Thus, the approximate increase in the area of the circular plate is 2kπ cm^2.

4. Question

Find the percentage error in calculating the surface area of a cubical box if an error of 1% is made in measuring the lengths of the edges of the cube.

Answer

Given the error in the measurement of the edge of a cubical box is 1%.

Let x be the edge of the cubical box, and Δx is the error in the value of x.

Hence, we have $\Delta x = \frac{1}{100} \times x$

∴ Δx = 0.01x

The surface area of a cubical box of radius x is given by

S = $6x^2$

On differentiating A with respect to x, we get

$$\frac{dS}{dx} = \frac{d}{dx}(6x^2)$$

$$\Rightarrow \frac{dS}{dx} = 6\frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 6(2x)$$

$\therefore \frac{dS}{dx} = 12x$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{dS}{dx} = 12x$ and Δx = 0.01x

⇒ ΔS = (12x)(0.01x)

∴ ΔS = $0.12x^2$

The percentage error is,

$\text{Error} = \frac{0.12x^2}{6x^2} \times 100\%$

⇒ Error = 0.02 × 100%

∴ Error = 2%

Thus, the error in calculating the surface area of the cubical box is 2%.

5. Question

If there is an error of 0.1% in the measurement of the radius of a sphere, find approximately the percentage error in the calculation of the volume of the sphere.

Answer

Given the error in the measurement of the radius of a sphere is 0.1%.

Let x be the radius of the sphere and Δx be the error in the value of x.

Hence, we have $\Delta x = \frac{0.1}{100} \times x$

∴ Δx = 0.001x

The volume of a sphere of radius x is given by

$V = \frac{4}{3}\pi x^3$

On differentiating V with respect to x, we get

$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$

$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$

$\therefore \frac{dV}{dx} = 4\pi x^2$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{dV}{dx} = 4\pi x^2$ and $\Delta x = 0.001x$

$\Rightarrow \Delta V = (4\pi x^2)(0.001x)$

$\therefore \Delta V = 0.004\pi x^3$

The percentage error is,

$$\text{Error} = \frac{0.004\pi x^3}{\frac{4}{3}\pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.004 \times 3}{4} \times 100\%$$

$\Rightarrow$ Error = $0.003 \times 100\%$

$\therefore$ Error = 0.3%

Thus, the error in calculating the volume of the sphere is 0.3%.

6. Question

The pressure p and the volume v of a gas are connected by the relation $pv^{1.4}$ = const. Find the percentage error in p corresponding to a decrease of $\frac{1}{2}\%$ in v.

Answer

Given $pv^{1.4}$ = constant and the decrease in v is $\frac{1}{2}\%$.

Hence, we have $\Delta v = -\frac{\frac{1}{2}}{100} \times v$

$\therefore \Delta v = -0.005v$

We have $pv^{1.4}$ = constant

Taking log on both sides, we get

$\log(pv^{1.4}) = \log(\text{constant})$

$\Rightarrow \log p + \log v^{1.4} = 0$ [$\because \log(ab) = \log a + \log b$]

$\Rightarrow \log p + 1.4 \log v = 0$ [$\because \log(a^m) = m \log a$]

On differentiating both sides with respect to v, we get

$$\frac{d}{dp}(\log p) \times \frac{dp}{dv} + \frac{d}{dv}(1.4 \log v) = 0$$

$$\Rightarrow \frac{d}{dp}(\log p) \times \frac{dp}{dv} + 1.4\frac{d}{dv}(\log v) = 0$$

We know $\frac{d}{dx}(\log x) = \frac{1}{x}$

$$\Rightarrow \frac{1}{p} \times \frac{dp}{dv} + 1.4 \times \frac{1}{v} = 0$$

$$\Rightarrow \frac{1}{p}\frac{dp}{dv} + \frac{1.4}{v} = 0$$

$$\Rightarrow \frac{1}{p}\frac{dp}{dv} = -\frac{1.4}{v}$$

$\therefore \frac{dp}{dv} = -\frac{1.4}{v}p$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{dp}{dv} = -\frac{1.4}{v}p$ and Δv = –0.005v

$\Rightarrow \Delta p = \left(-\frac{1.4}{v}p\right)(-0.005v)$

⇒ Δp = (–1.4p)(–0.005)

∴ Δp = 0.007p

The percentage error is,

$\text{Error} = \frac{0.007p}{p} \times 100\%$

⇒ Error = 0.007 × 100%

∴ Error = 0.7%

Thus, the error in p corresponding to the decrease in v is 0.7%.

7. Question

The height of a cone increases by k%, its semi-vertical angle remaining the same. What is the approximate percentage increase in (i) in total surface area, and (ii) in the volume, assuming that k is small.

Answer

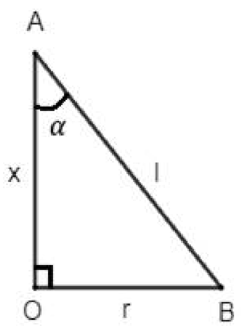
Given the height of a cone increases by k%.

Let x be the height of the cone and Δx be the change in the value of x.

Hence, we have $\Delta x = \frac{k}{100} \times x$

∴ Δx = 0.01kx

Let us assume the radius, the slant height and the semi-vertical angle of the cone to be r, l and α respectively as shown in the figure below.



From the above figure, using trigonometry, we have

$\tan\alpha = \frac{OB}{OA}$

$\Rightarrow \tan\alpha = \frac{r}{x}$

∴ r = x tan(α)

We also have

$$\cos\alpha = \frac{OA}{AB}$$

$$\Rightarrow \cos\alpha = \frac{x}{l}$$

$$\Rightarrow l = \frac{x}{\cos\alpha}$$

∴ $l = x \sec(\alpha)$

(i) The total surface area of the cone is given by

$S = \pi r^2 + \pi rl$

From above, we have $r = x \tan(\alpha)$ and $l = x \sec(\alpha)$.

$\Rightarrow S = \pi(x \tan(\alpha))^2 + \pi(x \tan(\alpha))(x \sec(\alpha))$

$\Rightarrow S = \pi x^2 \tan^2\alpha + \pi x^2 \tan(\alpha)\sec(\alpha)$

$\Rightarrow S = \pi x^2 \tan(\alpha)[\tan(\alpha) + \sec(\alpha)]$

On differentiating S with respect to x, we get

$$\frac{dS}{dx} = \frac{d}{dx}[\pi x^2 \tan\alpha(\tan\alpha + \sec\alpha)]$$

$$\Rightarrow \frac{dS}{dx} = \pi \tan\alpha(\tan\alpha + \sec\alpha)\frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = \pi \tan\alpha(\tan\alpha + \sec\alpha)(2x)$$

$$\therefore \frac{dS}{dx} = 2\pi x \tan\alpha(\tan\alpha + \sec\alpha)$$

Recall that if $y = f(x)$ and Δx is a small increment in x, then the corresponding increment in y, $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dS}{dx} = 2\pi x \tan\alpha(\tan\alpha + \sec\alpha)$ and $\Delta x = 0.01kx$

$\Rightarrow \Delta S = (2\pi x\tan(\alpha)[\tan(\alpha) + \sec(\alpha)])(0.01kx)$

∴ $\Delta S = 0.02k\pi x^2\tan(\alpha)[\tan(\alpha) + \sec(\alpha)]$

The percentage increase in S is,

$$\text{Increase} = \frac{\Delta S}{S} \times 100\%$$

$$\Rightarrow \text{Increase} = \frac{0.02k\pi x^2 \tan\alpha(\tan\alpha + \sec\alpha)}{\pi x^2 \tan\alpha(\tan\alpha + \sec\alpha)} \times 100\%$$

$\Rightarrow$ Increase = $0.02k \times 100\%$

∴ Increase = 2k%

Thus, the approximate increase in the total surface area of the cone is 2k%.

(ii) The volume of the cone is given by

$V = \frac{1}{3}\pi r^2 x$

From above, we have r = x tan(α).

$\Rightarrow V = \frac{1}{3}\pi(x\tan\alpha)^2 x$

$\Rightarrow V = \frac{1}{3}\pi(x^2\tan^2\alpha)x$

$\Rightarrow V = \frac{1}{3}\pi x^3\tan^2\alpha$

On differentiating V with respect to x, we get

$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{1}{3}\pi x^3\tan^2\alpha\right)$

$\Rightarrow \frac{dV}{dx} = \frac{1}{3}\pi\tan^2\alpha\frac{d}{dx}(x^3)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{dV}{dx} = \frac{1}{3}\pi\tan^2\alpha\,(3x^2)$

$\therefore \frac{dV}{dx} = \pi x^2\tan^2\alpha$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{dV}{dx} = \pi x^2\tan^2\alpha$ and Δx = 0.01kx

$\Rightarrow \Delta V = (\pi x^2\tan^2\alpha)(0.01kx)$

$\therefore \Delta V = 0.01k\pi x^3\tan^2\alpha$

The percentage increase in V is,

$\text{Increase} = \frac{\Delta V}{V} \times 100\%$

$\Rightarrow \text{Increase} = \frac{0.01k\pi x^3\tan^2\alpha}{\frac{1}{3}\pi x^3\tan^2\alpha} \times 100\%$

$\Rightarrow \text{Increase} = \frac{0.01k}{\frac{1}{3}} \times 100\%$

⇒ Increase = 0.03k × 100%

∴ Increase = 3k%

Thus, the approximate increase in the volume of the cone is 3k%.

8. Question

Show that the relative error in computing the volume of a sphere, due to an error in measuring the radius, is approximately equal to three times the relative error in the radius.

Answer

Let the error in measuring the radius of a sphere be k%.

Let x be the radius of the sphere and Δx be the error in the value of x.

Hence, we have $\Delta x = \frac{k}{100} \times x$

$\therefore \Delta x = 0.01kx$

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 4\pi x^2$ and $\Delta x = 0.01kx$

$\Rightarrow \Delta V = (4\pi x^2)(0.01kx)$

$\therefore \Delta V = 0.04k\pi x^3$

The percentage error is,

$$\text{Error} = \frac{0.04k\pi x^3}{\frac{4}{3}\pi x^3} \times 100\%$$

$$\Rightarrow \text{Error} = \frac{0.04k \times 3}{4} \times 100\%$$

$\Rightarrow$ Error = 0.03k × 100%

$\therefore$ Error = 3k%

Thus, the error in measuring the volume of the sphere is approximately three times the error in measuring its radius.

9 A. Question

Using differentials, find the approximate values of the following:

$$\sqrt{25.02}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 25 so that x + Δx = 25.02

⇒ 25 + Δx = 25.02

∴ Δx = 0.02

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(\sqrt{x}\right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When x = 25, we have $\frac{df}{dx} = \frac{1}{2\sqrt{25}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = \frac{1}{2 \times 5}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = 0.1$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.1$ and Δx = 0.02

⇒ Δf = (0.1)(0.02)

∴ Δf = 0.002

Now, we have f(25.02) = f(25) + Δf

$$\Rightarrow f(25.02) = \sqrt{25} + 0.002$$

⇒ f(25.02) = 5 + 0.002

∴ f(25.02) = 5.002

Thus, $\sqrt{25.02} \approx 5.002$

9 B. Question

Using differentials, find the approximate values of the following:

$(0.009)^{1/3}$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let x = 0.008 so that x + Δx = 0.009

⇒ 0.008 + Δx = 0.009

∴ Δx = 0.001

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{3}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When x = 0.008, we have $\frac{df}{dx} = \frac{1}{3(0.008)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{3\left((0.2)^3\right)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{3(0.2)^2}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{3(0.04)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{0.12}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = 8.3333$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 8.3333$ and Δx = 0.001

⇒ Δf = (8.3333)(0.001)

∴ Δf = 0.0083333

Now, we have f(0.009) = f(0.008) + Δf

$$\Rightarrow f(0.009) = (0.008)^{\frac{1}{3}} + 0.0083333$$

$$\Rightarrow f(0.009) = \left((0.2)^3\right)^{\frac{1}{3}} + 0.0083333$$

⇒ f(0.009) = 0.2 + 0.0083333

∴ f(0.009) = 0.2083333

Thus, $(0.009)^{1/3} \approx 0.2083333$

9 C. Question

Using differentials, find the approximate values of the following:

$(0.007)^{1/3}$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let x = 0.008 so that x + Δx = 0.007

⇒ 0.008 + Δx = 0.007

∴ Δx = –0.001

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{3}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When x = 0.008, we have $\frac{df}{dx} = \frac{1}{3(0.008)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{3\left((0.2)^3\right)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{3(0.2)^2}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{3(0.04)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = \frac{1}{0.12}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.008} = 8.3333$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 8.3333$ and Δx = 0.001

⇒ Δf = (8.3333)(–0.001)

∴ Δf = –0.0083333

Now, we have f(0.007) = f(0.008) + Δf

$$\Rightarrow f(0.007) = (0.008)^{\frac{1}{3}} - 0.0083333$$

$$\Rightarrow f(0.007) = \left((0.2)^3\right)^{\frac{1}{3}} - 0.0083333$$

⇒ f(0.007) = 0.2 – 0.0083333

∴ f(0.007) = 0.1916667

Thus, $(0.007)^{1/3} \approx 0.1916667$

9 D. Question

Using differentials, find the approximate values of the following:

$\sqrt{401}$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 400 so that x + Δx = 401

⇒ 400 + Δx = 401

∴ Δx = 1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When x = 400, we have $\frac{df}{dx} = \frac{1}{2\sqrt{400}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=400} = \frac{1}{2 \times 20}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=400} = \frac{1}{40}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=400} = 0.025$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.025$ and Δx = 1

⇒ Δf = (0.025)(1)

∴ Δf = 0.025

Now, we have f(401) = f(400) + Δf

$\Rightarrow f(401) = \sqrt{400} + 0.025$

⇒ f(401) = 20 + 0.025

∴ f(401) = 20.025

Thus, $\sqrt{401} \approx 20.025$

9 E. Question

Using differentials, find the approximate values of the following:

$(15)^{1/4}$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let x = 16 so that x + Δx = 15

⇒ 16 + Δx = 15

∴ Δx = –1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When x = 16, we have $\frac{df}{dx} = \frac{1}{4(16)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=16} = \frac{1}{4(2^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=16} = \frac{1}{4(2^3)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=16} = \frac{1}{4(8)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=16} = \frac{1}{32}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=16} = 0.03125$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.03125$ and Δx = –1

$\Rightarrow \Delta f = (0.03125)(-1)$

$\therefore \Delta f = -0.03125$

Now, we have f(15) = f(16) + Δf

$\Rightarrow f(15) = (16)^{\frac{1}{4}} - 0.03125$

$\Rightarrow f(15) = (2^4)^{\frac{1}{4}} - 0.03125$

$\Rightarrow f(15) = 2 - 0.03125$

$\therefore f(15) = 1.96875$

Thus, $(15)^{1/4} \approx 1.96875$

9 F. Question

Using differentials, find the approximate values of the following:

$(255)^{1/4}$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let x = 256 so that x + Δx = 255

$\Rightarrow 256 + \Delta x = 255$

$\therefore \Delta x = -1$

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When x = 256, we have $\frac{df}{dx} = \frac{1}{4(256)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=256} = \frac{1}{4(4^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=256} = \frac{1}{4(4^3)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=256} = \frac{1}{4(64)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=256} = \frac{1}{256}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=256} = 0.00390625$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.00390625$ and Δx = –1

⇒ Δf = (0.00390625)(–1)

∴ Δf = –0.00390625

Now, we have f(255) = f(256) + Δf

$$\Rightarrow f(255) = (256)^{\frac{1}{4}} - 0.00390625$$

$$\Rightarrow f(255) = (4^4)^{\frac{1}{4}} - 0.00390625$$

⇒ f(255) = 4 – 0.00390625

∴ f(255) = 3.99609375

Thus, $(255)^{1/4} \approx 3.99609375$

9 G. Question

Using differentials, find the approximate values of the following:

$$\frac{1}{(2.002)^2}$$

Answer

Let us assume that $f(x) = \frac{1}{x^2}$

Also, let x = 2 so that x + Δx = 2.002

⇒ 2 + Δx = 2.002

∴ Δx = 0.002

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(x^{-2})$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = -2x^{-2-1}$$

$$\Rightarrow \frac{df}{dx} = -2x^{-3}$$

$$\therefore \frac{df}{dx} = -\frac{2}{x^3}$$

When x = 2, we have $\frac{df}{dx} = -\frac{2}{2^3}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = -\frac{2}{8}$$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = -0.25$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = -0.25$ and Δx = 0.002

⇒ Δf = (–0.25)(0.002)

∴ Δf = –0.0005

Now, we have f(2.002) = f(2) + Δf

$\Rightarrow f(2.002) = \frac{1}{(2)^2} - 0.0005$

$\Rightarrow f(2.002) = \frac{1}{4} - 0.0005$

⇒ f(2.002) = 0.25 – 0.0005

∴ f(2.002) = 0.2495

Thus, $\frac{1}{(2.002)^2} \approx 0.2495$

9 H. Question

Using differentials, find the approximate values of the following:

$\log_e 4.04$, it being given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$

Answer

$\log_e 4.04$, it being given that $\log_{10} 4 = 0.6021$ and $\log_{10} e = 0.4343$

Let us assume that $f(x) = \log_e x$

Also, let x = 4 so that x + Δx = 4.04

⇒ 4 + Δx = 4.04

∴ Δx = 0.04

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(\log_e x)$

We know $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$\therefore \frac{df}{dx} = \frac{1}{x}$

When x = 4, we have $\frac{df}{dx} = \frac{1}{4}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=4} = 0.25$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0.25$ and $\Delta x = 0.04$

$\Rightarrow \Delta f = (0.25)(0.04)$

$\therefore \Delta f = 0.01$

Now, we have $f(4.04) = f(4) + \Delta f$

$\Rightarrow f(4.04) = \log_e 4 + 0.01$

$$\Rightarrow f(4.04) = \frac{\log_{10} 4}{\log_{10} e} + 0.01 \left[\because \log_b a = \frac{\log_c a}{\log_c b}\right]$$

$$\Rightarrow f(4.04) = \frac{0.6021}{0.4343} + 0.01$$

$\Rightarrow f(4.04) = 1.3863689 + 0.01$

$\therefore f(4.04) = 1.3963689$

Thus, $\log_e 4.04 \approx 1.3963689$

9 I. Question

Using differentials, find the approximate values of the following:

$\log_e 10.02$, it being given that $\log_e 10 = 2.3026$

Answer

$\log_e 10.02$, it being given that $\log_e 10 = 2.3026$

Let us assume that $f(x) = \log_e x$

Also, let $x = 10$ so that $x + \Delta x = 10.02$

$\Rightarrow 10 + \Delta x = 10.02$

$\therefore \Delta x = 0.02$

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\log_e x)$$

We know $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$$\therefore \frac{df}{dx} = \frac{1}{x}$$

When $x = 10$, we have $\frac{df}{dx} = \frac{1}{10}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=10} = 0.1$$

Recall that if $y = f(x)$ and Δx is a small increment in x, then the corresponding increment in y, $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.1$ and $\Delta x = 0.02$

$\Rightarrow \Delta f = (0.1)(0.02)$

$\therefore \Delta f = 0.002$

Now, we have $f(10.02) = f(10) + \Delta f$

$\Rightarrow f(10.02) = \log_e 10 + 0.002$

$\Rightarrow f(10.02) = 2.3026 + 0.002$

$\therefore f(10.02) = 2.3046$

Thus, $\log_e 10.02 \approx 2.3046$

9 J. Question

Using differentials, find the approximate values of the following:

$\log_{10} 10.1$, it being given that $\log_{10} e = 0.4343$

Answer

$\log_{10} 10.1$, it being given that $\log_{10} e = 0.4343$

Let us assume that $f(x) = \log_{10} x$

Also, let $x = 10$ so that $x + \Delta x = 10.1$

$\Rightarrow 10 + \Delta x = 10.1$

$\therefore \Delta x = 0.1$

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\log_{10} x)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(\frac{\log_e x}{\log_e 10}\right) \left[\because \log_b a = \frac{\log_c a}{\log_c b}\right]$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(\log_e x \times \log_{10} e) \left[\because \frac{1}{\log_a b} = \log_b a\right]$$

$$\Rightarrow \frac{df}{dx} = \log_{10} e \times \frac{d}{dx}(\log_e x)$$

$$\Rightarrow \frac{df}{dx} = 0.4343 \frac{d}{dx}(\log_e x)$$

We know $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$$\Rightarrow \frac{df}{dx} = 0.4343 \times \frac{1}{x}$$

$$\therefore \frac{df}{dx} = \frac{0.4343}{x}$$

When x = 10, we have $\frac{df}{dx} = \frac{0.4343}{10}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=10} = 0.04343$$

Recall that if $y = f(x)$ and Δx is a small increment in x, then the corresponding increment in y, $\Delta y = f(x + \Delta x) - f(x)$, is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$$

Here, $\frac{df}{dx} = 0.04343$ and $\Delta x = 0.1$

$\Rightarrow \Delta f = (0.04343)(0.1)$

$\therefore \Delta f = 0.004343$

Now, we have f(10.1) = f(10) + Δf

⇒ f(10.1) = $\log_{10}10$ + 0.004343

⇒ f(10.1) = 1 + 0.004343 [∵ $\log_a a = 1$]

∴ f(10.1) = 1.004343

Thus, $\log_{10}10.1 \approx 1.004343$

9 K. Question

Using differentials, find the approximate values of the following:

cos 61°, it being given that sin 60° = 0.86603 and 1° = 0.01745 radian

Answer

cos 61°, it being given that sin 60° = 0.86603 and 1° = 0.01745 radian

Let us assume that f(x) = cos x

Also, let x = 60° so that x + Δx = 61°

⇒ 60° + Δx = 61°

∴ Δx = 1° = 0.01745 radian

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\cos x)$$

We know $\frac{d}{dx}(\cos x) = -\sin x$

$$\therefore \frac{df}{dx} = -\sin x$$

When x = 60°, we have $\frac{df}{dx} = -\sin 60°$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=60°} = -0.86603$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = -0.86603$ and Δx = 0.01745

⇒ Δf = (–0.86603)(0.01745)

∴ Δf = –0.0151122

Now, we have f(61°) = f(60°) + Δf

⇒ f(61°) = cos(60°) – 0.0151122

⇒ f(61°) = 0.5 – 0.0151122

∴ f(61°) = 0.4848878

Thus, cos 61° ≈ 0.4848878

9 L. Question

Using differentials, find the approximate values of the following:

$$\frac{1}{\sqrt{25.1}}$$

Answer

Let us assume that $f(x) = \frac{1}{\sqrt{x}}$

Also, let x = 25 so that x + Δx = 25.1

⇒ 25 + Δx = 25.1

∴ Δx = 0.1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(\frac{1}{x^{\frac{1}{2}}}\right)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{-\frac{1}{2}}\right)$$

$$\Rightarrow \frac{df}{dx} = -\frac{1}{2}x^{-\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$$

$$\therefore \frac{df}{dx} = -\frac{1}{2x^{\frac{3}{2}}}$$

When x = 25, we have $\frac{df}{dx} = -\frac{1}{2(25)^{\frac{3}{2}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = -\frac{1}{2(5^2)^{\frac{3}{2}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = -\frac{1}{2(5^3)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = -\frac{1}{2(125)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = -\frac{1}{250}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = -0.004$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = -0.004$ and Δx = 0.1

⇒ Δf = (–0.004)(0.1)

∴ Δf = –0.0004

Now, we have f(25.1) = f(25) + Δf

$\Rightarrow f(25.1) = \frac{1}{\sqrt{25}} - 0.0004$

$\Rightarrow f(25.1) = \frac{1}{5} - 0.0004$

⇒ f(25.1) = 0.2 – 0.0004

∴ f(15) = 0.1996

Thus, $\frac{1}{\sqrt{25.1}} \approx 0.1996$

9 M. Question

Using differentials, find the approximate values of the following:

$$\sin\left(\frac{22}{14}\right)$$

Answer

Let us assume that f(x) = sin x

Let $x = \frac{\pi}{2}$ so that $x + \Delta x = \frac{22}{14}$

$\Rightarrow \frac{\pi}{2} + \Delta x = \frac{22}{14}$

$\therefore \Delta x = \frac{22}{14} - \frac{\pi}{2}$

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(\sin x)$

We know $\frac{d}{dx}(\sin x) = \cos x$

$\therefore \frac{df}{dx} = \cos x$

When $x = \frac{\pi}{2}$, we have $\frac{df}{dx} = \cos\left(\frac{\pi}{2}\right)$.

$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{\pi}{2}} = 0$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0$ and $\Delta x = \frac{22}{14} - \frac{\pi}{2}$

$\Rightarrow \Delta f = (0)\left(\frac{22}{14} - \frac{\pi}{2}\right)$

∴ Δf = 0

Now, we have $f\left(\frac{22}{14}\right) = f\left(\frac{\pi}{2}\right) + \Delta f$

$\Rightarrow f\left(\frac{22}{14}\right) = \sin\left(\frac{\pi}{2}\right) + 0$

$$\Rightarrow f\left(\frac{22}{14}\right) = \sin\left(\frac{\pi}{2}\right)$$

$$\therefore f\left(\frac{22}{14}\right) = 1$$

Thus, $\sin\left(\frac{22}{14}\right) \approx 1$

9 N. Question

Using differentials, find the approximate values of the following:

$$\cos\left(\frac{11\pi}{36}\right)$$

Answer

Let us assume that f(x) = cos x

Let $x = \frac{12\pi}{36} = \frac{\pi}{3}$ so that $x + \Delta x = \frac{11\pi}{36}$

$$\Rightarrow \frac{\pi}{3} + \Delta x = \frac{11\pi}{36}$$

$$\Rightarrow \Delta x = -\frac{\pi}{36}$$

$$\Rightarrow \Delta x = -\frac{\frac{22}{7}}{36}$$

$$\therefore \Delta x = -0.0873$$

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\cos x)$$

We know $\frac{d}{dx}(\cos x) = -\sin x$

$$\therefore \frac{df}{dx} = -\sin x$$

When $x = \frac{\pi}{3}$, we have $\frac{df}{dx} = -\sin\left(\frac{\pi}{3}\right)$.

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{\pi}{3}} = -0.86603$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = -0.86603$ and $\Delta x = -0.0873$

$\Rightarrow \Delta f = (-0.86603)(-0.0873)$

$\therefore \Delta f = 0.07560442$

Now, we have $f\left(\frac{11\pi}{36}\right) = f\left(\frac{\pi}{3}\right) + \Delta f$

$$\Rightarrow f\left(\frac{11\pi}{36}\right) = \cos\left(\frac{\pi}{3}\right) + 0.07560442$$

$\Rightarrow f\left(\frac{11\pi}{36}\right) = 0.5 + 0.07560442$

$\therefore f\left(\frac{11\pi}{36}\right) = 0.57560442$

Thus, $\cos\left(\frac{11\pi}{36}\right) \approx 0.57560442$

9 O. Question

Using differentials, find the approximate values of the following:

$(80)^{1/4}$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let x = 81 so that x + Δx = 80

⇒ 81 + Δx = 80

∴ Δx = –1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{\frac{1}{4}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$$

$$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$$

When x = 81, we have $\frac{df}{dx} = \frac{1}{4(81)^{\frac{3}{4}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^4)^{\frac{3}{4}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^3)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(27)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{108}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = 0.00926$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.00926$ and Δx = –1

⇒ Δf = (0.00926)(–1)

∴ Δf = –0.00926

Now, we have f(80) = f(81) + Δf

$\Rightarrow f(80) = (81)^{\frac{1}{4}} - 0.00926$

$\Rightarrow f(80) = (3^4)^{\frac{1}{4}} - 0.00926$

⇒ f(80) = 3 – 0.00926

∴ f(80) = 2.99074

Thus, $(80)^{1/4} \approx 2.99074$

9 P. Question

Using differentials, find the approximate values of the following:

$(29)^{1/3}$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let x = 27 so that x + Δx = 29

⇒ 27 + Δx = 29

∴ Δx = 2

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{3}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{\frac{1}{3}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$$

$$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$$

When x = 27, we have $\frac{df}{dx} = \frac{1}{3(27)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3(3^3)^{\frac{2}{3}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3 \times 3^2}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3(9)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{27}$$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = 0.03704$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$

Here, $\frac{df}{dx} = 0.03704$ and Δx = 2

⇒ Δf = (0.03704)(2)

∴ Δf = 0.07408

Now, we have f(29) = f(27) + Δf

$\Rightarrow f(29) = (27)^{\frac{1}{3}} + 0.07408$

$\Rightarrow f(29) = (3^3)^{\frac{1}{3}} + 0.07408$

⇒ f(29) = 3 + 0.07408

∴ f(29) = 3.07408

Thus, $(29)^{1/3} \approx 3.07408$

9 Q. Question

Using differentials, find the approximate values of the following:

$(66)^{1/3}$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let x = 64 so that x + Δx = 66

⇒ 64 + Δx = 66

∴ Δx = 2

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{3}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{\frac{1}{3}-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$

$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$

When x = 64, we have $\frac{df}{dx} = \frac{1}{3(64)^{\frac{2}{3}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{3(4^3)^{\frac{2}{3}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{3 \times 4^2}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{3(16)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = \frac{1}{48}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=64} = 0.02083$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.02083$ and Δx = 2

⇒ Δf = (0.02083)(2)

∴ Δf = 0.04166

Now, we have f(66) = f(64) + Δf

$$\Rightarrow f(66) = (64)^{\frac{1}{3}} + 0.04166$$

$$\Rightarrow f(66) = (4^3)^{\frac{1}{3}} + 0.04166$$

⇒ f(66) = 4 + 0.04166

∴ f(66) = 4.04166

Thus, $(66)^{1/3} \approx 4.04166$

9 R. Question

Using differentials, find the approximate values of the following:

$$\sqrt{26}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 25 so that x + Δx = 26

⇒ 25 + Δx = 26

∴ Δx = 1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$

When x = 25, we have $\frac{df}{dx} = \frac{1}{2\sqrt{25}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = \frac{1}{2 \times 5}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = \frac{1}{10}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=25} = 0.1$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0.1$ and Δx = 1

⇒ Δf = (0.1)(1)

∴ Δf = 0.1

Now, we have f(26) = f(25) + Δf

$\Rightarrow f(26) = \sqrt{25} + 0.1$

⇒ f(26) = 5 + 0.1

∴ f(26) = 5.1

Thus, $\sqrt{26} \approx 5.1$

9 S. Question

Using differentials, find the approximate values of the following:

$\sqrt{37}$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 36 so that x + Δx = 37

⇒ 36 + Δx = 37

∴ Δx = 1

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$

When x = 36, we have $\frac{df}{dx} = \frac{1}{2\sqrt{36}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{2 \times 6}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{12}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = 0.08333$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0.08333$ and Δx = 1

⇒ Δf = (0.08333)(1)

∴ Δf = 0.08333

Now, we have f(37) = f(36) + Δf

$\Rightarrow f(37) = \sqrt{36} + 0.08333$

⇒ f(37) = 6 + 0.08333

∴ f(37) = 6.08333

Thus, $\sqrt{37} \approx 6.08333$

9 T. Question

Using differentials, find the approximate values of the following:

$\sqrt{0.48}$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 0.49 so that x + Δx = 0.48

⇒ 0.49 + Δx = 0.48

∴ Δx = –0.01

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When x = 0.49, we have $\frac{df}{dx} = \frac{1}{2\sqrt{0.49}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.49} = \frac{1}{2 \times 0.7}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.49} = \frac{1}{1.4}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.49} = 0.7143$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.7143$ and Δx = –0.01

⇒ Δf = (0.7143)(–0.01)

∴ Δf = –0.007143

Now, we have f(0.48) = f(0.49) + Δf

$$\Rightarrow f(0.48) = \sqrt{0.49} + 0.08333$$

⇒ f(0.48) = 0.7 – 0.007143

∴ f(0.48) = 0.692857

Thus, $\sqrt{0.48} \approx 0.692857$

9 U. Question

Using differentials, find the approximate values of the following:

$(82)^{1/4}$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let x = 81 so that x + Δx = 82

⇒ 81 + Δx = 82

∴ Δx = 1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{\frac{1}{4}-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$

$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$

When x = 81, we have $\frac{df}{dx} = \frac{1}{4(81)^{\frac{3}{4}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^4)^{\frac{3}{4}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(3^3)}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{4(27)}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = \frac{1}{108}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=81} = 0.00926$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0.00926$ and Δx = 1

⇒ Δf = (0.00926)(1)

∴ Δf = 0.00926

Now, we have f(82) = f(81) + Δf

$\Rightarrow f(82) = (81)^{\frac{1}{4}} + 0.00926$

$\Rightarrow f(82) = (3^4)^{\frac{1}{4}} + 0.00926$

⇒ f(82) = 3 + 0.00926

∴ f(82) = 3.00926

Thus, $(82)^{1/4} \approx 3.00926$

9 V. Question

Using differentials, find the approximate values of the following:

$\left(\frac{17}{81}\right)^{1/4}$

Answer

Let us assume that $f(x) = x^{\frac{1}{4}}$

Also, let $x = \frac{16}{81}$ so that $x + \Delta x = \frac{17}{81}$

$\Rightarrow \frac{16}{81} + \Delta x = \frac{17}{81}$

$\therefore \Delta x = \frac{1}{81}$

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{4}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{\frac{1}{4}-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{4}x^{-\frac{3}{4}}$

$\therefore \frac{df}{dx} = \frac{1}{4x^{\frac{3}{4}}}$

When $x = \frac{16}{81}$, we have $\frac{df}{dx} = \frac{1}{4\left(\frac{16}{81}\right)^{\frac{3}{4}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{16}{81}} = \frac{1}{4\left(\left(\frac{2}{3}\right)^4\right)^{\frac{3}{4}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{16}{81}} = \frac{1}{4\left(\frac{2}{3}\right)^3}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{16}{81}} = \frac{1}{4\left(\frac{8}{27}\right)}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{16}{81}} = \frac{27}{32}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=\frac{16}{81}} = 0.84375$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0.84375$ and $\Delta x = \frac{1}{81}$

$\Rightarrow \Delta f = (0.84375)\left(\frac{1}{81}\right)$

∴ Δf = 0.0104166

Now, we have $f\left(\frac{17}{81}\right) = f\left(\frac{16}{81}\right) + \Delta f$

$\Rightarrow f\left(\frac{17}{81}\right) = \left(\frac{16}{81}\right)^{\frac{1}{4}} + 0.0104166$

$$\Rightarrow f\left(\frac{16}{81}\right) = \left(\left(\frac{2}{3}\right)^4\right)^{\frac{1}{4}} + 0.0104166$$

$$\Rightarrow f\left(\frac{16}{81}\right) = \frac{2}{3} + 0.0104166$$

$$\Rightarrow f\left(\frac{16}{81}\right) = 0.666666 + 0.0104166$$

$$\therefore f\left(\frac{16}{81}\right) = 0.6778026$$

Thus, $\left(\frac{17}{81}\right)^{1/4} \approx 0.6778026$

9 W. Question

Using differentials, find the approximate values of the following:

$(33)^{1/5}$

Answer

Let us assume that $f(x) = x^{\frac{1}{5}}$

Also, let x = 32 so that x + Δx = 33

⇒ 32 + Δx = 33

∴ Δx = 1

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{5}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{5}x^{\frac{1}{5}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{5}x^{-\frac{4}{5}}$$

$$\therefore \frac{df}{dx} = \frac{1}{5x^{\frac{4}{5}}}$$

When x = 32, we have $\frac{df}{dx} = \frac{1}{5(32)^{\frac{4}{5}}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=32} = \frac{1}{5(2^5)^{\frac{4}{5}}}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=32} = \frac{1}{5(2^4)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=32} = \frac{1}{5(16)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=32} = \frac{1}{80}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=32} = 0.0125$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.0125$ and Δx = 1

⇒ Δf = (0.0125)(1)

∴ Δf = 0.0125

Now, we have f(33) = f(32) + Δf

$$\Rightarrow f(33) = (32)^{\frac{1}{5}} + 0.0125$$

$$\Rightarrow f(33) = (2^5)^{\frac{1}{5}} + 0.0125$$

⇒ f(33) = 2 + 0.0125

∴ f(33) = 2.0125

Thus, $(33)^{1/5} \approx 2.0125$

9 X. Question

Using differentials, find the approximate values of the following:

$$\sqrt{36.6}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 36 so that x + Δx = 36.6

⇒ 36 + Δx = 36.6

∴ Δx = 0.6

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$$

When x = 36, we have $\frac{df}{dx} = \frac{1}{2\sqrt{36}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{2 \times 6}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = \frac{1}{12}$$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=36} = 0.0833333$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 0.0833333$ and Δx = 0.6

⇒ Δf = (0.0833333)(0.6)

∴ Δf = 0.05

Now, we have f(36.6) = f(36) + Δf

$\Rightarrow f(36.6) = \sqrt{36} + 0.05$

⇒ f(36.6) = 6 + 0.05

∴ f(36.6) = 6.05

Thus, $\sqrt{36.6} \approx 6.05$

9 Y. Question

Using differentials, find the approximate values of the following:

$25^{1/3}$

Answer

Let us assume that $f(x) = x^{\frac{1}{3}}$

Also, let x = 27 so that x + Δx = 25

⇒ 27 + Δx = 25

∴ Δx = –2

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{3}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{\frac{1}{3}-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$

$\therefore \frac{df}{dx} = \frac{1}{3x^{\frac{2}{3}}}$

When x = 27, we have $\frac{df}{dx} = \frac{1}{3(27)^{\frac{2}{3}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3(3^3)^{\frac{2}{3}}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3 \times 3^2}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{3(9)}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = \frac{1}{27}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=27} = 0.03704$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.03704$ and Δx = 2

⇒ Δf = (0.03704)(–2)

∴ Δf = –0.07408

Now, we have f(25) = f(27) + Δf

$$\Rightarrow f(25) = (27)^{\frac{1}{3}} - 0.07408$$

$$\Rightarrow f(25) = (3^3)^{\frac{1}{3}} - 0.07408$$

⇒ f(25) = 3 – 0.07408

∴ f(25) = 2.92592

Thus, $(25)^{1/3} \approx 2.92592$

9 Z. Question

Using differentials, find the approximate values of the following:

$$\sqrt{49.5}$$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 49 so that x + Δx = 49.5

⇒ 49 + Δx = 49.5

∴ Δx = 0.5

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$$

$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$

When x = 49, we have $\frac{df}{dx} = \frac{1}{2\sqrt{49}}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=49} = \frac{1}{2 \times 7}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=49} = \frac{1}{14}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=49} = 0.0714286$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$

Here, $\frac{df}{dx} = 0.0714286$ and Δx = 0.5

⇒ Δf = (0.0714286)(0.5)

∴ Δf = 0.0357143

Now, we have f(49.5) = f(49) + Δf

$\Rightarrow f(49.5) = \sqrt{49} + 0.0357143$

⇒ f(49.5) = 7 + 0.0357143

∴ f(49.5) = 7.0357143

Thus, $\sqrt{49.5} \approx 7.0357143$

9 A1. Question

Using differentials, find the approximate values of the following:

$(3.968)^{3/2}$

Answer

Let us assume that $f(x) = x^{\frac{3}{2}}$

Also, let x = 4 so that x + Δx = 3.968

⇒ 4 + Δx = 3.968

∴ Δx = –0.032

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{3}{2}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{3}{2}x^{\frac{3}{2}-1}$

$\Rightarrow \frac{df}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$\therefore \frac{df}{dx} = \frac{3}{2}\sqrt{x}$

When x = 4, we have $\frac{df}{dx} = \frac{3}{2}\sqrt{4}$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=4} = \frac{3}{2} \times 2$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=4} = 3$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 3$ and Δx = –0.032

⇒ Δf = (3)(–0.032)

∴ Δf = –0.096

Now, we have f(3.968) = f(4) + Δf

$\Rightarrow f(3.968) = (4)^{\frac{3}{2}} - 0.096$

$\Rightarrow f(3.968) = (2^2)^{\frac{3}{2}} - 0.096$

⇒ f(3.968) = 2^3– 0.096

⇒ f(3.968) = 8 – 0.096

∴ f(3.968) = 7.904

Thus, $(3.968)^{3/2} \approx 7.904$

9 B1. Question

Using differentials, find the approximate values of the following:

$(1.999)^5$

Answer

Let us assume that f(x) = x^5

Also, let x = 2 so that x + Δx = 1.999

⇒ 2 + Δx = 1.999

∴ Δx = –0.001

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(x^5)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = 5x^{5-1}$

$\therefore \frac{df}{dx} = 5x^4$

When x = 2, we have $\frac{df}{dx} = 5(2)^4$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = 5 \times 16$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = 80$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{df}{dx} = 80$ and Δx = –0.001

⇒ Δf = (80)(–0.001)

∴ Δf = –0.08

Now, we have f(1.999) = f(2) + Δf

⇒ f(1.999) = 2^5 – 0.08

⇒ f(1.999) = 32 – 0.08

∴ f(1.999) = 31.92

Thus, $(1.999)^5 \approx 31.92$

9 C1. Question

Using differentials, find the approximate values of the following:

$\sqrt{0.082}$

Answer

Let us assume that $f(x) = \sqrt{x}$

Also, let x = 0.09 so that x + Δx = 0.082

⇒ 0.09 + Δx = 0.082

∴ Δx = –0.008

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(x^{\frac{1}{2}}\right)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$

$\Rightarrow \frac{df}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$\therefore \frac{df}{dx} = \frac{1}{2\sqrt{x}}$

When x = 0.09, we have $\frac{df}{dx} = \frac{1}{2\sqrt{0.09}}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.09} = \frac{1}{2 \times 0.3}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.09} = \frac{1}{0.6}$$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=0.09} = 1.6667$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 1.6667$ and Δx = –0.008

⇒ Δf = (1.6667)(–0.008)

∴ Δf = –0.013334

Now, we have f(0.082) = f(0.09) + Δf

$$\Rightarrow f(0.082) = \sqrt{0.09} - 0.013334$$

⇒ f(0.082) = 0.3 – 0.013334

∴ f(0.082) = 0.286666

Thus, $\sqrt{0.082} \approx 0.286666$

10. Question

Find the approximate value of f(2.01), where f(x) = $4x^2$ + 5x + 2.

Answer

Given f(x) = $4x^2$ + 5x + 2

Let x = 2 so that x + Δx = 2.01

⇒ 2 + Δx = 2.01

∴ Δx = 0.01

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(4x^2 + 5x + 2)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(4x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(2)$$

$$\Rightarrow \frac{df}{dx} = 4\frac{d}{dx}(x^2) + 5\frac{d}{dx}(x) + \frac{d}{dx}(2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$$\Rightarrow \frac{df}{dx} = 4(2x) + 5(1) + 0$$

$$\therefore \frac{df}{dx} = 8x + 5$$

When x = 2, we have $\frac{df}{dx} = 8(2) + 5$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=2} = 21$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right) \Delta x$

Here, $\frac{df}{dx} = 21$ and Δx = 0.01

⇒ Δf = (21)(0.01)

∴ Δf = 0.21

Now, we have f(2.01) = f(2) + Δf

⇒ f(2.01) = $4(2)^2$ + 5(2) + 2 + 0.21

⇒ f(2.01) = 16 + 10 + 2 + 0.21

∴ f(2.01) = 28.21

Thus, f(2.01) = 28.21

11. Question

Find the approximate value of f(5.001), where f(x) = x^3 – $7x^2$ + 15.

Answer

Given f(x) = x^3 – $7x^2$ + 15

Let x = 5 so that x + Δx = 5.001

⇒ 5 + Δx = 5.001

∴ Δx = 0.001

On differentiating f(x) with respect to x, we get

$\frac{df}{dx} = \frac{d}{dx}(x^3 - 7x^2 + 15)$

$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(-7x^2) + \frac{d}{dx}(15)$

$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(x^3) - 7\frac{d}{dx}(x^2) + \frac{d}{dx}(15)$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$ and derivative of a constant is 0.

$\Rightarrow \frac{df}{dx} = 3x^2 - 7(2x) + 0$

$\therefore \frac{df}{dx} = 3x^2 - 14x$

When x = 5, we have $\frac{df}{dx} = 3(5)^2 - 14(5)$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=5} = 75 - 70$

$\Rightarrow \left(\frac{df}{dx}\right)_{x=5} = 5$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx)

– f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 5$ and Δx = 0.001

⇒ Δf = (5)(0.001)

∴ Δf = 0.005

Now, we have f(5.001) = f(5) + Δf

⇒ f(5.001) = 5^3 – $7(5)^2$ + 15 + 0.005

⇒ f(5.001) = 125 – 175 + 15 + 0.005

⇒ f(5.001) = –35 + 0.005

∴ f(5.001) = –34.995

Thus, f(5.001) = –34.995

12. Question

Find the approximate value of $\log_{10}1005$, given that $\log_{10}e = 0.4343$.

Answer

Let us assume that $f(x) = \log_{10}x$

Also, let x = 1000 so that x + Δx = 1005

⇒ 1000 + Δx = 1005

∴ Δx = 5

On differentiating f(x) with respect to x, we get

$$\frac{df}{dx} = \frac{d}{dx}(\log_{10}x)$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(\frac{\log_e x}{\log_e 10}\right)\ \left[\because \log_b a = \frac{\log_c a}{\log_c b}\right]$$

$$\Rightarrow \frac{df}{dx} = \frac{d}{dx}(\log_e x \times \log_{10} e)\ \left[\because \frac{1}{\log_a b} = \log_b a\right]$$

$$\Rightarrow \frac{df}{dx} = \log_{10} e \times \frac{d}{dx}(\log_e x)$$

$$\Rightarrow \frac{df}{dx} = 0.4343\frac{d}{dx}(\log_e x)$$

We know $\frac{d}{dx}(\log_e x) = \frac{1}{x}$

$$\Rightarrow \frac{df}{dx} = 0.4343 \times \frac{1}{x}$$

$$\therefore \frac{df}{dx} = \frac{0.4343}{x}$$

When x = 1000, we have $\frac{df}{dx} = \frac{0.4343}{1000}$

$$\Rightarrow \left(\frac{df}{dx}\right)_{x=1000} = 0.0004343$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{df}{dx} = 0.0004343$ and Δx = 5

⇒ Δf = (0.0004343)(5)

∴ Δf = 0.0021715

Now, we have f(1005) = f(1000) + Δf

⇒ f(1005) = $\log_{10}1000$ + 0.0021715

⇒ f(1005) = $\log_{10}10^3$ + 0.0021715

⇒ f(1005) = 3 × $\log_{10}10$ + 0.0021715

⇒ f(1005) = 3 + 0.0021715 [∵ $\log_a a = 1$]

∴ f(1005) = 3.0021715

Thus, $\log_{10}1005$ = 3.0021715

13. Question

If the radius of a sphere is measured as 9 cm with an error of 0.03 m, find the approximate error in calculating its surface area.

Answer

Given the radius of a sphere is measured as 9 cm with an error of 0.03 m = 3 cm.

Let x be the radius of the sphere and Δx be the error in measuring the value of x.

Hence, we have x = 9 and Δx = 3

The surface area of a sphere of radius x is given by

S = $4\pi x^2$

On differentiating S with respect to x, we get

$$\frac{dS}{dx} = \frac{d}{dx}(4\pi x^2)$$

$$\Rightarrow \frac{dS}{dx} = 4\pi\frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 4\pi(2x)$$

$$\therefore \frac{dS}{dx} = 8\pi x$$

When x = 9, we have $\frac{dS}{dx} = 8\pi(9)$.

$$\Rightarrow \left(\frac{dS}{dx}\right)_{x=9} = 72\pi$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$

Here, $\frac{dS}{dx} = 72\pi$ and Δx = 3

⇒ ΔS = (72π)(3)

∴ ΔS = 216π

Thus, the approximate error in calculating the surface area of the sphere is 216π cm^2.

14. Question

Find the approximate change in the surface area of cube of side x meters caused by decreasing the side by 1%.

Answer

Given a cube whose side x is decreased by 1%.

Let Δx be the change in the value of x.

Hence, we have $\Delta x = -\frac{1}{100} \times x$

∴ Δx = –0.01x

The surface area of a cube of radius x is given by

S = $6x^2$

On differentiating A with respect to x, we get

$$\frac{dS}{dx} = \frac{d}{dx}(6x^2)$$

$$\Rightarrow \frac{dS}{dx} = 6\frac{d}{dx}(x^2)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dS}{dx} = 6(2x)$$

$$\therefore \frac{dS}{dx} = 12x$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dS}{dx} = 12x$ and Δx = –0.01x

⇒ ΔS = (12x)(–0.01x)

∴ ΔS = $-0.12x^2$

Thus, the approximate change in the surface area of the cube is $0.12x^2$ m^2.

15. Question

If the radius of a sphere is measured as 7 m with an error of 0.02m, find the approximate error in calculating its volume.

Answer

Given the radius of a sphere is measured as 7 m with an error of 0.02 m.

Let x be the radius of the sphere and Δx be the error in measuring the value of x.

Hence, we have x = 7 and Δx = 0.02

The volume of a sphere of radius x is given by

$$V = \frac{4}{3}\pi x^3$$

On differentiating V with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}\left(\frac{4}{3}\pi x^3\right)$$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}\frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = \frac{4\pi}{3}(3x^2)$$

$$\therefore \frac{dV}{dx} = 4\pi x^2$$

When x = 7, we have $\frac{dV}{dx} = 4\pi(7)^2$.

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=7} = 4\pi \times 49$$

$$\Rightarrow \left(\frac{dV}{dx}\right)_{x=7} = 196\pi$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 196\pi$ and Δx = 0.02

⇒ ΔV = (196π)(0.02)

∴ ΔV = 3.92π

Thus, the approximate error in calculating the volume of the sphere is 3.92π m^3.

16. Question

Find the approximate change in the volume of a cube of side x meters caused by increasing the side by 1%.

Answer

Given a cube whose side x is increased by 1%.

Let Δx be the change in the value of x.

Hence, we have $\Delta x = \frac{1}{100} \times x$

∴ Δx = 0.01x

The volume of a cube of radius x is given by

$V = x^3$

On differentiating A with respect to x, we get

$$\frac{dV}{dx} = \frac{d}{dx}(x^3)$$

We know $\frac{d}{dx}(x^n) = nx^{n-1}$

$$\Rightarrow \frac{dV}{dx} = 3x^{3-1}$$

$$\therefore \frac{dV}{dx} = 3x^2$$

Recall that if y = f(x) and Δx is a small increment in x, then the corresponding increment in y, Δy = f(x + Δx) – f(x), is approximately given as

$$\Delta y = \left(\frac{dy}{dx}\right)\Delta x$$

Here, $\frac{dV}{dx} = 3x^2$ and Δx = 0.01x

$\Rightarrow \Delta V = (3x^2)(0.01x)$

$\therefore \Delta V = 0.03x^3$

Thus, the approximate change in the volume of the cube is $0.03x^3$ m^3.

MCQ

1. Question

Mark the correct alternative in the following:

If there is an error of 2% in measuring the length of a simple pendulum, then percentage error in its period is:

A. 1%

B. 2%

C. 3%

D. 4%

Answer

given (ΔL/L)×100 = 2 (if we let the length of pendulum is L)

we all know the formula of period of a pendulum is T=2π×√(l/g)

By the formula of approximation in derivation ,we get-

$$\left(\frac{\Delta T}{T}\right) \times 100 = \frac{1}{2} \times \left(\frac{\Delta L}{L}\right) \times 100$$

$$= \left(\frac{1}{2}\right) \times (2)$$

=1%

2. Question

Mark the correct alternative in the following:

If there is an error of a% in measuring the edge of a cube, then percentage error in its surface is:

A. 2a%

B. $\frac{a}{2}\%$

C. 3a%

D. none of these

Answer

given that

% Error in measuring the edge of a cube [(ΔL/L)×100] is = a (if L is edge of the cube)

We have to find out (ΔA/A)×100 = ? (IF let the surface of the cube is A)

By the formula of approximation of derivation we get,

$$\left(\frac{\Delta A}{A}\right) \times 100 = 2 \times \left(\frac{\Delta L}{L}\right) \times 100$$

=2×a

=2a

3. Question

Mark the correct alternative in the following:

If an error of k% is made in measuring the radius of a sphere, then percentage error in its volume is

A. k%

B. 3k%

C. 2k%

D. $\frac{k}{3}\%$

Answer

given % error in measuring the radius of a sphere Δr/r×100 =k (if let r is radius)

Find out : (Δv/v)×100 = ?

We know by the formula of the volume of the sphere

$$V = \frac{4}{3}\pi r^3$$

$$\text{So, } dV = \frac{4}{3}\pi \times 3r^2 dr$$

$$\text{So, } \frac{\Delta V}{V} = \frac{\frac{\frac{4}{3}\pi \times 3r^2 dr}{\frac{4}{3}}}{\frac{4}{3}}\pi r^3$$

$$\text{So, } \left(\frac{\Delta v}{v}\right) \times 100 = 3 \times \left(\frac{\Delta r}{r}\right) \times 100$$

=3×k

=3k%

4. Question

Mark the correct alternative in the following:

The height of a cylinder is equal to the radius. If an error of α% is made in the height, then percentage error in its volume is:

A. α%

B. 2α%

C. 3α%

D. none of these

Answer

let height of a cylinder=h=radius of that cylinder=r

% error in height Δh/h×100 = a (given)

Volume of cylinder= v =(1/3)×πr^{2}h

We have given that h=r

Then

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3$$

So,$\Delta V = \frac{1}{3}\pi h^2 dh$

Finally

$$\left(\frac{\Delta v}{v}\right) \times 100 \ = 3 \times \left(\frac{\Delta h}{h}\right) \times 100$$

=3×a

=3a%

5. Question

Mark the correct alternative in the following:

While measuring the side of an equilateral triangle an error of k% is made, the percentage error in its area is

A. k%

B. 2k%

C. $\frac{k}{2}\%$

D. 3k%

Answer

we know that the area of a equilateral traiangle is =A =(√3/4)×a^2

Where a= side of equilateral triangle

So by the formula of approximation of derivation, we get,

$$\left(\frac{\Delta A}{A}\right) \times 100 = 2 \times \left(\frac{\Delta a}{a}\right) \times 100$$

=2×k

=2k% ans

6. Question

Mark the correct alternative in the following:

If $\log_e 4$ = 1.3868, then $\log_e 4.01$ =

A. 1.3968

B. 1.3898

C. 1.3893

D. none of these

Answer

let y=f(x)=logx

Let x=4,

X+Δx=4.01,

Δx=0.01,

For x=4,

Y=log4=1.3868,

y=logx

$$\frac{dy}{dx} = \frac{1}{x} = \frac{1}{4}$$

Δy=dy

$$= \left(\frac{dy}{dx}\right) . dx$$

$$= \left(\frac{1}{4}\right) \times 0.01$$

Δy=0.0025

So, log(4.01)=y+Δy

=1.3893

7. Question

Mark the correct alternative in the following:

A sphere of radius 100 mm shrinks to radius 98 mm, then the approximate decrease in its volume is

A. 12000π mm^3

B. 800π mm^3

C. 80000π mm^3

D. 120π mm^3

Answer

we know that volume of sphere = v = (4/3)×πr^3 (r is radius of sphere)

r = 100mm

$$\Delta v \ = \left(\frac{4}{3}\right) \times \pi \times 3r^2 \Delta r$$

=4πr^2Δr

Δr = (98-100)

=-2

Δv=4π$(100)^2$×(-2)

Δv = - 80,000π mm^3ans

8. Question

Mark the correct alternative in the following:

If the ratio of base radius and height of a cone is 1 : 2 and percentage error in radius is λ%, then the error in its volume is:

A. λ%

B. 2λ%

C. 3λ%

D. none of these

Answer

given that the radius is half then the height of the cone so

Let h = 2r (where r is radius and h is height of the cone)

Volume of the cone = v

$= \left(\frac{1}{3}\right) \times \pi r^2 \times h$

$= \left(\frac{2}{3}\right) \times \pi r^3$ (because h = 2r)

$\Delta v = \left(\frac{2}{3}\right) \pi \times 3r^2 \Delta r$

$\Delta v = 2\pi r^2 \Delta r$

So finally ,

$\left(\frac{\Delta v}{v}\right) \times 100 = 3.\left(\frac{\Delta r}{r}\right) \times 100$

= 3×λ

= 3λ%

9. Question

Mark the correct alternative in the following:

The pressure P and volume V of a gas are connected by the relation $PV^{1/4}$ = constant. The percentage increase in the pressure corresponding to a deminition of 1/2% in the volume is

A. $\frac{1}{2}\%$

B. $\frac{1}{4}\%$

C. $\frac{1}{8}\%$

D. none of these

Answer

let $pv^{1/4}$=k (constant)

$Pv^{1/4}$=k

$P=k.v^{-1/4}$

$\log(p) = \log(k.v^{-1/4})$

log(p) = log(k) – (1/4)log(v)

$$\frac{dP}{P} = 0 - \frac{1}{4} \times \frac{dV}{V}$$

$$\frac{dP}{P} = -\frac{1}{4} \times -\frac{1}{2}\%$$

$$= \frac{1}{8}\%$$

10. Question

Mark the correct alternative in the following:

If $y = x^n$, then the ratio of relative errors in y and x is

A. 1 : 1

B. 2 : 1

C. 1 : n

D. n : 1

Answer

given $y=x^n$

$\Delta y = n.x^{n-1}.\Delta x = x$

$$\frac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \frac{x}{y}.\frac{\Delta y}{\Delta x}$$

$$= \frac{x}{y} \times \frac{n.x^{n-1}.\Delta x}{\Delta x}$$

$$= \frac{n.x^n}{x^n}$$

$$= \frac{n}{1}$$

So finally ratio is = n:1

11. Question

Mark the correct alternative in the following:

The approximate value of $(33)^{1/5}$ is

A. 2.0125

B. 2.1

C. 2.01

D. none of these

Answer

$f(x) = x^{1/5}$

$F'(x) = (1/5).x^{-4/5}$

$F(a+h) = f(a) + h \times f'(a)$

$$(a+h)^{\frac{1}{5}} = a^{\frac{1}{5}} + h \times \left(\frac{1}{5}\right) \times (a)^{-\frac{4}{5}}$$

Now

Let a = 32 & h=1

$$(32+1)^{\frac{1}{5}} = (32)^{\frac{1}{5}} + 1 \times \left(\frac{1}{5}\right) \times (32)^{-\frac{4}{5}}$$

$$= 2 + 1 \times \left(\frac{1}{5}\right) \times (2)^{-4}$$

$$= 2 + \left(\frac{1}{80}\right)$$

$$= \frac{161}{80}$$

=2.0125

12. Question

Mark the correct alternative in the following:

The circumference of a circle is measured as 28 cm with an error of 0.01 cm. The percentage error in the area is

A. $\frac{1}{14}$

B. 0.01

C. $\frac{1}{7}$

D. none of these

Answer

given that circumference is = C = 2πr = 28 cm

That's mean r=14/π

ΔC = 2πΔr = 0.01

Δr = (0.01/2π)

We all know that area of a circle is = A =πr^2

ΔA= 2πr×dr

So finally,

$$\left(\frac{\Delta A}{A}\right) \times 100 = 2 \times \frac{\frac{0.01}{2\pi}}{\frac{14}{\pi}} \times 100$$

= 1/14

Very short answer

1. Question

For the function y = x^2, if x = 10 and Δx = 0.1. Find Δy.

Answer

by the formula of differentiation we all know that-

$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$.........eq(1)

Ify=x^2 then

$\frac{dy}{dx} = 2x$,so put the value of $\frac{dy}{dx}$ in eq(1),we get-

$2x = \frac{\Delta y}{0.1}$

Δy=2×10×(0.1)

Δy=2

2. Question

If y = $\log_e$ x, then find Δy when x = 3 and Δx = 0.03.

Answer

given that

Y=logx then y'=1/x

Δy=?

X=3

Δx=0.03

By putting the values of above in the formula $\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$ we get

$\frac{1}{x} = \frac{\Delta y}{0.03}$

$\frac{1}{3} = \frac{\Delta y}{0.03}$

Δy=0.01

3. Question

If the relative error in measuring the radius of a circular plane is α, find the relative error measuring its area.

Answer

given that

$\frac{\Delta r}{r} = a$ (if let r is radius)

$\frac{\Delta A}{A} = ?$ (if let A is area of circle)

We know that the area of a circle(A)=πr^2 then

dA=2πr×dr

now

$\frac{dA}{A} = \frac{2\pi r \times dr}{A}$

$\frac{dA}{A} = \frac{2\pi r \times dr}{\pi r^2}$

$\frac{dA}{A} = 2 \times \frac{dr}{r}$

we know that if there is a little approximation in variables then,

$\frac{dA}{A} = \frac{\Delta A}{A}$

$= 2 \times \frac{\Delta r}{r}$

=2×a

=2a

4. Question

If the percentage error in the radius of a sphere is α, find the percentage error in its volume.

Answer

given that

$\left(\frac{\Delta r}{r}\right)\times 100 = a$ (if let r is a radius of a sphere)

$\left(\frac{\Delta v}{v}\right)\times 100 = ?$

We know that $v = \left(\frac{4}{3}\right)\pi r^3$

Then, dv = $(4\pi r^2)\times dr$

Finally $\left(\frac{\Delta v}{v}\right)\times 100 = 3\times\left(\frac{\Delta r}{r}\right)\times 100$

=3×a

=3a%

5. Question

A piece of ice is in the from of a cube melts so that the percentage error in the edge of cube is a, then find the percentage error in its volume.

Answer

given that cube edge error %[(Δx/x)×100]=a

Volume %=?

Let the edge of cube is x,

Volume; $v=x^3$

Then, $dv=3x^2.dx$

So finally $\frac{\Delta v}{v}\times 100 = \frac{(3x^2)\Delta x}{x^3}\times 100$

$= 3.\left(\frac{\Delta x}{x}\right)\times 100$

=3×a

=3a